# Set theory 4

- **null event**: no outcomes $\varnothing$ or {}
  $C = \{1, 2\}$
  $D = \{3, 4\}$
  $C \cap D = \varnothing$
- **mutually exclusive** – events that cannot occur at the same time
  if $A \cap B = \varnothing$, then A and B are *mutually exclusive* or *disjoint*

# Set theory: more than two events 1

- $A \cup B \cup C$: all outcomes in at least one of A, B, & C

  $A = \{1, 2, 3\}$
  $B = \{5\}$
  $C = \{1, 5, 10\}$

  $A \cup B \cup C = \{1, 2, 3, 5, 10\}$

- $A \cap B \cap C$: all outcomes in A, B, *and* C
  $A \cap B \cap C = \{\}$

# Set theory: more than two events 2

- **mutually exclusive** or **pairwise disjoint** – no *two* events have any outcomes in common

  A = {1, 3}

  B = {2, 4}

  C = {5, 6}

  A, B, & C are mutually exclusive

- Are the following events mutually exclusive?

  D = {H, T}

  E = {HH, TT, TH, HT}

  F = {T, TT}

# Venn diagrams

# Denzel Washington Venn diagram

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- **Sample Space**: all Denzel Washington movies
- **Events**: Hat, Glasses, Facial Hair
- Hat ∩ Glasses ∩ Facial Hair = {“Malcolm X”}

# Venn diagrams of **events**

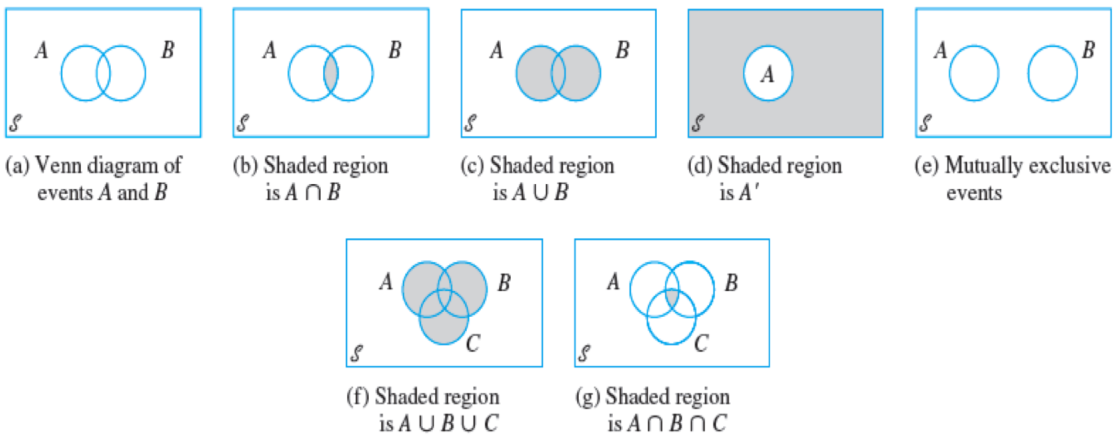
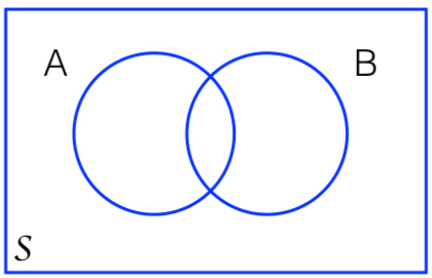


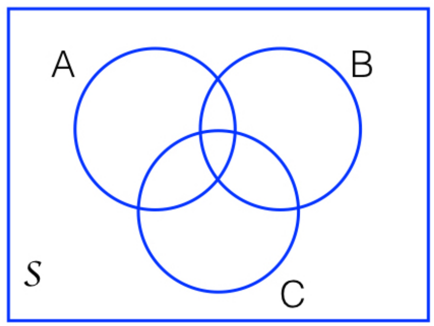

Figure 2.1 Venn diagrams

# EXERCISE

Shade:

1. $(A \cap B) \cup (A' \cap B')$ 2. $(A \cup B) \cap C \cap (A \cap B)'$





# Basic properties of probability (axioms) (2.2)

P(A) = *measure of the chance that A will occur* (multiple interpretations)

1. For any event A, P(A) $\geq$ 0.
2. P(S) = $1$
3. If $A_1, A_2, A_3, \ldots$ is an infinite collection of disjoint events, then $P(A_1 \cup A_2 \cup A_3 \cup \ldots) = \sum_{i=1}^{\infty} P(A_i)$

# Propositions

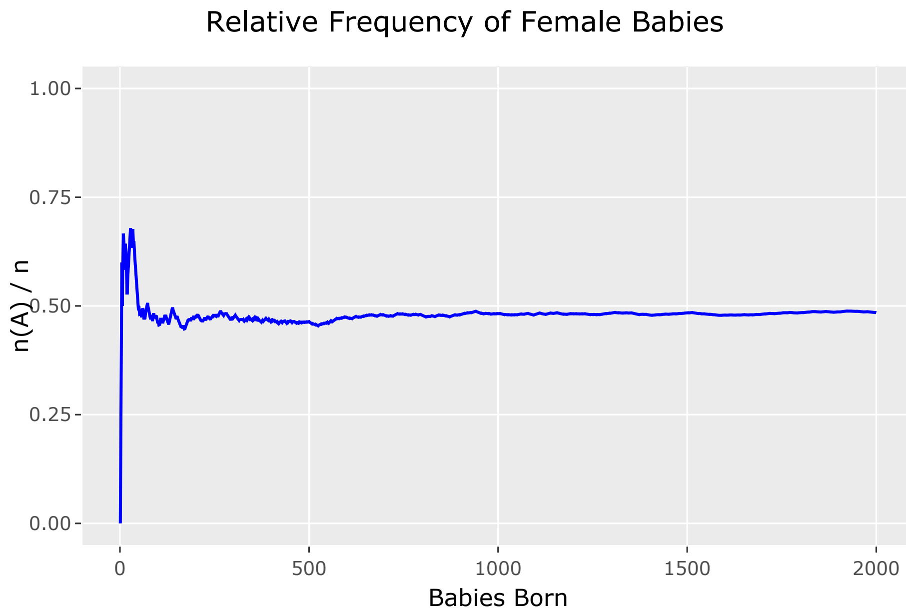
- $P(\emptyset) = 0$
- Axiom 3 is valid for **finite** disjoint events
  $P(A \cup B) = P(A) + P(B)$

# Relative frequency

- probability = relative frequency = $\frac{n(A)}{n}$ = number of times $A$ occurs in $n$ replications
- Experiments that can be repeated
- Example: What is the probability that a baby born in the U.S. is female?

# Observation

| BabiesBorn | Females | RelFreq |
|---:|---:|---:|
| 1 | 0 | 0.000 |
| 2 | 1 | 0.500 |
| 3 | 1 | 0.333 |
| 4 | 2 | 0.500 |
| 5 | 3 | 0.600 |
| 10 | 6 | 0.600 |
| 50 | 25 | 0.500 |
| 100 | 46 | 0.460 |
| 500 | 232 | 0.464 |
| 1000 | 482 | 0.482 |
| 2000 | 969 | 0.484 |



# Objective vs. subjective interpretations of probability

- objective interpretation of probability is based on long-run relative frequency
- subjective interpretation – assignment of probability to nonrepeatable events

# More probability properties

- For any event A, P(A) + P(A’) = 1, from which P(A) = 1 - P(A’).
- For any event A, P(A) $\leq$ 1.
- For any two events A and B, P($A \cup B$) = P(A) + P(B) - P($A \cap B$)

# Counting techniques (2.3)

- Are items being replaced?
- Does order matter?
- Add (“or”, alternatives) or multiply (“and”)?

# Counting techniques

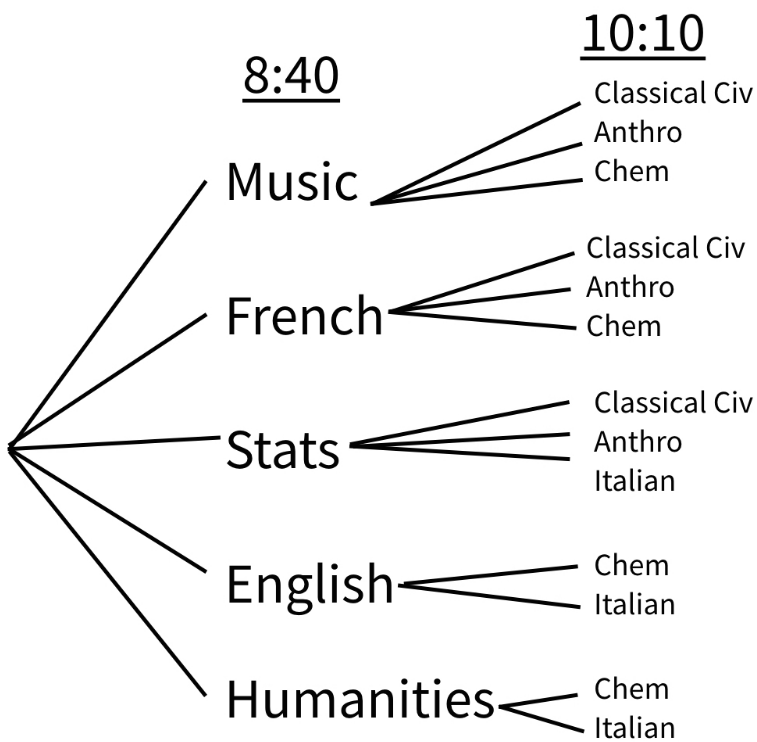
# The Product Rule

If one element of a pair can be selected in $n_1$ ways, and for each of these $n_1$ ways the other element of the pair can be selected $n_2$ ways, then the number of pairs is $n_1 n_2$.

Example

| **TuTh 8:40 classes** | **TuTh 10:10 classes** |
|---|---|
| Music BC1002 | Classical Civilization UN3230 |
| French UN2102 | Anthropology UN2003 |
| Statistics UN1201 | Chemistry S1404 |
| English BC1211 | Italian UN1102 |
| Humanities UN1123 | |

# Tree diagram



# Permutations

*order matters*

$$P_{k,n} = \frac{n!}{(n-k)!}$$

- 10 people, 1st, 2nd, 3rd place
- $P_{3,10} = \frac{10!}{(10-3)!} = \frac{10!}{7!} = 10(9)(8) = 720$

# Combinations

*order doesn't matter*

Handshake problem: 5 people, everyone must shake everyone else's hand, how many handshakes?

- $C_{k,n} = \binom{n}{k} = \frac{P_{k,n}}{k!} = \frac{n!}{k!(n-k)!}$
- 10 people, how many distinct groups of 3 can be formed?
- $C_{3,10} = \binom{10}{3} = \frac{10!}{3!(10-3)!} = \frac{10(9)(8)}{3(2)(1)} = 120$
- Permutations: ABC, ACB, BAC, BCA, CAB, CBA are all different, for combinations, all the same