Set theory 1

- **complement** of an event – all outcomes in the sample space that are *not* in the event
  S = {1, 2, 3, 4, 5, 6}
  A = {1, 3}
  A' (“not A”) = {2, 4, 5, 6}

Set theory 2

- **union** of two events: all outcomes in *either* event or in *both* $A \cup B$ (“A *or* B”)
  $A = \{1, 3\}$
  $B = \{3, 5\}$
  $A \cup B = \{1, 3, 5\}$

Set theory 3

- **intersection** of two events: all outcomes in *both* events
  $A \cap B$ ("A *and* B")
  $A = \{1, 3\}$
  $B = \{3, 5\}$
  $A \cap B = \{3\}$

Set theory 4

- **null event**: no outcomes $\varnothing$ or {}
  C = {1, 2}
  D = {3, 4}
  $C \cap D = \varnothing$
- **mutually exclusive** – events that cannot occur at the same time
  if $A \cap B = \varnothing$, then A and B are *mutually exclusive* or *disjoint*

Set theory: more than two events 1

- $A \cup B \cup C$: all outcomes in at least one of A, B, & C

  $A = \{1, 2, 3\}$
  $B = \{5\}$
  $C = \{1, 5, 10\}$

  $A \cup B \cup C = \{1, 2, 3, 5, 10\}$

- $A \cap B \cap C$: all outcomes in A, B, *and* C
  $A \cap B \cap C = \{\}$

Set theory: more than two events 2

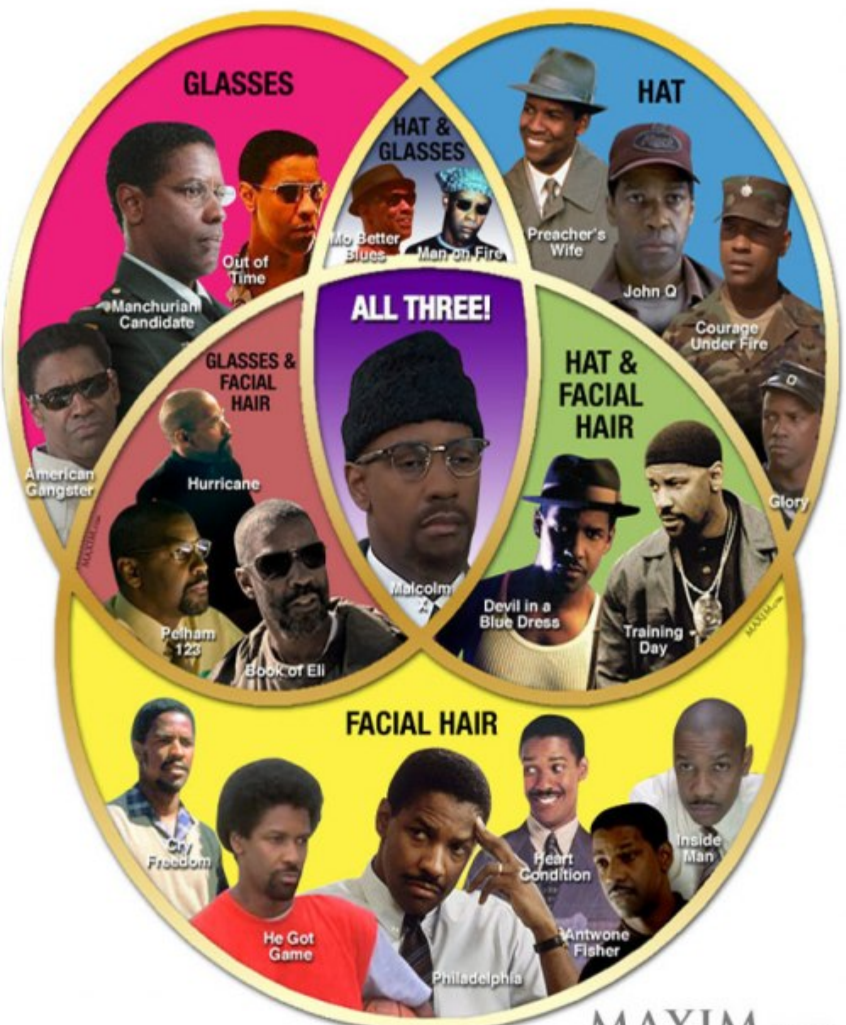
- **mutually exclusive** or **pairwise disjoint** – no *two* events have any outcomes in common
  A = {1, 3}
  B = {2, 4}
  C = {5, 6}
  A, B, & C are mutually exclusive
- Are the following events mutually exclusive?
  D = {H, T}
  E = {HH, TT, TH, HT}
  F = {T, TT}

Venn diagrams

Denzel Washington Venn diagram

THE DENZEL WASHINGTON VENN DIAGRAM

GLASSES | HAT | FACIAL HAIR | HAT & GLASSES | GLASSES & FACIAL HAIR | HAT & FACIAL HAIR | ALL THREE!



Denzel Washington Venn diagram

- **Sample Space**: all Denzel Washington movies
- **Events**: Hat, Glasses, Facial Hair
- Hat ∩ Glasses ∩ Facial Hair = {“Malcolm X”}

Venn diagrams of **events**

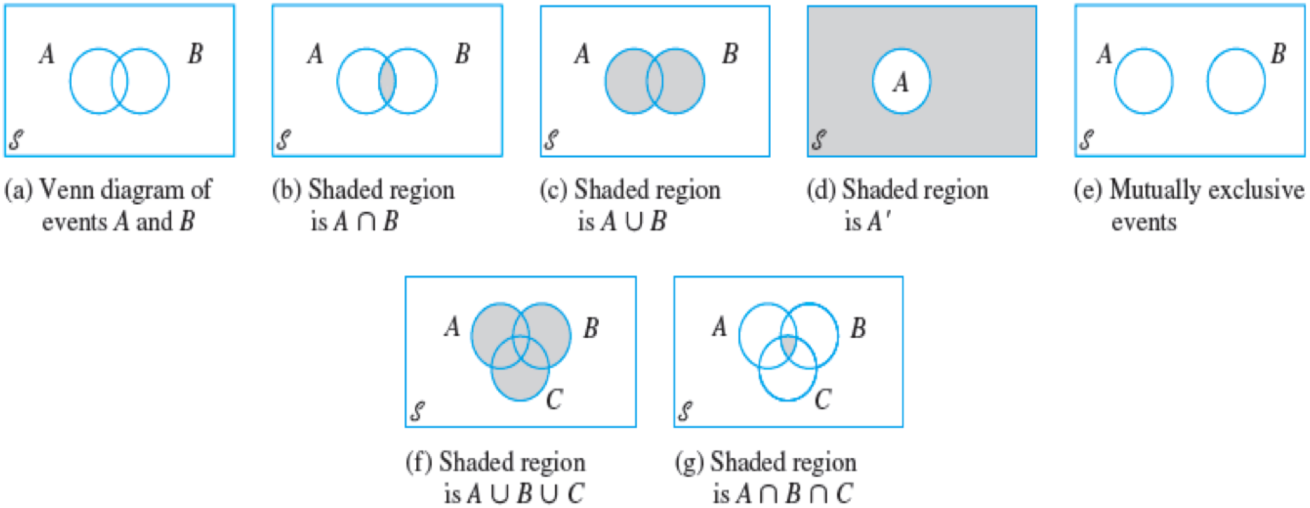
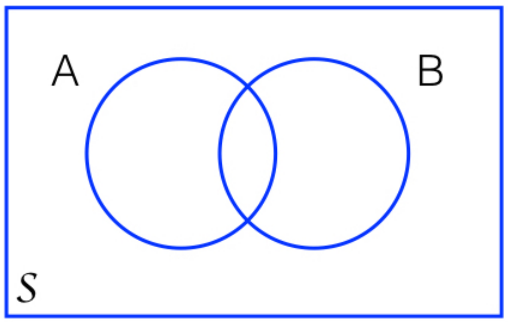


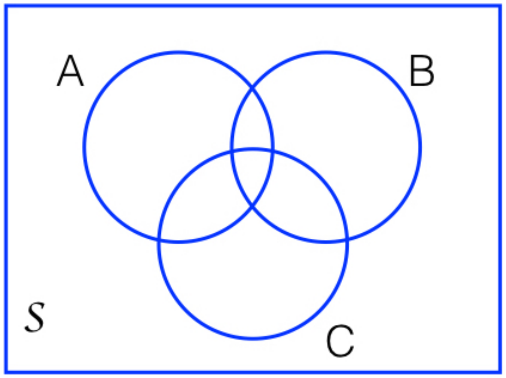
Figure 2.1 Venn diagrams

EXERCISE

Shade:

1. $(A \cap B) \cup (A' \cap B')$ 2. $(A \cup B) \cap C \cap (A \cap B)'$





Basic properties of probability (axioms) (2.2)

P(A) = *measure of the chance that A will occur*
(multiple interpretations)

1. For any event A, P(A) $\geq$ 0.
2. P(S) = 1
3. If $A_1, A_2, A_3, \ldots$ is an infinite collection of disjoint events, then $P(A_1 \cup A_2 \cup A_3 \cup \ldots) = \sum_{i=1}^{\infty} P(A_i)$

Propositions

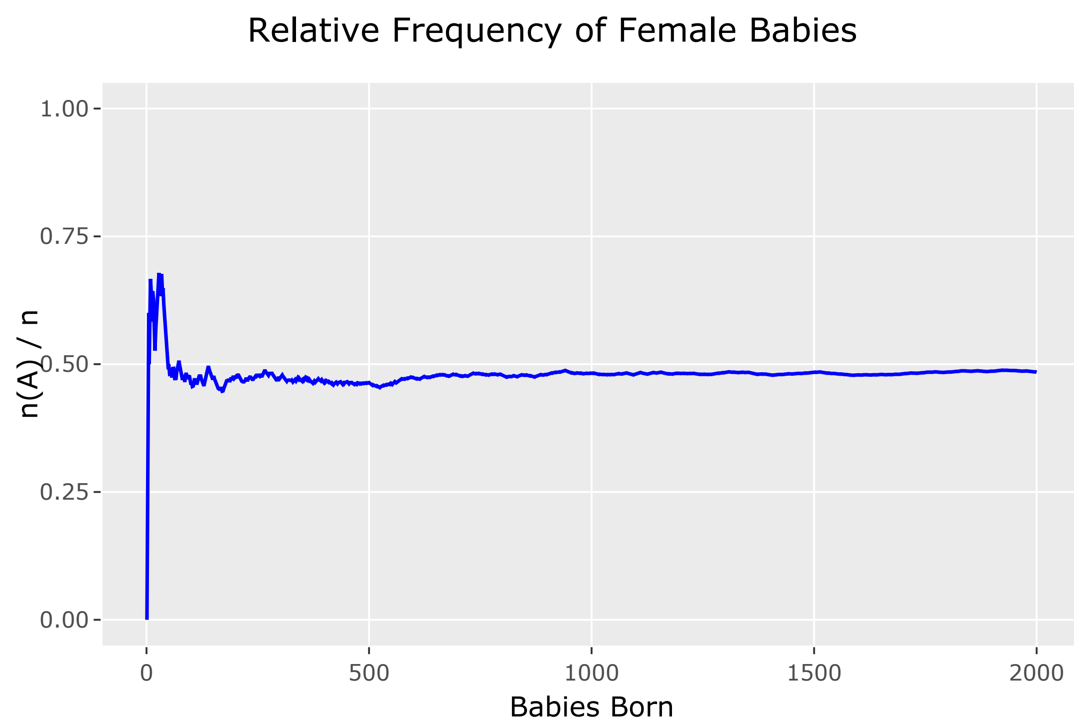
- $P(\varnothing) = 0$
- Axiom 3 is valid for **finite** disjoint events
  $P(A \cup B) = P(A) + P(B)$

Relative frequency

- probability = relative frequency = $\frac{n(A)}{n}$ = number of times A occurs in n replications
- Experiments that can be repeated
- Example: What is the probability that a baby born in the U.S. is female?

Observation

BabiesBorn	Females	RelFreq
1	0	0.000
2	1	0.500
3	1	0.333
4	2	0.500
5	3	0.600
10	6	0.600
50	25	0.500
100	46	0.460
500	232	0.464
1000	482	0.482
2000	969	0.484



Objective vs. subjective interpretations of probability

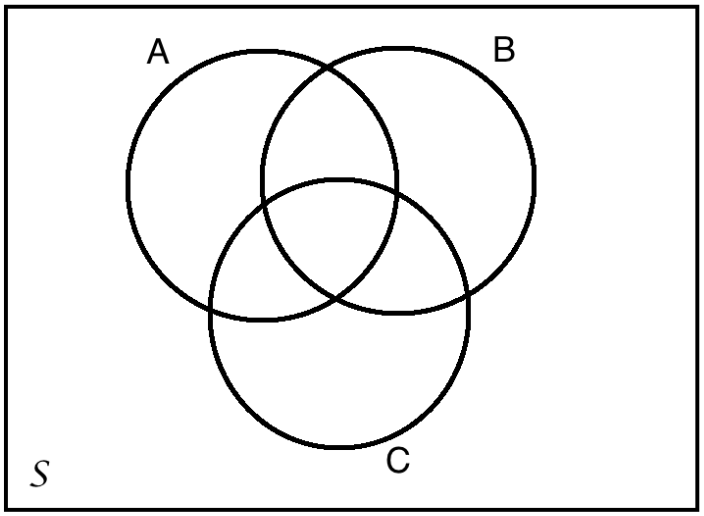
- objective interpretation of probability is based on long-run relative frequency
- subjective interpretation – assignment of probability to nonrepeatable events

More probability properties

- For any event A, P(A) + P(A’) = 1, from which P(A) = 1 - P(A’).
- For any event A, P(A) $\leq$ 1.
- For any two events A and B, P($A \cup B$) = P(A) + P(B) - P($A \cap B$)

Union of three events

- What is P($A \cup B \cup$ C) expressed in terms of intersection rather than union?
  (Extending $P(A \cup B) = P(A) + P(B) - P(A \cap B)$to three events)



Union of three events

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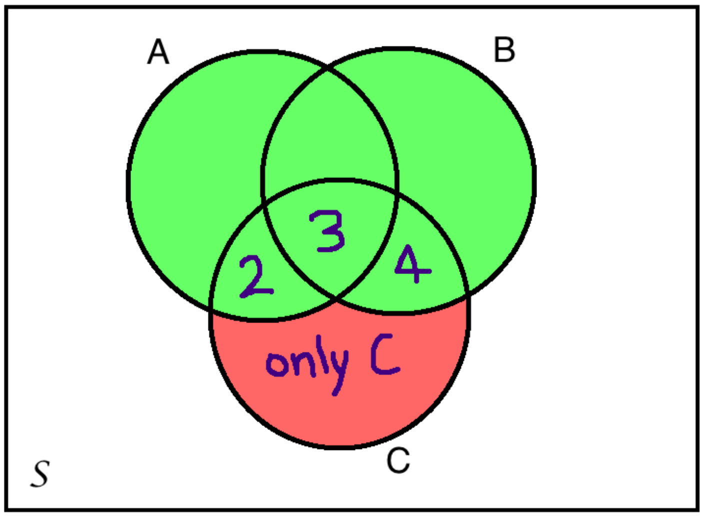


Counting techniques (2.3)

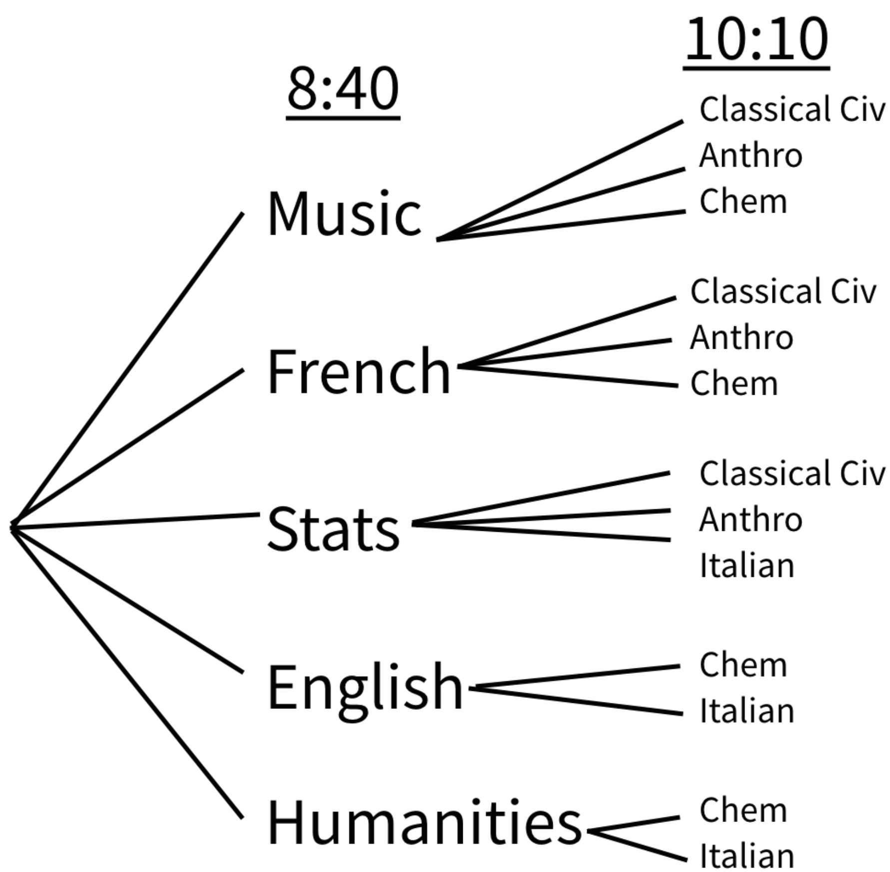
The Product Rule for Ordered Pairs

If the first element or object of an ordered pair can be selected in n_1 ways, and for each of these n_1 ways the second element of the pair can be selected n_2 ways, then the number of pairs is $n_1 n_2$.

Example

TuTh 8:40 classes	**TuTh 10:10 classes**
Music BC1002	Classical Civilization UN3230
French UN2102	Anthropology UN2003
Statistics UN1201	Chemistry S1404
English BC1211	Italian UN1102
Humanities UN1123	

Tree diagram



Permutations

order matters

$P_{k,n} = \frac{n!}{(n-k)!}$

- 10 people, 1st, 2nd, 3rd place
- $P_{3,10} = \frac{10!}{(10-3)!} = \frac{10!}{7!} = 10(9)(8) = 720$

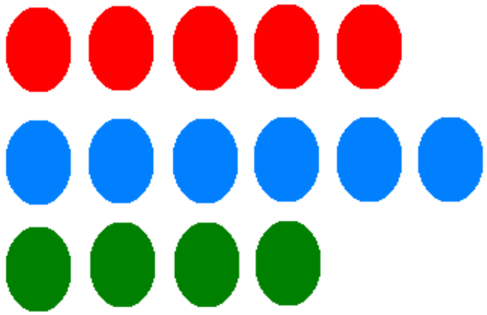
Combinations

order doesn't matter

Handshake problem: 5 people, everyone must shake everyone else's hand, how many handshakes?

- $\binom{n}{k} = \frac{P_{k,n}}{k!} = \frac{n!}{k!(n-k)!}$
- 10 people, how many distinct groups of 3 can be formed?
- $\binom{10}{3} = \frac{10!}{3!(10-3)!} = \frac{10(9)(8)}{3(2)(1)} = 120$
- Permutations: ABC, ACB, BAC, BCA, CAB, CBA are all different, for combinations, all the same

EXERCISE



(based on #39)

A box has 5 red, 6 blue, and 4 green lightbulbs. **Three** are randomly selected.

1. What is the probability that exactly two are green?
2. What is the probability that all three are the same color?
3. What is the probability that one of each color is selected?
4. If bulbs are selected one by one until a green one is obtained, what is the probability that it is necessary to examine at least 6 bulbs?