Assignment2: Viterbi Algorithm

1. Problem Description

- **Teacher-mood-model**

One week, your teacher gave the following homework assignments:

Monday	Tuesday	Wednesday	Thursday	Friday
A	C	B	A	C

- **Questions**

What did his mood curve look like most likely that week?

Give the full process of computation in your report.

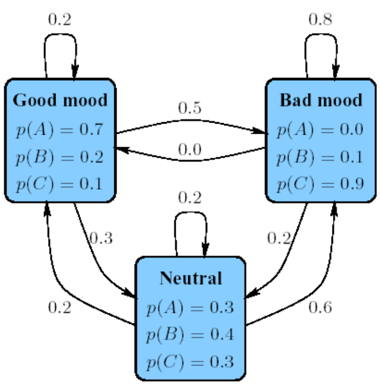
You can refer to the Result Table in P. 60 in ch3 HMM.pptx. p.s. Some numbers in the table are incorrect.

2. Computing Process

Following the prompts on the PPT, we can set the following parameters and draw the following table:

Model parameters:

- Observation $\Sigma = \{A, B, C\}$
- Set of states $S = \{\,good, neutral, bad\}$
- Transition probabilities between any two states a_{ij}
- Emission probabilities within each state $b_i(x)$



According to the above chart of PPT, the state transfer probability distribution matrix is:

$$A = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.2 & 0.2 & 0.6 \\ 0.0 & 0.2 & 0.8 \end{bmatrix}$$

and the observed state probability matrix is: $B = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.0 & 0.1 & 0.9 \end{bmatrix}$

I define $\Phi_t(j) = arg\ \underset{i \in S}{max}(V_{t-1}^{i} * a_{ij})$,$(i, j, \Phi_t(j) \in S = \{good, neutral, bad\})$, is the node passing at the previous moment of the probability maximization path from time t to state j, which retains the node passing through the shortest path.I set it here for the mood of the one day before.

- **Empty table**

	A	C	B	A	C
good					
neutral					
bad					

I assume here that the three states are equally likely to appear.

So the initial state is set to: $\Pi = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$.

- **Initialization:** $V_1^j = b_j(x_1)p(q_1 = j) = \frac{b_j(x_1)}{\#states}$(The formula in PPT in class)

so, we can get that:

$b_{good}(A) = 0.7, b_{neutral}(A) = 0.3, b_{bad}(A) = 0.0$

$V_1^{good} = b_{good}(A)p(q_1 = good) = \frac{b_{good}(A)}{\#states} = \frac{0.7}{3} = 0.2\dot{3} \approx 0.23$ (After it, I calculated it at 0.23)

$V_1^{neutral} = b_{neutral}(A)p(q_1 = neutral) = \frac{b_{neutral}(A)}{\#states} = \frac{0.3}{3} = 0.1$

$V_1^{bad} = b_{bad}(A)p(q_1 = bad) = \frac{b_{bad}(A)}{\#states} = \frac{0.0}{3} = 0.0$

Here we might as well make $\Phi_1(good) = \Phi_1(neutral) = \Phi_1(bad) = 0$. There is no meaning here.

Fill in the form:

	A	C	B	A	C
good	0.23				
neutral	0.1				
bad	0.0				

- **Iteration:** $V_t^j = b_j(x_t)max(V_{t-1}^i * a_{ij})$ **for all states** $i, j \in S, t \geq 2$. (The formula in PPT in class)
- **When t = 2:** $V_2^j = b_j(C)max(V_1^i * a_{ij})$

$b_{good}(C) = 0.1, b_{neutral}(C) = 0.3, b_{bad}(C) = 0.9$

$V_2^{good} = b_{good}(C)max(0.23 * 0.2, 0.1 * 0.2, 0.0 * 0.0) = 0.1 * 0.046 = 0.0046$

$i = 1$, so $\Phi_2(good) = good.$

$V_2^{neutral} = b_{neutral}(C)max(0.23 * 0.3, 0.1 * 0.2, 0.0 * 0.2) = 0.3 * 0.069 = 0.0207$

$i = 1$, so $\Phi_2(neutral) = good.$

$V_2^{bad} = b_{bad}(C)max(0.23 * 0.5, 0.1 * 0.6, 0.0 * 0.8) = 0.9 * 0.115 = 0.1035$

$i = 1$, so $\Phi_2(bad) = good.$

Fill in the form:

	A	C	B	A	C
good	0.23	0.0046			
neutral	0.1	0.0207			
bad	0.0	0.1035			

- **When t = 3:** $V_3^j = b_j(B)max(V_2^i * a_{ij})$

$b_{good}(B) = 0.2, b_{neutral}(B) = 0.4, b_{bad}(B) = 0.1$

$V_3^{good} = b_{good}(B)max(0.0046 * 0.2, 0.0207 * 0.2, 0.105 * 0.0) = 0.2 * 0.0207 * 0.2 = 0.000828$

$i = 2$, so $\Phi_3(good) = neutral.$

$V_3^{neutral} = b_{neutral}(B)max(0.0046 * 0.3, 0.0207 * 0.2, 0.1035 * 0.2) = 0.4 * 0.1035 * 0.2 = 0.00828$

$i = 3$, so $\Phi_3(neutral) = bad.$

$V_3^{bad} = b_{bad}(B)max(0.0046 * 0.5, 0.0207 * 0.6, 0.1035 * 0.8) = 0.1 * 0.1035 * 0.8 = 0.00828$

$i = 3$, so $\Phi_3(bad) = bad.$

Fill in the form:

	A	C	B	A	C
good	0.23	0.0046	0.000828		
neutral	0.1	0.0207	0.00828		
bad	0.0	0.1035	0.00828		

- **When t = 4:** $V_4^j = b_j(A)max(V_3^i * a_{ij})$

$b_{good}(A) = 0.7, b_{neutral}(A) = 0.3, b_{bad}(A) = 0.0$

$V_4^{good} = b_{good}(A)max(0.000828 * 0.2, 0.00828 * 0.2, 0.00828 * 0.0) = 0.7 * 0.00828 * 0.2 = 0.0011592$

$i = 2$, so $\Phi_4(good) = neutral.$

$V_4^{neutral} = b_{neutral}(A)max(0.000828 * 0.3, 0.00828 * 0.2, 0.00828 * 0.2) = 0.3 * 0.00828 * 0.2 = 0.0004968$

$i = 2$, so $\Phi_4(neutral) = neutral$

$V_4^{bad} = b_{bad}(A)max(0.000828 * 0.5, 0.00828 * 0.6, 0.00828 * 0.8) = 0.0$

$i = 3$, so $\Phi_4(bad) = bad.$

Fill in the form:

	A	C	B	A	C
good	0.23	0.0046	0.000828	0.0011592	
neutral	0.1	0.0207	0.00828	0.0004968	
bad	0.0	0.1035	0.00828	0.0	

- **When t = 5:** $V_5^j = b_j(A)max(V_4^i * a_{ij})$

$b_{good}(C) = 0.1, b_{neutral}(C) = 0.3, b_{bad}(C) = 0.9$

$V_5^{good} = b_{good}(C)max(0.0011592 * 0.2, 0.0004968 * 0.2, 0.0 * 0.0) = 0.1 * 0.0011592 * 0.2 = 0.000023184$

$i = 1$, so $\Phi_5(good) = good$.

$V_5^{neutral} = b_{neutral}(C)max(0.0011592 * 0.3, 0.0004968 * 0.2, 0.0 * 0.2) = 0.3 * 0.0011592 * 0.3 = 0.000104328$

$i = 1$, so $\Phi_5(neutral) = good$.

$V_5^{bad} = b_{bad}(C)max(0.0011592 * 0.5, 0.0004968 * 0.6, 0.0 * 0.8) = 0.9 * 0.0011592 * 0.5 = 0.00052164$

$i = 1$, so $\Phi_5(bad) = good$.

Fill in the form:

	A	C	B	A	C
good	0.23	0.0046	0.000828	0.0011592	0.000023184
neutral	0.1	0.0207	0.00828	0.0004968	0.000104328
bad	0.0	0.1035	0.00828	0.0	0.00052164

The maximum probability at $t = 5$ is at V_5^{bad}(**bad-C** is the last state), Let's look back:

- $\Phi_5(bad) = good$, that is at $t = 4$, the mood is most probably **good**;
- $\Phi_4(good) = neutral$, that is at $t = 3$, the mood is most probably **neutral**;
- $\Phi_3(neutral) = bad$, that is at $t = 2$, the mood is most probably **bad**;
- $\Phi_2(bad) = good$, that is at $t = 1$, the mood is most probably **good**;

At last, Reconstruct path along pointers, we can get(The path is a bold font):

	A	C	B	A	C
good	**0.23**	0.0046	0.000828	**0.0011592**	0.000023184
neutral	0.1	0.0207	**0.00828**	0.0004968	0.000104328
bad	0.0	**0.1035**	0.00828	0.0	**0.00052164**

His mood curve look like most likely that week:

Monday	Tuesday	Wednesday	Thursday	Friday
A	C	B	A	C
good	bad	neutral	good	bad