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Problem1

In the augmented Euclidean plane, there is a line $x - 3y + 4 = 0$, what is the homogeneous coordinate of the infinity point of this line?

Solution

On the augmented Euclidean plane, a line and infinity intersect at infinity.

That means we have to calculate the intersection of this line and the infinite line.

Let the line $l : x - 3y + 4 = 0$, we can easily conclude that the homogeneous coordinate of the line l is $(1, -3, 4)$.

The homogeneous coordinates of the infinity line l_∞ is $(0, 0, 1)$.

So the homogeneous coordinates of the intersection of these two straight lines is:

$$l \times l_\infty = (1, -3, 4) \times (0, 0, 1) = (\begin{bmatrix} -3 & 4 \\ 0 & 1 \end{bmatrix}, -\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -3 \\ 0 & 0 \end{bmatrix}) = (-3, -1, 0)$$

Therefore, the homogeneous coordinates of the infinite point of the line $l : x - 3y + 4 = 0$ are $k(-3, -1, 0)^T$and $k \neq 0$.

Problem2

For performing nonlinear optimization in the pipeline of camera calibration, we need to compute the Jacobian matrix of $\mathbf{p}_d$ w.r.t $\mathbf{p}_n$, i.e., $\frac{d\mathbf{p}_d}{d\mathbf{p}_n^T}$.

It should be noted that in this question $\mathbf{p}_d$ is the function of $\mathbf{p}_n$ and all the other parameters can be regarded as

constants.

Solution

We know that:

$$\mathbf{p}_d = (x_d, y_d)^T, \mathbf{p}_n = (x_n, y_n)^T$$

$$\begin{cases} x_d = x_n(1 + k_1 r^2 + k_2 r^4) + 2\rho_1 x_n y_n + \rho_2(r^2 + 2x_n^2) + x_n k_3 r^6 \\ y_d = y_n(1 + k_1 r^2 + k_2 r^4) + 2\rho_2 x_n y_n + \rho_1(r^2 + 2y_n^2) + y_n k_3 r^6 \end{cases}$$

$$where\ r^2 = x_n^2 + y_n^2$$

$$\frac{d\mathbf{p}_d}{d\mathbf{p}_n^T} = \begin{bmatrix} \frac{\partial x_d}{\partial x_n} & \frac{\partial x_d}{\partial y_n} \\ \frac{\partial y_d}{\partial x_n} & \frac{\partial y_d}{\partial y_n} \end{bmatrix}$$

First, we should know that:

$$\frac{\partial r}{\partial x_n} = x_n(x_n^2 + y_n^2)^{-\frac{1}{2}} = x_n r^{-1}$$
$$\frac{\partial r}{\partial y_n} = y_n(x_n^2 + y_n^2)^{-\frac{1}{2}} = y_n r^{-1}$$

Next, I will calculate the four formulas: $\frac{\partial x_d}{\partial x_n}, \frac{\partial x_d}{\partial y_n}, \frac{\partial y_d}{\partial x_n}, \frac{\partial y_d}{\partial y_n}$one by one:

Caculate $\frac{\partial x_d}{\partial x_n}$:

$$
\begin{aligned}
\frac{\partial x_d}{\partial x_n} &= (1+k_1r^2+k_2r^4)+x_n(2k_1r\frac{\partial r}{\partial x_n}+4k_2r^3\frac{\partial r}{\partial x_n})+2\rho_1 y_n+\rho_2(2r\frac{\partial r}{\partial x_n}+4x_n)+k_3r^6+6x_nk_3r^5\frac{\partial r}{\partial x_n} \\
&= (1+k_1r^2+k_2r^4)+x_n(2k_1x_n+4k_2r^2x_n)+2\rho_1y_n+\rho_2(2x_n+4x_n)+k_3r^6+x_n(6k_3r^4x_n) \\
&= (2k_1+4k_2r^2+6k_3r^4)x_n^2+6\rho_2x_n+2\rho_1y_n+1+k_1r^2+k_2r^4+k_3r^6
\end{aligned}
$$

Caculate $\frac{\partial x_d}{\partial y_n}$:

$$
\begin{aligned}
\frac{\partial x_d}{\partial y_n} &= x_n(2k_1r\frac{\partial r}{\partial y_n}+4k_2r^3\frac{\partial r}{\partial y_n})+2\rho_1x_n+2\rho_2r\frac{\partial r}{\partial y_n}+6x_nk_3r^5\frac{\partial r}{\partial y_n} \\
&= x_n(2k_1y_n+4k_2r^2y_n)+2\rho_1x_n+2\rho_2y_n+6k_3r^4x_ny_n \\
&= (2k_1+4k_2r^2+6k_3r^4)x_ny_n+2\rho_1x_n+2\rho_2y_n
\end{aligned}
$$

Caculate $\frac{\partial y_d}{\partial x_n}$:

$$
\begin{aligned}
\frac{\partial y_d}{\partial x_n} &= y_n(2k_1r\frac{\partial r}{\partial x_n}+4k_2r^3\frac{\partial r}{\partial x_n})+2\rho_2y_n+2\rho_1r\frac{\partial r}{\partial x_n}+6y_nk_3r^5\frac{\partial r}{\partial x_n} \\
&= (2k_1+4k_2r^2+6k_3r^4)x_ny_n+2\rho_1x_n+2\rho_2y_n
\end{aligned}
$$

Caculate $\frac{\partial y_d}{\partial y_n}$:

$$
\begin{aligned}
\frac{\partial y_d}{\partial y_n} &= (1+k_1r^2+k_2r^4)+y_n(2k_1r\frac{\partial r}{\partial y_n}+4k_2r^3\frac{\partial r}{\partial y_n})+2\rho_2x_n+\rho_1(2r\frac{\partial r}{\partial y_n}+4y_n)+k_3r^6+6y_nk_3r^5\frac{\partial r}{\partial y_n} \\
&= (1+k_1r^2+k_2r^4)+x_n(2k_1x_n+4k_2r^2x_n)+2\rho_1y_n+\rho_2(2x_n+4x_n)+k_3r^6+x_n(6k_3r^4x_n) \\
&= (2k_1+4k_2r^2+6k_3r^4)y_n^2+6\rho_1y_n+2\rho_2x_n+1+k_1r^2+k_2r^4+k_3r^6
\end{aligned}
$$

So the Jacobian matrix is:

$$
\frac{d\mathbf{p}_d}{d\mathbf{p}_n^T}=\begin{bmatrix}\frac{\partial x_d}{\partial x_n} & \frac{\partial x_d}{\partial y_n} \\ \frac{\partial y_d}{\partial x_n} & \frac{\partial y_d}{\partial y_n}\end{bmatrix}
$$

$$
=\begin{bmatrix}(2k_1+4k_2r^2+6k_3r^4)x_n^2+6\rho_2x_n+2\rho_1y_n+1+k_1r^2+k_2r^4+k_3r^6 & (2k_1+4k_2r^2+6k_3r^4)x_ny_n+2\rho_1x_n+2\rho_2y_n \\ (2k_1+4k_2r^2+6k_3r^4)x_ny_n+2\rho_1x_n+2\rho_2y_n & (2k_1+4k_2r^2+6k_3r^4)y_n^2+6\rho_1y_n+2\rho_2x_n+1+k_1r^2+k_2r^4+k_3r^6\end{bmatrix}
$$

Problem3

Please give the concrete form of Jacobian matrix of $\mathbf{r}$ w.r.t $\mathbf{d}$, i.e., $\frac{d\mathbf{r}}{d\mathbf{d}^T}\in R^{9\times 3}$.

In order to make it easy to check your result, please follow the following notation requirements,

$\alpha \overset{\triangle}{=} sin\theta, \beta \overset{\triangle}{=} cos\theta, \gamma \overset{\triangle}{=} 1-cos\theta$

In other words, the ingredients appearing in your formula are restricted to $\alpha,\beta,\gamma,\theta,n_1,n_2,n_3$.

Solution

$\mathbf{n}=[n_1\ n_2\ n_3]^T, \mathbf{n}=[n_1\ n_2\ n_3]$,so $\mathbf{nn}^T=\begin{bmatrix}n_1n_1 & n_1n_2 & n_1n_3 \\ n_2n_1 & n_2n_2 & n_2n_3 \\ n_3n_1 & n_3n_2 & n_3n_3\end{bmatrix}$,$\mathbf{n}$^$=\begin{bmatrix}0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0\end{bmatrix}$.

According to the Rodriguez formula, we can conclude that:

$$
\begin{aligned}
\mathbf{R} &= \begin{bmatrix}\cos\theta & & \\ & \cos\theta & \\ & & \cos\theta\end{bmatrix}+(1-\cos\theta)\begin{bmatrix}n_1n_1 & n_1n_2 & n_1n_3 \\ n_2n_1 & n_2n_2 & n_2n_3 \\ n_3n_1 & n_3n_2 & n_3n_3\end{bmatrix}+\sin\theta\begin{bmatrix}0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0\end{bmatrix} \\
&= \begin{bmatrix}\beta & & \\ & \beta & \\ & & \beta\end{bmatrix}+\gamma\begin{bmatrix}n_1n_1 & n_1n_2 & n_1n_3 \\ n_2n_1 & n_2n_2 & n_2n_3 \\ n_3n_1 & n_3n_2 & n_3n_3\end{bmatrix}+\alpha\begin{bmatrix}0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0\end{bmatrix} \\
&= \begin{bmatrix}\beta+\gamma n_1^2 & \gamma n_1n_2-\alpha n_3 & \gamma n_1n_3+\alpha n_2 \\ \gamma n_1n_2+\alpha n_3 & \beta+\gamma n_2^2 & \gamma n_2n_3-\alpha n_1 \\ \gamma n_1n_3-\alpha n_2 & \gamma n_2n_3+\alpha n_1 & \beta+\gamma n_3^2\end{bmatrix}
\end{aligned}
$$

So we can get $\mathbf{r}$:

$$
\mathbf{r}=\begin{bmatrix}\beta+\gamma n_1^2\\ \gamma n_1n_2-\alpha n_3\\ \gamma n_1n_3+\alpha n_2\\ \gamma n_1n_2+\alpha n_3\\ \beta+\gamma n_2^2\\ \gamma n_2n_3-\alpha n_1\\ \gamma n_1n_3-\alpha n_2\\ \gamma n_2n_3+\alpha n_1\\ \beta+\gamma n_3^2\end{bmatrix}
$$

According to the relationship between $\mathbf{d},\theta$ and $\mathbf{n}:\mathbf{d}=\begin{bmatrix}d_1\\d_2\\d_3\end{bmatrix}=\theta\mathbf{n}=\theta\begin{bmatrix}n_1\\n_2\\n_3\end{bmatrix}$.

Beacause $\mathbf{n}$ is a unit vector, we have $\theta=||\mathbf{d}||_2=\sqrt{d_1^2+d_2^2+d_3^2}$.

Since the required derivative contains θ term, so here's a unified calculation:

$$
\begin{aligned}
\frac{\partial\theta}{\partial d_i}&=\frac{d_i}{\sqrt{d_1^2+d_2^2+d_3^2}}=\frac{d_i}{\theta}=n_i\\
\frac{\partial\alpha}{\partial d_i}&=\frac{\partial sin\theta}{\partial d_i}=cos\theta\frac{\partial\theta}{\partial d_i}=\beta n_i\\
\frac{\partial\beta}{\partial d_i}&=\frac{\partial cos\theta}{\partial d_i}=-sin\theta\frac{\partial\theta}{\partial d_i}=-\alpha n_i\\
\frac{\partial\gamma}{\partial d_i}&=\frac{\partial(1-cos\theta)}{\partial d_i}=sin\theta\frac{\partial\theta}{\partial d_i}=\alpha n_i
\end{aligned}
$$

Since the required derivative contains n_i term, so here's a unified calculation:

$$
\begin{gathered}
According\ to\ the\ relationship\ between\ \mathbf{d},\theta\ and\ \mathbf{n}:\\
n_i=\frac{d_i}{\theta}=\frac{d_i}{\sqrt{d_1^2+d_2^2+d_3^2}}\\
\frac{\partial n_i}{\partial d_i}=\frac{\sqrt{d_1^2+d_2^2+d_3^2}-d_i\frac{d_i}{\sqrt{d_1^2+d_2^2+d_3^2}}}{d_1^2+d_2^2+d_3^2}=\frac{1-\frac{{d_i}^2}{d_1^2+d_2^2+d_3^2}}{\sqrt{d_1^2+d_2^2+d_3^2}}=\frac{1-n_i^2}{\theta}\\
\frac{\partial n_i}{\partial d_j}=\frac{-d_i\frac{d_j}{\sqrt{d_1^2+d_2^2+d_3^2}}}{d_1^2+d_2^2+d_3^2}=-\frac{n_in_j}{\theta},i\neq j
\end{gathered}
$$

The Jacobian matrix is :

$$
\frac{d\mathbf{r}}{d\mathbf{d}^T}=\begin{bmatrix}\frac{\partial r_{11}}{\partial d_1}&\frac{\partial r_{11}}{\partial d_2}&\frac{\partial r_{11}}{\partial d_3}\\ \frac{\partial r_{12}}{\partial d_1}&\frac{\partial r_{12}}{\partial d_2}&\frac{\partial r_{12}}{\partial d_3}\\ \vdots&\vdots&\vdots\\ \frac{\partial r_{33}}{\partial d_1}&\frac{\partial r_{33}}{\partial d_2}&\frac{\partial r_{33}}{\partial d_3}\end{bmatrix}
$$

I then found that the derivative of this matrix is very complex, but can observe that some classes of r_{ij} have similar formula substructures, so I can calculate them roughly first:

Equations like structures like $\beta+\gamma n_i^2(r_{11},r_{22},r_{33})$:

$$\frac{\partial(\beta+\gamma n_i^2)}{\partial d_i}=\frac{\partial\beta}{\partial d_i}+\frac{\partial(\gamma n_i^2)}{\partial d_i}=-\alpha n_i+\alpha n_i n_i^2+2\gamma n_i\frac{1-n_i^2}{\theta}$$
$$=\alpha n_i(n_i^2-1)+\frac{2\gamma n_i(1-n_i^2)}{\theta}$$
$$\frac{\partial(\beta+\gamma n_i^2)}{\partial d_j}=\frac{\partial\beta}{\partial d_j}+\frac{\partial(\gamma n_i^2)}{\partial d_j}=-\alpha n_j+\alpha n_j n_i^2-2\gamma n_i\frac{n_i n_j}{\theta}$$
$$=an_j(n_i^2-1)-\frac{2\gamma n_i^2 n_j}{\theta}, where\ i\neq j$$

Equations like structures like $\gamma n_j n_k \pm \alpha n_i (r_{12}, r_{13}, r_{21}, r_{23}, r_{31}, r_{32})$:

$$\frac{\partial(\gamma n_j n_k\pm\alpha n_i)}{\partial d_i}=\frac{\partial(\gamma n_j n_k)}{\partial d_i}+\frac{\partial(\pm\alpha n_i)}{\partial d_i}$$
$$=\alpha n_i n_j n_k-\gamma\frac{n_i n_j}{\theta}n_k-\gamma\frac{n_i n_k}{\theta}n_j\pm\beta n_i^2\pm\alpha\frac{1-n_i^2}{\theta}$$
$$=n_i(\alpha n_j n_k\pm\beta n_i)+\frac{\pm\alpha(1-n_i^2)-2\gamma n_i n_j n_k}{\theta}$$
$$\frac{\partial(\gamma n_j n_k\pm\alpha n_i)}{\partial d_j}=\frac{\partial(\gamma n_j n_k)}{\partial d_j}+\frac{\partial(\pm\alpha n_i)}{\partial d_j}$$
$$=\alpha n_j^2 n_k+\gamma\frac{1-n_j^2}{\theta}n_k-\gamma n_j\frac{n_j n_k}{\theta}\pm\beta n_i n_j\mp\alpha\frac{n_i n_j}{\theta}$$
$$=n_j(\alpha n_j n_k\pm\beta n_i)+\frac{\gamma n_k(1-2n_j^2)\mp\alpha n_i n_j}{\theta}, where\ i\neq j$$

So we can finally sort it out:

$$\frac{\partial r_{11}}{\partial d_1}=\frac{\partial(\beta+\gamma n_1^2)}{\partial d_1}=\frac{2\gamma n_1(1-n_1^2)}{\theta}+\alpha n_1(n_1^2-1)$$
$$\frac{\partial r_{11}}{\partial d_2}=\frac{\partial(\beta+\gamma n_1^2)}{\partial d_2}=-\frac{2\gamma n_1^2 n_2}{\theta}+\alpha n_2(n_1^2-1)$$
$$\frac{\partial r_{11}}{\partial d_3}=\frac{\partial(\beta+\gamma n_1^2)}{\partial d_3}=-\frac{2\gamma n_1^2 n_3}{\theta}+\alpha n_3(n_1^2-1)$$

$$\frac{\partial r_{12}}{\partial d_1}=\frac{\partial(\gamma n_1 n_2-\alpha n_3)}{\partial d_1}=n_1(\alpha n_1 n_2-\beta n_3)+\frac{\gamma n_2(1-2n_1^2)+\alpha n_1 n_3}{\theta}$$
$$\frac{\partial r_{12}}{\partial d_2}=\frac{\partial(\gamma n_1 n_2-\alpha n_3)}{\partial d_2}=n_2(\alpha n_1 n_2-\beta n_3)+\frac{\gamma n_1(1-2n_2^2)+\alpha n_2 n_3}{\theta}$$
$$\frac{\partial r_{12}}{\partial d_3}=\frac{\partial(\gamma n_1 n_2-\alpha n_3)}{\partial d_3}=n_3(\alpha n_1 n_2-\beta n_3)+\frac{\alpha(n_3^2-1)-2\gamma n_1 n_2 n_3}{\theta}$$

$$\frac{\partial r_{13}}{\partial d_1}=\frac{\partial(\gamma n_1 n_3+\alpha n_2)}{\partial d_1}=n_1(\alpha n_1 n_3+\beta n_2)+\frac{\gamma n_3(1-2n_1^2)-\alpha n_1 n_2}{\theta}$$
$$\frac{\partial r_{13}}{\partial d_2}=\frac{\partial(\gamma n_1 n_3+\alpha n_2)}{\partial d_2}=n_2(\alpha n_1 n_3+\beta n_2)+\frac{\alpha(1-n_2^2)-2\gamma n_1 n_2 n_3}{\theta}$$
$$\frac{\partial r_{13}}{\partial d_3}=\frac{\partial(\gamma n_1 n_3+\alpha n_2)}{\partial d_3}=n_3(\alpha n_1 n_3+\beta n_2)+\frac{\gamma n_1(1-2n_3^2)-\alpha n_2 n_3}{\theta}$$

$$\frac{\partial r_{21}}{\partial d_1}=\frac{\partial(\gamma n_1 n_2+\alpha n_3)}{\partial d_1}=n_1(\alpha n_1 n_2+\beta n_3)+\frac{\gamma n_2(1-2n_1^2)-\alpha n_1 n_3}{\theta}$$
$$\frac{\partial r_{21}}{\partial d_2}=\frac{\partial(\gamma n_1 n_2+\alpha n_3)}{\partial d_2}=n_2(\alpha n_1 n_2+\beta n_3)+\frac{\gamma n_1(1-2n_2^2)-\alpha n_2 n_3}{\theta}$$

$$\frac{\partial r_{21}}{\partial d_3} = \frac{\partial(\gamma n_1 n_2 + \alpha n_3)}{\partial d_3} = n_3(\alpha n_1 n_2 + \beta n_3) + \frac{\alpha(1-n_3^2) - 2\gamma n_1 n_2 n_3}{\theta}$$

$$\frac{\partial r_{22}}{\partial d_1} = \frac{\partial(\beta + \gamma n_2^2)}{\partial d_1} = -\frac{2\gamma n_1 n_2^2}{\theta} + \alpha n_1(n_2^2 - 1)$$

$$\frac{\partial r_{22}}{\partial d_2} = \frac{\partial(\beta + \gamma n_2^2)}{\partial d_2} = \frac{2\gamma n_2(1-n_2^2)}{\theta} + \alpha n_2(n_2^2 - 1)$$

$$\frac{\partial r_{22}}{\partial d_3} = \frac{\partial(\beta + \gamma n_2^2)}{\partial d_3} = -\frac{2\gamma n_2^2 n_3}{\theta} + \alpha n_3(n_2^2 - 1)$$

$$\frac{\partial r_{23}}{\partial d_1} = \frac{\partial(\gamma n_2 n_3 - \alpha n_1)}{\partial d_1} = n_1(\alpha n_2 n_3 - \beta n_1) - \frac{\alpha(1-n_1^2) + 2\gamma n_1 n_2 n_3}{\theta}$$

$$\frac{\partial r_{23}}{\partial d_2} = \frac{\partial(\gamma n_2 n_3 - \alpha n_1)}{\partial d_2} = n_2(\alpha n_2 n_3 - \beta n_1) + \frac{\gamma n_3(1-2n_2^2) + \alpha n_1 n_2}{\theta}$$

$$\frac{\partial r_{23}}{\partial d_3} = \frac{\partial(\gamma n_2 n_3 - \alpha n_1)}{\partial d_3} = n_3(\alpha n_2 n_3 - \beta n_1) + \frac{\gamma n_2(1-2n_3^2) + \alpha n_1 n_3}{\theta}$$

$$\frac{\partial r_{31}}{\partial d_1} = \frac{\partial(\gamma n_1 n_3 - \alpha n_2)}{\partial d_1} = n_1(\alpha n_1 n_3 - \beta n_2) + \frac{\gamma n_3(1-2n_1^2) + \alpha n_1 n_2}{\theta}$$

$$\frac{\partial r_{31}}{\partial d_2} = \frac{\partial(\gamma n_1 n_3 - \alpha n_2)}{\partial d_2} = n_2(\alpha n_1 n_3 - \beta n_2) - \frac{\alpha(1-n_2^2) + 2\gamma n_1 n_2 n_3}{\theta}$$

$$\frac{\partial r_{31}}{\partial d_3} = \frac{\partial(\gamma n_1 n_3 - \alpha n_2)}{\partial d_3} = n_3(\alpha n_1 n_3 - \beta n_2) + \frac{\gamma n_1(1-2n_3^2) + \alpha n_2 n_3}{\theta}$$

$$\frac{\partial r_{32}}{\partial d_1} = \frac{\partial(\gamma n_2 n_3 + \alpha n_1)}{\partial d_1} = n_1(\alpha n_2 n_3 + \beta n_1) + \frac{\alpha(1-n_1^2) - 2\gamma n_1 n_2 n_3}{\theta}$$

$$\frac{\partial r_{32}}{\partial d_2} = \frac{\partial(\gamma n_2 n_3 + \alpha n_1)}{\partial d_2} = n_2(\alpha n_2 n_3 + \beta n_1) + \frac{\gamma n_3(1-2n_2^2) - \alpha n_1 n_2}{\theta}$$

$$\frac{\partial r_{32}}{\partial d_3} = \frac{\partial(\gamma n_2 n_3 + \alpha n_1)}{\partial d_3} = n_3(\alpha n_2 n_3 + \beta n_1) + \frac{\gamma n_2(1-2n_3^2) - \alpha n_1 n_3}{\theta}$$

$$\frac{\partial r_{33}}{\partial d_1} = \frac{\partial(\beta + \gamma n_3^2)}{\partial d_1} = -\frac{2\gamma n_1 n_3^2}{\theta} + \alpha n_1(n_3^2 - 1)$$

$$\frac{\partial r_{33}}{\partial d_2} = \frac{\partial(\beta + \gamma n_3^2)}{\partial d_2} = -\frac{2\gamma n_2 n_3^2}{\theta} + \alpha n_2(n_3^2 - 1)$$

$$\frac{\partial r_{33}}{\partial d_3} = \frac{\partial(\beta + \gamma n_3^2)}{\partial d_3} = \frac{2\gamma n_3(1-n_3^2)}{\theta} + \alpha n_3(n_3^2 - 1)$$

So the final answer is:

$$
\frac{d\mathbf{r}}{d\mathbf{d}^T} = \begin{bmatrix}
\frac{2\gamma n_1(1-n_1^2)}{\theta} + \alpha n_1(n_1^2-1) & -\frac{2\gamma n_1^2 n_2}{\theta} + \alpha n_2(n_1^2-1) & -\frac{2\gamma n_1^2 n_3}{\theta} + \alpha n_3(n_1^2-1) \\
n_1(\alpha n_1 n_2 - \beta n_3) + \frac{\gamma n_2(1-2n_1^2)+\alpha n_1 n_3}{\theta} & n_2(\alpha n_1 n_2 - \beta n_3) + \frac{\gamma n_1(1-2n_2^2)+\alpha n_2 n_3}{\theta} & n_3(\alpha n_1 n_2 - \beta n_3) + \frac{\alpha(n_3^2-1)-2\gamma n_1 n_2 n_3}{\theta} \\
n_1(\alpha n_1 n_3 + \beta n_2) + \frac{\gamma n_3(1-2n_1^2)-\alpha n_1 n_2}{\theta} & n_2(\alpha n_1 n_3 + \beta n_2) + \frac{\alpha(1-n_2^2)-2\gamma n_1 n_2 n_3}{\theta} & n_3(\alpha n_1 n_3 + \beta n_2) + \frac{\gamma n_1(1-2n_3^2)-\alpha n_2 n_3}{\theta} \\
n_1(\alpha n_1 n_2 + \beta n_3) + \frac{\gamma n_2(1-2n_1^2)-\alpha n_1 n_3}{\theta} & n_2(\alpha n_1 n_2 + \beta n_3) + \frac{\gamma n_1(1-2n_2^2)-\alpha n_2 n_3}{\theta} & n_3(\alpha n_1 n_2 + \beta n_3) + \frac{\alpha(1-n_3^2)-2\gamma n_1 n_2 n_3}{\theta} \\
-\frac{2\gamma n_1 n_2^2}{\theta} + \alpha n_1(n_2^2-1) & \frac{2\gamma n_2(1-n_2^2)}{\theta} + \alpha n_2(n_2^2-1) & -\frac{2\gamma n_2^2 n_3}{\theta} + \alpha n_3(n_2^2-1) \\
n_1(\alpha n_2 n_3 - \beta n_1) - \frac{\alpha(1-n_1^2)+2\gamma n_1 n_2 n_3}{\theta} & n_2(\alpha n_2 n_3 - \beta n_1) + \frac{\gamma n_3(1-2n_2^2)+\alpha n_1 n_2}{\theta} & n_3(\alpha n_2 n_3 - \beta n_1) + \frac{\gamma n_2(1-2n_3^2)+\alpha n_1 n_3}{\theta} \\
n_1(\alpha n_1 n_3 - \beta n_2) + \frac{\gamma n_3(1-2n_1^2)+\alpha n_1 n_2}{\theta} & n_2(\alpha n_1 n_3 - \beta n_2) - \frac{\alpha(1-n_2^2)+2\gamma n_1 n_2 n_3}{\theta} & n_3(\alpha n_1 n_3 - \beta n_2) + \frac{\gamma n_1(1-2n_3^2)+\alpha n_2 n_3}{\theta} \\
n_1(\alpha n_2 n_3 + \beta n_1) + \frac{\alpha(1-n_1^2)-2\gamma n_1 n_2 n_3}{\theta} & n_2(\alpha n_2 n_3 + \beta n_1) + \frac{\gamma n_3(1-2n_2^2)-\alpha n_1 n_2}{\theta} & n_3(\alpha n_2 n_3 + \beta n_1) + \frac{\gamma n_2(1-2n_3^2)-\alpha n_1 n_3}{\theta} \\
-\frac{2\gamma n_1 n_3^2}{\theta} + \alpha n_1(n_3^2-1) & -\frac{2\gamma n_2 n_3^2}{\theta} + \alpha n_2(n_3^2-1) & \frac{2\gamma n_3(1-n_3^2)}{\theta} + \alpha n_3(n_3^2-1)
\end{bmatrix}
$$

That is the same result as that found in the textbook.