

$$\leq\text{-refl} : \forall \{n : \mathbb{N}\} \rightarrow n \leq n$$

$$\leq\text{-refl} \{\text{zero}\} = \text{z}\leq\text{n}$$

$$\leq\text{-refl} \{\text{suc } n\} = \text{s}\leq\text{s } \leq\text{-refl}$$

$$\leq\text{-trans} : \forall \{m \ n \ p : \mathbb{N}\} \rightarrow m \leq n \rightarrow n \leq p \rightarrow m \leq p$$

$$\leq\text{-trans } \text{z}\leq\text{n } \_ = \text{z}\leq\text{n}$$

$$\leq\text{-trans } (\text{s}\leq\text{s } m\leq n) (\text{s}\leq\text{s } n\leq p) = \text{s}\leq\text{s } (\leq\text{-trans } m\leq n \ n\leq p)$$

$$<\text{-trans} : \forall \{m \ n \ p : \mathbb{N}\} \rightarrow m < n \rightarrow n < p \rightarrow m < p$$

$$<\text{-trans } \text{z}<\text{s } (\text{s}<\text{s } \_) = \text{z}<\text{s}$$

$$<\text{-trans } (\text{s}<\text{s } m< n) (\text{s}<\text{s } n< p) = \text{s}<\text{s } (<\text{-trans } m< n \ n< p)$$