

-- Simultaneous substitution

exts : $\forall\{\Gamma \Delta\} \rightarrow (\forall\{A\} \rightarrow A \in \Gamma \rightarrow \Delta \vdash A)$
 $\rightarrow (\forall\{A B\} \rightarrow B \in \Gamma, A \rightarrow \Delta, A \vdash B)$

exts σ Zero = Var Zero

exts σ (Suc x) = rename Suc (σx)

subst : $\forall\{\Gamma \Delta\} \rightarrow (\forall\{A\} \rightarrow A \in \Gamma \rightarrow \Delta \vdash A)$
 $\rightarrow (\forall\{A\} \rightarrow \Gamma \vdash A \rightarrow \Delta \vdash A)$

subst σ (Var a) = σa

subst σ (Sub $a A \leq B$) = Sub (subst σa) $A \leq B$

subst σ (Lambda f) = Lambda (subst (exts σ) f)

subst σ (App $f e$) = App (subst σf) (subst σe)

subst σ Skip = Skip

subst σ (Seq $c_1 c_2$) = Seq (subst σc_1) (subst σc_2)

subst σ (NewVar c) = NewVar (subst (exts σ) c)

subst σ (Assign $a e$) = Assign (subst σa) (subst σe)

subst σ (Lit i) = Lit i

subst σ (Neg e) = Neg (subst σe)

subst σ (Plus $e_1 e_2$) = Plus (subst σe_1) (subst σe_2)

-- Single substitution

$_[_] : \forall\{\Gamma A B\} \rightarrow \Gamma, B \vdash A \rightarrow \Gamma \vdash B \rightarrow \Gamma \vdash A$

$_[_] \{\Gamma\} \{A\} \{B\} N M = \text{subst } \{\Gamma, B\} \{\Gamma\} \sigma \{A\} N$

where

$\sigma : \forall\{A\} \rightarrow A \in \Gamma, B \rightarrow \Gamma \vdash A$

σ Zero = M

σ (Suc x) = Var x

-- Reduction

data $_ \longrightarrow _ : \forall\{\Gamma A\} \rightarrow (\Gamma \vdash A) \rightarrow (\Gamma \vdash A) \rightarrow \text{Set where}$

App-cong₁ : $\forall\{\Gamma A B\} \{F F' : \Gamma \vdash A \Rightarrow B\} \{E : \Gamma \vdash A\}$
 $\rightarrow F \longrightarrow F' \rightarrow \text{App } F E \longrightarrow \text{App } F' E$

App-cong₂ : $\forall\{\Gamma A B\} \{V : \Gamma \vdash A \Rightarrow B\} \{E E' : \Gamma \vdash A\}$
 $\rightarrow \text{Value } V \rightarrow E \longrightarrow E' \rightarrow \text{App } V E \longrightarrow \text{App } V E'$

Lambda- β : $\forall\{\Gamma A B\} \{F : \Gamma, A \vdash B\} \{V : \Gamma \vdash A\}$
 $\rightarrow \text{Value } V \rightarrow \text{App (Lambda } F) V \longrightarrow F [V]$