```
module source where
open import lib
-- Operator precedence and associativity
infix 1 <u>_</u>≤:_
-- infix 2 \longrightarrow_
infix 4 _⊢_
infix 4 _∈_
infixl 5 _,_
infixr 7 _⇒_
-- Types
data Type : Set where
   comm : Type
intexp : Type
intacc : Type
intvar : Type
     \Rightarrow_: Type \rightarrow Type \rightarrow Type
-- Subtype relation data \_\le:\_: Type \to Type \to Set where
   \leq:-refl : \forall \{A\} \rightarrow A \leq: A
     \stackrel{-}{\leq} : -\mathsf{trans} : \ \forall \{ \stackrel{\cdot}{A} \ A \ ' \ A \ '' \} \rightarrow A \leq : \ A \ ' \rightarrow A \ ' \leq : \ A \ '' \rightarrow A \leq : \ A \ '' 
   \leq:-fn : \forall \{A \ A' \ B \ B'\} \rightarrow A' \leq: A \rightarrow B \leq: B' \rightarrow A \Rightarrow B \leq: A' \Rightarrow B'
   \mathsf{var}\text{-}\underline{\leq}\text{:-}\mathsf{exp}\,:\,\mathsf{intvar}\,\underline{\leq}\text{:}\,\,\mathsf{intexp}
   var-≤:-acc : intvar ≤: intacc
-- Contexts
data Context : Set where
    · : Context
    \_,\_: Context \rightarrow Type \rightarrow Context
-- Variables and the lookup judgement
data \subseteq: Type \rightarrow Context \rightarrow Set where
    Zero : \forall \{ \Gamma \ A \} \rightarrow A \in \Gamma , A
    Suc : \forall \{ \Gamma \ A \ B \} \rightarrow B \in \Gamma \rightarrow B \in \Gamma , A
-- Terms and the typing judgement data \_\vdash\_ : Context \to Type \to Set where
    \mathsf{Var}:\,\forall\{\varGamma\ A\}\to A\in\varGamma\to\varGamma\vdash A
    -- subtyping
   \mathsf{Sub}:\,\forall\{\varGamma\ A\ B\}\to\varGamma\vdash A\to A\leq:\ B\to\varGamma\vdash B
    -- lambda function and application
   Lambda : \forall \{ \Gamma \ A \ B \} \to \Gamma , A \vdash B \to \Gamma \vdash A \Rightarrow B
   \mathsf{App} : \forall \{\varGamma \ A \ B\} \to \varGamma \vdash A \Rightarrow B \to \varGamma \vdash A \to \varGamma \vdash B
    -- command
   \mathsf{Skip}:\,\forall\{\varGamma\}\to\varGamma\vdash\mathsf{comm}
   \begin{array}{l} \mathsf{Seq} : \, \forall \{\varGamma\} \to \varGamma \vdash \mathsf{comm} \to \varGamma \vdash \mathsf{comm} \to \varGamma \vdash \mathsf{comm} \\ \mathsf{NewVar} : \, \forall \{\varGamma\} \to \varGamma \text{ , intvar} \vdash \mathsf{comm} \to \varGamma \vdash \mathsf{comm} \\ \mathsf{Assign} : \, \forall \{\varGamma\} \to \varGamma \vdash \mathsf{intacc} \to \varGamma \vdash \mathsf{intexp} \to \varGamma \vdash \mathsf{comm} \end{array}
    -- intexp
   Lit : \forall \{\Gamma\} \rightarrow \mathbb{Z} \rightarrow \Gamma \vdash \text{intexp}
    \mathsf{Neg} : \, \dot{\forall} \{ \dot{\varGamma} \} \to \varGamma \vdash \mathsf{intexp} \to \varGamma \vdash \mathsf{intexp}
    Plus : \forall \{\Gamma\} \rightarrow \Gamma \vdash \text{intexp} \rightarrow \Gamma \vdash \text{intexp} \rightarrow \Gamma \vdash \text{intexp}
data Value : \forall \{ \Gamma A \} \rightarrow \Gamma \vdash A \rightarrow \mathsf{Set} \ \mathsf{where}
   \mathsf{V-Lit}:\,\forall\{\varGamma\}\,\,\{i:\,\mathbb{Z}\}\,\rightarrow\,\mathsf{Value}\,\,(\mathsf{Lit}\,\,\{\varGamma\}\,\,i)
    V-Lambda : \forall \{ \Gamma \ A \ B \} \ \{ F : \Gamma \ , \ A \vdash B \} \rightarrow \mathsf{Value} \ (\mathsf{Lambda} \ \{ \Gamma \} \ F )
-- -- Renaming
-- ext : \forall{Γ Δ} → (\forall{A} → A ∈ Γ → A ∈ Δ) → (\forall{A B} → B ∈
-- ext
                 Zero = Zero
                  (Suc x) = Suc (x)
-- rename : \forall \{\Gamma \ \Delta\} \ \rightarrow \ (\forall \{A\} \ \rightarrow \ A \ \in \ \Gamma \ \rightarrow \ A \ \in \ \Delta) \ \rightarrow \ (\forall \{A\} \ \rightarrow \ \Gamma \ )
-- rename (Var x) = Var ( x)
-- rename (Lambda F) = Lambda (rename (ext ) F)
-- rename
                      (App F E) = App (rename F) (rename E)
-- rename Skip = Skip
-- rename
                         (Seq c c) = Seq (rename c) (rename
                        (Lit i) = Lit i
 -- rename
-- rename
                        (Neg I) = Neg (rename I)
-- rename (Plus I I) = Plus (rename I) (rename I)
-- -- Simultaneous substitution
-- exts : \forall \{\Gamma \ \Delta\} \ \rightarrow \ (\forall \{A\} \ \rightarrow \ A \ \in \ \Gamma \ \rightarrow \ \Delta \ \vdash \ A) \ \rightarrow \ (\forall \{A \ B\} \ \rightarrow \ B \ )
-- exts Zero = Var Zero
                    (Suc x) = rename Suc (x)
-- subst : \forall{Γ Δ} → (\forall{A} → A ∈ Γ → Δ ⊢ A) → (\forall{A} → Γ ⊢
                       (Var x) =
                                                X
                       (Lambda F) = Lambda (subst (exts
                        (App F E) = App (subst F) (subst
-- subst
                      Skip = Skip
-- subst
                       (Seq c c) = Seq (subst c) (subst c
(Lit i) = Lit i
(Neg I) = Neg (subst I)
(Plus I I) = Plus (subst I) (subst
-- subst
-- subst
-- subst
-- subst
-- -- Single substitution
-- _[_] : \forall \{\Gamma \ A \ B\} \ \to \ \Gamma , B \vdash A \ \to \ \Gamma \vdash B \ \to \ \Gamma \vdash A
-- _[_] {\Gamma} {A} {B} N M = subst {\Gamma , B} {\Gamma} {A} N
--
                  : \forall \{A\} \rightarrow A \in \Gamma , B \rightarrow \Gamma \vdash A
__
                  Zero = M
                   (Suc x) = Var x
-- -- Reduction
-- data \_\longrightarrow\_ : \forall \{\Gamma \ A\} \rightarrow (\Gamma \vdash A) \rightarrow (\Gamma \vdash A) \rightarrow Set where -- App-cong : \forall \{\Gamma \ A \ B\} \ \{F \ F \ : \Gamma \vdash A \Rightarrow B\} \ \{E \ : \Gamma \vdash A\} \rightarrow App-cong : \forall \{\Gamma \ A \ B\} \ \{V \ : \Gamma \vdash A \Rightarrow B\} \ \{E \ E \ : \Gamma \vdash A\} \rightarrow App-cong : \forall \{\Gamma \ A \ B\} \ \{V \ : \Gamma \vdash A \Rightarrow B\} \ \{E \ E \ : \Gamma \vdash A\} \rightarrow App-cong
               Lambda- : \forall \{\Gamma \ A \ B\} \ \{F \ : \ \Gamma \ , \ A \ \vdash \ B\} \ \{V \ : \ \Gamma \ \vdash \ A\} \ \to \ Val
```