```
-- Operator precedence and associativity
infix 4 _≤<sub>s</sub>_
infixl 6_
open import lib
-- Stack descriptor: (frames, displacement)
record SD: Set where
   constructor (_,_)
   \mathsf{field}\;\mathsf{f}\;\mathsf{d}:\mathbb{N}
-- Stack descriptor operations
  _+,_s:SD \to \mathbb{N} \to SD
\langle f, d \rangle +_{s} n = \langle f, d + n \rangle
      \dot{} : SD \rightarrow \mathbb{N} \rightarrow SD
-- \langle f , d \rangle \dot{} n = \langle f , d \dot{} n \rangle
-- _- : (sd : SD) \rightarrow (n : \mathbb{N}) \rightarrow n SD.d sd \rightarrow SD -- (\langle S_f , S_d \rangle - n) p = \langle S_f , (S_d - n) p \rangle
  [-s]: (sd: SD) \to (n: \mathbb{N}) \to (n \le d: n \le SD.d \ sd) \to SD
(\langle f, d \rangle -_{s} n) \ n \leq d = \langle f, (d-n) \ n \leq d \rangle
\neg_{s} \equiv : \forall \{f \ d \ d' \ n\} \rightarrow \{n \leq d' : \ n \leq d'\} \rightarrow (d' - n) \ n \leq d' \equiv d
             \rightarrow \langle f, d \rangle = (\langle f, d' \rangle -_{s} n) \ n \leq d'
-s = p \text{ rewrite } p = \text{refl}
-- Stack descriptor lexicographic ordering
data \_\leq_s \_: SD \to SD \to Set where
        <-f : \forall \{f \ f' \ d \ d'\} \rightarrow f < f' \rightarrow \langle f \ , \ d \ \rangle \leq_s \langle f' \ , \ d' \ \rangle      <-d : \forall \{f \ d \ d'\} \rightarrow d \leq d' \rightarrow \langle f \ , \ d \ \rangle \leq_s \langle f \ , \ d' \ \rangle 
\leq_s-refl: \forall \{sd: SD\} \rightarrow sd \leq_s sd
\leq_s-refl \{\langle f, d \rangle\} = \leq-d \leq-refl
\leq_s-trans: \forall \{sd \ sd' \ sd'' : SD\} \rightarrow sd \leq_s sd' \rightarrow sd' \leq_s sd'' \rightarrow sd \leq_s sd''
\leq_s-trans (<-f f < f') (\leq-d _) = <-f f < f'
\leq_s-trans (<-f f < f') (<-f f' < f'') = <-f (<-trans f < f'' f' < f'') \leq_s-trans (\leq-d _) (<-f f' < f'') = <-f f' < f''
\leq_s-trans (\leq-d d\leq d') (\leq-d d'\leq d'') = \leq-d (\leq-trans d\leq d' d'\leq d'')
+_s \longrightarrow \leq_s : \forall \{sd : SD\} \longrightarrow \forall \{n : \mathbb{N}\} \longrightarrow sd \leq_s sd +_s n
+_s \rightarrow \leq_s = \leq -d + \rightarrow \leq
sub-sd \leq_s : \forall \{sd \ sd' \ sd''\} \rightarrow sd' \equiv sd'' \rightarrow sd \leq_s sd' \rightarrow sd \leq_s sd''
sub-sd \le_s sd' = sd'' sd \le_s sd' rewrite sd' = sd \le_s sd'
-- Operator
data UnaryOp: Set where
    UNeg: UnaryOp
data BinaryOp: Set where
    BPlus BMinus BTimes : BinaryOp
data RelOp : Set where
    RLeq RLt : RelOp
-- Nonterminals
-- Lefthand sides
data L (sd: SD): Set where
   l\text{-var}: (sd^{v}: SD) \rightarrow sd^{v} \leq_{s} sd \rightarrow L sd
   l-sbrs : L sd
 -- Simple righthand sides
data S (sd: SD): Set where
   s-1: L sd \rightarrow S sd
   s-lit : \mathbb{Z} \to S sd
-- Righthand sides
data R(sd : SD) : Set where
    r-s: S sd \rightarrow R sd
    r\text{-unary}: \mathsf{UnaryOp} \to \mathsf{S} \; \mathit{sd} \to \mathsf{R} \; \mathit{sd}
   r-binary : S sd \rightarrow BinaryOp \rightarrow S sd \rightarrow R sd
-- Instruction sequences
data I(sd : SD) : Set where
   stop: I sd
    assign-inc : (\delta : \mathbb{N}) \to \mathsf{L} (sd +_s \delta) \to \mathsf{R} sd \to \mathsf{I} (sd +_s \delta) \to \mathsf{I} sd
   assign-dec : (\delta : \mathbb{N}) → (\delta \le d : \delta \le \text{SD.d } sd) → L ((sd -_s \delta) \delta \le d)
                               \rightarrow \mathsf{R} \ sd \rightarrow \mathsf{I} \ ((sd -_{\mathsf{s}} \delta) \ \delta \leq d) \rightarrow \mathsf{I} \ sd
    if-then-else-inc : (\delta : \mathbb{N}) \to S \ sd \to \mathsf{RelOp} \to S \ sd
                                           \rightarrow I (sd +_s \delta) \rightarrow I (sd +_s \delta) \rightarrow I sd
   if-then-else-dec : (\delta : \mathbb{N}) \rightarrow (\delta \leq d : \delta \leq \text{SD.d } sd)
                                           \rightarrow S sd \rightarrow RelOp \rightarrow S sd
                                           \rightarrow I ((sd -_s \delta) \delta \leq d)
                                           \rightarrow I ((sd -_s \delta) \delta \leq d) \rightarrow I sd
   adjustdisp-inc : (\delta : \mathbb{N}) \to \mathsf{I}(sd +_s \delta) \to \mathsf{I}sd
   adjustdisp-dec : (\delta : \mathbb{N}) \rightarrow (\delta \leq d : \delta \leq SD.d \ sd)
                                      \rightarrow I ((sd -_s \delta) \delta \leq d) \rightarrow I sd
    popto : (sd' : SD) \rightarrow sd' \leq_s sd \rightarrow I sd' \rightarrow I sd
\mathsf{I\text{-}sub}: \forall \{f \ d \ d' \ n\} \rightarrow \{n \leq d' : \ n \leq d'\} \rightarrow (d' - n) \ n \leq d' \equiv d
                    \rightarrow \mathsf{I}\left(\left\langle\,f\;,\;d\;\right\rangle\right) \rightarrow \mathsf{I}\left(\left(\left\langle\,f\;,\;d'\;\right\rangle\,\mathsf{-_s}\;n\right)\;n{\leq}d'\right)
I-sub \{n = n\} d'-n=d c = \text{sub I } (-s = \{n = n\} \ d'-n=d) c
```

module target where