

--irrelevant : $\forall \{n\ m\} \rightarrow (p_1\ p_2 : m \leq n) \rightarrow (n - m)\ p_1 \equiv (n - m)\ p_2$
 --irrelevant $\{n\} \{m\}\ p_1\ p_2 =$
 cong $(\lambda\ p \rightarrow (n - m)\ p)\ (\leq\text{-irrelevant}\ p_1\ p_2)$

-- n - m ≤ n

--→≤ : $\forall \{n\ m\} \rightarrow (m \leq n : m \leq n) \rightarrow (n - m)\ m \leq n \leq n$

--→≤ z≤n = ≤-refl

--→≤ (s≤s m≤n) = ≤-trans (--→≤ m≤n) n≤suc-n

n-n≡0 : $\forall \{n\} \rightarrow (n - n)\ (\leq\text{-refl}\ \{n\}) \equiv 0$

n-n≡0 {zero} = refl

n-n≡0 {suc n} = n-n≡0 {n}

-- suc (n - m) ≡ suc n - m

--suc : $\forall \{n\ m\} \rightarrow \{m \leq n : m \leq n\}$

$\rightarrow \text{suc}\ ((n - m)\ m \leq n) \equiv (\text{suc}\ n - m)\ (\leq\text{-trans}\ m \leq n\ n \leq \text{suc}\ n)$

--suc { } {zero} {z≤n} = refl

--suc {suc n} {suc m} {s≤s m≤n} = --suc {n} {m} {m≤n}

n-[n-m]≡m : $\forall \{m\ n\} \rightarrow (m \leq n : m \leq n)$

$\rightarrow (n - ((n - m)\ m \leq n))\ (--→≤\ m \leq n) \equiv m$

n-[n-m]≡m {zero} {n} z≤n = n-n≡0 {n}

n-[n-m]≡m {suc m} {suc n} (s≤s m≤n) =

trans (sym (--suc {n} {(n - m)\ m≤n}))

(cong suc (n-[n-m]≡m {m} {n} m≤n))

n+m-m≡n : $\forall \{m\ n\} \rightarrow (n + m - m)\ (+→≤') \equiv n$

n+m-m≡n {m} {zero} =

trans (--irrelevant {m} {m} +→≤' ≤-refl) (n-n≡0 {m})

n+m-m≡n {m} {suc n} =

trans

(--irrelevant {suc n + m} {m} +→≤' (≤-trans +→≤' n≤suc-n))

(trans (sym (--suc {n + m} {m})))

(cong suc (n+m-m≡n {m} {n})))

-- m ≤ n → m - p ≤ n - p

sub-mono^r-≤ : $\forall \{p\ m\ n\} \rightarrow (p \leq m : p \leq m) \rightarrow (m \leq n : m \leq n)$

$\rightarrow (m - p)\ p \leq m \leq (n - p)\ (\leq\text{-trans}\ p \leq m\ m \leq n)$

sub-mono^r-≤ z≤n m≤n = m≤n

sub-mono^r-≤ (s≤s p≤m) (s≤s m≤n) = sub-mono^r-≤ p≤m m≤n

-- suc d ≤ d' → d ≤ d' - (d' - (suc d))

suc-d≤d'→d≤d'-[d'-(suc-d)] : $\forall \{d\ d'\} \rightarrow (\delta_1 \leq \delta_2 : \text{suc}\ d \leq d')$

$\rightarrow d \leq ((d' - ((d' - (\text{suc}\ d))\ \delta_1 \leq \delta_2))\ (--→≤\ \delta_1 \leq \delta_2))$

suc-d≤d'→d≤d'-[d'-(suc-d)] $\delta_1 \leq \delta_2 =$

$m \equiv n, p \leq n \rightarrow p \leq m\ (n - [n - m] \equiv m\ \delta_1 \leq \delta_2)\ n \leq \text{suc}\ n$