

$\leq\text{-irrelevant} : \forall \{m\ n\} \rightarrow (p_1\ p_2 : m \leq n) \rightarrow p_1 = p_2$

$\leq\text{-irrelevant}\ z \leq n\ z \leq n = \text{refl}$

$\leq\text{-irrelevant}\ (s \leq s\ p_1)\ (s \leq s\ p_2) = \text{cong}\ s \leq s\ (\leq\text{-irrelevant}\ p_1\ p_2)$

$\leq\text{-refl} : \forall \{n : \mathbb{N}\} \rightarrow n \leq n$

$\leq\text{-refl}\ \{\text{zero}\} = z \leq n$

$\leq\text{-refl}\ \{\text{suc}\ n\} = s \leq s\ \leq\text{-refl}$

$\leq\text{-trans} : \forall \{m\ n\ p : \mathbb{N}\} \rightarrow m \leq n \rightarrow n \leq p \rightarrow m \leq p$

$\leq\text{-trans}\ z \leq n\ _ = z \leq n$

$\leq\text{-trans}\ (s \leq s\ m \leq n)\ (s \leq s\ n \leq p) = s \leq s\ (\leq\text{-trans}\ m \leq n\ n \leq p)$

$n \leq \text{suc}\text{-}n : \forall \{n : \mathbb{N}\} \rightarrow n \leq \text{suc}\ n$

$n \leq \text{suc}\text{-}n\ \{\text{zero}\} = z \leq n$

$n \leq \text{suc}\text{-}n\ \{\text{suc}\ n\} = s \leq s\ n \leq \text{suc}\text{-}n$

data **Order** : $\mathbb{N} \rightarrow \mathbb{N} \rightarrow \text{Set} **where**$

leq : $\forall \{m\ n : \mathbb{N}\} \rightarrow m \leq n \rightarrow \text{Order}\ m\ n$

geq : $\forall \{m\ n : \mathbb{N}\} \rightarrow n \leq m \rightarrow \text{Order}\ m\ n$

$\leq\text{-compare} : \forall \{m\ n : \mathbb{N}\} \rightarrow \text{Order}\ m\ n$

$\leq\text{-compare}\ \{\text{zero}\}\ \{n\} = \text{leq}\ z \leq n$

$\leq\text{-compare}\ \{\text{suc}\ m\}\ \{\text{zero}\} = \text{geq}\ z \leq n$

$\leq\text{-compare}\ \{\text{suc}\ m\}\ \{\text{suc}\ n\} \text{ with } \leq\text{-compare}\ \{m\}\ \{n\}$

$\dots \mid \text{leq}\ m \leq n = \text{leq}\ (s \leq s\ m \leq n)$

$\dots \mid \text{geq}\ n \leq m = \text{geq}\ (s \leq s\ n \leq m)$

data **_<_** : $\mathbb{N} \rightarrow \mathbb{N} \rightarrow \text{Set} **where**$

z< s : $\forall \{n : \mathbb{N}\} \rightarrow \text{zero} < \text{suc}\ n$

s< s : $\forall \{m\ n : \mathbb{N}\} \rightarrow m < n \rightarrow \text{suc}\ m < \text{suc}\ n$

$<\rightarrow s \leq : \forall \{m\ n : \mathbb{N}\} \rightarrow m < n \rightarrow \text{suc}\ m \leq n$

$<\rightarrow s \leq\ (z < s) = s \leq s\ z \leq n$

$<\rightarrow s \leq\ (s < s\ m < n) = s \leq s\ (<\rightarrow s \leq\ m < n)$

$<\rightarrow \leq : \forall \{m\ n : \mathbb{N}\} \rightarrow m < n \rightarrow m \leq n$

$<\rightarrow \leq\ m < n = \leq\text{-trans}\ n \leq \text{suc}\text{-}n\ (<\rightarrow s \leq\ m < n)$

$<\text{-trans} : \forall \{m\ n\ p : \mathbb{N}\} \rightarrow m < n \rightarrow n < p \rightarrow m < p$

$<\text{-trans}\ z < s\ (s < s\ _) = z < s$

$<\text{-trans}\ (s < s\ m < n)\ (s < s\ n < p) = s < s\ (<\text{-trans}\ m < n\ n < p)$