

$$\begin{aligned}
-&_s \equiv : \forall \{f \ d \ d' \ n\} \rightarrow \{n \leq d' : n \leq d'\} \rightarrow (d' - n) \ n \leq d' = d \\
&\rightarrow \langle f \ , \ d \rangle \equiv (\langle f \ , \ d' \rangle -_s n) \ n \leq d' \\
-&_s \equiv p \text{ \textcolor{brown}{rewrite}} \ p = \text{refl}
\end{aligned}$$

$$I\text{-sub} \{n = n\} \ d' - n \equiv d \ c = \text{sub } I \ (-_s \equiv \{n = n\} \ d' - n \equiv d) \ c$$

$$\leq_s\text{-refl} : \forall \{sd : \text{SD}\} \rightarrow sd \leq_s sd$$

$$\leq_s\text{-refl} \{\langle f \ , \ d \rangle\} = \leq\text{-d} \leq\text{-refl}$$

$$\leq_s\text{-trans} : \forall \{sd \ sd' \ sd'' : \text{SD}\} \rightarrow sd \leq_s sd' \rightarrow sd' \leq_s sd'' \rightarrow sd \leq_s sd''$$

$$\leq_s\text{-trans} (<\text{-f} \ f < f') (\leq\text{-d} \ _) = <\text{-f} \ f < f'$$

$$\leq_s\text{-trans} (<\text{-f} \ f < f') (<\text{-f} \ f' < f'') = <\text{-f} \ (<\text{-trans} \ f < f' \ f' < f'')$$

$$\leq_s\text{-trans} (\leq\text{-d} \ _) (<\text{-f} \ f' < f'') = <\text{-f} \ f' < f''$$

$$\leq_s\text{-trans} (\leq\text{-d} \ d \leq d') (\leq\text{-d} \ d' \leq d'') = \leq\text{-d} \ (\leq\text{-trans} \ d \leq d' \ d' \leq d'')$$

$$+_s \rightarrow \leq_s : \forall \{sd : \text{SD}\} \rightarrow \forall \{n : \mathbb{N}\} \rightarrow sd \leq_s sd +_s n$$

$$+_s \rightarrow \leq_s = \leq\text{-d} \ + \rightarrow \leq$$

$$\text{sub-sd}_{\leq_s} : \forall \{sd \ sd' \ sd''\} \rightarrow sd' \equiv sd'' \rightarrow sd \leq_s sd' \rightarrow sd \leq_s sd''$$

$$\text{sub-sd}_{\leq_s} \ sd' \equiv sd'' \ sd \leq_s sd' \text{ \textcolor{brown}{rewrite}} \ sd' \equiv sd'' = sd \leq_s sd'$$

$$\text{sd} \leq_s \text{sd}' \rightarrow \text{sd} \leq_s \text{sd}' -_s [d' - [\text{suc-d}]] : \forall \{sd \ sd'\} \rightarrow sd \leq_s sd'$$

$$\rightarrow (\delta_1 \leq \delta_2 : \text{succ}(\text{SD.d} \ sd) \leq \text{SD.d} \ sd')$$

$$\rightarrow sd \leq_s ((sd' -_s ((\text{SD.d} \ sd' - (\text{succ}(\text{SD.d} \ sd)))) \ \delta_1 \leq \delta_2)) \ (-\rightarrow \leq \ \delta_1 \leq \delta_2))$$

$$\text{sd} \leq_s \text{sd}' \rightarrow \text{sd} \leq_s \text{sd}' -_s [d' - [\text{suc-d}]] \{\langle f \ , \ _ \rangle\} \{\langle f' \ , \ _ \rangle\} (<\text{-f} \ f < f') \ _$$

$$= <\text{-f} \ f < f'$$

$$\text{sd} \leq_s \text{sd}' \rightarrow \text{sd} \leq_s \text{sd}' -_s [d' - [\text{suc-d}]] \{\langle f \ , \ d \rangle\} \{\langle f' \ , \ d' \rangle\} (\leq\text{-d} \ d \leq d') \ \delta_1 \leq \delta_2$$

$$= \leq\text{-d} \ (\text{suc-d} \leq d' \rightarrow d \leq d' - [d' - [\text{suc-d}]] \ \delta_1 \leq \delta_2)$$