compiler.md 2025-04-09

```
module compiler where
open import source
open import target
open import lib
infixr 1 _⇒s_
infixl 2 _x_
-- Product and projection function
data _x_ (A B : Set) : Set where
      _{-\prime} : A \rightarrow B \rightarrow A \times B
\pi_1: \forall \{A B\} \rightarrow A \times B \rightarrow A
\pi_1 (a , _) = a
\pi_2: \forall \{A B\} \rightarrow A \times B \rightarrow B
\pi_2 (_ , b) = b
-- Type Interpretation
Compl : SD \rightarrow Set
Compl sd = I sd
\_x_s\_ : (SD \rightarrow Set) \rightarrow (SD \rightarrow Set) \rightarrow SD \rightarrow Set
(P \times_s Q) sd = P sd \times Q sd
\_\Rightarrow_s\_ : (SD \rightarrow Set) \rightarrow (SD \rightarrow Set) \rightarrow SD \rightarrow Set
(P \Rightarrow_s Q) sd = \forall \{sd'\} \rightarrow (sd \leq_s sd') \rightarrow P sd' \rightarrow Q sd'
Intcompl : SD → Set
Intcompl = R \Rightarrow_s Compl
[\![ ]\!]ty : Type \rightarrow SD \rightarrow Set
[ comm ]ty = Compl \Rightarrow_s Compl
\llbracket intexp \rrbracketty = Intcompl ⇒<sub>s</sub> Compl
[ intacc ]ty = Compl \Rightarrow_s Intcompl
[ intvar ]ty = [ intexp ]ty xs [ intacc ]ty
\llbracket \theta_1 \Rightarrow \theta_2 \rrbracket ty = \llbracket \theta_1 \rrbracket ty \Rightarrow_s \llbracket \theta_2 \rrbracket ty
-- Unit type for empty context
data \emptyset : Set where
      unit : Ø
-- Context Interpretation
[_]ctx : Context → SD → Set
[ \cdot ] ctx _ = \emptyset
\llbracket \Gamma , A \rrbracket ctx sd = \llbracket \Gamma \rrbracket ctx sd \times \llbracket A \rrbracket ty sd
get-var : \forall \{\Gamma \land sd\} \rightarrow A \in \Gamma \rightarrow [\Gamma ] ctx sd \rightarrow [A] ty sd
get-var Z \qquad (\_, a) = a
```

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get-var(S x)(y, _) = get-var x y
-- get-num : \forall {\Gamma} → (e : \Gamma \vdash source.\mathbb{N}) → target.\mathbb{N}
-- get-num Zero = target.zero
-- get-num (Suc m) = target.suc (get-num m)
fmap \rightarrow : \forall \{P \ Q \ sd \ sd'\} \rightarrow (P \Rightarrow_s Q) \ sd \rightarrow sd \leq_s sd' \rightarrow (P \Rightarrow_s Q) \ sd'
fmap-\Rightarrow \theta p p' x = \theta (\leq_s-trans p p') x
fmap\text{-}Compl : \ \forall \ \{sd\ sd'\} \ \rightarrow \ Compl\ sd \ \rightarrow \ sd \ \leq_s \ sd' \ \rightarrow \ Compl\ sd'
fmap-Compl \{sd\} c (<-f f<f') = popto sd (<-f f<f') c
fmap-Compl \{\langle \ \_\ ,\ d\ \rangle\}\ \{\langle \ \_\ ,\ d'\ \rangle\}\ c\ (\leq -d\ d\leq d') = adjustdisp\_dec\ (\leq \to Fin\ (-d\ d'))
\rightarrow \le \{d'\} \{\le \rightarrow Fin \ d \le d'\}\} ) (sub I \{! \ !\} \ c)
-- fmap-x : \forall {P Q sd sd'} \rightarrow (P \times_s Q) sd \rightarrow sd \leq_s sd' \rightarrow (P \times_s Q) sd'
fmap-A : \forall {A sd sd'} \rightarrow [ A ]ty sd \rightarrow sd \leqs sd' \rightarrow [ A ]ty sd'
fmap-A \{comm\} c p i = \{! !\}
fmap-A \{intexp\} = \{! !\}
fmap-A {intacc} = {! !}
fmap-A {intvar} = {! !}
fmap-A \{A \Rightarrow B\} = fmap- \Rightarrow \{ [A]ty \} \{ [B]ty \}
fmap-\Gamma : \forall \{\Gamma \ sd \ sd'\} \rightarrow \llbracket \Gamma \ \llbracket ctx \ sd \rightarrow \ sd \le_s \ sd' \rightarrow \llbracket \Gamma \ \llbracket ctx \ sd' \ \end{bmatrix}
fmap-\Gamma {·} unit _ = unit
fmap-\Gamma \{\Gamma, A\} (\gamma, a) p = fmap-\Gamma \gamma p, fmap-A \{A\} a p
\llbracket \_ \rrbracket : \forall {\Gamma A} \rightarrow (e : \Gamma \vdash A) \rightarrow (sd : SD) \rightarrow \llbracket \Gamma \rrbracketctx sd \rightarrow \llbracket A \rrbracketty sd
[Var x]_y = get-var x y
\llbracket \text{ Lambda f } \rrbracket \text{ sd y } \{\text{sd'} = \text{sd'}\} \text{ p a = } \llbracket \text{ f } \rrbracket \text{ sd'} \text{ (fmap-}\Gamma \text{ y p , a)} 
\llbracket \text{ App f e } \rrbracket \text{ sd y } = \llbracket \text{ f } \rrbracket \text{ sd y } (\leq -d \leq -\text{refl}) (\llbracket \text{ e } \rrbracket \text{ sd y})
[ Skip ] _ _ _ y = y
[Lit i] _ _ _ \kappa = \kappa \leq_s-refl (r-s (s-lit i))
[ Neg e ] sd \gamma p \kappa = {! !}
\llbracket \text{ Plus e } e_1 \rrbracket \_ \gamma = \{! !\}
```