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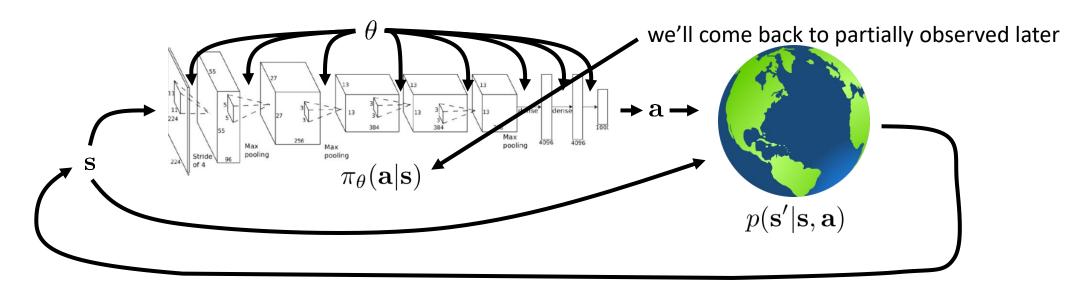
# Policy Gradients

CS 285

Instructor: Sergey Levine UC Berkeley



# The goal of reinforcement learning



$$\underbrace{p_{\theta}(\mathbf{s}_1, \mathbf{a}_1, \dots, \mathbf{s}_T, \mathbf{a}_T)}_{p_{\theta}(\tau)} = p(\mathbf{s}_1) \prod_{t=1}^{T} \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)$$

$$\theta^* = \arg\max_{\theta} E_{\tau \sim p_{\theta}(\tau)} \left[ \sum_{t} r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

# The goal of reinforcement learning

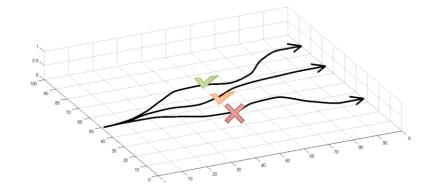
$$\theta^* = \arg\max_{\theta} E_{\tau \sim p_{\theta}(\tau)} \left[ \sum_{t} r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

$$\theta^{\star} = \arg\max_{\theta} E_{(\mathbf{s}, \mathbf{a}) \sim p_{\theta}(\mathbf{s}, \mathbf{a})}[r(\mathbf{s}, \mathbf{a})] \qquad \qquad \theta^{\star} = \arg\max_{\theta} \sum_{t=1}^{r} E_{(\mathbf{s}_{t}, \mathbf{a}_{t}) \sim p_{\theta}(\mathbf{s}_{t}, \mathbf{a}_{t})}[r(\mathbf{s}_{t}, \mathbf{a}_{t})]$$
 infinite horizon case

# Evaluating the objective

$$\theta^* = \arg \max_{\theta} E_{\tau \sim p_{\theta}(\tau)} \left[ \sum_{t} r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

$$J(\theta)$$



$$J(\theta) = E_{\tau \sim p_{\theta}(\tau)} \left[ \sum_{t} r(\mathbf{s}_{t}, \mathbf{a}_{t}) \right] \approx \frac{1}{N} \sum_{i} \sum_{t} r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

sum over samples from  $\pi_{\theta}$ 

# Direct policy differentiation

$$\theta^* = \arg\max_{\theta} E_{\tau \sim p_{\theta}(\tau)} \left[ \sum_{t} r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

$$J(\theta)$$

$$J(\theta) = E_{\tau \sim p_{\theta}(\tau)}[r(\tau)] = \int p_{\theta}(\tau)r(\tau)d\tau$$

$$\sum_{t=1}^{T} r(\mathbf{s}_{t}, \mathbf{a}_{t})$$

a convenient identity

$$p_{\theta}(\tau)\nabla_{\theta}\log p_{\theta}(\tau) = p_{\theta}(\tau)\frac{\nabla_{\theta}p_{\theta}(\tau)}{p_{\theta}(\tau)} = \nabla_{\theta}p_{\theta}(\tau)$$

$$\nabla_{\theta} J(\theta) = \int \underline{\nabla_{\theta} p_{\theta}(\tau)} r(\tau) d\tau = \int \underline{p_{\theta}(\tau)} \nabla_{\theta} \log p_{\theta}(\tau) r(\tau) d\tau = E_{\tau \sim p_{\theta}(\tau)} [\nabla_{\theta} \log p_{\theta}(\tau) r(\tau)]$$

# Direct policy differentiation

$$\theta^* = \arg \max_{\theta} J(\theta)$$

$$J(\theta) = E_{\tau \sim p_{\theta}(\tau)}[r(\tau)]$$

$$\log \text{ of both sides } p_{\theta}(\tau)$$

$$\log p_{\theta}(\tau) = \log p(\mathbf{s}_1) + \sum_{t=1}^{T} \log \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) + \log p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)$$

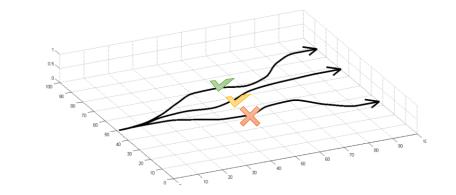
$$\nabla_{\theta} J(\theta) = E_{\tau \sim p_{\theta}(\tau)}[\nabla_{\theta} \log p_{\theta}(\tau) r(\tau)]$$

$$\nabla_{\theta} \left[ \log p(\mathbf{s}_1) + \sum_{t=1}^{T} \log \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) + \log p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t) \right]$$

$$\nabla_{\theta} J(\theta) = E_{\tau \sim p_{\theta}(\tau)} \left[ \left( \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{t} | \mathbf{s}_{t}) \right) \left( \sum_{t=1}^{T} r(\mathbf{s}_{t}, \mathbf{a}_{t}) \right) \right]$$

# Evaluating the policy gradient

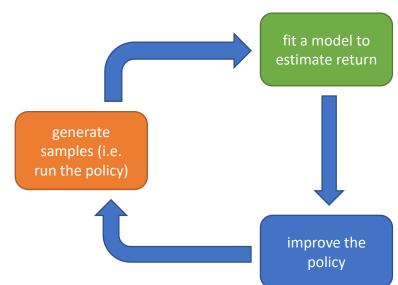
recall: 
$$J(\theta) = E_{\tau \sim p_{\theta}(\tau)} \left[ \sum_{t} r(\mathbf{s}_{t}, \mathbf{a}_{t}) \right] \approx \frac{1}{N} \sum_{i} \sum_{t} r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$



$$\nabla_{\theta} J(\theta) = E_{\tau \sim p_{\theta}(\tau)} \left[ \left( \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{t} | \mathbf{s}_{t}) \right) \left( \sum_{t=1}^{T} r(\mathbf{s}_{t}, \mathbf{a}_{t}) \right) \right]$$

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left( \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \right) \left( \sum_{t=1}^{T} r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$$

$$\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$$



#### REINFORCE algorithm:

- 1. sample  $\{\tau^i\}$  from  $\pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t)$  (run the policy)
- 2.  $\nabla_{\theta} J(\theta) \approx \sum_{i} \left( \sum_{t} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{t}^{i} | \mathbf{s}_{t}^{i}) \right) \left( \sum_{t} r(\mathbf{s}_{t}^{i}, \mathbf{a}_{t}^{i}) \right)$
- 3.  $\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$

# Understanding Policy Gradients

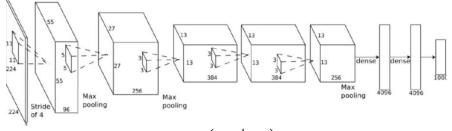
# Evaluating the policy gradient

recall: 
$$J(\theta) = E_{\tau \sim p_{\theta}(\tau)} \left[ \sum_{t} r(\mathbf{s}_{t}, \mathbf{a}_{t}) \right] \approx \frac{1}{N} \sum_{i} \sum_{t} r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

$$\nabla_{\theta} J(\theta) = E_{\tau \sim p_{\theta}(\tau)} \left[ \left( \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{t} | \mathbf{s}_{t}) \right) \left( \sum_{t=1}^{T} r(\mathbf{s}_{t}, \mathbf{a}_{t}) \right) \right]$$

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left( \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \right) \left( \sum_{t=1}^{T} r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$$
what is this?







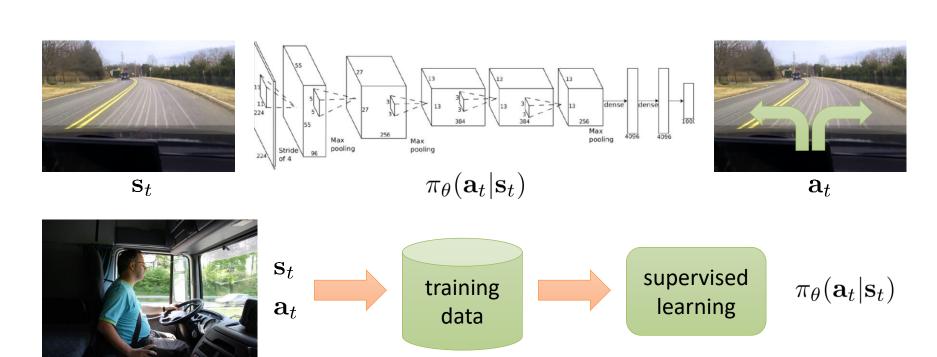
 $\pi_{ heta}(\mathbf{a}_t|\mathbf{s}_t)$ 

 $\mathbf{a}_t$ 

## Comparison to maximum likelihood

policy gradient: 
$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left( \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \right) \left( \sum_{t=1}^{T} r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$$

maximum likelihood: 
$$\nabla_{\theta} J_{\text{ML}}(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left( \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \right)$$



## Example: Gaussian policies

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left( \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \right) \left( \sum_{t=1}^{T} r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$$

example:  $\pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t) = \mathcal{N}(f_{\text{neural network}}(\mathbf{s}_t); \Sigma)$ 

$$\log \pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t) = -\frac{1}{2} ||f(\mathbf{s}_t) - \mathbf{a}_t||_{\Sigma}^2 + \text{const}$$

$$\nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) = -\frac{1}{2} \Sigma^{-1} (f(\mathbf{s}_t) - \mathbf{a}_t) \frac{df}{d\theta}$$

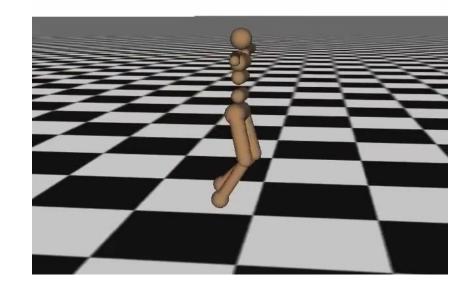
#### REINFORCE algorithm:



2.  $\nabla_{\theta} J(\theta) \approx \sum_{i} \left( \sum_{t} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{t}^{i} | \mathbf{s}_{t}^{i}) \right) \left( \sum_{t} r(\mathbf{s}_{t}^{i}, \mathbf{a}_{t}^{i}) \right)$ 

3.  $\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$ 

#### Iteration 2000



## What did we just do?

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left( \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \right) \left( \sum_{t=1}^{T} r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$$

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_{\theta} \log \pi_{\theta}(\tau_{i}) r(\tau_{i})$$
$$\sum_{t=1}^{T} \nabla_{\theta} \log_{\theta} \pi_{\theta}(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})$$

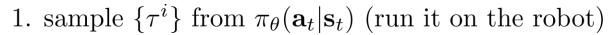
maximum likelihood: 
$$\nabla_{\theta} J_{\mathrm{ML}}(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_{\theta} \log \pi_{\theta}(\tau_{i})$$

good stuff is made more likely

bad stuff is made less likely

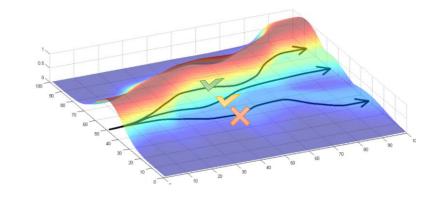
simply formalizes the notion of "trial and error"!

#### REINFORCE algorithm:

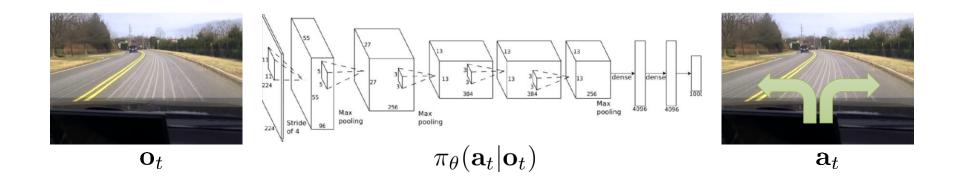


2. 
$$\nabla_{\theta} J(\theta) \approx \sum_{i} \left( \sum_{t} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{t}^{i} | \mathbf{s}_{t}^{i}) \right) \left( \sum_{t} r(\mathbf{s}_{t}^{i}, \mathbf{a}_{t}^{i}) \right)$$

3. 
$$\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$$



## Partial observability

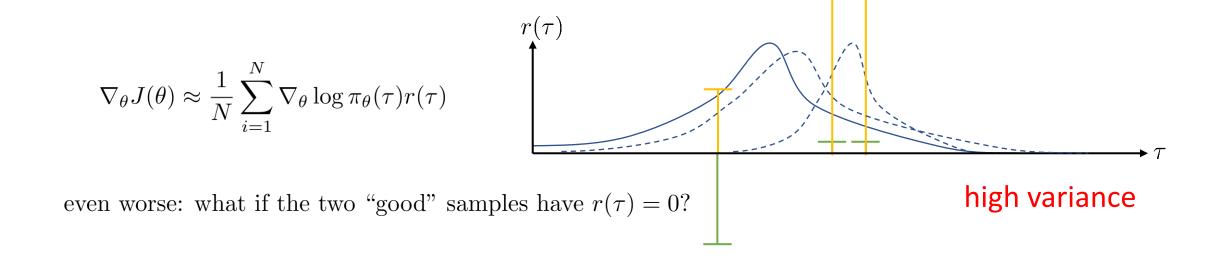


$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left( \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{o}_{i,t}) \right) \left( \sum_{t=1}^{T} r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$$

Markov property is not actually used!

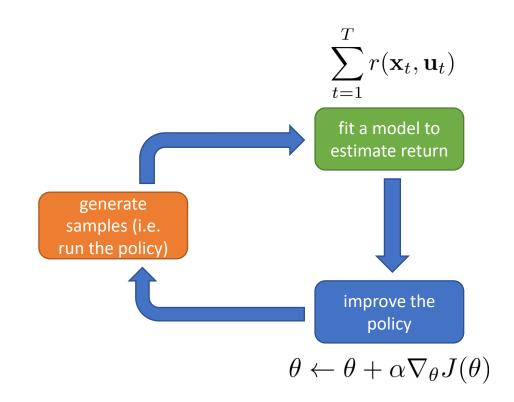
Can use policy gradient in partially observed MDPs without modification

# What is wrong with the policy gradient?



### Review

- Evaluating the RL objective
  - Generate samples
- Evaluating the policy gradient
  - Log-gradient trick
  - Generate samples
- Understanding the policy gradient
  - Formalization of trial-and-error
- Partial observability
  - Works just fine
- What is wrong with policy gradient?



# Reducing Variance

### Reducing variance

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left( \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \right) \left( \sum_{t=1}^{T} r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$$

Causality: policy at time t' cannot affect reward at time t when t < t'

$$abla_{ heta} J( heta) pprox rac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} 
abla_{ heta} \log \pi_{ heta}(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \Phi_{i,t'} \mathbf{r}(\mathbf{s}_{i,t''}, \mathbf{a}_{i,t''})$$

"reward to go"

$$\hat{Q}_{i,t}$$

### Baselines

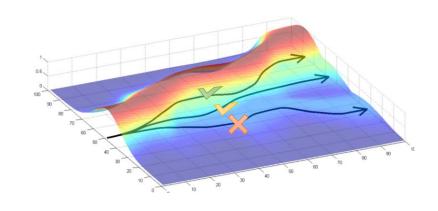
#### a convenient identity

$$p_{\theta}(\tau)\nabla_{\theta}\log p_{\theta}(\tau) = \nabla_{\theta}p_{\theta}(\tau)$$

$$abla_{ heta}J( heta)pproxrac{1}{N}\sum_{i=1}^{N}
abla_{ heta}\log p_{ heta}( au)[n( au)-b]$$

$$b = \frac{1}{N} \sum_{i=1}^{N} r(\tau)$$

 $b = \frac{1}{N} \sum_{i=1}^{N} r(\tau)$  but... are we *allowed* to do that??



$$E[\nabla_{\theta} \log p_{\theta}(\tau)b] = \int p_{\theta}(\tau)\nabla_{\theta} \log p_{\theta}(\tau)b \,d\tau = \int \nabla_{\theta} p_{\theta}(\tau)b \,d\tau = b\nabla_{\theta} \int p_{\theta}(\tau)d\tau = b\nabla_{\theta} 1 = 0$$

subtracting a baseline is *unbiased* in expectation!

average reward is *not* the best baseline, but it's pretty good!

### Analyzing variance

can we write down the variance?

$$Var[x] = E[x^2] - E[x]^2$$

$$\nabla_{\theta} J(\theta) = E_{\tau \sim p_{\theta}(\tau)} [\nabla_{\theta} \log p_{\theta}(\tau) (r(\tau) - b)]$$

$$Var = E_{\tau \sim p_{\theta}(\tau)} [(\nabla_{\theta} \log p_{\theta}(\tau)(r(\tau) - b))^{2}] - E_{\tau \sim p_{\theta}(\tau)} [\nabla_{\theta} \log p_{\theta}(\tau)(r(\tau) - b)]^{2}$$

this bit is just  $E_{\tau \sim p_{\theta}(\tau)}[\nabla_{\theta} \log p_{\theta}(\tau)r(\tau)]$  (baselines are unbiased in expectation)

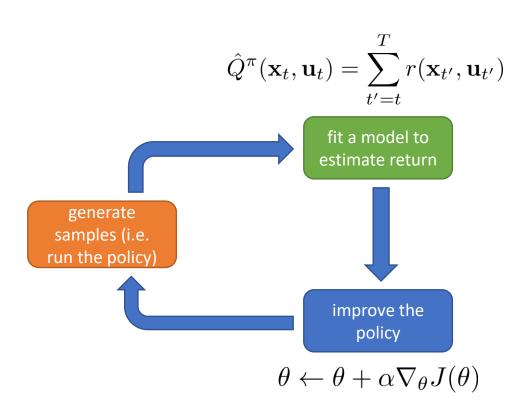
$$\frac{d\text{Var}}{db} = \frac{d}{db}E[g(\tau)^2(r(\tau) - b)^2] = \frac{d}{db}\left(E[g(\tau)^2r(\tau)^2] - 2E[g(\tau)^2r(\tau)b] + b^2E[g(\tau)^2]\right)$$
$$= -2E[g(\tau)^2r(\tau)] + 2bE[g(\tau)^2] = 0$$

$$b = \frac{E[g(\tau)^2 r(\tau)]}{E[g(\tau)^2]} \quad \longleftarrow$$

This is just expected reward, but weighted by gradient magnitudes!

### Review

- The high variance of policy gradient
- Exploiting causality
  - Future doesn't affect the past
- Baselines
  - Unbiased!
- Analyzing variance
  - Can derive optimal baselines



# Off-Policy Policy Gradients

# Policy gradient is on-policy

$$\theta^{\star} = \arg\max_{\theta} J(\theta)$$

$$J(\theta) = E_{\tau \sim p_{\theta}(\tau)}[r(\tau)]$$

$$\nabla_{\theta} J(\theta) = E_{\underline{\tau \sim p_{\theta}(\tau)}} [\nabla_{\theta} \log p_{\theta}(\tau) r(\tau)]$$
this is trouble...

- Neural networks change only a little bit with each gradient step
- On-policy learning can be extremely inefficient!

REINFORCE algorithm:



can't just skip this!

- 1. sample  $\{\tau^i\}$  from  $\pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t)$  (run it on the robot)
- 2.  $\nabla_{\theta} J(\theta) \approx \sum_{i} \left( \sum_{t} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{t}^{i}|\mathbf{s}_{t}^{i}) \right) \left( \sum_{t} r(\mathbf{s}_{t}^{i}, \mathbf{a}_{t}^{i}) \right)$
- 3.  $\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$

# Off-policy learning & importance sampling

$$\theta^* = \arg\max_{\theta} J(\theta)$$

$$J(\theta) = E_{\tau \sim p_{\theta}(\tau)}[r(\tau)]$$

what if we don't have samples from  $p_{\theta}(\tau)$ ? (we have samples from some  $\bar{p}(\tau)$  instead)

$$J(\theta) = E_{\tau \sim \bar{p}(\tau)} \left[ \frac{p_{\theta}(\tau)}{\bar{p}(\tau)} r(\tau) \right]$$

$$p_{\theta}(\tau) = p(\mathbf{s}_1) \prod_{t=1}^{T} \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)$$

$$\frac{p_{\theta}(\tau)}{\bar{p}(\tau)} = \frac{p(\mathbf{s}_1) \prod_{t=1}^{T} \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)}{p(\mathbf{s}_1) \prod_{t=1}^{T} \bar{\pi}(\mathbf{a}_t | \mathbf{s}_t) p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)} = \frac{\prod_{t=1}^{T} \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)}{\prod_{t=1}^{T} \bar{\pi}(\mathbf{a}_t | \mathbf{s}_t)}$$

#### importance sampling

$$E_{x \sim p(x)}[f(x)] = \int p(x)f(x)dx$$

$$= \int \frac{q(x)}{q(x)}p(x)f(x)dx$$

$$= \int q(x)\frac{p(x)}{q(x)}f(x)dx$$

$$= E_{x \sim q(x)}\left[\frac{p(x)}{q(x)}f(x)\right]$$

## Deriving the policy gradient with IS

$$\theta^{\star} = \arg\max_{\theta} J(\theta)$$

$$J(\theta) = E_{\tau \sim p_{\theta}(\tau)}[r(\tau)]$$

can we estimate the value of some new parameters  $\theta'$ ?

$$J( heta') = E_{ au \sim p_{ heta}( au)} \left[ rac{p_{ heta'}( au)}{p_{ heta}( au)} r( au) 
ight]$$
 the only bit that depends on  $heta'$ 

$$\nabla_{\theta'} J(\theta') = E_{\tau \sim p_{\theta}(\tau)} \left[ \frac{\nabla_{\theta'} p_{\theta'}(\tau)}{p_{\theta}(\tau)} r(\tau) \right] = E_{\tau \sim p_{\theta}(\tau)} \left[ \frac{p_{\theta'}(\tau)}{p_{\theta}(\tau)} \nabla_{\theta'} \log p_{\theta'}(\tau) r(\tau) \right]$$

now estimate locally, at  $\theta = \theta'$ :  $\nabla_{\theta} J(\theta) = E_{\tau \sim p_{\theta}(\tau)} [\nabla_{\theta} \log p_{\theta}(\tau) r(\tau)]$ 

a convenient identity

$$p_{\theta}(\tau)\nabla_{\theta}\log p_{\theta}(\tau) = \nabla_{\theta}p_{\theta}(\tau)$$

# The off-policy policy gradient

$$\theta^* = \arg\max_{\theta} J(\theta) \qquad J(\theta) = E_{\tau \sim p_{\theta}(\tau)}[r(\tau)]$$

$$\nabla_{\theta'} J(\theta') = E_{\tau \sim p_{\theta}(\tau)} \left[ \frac{p_{\theta'}(\tau)}{p_{\theta}(\tau)} \nabla_{\theta'} \log \pi_{\theta'}(\tau) r(\tau) \right] \quad \text{when } \theta \neq \theta'$$

$$\frac{p_{\theta'}(\tau)}{p_{\theta}(\tau)} = \frac{\prod_{t=1}^{T} \pi_{\theta'}(\mathbf{a}_t | \mathbf{s}_t)}{\prod_{t=1}^{T} \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)}$$

$$= E_{\tau \sim p_{\theta}(\tau)} \left[ \left( \prod_{t=1}^{T} \frac{\pi_{\theta'}(\mathbf{a}_{t}|\mathbf{s}_{t})}{\pi_{\theta}(\mathbf{a}_{t}|\mathbf{s}_{t})} \right) \left( \sum_{t=1}^{T} \nabla_{\theta'} \log \pi_{\theta'}(\mathbf{a}_{t}|\mathbf{s}_{t}) \right) \left( \sum_{t=1}^{T} r(\mathbf{s}_{t}, \mathbf{a}_{t}) \right) \right]$$
 what about causality?

$$= E_{\tau \sim p_{\theta}(\tau)} \left[ \sum_{t=1}^{T} \nabla_{\theta'} \log \pi_{\theta'}(\mathbf{a}_{t}|\mathbf{s}_{t}) \left( \prod_{\underline{t'=1}}^{t} \frac{\pi_{\theta'}(\mathbf{a}_{t'}|\mathbf{s}_{t'})}{\pi_{\theta}(\mathbf{a}_{t'}|\mathbf{s}_{t'})} \right) \left( \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) \left( \prod_{\underline{t''=t}}^{t'} \frac{\pi_{\theta'}(\mathbf{a}_{t''}|\mathbf{s}_{t''})}{\pi_{\theta}(\mathbf{a}_{t''}|\mathbf{s}_{t''})} \right) \right) \right]$$

future actions don't affect current weight

if we ignore this, we get a policy iteration algorithm (more on this in a later lecture)

# A first-order approximation for IS (preview)

$$\nabla_{\theta'} J(\theta') = E_{\tau \sim p_{\theta}(\tau)} \left[ \sum_{t=1}^{T} \nabla_{\theta'} \log \pi_{\theta'}(\mathbf{a}_{t}|\mathbf{s}_{t}) \left( \underbrace{\prod_{t'=1}^{t} \frac{\pi_{\theta'}(\mathbf{a}_{t'}|\mathbf{s}_{t'})}{\pi_{\theta}(\mathbf{a}_{t'}|\mathbf{s}_{t'})}} \right) \left( \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) \right) \right]$$

let's write the objective a bit differently...

on-policy prolicy gradient: 
$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \hat{Q}_{i,t}$$

$$(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \sim \pi_{\theta}(\mathbf{s}_{t}, \mathbf{a}_{t})$$

off-policy policy gradient:  $\nabla_{\theta'} J(\theta') \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \frac{\pi_{\theta'}(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})}{\pi_{\theta}(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})} \nabla_{\theta'} \log \pi_{\theta'}(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \hat{Q}_{i,t}$ 

We'll see why this is reasonable later in the course!

$$= \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \frac{\pi_{\theta'}(\mathbf{s}_{i,t})}{\pi_{\theta}(\mathbf{s}_{i,t})} \frac{\pi_{\theta'}(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})}{\pi_{\theta}(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})} \nabla_{\theta'} \log \pi_{\theta'}(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \hat{Q}_{i,t}$$
ignore this part

exponential in T...

# Implementing Policy Gradients

## Policy gradient with automatic differentiation

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \hat{Q}_{i,t}$$
 pretty inefficient to compute these explicitly!

How can we compute policy gradients with automatic differentiation?

We need a graph such that its gradient is the policy gradient!

maximum likelihood: 
$$\nabla_{\theta} J_{\text{ML}}(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \qquad J_{\text{ML}}(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \log \pi_{\theta}(\mathbf{a}_{i,t}|\mathbf{s}_{i,t})$$

Just implement "pseudo-loss" as a weighted maximum likelihood:

$$\tilde{J}(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \log \pi_{\theta}(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}) \hat{Q}_{i,t}$$
 cross entropy (discrete) or squared error (Gaussian)

### Policy gradient with automatic differentiation

Pseudocode example (with discrete actions):

#### Maximum likelihood:

```
# Given:
# actions - (N*T) x Da tensor of actions
# states - (N*T) x Ds tensor of states
# Build the graph:
logits = policy.predictions(states) # This should return (N*T) x Da tensor of action logits
negative_likelihoods = tf.nn.softmax_cross_entropy_with_logits(labels=actions, logits=logits)
loss = tf.reduce_mean(negative_likelihoods)
gradients = loss.gradients(loss, variables)
```

### Policy gradient with automatic differentiation

Pseudocode example (with discrete actions):

#### Policy gradient:

```
# Given:
# actions - (N*T) x Da tensor of actions
# states - (N*T) x Ds tensor of states
# q_values - (N*T) x 1 tensor of estimated state-action values
# Build the graph:
logits = policy.predictions(states) # This should return (N*T) x Da tensor of action logits
negative_likelihoods = tf.nn.softmax_cross_entropy_with_logits(labels=actions, logits=logits)
weighted_negative_likelihoods = tf.multiply(negative_likelihoods, q_values)
loss = tf.reduce_mean(weighted_negative_likelihoods)
gradients = loss.gradients(loss, variables)
```

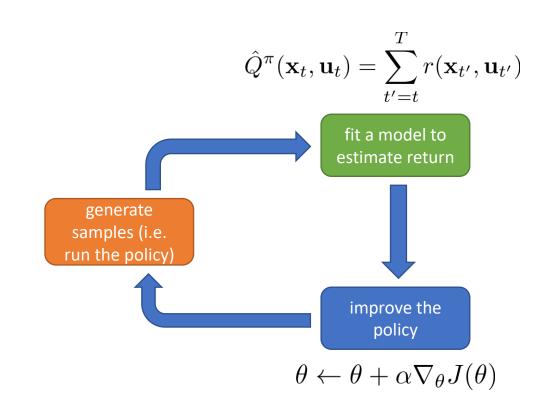
$$ilde{J}( heta) pprox rac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \log \pi_{ heta}(\mathbf{a}_{i,t}|\mathbf{s}_{i,t}|\hat{Q}_{i,t})$$
q\_values

### Policy gradient in practice

- Remember that the gradient has high variance
  - This isn't the same as supervised learning!
  - Gradients will be really noisy!
- Consider using much larger batches
- Tweaking learning rates is very hard
  - Adaptive step size rules like ADAM can be OK-ish
  - We'll learn about policy gradient-specific learning rate adjustment methods later!

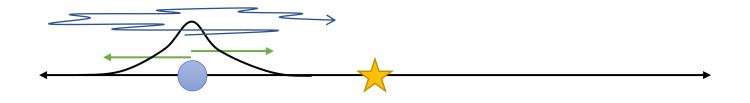
### Review

- Policy gradient is on-policy
- Can derive off-policy variant
  - Use importance sampling
  - Exponential scaling in T
  - Can ignore state portion (approximation)
- Can implement with automatic differentiation – need to know what to backpropagate
- Practical considerations: batch size, learning rates, optimizers



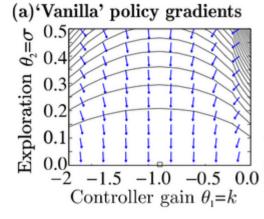
# Advanced Policy Gradients

# What else is wrong with the policy gradient?



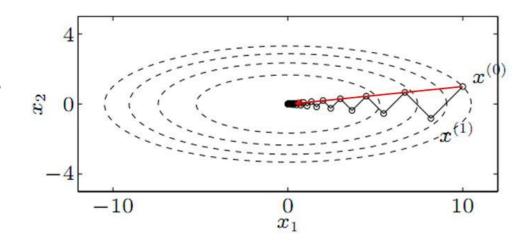
$$r(\mathbf{s}_t, \mathbf{a}_t) = -\mathbf{s}_t^2 - \mathbf{a}_t^2$$

$$\log \pi_{\theta}(\mathbf{a}_t|\mathbf{s}_t) = -\frac{1}{2\sigma^2}(k\mathbf{s}_t - \mathbf{a}_t)^2 + \text{const} \qquad \theta = (k, \sigma)$$



(image from Peters & Schaal 2008)

Essentially the same problem as this:



# Covariant/natural policy gradient

$$\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$$
  $\pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)$ 

some parameters change probabilities a lot more than others!

$$\theta' \leftarrow \arg\max_{\theta'} (\theta' - \theta)^T \nabla_{\theta} J(\theta) \text{ s.t. } \underline{\|\theta' - \theta\|^2 \le \epsilon}$$

controls how far we go

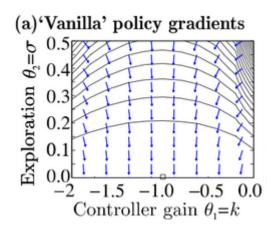
can we *rescale* the gradient so this doesn't happen?

$$\theta' \leftarrow \arg\max_{\theta'} (\theta' - \theta)^T \nabla_{\theta} J(\theta) \text{ s.t. } D(\pi_{\theta'}, \pi_{\theta}) \leq \epsilon$$

parameterization-independent divergence measure

usually KL-divergence: 
$$D_{\text{KL}}(\pi_{\theta'} || \pi_{\theta}) = E_{\pi_{\theta'}}[\log \pi_{\theta} - \log \pi_{\theta'}]$$

$$D_{\mathrm{KL}}(\pi_{\theta'} \| \pi_{\theta}) \approx (\theta' - \theta)^T \mathbf{F}(\theta' - \theta) \qquad \mathbf{F} = E_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta} (\mathbf{a} | \mathbf{s}) \nabla_{\theta} \log \pi_{\theta} (\mathbf{a} | \mathbf{s})^T]$$
Fisher-information matrix can estimate with samples



# Covariant/natural policy gradient

$$D_{\mathrm{KL}}(\pi_{\theta'} \| \theta_{\pi}) \approx (\theta' - \theta)^T \mathbf{F} (\theta' - \theta)$$

$$\mathbf{F} = E_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(\mathbf{a}|\mathbf{s}) \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}|\mathbf{s})^{T}]$$

$$\theta' \leftarrow \arg\max_{\theta'} (\theta' - \theta)^T \nabla_{\theta} J(\theta) \text{ s.t. } \mathcal{D}(\pi_{\theta'} \theta | \mathbb{P}) \leq \epsilon$$

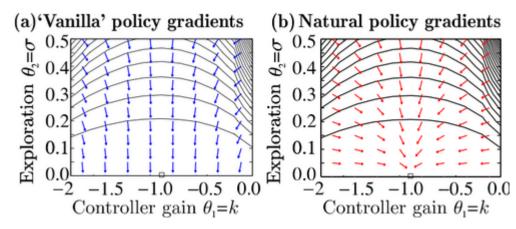
$$\theta \leftarrow \theta + \alpha \mathbf{F}^{-1} \nabla_{\theta} J(\theta)$$

natural gradient: pick  $\alpha$ 

trust region policy optimization: pick  $\epsilon$ 

can solve for optimal  $\alpha$  while solving  $\mathbf{F}^{-1}\nabla_{\theta}J(\theta)$ 

conjugate gradient works well for this



(figure from Peters & Schaal 2008)

see Schulman, L., Moritz, Jordan, Abbeel (2015) Trust region policy optimization

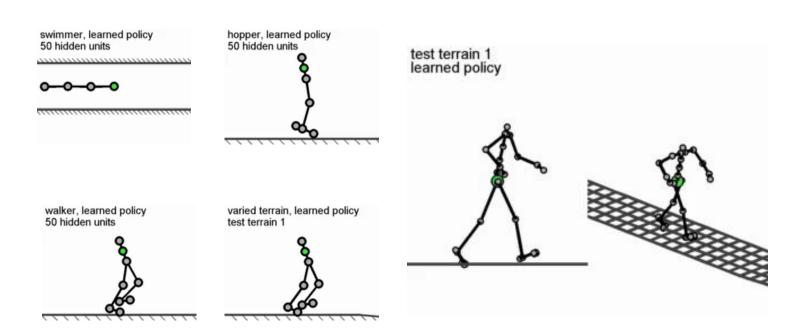
## Advanced policy gradient topics

- What more is there?
- Next time: introduce value functions and Q-functions
- Later in the class: more on natural gradient and automatic step size adjustment

# Example: policy gradient with importance sampling

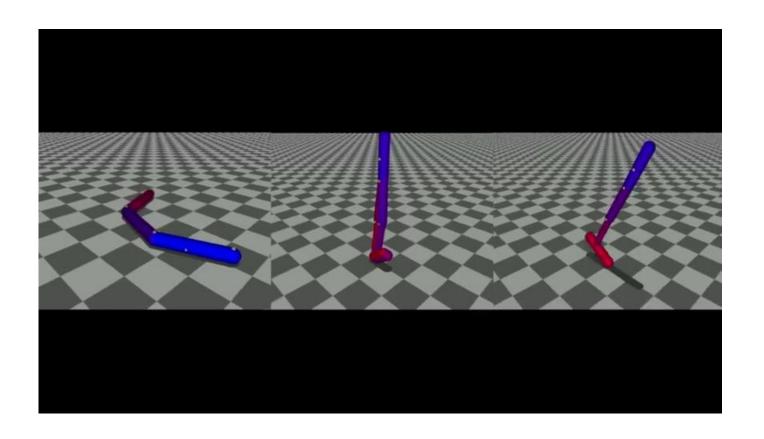
$$\nabla_{\theta'} J(\theta') = E_{\tau \sim \pi_{\theta}(\tau)} \left[ \sum_{t=1}^{T} \nabla_{\theta'} \log \pi_{\theta'}(\mathbf{a}_{t}|\mathbf{s}_{t}) \left( \prod_{t'=1}^{t} \frac{\pi_{\theta'}(\mathbf{a}_{t'}|\mathbf{s}_{t'})}{\pi_{\theta}(\mathbf{a}_{t'}|\mathbf{s}_{t'})} \right) \left( \sum_{t'=t}^{T} r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) \right) \right]$$

- Incorporate example demonstrations using importance sampling
- Neural network policies



### Example: trust region policy optimization

- Natural gradient with automatic step adjustment
- Discrete and continuous actions
- Code available (see Duan et al. '16)



## Policy gradients suggested readings

#### Classic papers

- Williams (1992). Simple statistical gradient-following algorithms for connectionist reinforcement learning: introduces REINFORCE algorithm
- Baxter & Bartlett (2001). Infinite-horizon policy-gradient estimation: temporally decomposed policy gradient (not the first paper on this! see actor-critic section later)
- Peters & Schaal (2008). Reinforcement learning of motor skills with policy gradients: very accessible overview of optimal baselines and natural gradient
- Deep reinforcement learning policy gradient papers
  - Levine & Koltun (2013). Guided policy search: deep RL with importance sampled policy gradient (unrelated to later discussion of guided policy search)
  - Schulman, L., Moritz, Jordan, Abbeel (2015). Trust region policy optimization: deep RL with natural policy gradient and adaptive step size
  - Schulman, Wolski, Dhariwal, Radford, Klimov (2017). Proximal policy optimization algorithms: deep RL with importance sampled policy gradient

### UC Berkeley CS285 | Deep Reinforcement Learning (2020)

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