

# MIT · 6.036 | Introduction to Machine Learning (2020)

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# 6.036/6.862: Introduction to Machine Learning

**Lecture:** starts Tuesdays 9:35am (Boston time zone)

**Course website:** [introml.odl.mit.edu](http://introml.odl.mit.edu)

**Who's talking?** Prof. Tamara Broderick

**Questions?** [discourse.odl.mit.edu](http://discourse.odl.mit.edu) ("Lecture 2" category)

**Materials:** Will all be available at course website

## Last Time

- I. Machine learning setup
- II. Linear classifiers
- III. Learning algorithms

## Today's Plan

- I. Perceptron algorithm
- II. Harder and easier linear classification
- III. Perceptron theorem

# Recall: Classifiers

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- A linear classifier:

$$h(x; \theta, \theta_0)$$

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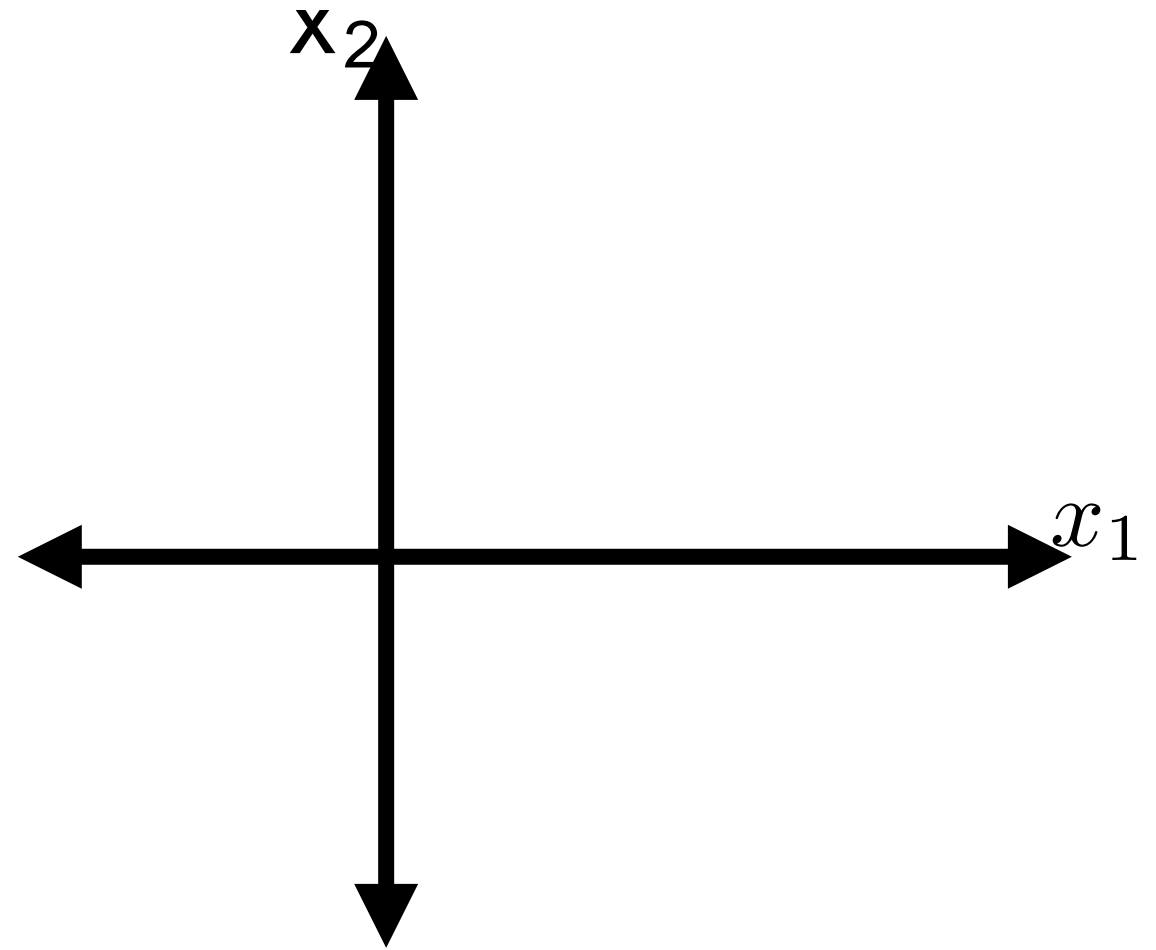
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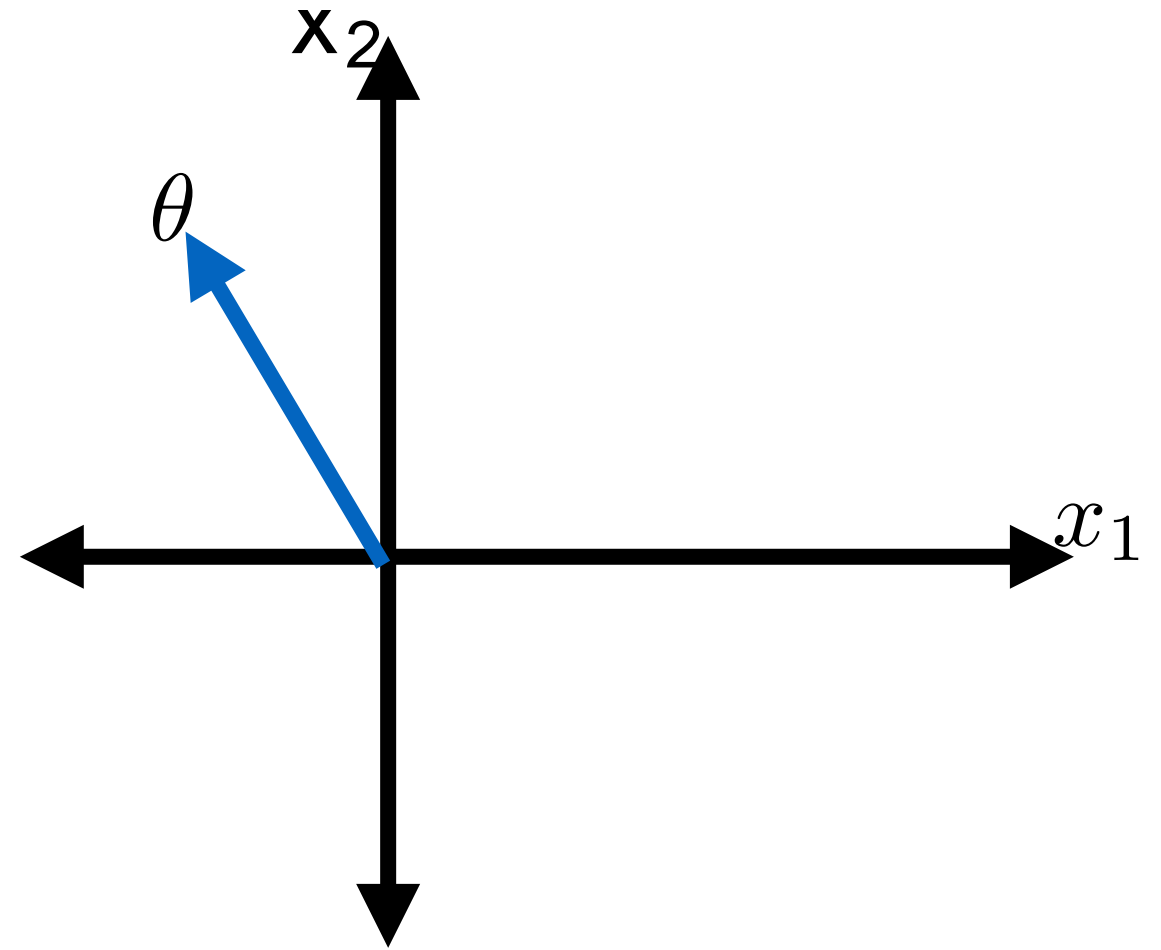
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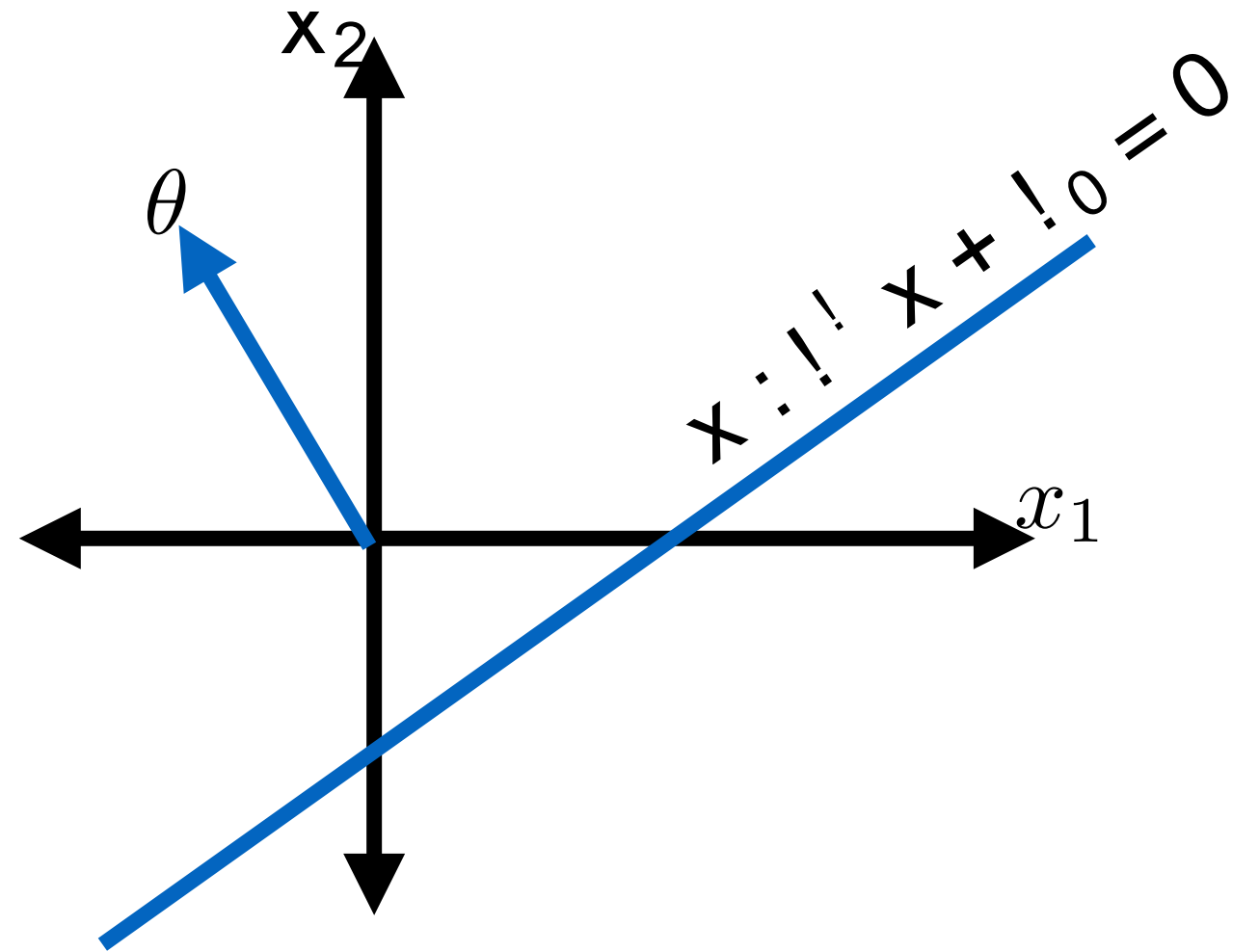




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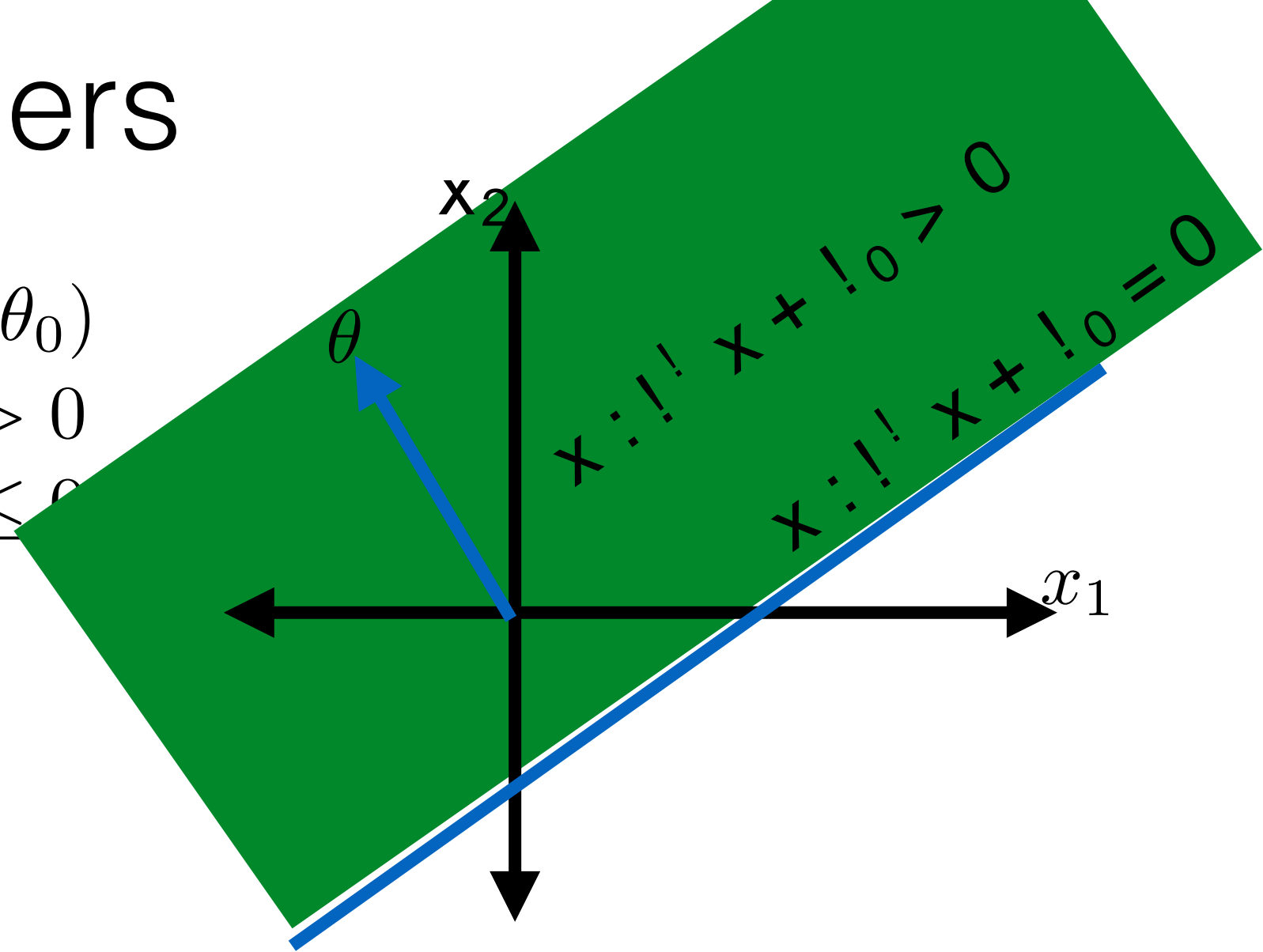


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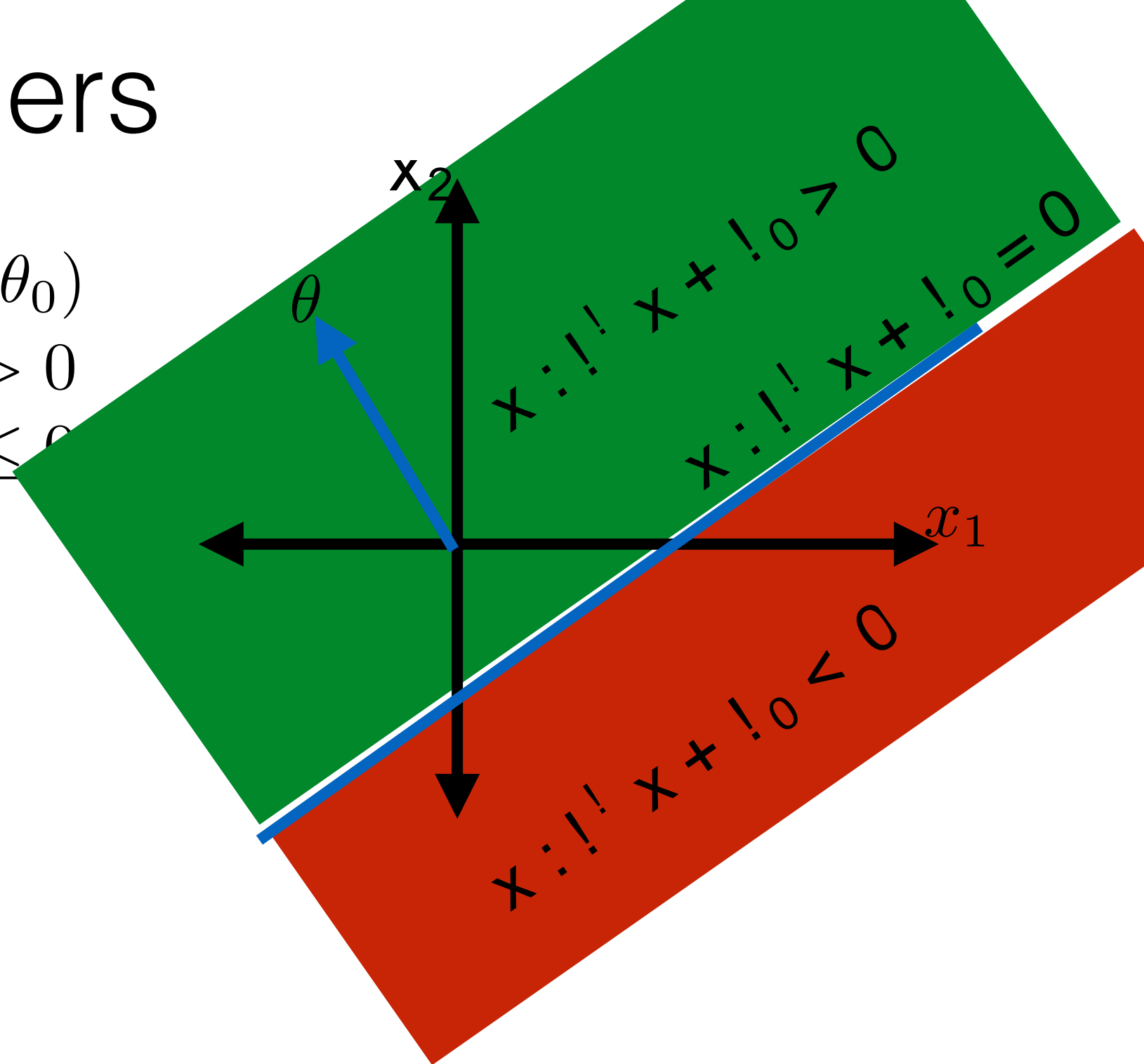


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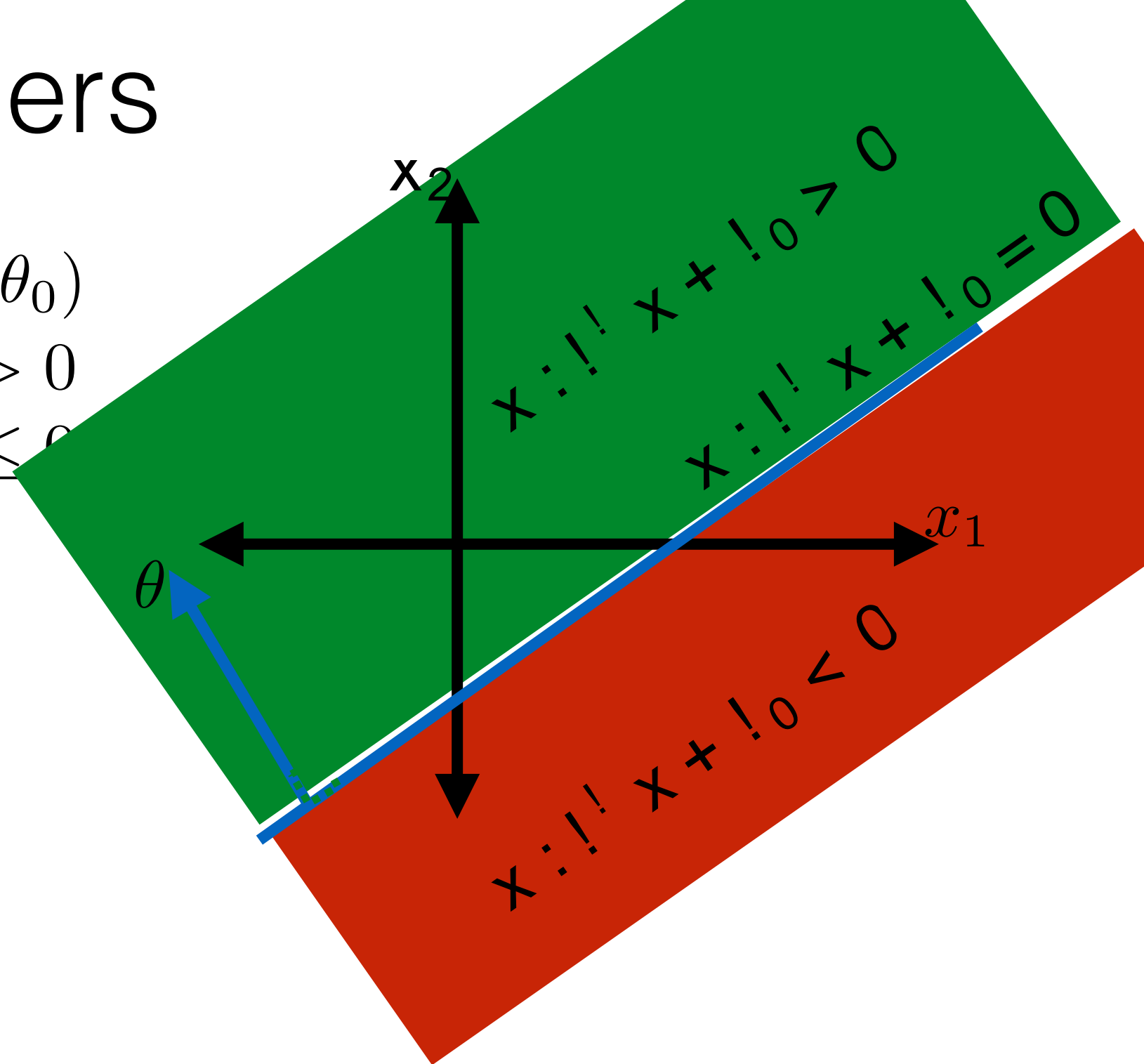


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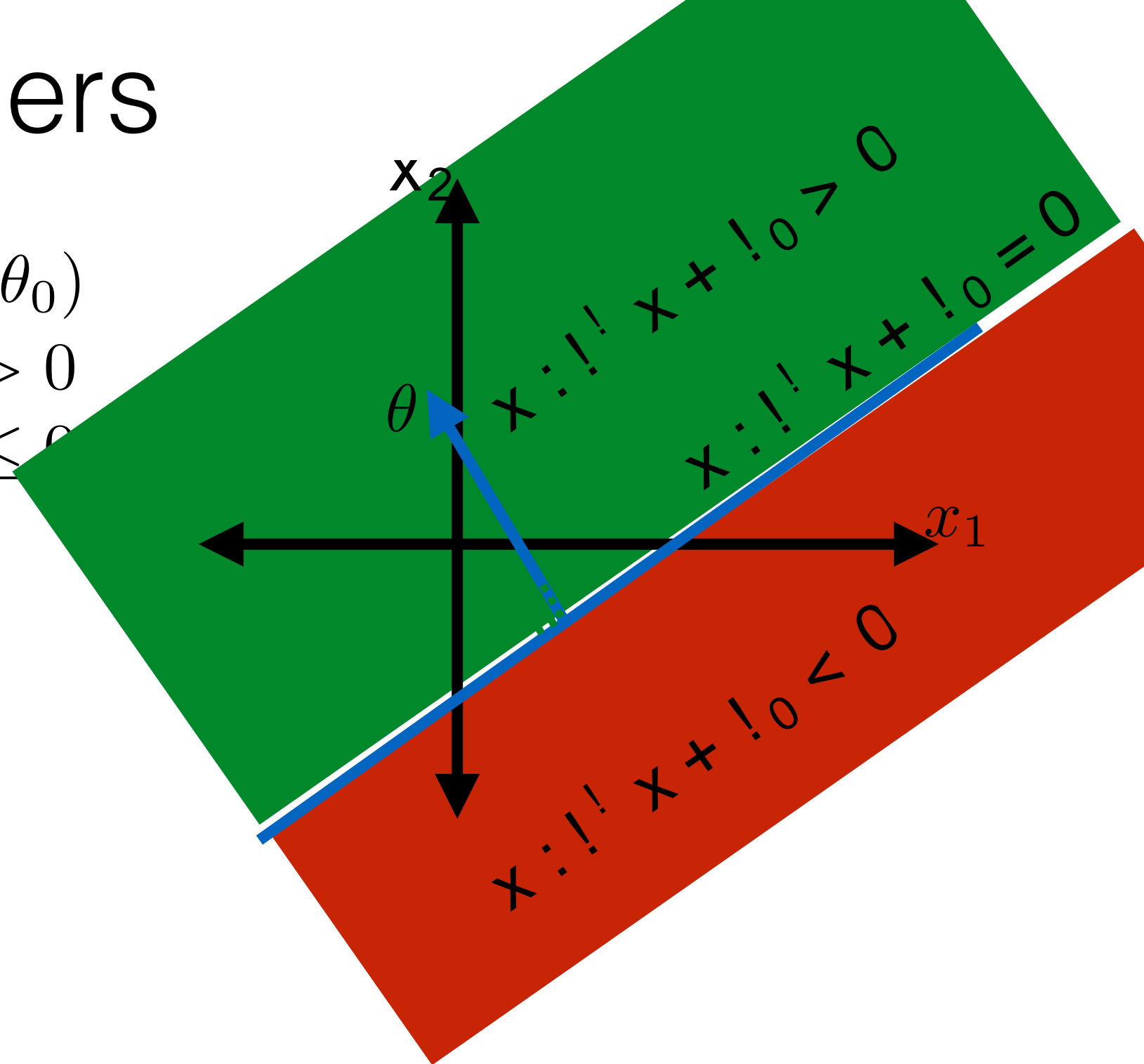


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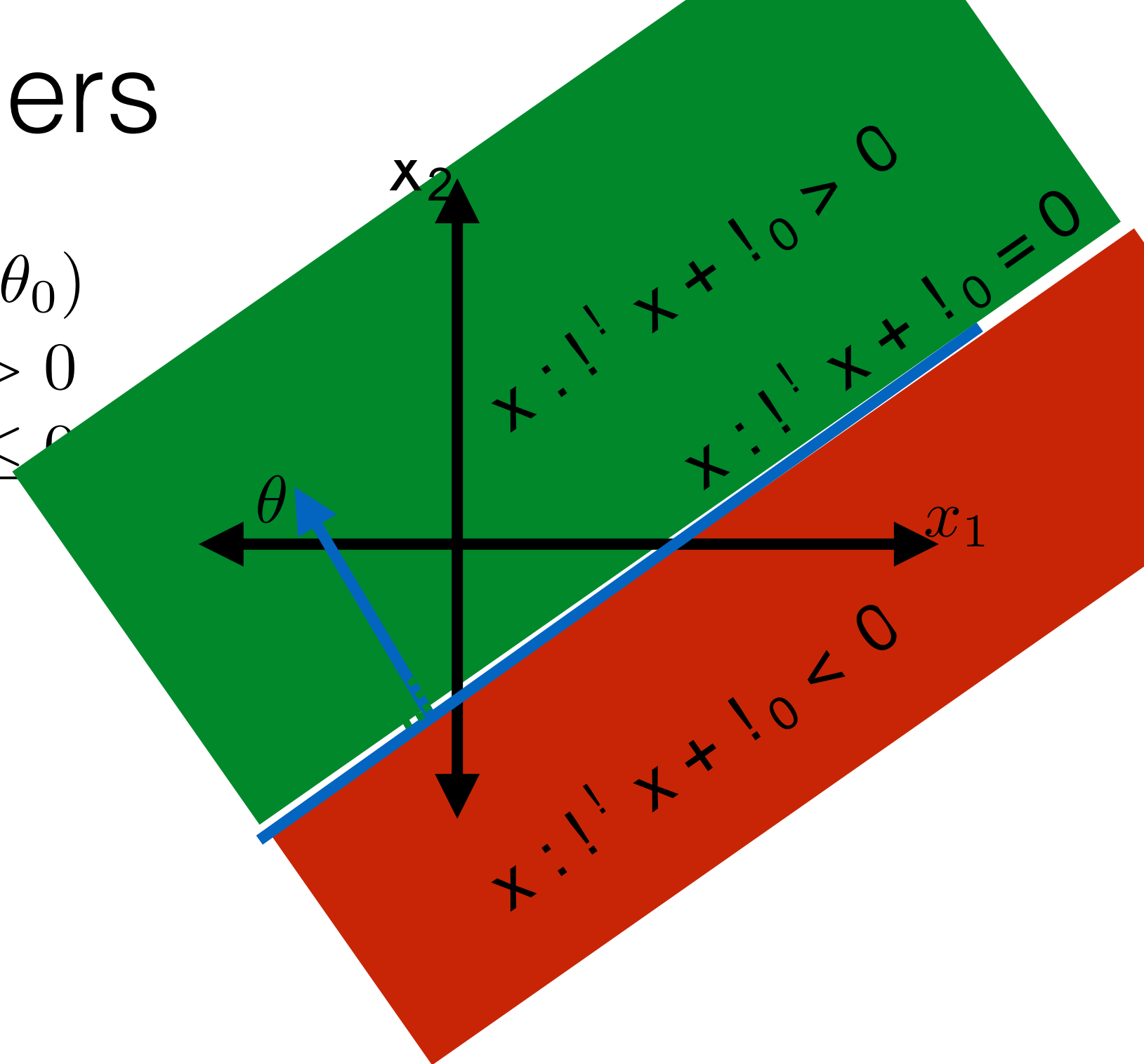


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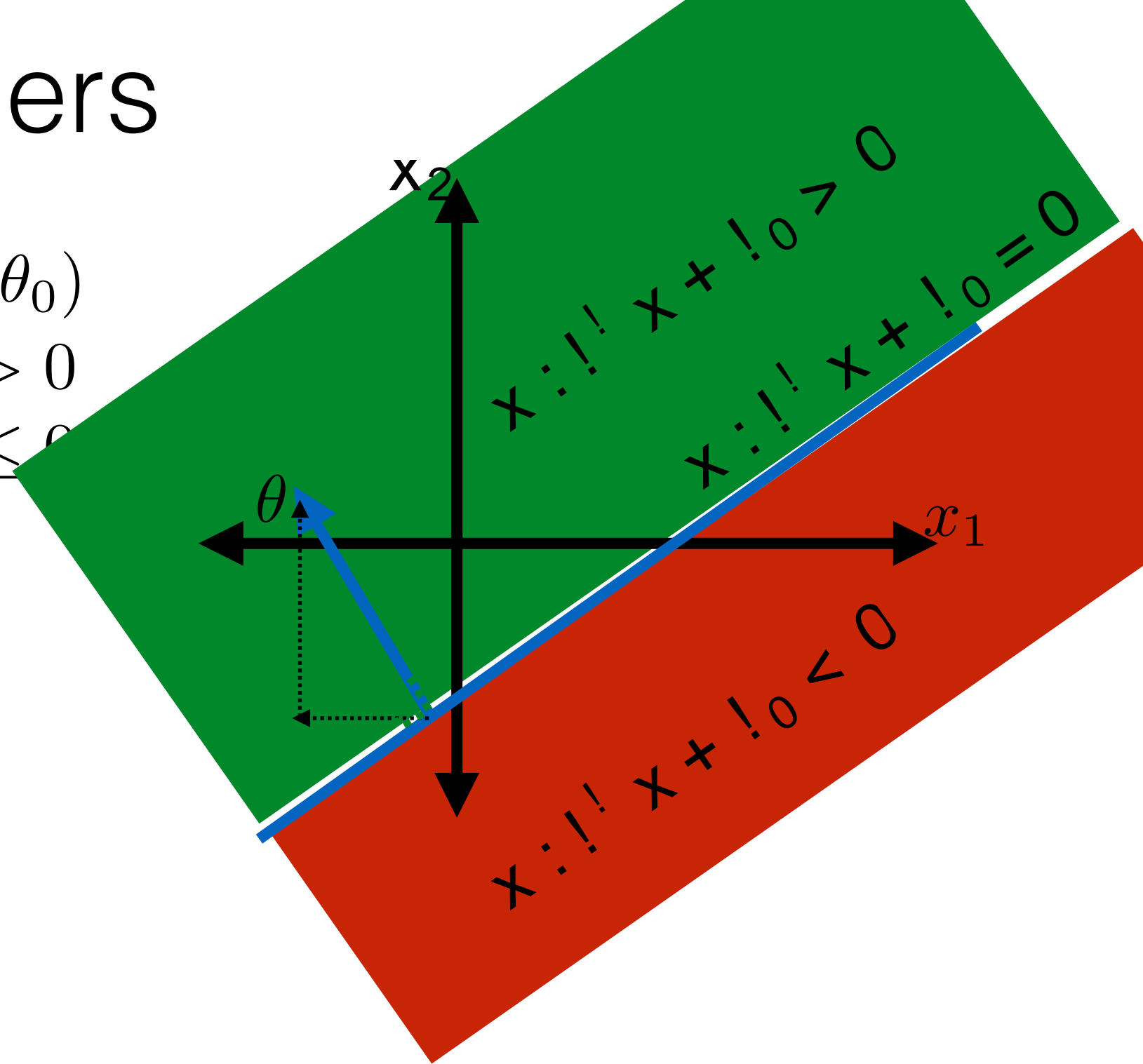


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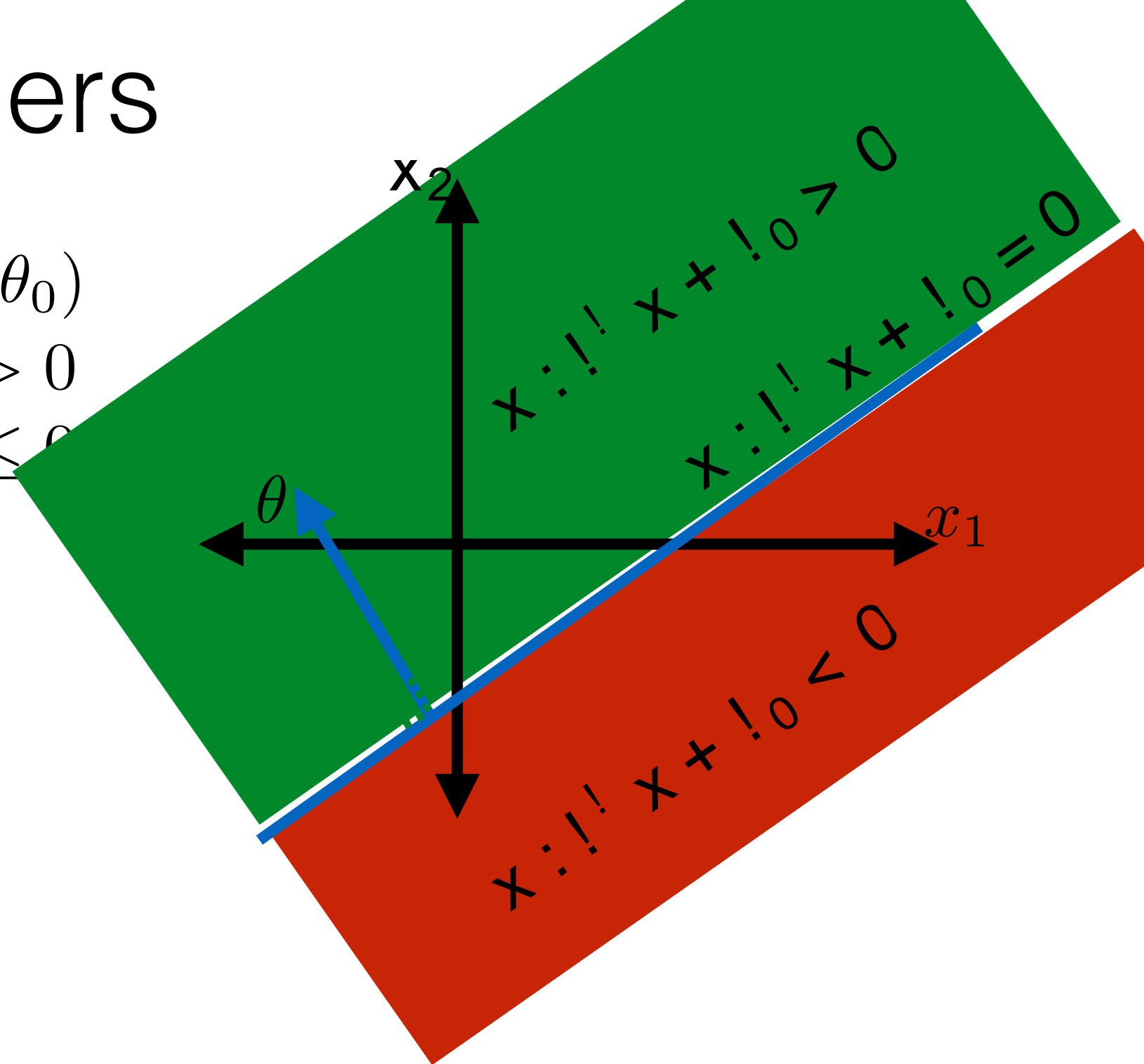


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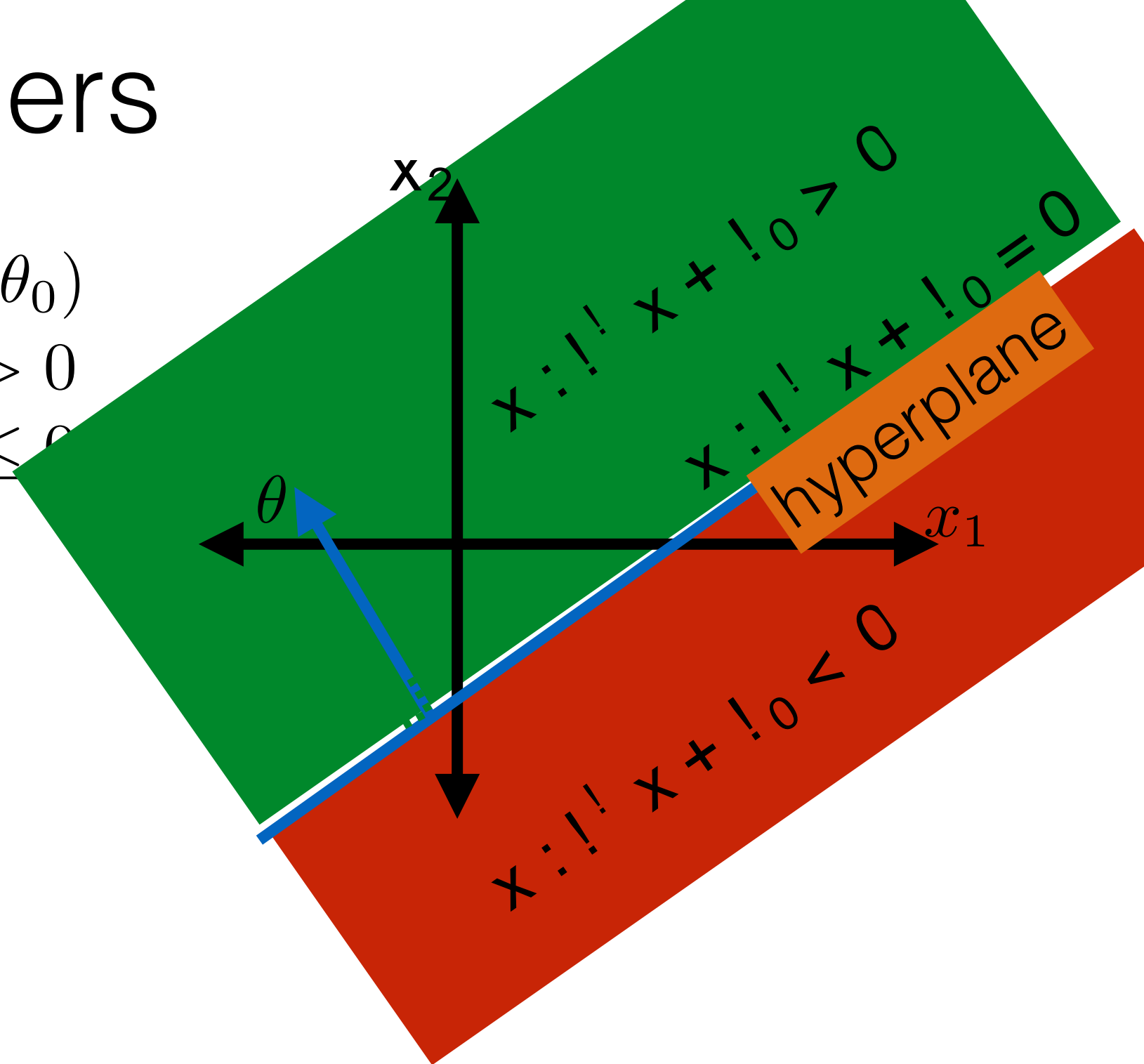


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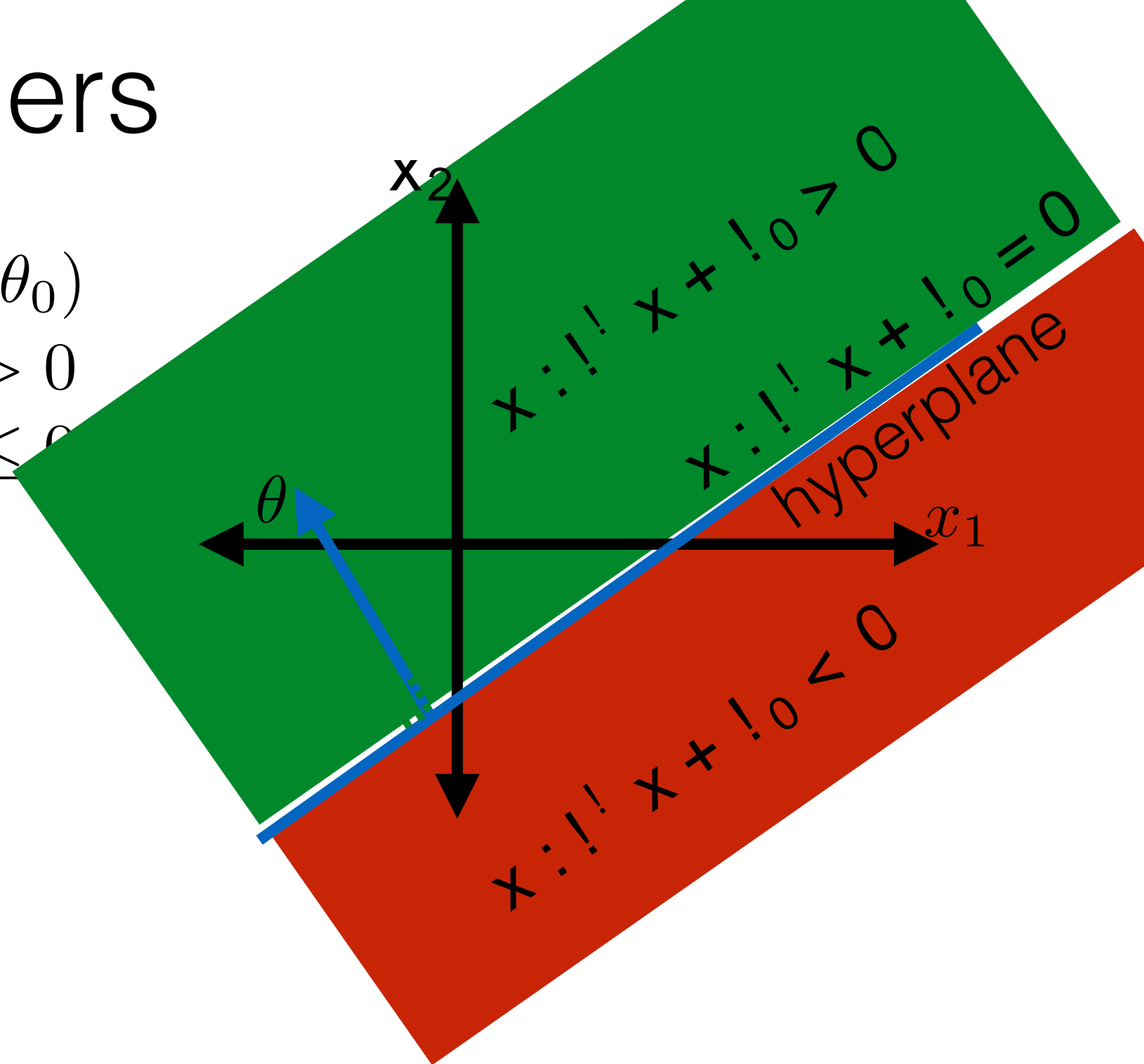


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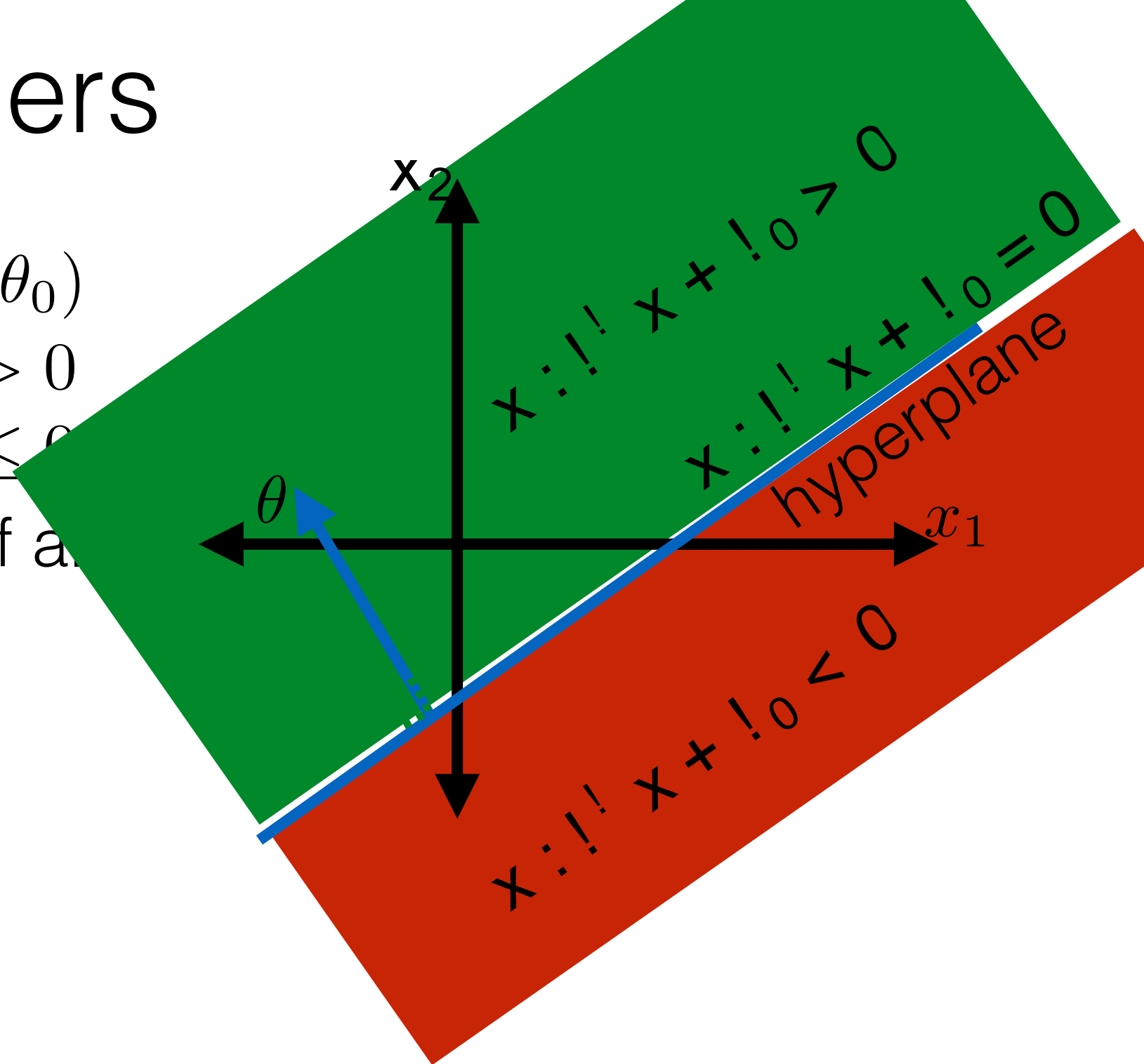
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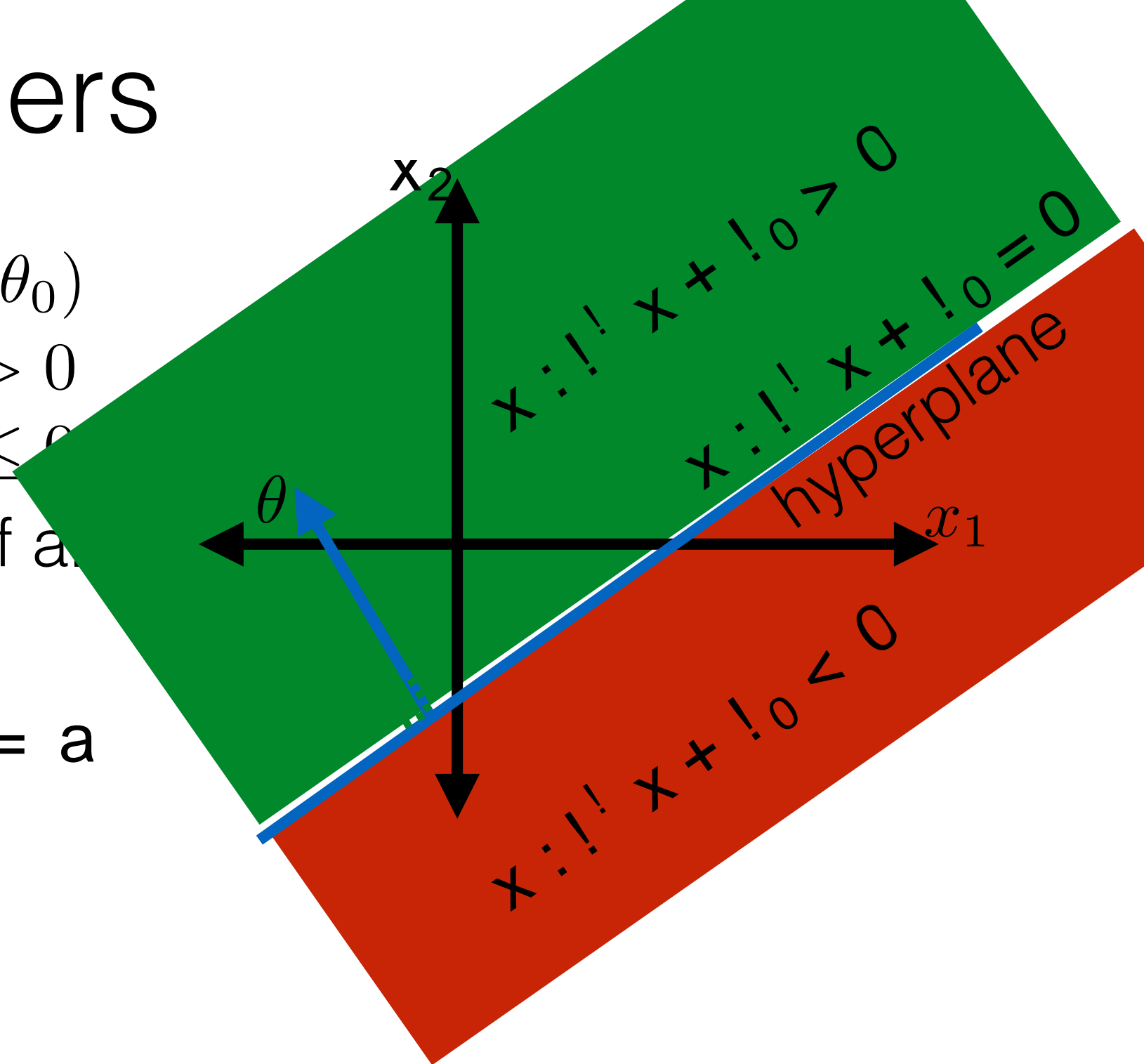
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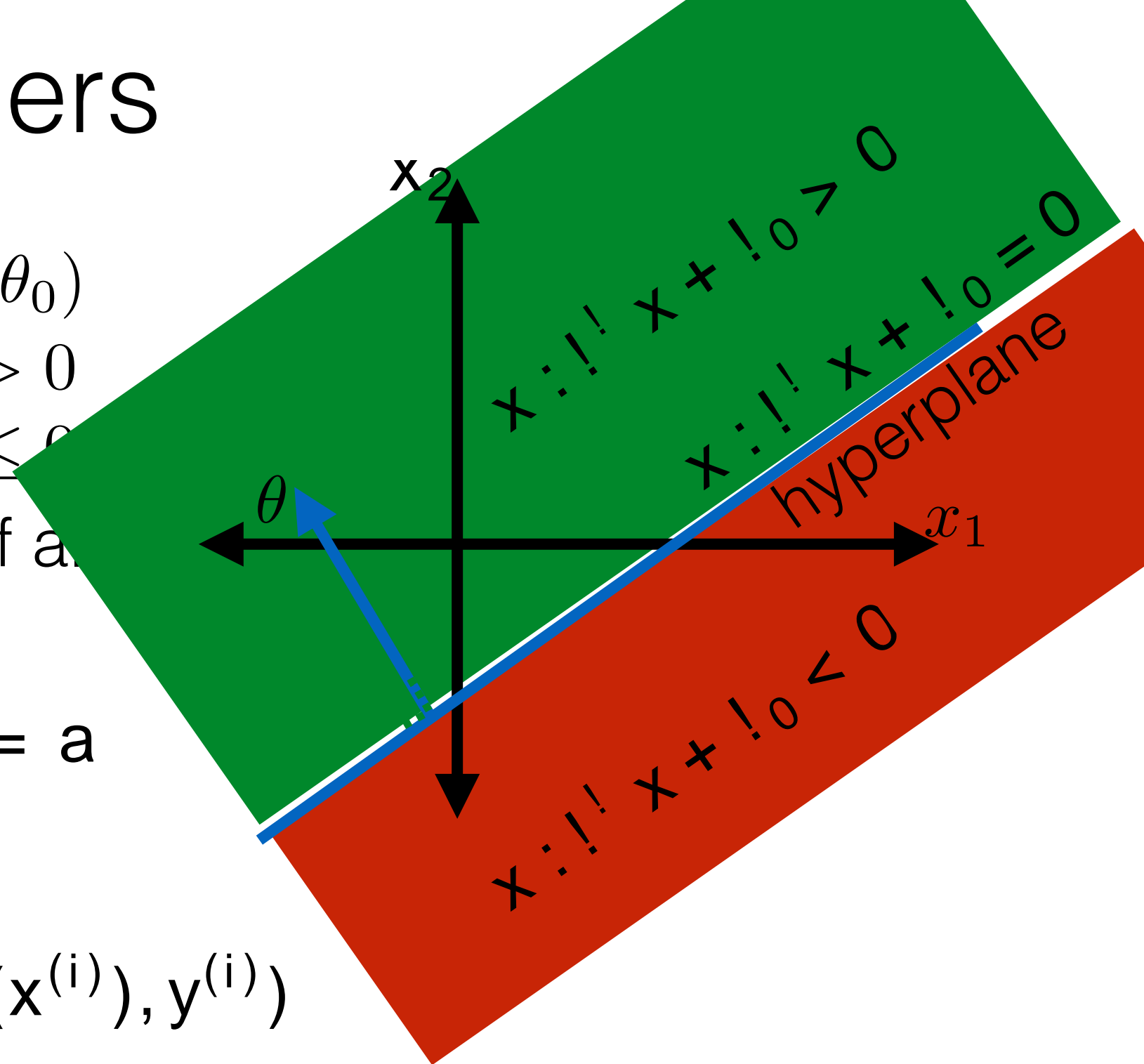
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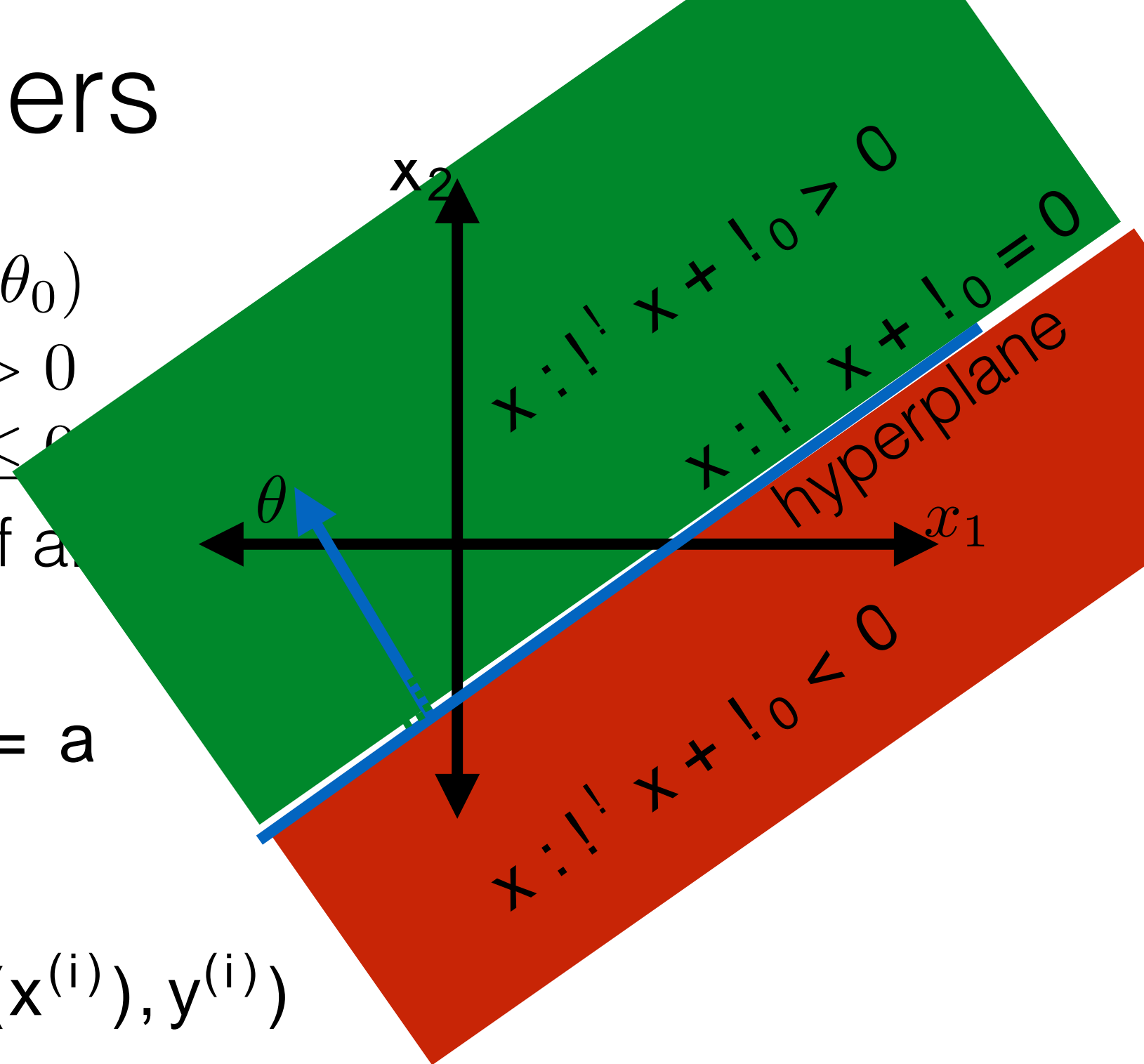
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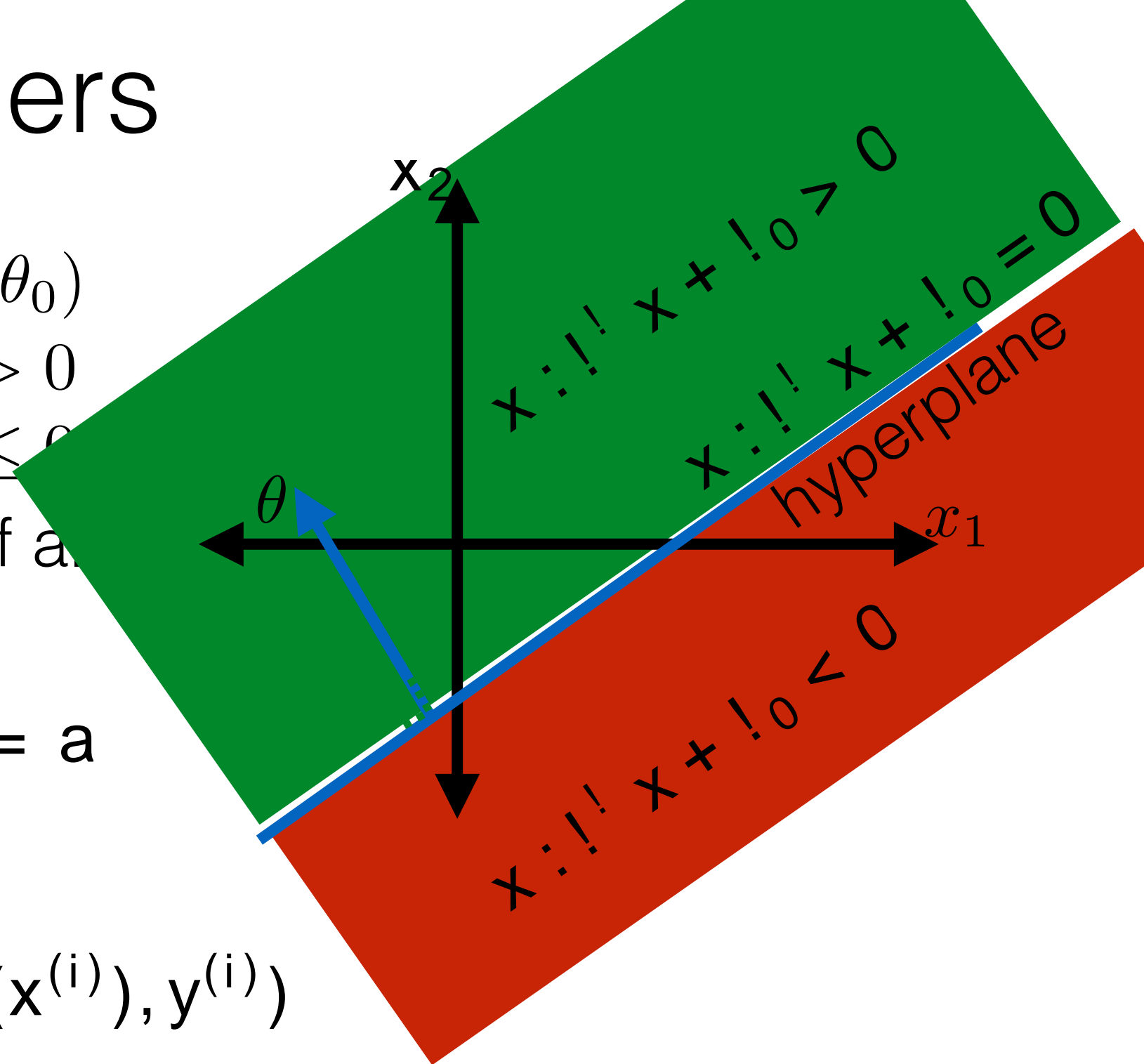
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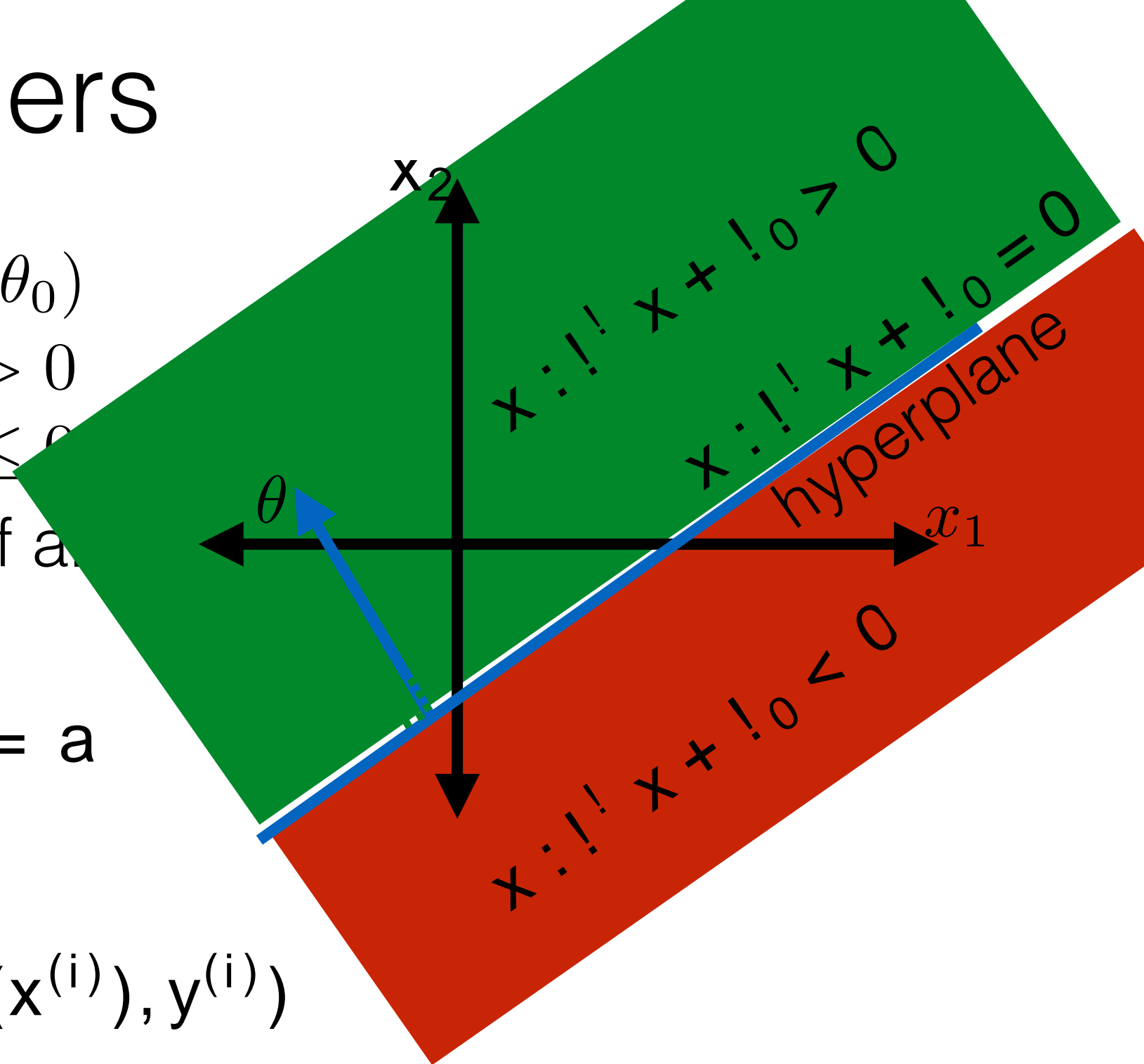
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[demo]





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Set  $w_0 = w_0 + y^{(i)}$

changed = True

**if** not changed

**break**

# Perceptron Algorithm

Perceptron (  $\mathcal{D}_n$  ;  $\epsilon$  )

Initialize  $\mathbf{w} = [0 \ 0 \ \dots \ 0]$

[How many 0s?]

Initialize  $w_0 = 0$

**for**  $t = 1$  to  $\epsilon$

[i.e. True if either:

changed = False

A. point is not on the line  
& prediction is wrong

**for**  $i = 1$  to  $n$

B. point is on the line

**if**  $y^{(i)} (\mathbf{w} \cdot \mathbf{x}^{(i)} + w_0) \leq 0$

C. initial step]

Set  $\mathbf{w} = \mathbf{w} + y^{(i)} \mathbf{x}^{(i)}$

Set  $w_0 = w_0 + y^{(i)}$

changed = True

**if** not changed

**break**

**Return**  $\mathbf{w}, w_0$



# Perceptron Algorithm

Perceptron (  $\mathcal{D}_n$  ; ! )

Initialize ! = [0 0 ... 0]

[How many 0s?]

Initialize !<sub>0</sub> = 0

**for** t = 1 to !

[i.e. True if either:

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**for** i = 1 to n

B. point is on the line

**if**  $y^{(i)} (! \cdot x^{(i)} + !_0) \leq 0$

C. initial step]

Set ! = ! +  $y^{(i)} x^{(i)}$

Set !<sub>0</sub> = !<sub>0</sub> +  $y^{(i)}$

changed = True

**if** not changed

**break**

**Return** !, !<sub>0</sub>

# Perceptron Algorithm

Perceptron (  $\mathcal{D}_n$  ; ! )

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Set ! = ! +  $y^{(i)} x^{(i)}$

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What does an update do?

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What does an update do?

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Set ! = ! +  $y^{(i)} x^{(i)}$

Set !<sub>0</sub> = !<sub>0</sub> +  $y^{(i)}$

changed = True

**if** not changed

**break**

**Return** !, !<sub>0</sub>

[i.e. True if either:

A. point is not on the line  
& prediction is wrong

B. point is on the line

C. initial step]

What does an update do?

$y^{(i)}$

! updated

$x^{(i)}$

+ (!<sub>0,updated</sub>)

# Perceptron Algorithm

Perceptron (  $\mathcal{D}_n$  ; ! )

Initialize ! = [0 0 ... 0]! [How many 0s?]

Initialize !<sub>0</sub> = 0

**for** t = 1 to !

changed = False

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**if**  $y^{(i)} (! \cdot x^{(i)} + !_0) \leq 0$

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A. point is not on the line  
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B. point is on the line

C. initial step]

What does an update do?

$$y^{(i)} \cdot (! + y^{(i)} x^{(i)}) \cdot x^{(i)} + (!_0 + y^{(i)})$$

# Perceptron Algorithm

Perceptron (  $\mathcal{D}_n$  ; ! )

Initialize ! = [0 0 ... 0]!

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**for** i = 1 to n

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**if**  $y^{(i)} (! ! x^{(i)} + !_0) ! 0$

C. initial step]

Set ! = ! +  $y^{(i)} x^{(i)}$

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**if** not changed

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**Return** !, !<sub>0</sub>

What does an update do?

$$y^{(i)} \cdot (! + y^{(i)} x^{(i)}) ! x^{(i)} + (!_0 + y^{(i)})$$

$$= y^{(i)} (! ! x^{(i)} + !_0) + (y^{(i)})^2 (x^{(i)} ! x^{(i)} + 1)$$

# Perceptron Algorithm

Perceptron (  $\mathcal{D}_n$  ; ! )

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Perceptron (  $\mathcal{D}_n$  ; ! )

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[demo]

[i.e. True if either:

A. point is not on the line  
& prediction is wrong

B. point is on the line

C. initial step]

What does an update do?

$$y^{(i)} \cdot (! + y^{(i)} x^{(i)}) \cdot x^{(i)} + (!_0 + y^{(i)})$$

$$= y^{(i)} (! \cdot x^{(i)} + !_0) + (y^{(i)})^2 (x^{(i)} \cdot x^{(i)} + 1)$$

$$= y^{(i)} (! \cdot x^{(i)} + !_0) + (! \cdot x^{(i)})^2 + 1$$

# Let's Talk About Classifier Quality

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- *Definition:* A training set  $\mathcal{D}_n$  is **linearly separable** if there exist  $\gamma, \gamma_0$  such that, for every point index  $i \in \{1, \dots, n\}$ , we have

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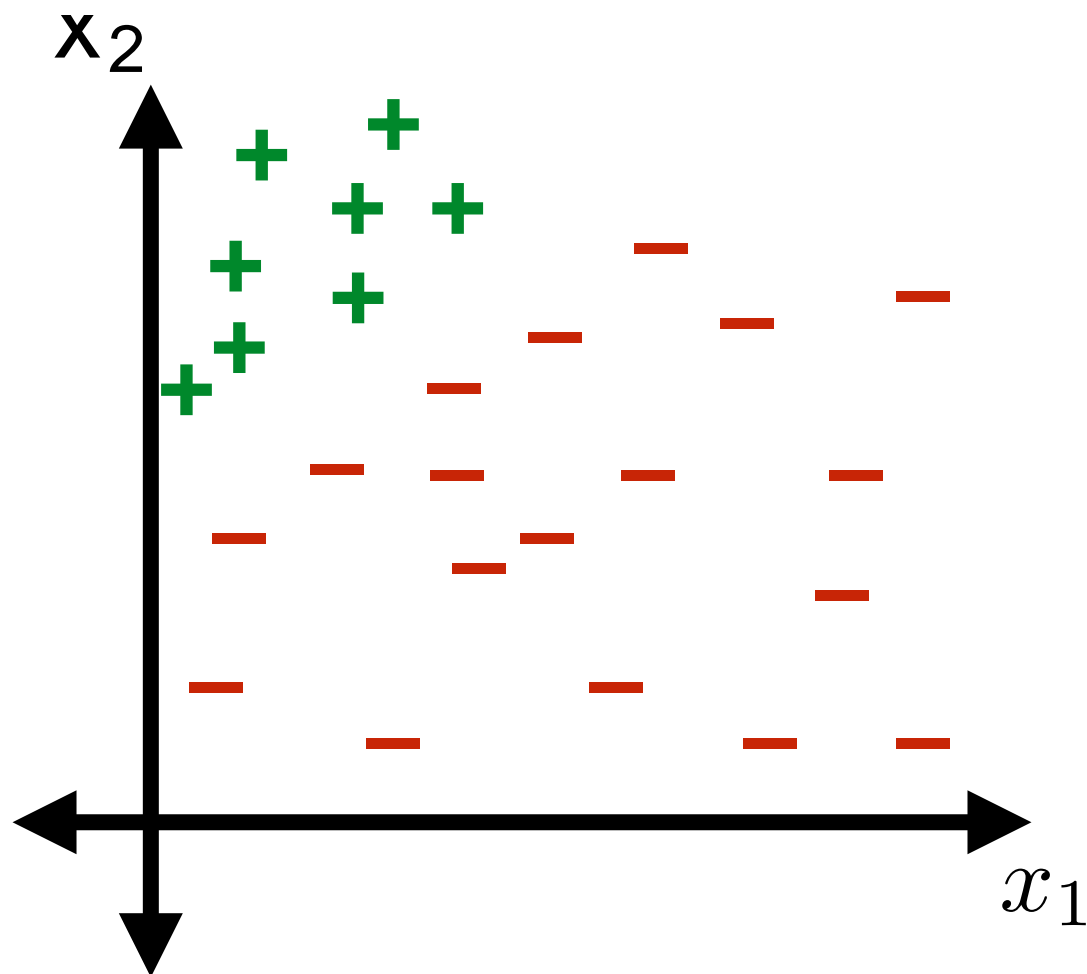
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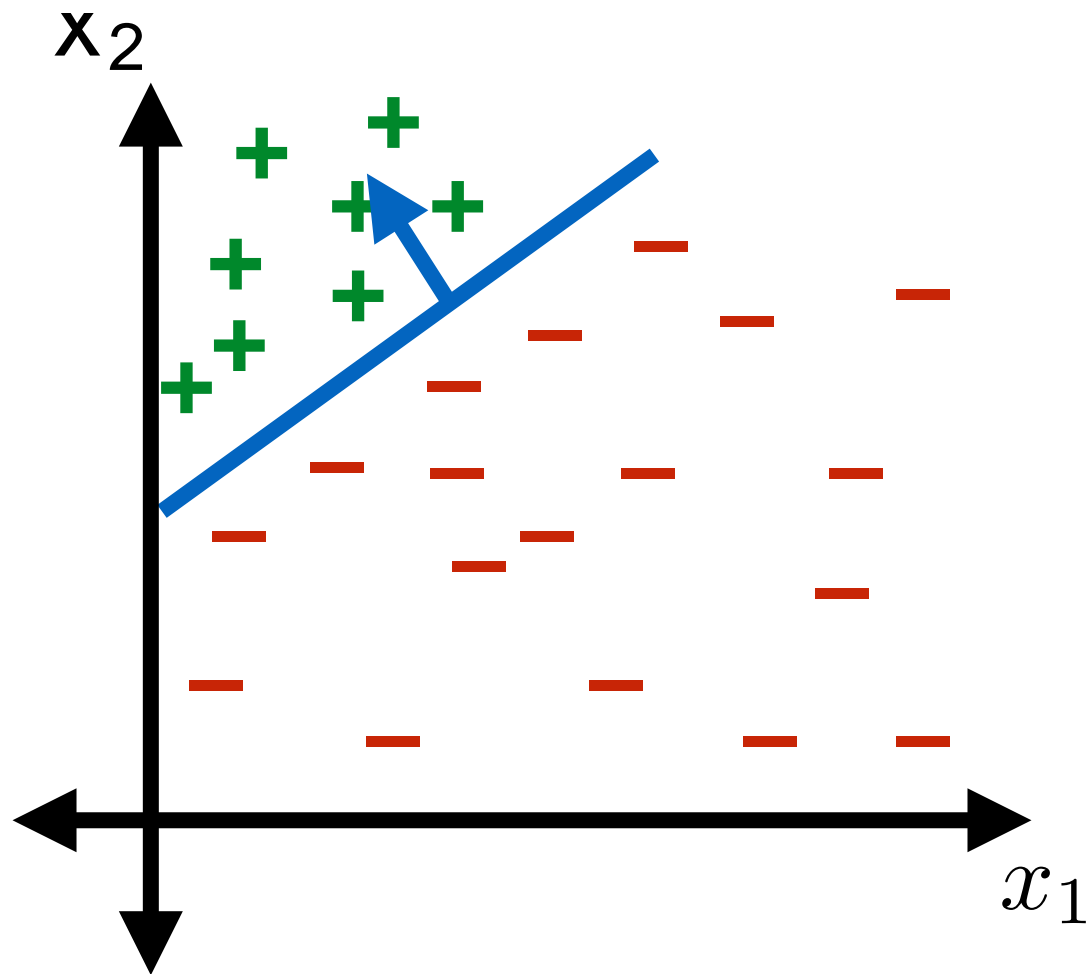
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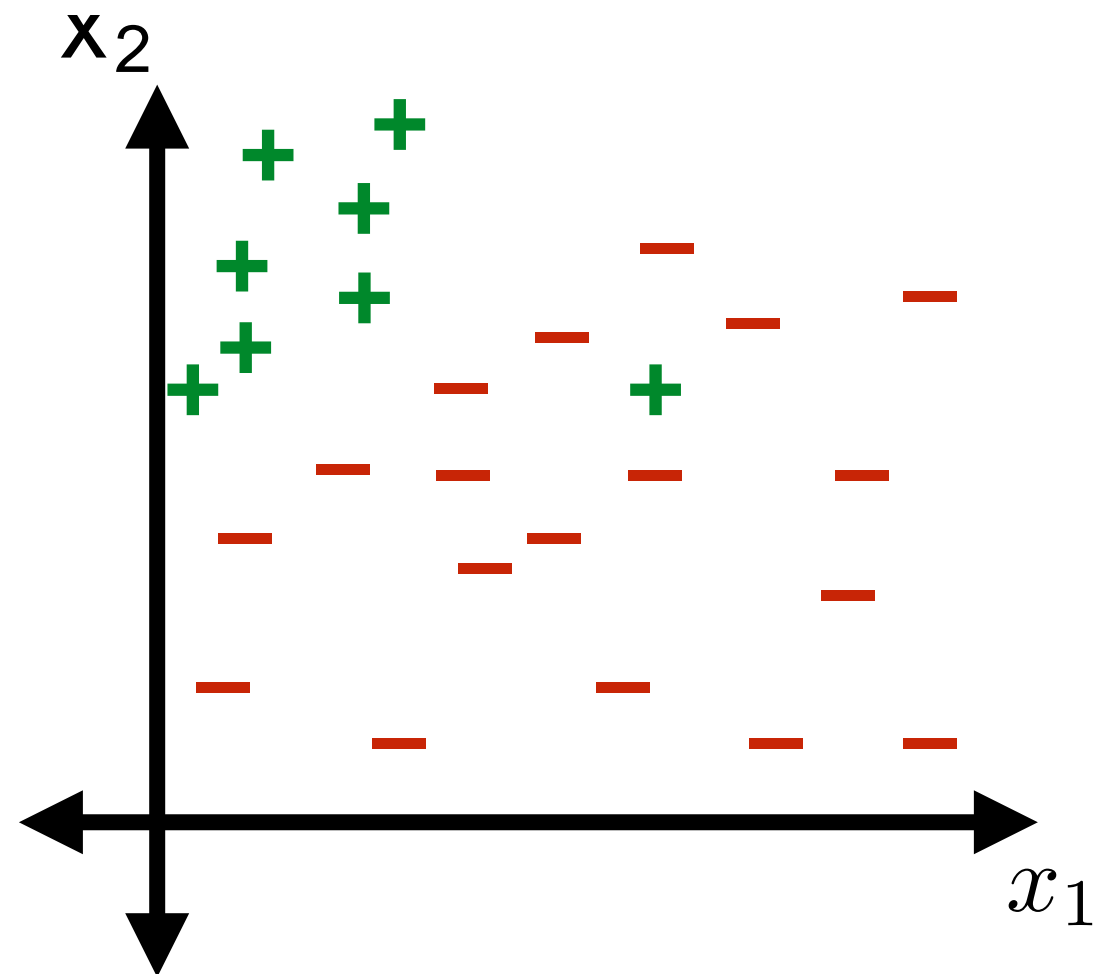
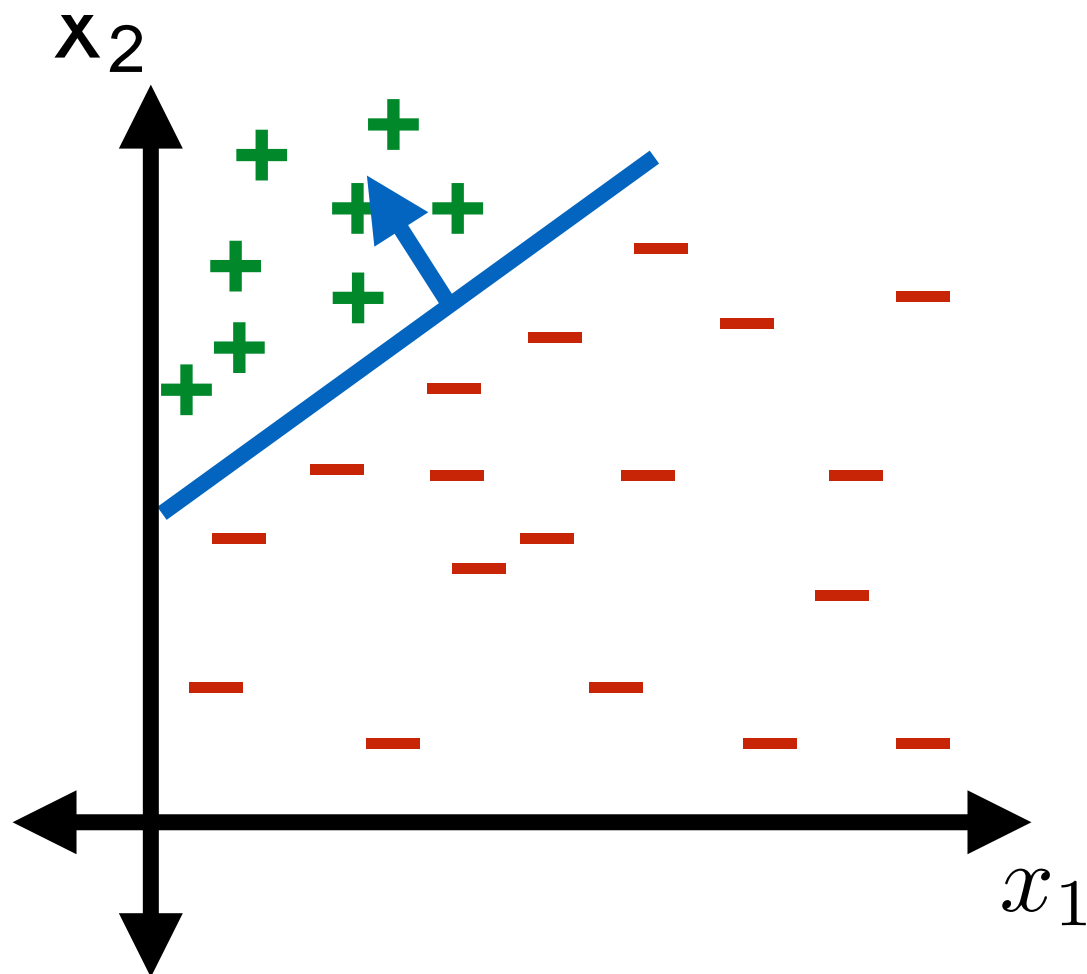




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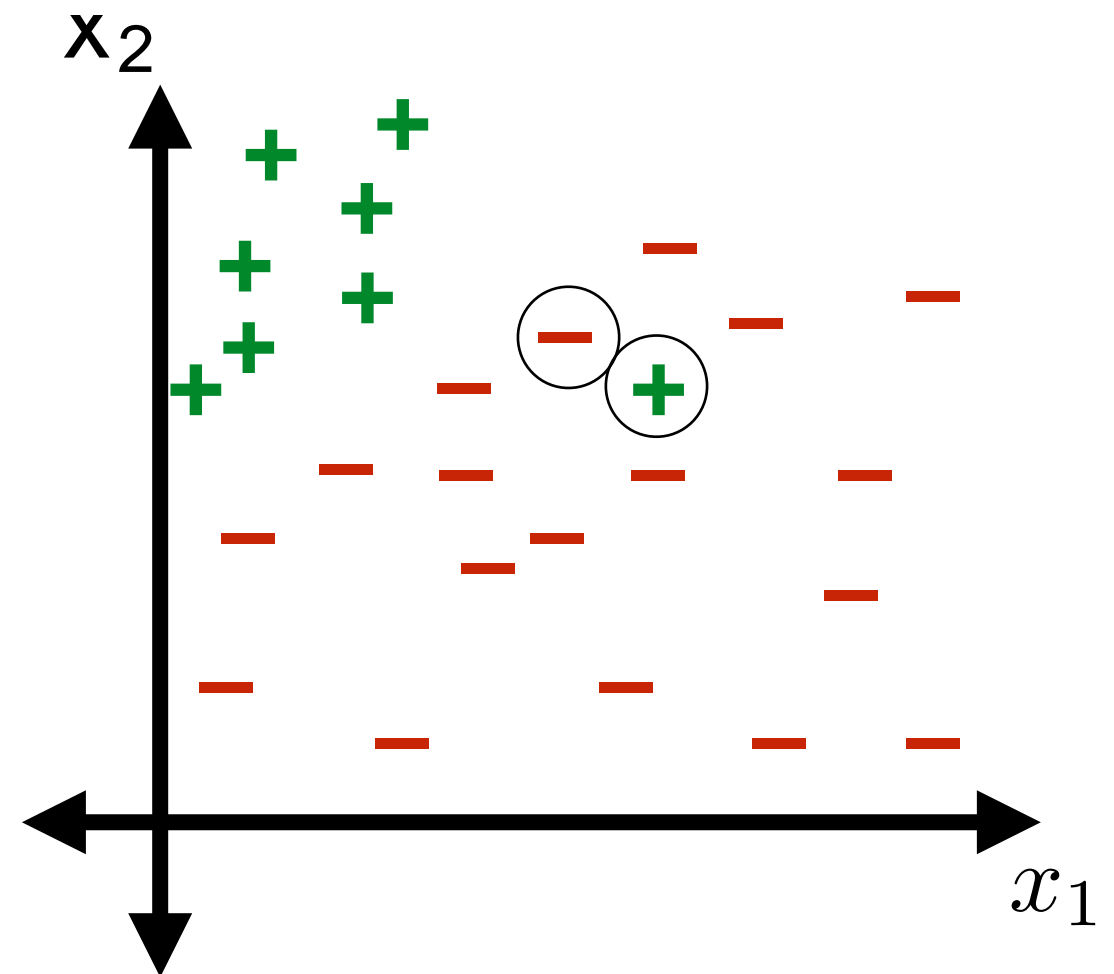
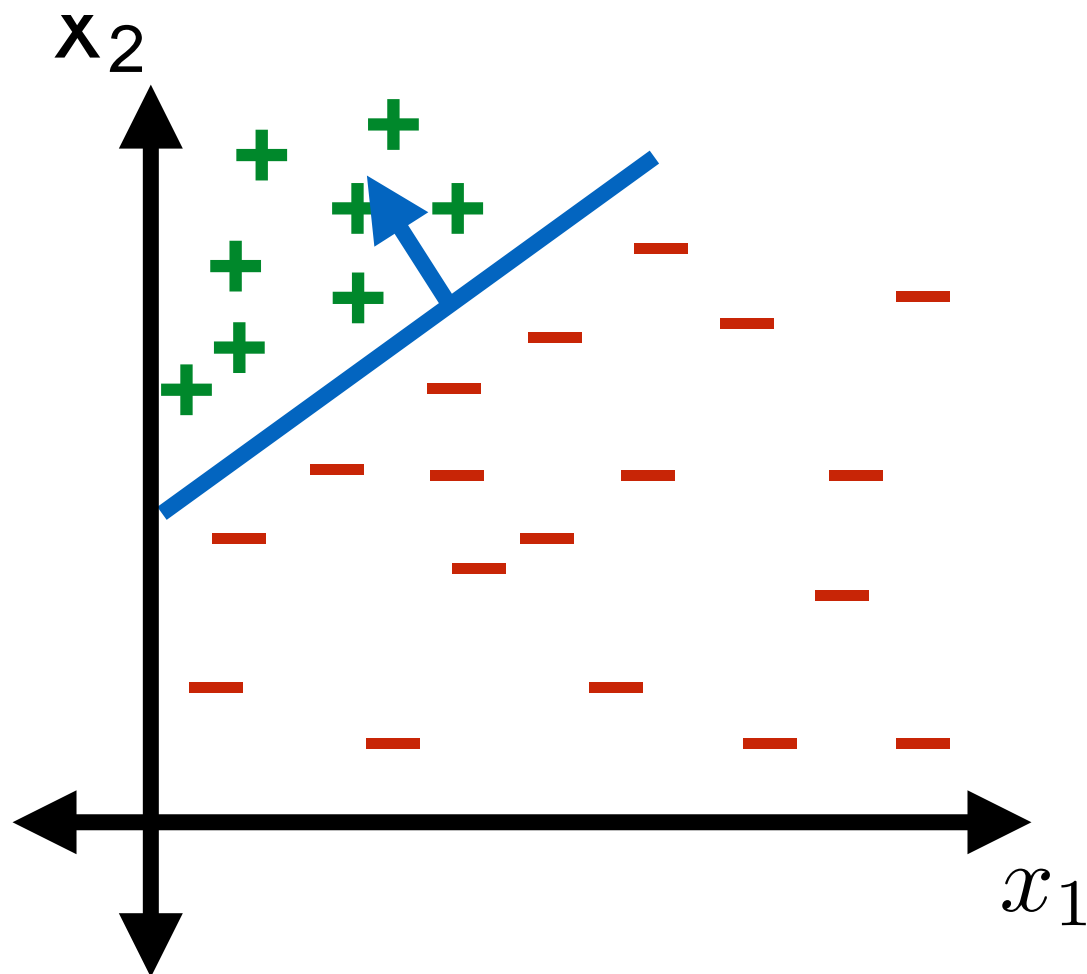
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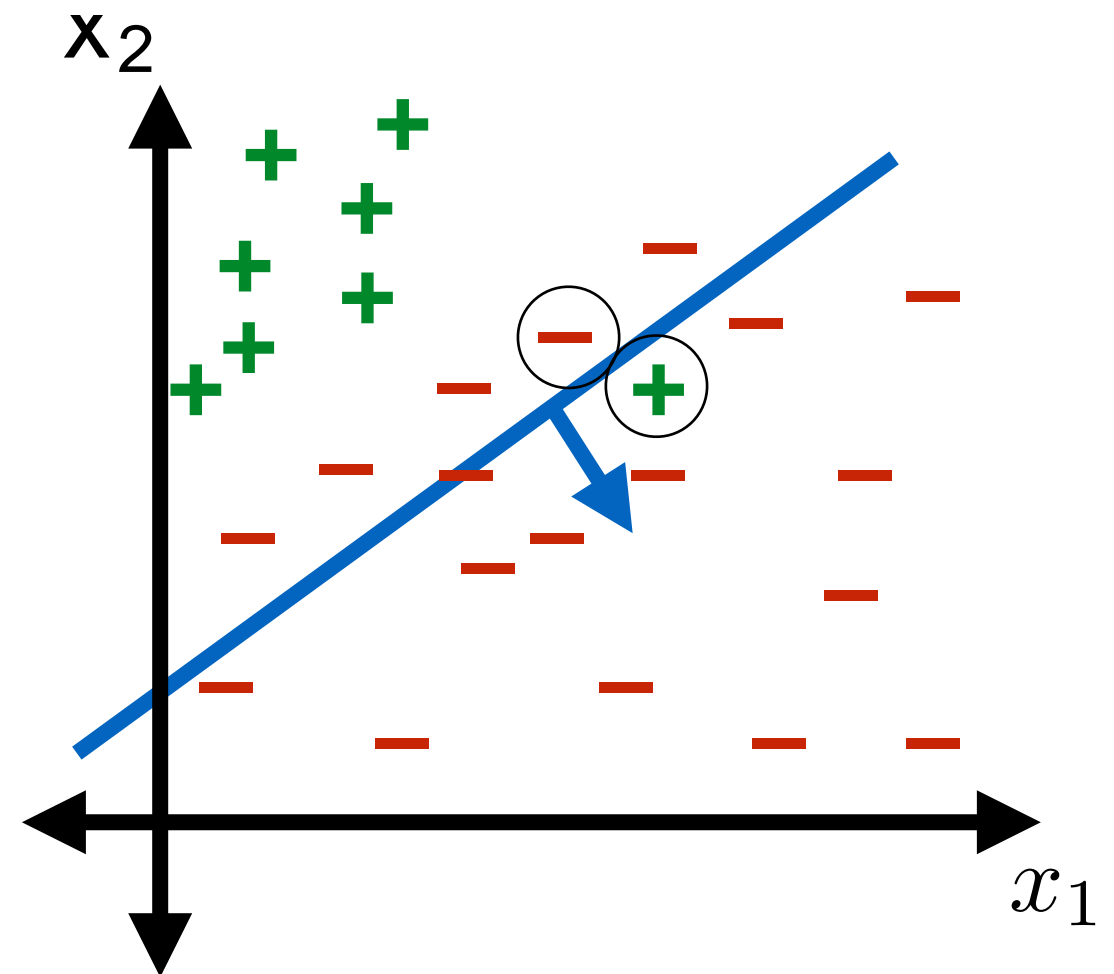
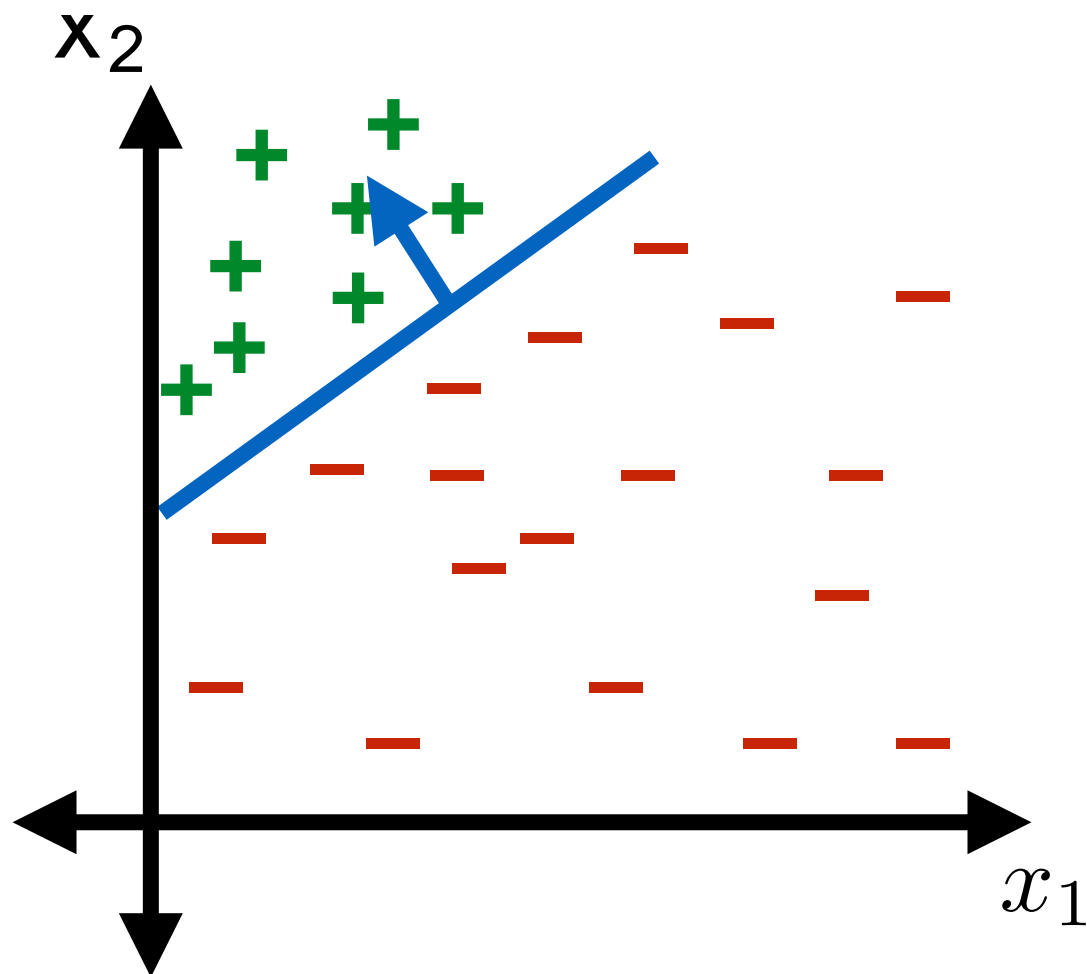
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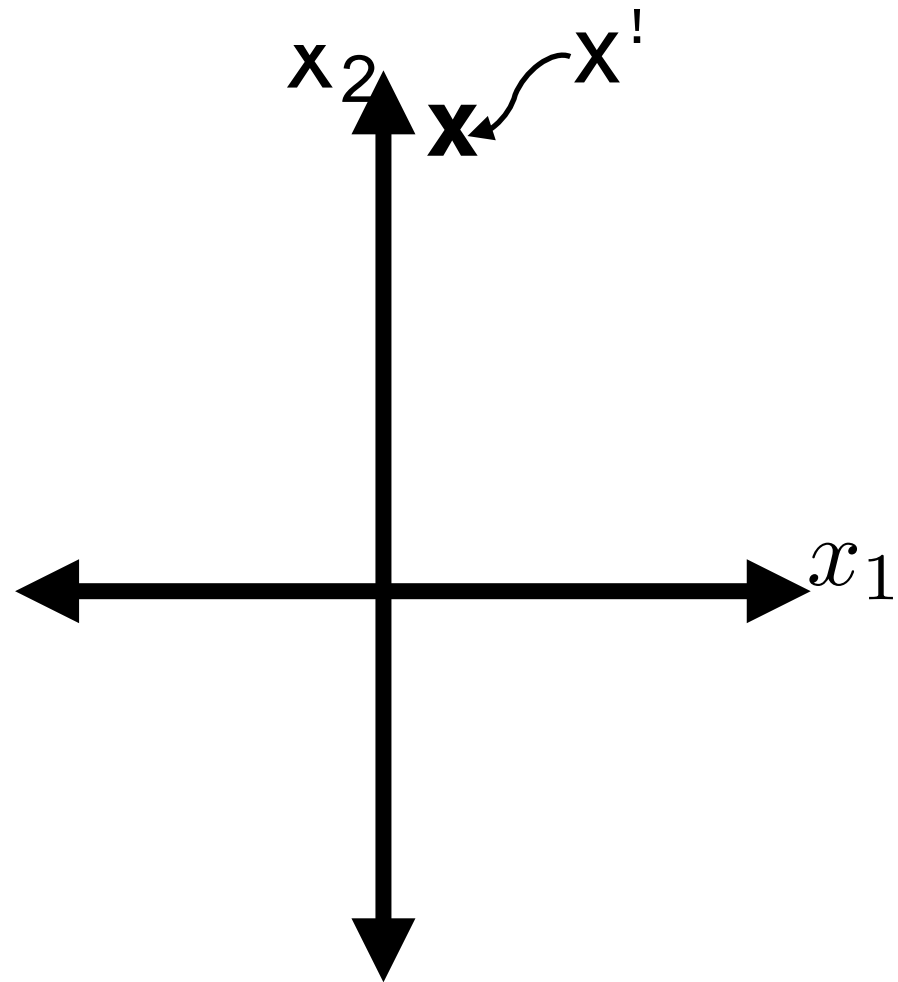
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**Math facts!**

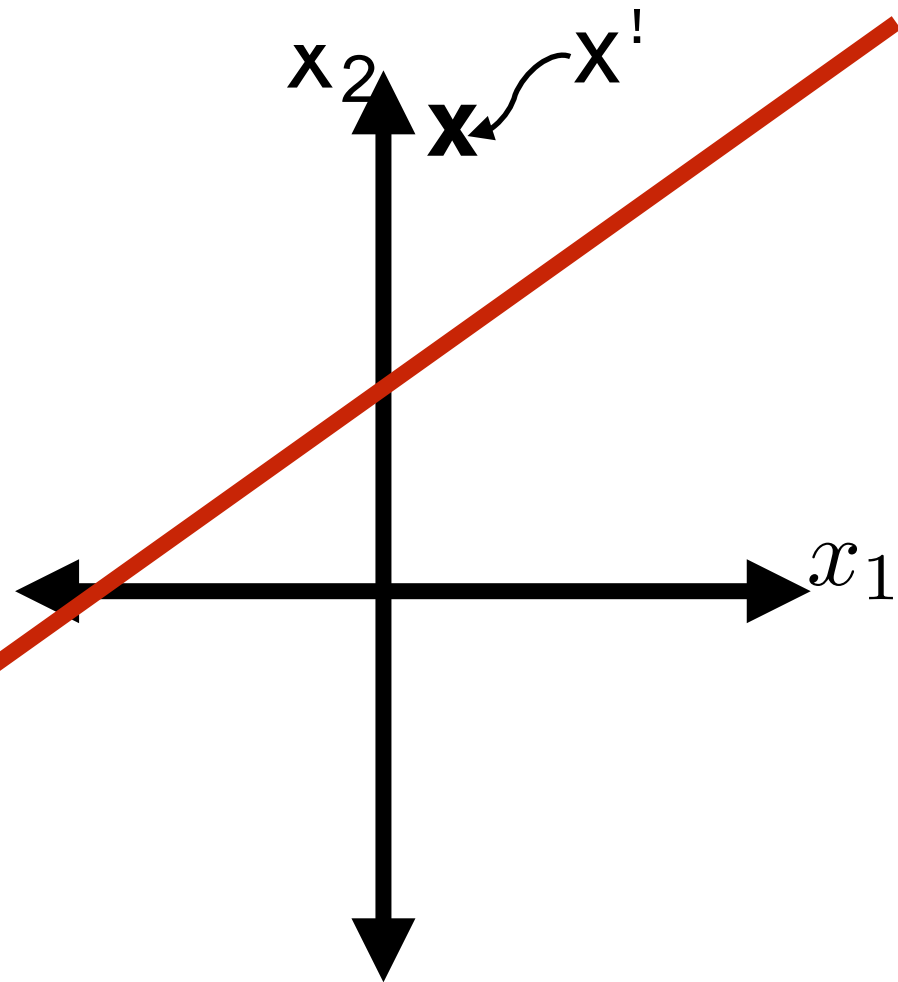
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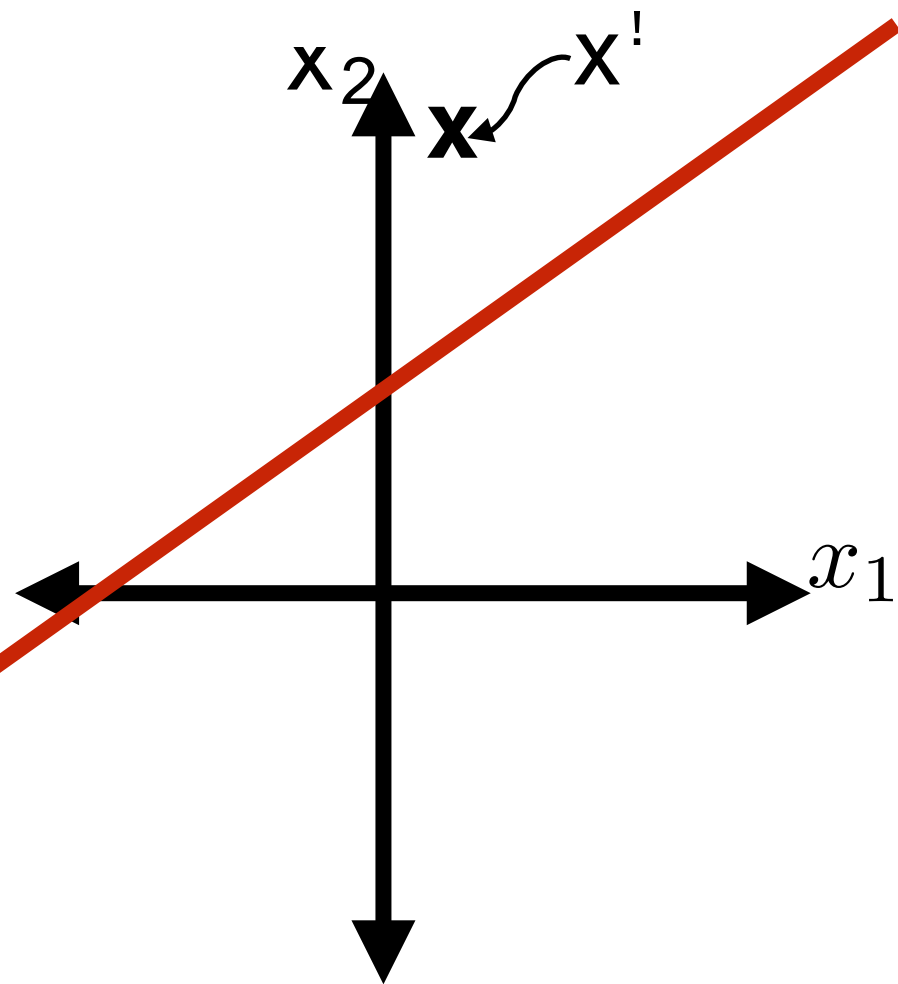
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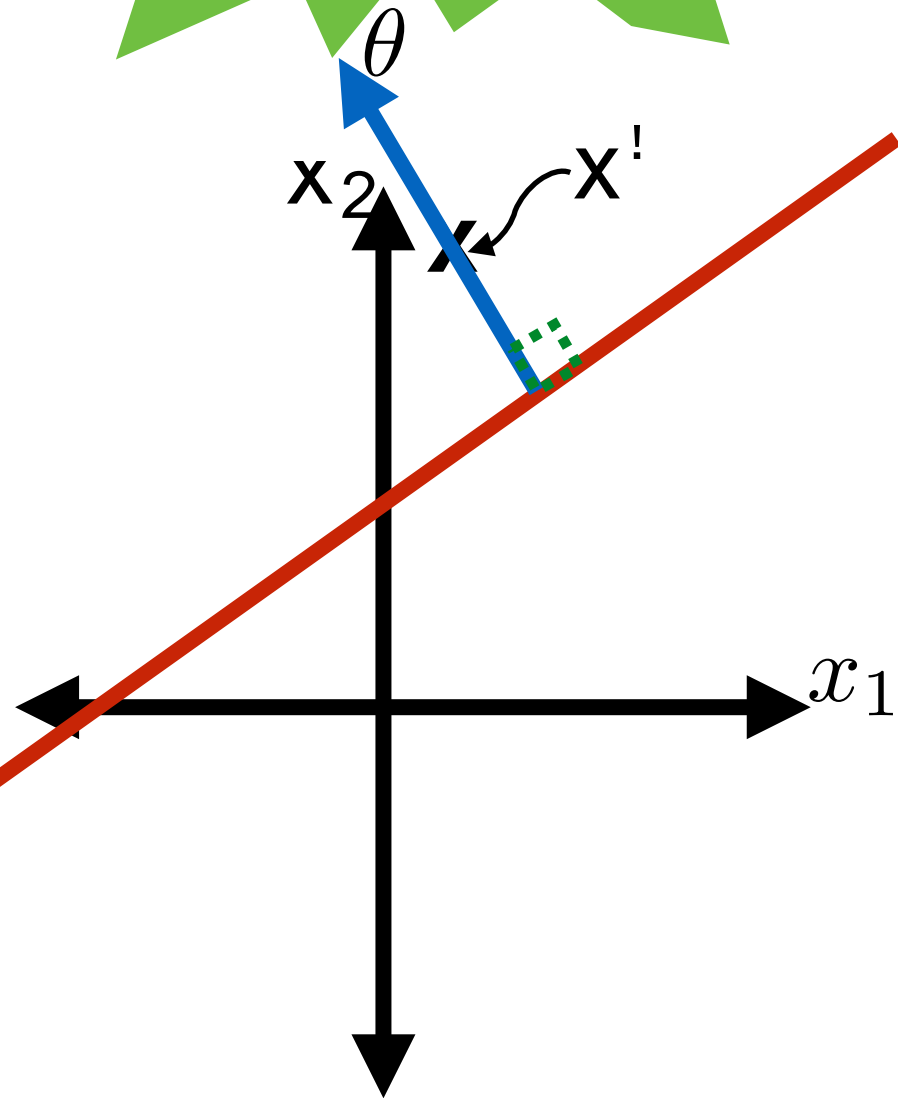
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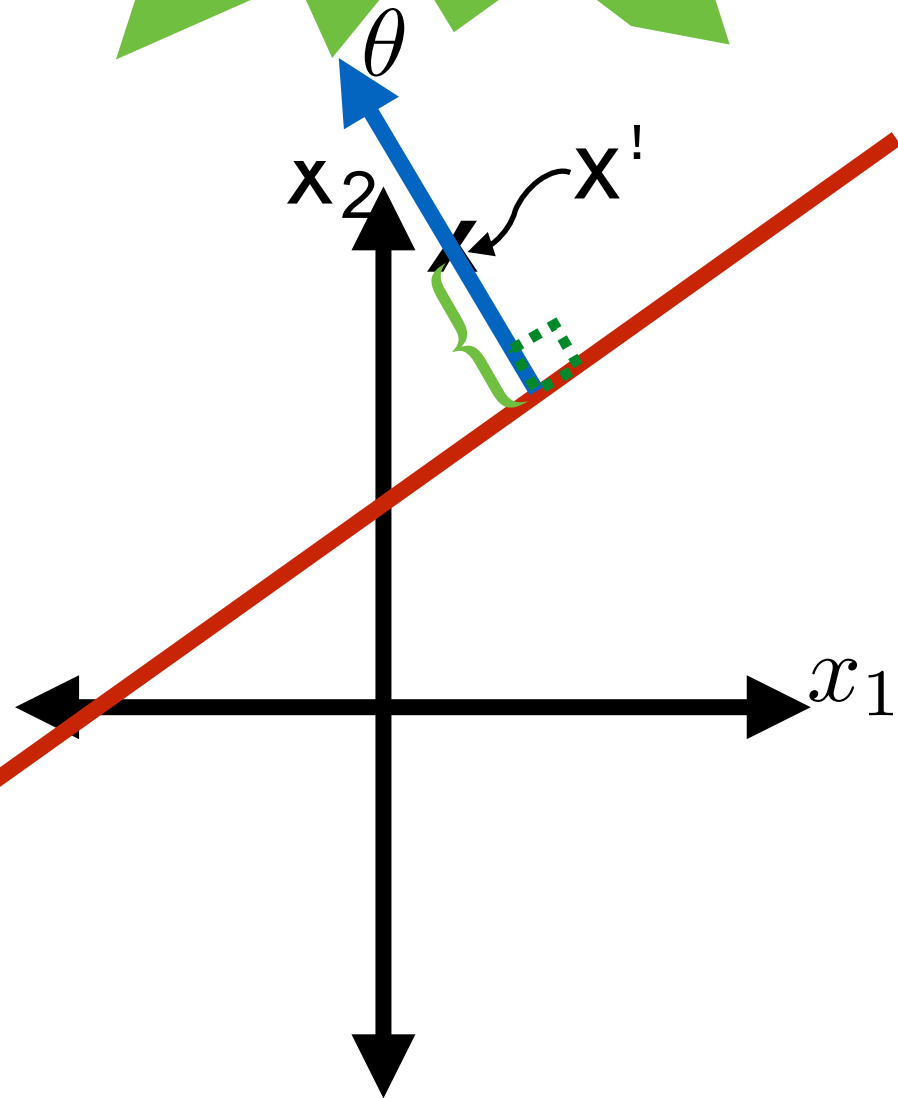




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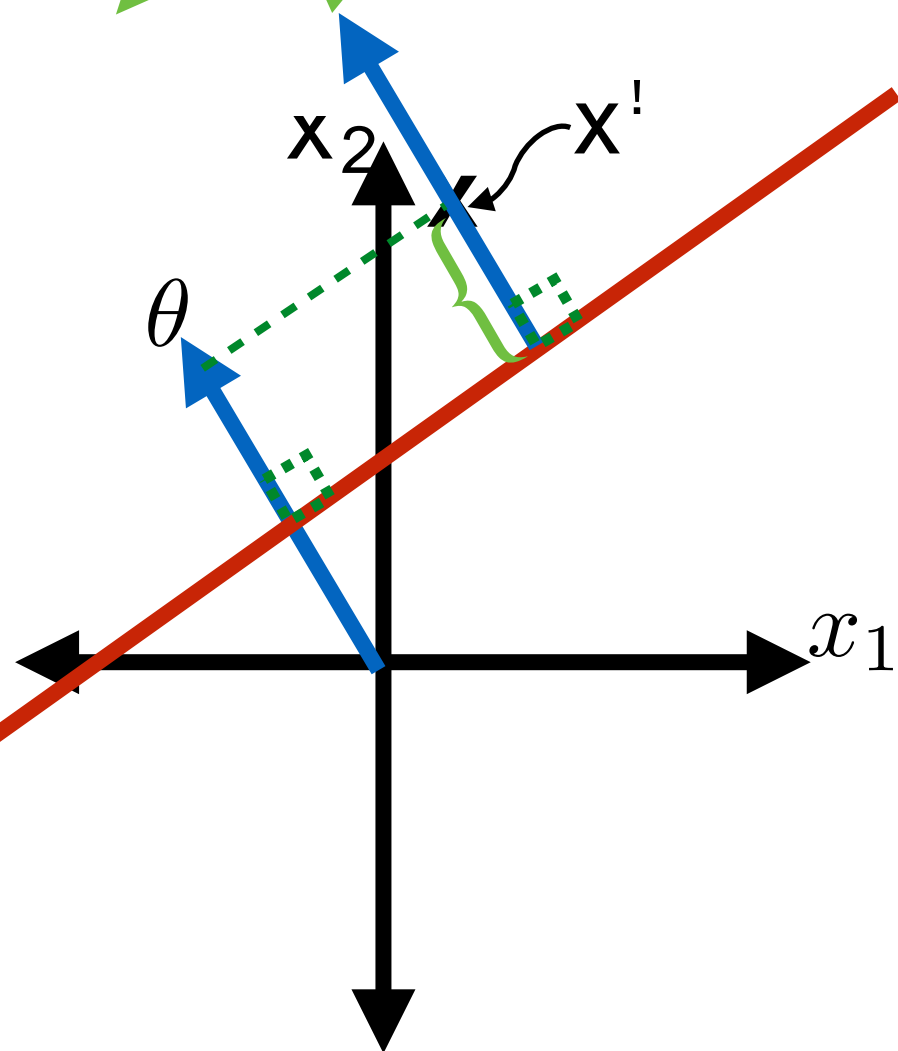
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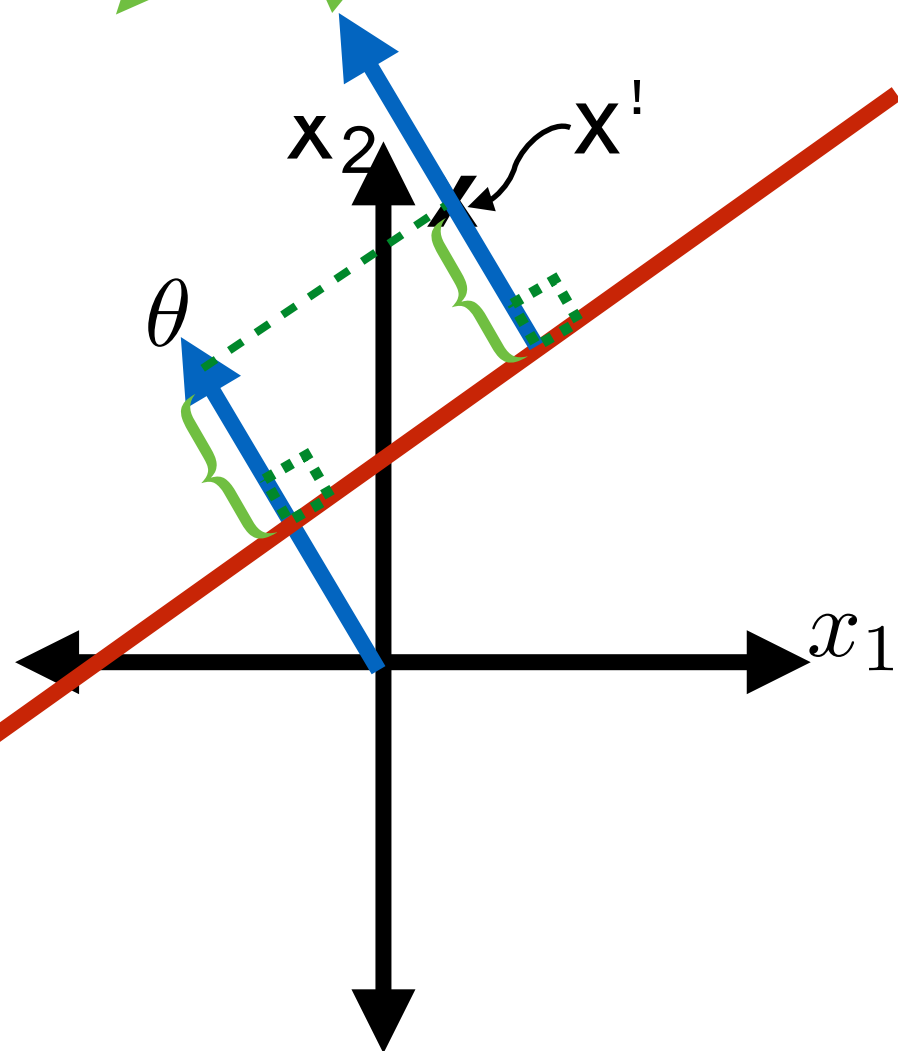
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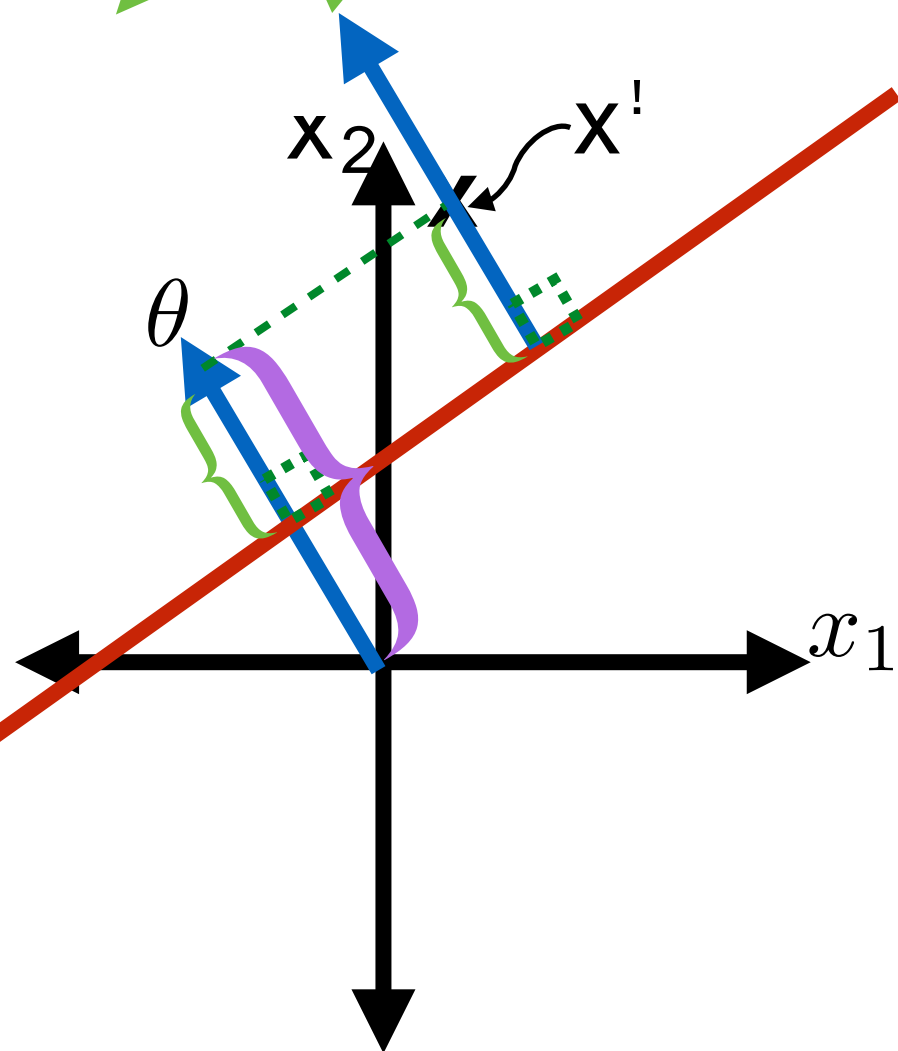
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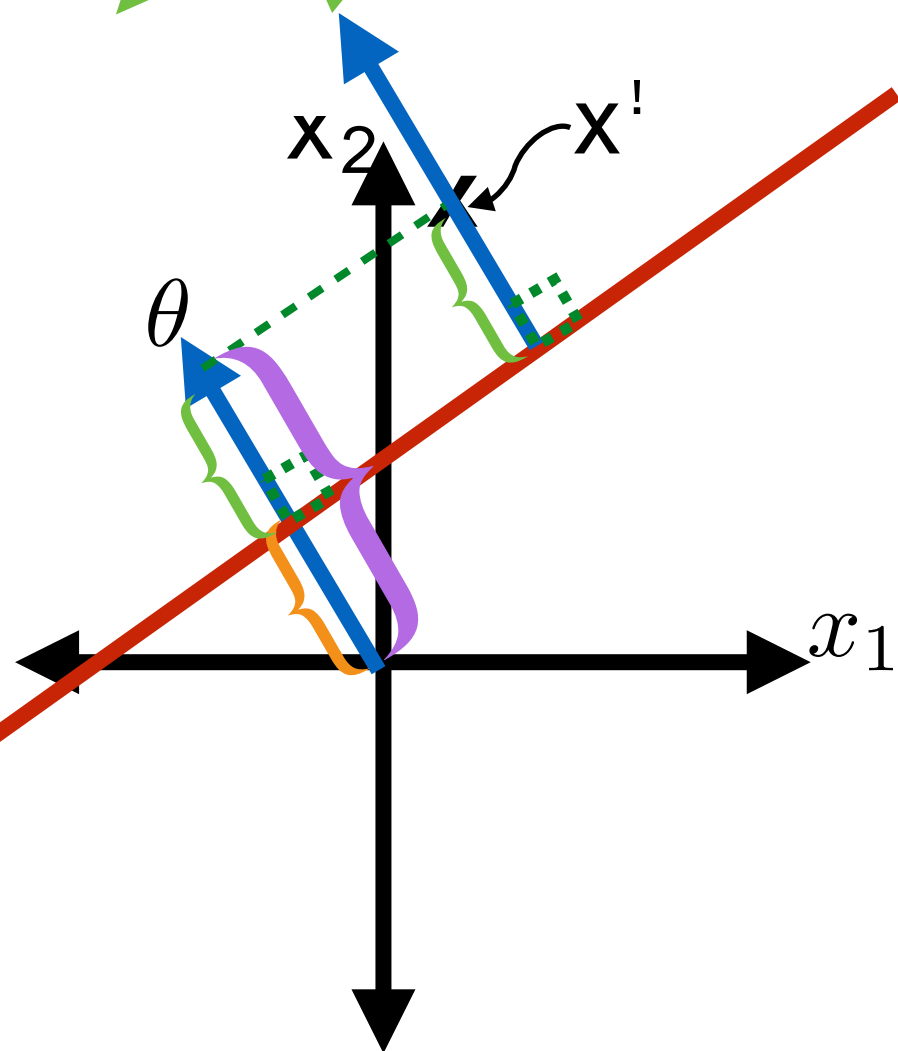
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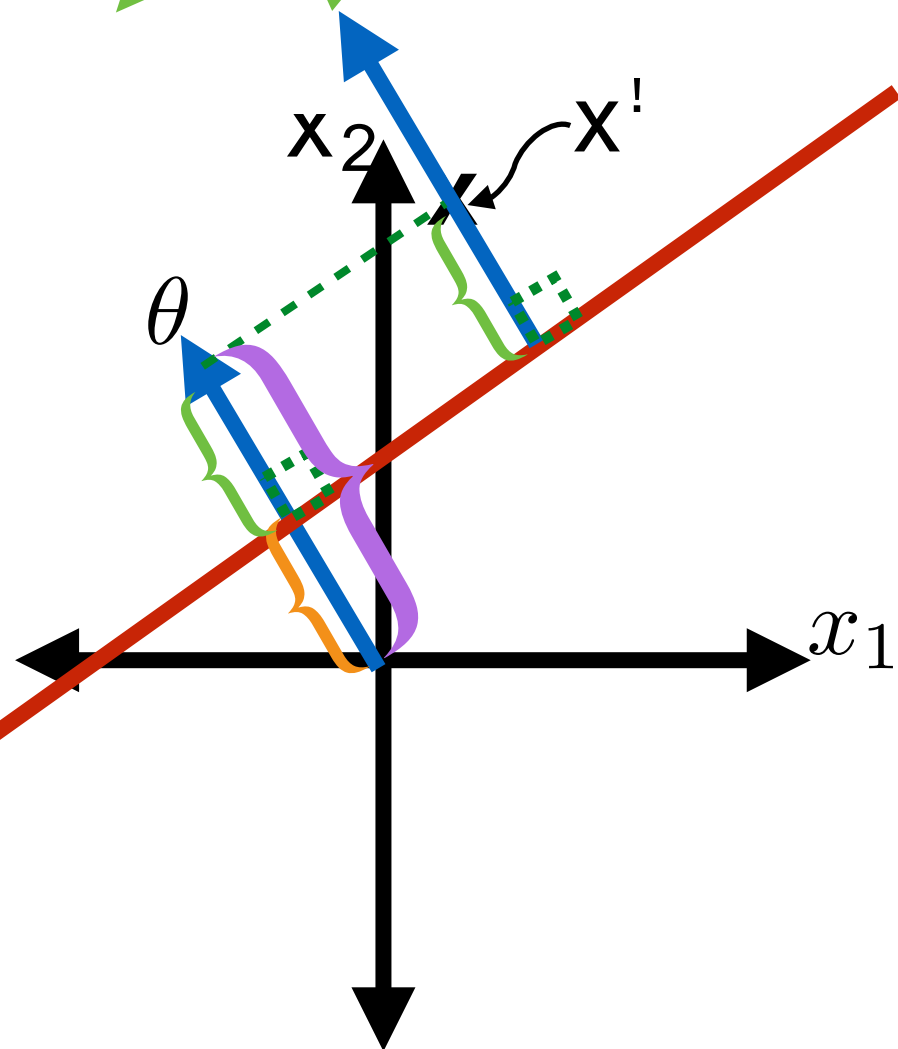
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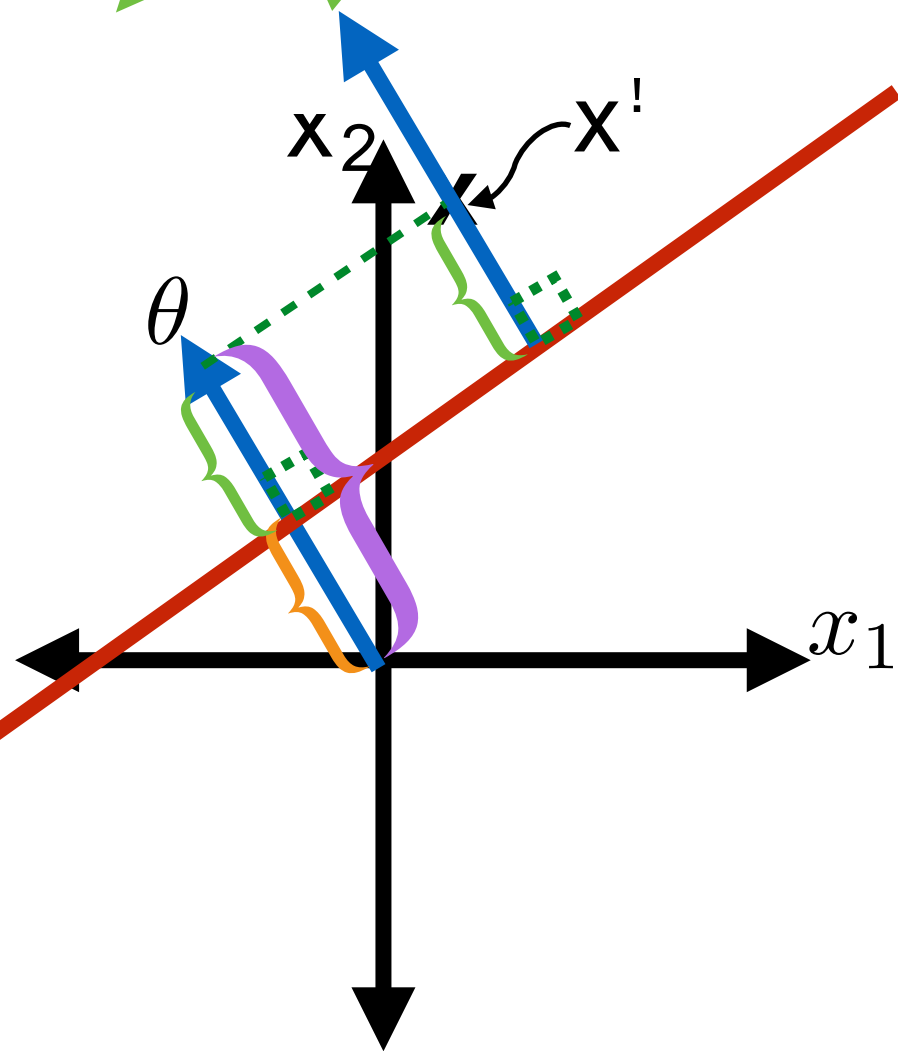
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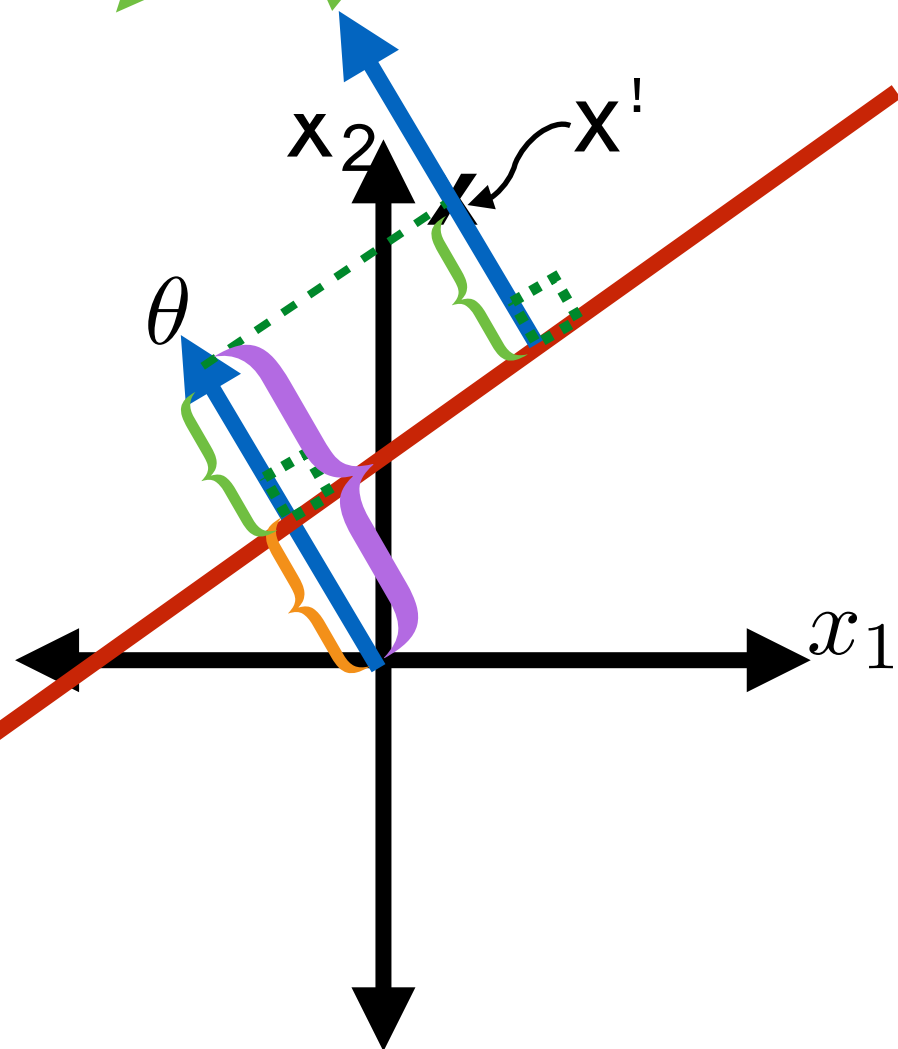
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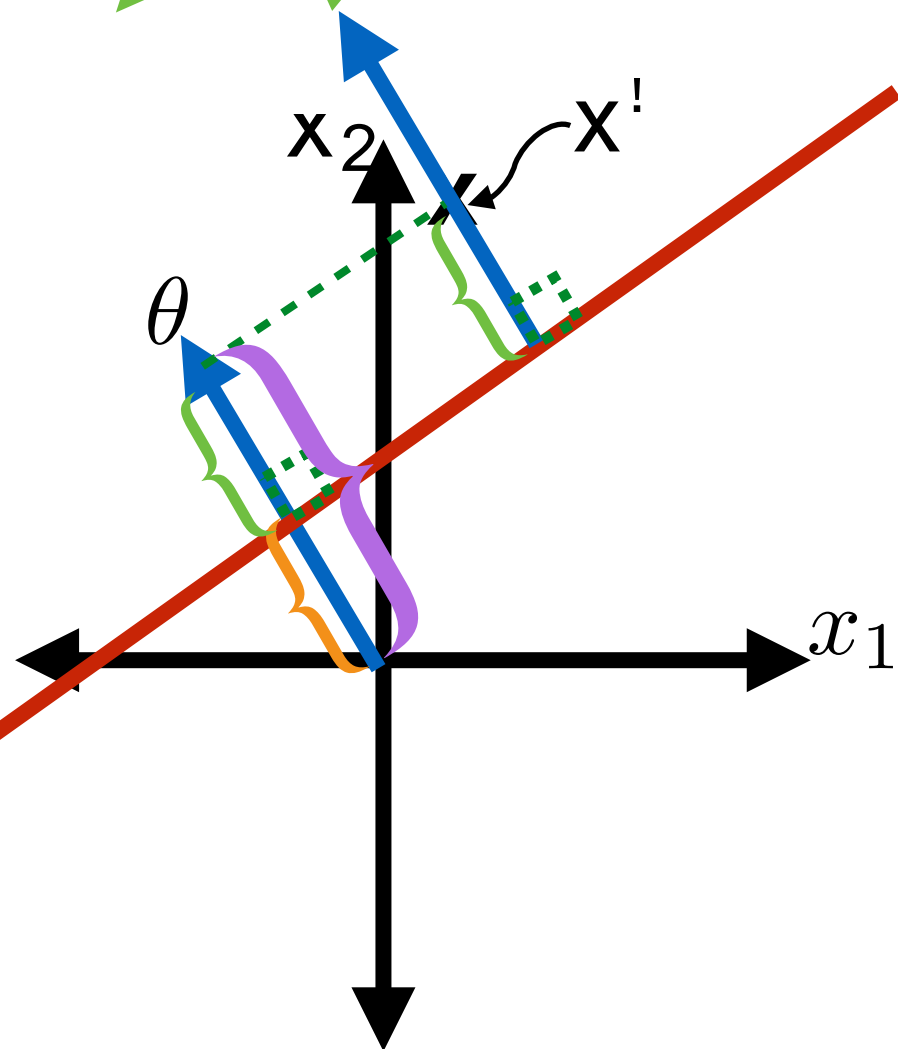
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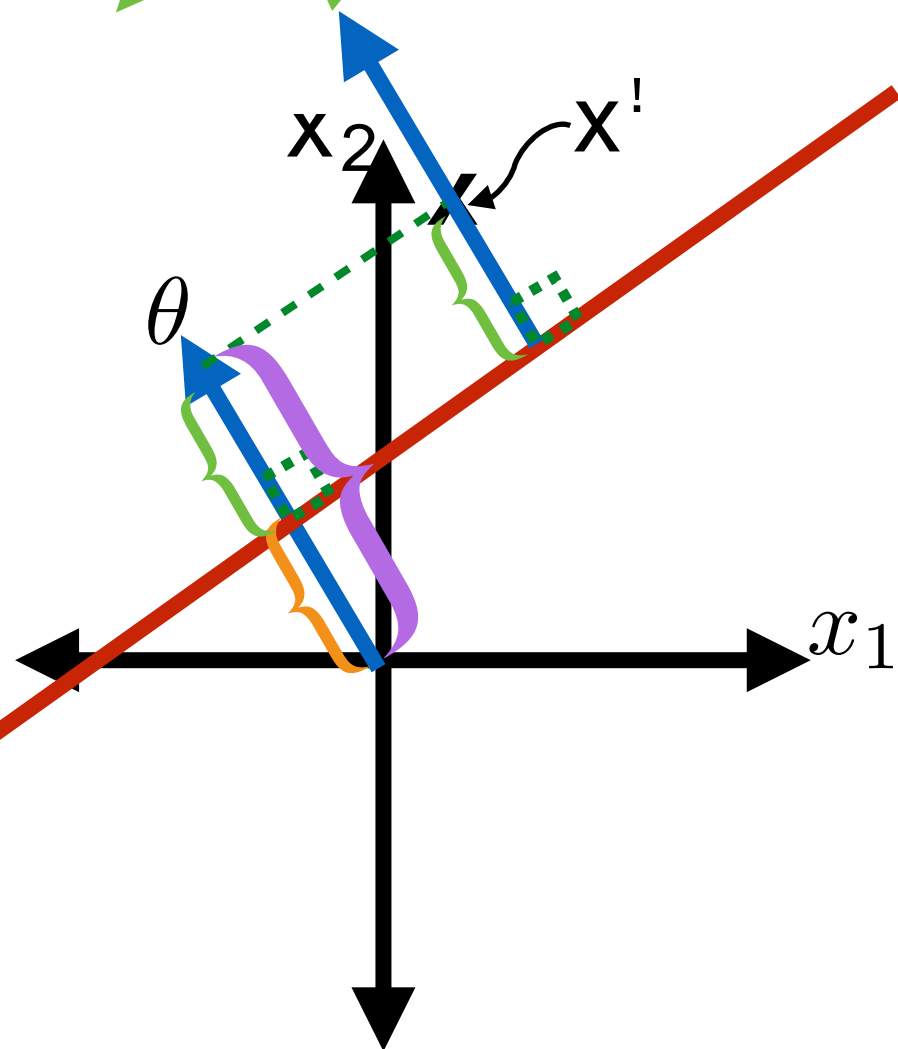
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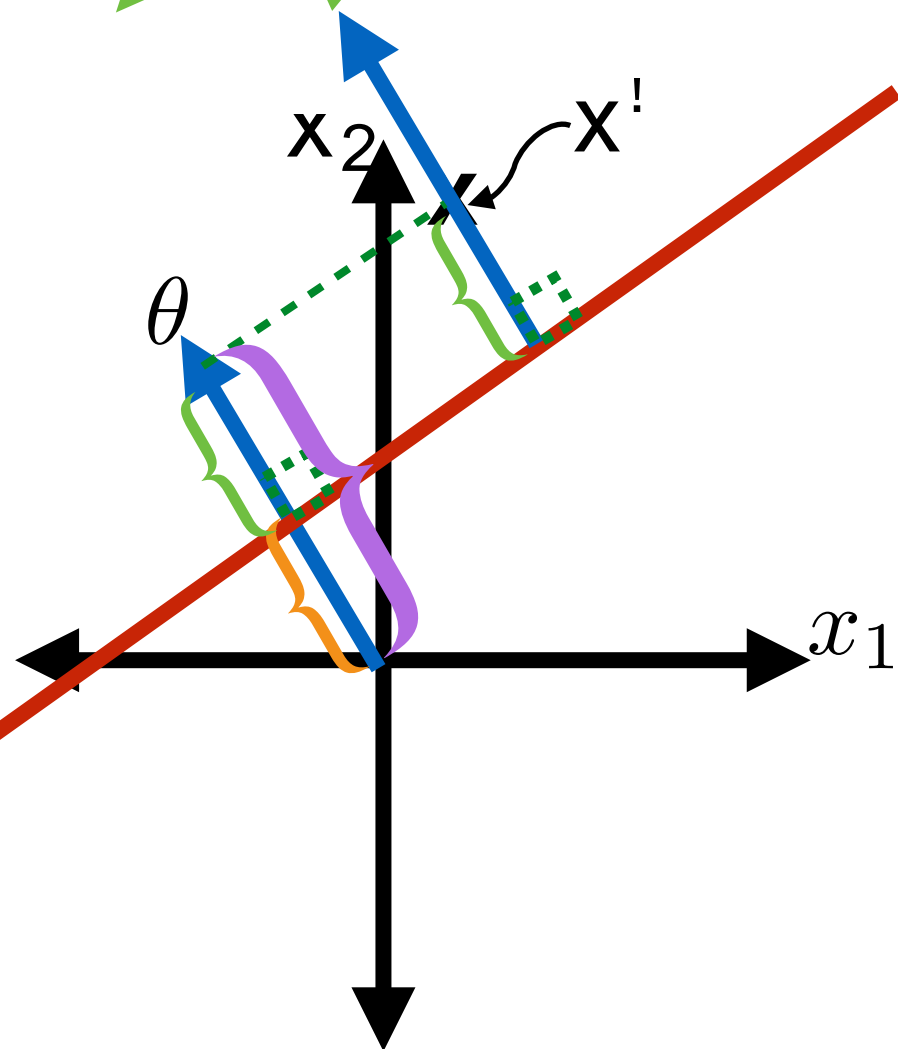
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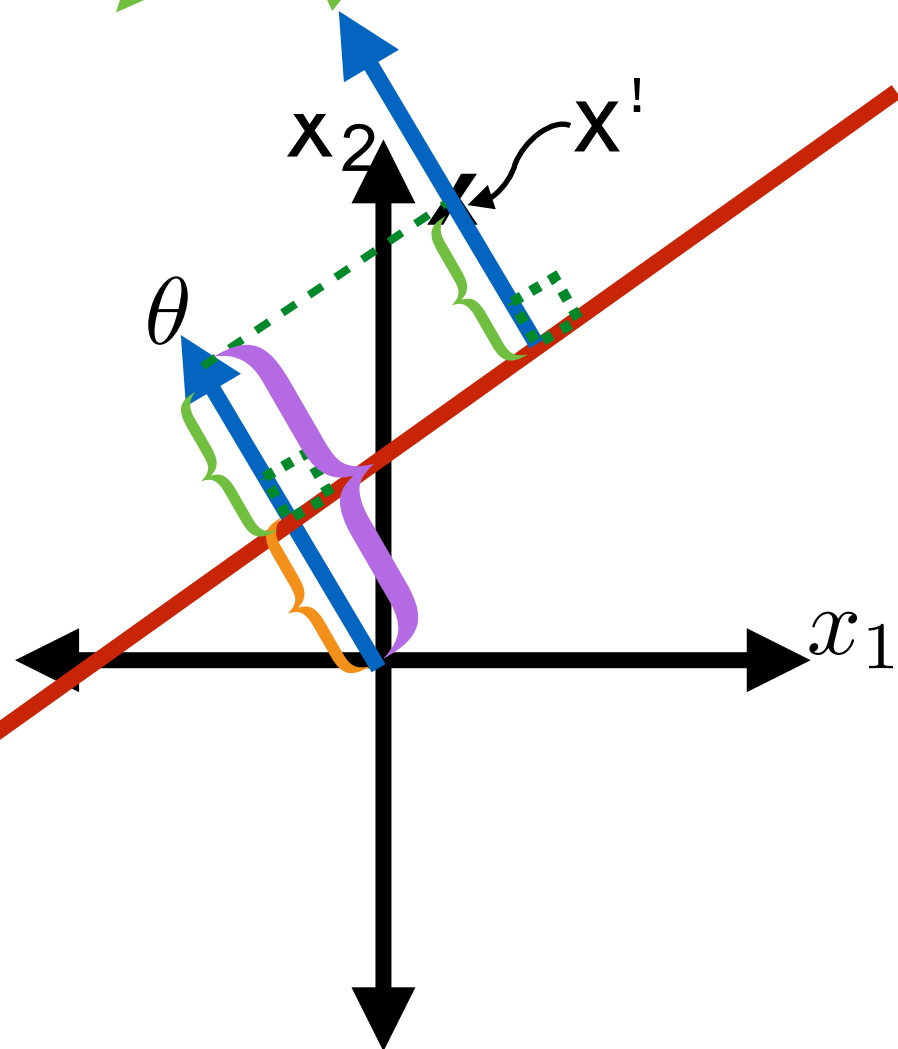
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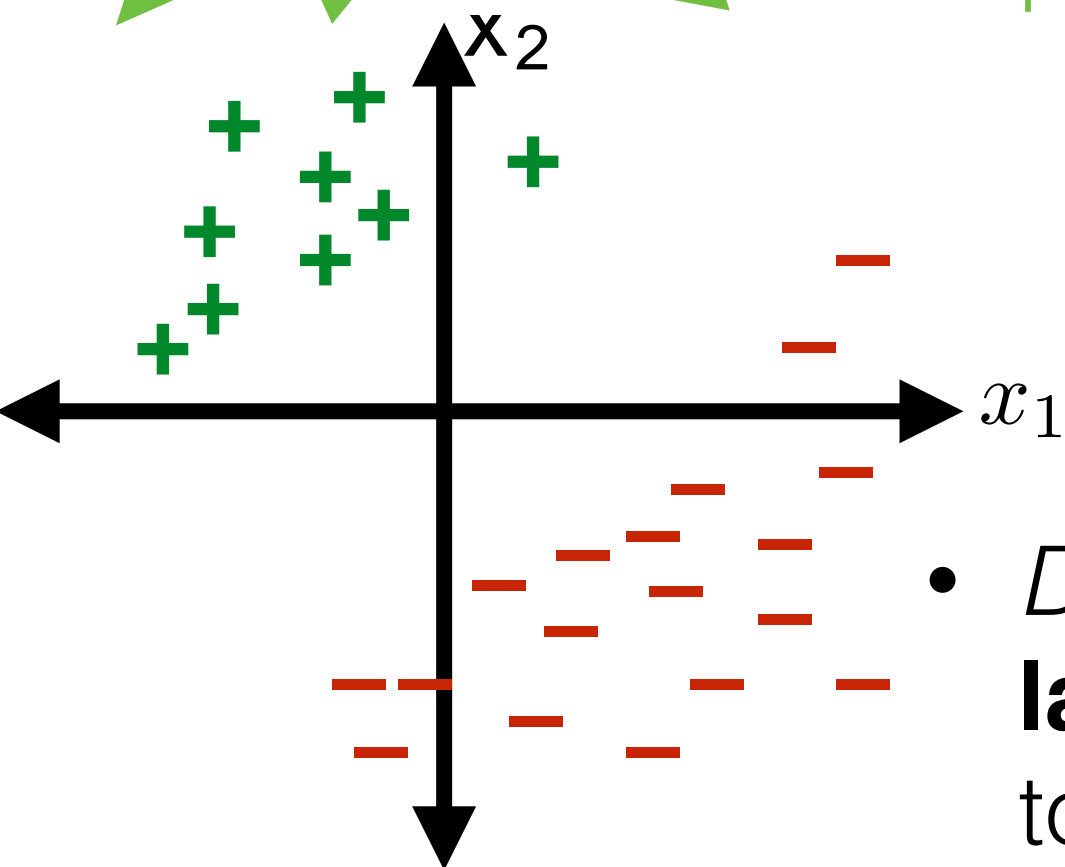
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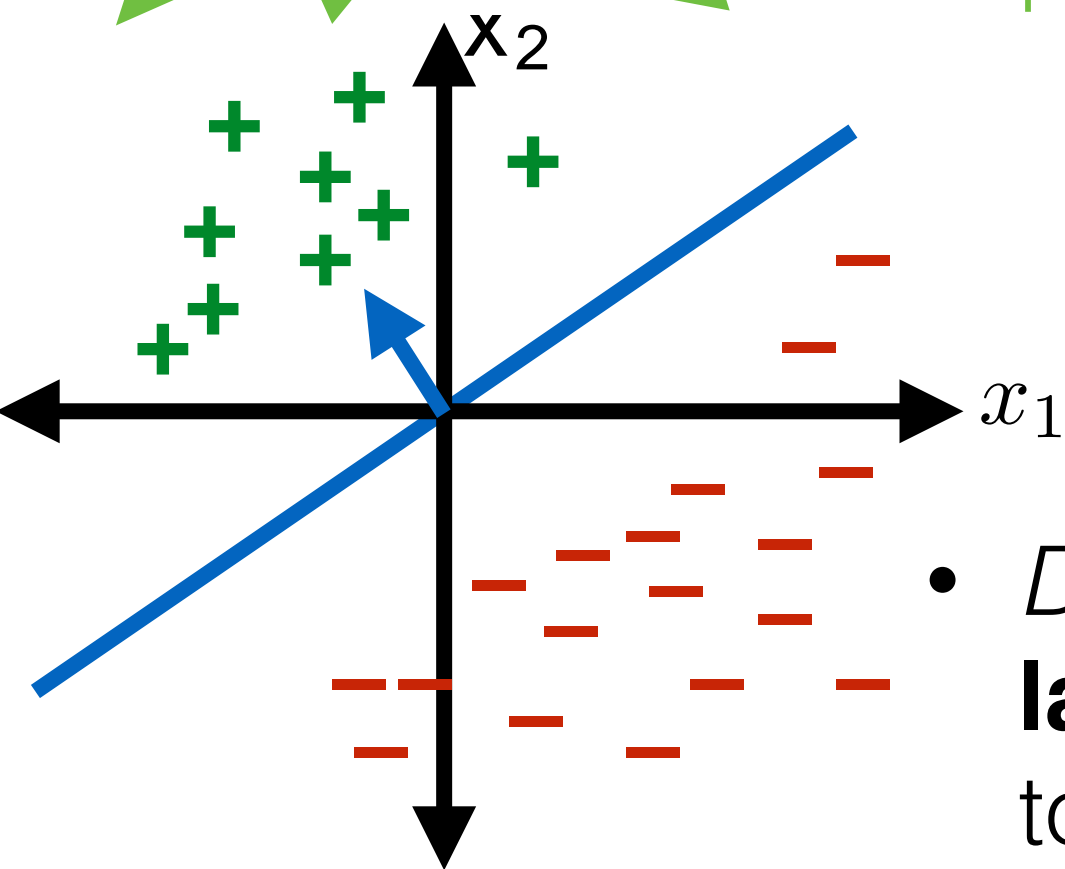
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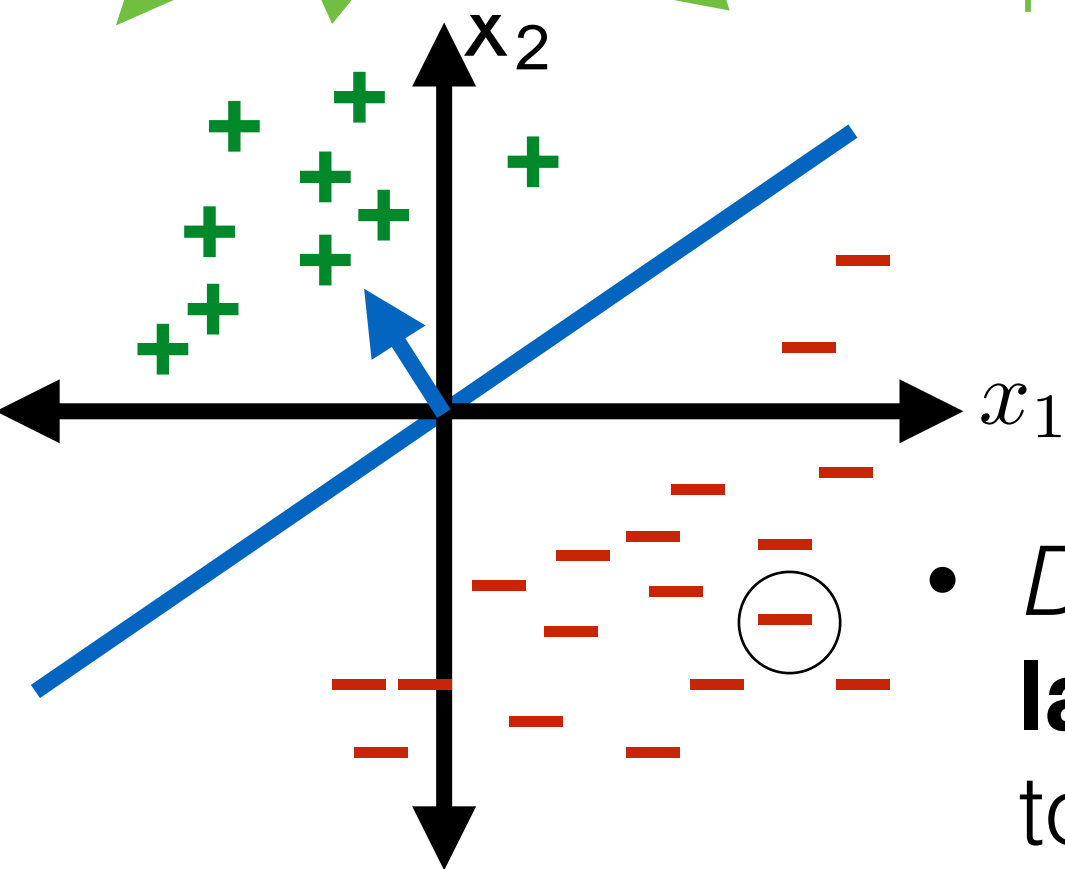
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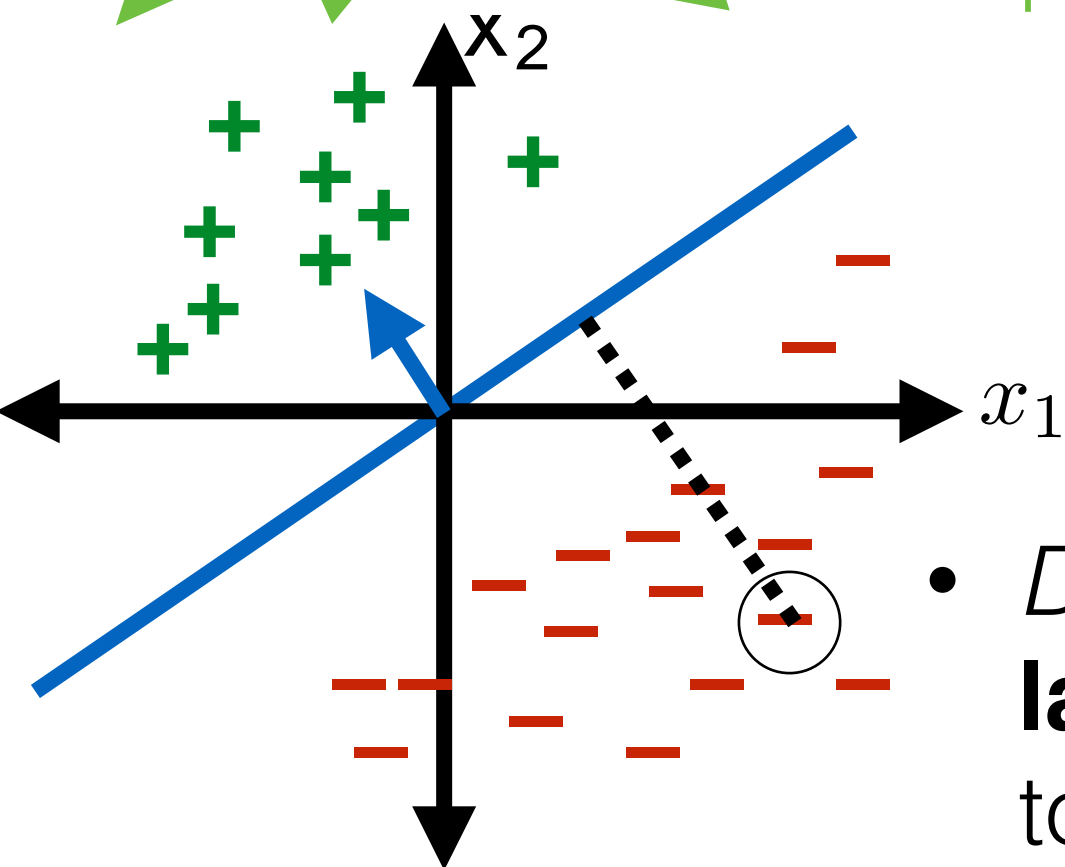
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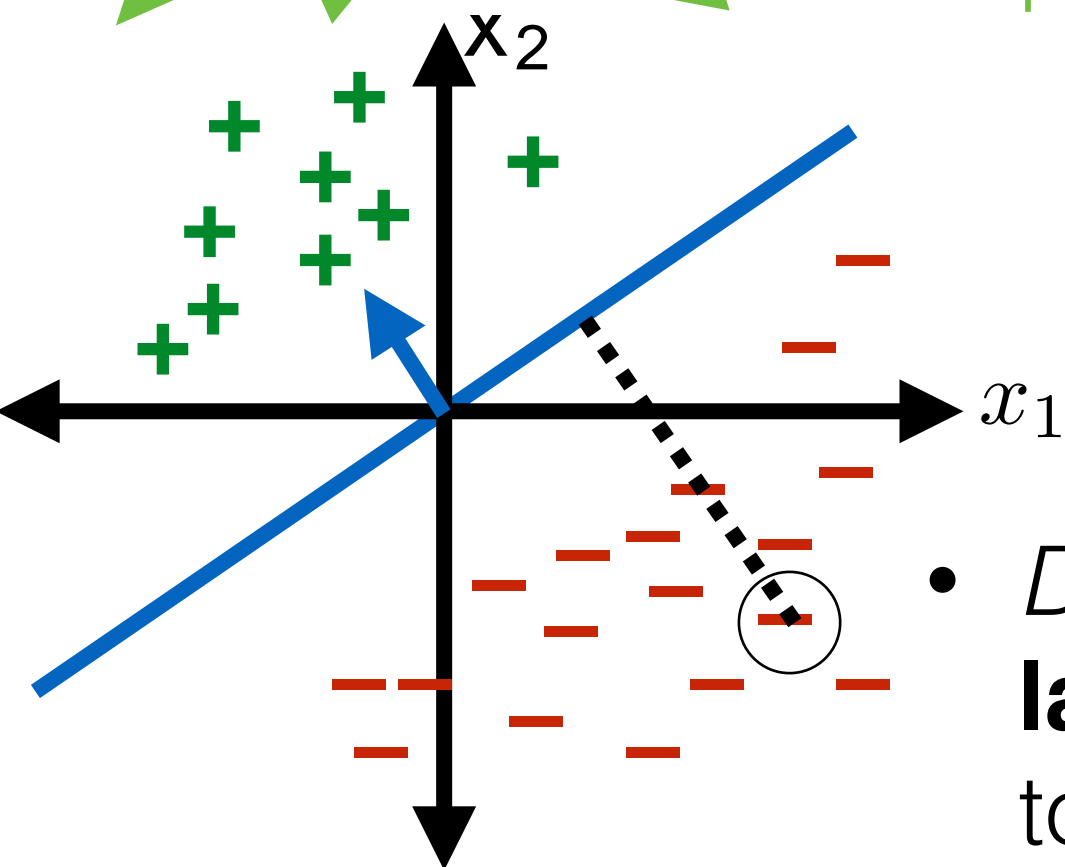
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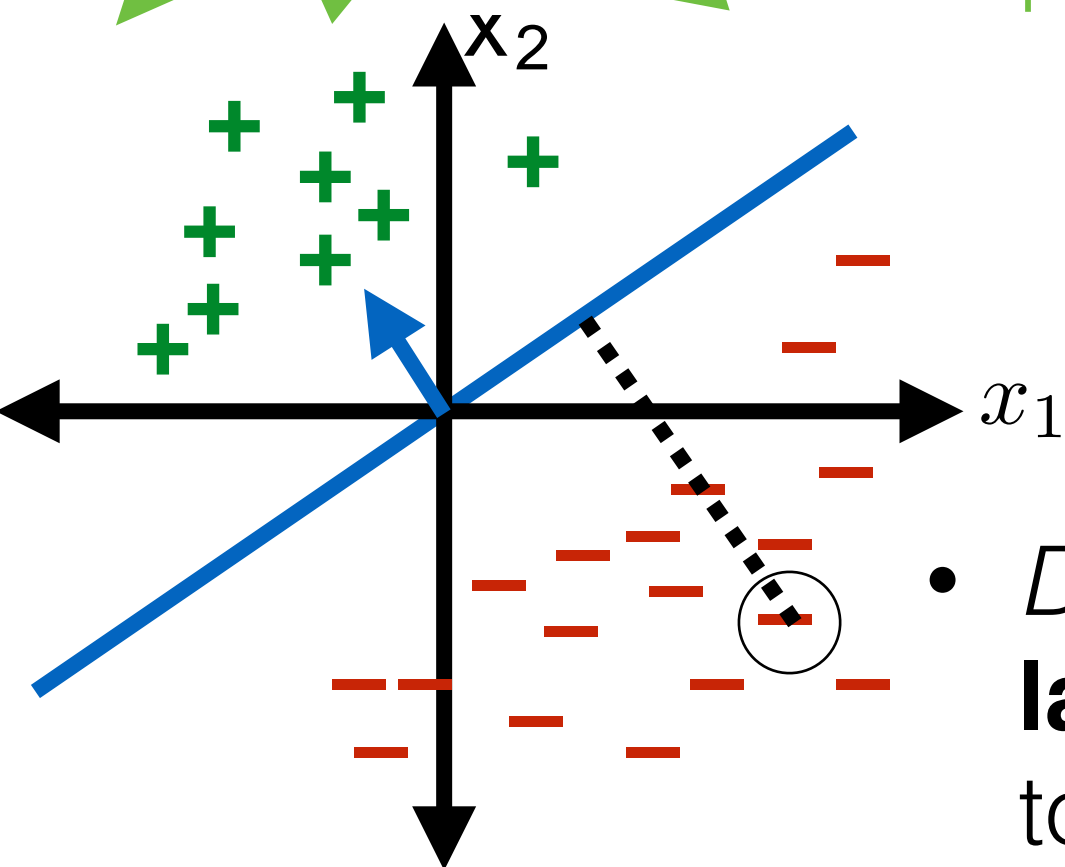
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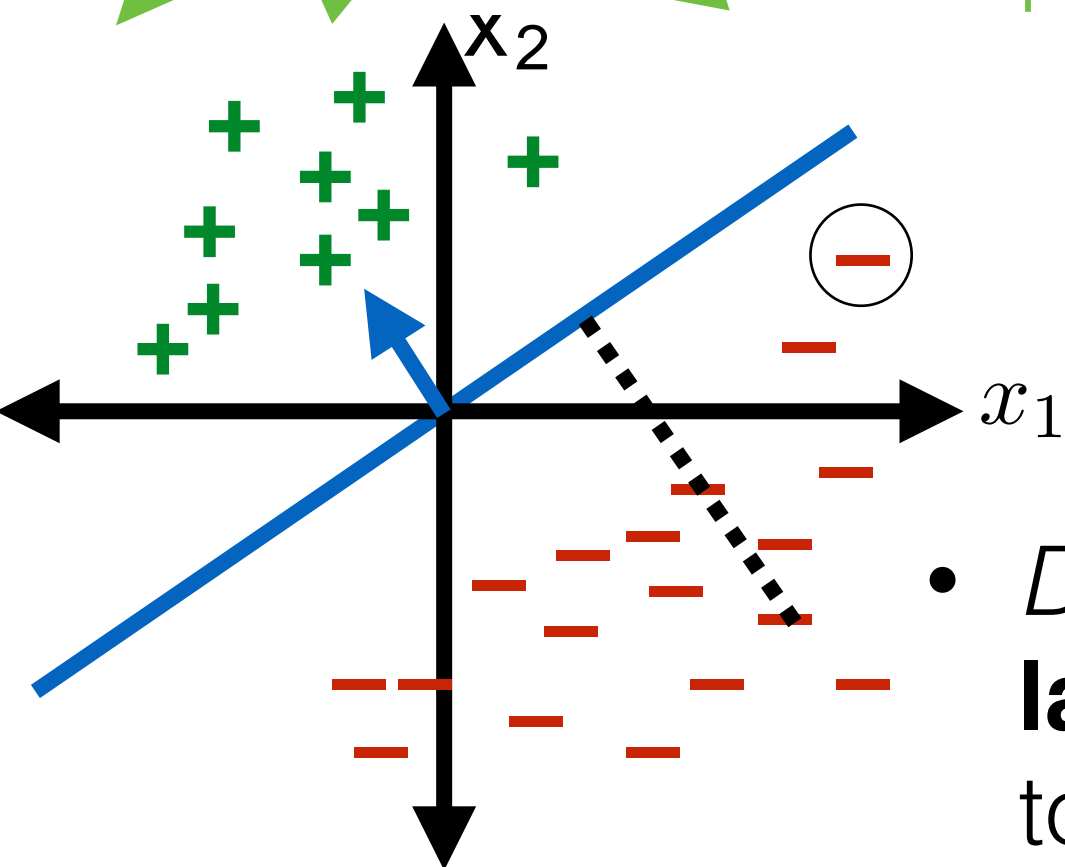
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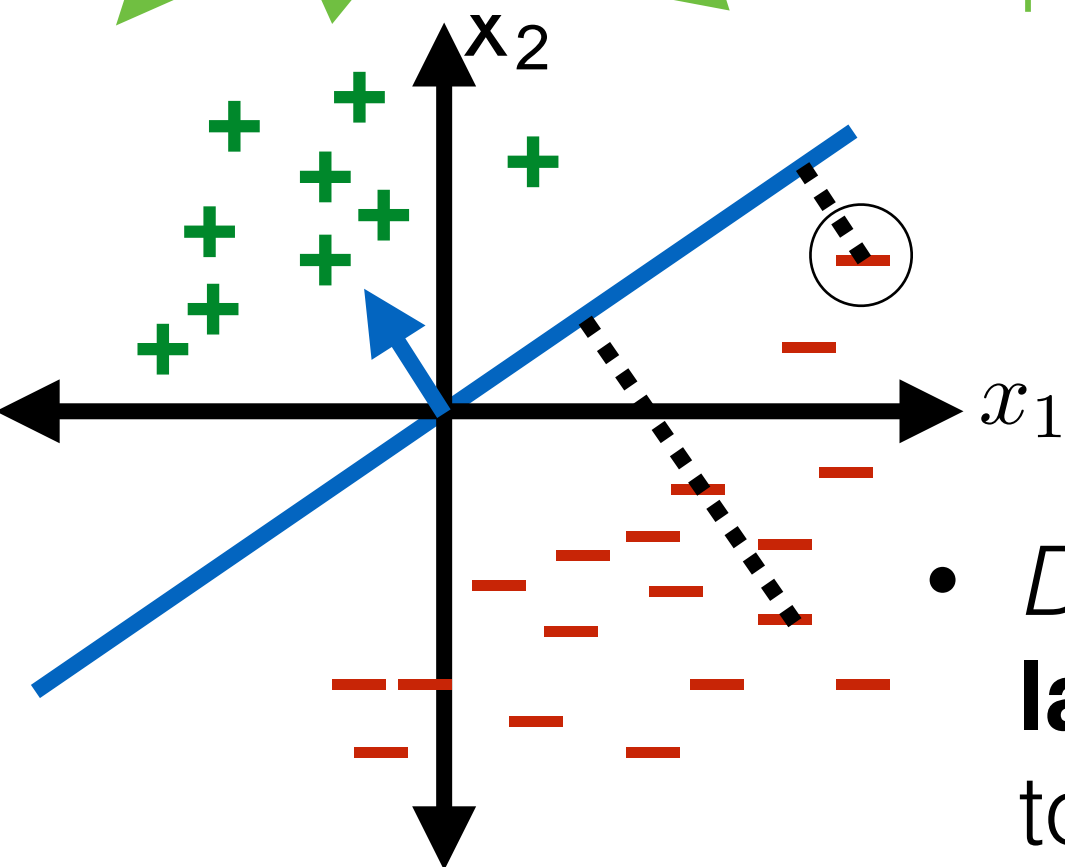
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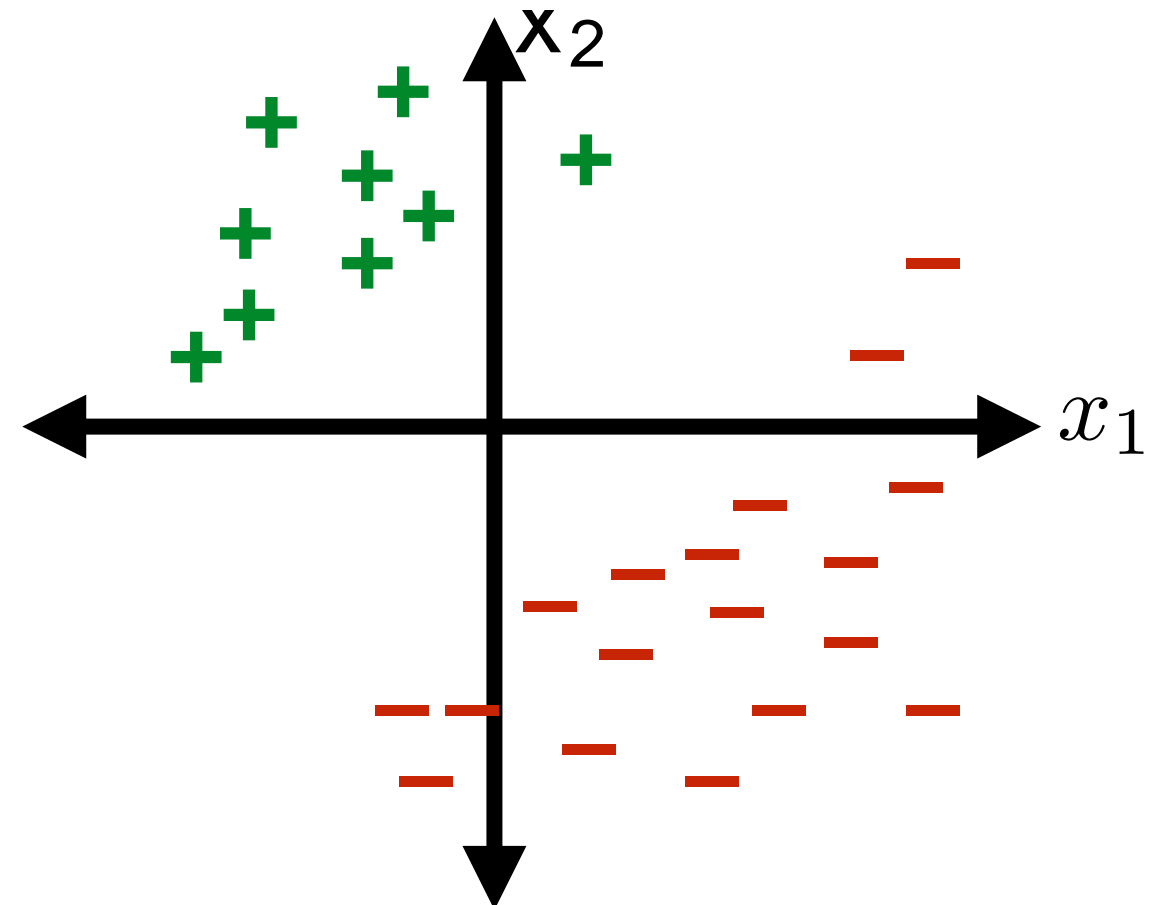
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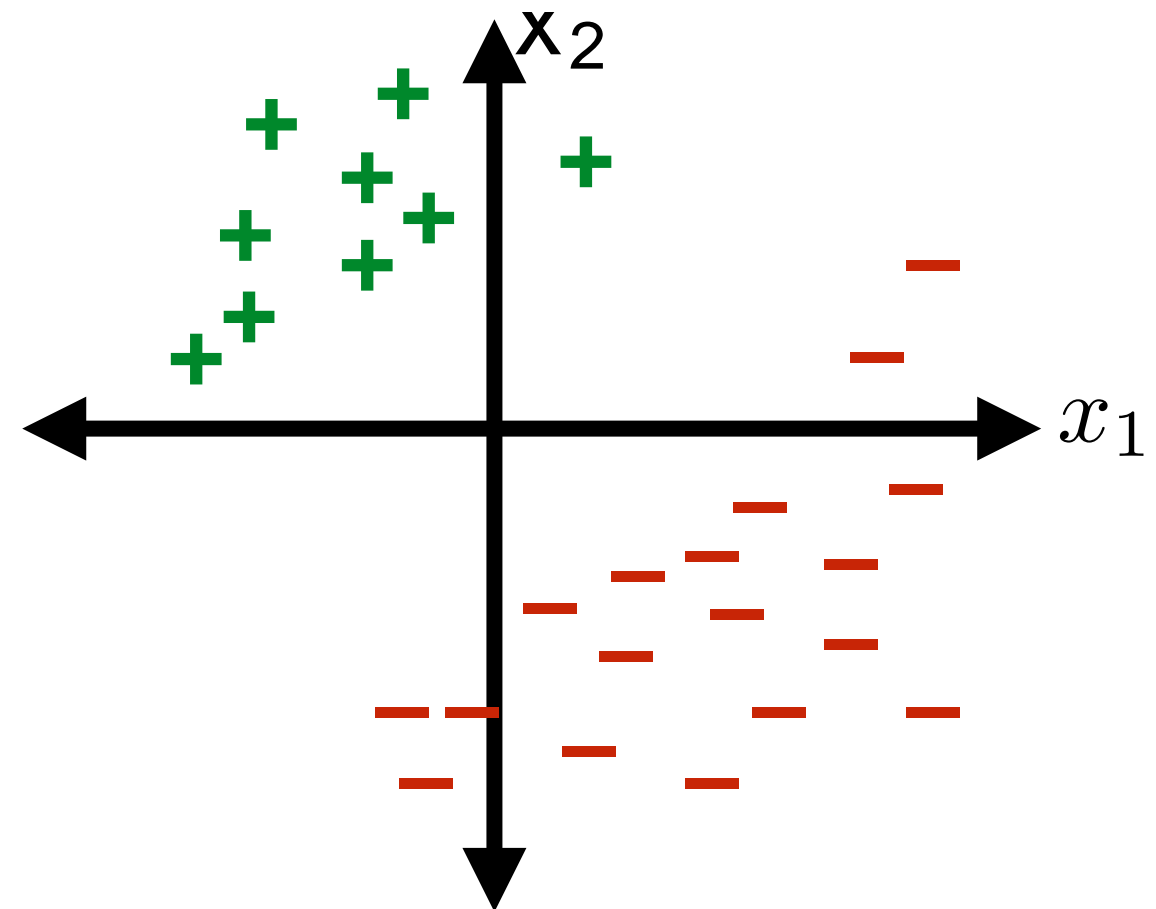




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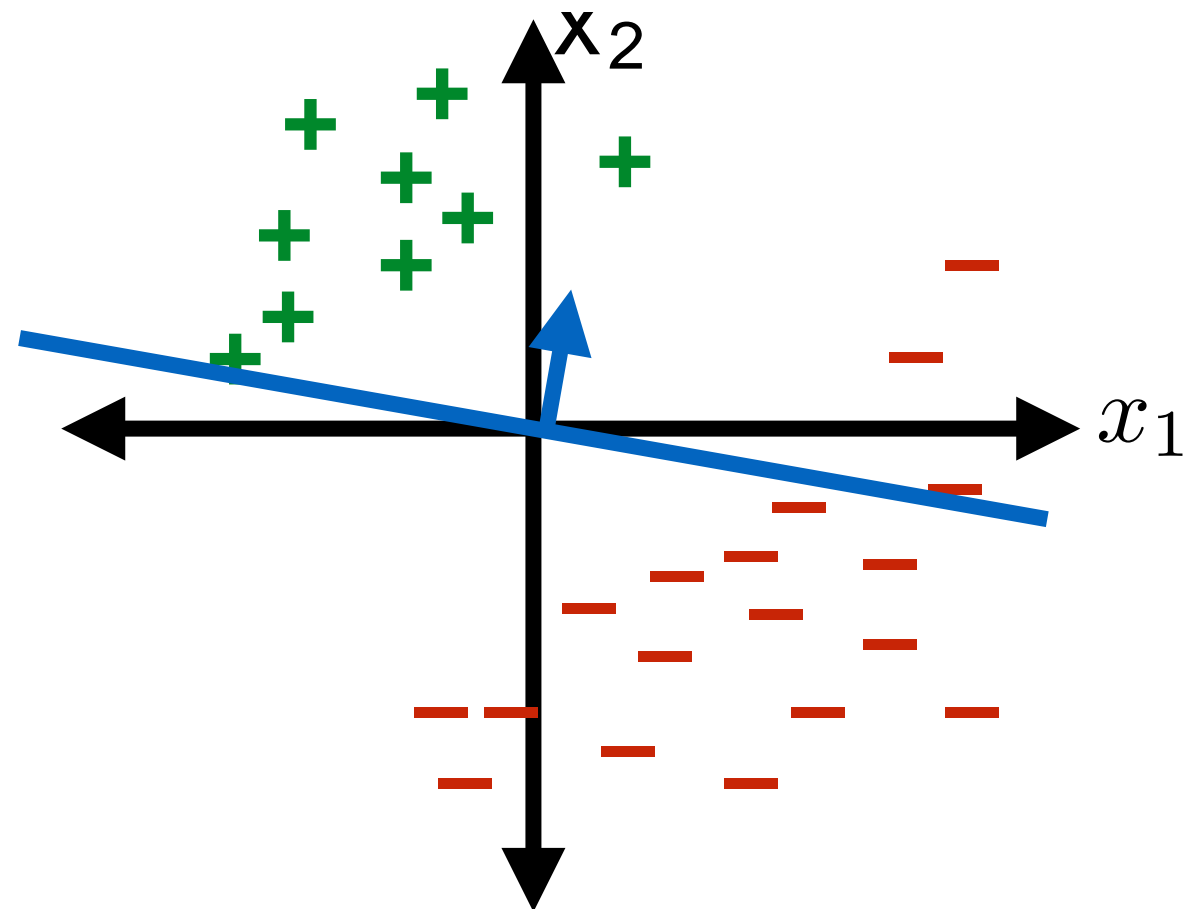
A. Our hypothesis class = classifiers with separating hyperplanes that pass through the origin (i.e.  $\theta_0 = 0$ )



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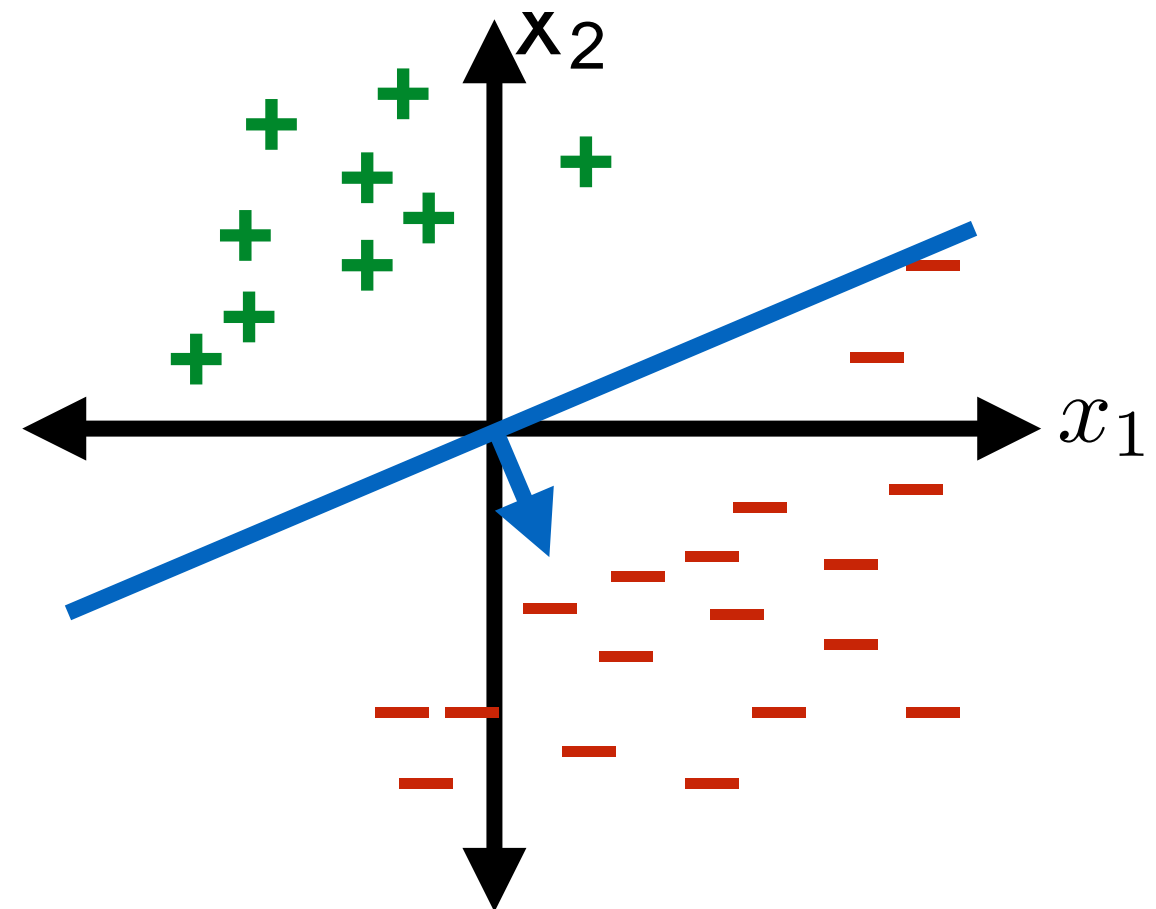
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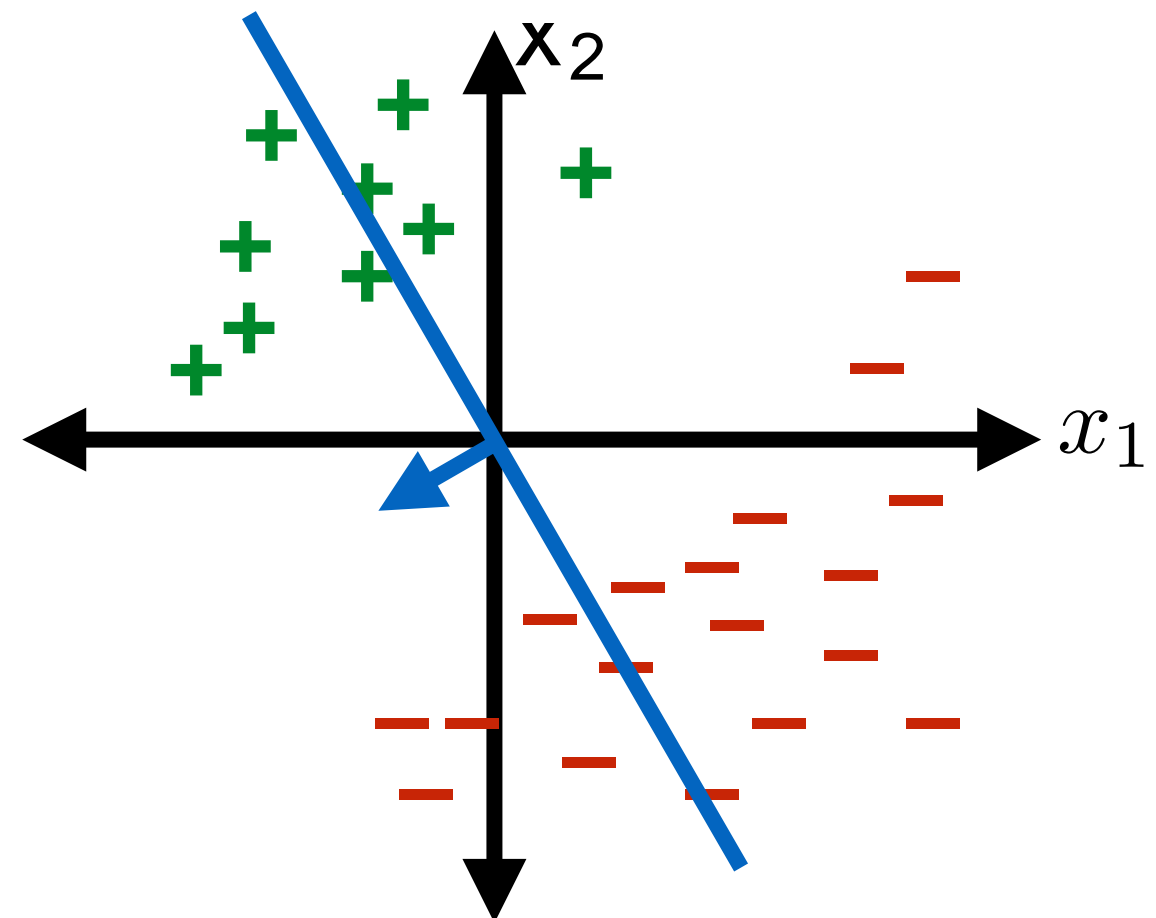
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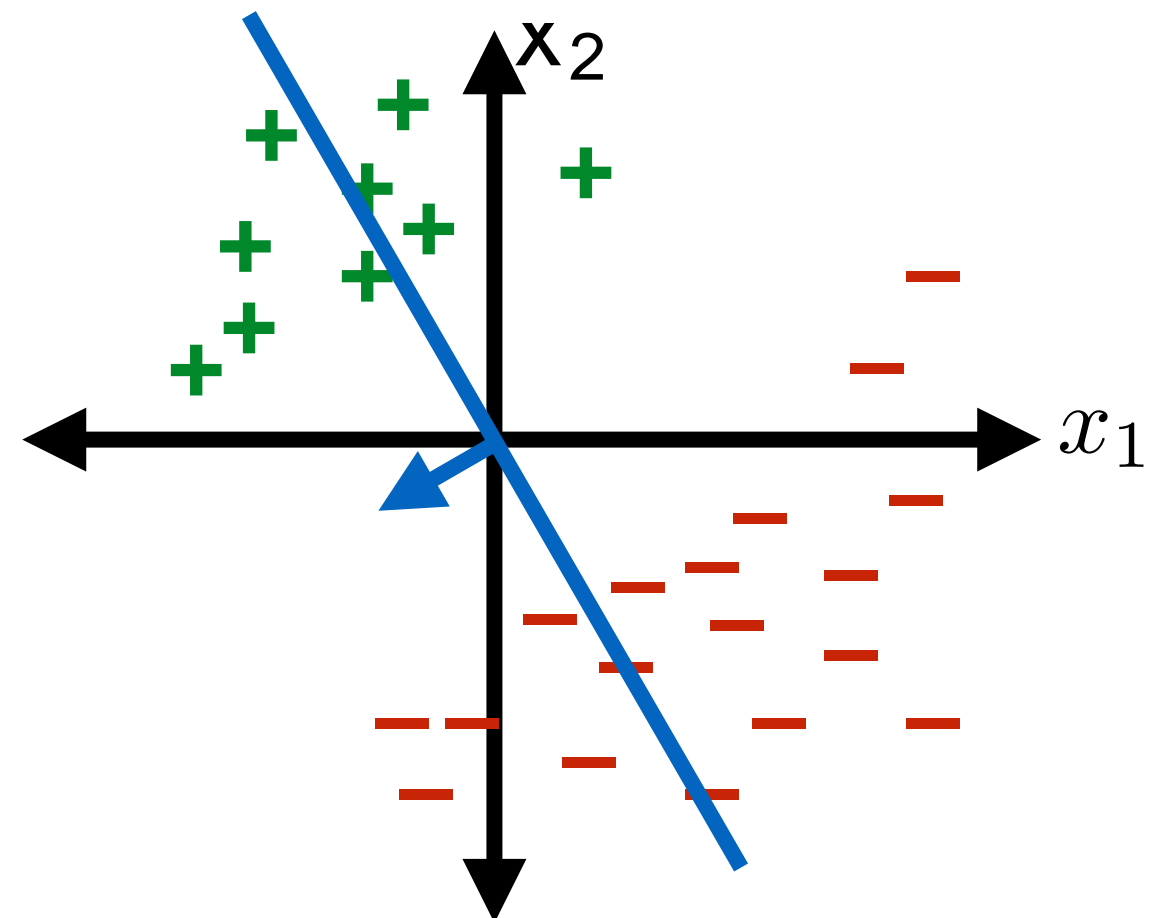


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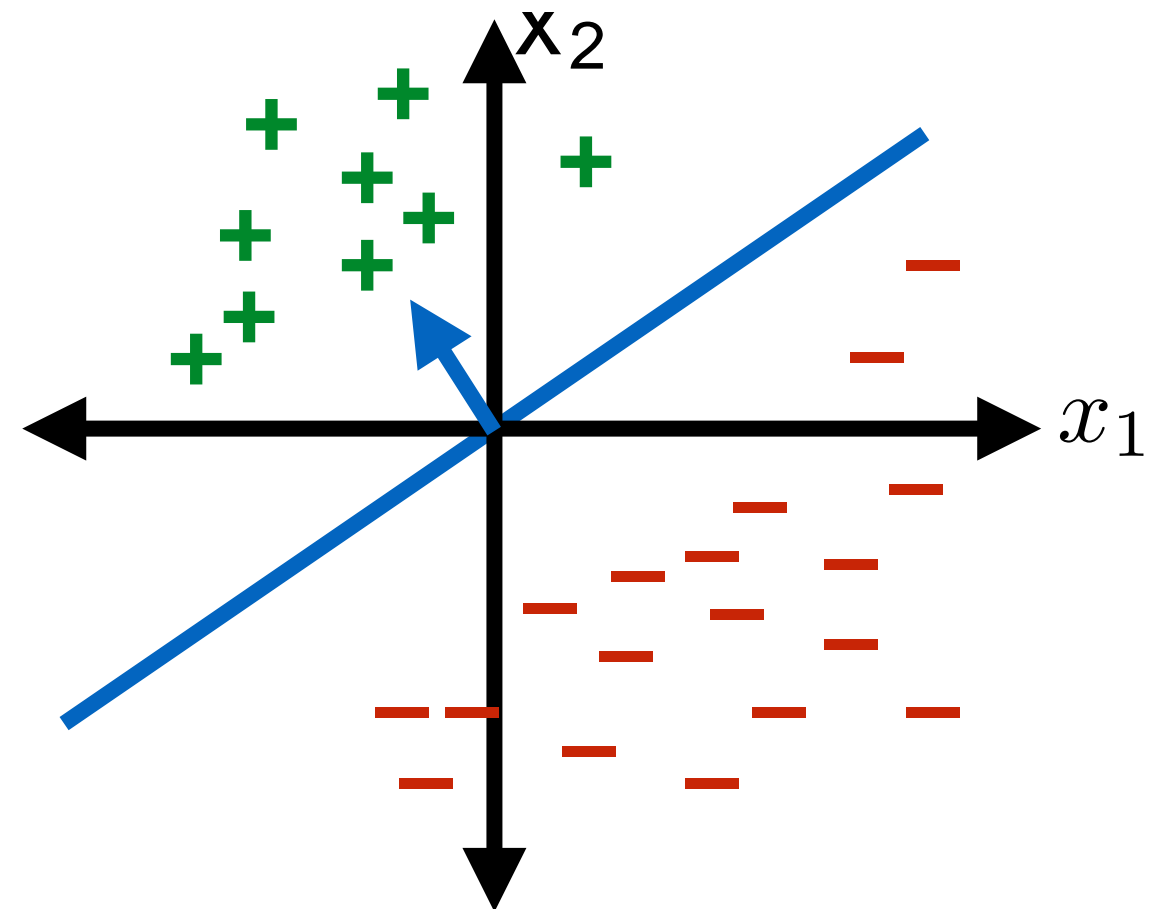


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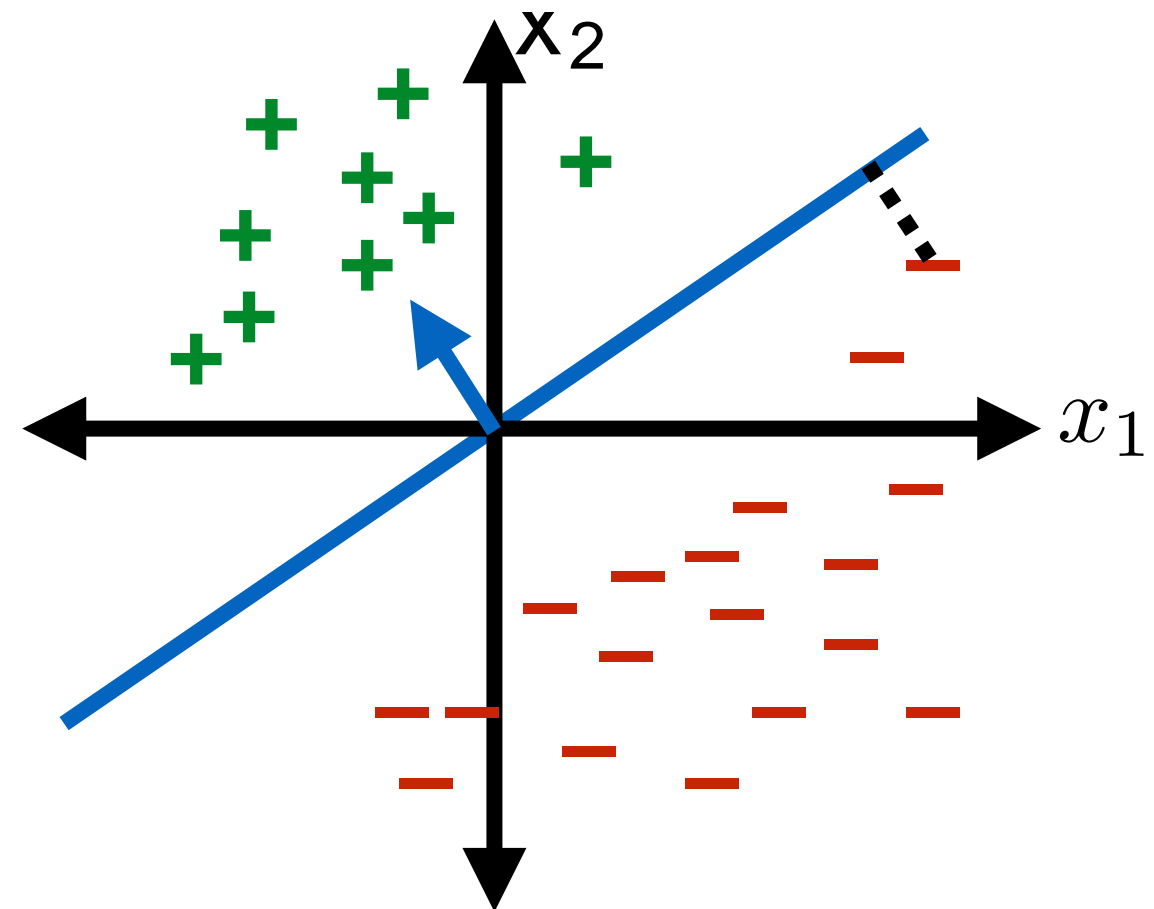


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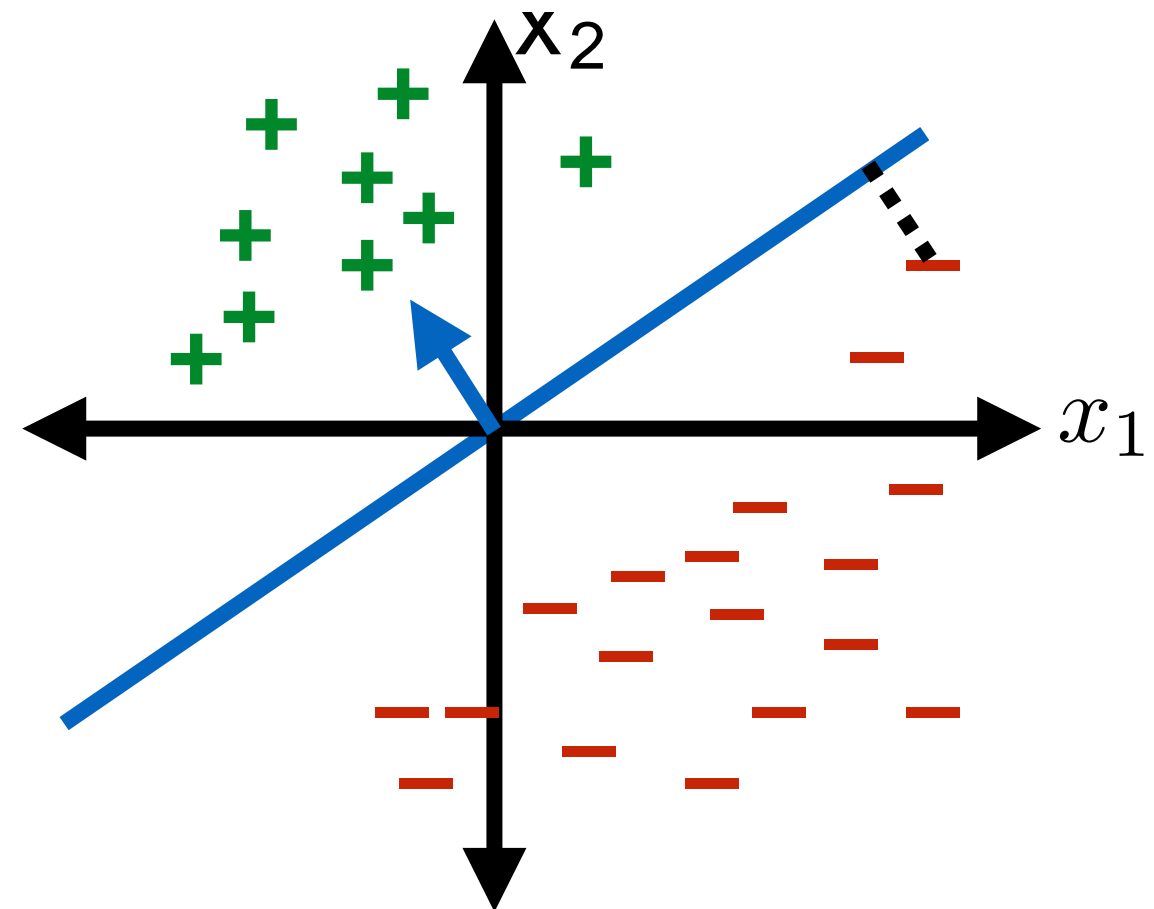
# Theorem: Perceptron Performance

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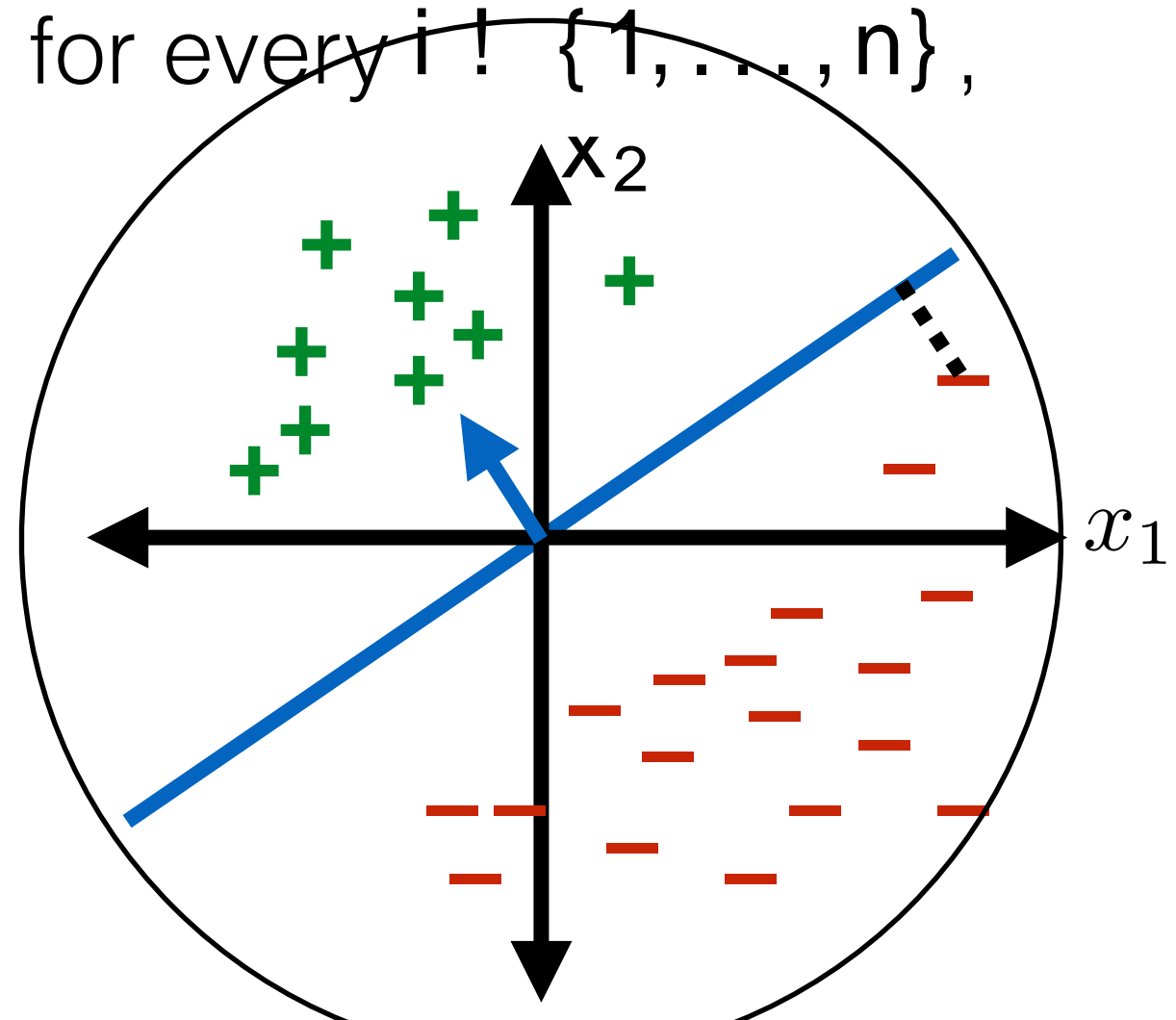
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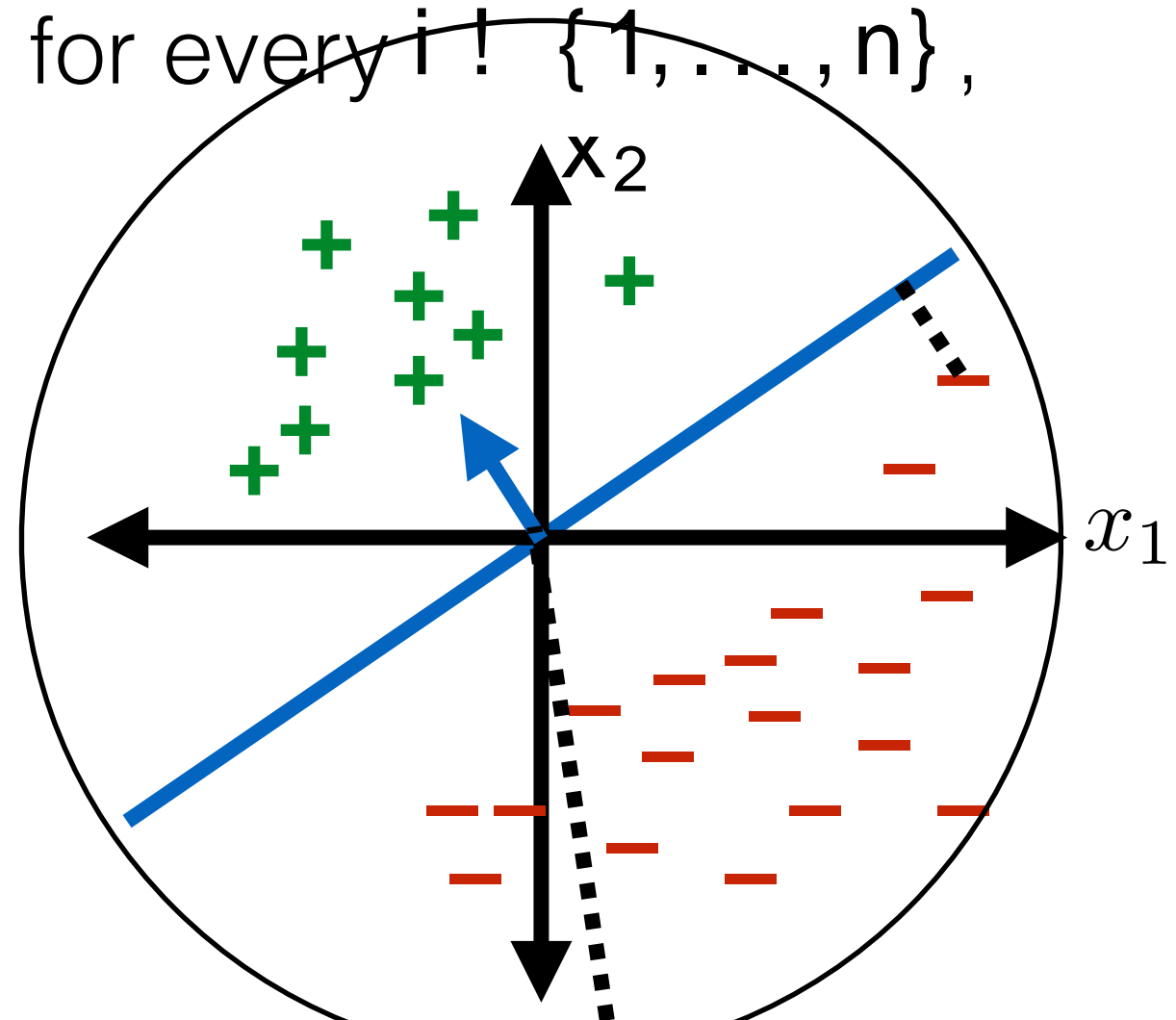
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# Theorem: Perceptron Performance

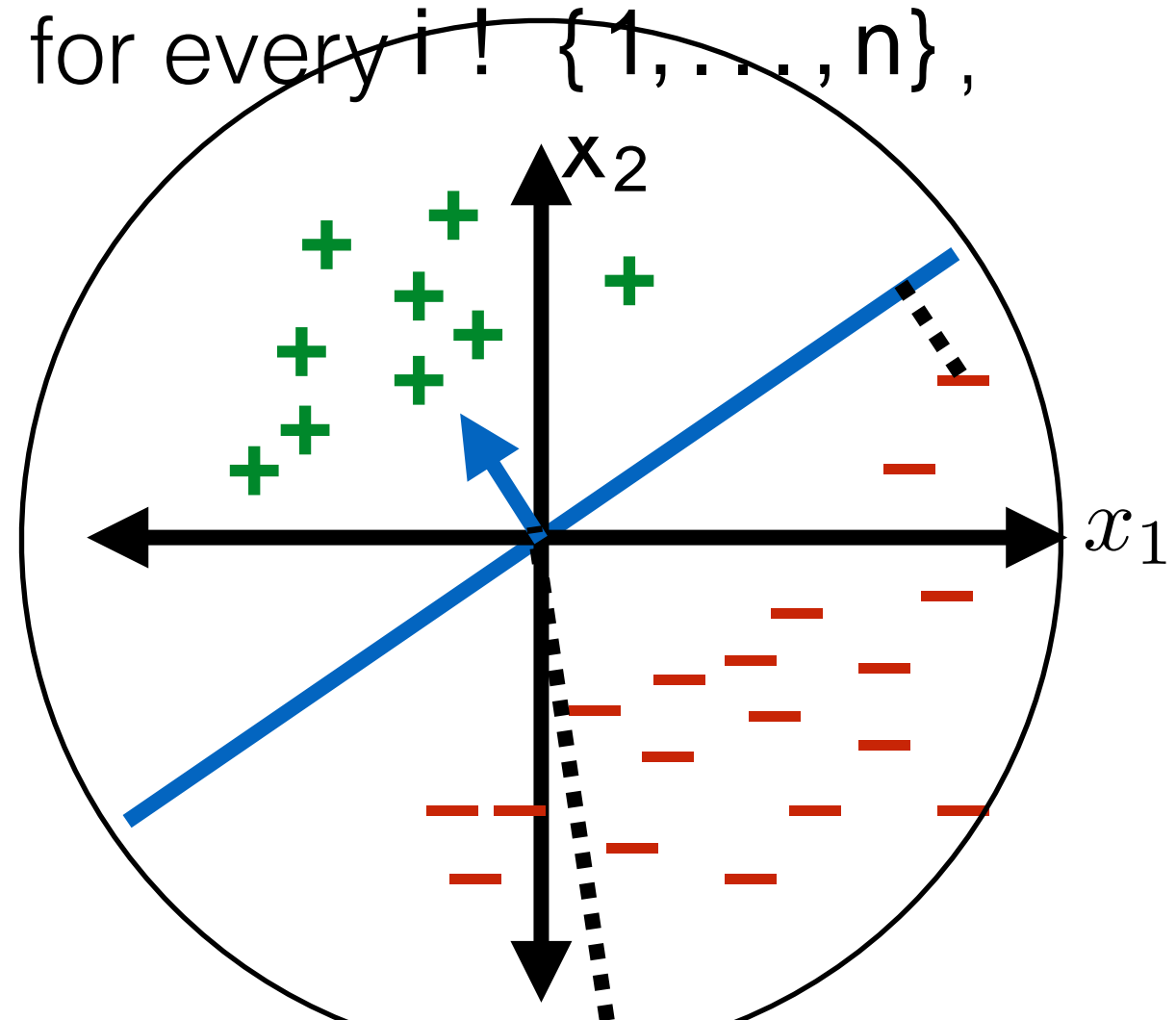
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- **Conclusion:** Then the perceptron algorithm will make at most  $(R/\gamma)^2$  updates to  $w$ . Once it goes through a pass of  $n$  without changes, the training error of its hypothesis will be 0.



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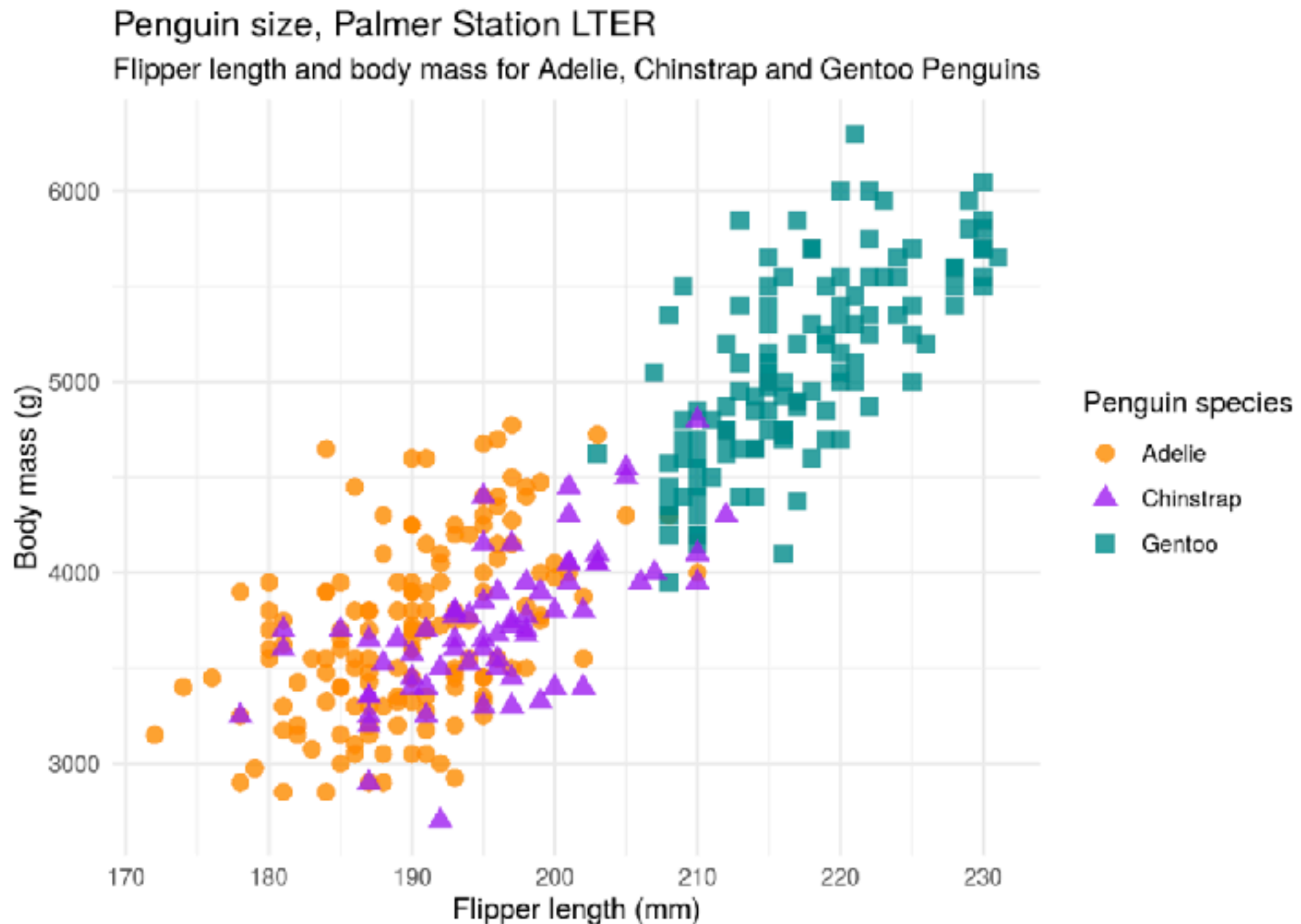
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- Can first convert to “expanded” feature space, then apply theorem

# Problem: data not linearly separable

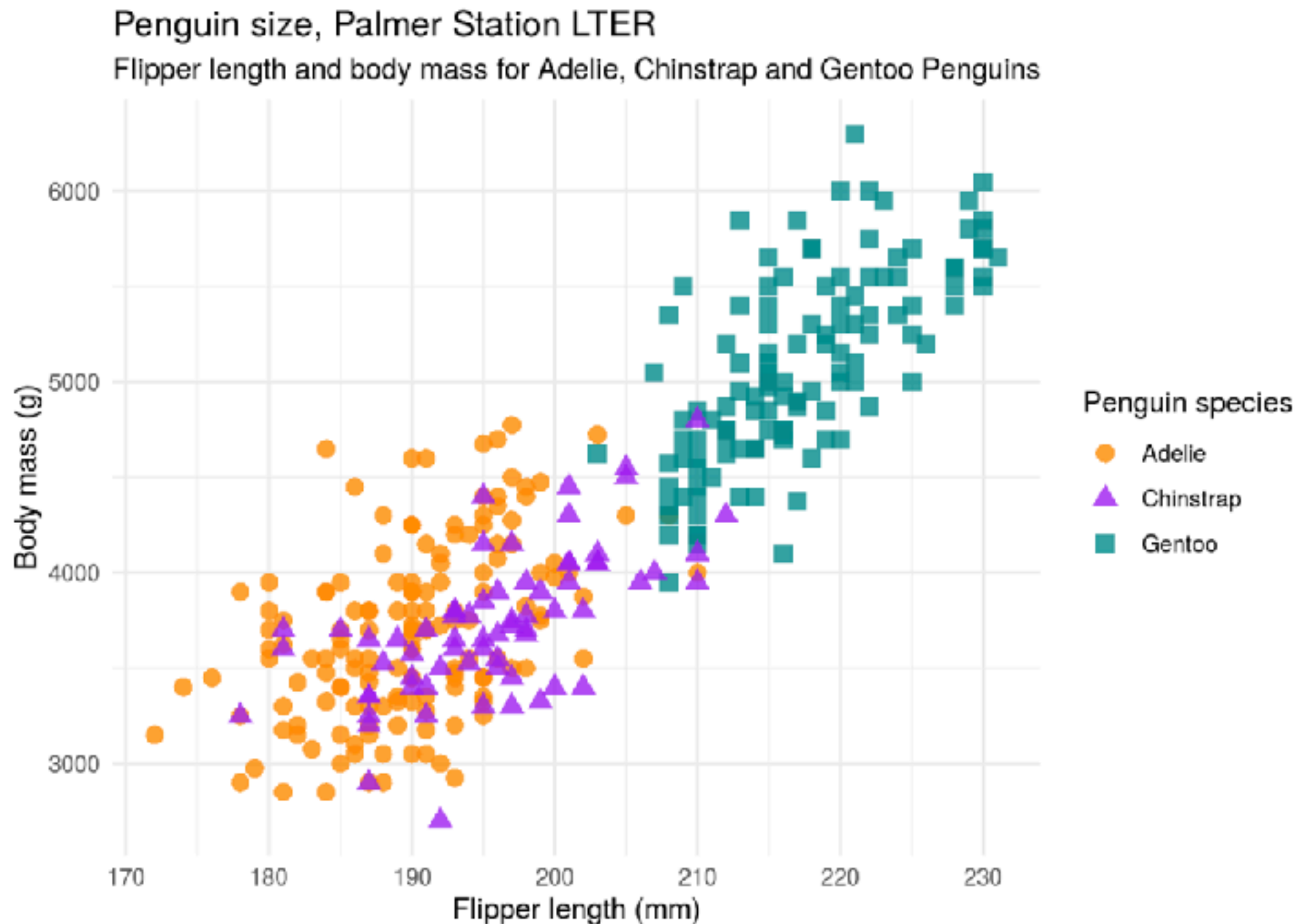
- Typical real data sets aren't linearly separable





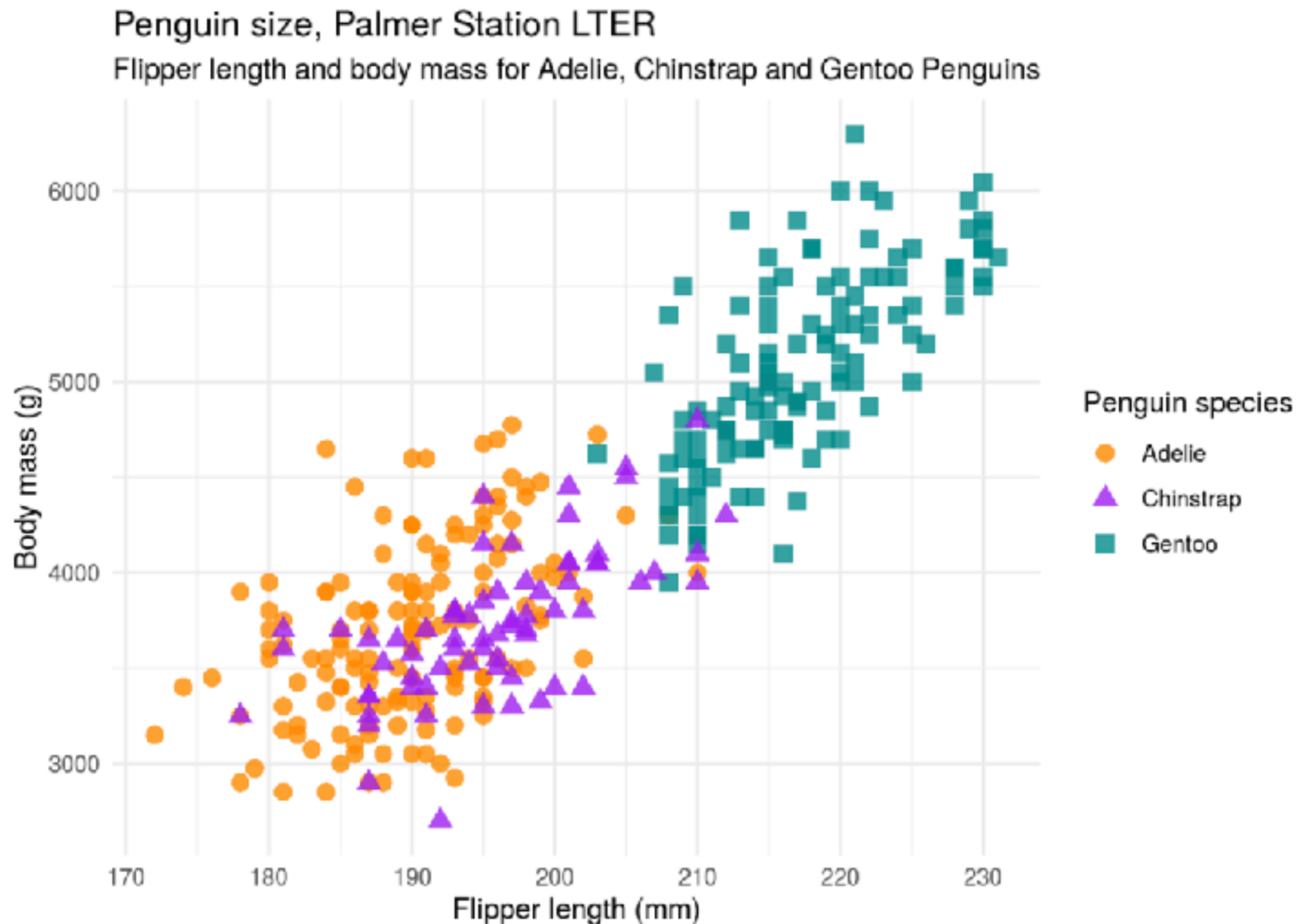
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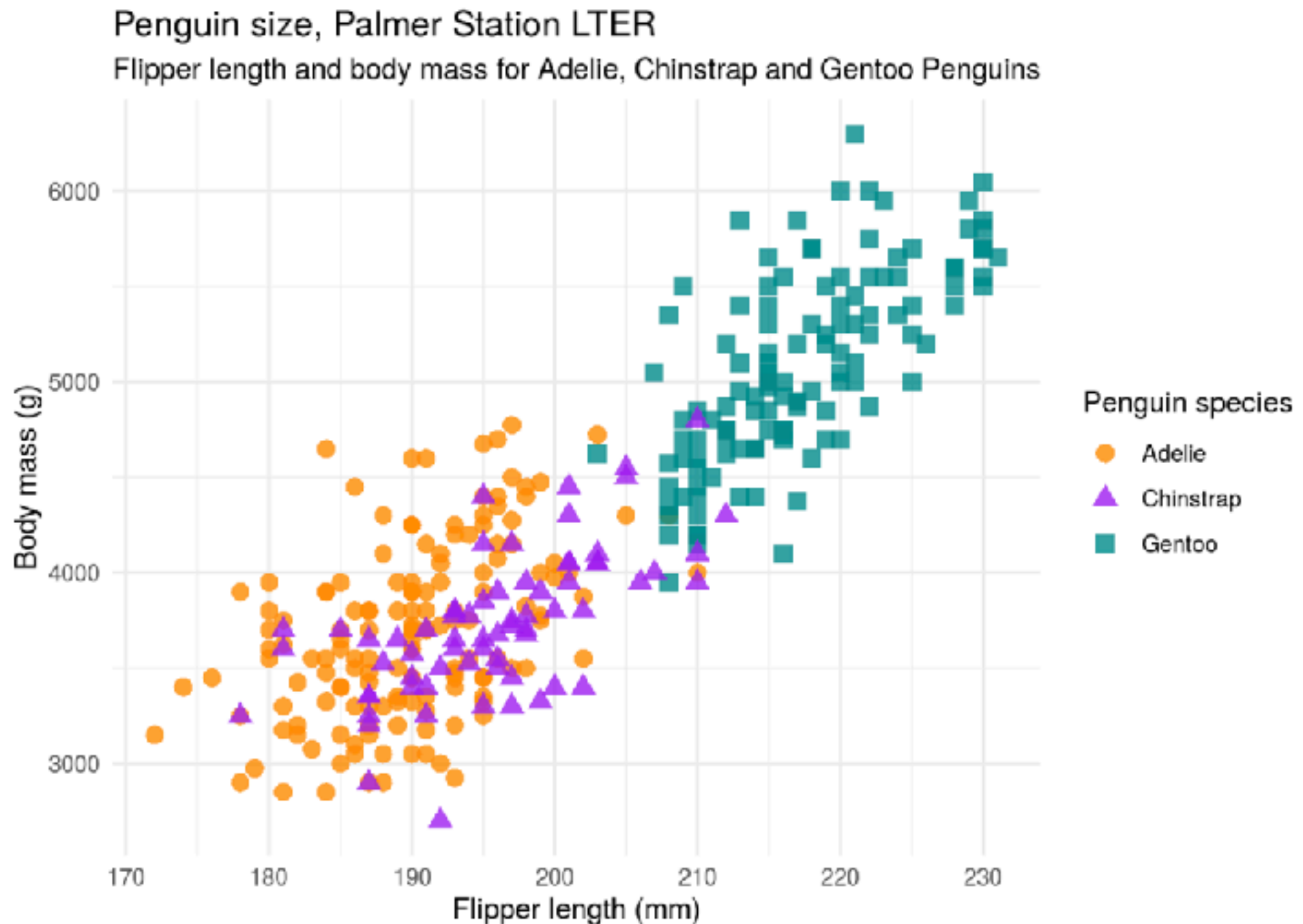
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- What can we do?

# Problem: data not linearly separable

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- What can we do? See upcoming lectures!

# Machine Learning Tasks

# Machine Learning Tasks

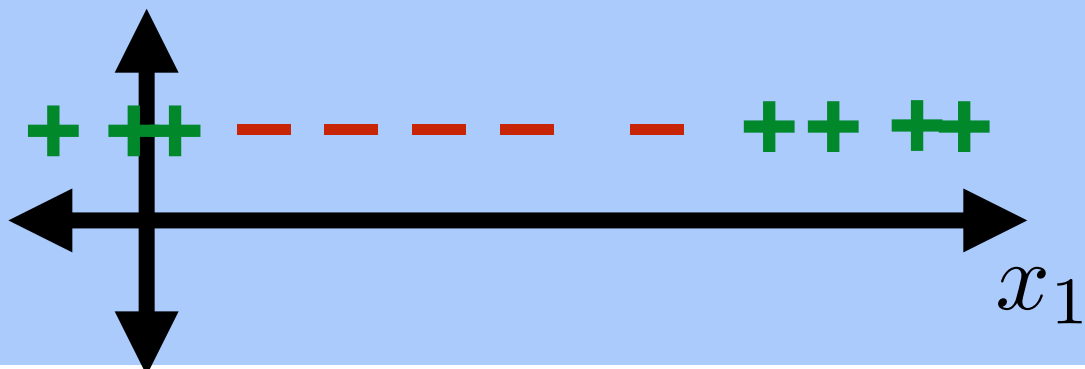
- **Binary/two-class classification**

# Machine Learning Tasks

- **Binary/two-class classification:**  
Learn a mapping:  $\mathbb{R}^d \rightarrow \{-1, +1\}$

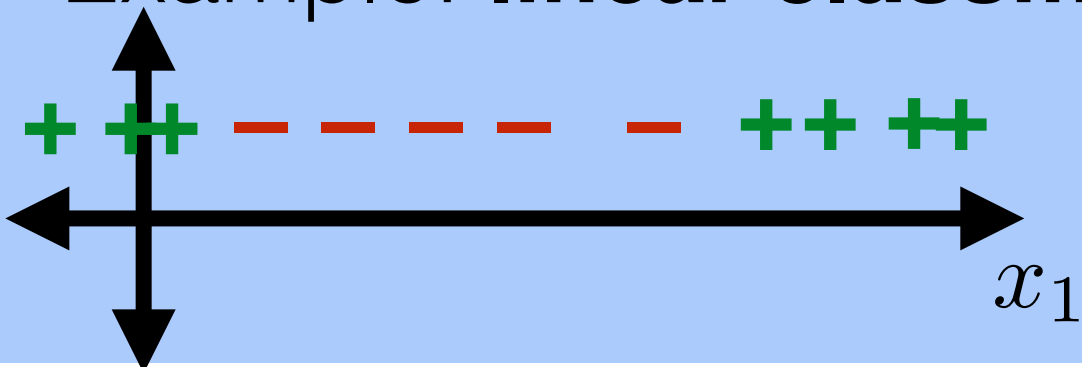
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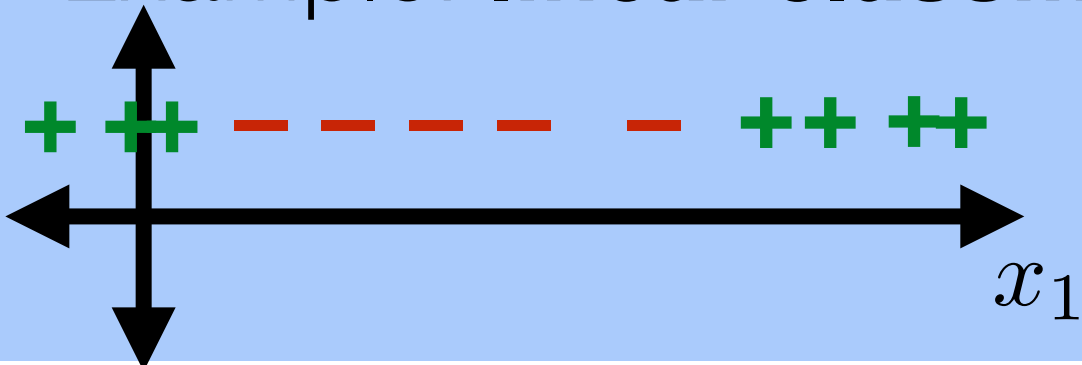
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# Machine Learning Tasks

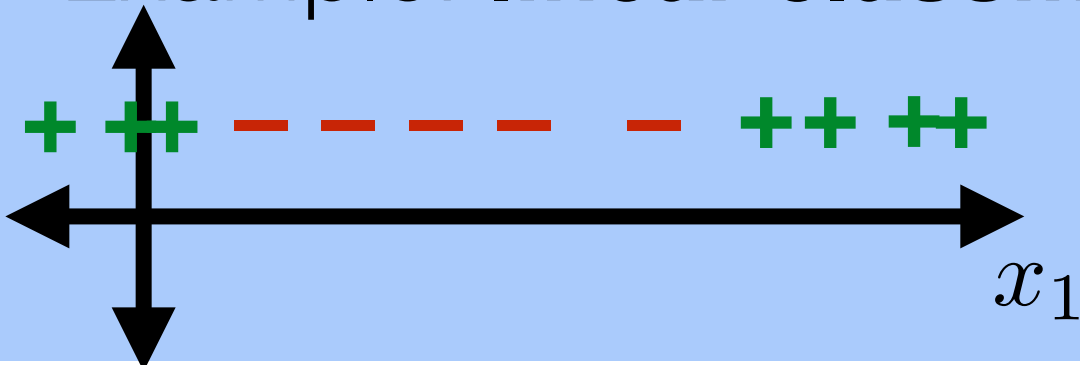
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- **Multi-class classification:**

# Machine Learning Tasks

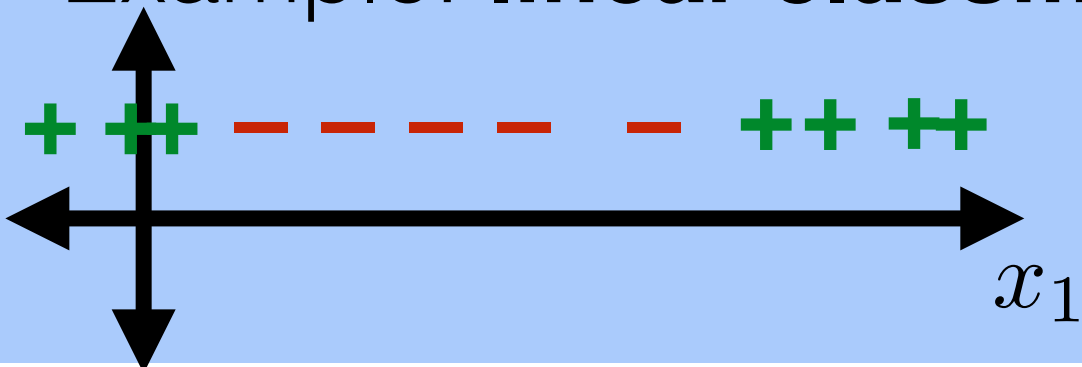
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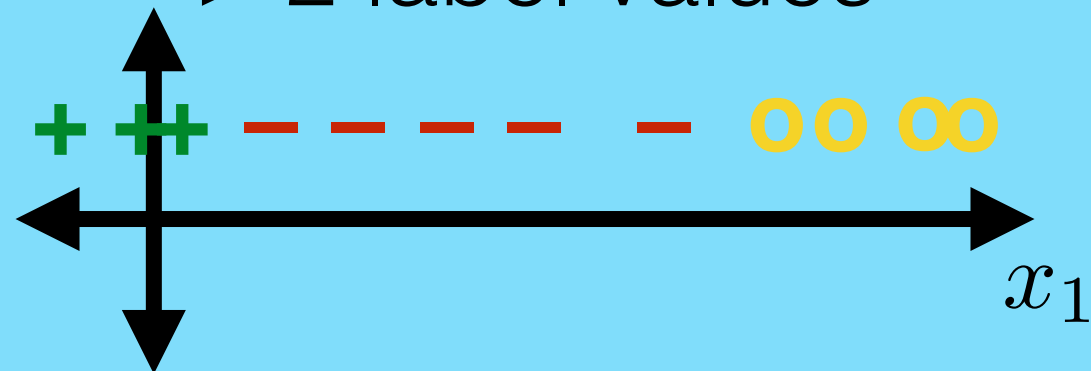
- **Multi-class classification:**  
> 2 label values

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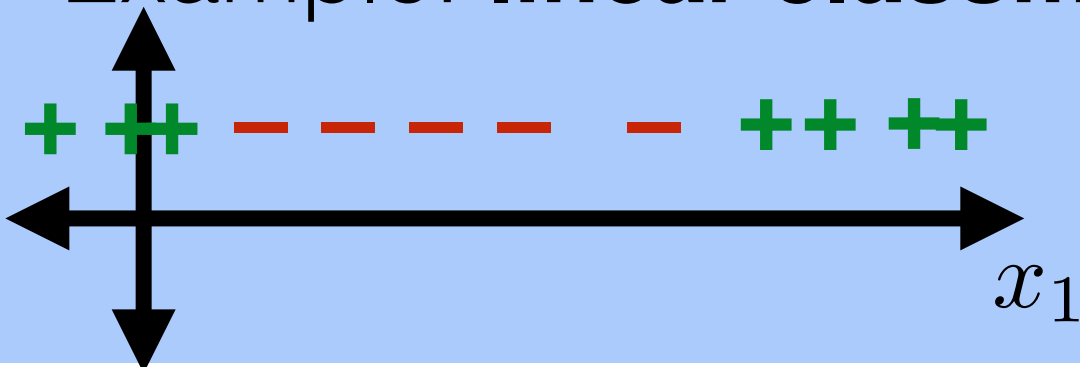


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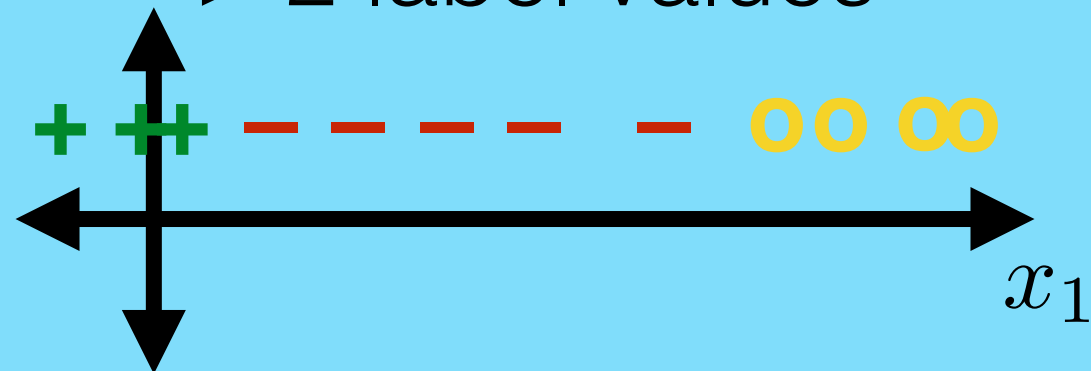


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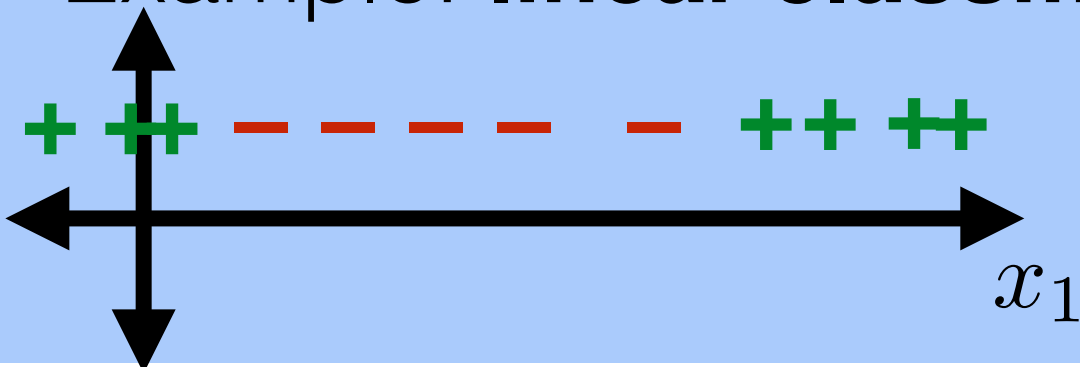


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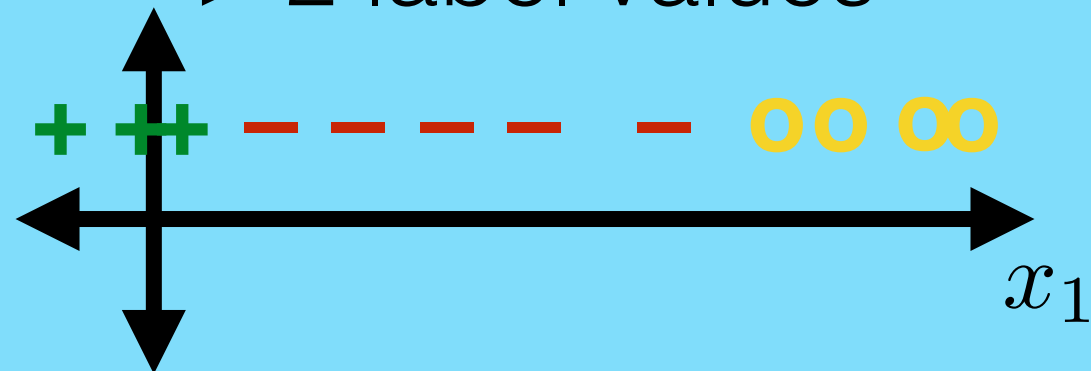
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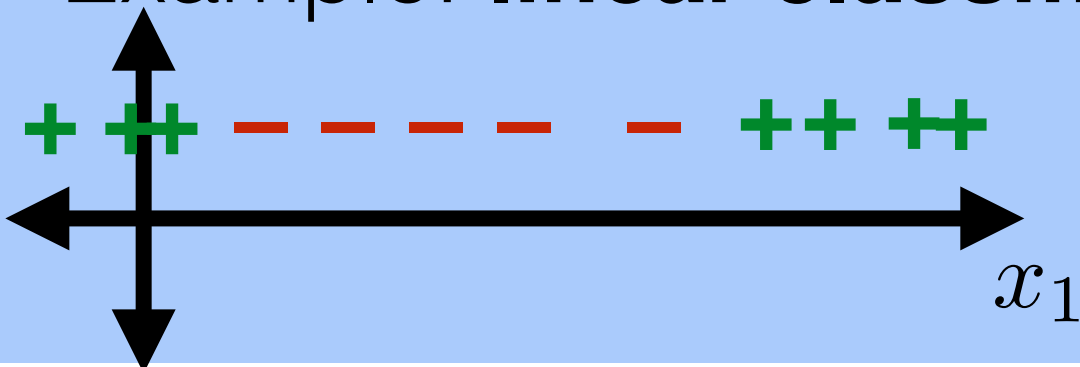
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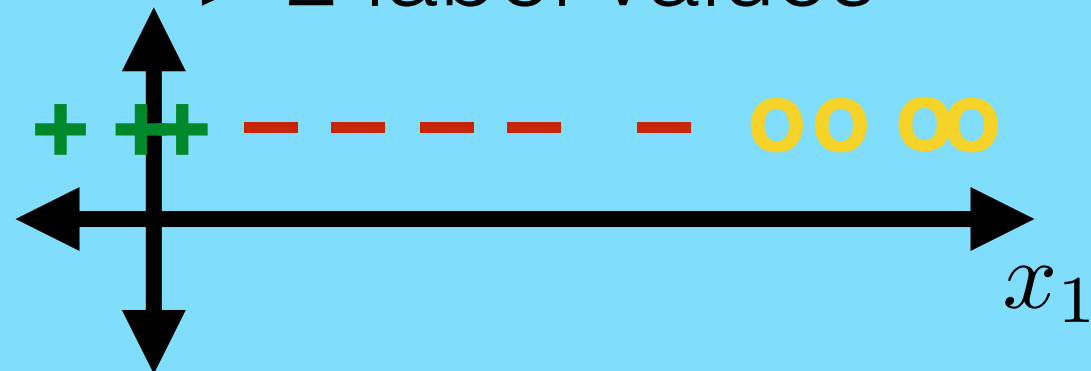
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- **Classification:**  
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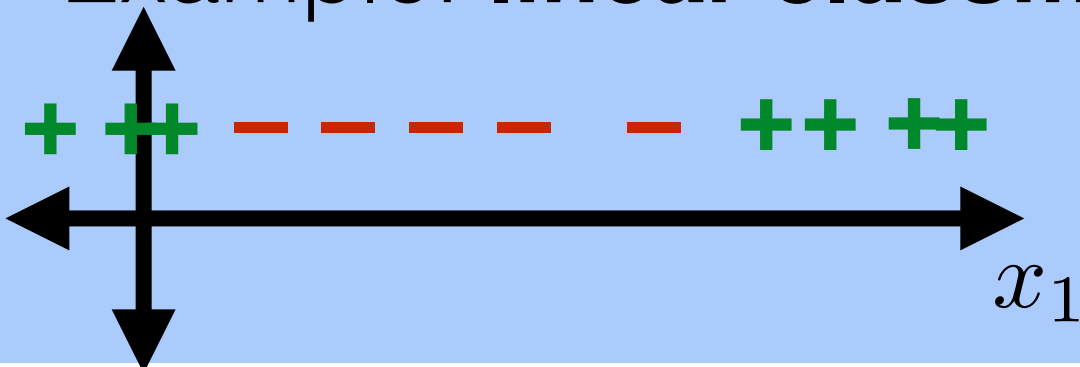


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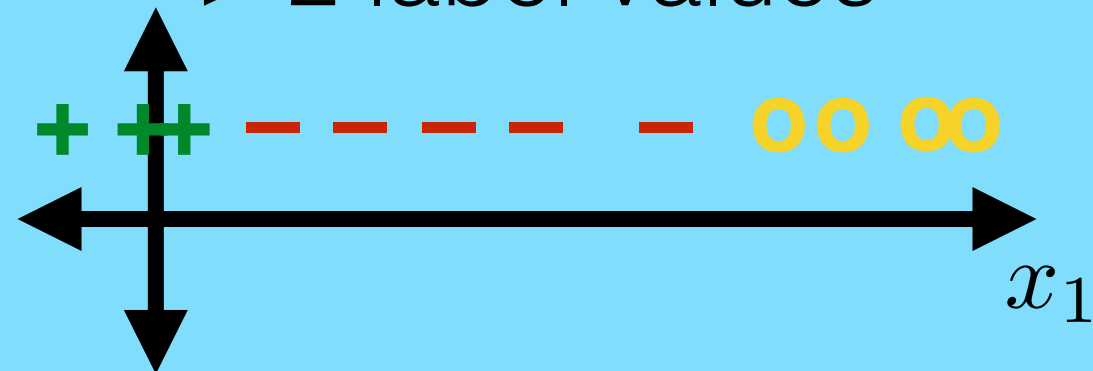
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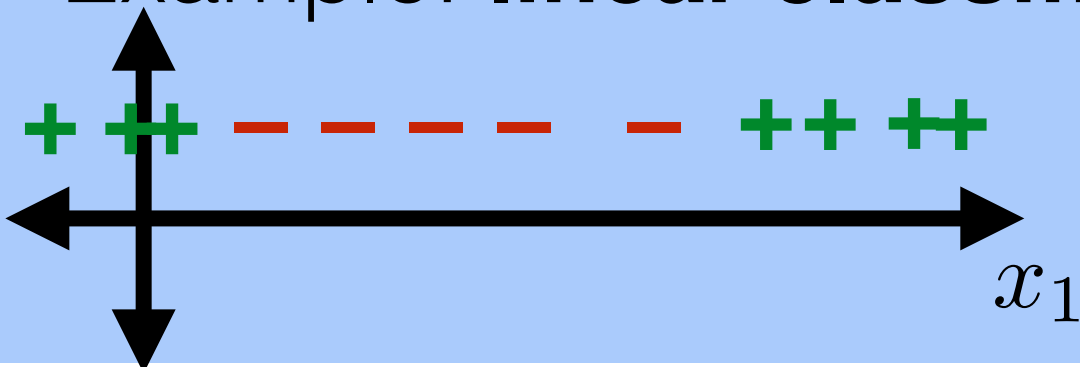


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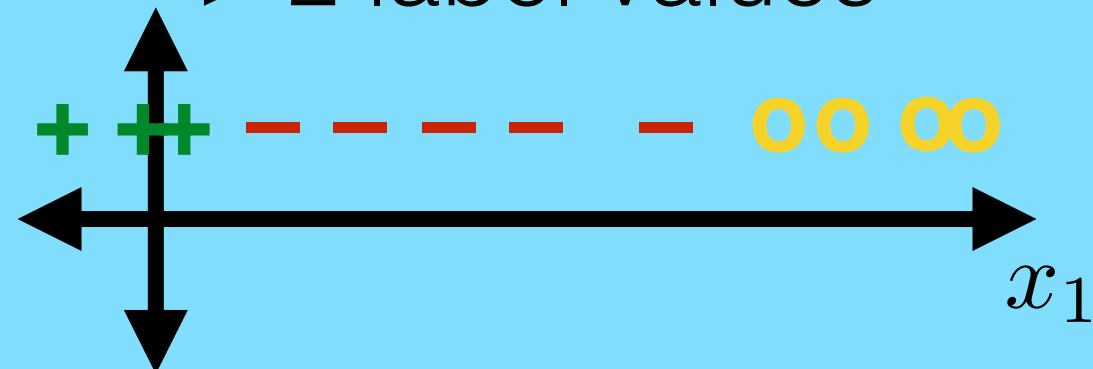
- **Regression:** Learn a mapping to continuous values:  $\mathbb{R}^d \rightarrow \mathbb{R}^k$

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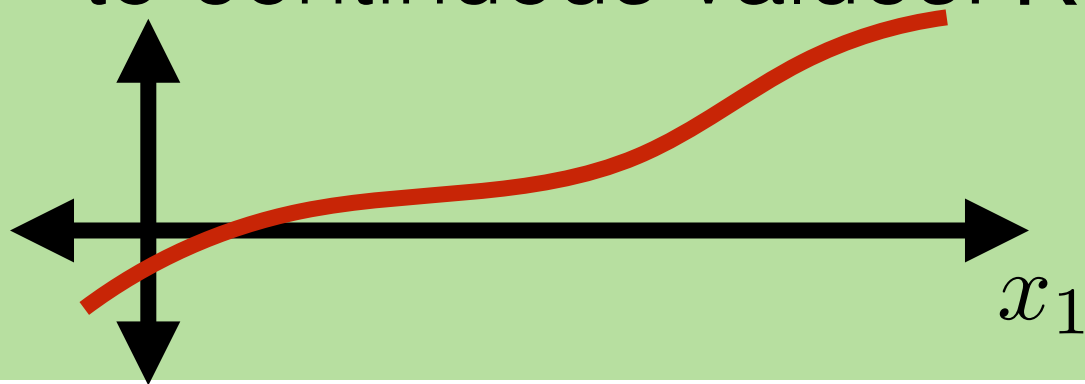
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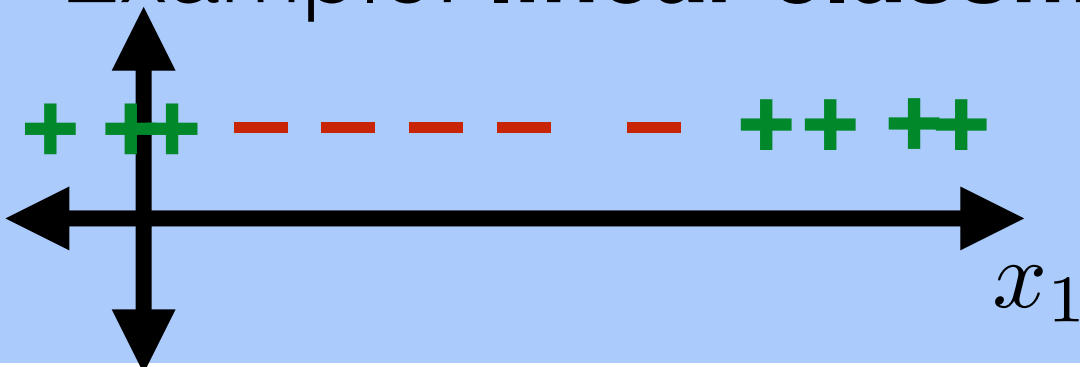
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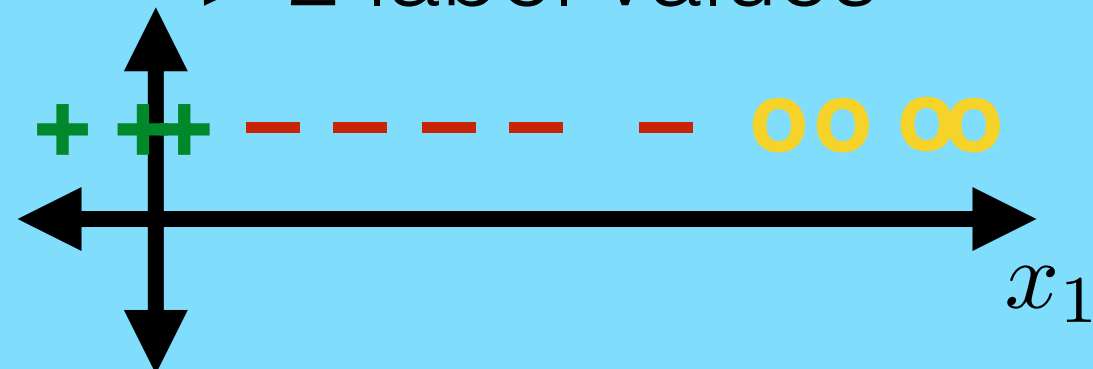
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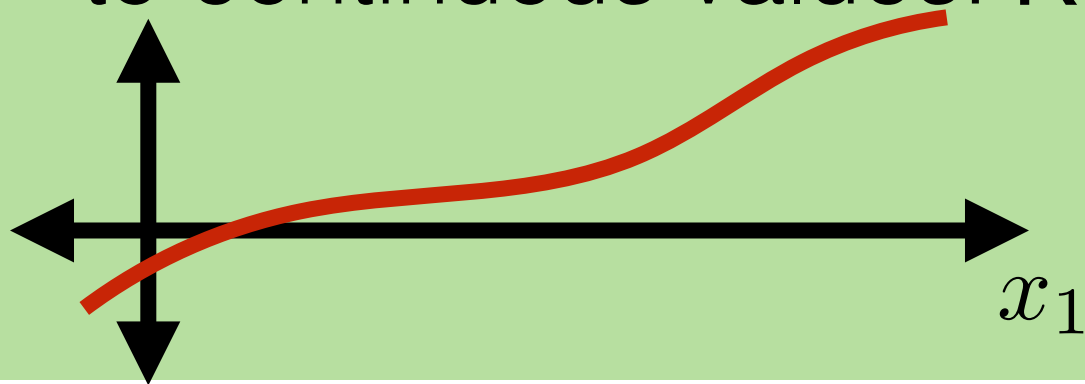
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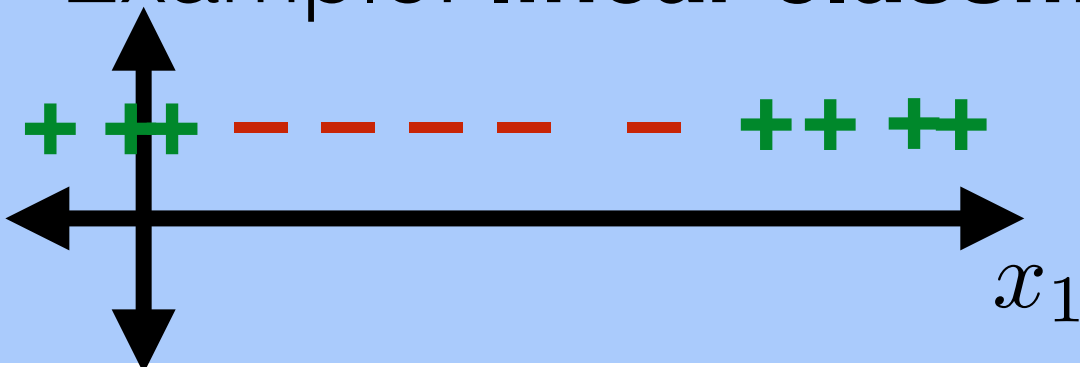
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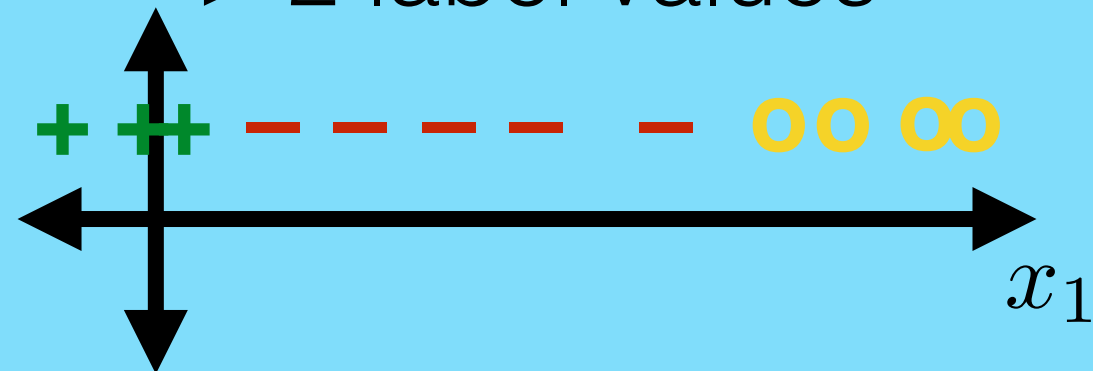
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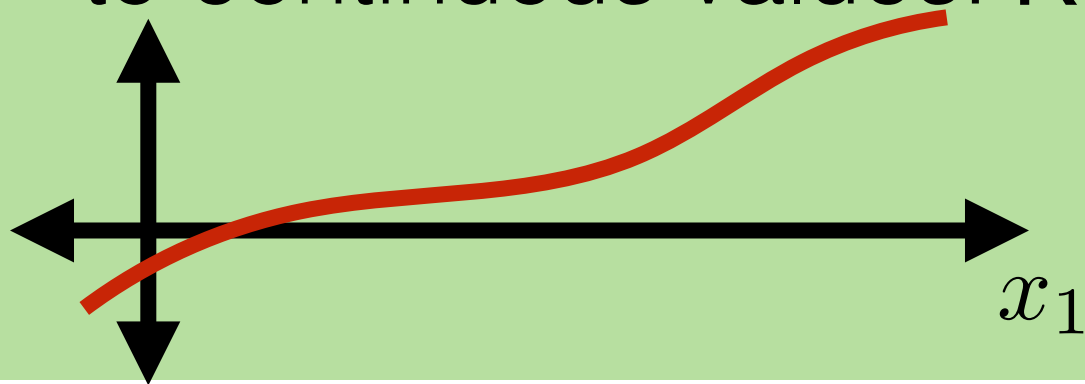
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# Machine Learning Tasks

- **Supervised learning**

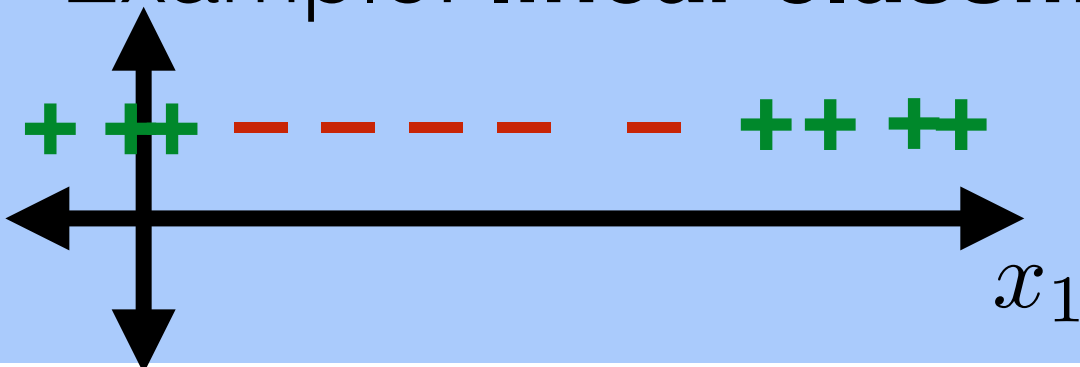
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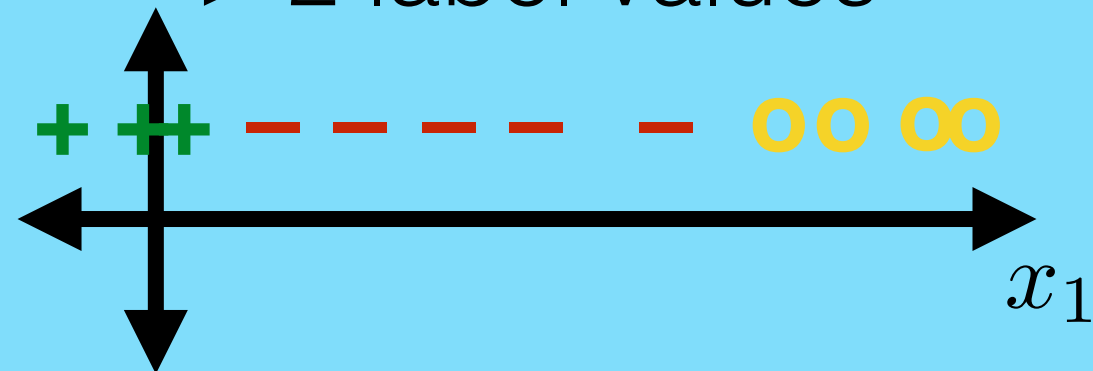
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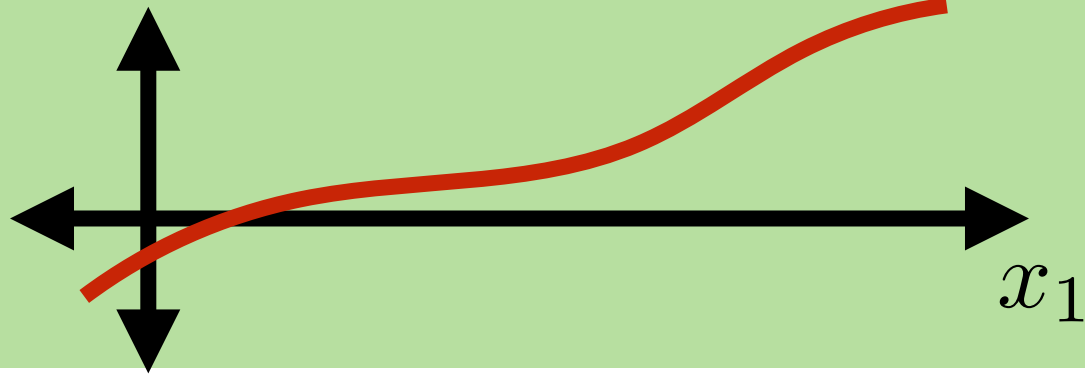
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# Machine Learning Tasks

- **Supervised learning:** Learn a mapping from features to labels

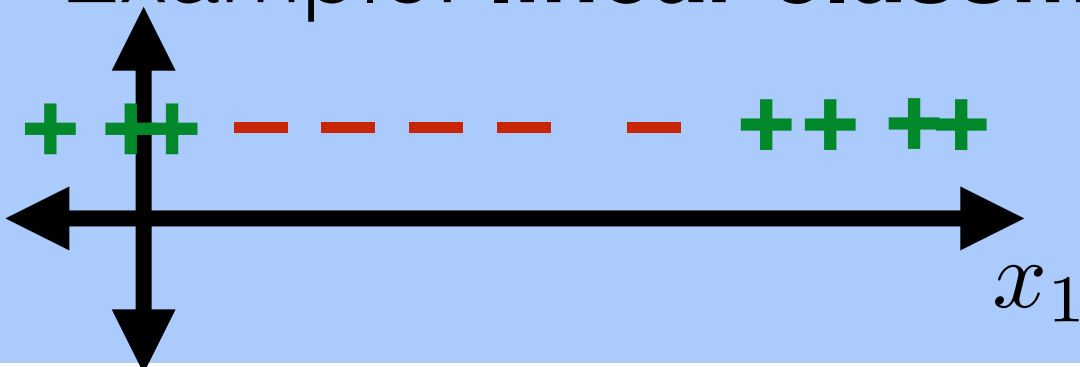
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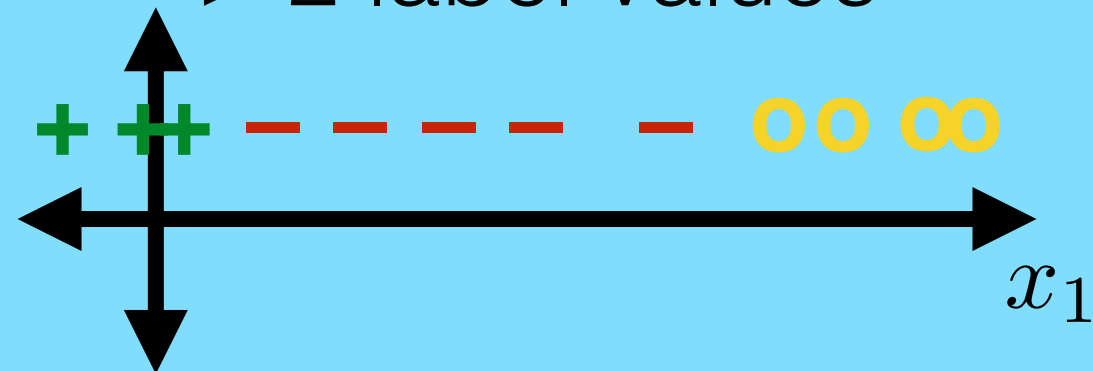
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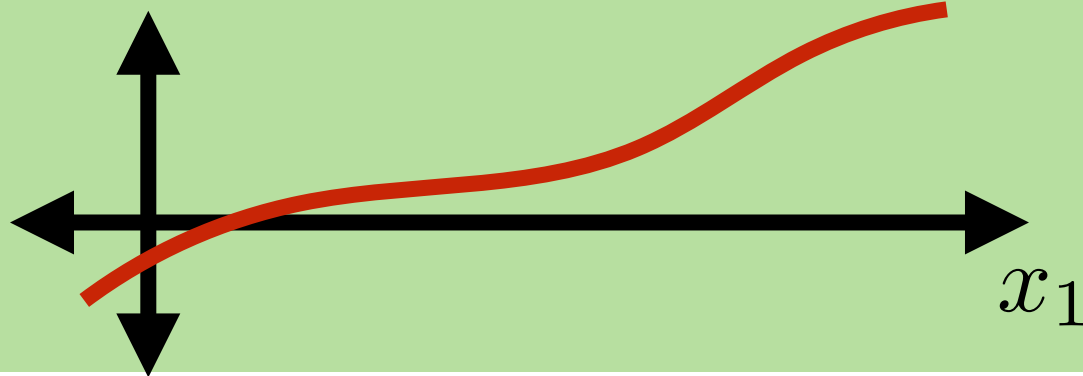


# Machine Learning Tasks

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- **Unsupervised learning**

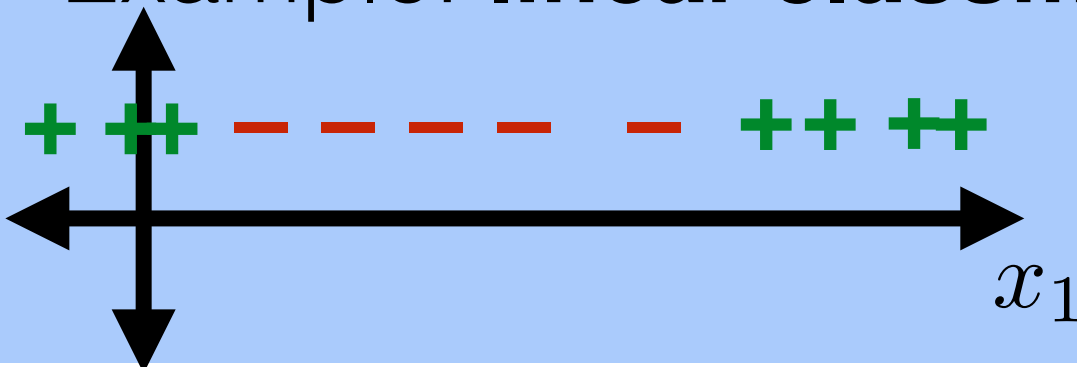
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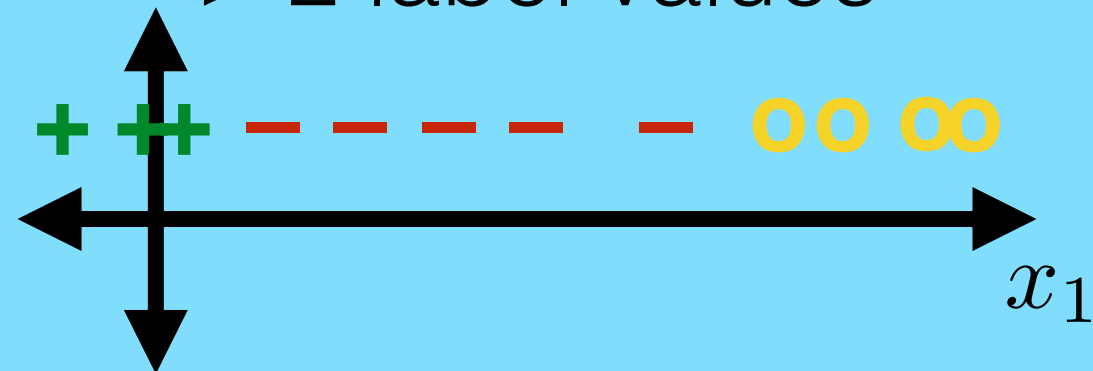
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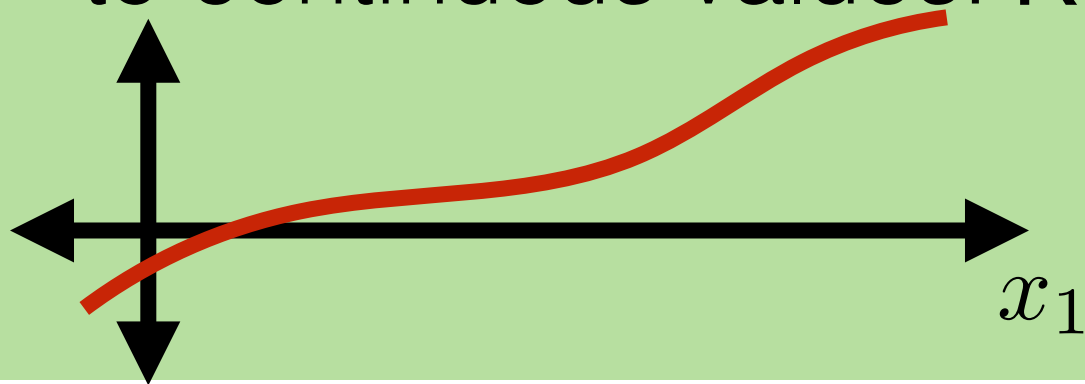


# Machine Learning Tasks

- **Supervised learning:** Learn a mapping from features to labels

- **Unsupervised learning:** No labels; find patterns

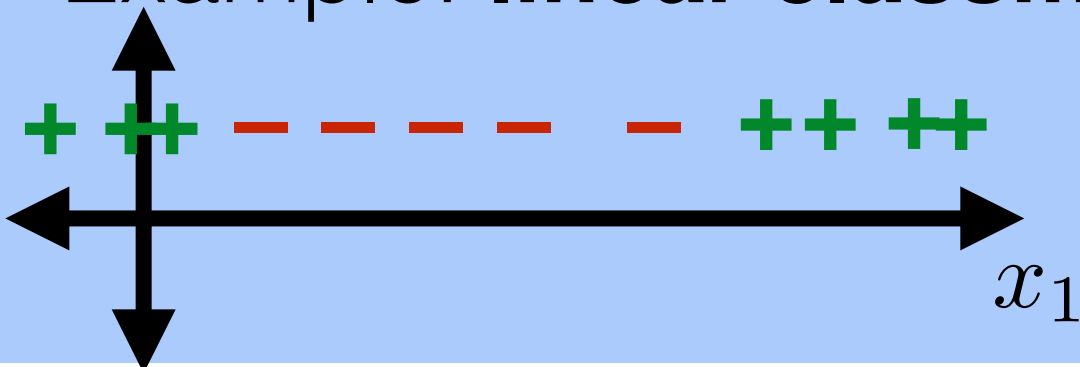
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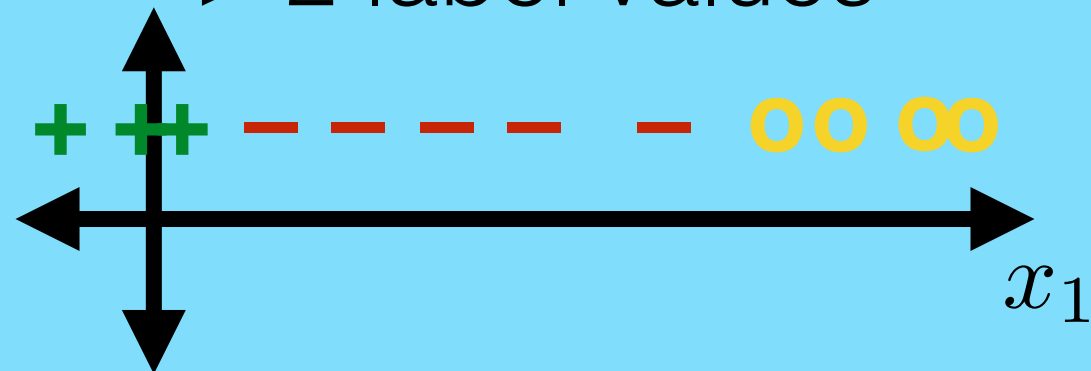
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