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# Logistic Regression

Background: Generative and  
Discriminative Classifiers

# Logistic Regression

Important analytic tool in natural and social sciences

Baseline supervised machine learning tool for classification

Is also the foundation of neural networks

# Generative and Discriminative Classifiers

Naive Bayes is a **generative** classifier

by contrast:

Logistic regression is a **discriminative** classifier

# Generative and Discriminative Classifiers

Suppose we're distinguishing cat from dog images



imagenet



imagenet



# Generative Classifier:

- Build a model of what's in a cat image
  - Knows about whiskers, ears, eyes
  - Assigns a probability to any image:
    - how cat-y is this image?



Also build a model for dog images

Now given a new image:

**Run both models and see which one fits better**

# Discriminative Classifier

Just try to distinguish dogs from cats



Oh look, dogs have collars!  
Let's ignore everything else

# Finding the correct class $c$ from a document $d$ in Generative vs Discriminative Classifiers

## Naive Bayes

$$\hat{c} = \operatorname{argmax}_{c \in C} \underbrace{P(d|c)}_{\text{likelihood}} \underbrace{P(c)}_{\text{prior}}$$

## Logistic Regression

$$\hat{c} = \operatorname{argmax}_{c \in C} \underbrace{P(c|d)}_{\text{posterior}}$$



# Components of a probabilistic machine learning classifier

Given  $m$  input/output pairs  $(x^{(i)}, y^{(i)})$ :

1. A **feature representation** of the input. For each input observation  $x^{(i)}$ , a vector of features  $[x_1, x_2, \dots, x_n]$ . Feature  $j$  for input  $x^{(i)}$  is  $x_j$ , more completely  $x_j^{(i)}$ , or sometimes  $f_j(x)$ .
2. A **classification function** that computes  $\hat{y}$ , the estimated class, via  $p(y|x)$ , like the **sigmoid** or **softmax** functions.
3. An objective function for learning, like **cross-entropy loss**.
4. An algorithm for optimizing the objective function: **stochastic gradient descent**.

# The two phases of logistic regression

**Training:** we learn weights  $w$  and  $b$  using **stochastic gradient descent** and **cross-entropy loss**.

**Test:** Given a test example  $x$  we compute  $p(y|x)$  using learned weights  $w$  and  $b$ , and return whichever label ( $y = 1$  or  $y = 0$ ) is higher probability

# Logistic Regression

Background: Generative and  
Discriminative Classifiers

# Classification in Logistic Regression

Logistic  
Regression

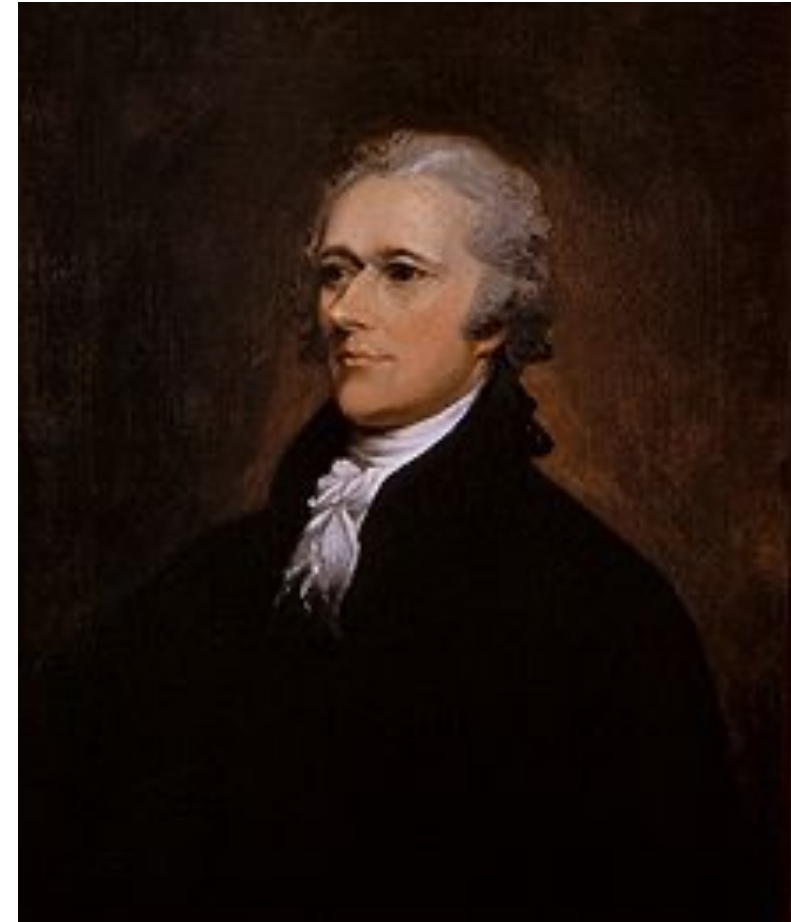


# Classification Reminder

Positive/negative sentiment

Spam/not spam

Authorship attribution  
(Hamilton or Madison?)



Alexander Hamilton

# Text Classification: definition

*Input:*

- a document  $x$
- a fixed set of classes  $C = \{c_1, c_2, \dots, c_J\}$

*Output:* a predicted class  $\hat{y} \in C$

# Binary Classification in Logistic Regression

Given a series of input/output pairs:

- $(x^{(i)}, y^{(i)})$

For each observation  $x^{(i)}$

- We represent  $x^{(i)}$  by a **feature vector**  $[x_1, x_2, \dots, x_n]$
- We compute an output: a predicted class  $\hat{y}^{(i)} \in \{0, 1\}$

# Features in logistic regression

- For feature  $x_i$ , weight  $w_i$  tells is how important is  $x_i$ 
  - $x_i$  = "review contains 'awesome'":  $w_i = +10$
  - $x_j$  = "review contains 'abysmal'":  $w_j = -10$
  - $x_k$  = "review contains 'mediocre'":  $w_k = -2$



# Logistic Regression for one observation $x$

Input observation: vector  $x = [x_1, x_2, \dots, x_n]$

Weights: one per feature:  $W = [w_1, w_2, \dots, w_n]$

- Sometimes we call the weights  $\theta = [\theta_1, \theta_2, \dots, \theta_n]$

Output: a predicted class  $\hat{y} \in \{0, 1\}$

(multinomial logistic regression:  $\hat{y} \in \{0, 1, 2, 3, 4\}$ )

# How to do classification

For each feature  $x_i$ , weight  $w_i$  tells us importance of  $x_i$

- (Plus we'll have a bias  $b$ )

We'll sum up all the weighted features and the bias

$$z = \left( \sum_{i=1}^n w_i x_i \right) + b$$

$$z = w \cdot x + b$$

If this sum is high, we say  $y=1$ ; if low, then  $y=0$

But we want a probabilistic classifier

We need to formalize “sum is high”.

We’d like a principled classifier that gives us a probability, just like Naive Bayes did

We want a model that can tell us:

$$p(y=1 | x; \theta)$$

$$p(y=0 | x; \theta)$$

The problem:  $z$  isn't a probability, it's just a number!

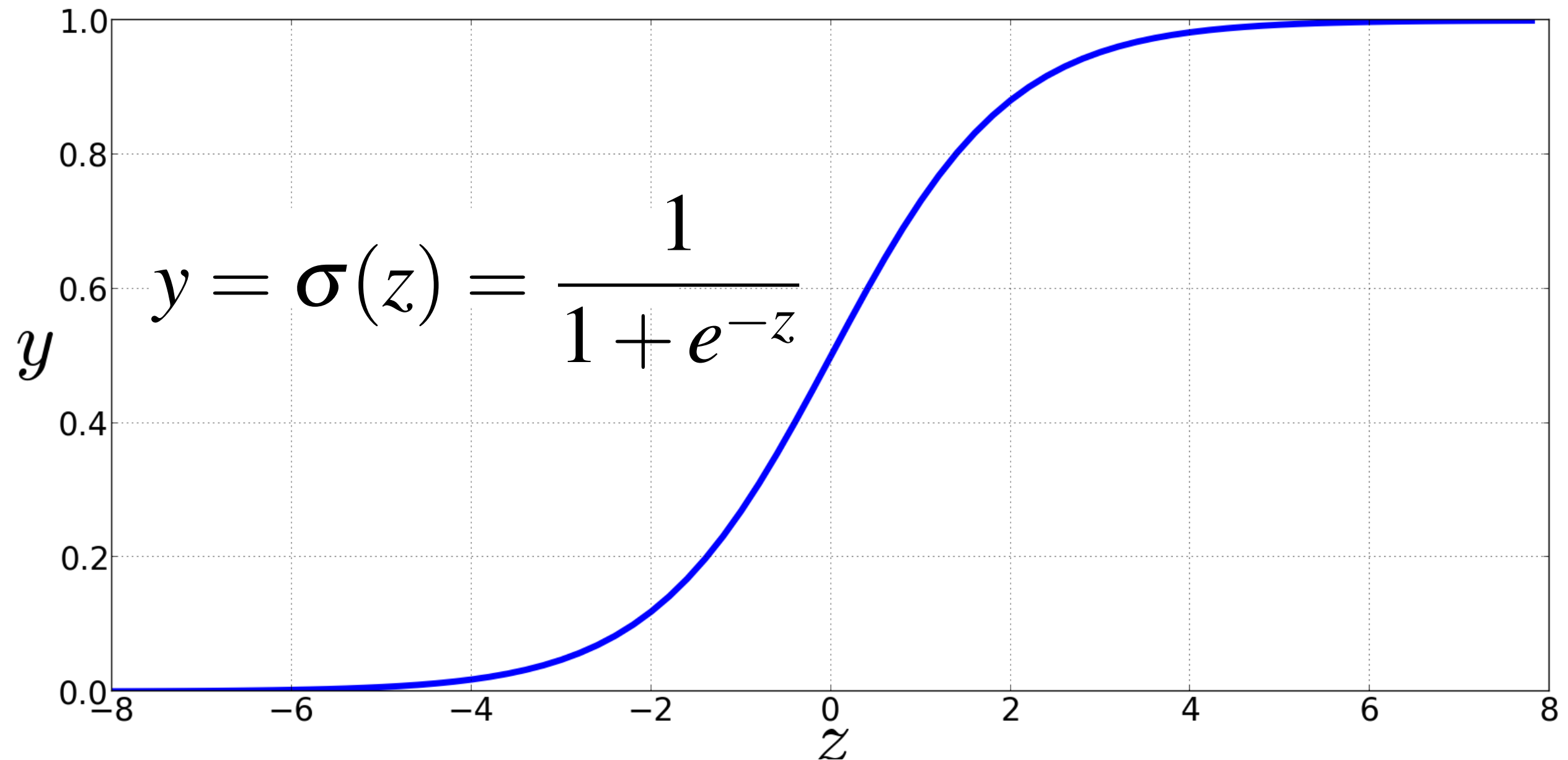
$$z = w \cdot x + b$$

Solution: use a function of  $z$  that goes from 0 to 1

$$y = \sigma(z) = \frac{1}{1 + e^{-z}} = \frac{1}{1 + \exp(-z)}$$



# The very useful sigmoid or logistic function



Idea of logistic regression

We'll compute  $w \cdot x + b$

And then we'll pass it through the sigmoid function:

$$\sigma(w \cdot x + b)$$

And we'll just treat it as a probability

# Making probabilities with sigmoids

$$\begin{aligned} P(y = 1) &= \sigma(w \cdot x + b) \\ &= \frac{1}{1 + \exp(-(w \cdot x + b))} \end{aligned}$$

$$\begin{aligned} P(y = 0) &= 1 - \sigma(w \cdot x + b) \\ &= 1 - \frac{1}{1 + \exp(-(w \cdot x + b))} \\ &= \frac{\exp(-(w \cdot x + b))}{1 + \exp(-(w \cdot x + b))} \end{aligned}$$

By the way:

$$P(y = 0) = 1 - \sigma(w \cdot x + b) = \sigma(-(w \cdot x + b))$$

$$= 1 - \frac{1}{1 + \exp(-(w \cdot x + b))}$$

$$= \frac{\exp(-(w \cdot x + b))}{1 + \exp(-(w \cdot x + b))}$$

Because

$$1 - \sigma(x) = \sigma(-x)$$

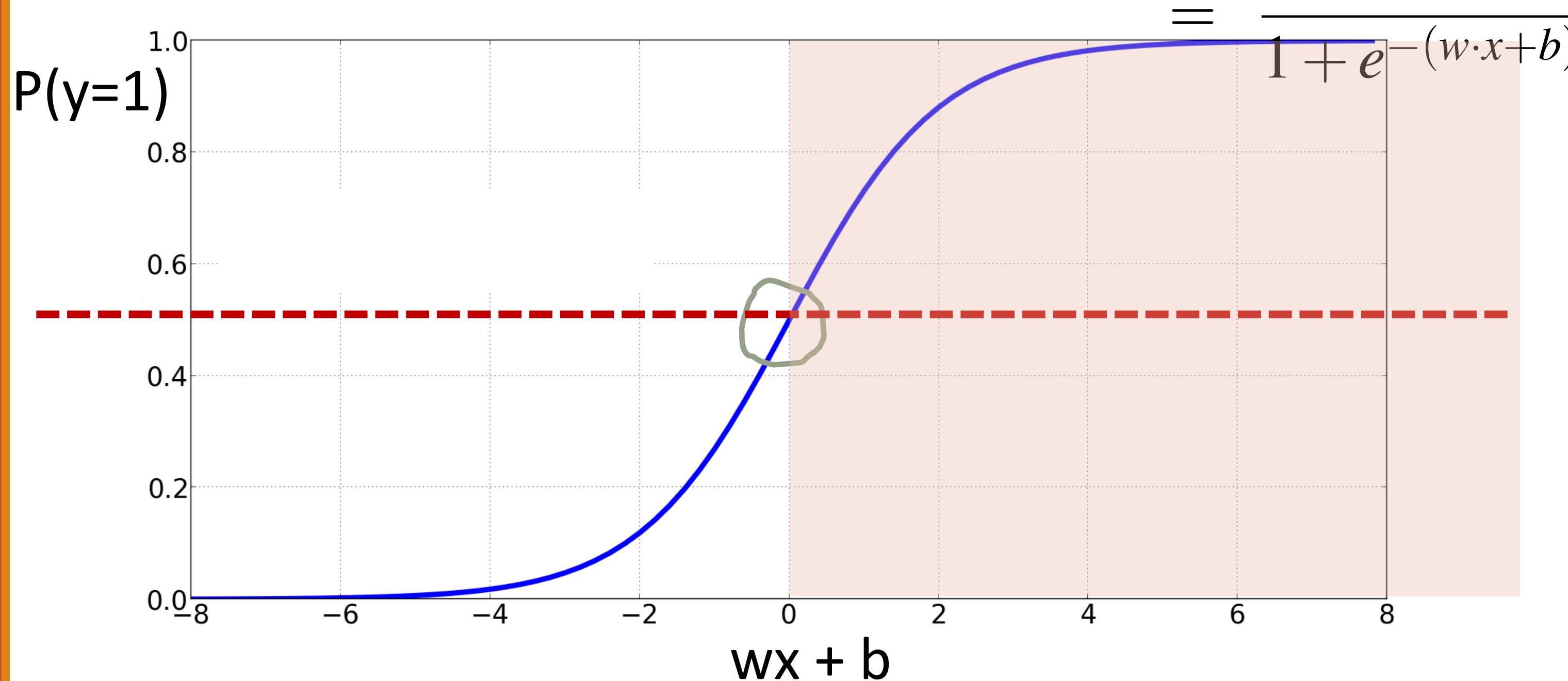


# Turning a probability into a classifier

$$\hat{y} = \begin{cases} 1 & \text{if } P(y = 1|x) > 0.5 \\ 0 & \text{otherwise} \end{cases}$$

0.5 here is called the **decision boundary**

The probabilistic classifier  $P(y = 1) = \sigma(w \cdot x + b)$



# Turning a probability into a classifier

$$\hat{y} = \begin{cases} 1 & \text{if } P(y = 1|x) > 0.5 \\ 0 & \text{otherwise} \end{cases} \quad \begin{array}{l} \text{if } w \cdot x + b > 0 \\ \text{if } w \cdot x + b \leq 0 \end{array}$$

# Classification in Logistic Regression

Logistic  
Regression

# Logistic Regression

Logistic Regression: a text example  
on sentiment classification

# Sentiment example: does $y=1$ or $y=0$ ?

It's hokey . There are virtually no surprises , and the writing is second-rate .  
So why was it so enjoyable ? For one thing , the cast is  
great . Another nice touch is the music . I was overcome with the urge to get off  
the couch and start dancing . It sucked me in , and it'll do the same to you .

It's **hokey**. There are virtually **no** surprises, and the writing is **second-rate**. So why was it so **enjoyable**? For one thing, the cast is **great**. Another **nice** touch is the music. **I** was overcome with the urge to get off the couch and start dancing. It sucked **me** in, and it'll do the same to **you**.

$x_2=2$   
 $x_3=1$   
 $x_1=3$   
 $x_5=0$   
 $x_6=4.19$   
 $x_4=3$

Var	Definition	Value in Fig. 5.2
$x_1$	count(positive lexicon) $\in$ doc)	3
$x_2$	count(negative lexicon) $\in$ doc)	2
$x_3$	$\begin{cases} 1 & \text{if "no"} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	1
$x_4$	count(1st and 2nd pronouns $\in$ doc)	3
$x_5$	$\begin{cases} 1 & \text{if "!"} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	0
$x_6$	log(word count of doc)	$\ln(66) = 4.19$

# Classifying sentiment for input x

Var	Definition	Val	5.2
$x_1$	count(positive lexicon) $\in$ doc)	3	
$x_2$	count(negative lexicon) $\in$ doc)	2	
$x_3$	$\begin{cases} 1 & \text{if "no"} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	1	
$x_4$	count(1st and 2nd pronouns $\in$ doc)	3	
$x_5$	$\begin{cases} 1 & \text{if "!"} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	0	
$x_6$	log(word count of doc)	$\ln(66) = 4.19$	

Suppose  $w = [2.5, -5.0, -1.2, 0.5, 2.0, 0.7]$

$$b = 0.1$$



# Classifying sentiment for input $x$

$$\begin{aligned} p(+|x) = P(Y = 1|x) &= \sigma(w \cdot x + b) \\ &= \sigma([2.5, -5.0, -1.2, 0.5, 2.0, 0.7] \cdot [3, 2, 1, 3, 0, 4.19] + 0.1) \\ &= \sigma(.833) \\ &= 0.70 \end{aligned}$$

$$\begin{aligned} p(-|x) = P(Y = 0|x) &= 1 - \sigma(w \cdot x + b) \\ &= 0.30 \end{aligned}$$

We can build features for logistic regression for any classification task: period disambiguation

End of sentence

This ends in a period.  
The house at 465 Main St. is new.

Not end

$$x_1 = \begin{cases} 1 & \text{if } \textit{Case}(w_i) = \textit{Lower} \\ 0 & \text{otherwise} \end{cases}$$

$$x_2 = \begin{cases} 1 & \text{if } w_i \in \textit{AcronymDict} \\ 0 & \text{otherwise} \end{cases}$$

$$x_3 = \begin{cases} 1 & \text{if } w_i = \textit{St.} \ \& \ \textit{Case}(w_{i-1}) = \textit{Cap} \\ 0 & \text{otherwise} \end{cases}$$

# Classification in (binary) logistic regression: summary

Given:

- a set of classes: (+ sentiment, - sentiment)
- a vector  $\mathbf{x}$  of features  $[x_1, x_2, \dots, x_n]$ 
  - $x_1 = \text{count}(\text{"awesome"})$
  - $x_2 = \log(\text{number of words in review})$
- A vector  $\mathbf{w}$  of weights  $[w_1, w_2, \dots, w_n]$ 
  - $w_i$  for each feature  $f_i$

$$\begin{aligned} P(y = 1) &= \sigma(w \cdot x + b) \\ &= \frac{1}{1 + e^{-(w \cdot x + b)}} \end{aligned}$$

# Logistic Regression

Logistic Regression: a text example  
on sentiment classification

# Learning: Cross-Entropy Loss

Logistic  
Regression

# Wait, where did the $W$ 's come from?

Supervised classification:

- We know the correct label  $y$  (either 0 or 1) for each  $x$ .
- But what the system produces is an estimate,  $\hat{y}$

We want to set  $w$  and  $b$  to minimize the **distance** between our estimate  $\hat{y}^{(i)}$  and the true  $y^{(i)}$ .

- We need a distance estimator: a **loss function** or a **cost function**
- We need an optimization algorithm to update  $w$  and  $b$  to minimize the loss.

# Learning components

A loss function:

- **cross-entropy loss**

An optimization algorithm:

- **stochastic gradient descent**

# The distance between $\hat{y}$ and $y$

We want to know how far is the classifier output:

$$\hat{y} = \sigma(w \cdot x + b)$$

from the true output:

$$y \quad [= \text{either } 0 \text{ or } 1]$$

We'll call this difference:

$$L(\hat{y}, y) = \text{how much } \hat{y} \text{ differs from the true } y$$



Intuition of negative log likelihood loss  
= cross-entropy loss

A case of conditional maximum likelihood estimation

We choose the parameters  $w, b$  that maximize

- the log probability
- of the true  $y$  labels in the training data
- given the observations  $x$

# Deriving cross-entropy loss for a single observation $x$

**Goal:** maximize probability of the correct label  $p(y|x)$

Since there are only 2 discrete outcomes (0 or 1) we can express the probability  $p(y|x)$  from our classifier (the thing we want to maximize) as

$$p(y|x) = \hat{y}^y (1 - \hat{y})^{1-y}$$

noting:

if  $y=1$ , this simplifies to  $\hat{y}$

if  $y=0$ , this simplifies to  $1 - \hat{y}$

# Deriving cross-entropy loss for a single observation $x$

**Goal:** maximize probability of the correct label  $p(y|x)$

Maximize:  $p(y|x) = \hat{y}^y (1 - \hat{y})^{1-y}$

Now take the log of both sides (mathematically handy)

Maximize:  $\log p(y|x) = \log [\hat{y}^y (1 - \hat{y})^{1-y}]$   
 $= y \log \hat{y} + (1 - y) \log(1 - \hat{y})$

Whatever values maximize  $\log p(y|x)$  will also maximize  $p(y|x)$

Deriving cross-entropy loss for a single observation  $x$

**Goal:** maximize probability of the correct label  $p(y|x)$

**Maximize:**

$$\begin{aligned}\log p(y|x) &= \log [\hat{y}^y (1 - \hat{y})^{1-y}] \\ &= y \log \hat{y} + (1 - y) \log(1 - \hat{y})\end{aligned}$$

Now flip sign to turn this into a loss: something to minimize

**Cross-entropy loss** (because is formula for cross-entropy( $y, \hat{y}$ ))

**Minimize:**

$$L_{\text{CE}}(\hat{y}, y) = -\log p(y|x) = -[y \log \hat{y} + (1 - y) \log(1 - \hat{y})]$$

Or, plugging in definition of  $\hat{y}$ :

$$L_{\text{CE}}(\hat{y}, y) = -[y \log \sigma(w \cdot x + b) + (1 - y) \log (1 - \sigma(w \cdot x + b))]$$

# Let's see if this works for our sentiment example

We want loss to be:

- smaller if the model estimate is close to correct
- bigger if model is confused

Let's first suppose the true label of this is  $y=1$  (positive)

It's hokey . There are virtually no surprises , and the writing is second-rate . So why was it so enjoyable ? For one thing , the cast is great . Another nice touch is the music . I was overcome with the urge to get off the couch and start dancing . It sucked me in , and it'll do the same to you .

# Let's see if this works for our sentiment example

True value is  $y=1$ . How well is our model doing?

$$\begin{aligned} p(+|x) = P(Y = 1|x) &= \sigma(w \cdot x + b) \\ &= \sigma([2.5, -5.0, -1.2, 0.5, 2.0, 0.7] \cdot [3, 2, 1, 3, 0, 4.19] + 0.1) \\ &= \sigma(.833) \\ &= 0.70 \end{aligned} \tag{5.6}$$

Pretty well! What's the loss?

$$\begin{aligned} L_{\text{CE}}(\hat{y}, y) &= -[y \log \sigma(w \cdot x + b) + (1 - y) \log (1 - \sigma(w \cdot x + b))] \\ &= -[\log \sigma(w \cdot x + b)] \\ &= -\log(.70) \\ &= .36 \end{aligned}$$

# Let's see if this works for our sentiment example

Suppose true value instead was  $y=0$ .

$$\begin{aligned} p(-|x) = P(Y = 0|x) &= 1 - \sigma(w \cdot x + b) \\ &= 0.30 \end{aligned}$$

## What's the loss?

$$\begin{aligned} L_{\text{CE}}(\hat{y}, y) &= -[y \log \sigma(w \cdot x + b) + (1 - y) \log (1 - \sigma(w \cdot x + b))] \\ &= -[\log (1 - \sigma(w \cdot x + b))] \\ &= -\log (.30) \\ &= 1.2 \end{aligned}$$

# Let's see if this works for our sentiment example

## The loss when model was right (if true $y=1$ )

$$\begin{aligned} L_{\text{CE}}(\hat{y}, y) &= -[y \log \sigma(w \cdot x + b) + (1 - y) \log (1 - \sigma(w \cdot x + b))] \\ &= -[\log \sigma(w \cdot x + b)] \\ &= -\log(.70) \\ &= .36 \end{aligned}$$

## Is lower than the loss when model was wrong (if true $y=0$ ):

$$\begin{aligned} L_{\text{CE}}(\hat{y}, y) &= -[y \log \sigma(w \cdot x + b) + (1 - y) \log (1 - \sigma(w \cdot x + b))] \\ &= -[\log (1 - \sigma(w \cdot x + b))] \\ &= -\log (.30) \\ &= 1.2 \end{aligned}$$

Sure enough, loss was bigger when model was wrong!



# Cross-Entropy Loss

Logistic  
Regression

# Stochastic Gradient Descent

Logistic  
Regression

Our goal: minimize the loss

Let's make explicit that the loss function is parameterized by weights  $\theta=(w,b)$

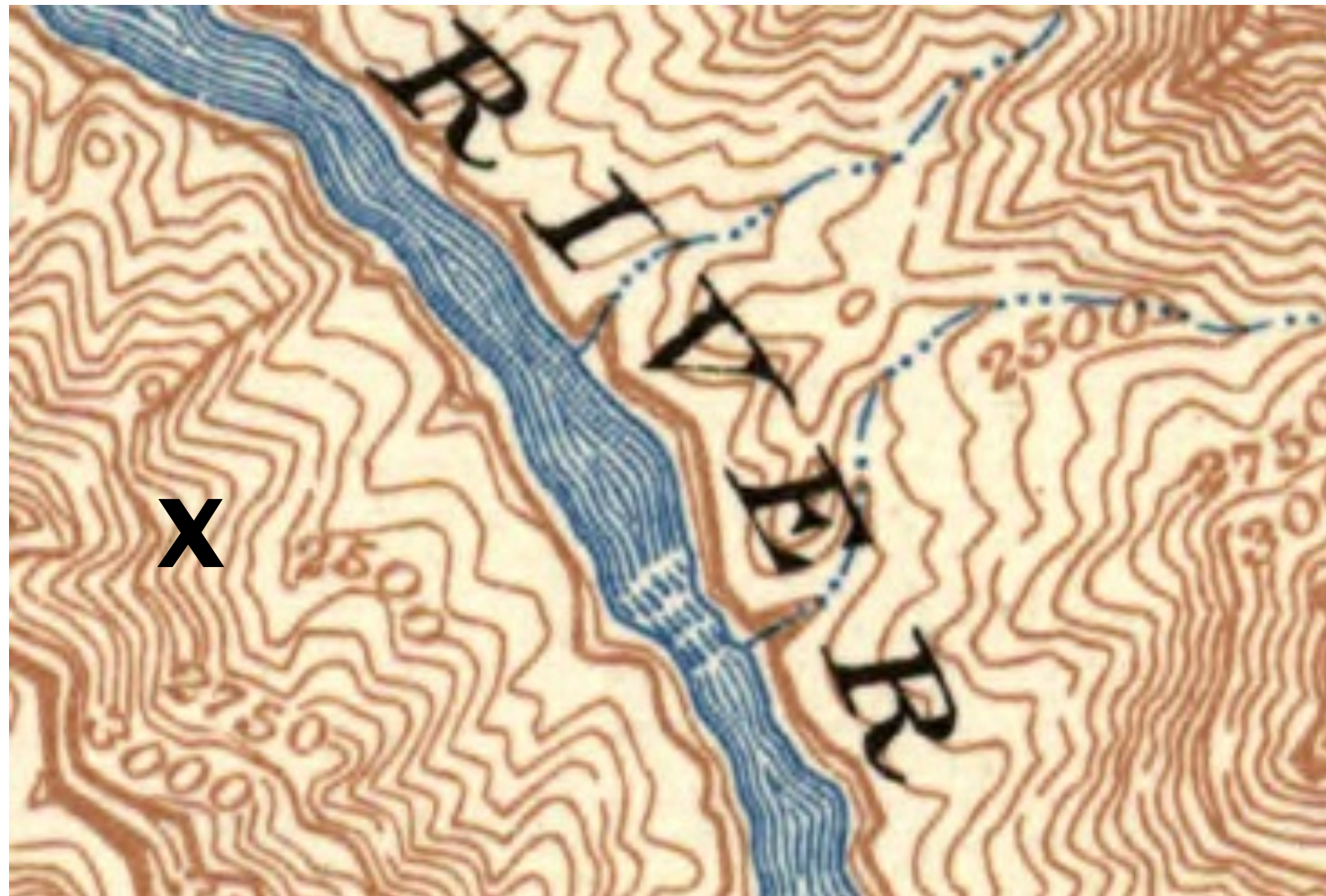
- And we'll represent  $\hat{y}$  as  $f(x; \theta)$  to make the dependence on  $\theta$  more obvious

We want the weights that minimize the loss, averaged over all examples:

$$\hat{\theta} = \operatorname{argmin}_{\theta} \frac{1}{m} \sum_{i=1}^m L_{\text{CE}}(f(x^{(i)}; \theta), y^{(i)})$$

# Intuition of gradient descent

How do I get to the bottom of this river canyon?



Look around me 360°

Find the direction of  
steepest slope down

Go that way

# Our goal: minimize the loss

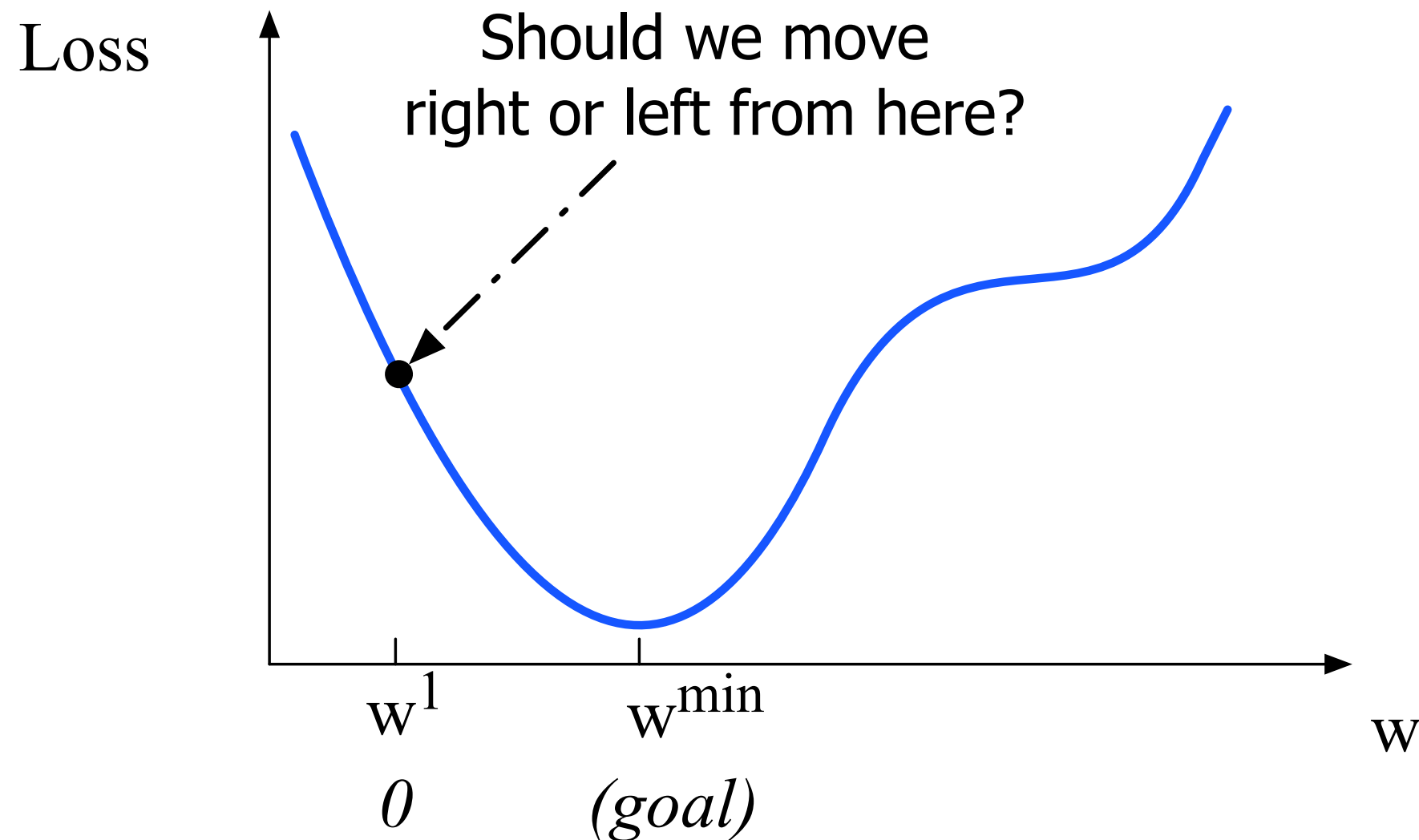
For logistic regression, loss function is **convex**

- A convex function has just one minimum
- Gradient descent starting from any point is guaranteed to find the minimum
  - (Loss for neural networks is non-convex)

# Let's first visualize for a single scalar $w$

Q: Given current  $w$ , should we make it bigger or smaller?

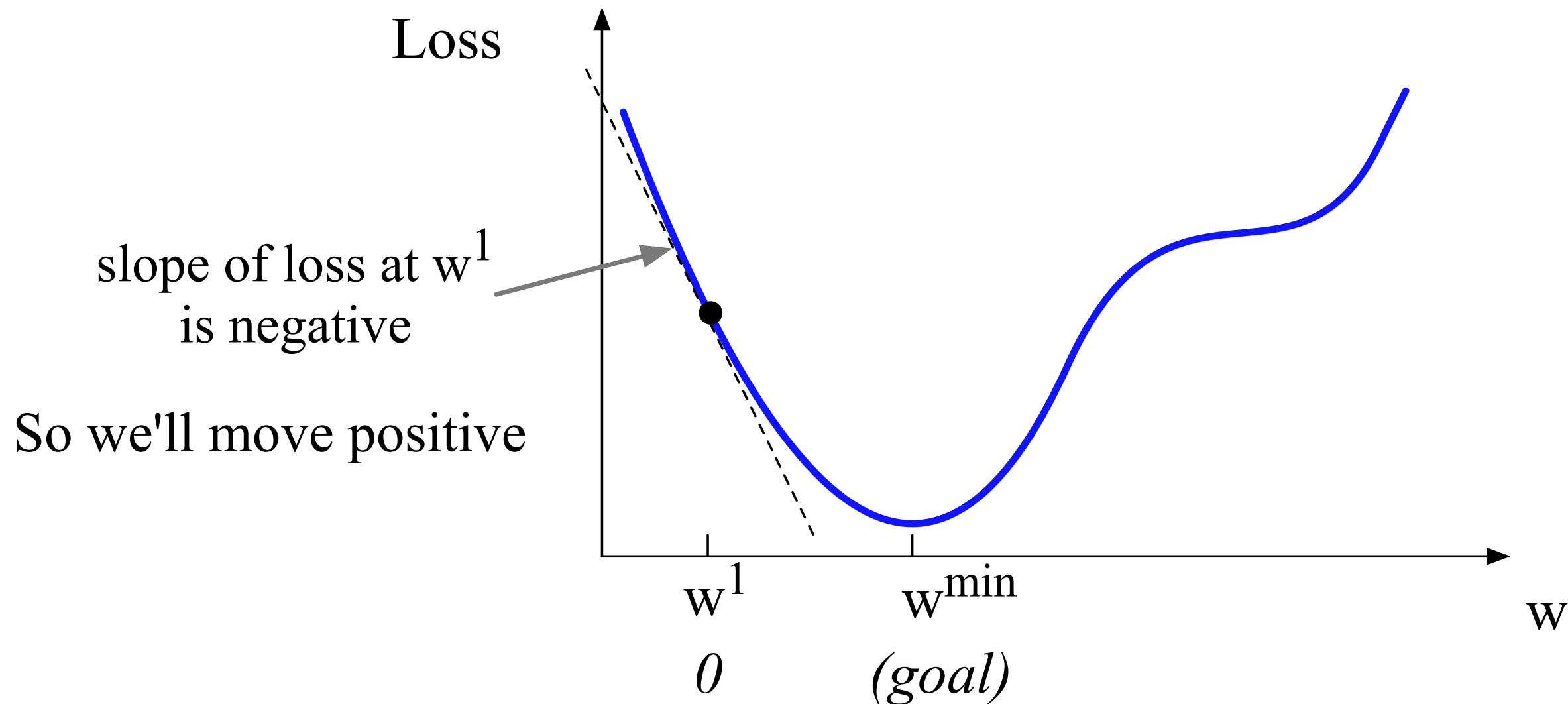
A: Move  $w$  in the reverse direction from the slope of the function



# Let's first visualize for a single scalar $w$

Q: Given current  $w$ , should we make it bigger or smaller?

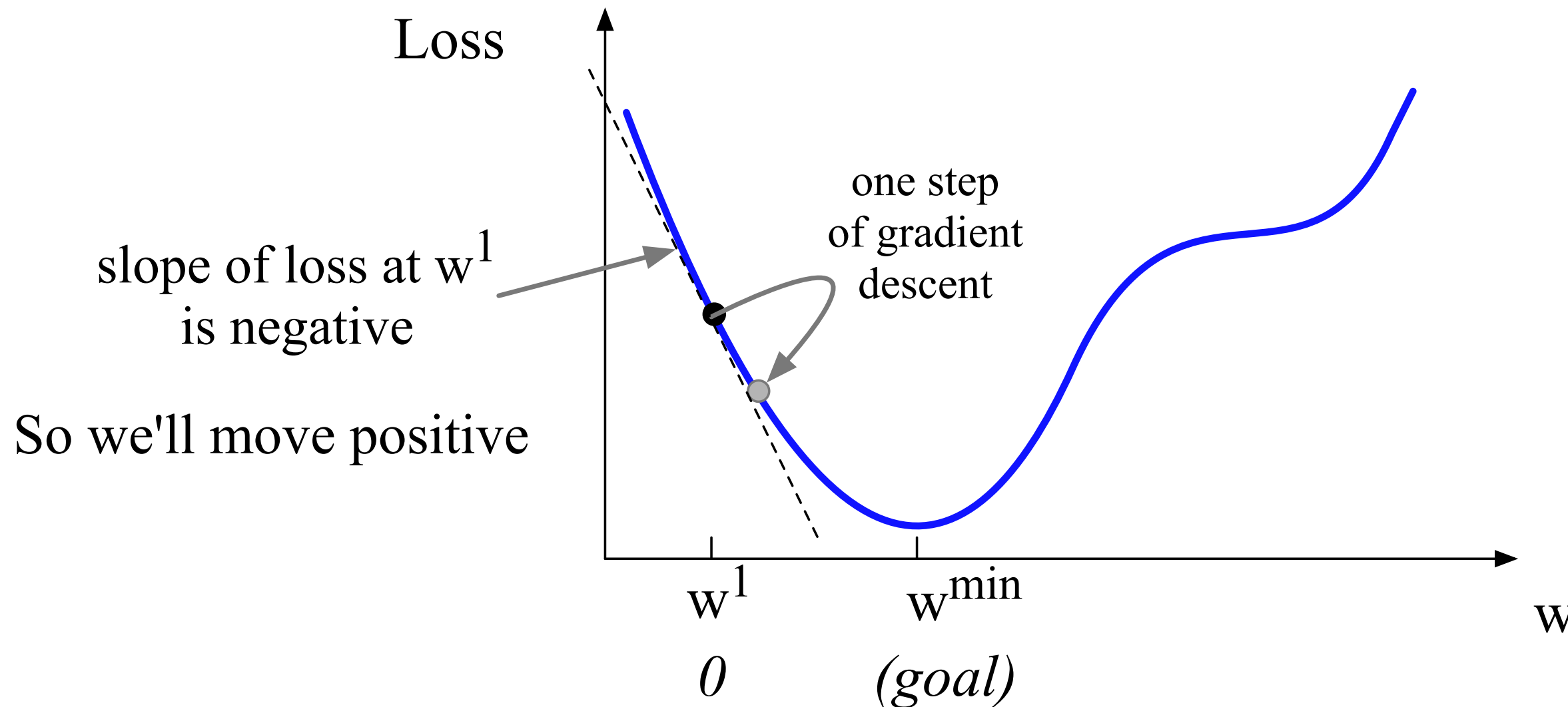
A: Move  $w$  in the reverse direction from the slope of the function



# Let's first visualize for a single scalar $w$

Q: Given current  $w$ , should we make it bigger or smaller?

A: Move  $w$  in the reverse direction from the slope of the function





# Gradients

The **gradient** of a function of many variables is a vector pointing in the direction of the greatest increase in a function.

**Gradient Descent:** Find the gradient of the loss function at the current point and move in the **opposite** direction.

How much do we move in that direction ?

- The value of the gradient (slope in our example)  $\frac{d}{dw} L(f(x; w), y)$  weighted by a **learning rate**  $\eta$
- Higher learning rate means move  $w$  faster

$$w^{t+1} = w^t - \eta \frac{d}{dw} L(f(x; w), y)$$

# Now let's consider $N$ dimensions

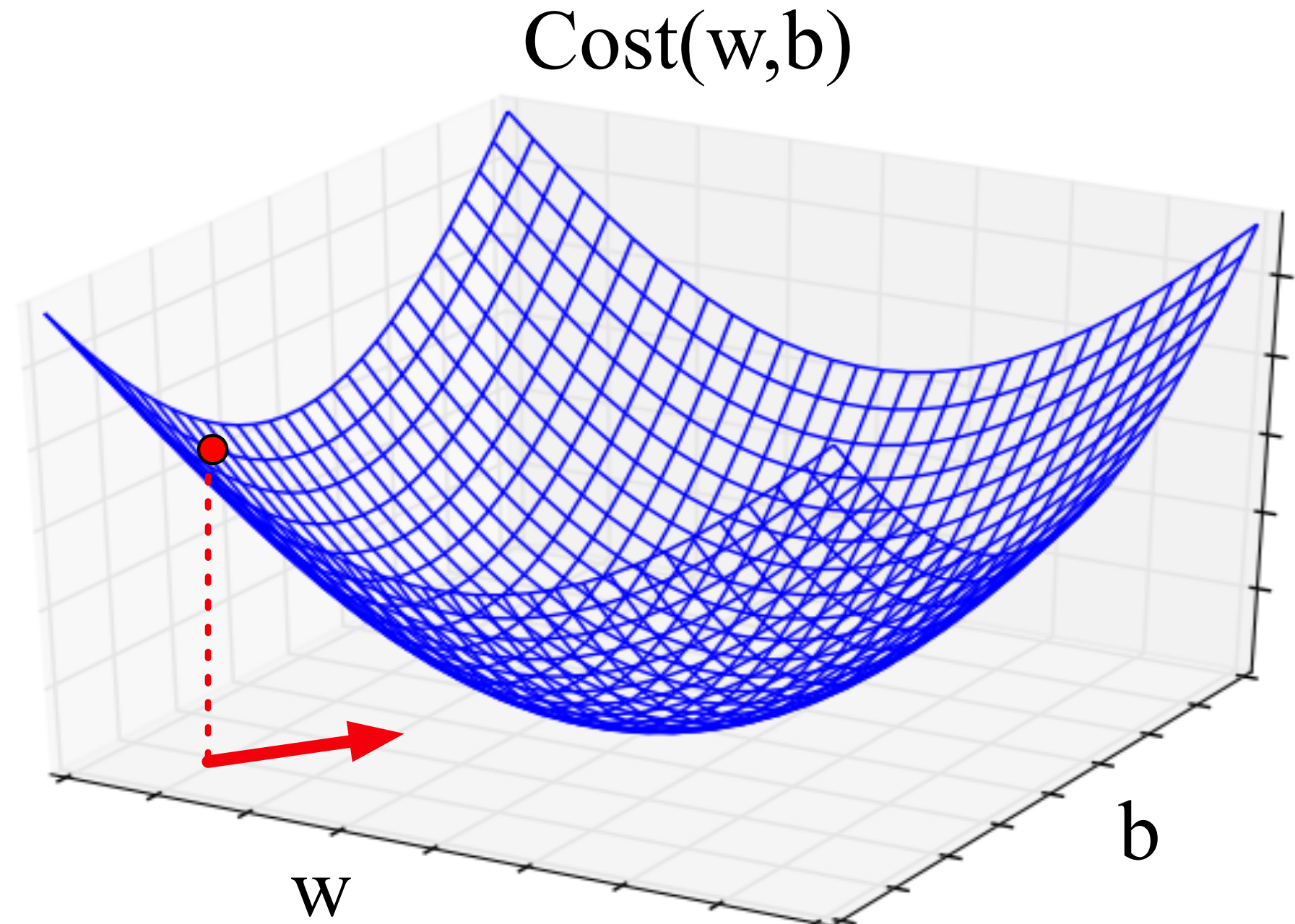
We want to know where in the  $N$ -dimensional space (of the  $N$  parameters that make up  $\theta$ ) we should move.

The gradient is just such a vector; it expresses the directional components of the sharpest slope along each of the  $N$  dimensions.

Imagine 2 dimensions,  $w$  and  $b$

Visualizing the  
gradient vector at  
the red point

It has two  
dimensions shown  
in the  $x$ - $y$  plane



# Real gradients

Are much longer; lots and lots of weights

For each dimension  $w_i$ , the gradient component  $i$  tells us the slope with respect to that variable.

- “How much would a small change in  $w_i$  influence the total loss function  $L$ ?”
- We express the slope as a partial derivative  $\partial$  of the loss  $\partial w_i$

The gradient is then defined as a vector of these partials.

# The gradient

We'll represent  $\hat{y}$  as  $f(x; \theta)$  to make the dependence on  $\theta$  more obvious:

$$\nabla_{\theta} L(f(x; \theta), y) = \begin{bmatrix} \frac{\partial}{\partial w_1} L(f(x; \theta), y) \\ \frac{\partial}{\partial w_2} L(f(x; \theta), y) \\ \vdots \\ \frac{\partial}{\partial w_n} L(f(x; \theta), y) \end{bmatrix}$$

The final equation for updating  $\theta$  based on the gradient is thus

$$\theta_{t+1} = \theta_t - \eta \nabla L(f(x; \theta), y)$$

# What are these partial derivatives for logistic regression?

The loss function

$$L_{\text{CE}}(\hat{y}, y) = -[y \log \sigma(w \cdot x + b) + (1 - y) \log (1 - \sigma(w \cdot x + b))]$$

The elegant derivative of this function (see textbook 5.8 for derivation)

$$\frac{\partial L_{\text{CE}}(\hat{y}, y)}{\partial w_j} = [\sigma(w \cdot x + b) - y]x_j$$

**function** STOCHASTIC GRADIENT DESCENT( $L()$ ,  $f()$ ,  $x$ ,  $y$ ) **returns**  $\theta$

# where:  $L$  is the loss function

#  $f$  is a function parameterized by  $\theta$

#  $x$  is the set of training inputs  $x^{(1)}, x^{(2)}, \dots, x^{(m)}$

#  $y$  is the set of training outputs (labels)  $y^{(1)}, y^{(2)}, \dots, y^{(m)}$

$\theta \leftarrow 0$

**repeat** til done

For each training tuple  $(x^{(i)}, y^{(i)})$  (in random order)

1. Optional (for reporting): # How are we doing on this tuple?

    Compute  $\hat{y}^{(i)} = f(x^{(i)}; \theta)$  # What is our estimated output  $\hat{y}$ ?

    Compute the loss  $L(\hat{y}^{(i)}, y^{(i)})$  # How far off is  $\hat{y}^{(i)}$  from the true output  $y^{(i)}$ ?

2.  $g \leftarrow \nabla_{\theta} L(f(x^{(i)}; \theta), y^{(i)})$  # How should we move  $\theta$  to maximize loss?

3.  $\theta \leftarrow \theta - \eta g$  # Go the other way instead

**return**  $\theta$



# Hyperparameters

The learning rate  $\eta$  is a **hyperparameter**

- too high: the learner will take big steps and overshoot
- too low: the learner will take too long

Hyperparameters:

- Briefly, a special kind of parameter for an ML model
- Instead of being learned by algorithm from supervision (like regular parameters), they are chosen by algorithm designer.

# Stochastic Gradient Descent

Logistic  
Regression

# Logistic Regression

Stochastic Gradient Descent:  
An example and more details

# Working through an example

One step of gradient descent

A mini-sentiment example, where the true  $y=1$  (positive)

Two features:

$x_1 = 3$  (count of positive lexicon words)

$x_2 = 2$  (count of negative lexicon words)

Assume 3 parameters (2 weights and 1 bias) in  $\Theta^0$  are zero:

$$w_1 = w_2 = b = 0$$

$$\eta = 0.1$$

# Example of gradient descent

Update step for update  $\theta$  is:

$$w_1 = w_2 = b = 0;$$

$$x_1 = 3; \quad x_2 = 2$$

$$\theta_{t+1} = \theta_t - \eta \nabla L(f(x; \theta), y)$$

where 
$$\frac{\partial L_{\text{CE}}(\hat{y}, y)}{\partial w_j} = [\sigma(w \cdot x + b) - y]x_j$$

Gradient vector has 3 dimensions:

$$\nabla_{w,b} = \begin{bmatrix} \frac{\partial L_{\text{CE}}(\hat{y}, y)}{\partial w_1} \\ \frac{\partial L_{\text{CE}}(\hat{y}, y)}{\partial w_2} \\ \frac{\partial L_{\text{CE}}(\hat{y}, y)}{\partial b} \end{bmatrix}$$

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Note that enough negative examples would eventually make  $w_2$  negative

# Mini-batch training

Stochastic gradient descent chooses a single random example at a time.

That can result in choppy movements

More common to compute gradient over batches of training instances.

**Batch training:** entire dataset

**Mini-batch training:**  $m$  examples (512, or 1024)

# Logistic Regression

Stochastic Gradient Descent:  
An example and more details

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