$$\left[L_{\beta^i \otimes c_j}\right] = \left[\left[L_{\beta^i \otimes c_j}(b_1)\right]^C, \left[L_{\beta^i \otimes c_j}(b_2)\right]^C, \ldots, \left[L_{\beta^i \otimes c_j}(b_n)\right]^C\right]$$

$$L_{\beta^i \otimes c_j}(b_1) = \beta^i(b_1)\, c_j = \delta^i_1 c_j$$

$$= \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \delta^i_1 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \begin{matrix} \leftarrow 1 \\ \leftarrow 2 \\ \vdots \\ \leftarrow j \\ \vdots \\ \leftarrow m \end{matrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \left[L_{\beta^i \otimes c_j}\right]^k_1$$

$\left[L_{\beta^i \otimes c_j}\right]^k_\ell$ ← k: fila, ℓ: columna

$$= \gamma^k\left(L_{\beta^i \otimes c_j}(b_\ell)\right) = \gamma^k\left(\beta^i(b_\ell)\, c_j\right)$$
$$= \gamma^k(\delta^i_\ell c_j)$$
$$= \delta^i_\ell\, \gamma^k(c_j)$$
$$= \delta^i_\ell \delta^k_j$$

$v \in V \quad \{b_i\}, \{\beta^i\}$

$$v = \lambda^i b_i$$
$$\beta^j(v) = \lambda^i \beta^j(b_i) = \lambda^j$$

$$\left[L_{\beta^i \otimes c_j}\right] = \begin{bmatrix} \delta^i_1\delta^1_j & \delta^i_1\delta^2_j & \cdots & \delta^i_1\delta^m_j \\ \delta^i_2\delta^1_j & \delta^i_2\delta^2_j & \cdots & \delta^i_2\delta^m_j \\ \vdots & \vdots & \ddots & \vdots \\ \delta^i_n\delta^1_j & \delta^i_n\delta^2_j & \cdots & \delta^i_n\delta^m_j \end{bmatrix}$$

$\dim V = n = \dim V^*$

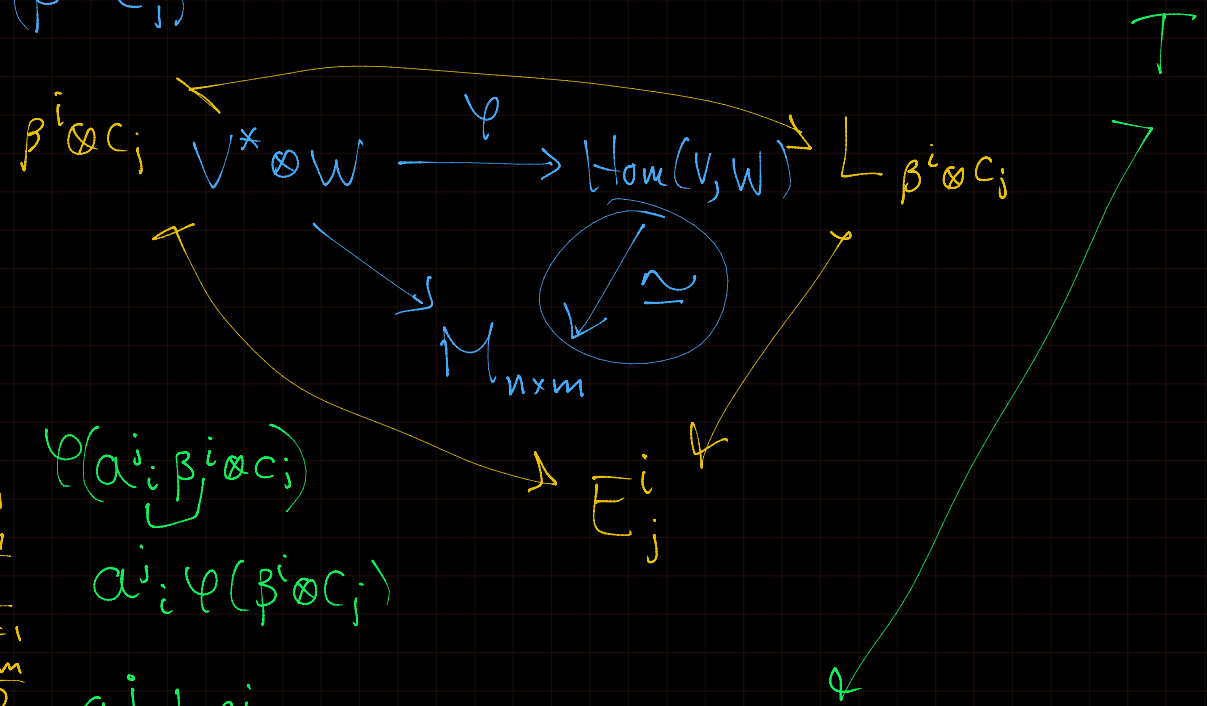
$\dim W = m$

$$[L_{\beta^1 \otimes c_1}] = \begin{bmatrix} \delta_1^1\delta_1^1 & \delta_1^1\delta_1^2 & \cdots & \delta_1^1\delta_1^m \\ \delta_2^1\delta_1^1 & \delta_2^1\delta_1^2 & \cdots & \delta_2^1\delta_1^m \\ \vdots & \vdots & \ddots & \vdots \\ \delta_n^1\delta_1^1 & \delta_n^1\delta_1^2 & \cdots & \delta_n^1\delta_1^m \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} = E_1^1$$

$\beta^i \otimes c_j$

$$[L_{\beta^1 \otimes c_2}] = \begin{bmatrix} \delta_1^1\delta_2^1 & \delta_1^1\delta_2^2 & \cdots & \delta_1^1\delta_2^m \\ \delta_2^1\delta_2^1 & \delta_2^1\delta_2^2 & \cdots & \delta_2^1\delta_2^m \\ \vdots & \vdots & \ddots & \vdots \\ \delta_n^1\delta_2^1 & \delta_n^1\delta_2^2 & \cdots & \delta_n^1\delta_2^m \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} = E_2^1$$

$$\chi = \sum_{ij} a^j_i (\beta^i \otimes c_j)$$



$$\varphi(\chi) = \sum_{i=1}^{n}\sum_{j=1}^{m} \varphi(a^j_i \beta^i \otimes c_j)$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{m} a^j_i \varphi(\beta^i \otimes c_j)$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{m} a^j_i L_{\beta^i \otimes c_j}$$

$$[T] = (a^j_i)$$

$$[\varphi(\chi)] = \sum\sum a^j_i [L_{\beta^i \otimes c_j}]$$

$$= \sum\sum a^j_i E^i_j$$

$\chi \in V^* \otimes W$

$$= a^1_1 \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} + a^2_1 \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} + \cdots = (a^j_i) = [T]$$

$\therefore \quad \varphi(\chi) = T$

$$\chi = a^j_i \, \beta^i \otimes c_j$$

$$\xi = \lambda^j_i \, \beta^i \otimes c_j$$

$$\varphi(\chi) = \varphi(\xi) \in \mathrm{Hom}(V, W)$$

$$[\varphi(\chi)] = [\varphi(\xi)] \in M_{m \times n}$$

$$[a^j_i L_{\beta^i \otimes c_j}] \qquad [\lambda^j_i L_{\beta^i \otimes c_j}]$$

$$a^j_i E^i_j \qquad \lambda^j_i E^i_j$$

$$(a^j_i) = (\lambda^j_i)$$

$$a^j_i = \lambda^j_i \qquad \forall i = 1, \dots, n \; ; j = 1, \dots, m.$$

$$\chi = a^j_i \, \beta^i \otimes c_j = \lambda^j_i \, \beta^i \otimes c_j = \xi$$