$$\phi : \underset{\mathbb{R}^2}{\overset{\cup\!\!\shortmid}{U}} \longrightarrow \mathbb{R}^3$$

$$\phi\begin{pmatrix}u\\v\end{pmatrix} = \begin{pmatrix}u\\v\\f(u,v)\end{pmatrix} = \begin{pmatrix}\phi^1\\\phi^2\\\phi^3\end{pmatrix}$$

$$\partial_i = \begin{pmatrix}\partial_i\phi^1\\\partial_i\phi^2\\\partial_i\phi^3\end{pmatrix}$$

$$g_{ij} = \partial_i \cdot \partial_j$$

$$(g^{ij}) = (g_{ij})^{-1} = \begin{pmatrix} g^{11} & g^{12} \\ g^{21} & g^{22}\end{pmatrix}$$

Base del espacio tangente $T_p\Sigma$

$$\partial_1 = \partial_1\phi = \begin{pmatrix}1\\0\\\partial_1 f\end{pmatrix}$$

$$\partial_2 = \partial_2\phi = \begin{pmatrix}0\\1\\\partial_2 f\end{pmatrix}$$

$\beta^1 \quad \beta^2$ Base dual de $T_p^*\Sigma$

$$\beta^i(\partial_j) = \delta^i_j$$

$$\partial^1 = g^{11}\partial_1 + g^{12}\partial_2 = \begin{pmatrix}\cdot\\\cdot\\\cdot\end{pmatrix}$$

$$\partial^2 = g^{21}\partial_1 + g^{22}\partial_2 = \begin{pmatrix}\cdot\\\cdot\\\cdot\end{pmatrix}$$

Rep. de Riesz de β^i

$$\beta^1(v) = \partial^1 \cdot v$$
$$\beta^2(v) = \partial^2 \cdot v$$

$$\boxed{\Gamma^i_{jk} = \partial^i \cdot D_{\partial_j}\partial_k}$$

$$= \partial^i \cdot \partial_j\partial_k\phi^\mu e_\mu = \partial_j\partial_k\phi^\mu \, \partial^i\cdot e_\mu$$

$$\left(\Gamma^i_{jk}\right) = \left(\partial_j\partial_k\phi^\mu\right) \underline{\partial^i \cdot e_\mu}$$

$$= \cancel{\left(\partial_j\partial_k\underset{u}{\phi^1}\right)}^{\,0} \underline{\partial^i\cdot e_1} + \cancel{\left(\partial_j\partial_k\underset{v}{\phi^2}\right)}^{\,0} \underline{\partial^i\cdot e_2} + \left(\partial_j\partial_k\underset{f}{\phi^3}\right) \underline{\partial^i\cdot e_3}$$

$$\begin{pmatrix}\Gamma^i_{11} & \Gamma^i_{12}\\ \Gamma^i_{21} & \Gamma^i_{22}\end{pmatrix} = \begin{pmatrix}\partial_1\partial_1\phi^3 & \partial_1\partial_2\phi^3\\ \partial_2\partial_1\phi^3 & \partial_2\partial_2\phi^3\end{pmatrix}\partial^i\cdot e_3 = \begin{pmatrix}\partial_1\partial_1 f & \partial_1\partial_2 f\\ \partial_2\partial_1 f & \partial_2\partial_2 f\end{pmatrix}\partial^i\cdot e_3$$

$$\underset{\phi^3 = f}{=} \begin{pmatrix}\partial_1\partial_1 f & \partial_1\partial_2 f\\ \partial_2\partial_1 f & \partial_2\partial_2 f\end{pmatrix} g^{im}\partial_m f = \begin{pmatrix}\partial_1\partial_1 f & \partial_1\partial_2 f\\ \partial_2\partial_1 f & \partial_2\partial_2 f\end{pmatrix} \begin{bmatrix}\partial_1 f & \partial_2 f\end{bmatrix} \begin{bmatrix} g^{i1} \\ g^{i2}\end{bmatrix}$$

$$\partial^i = g^{im}\partial_m = g^{im}\begin{pmatrix}\partial_m\phi^1\\\partial_m\phi^2\\\partial_m\phi^3\end{pmatrix}$$

$$\partial^i \cdot e_3 = g^{im}\begin{pmatrix}\partial_m\phi^1\\\partial_m\phi^2\\\underbrace{\partial_m\phi^3}_{f}\end{pmatrix}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix} = g^{im}\partial_m f$$