23 sep 2021.

Consideremos el caso general de una EDO lineal hom. de 2do orden:

$$ax''+bx'+cx=0$$

Haciendo $y=x'$, podemos expresarla como el sistema de 1er orden

$$\begin{cases} x'=y \\ y'=\dfrac{-bx'-cx}{a}. \end{cases}$$

Para una EDO lineal de orden n

$$y^{(n)} = g(t, y, y', \ldots, y^{(n-1)})$$

podemos expresarlo como un sistema de 1er orden tomando

$$x_1 = y$$

$$x_2 = y'$$

$$\vdots$$

$$x_{n-1} = y^{(n-2)}$$

$$x_n = y^{(n-1)}$$

Al derivar, tenemos

$$x_1' = y' = x_2$$

$$x_2' = y'' = x_3$$

$$\vdots$$

$$x_{n-1}' = y^{(n-1)} = x_{n-2}$$

$$x_n' = y^{(n)} = g(t, y, y', y'', \ldots, y^{(n-1)})$$

$$= g(t, x_1, x_2, x_3, \ldots, x_n).$$

$$\lambda^3 + 6\lambda^2 + 11\lambda + 6 = 0$$

$$\lambda(\lambda^2 + 6\lambda + 11) + 6$$

$$\lambda(\lambda(\lambda+6) + 11) + 6$$

$\lambda^3+2\lambda^2-3\lambda-10$ $\qquad\qquad$ $\lambda^2+(5-1)\lambda+1$

$8+8-6-10$

$$\begin{array}{c|cccc} & 1 & 2 & -3 & -10 \\ 2 & & 2 & 8 & 10 \\ \hline & 1 & 4 & 5 & 0 \end{array}$$

$$\lambda^3+2\lambda^2-3\lambda-10=(\lambda-2)(\lambda^2+4\lambda+5)$$

$$\lambda^2+2\cdot2\lambda+5$$

$$\lambda=-2\pm\sqrt{2^2-5}$$

$$=-2\pm i$$

ent. $\quad y=c_1e^{-2t}+c_2e^{-2t}\cos t+c_3e^{-2t}\sin t$

Sistemas dinámicos y soluciones.

Una ec. dif. de 1er orden en t y en las variables dependientes $x_1, \dots, x_n$ se llama sistema dinámico.

$$\left\{\begin{array}{l} x_1' = f_1(t, x_1, x_2, \dots, x_n) \\ x_2' = f_2(t, x_1, x_2, \dots, x_n) \\ \quad \vdots \qquad\quad \vdots \\ x_n' = f_n(t, x_1, x_2, \dots, x_n) \end{array}\right.$$

$$\mathbb{R}^n \longrightarrow \mathbb{R} \times \mathbb{R}^n$$

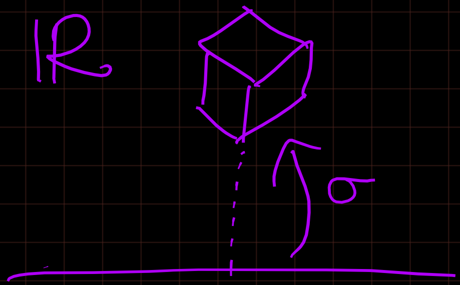
$$\big\downarrow$$

$$\mathbb{R}$$

Un campo vectorial vertical en el haz fibrado trivial.

Las x_i se llaman variables de estado y las f_i son funciones

Una solución es una sección



Dada una sección $\sigma : \mathbb{R} \to \mathbb{R}^n$,

$\sigma' : \mathbb{R} \to T\mathbb{R}^n$

de tasa de cambio.

Se busca una curva

$\gamma = (x_1, \ldots, x_n)$

que satisfaga la ecuación

Queda como

$$\begin{cases} X' = F(t, x) \\ x(t_0) = X_0. \end{cases}$$

Se clasifican en:

- continuos
- discretos

- autónomos
- no autónomos

F no depende de t.
(el campo vectorial es "constante" a lo largo del espacio base)

- lineales
- no lineales.

ej Resolver

$$
\begin{aligned}
x' &= -4x + y + z \\
y' &= x + 5y - z \\
z' &= y - 3z
\end{aligned}
$$

$$A = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 5 & -1 \\ 0 & 1 & -3 \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda + 4 & -1 & -1 \\ -1 & \lambda - 5 & 1 \\ 0 & -1 & \lambda + 3 \end{bmatrix}$$

$$\det(\lambda I - A) = (\lambda + 4)\left[(\lambda - 5)(\lambda + 3) + 1\right]$$

$$+ 1 \cdot \left[-(\lambda+3) - 1\right]$$

$$= (\lambda+4)\left[(\lambda-5)(\lambda+3)+1\right]$$

$$(\lambda+4)\left[-1\right]$$

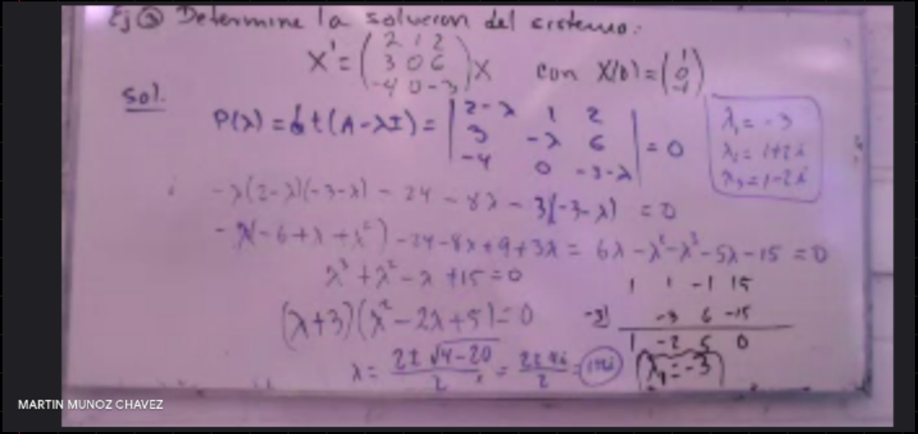
$$= (\lambda+4)(\lambda-5)(\lambda+3)$$

$$\lambda_{1,2,3} = \begin{cases} 5 \\ -4 \\ -3 \end{cases}$$

$$(A - 5I) = \begin{bmatrix} -9 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -8 \end{bmatrix} \qquad A+4I = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 5 & -1 \\ 0 & 1 & -3 \end{bmatrix} \qquad A+3I = \begin{bmatrix} -1 & 1 & 1 \\ 1 & 8 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -9 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -8 \end{bmatrix}\begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix} = 0 \qquad \begin{bmatrix} 0 & 1 & 1 \\ 1 & 9 & -1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 10 \\ -1 \\ 1 \end{bmatrix} = 0 \qquad \begin{bmatrix} -1 & 1 & 1 \\ 1 & 8 & -1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$\therefore X(t) = \begin{bmatrix} 1 \\ 8 \\ 1 \end{bmatrix} e^{5t} + \begin{bmatrix} 10 \\ -1 \\ 1 \end{bmatrix} e^{-4t} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} e^{-3t}$$



Ej ② Determine la solucion del sistema:

$$X' = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 0 & 6 \\ -4 & 0 & -3 \end{pmatrix} X \quad \text{con } X(0) = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

Sol.

$$P(\lambda) = \det(A - \lambda I) = \begin{vmatrix} 2-\lambda & 1 & 2 \\ 3 & -\lambda & 6 \\ -4 & 0 & -3-\lambda \end{vmatrix} = 0$$

$\lambda_1 = -3$
$\lambda_2 = 1+2i$
$\lambda_3 = 1-2i$

$$-\lambda(2-\lambda)(-3-\lambda) - 24 - 8\lambda - 3(-3-\lambda) = 0$$

$$-\lambda(-6+\lambda+\lambda^2) - 24 - 8\lambda + 9 + 3\lambda = 6\lambda - \lambda^2 - \lambda^3 - 5\lambda - 15 = 0$$

$$\lambda^3 + \lambda^2 - \lambda + 15 = 0$$

$$\begin{array}{r|rrrr} & 1 & 1 & -1 & 15 \\ -3 & & -3 & 6 & -15 \\ \hline & 1 & -2 & 5 & 0 \end{array}$$

$$(\lambda+3)(\lambda^2 - 2\lambda + 5) = 0$$

$$\lambda_1 = -3$$

$$\lambda = \frac{2 \pm \sqrt{4-20}}{2} = \frac{2 \pm 4i}{2} = 1 \pm 2i$$

$$A = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 0 & 6 \\ -4 & 0 & -3 \end{pmatrix} \qquad \sigma(A) = \{-3,\ 1 \pm 2i\}$$

$$A - (-3)I = \begin{pmatrix} 5 & 1 & 2 \\ 3 & 3 & 6 \\ -4 & 0 & 0 \end{pmatrix} \quad , \quad \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$$

$$A - (1+2i)I = \begin{pmatrix} 1-2i & 1 & 2 \\ 3 & -1-2i & 6 \\ -4 & 0 & -4-2i \end{pmatrix} \begin{pmatrix} 2+i \\ 3i \\ -2 \end{pmatrix}$$

$$6 + 3i - 12 = -6 + 3i$$

$$\frac{-6+3i}{1+2i} = \frac{(-6+3i)(1-2i)}{5} = \frac{-6+6+i(3+12)}{5} = 3i$$

$$c_1 \begin{pmatrix} 2+i \\ 3i \\ -2 \end{pmatrix} e^{(1+2i)t} + c_2 \begin{pmatrix} 2-i \\ -3i \\ -2 \end{pmatrix} e^{(1-2i)t}$$

$$= c_1 \left[\begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} + i \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} \right] \left(e^t \cos(2t) + i e^t \sin(2t) \right)$$

$$+ c_2 \left[\begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} - i \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} \right] \left(e^t \cos(2t) - i e^t \sin(2t) \right)$$

$$= c_1 \left[\begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} e^t \cos(2t) - \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} e^t \sin(2t) \right] + i c_1 \left[\begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} e^t \sin(2t) + \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} e^t \cos(2t) \right)$$

$$+c_2\left[\begin{pmatrix}2\\0\\-2\end{pmatrix}e^t\cos(2t)-\begin{pmatrix}1\\3\\0\end{pmatrix}e^t\sin(2t)\right]-ic_2\left[\begin{pmatrix}2\\0\\-2\end{pmatrix}e^t\sin(2t)+\begin{pmatrix}1\\3\\0\end{pmatrix}e^t\cos(2t)\right]$$

$$=(c_1+c_2)\left[\begin{pmatrix}2\\0\\-2\end{pmatrix}e^t\cos(2t)-\begin{pmatrix}1\\3\\0\end{pmatrix}e^t\sin(2t)\right]$$

$$+i(c_1-c_2)\left[\begin{pmatrix}2\\0\\-2\end{pmatrix}e^t\sin(2t)+\begin{pmatrix}1\\3\\0\end{pmatrix}e^t\cos(2t)\right]$$

$$=C_1\left[\begin{pmatrix}2\\0\\2\end{pmatrix}e^t\cos(2t)-\begin{pmatrix}1\\3\\0\end{pmatrix}e^t\sin(2t)\right]+C_2\left[\begin{pmatrix}2\\0\\2\end{pmatrix}e^t\sin(2t)+\begin{pmatrix}1\\3\\0\end{pmatrix}e^t\cos(2t)\right]$$

$$= \left[c_1 v_1 + c_2 v_2\right] e^{at}\cos(bt) + \left[-c_1 v_1 + c_2 v_2\right] e^{at}\cos(bt)$$

Resolver por diagonalización

$$X' = \begin{pmatrix} 5 & -4 & 4 \\ 12 & -11 & 12 \\ 4 & -4 & 5 \end{pmatrix} X$$

$$\det \begin{pmatrix} 5-\lambda & -4 & 4 \\ 12 & -11-\lambda & 12 \\ 4 & -4 & 5-\lambda \end{pmatrix} = 4\left[12(5-\lambda) - 12\cdot 4\right]$$
$$-(11+\lambda)\left[(5-\lambda)^2 - 16\right]$$
$$+4\left[(5-\lambda)12 - 12\cdot 4\right]$$

$$= 8\cdot 12(1-\lambda) - (11+\lambda)(5-\lambda-4)(5-\lambda+4)$$
$$= 8\cdot 12(1-\lambda) - (11+\lambda)(1-\lambda)(9-\lambda)$$
$$= (1-\lambda)\left[8\cdot 12 - (11+\lambda)(9-\lambda)\right]$$

$$= (1-\lambda)\left[8\cdot 12-(11+\lambda)(9-\lambda)\right]$$

$$= (1-\lambda)\left[96-99+\lambda^2+2\lambda\right]$$

$$= (1-\lambda)(\lambda^2+2\lambda-3)$$

$$= (1-\lambda)(\lambda-1)(\lambda+3)$$

$$= -(\lambda-1)^2(\lambda+3)$$

$$A = \begin{pmatrix} 5 & -4 & 4 \\ 12 & -11 & 12 \\ 4 & -4 & 5 \end{pmatrix} \qquad \sigma A = \{1, -3\}$$

$$A-(1)I = \begin{pmatrix} 4 & -4 & 4 \\ 12 & -12 & 12 \\ 4 & -4 & 4 \end{pmatrix} \qquad \left\langle \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\rangle$$

$$A-(-3)I = \begin{pmatrix} 8 & -4 & 4 \\ 12 & -8 & 12 \\ 4 & -4 & 8 \end{pmatrix} \qquad \left\langle \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \right\rangle$$

$$\left[\begin{pmatrix}1\\1\\0\end{pmatrix}\begin{pmatrix}0\\1\\1\end{pmatrix}\begin{pmatrix}1\\3\\1\end{pmatrix}\right] = \left[[v_1]^C\ [v_2]^C\ [v_3]^C\right] = [id]^C_E$$

canonical

eigen basis

$$A = [\phi]^C_C$$

Resolver $X' = AX$ en la base propia:

diagonal

$$[X']^E = [\phi]^E_E [X]^E$$

y luego

$$[X]^C = [id]^C_E [X]^E.$$

$\begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix}$ prod es $-1+2=1$

suma es 0

$$\begin{pmatrix} -1-i & 2 \\ -1 & 1-i \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1-i \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + i \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$C_1\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t \cos(-t) - \begin{pmatrix} -1 \\ 0 \end{pmatrix} e^t \sin(-t)\right] + C_2\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t \sin(-t) + \begin{pmatrix} -1 \\ 0 \end{pmatrix} e^t \cos(-t)\right]$$

$$= \left[c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix}\right] e^t \cos(-t)$$

$$\begin{pmatrix} -a_1+2a_2 \\ -a_1+a_2+3 \end{pmatrix} + \begin{pmatrix} -b_1+2b_2+8 \\ -b_1+b_2 \end{pmatrix} t$$

$$b_1 = b_2$$

$$b_1 + 8 = 0$$

$$b_1 = -8 \qquad a_2 = 3$$

$$\begin{pmatrix} -8 \\ -8 \end{pmatrix} = \begin{pmatrix} -a_1+2a_2 \\ -a_1+a_2+3 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} -1 & 2 & -8 \\ -1 & 1 & -11 \end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c} 1 & -2 & 8 \\ -1 & 1 & -11 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{cc|c} 1 & -2 & 8 \\ 0 & -1 & -3 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{cc|c} 1 & 0 & 14 \\ 0 & 1 & 3 \end{array}\right)$$

Ej 2. Resolver

$$X' = \begin{pmatrix} -3 & 1 \\ 2 & -4 \end{pmatrix} X + \begin{pmatrix} e^{-2t} \\ \sin 3t \end{pmatrix}$$

suma -7

prod 10

spec $= \{-5, -2\}$

$$\begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 \\ -2 \end{pmatrix} \qquad \begin{pmatrix} -1 & 1 \\ 2 & -2 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$X_h = c_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-5t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-2t}$$

Sol particular

$$F(t) = \begin{pmatrix} e^{-2t} \\ \sin 3t \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-2t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin 3t$$

Para $F_1(t) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-2t}$

$$x_{p,1} = \begin{pmatrix} a \\ b \end{pmatrix} t e^{-2t} + \begin{pmatrix} c \\ 0 \end{pmatrix} e^{-2t}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} e^{-2t} + \begin{pmatrix} a \\ b \end{pmatrix} t(-2)e^{-2t} + \begin{pmatrix} -2c \\ 0 \end{pmatrix} e^{-2t} = \begin{pmatrix} -3 & 1 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} t e^{-2t} + \begin{pmatrix} -3 & 1 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} c \\ 0 \end{pmatrix} e^{-2t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-2t}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} -2a \\ -2b \end{pmatrix} t + \begin{pmatrix} -2c \\ 0 \end{pmatrix} = \begin{pmatrix} -3 & 1 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} t + \begin{pmatrix} -3c \\ 2c \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a - 2at - 2c \\ b - 2bt \end{pmatrix} = \begin{pmatrix} -3at + bt + 1 - 3c \\ 2at - 4bt + 2c \end{pmatrix}$$

$$a + at - bt + c = 1$$

$$b + 2bt - 2at - 2c = 0$$

$$(a + c) + 2(a - b)t = 1$$

$$(b - 2c) + 2(b - a)t = 0$$

$$a = b$$

$$a + c = 1$$

$$a - 2c = 0$$

$$a = 2c$$

$$3c = 1$$

$$c = \frac{1}{3}$$

Para $F_2(t) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin 3t$

$$X_{p,2} = \begin{pmatrix} a \\ b \end{pmatrix} \sin(3t) + \begin{pmatrix} c \\ d \end{pmatrix} \cos(3t)$$

$$X' = \begin{pmatrix} -3 & 1 \\ 2 & -4 \end{pmatrix} X + \begin{pmatrix} 0 \\ \sin 3t \end{pmatrix}$$

$$\begin{pmatrix} 3a \\ 3b \end{pmatrix} \cos(3t) - \begin{pmatrix} 3c \\ 3d \end{pmatrix} \sin(3t)$$

$$= \begin{pmatrix} -3a+b \\ 2a-4b \end{pmatrix} \sin(3t) + \begin{pmatrix} -3c+d \\ 2c-4d \end{pmatrix} \cos(3t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin(3t)$$

$$\begin{pmatrix} 3a+3c-d \\ 3b-2c+4d \end{pmatrix}\cos(3t) + \begin{pmatrix} -3c+3a-b \\ -3d-2a+4b-1 \end{pmatrix}\sin(3t) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$3a+3c-d=0$$
$$3b-2c+4d=0$$
$$-3c+3a-b=0$$
$$-3d-2a+4b=1$$

$$\left(\begin{array}{cccc|c} 3 & 0 & 3 & -1 & 0 \\ 0 & 3 & -2 & 4 & 0 \\ 3 & -1 & -3 & 0 & 0 \\ -2 & 4 & 0 & -3 & 1 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 6 & 0 & 6 & -2 & 0 \\ 0 & 3 & -2 & 4 & 0 \\ 0 & -1 & -6 & 1 & 0 \\ -6 & 12 & 0 & -9 & 3 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 6 & 0 & 6 & -2 & 0 \\ 0 & 3 & -2 & 4 & 0 \\ 0 & -3 & -18 & 3 & 0 \\ 0 & 12 & 6 & -11 & 3 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 6 & 0 & 6 & -2 & 0 \\ 0 & 3 & -2 & 4 & 0 \\ 0 & 0 & -20 & 7 & 0 \\ 0 & 0 & 14 & -27 & 3 \end{array}\right) \qquad \left(\begin{array}{cccc|c} 6 & 0 & 6 & -2 & 0 \\ 0 & 3 & -2 & 4 & 0 \\ 0 & 0 & -20 & 7 & 0 \\ 0 & 0 & 0 & -27+\frac{49}{10} & 3 \end{array}\right)$$

$$-20\,\frac{14}{20} \qquad 7\,\frac{14}{20} = \frac{98}{20} = \frac{49}{10}$$

$$221$$

$$d = \frac{3}{-27+\frac{49}{10}} = \frac{30}{-270+49} = -\frac{30}{221}$$

$\lambda = 5$

$$A = \begin{pmatrix} -4 & 1 & 1 \\ 1 & 5 & -1 \\ 0 & 1 & 3 \end{pmatrix}$$

$$P(\lambda) = \lambda^3 + 2\lambda^2 - 23\lambda - 60$$

$\lambda_1 = 5$

$\lambda_2 = -4$

$\lambda_3 = -3$

$$e^{At} = \sum_{j=0}^{2} A^j b_j = A^0 b_0 + A^1 b_1 + A^2 b_2$$

$$e^{\lambda t} = \sum_{j=0}^{2} \lambda^j b_j = \lambda^0 b_0 + \lambda^1 b_1 + \lambda^2 b_2$$

$$e^{5t} = b_0 + 5b_1 + 25b_2$$

$$e^{-4t} = b_0 - 4b_1 + 16b_2$$

$$e^{-3t} = b_0 - 3b_1 + 9b_2$$

$$\left(\begin{array}{ccc|c} 1 & 5 & 25 & e^{5t} \\ 1 & -4 & 16 & e^{-4t} \\ 1 & -3 & 9 & e^{-3t} \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 5 & 25 & e^{5t} \\ 1 & -4 & 16 & e^{-4t} \\ 1 & -3 & 9 & e^{-3t} \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 5 & 25 & e^{5t} \\ 0 & 9 & 9 & e^{5t}-e^{-4t} \\ 0 & 8 & 16 & e^{5t}-e^{-3t} \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 5 & 25 & e^{5t} \\ 0 & 9 & 9 & e^{5t}-e^{-4t} \\ 0 & 1 & -7 & e^{-3t}-e^{-4t} \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 5 & 25 & e^{5t} \\ 0 & 0 & -72 & 9e^{-3t}-e^{5t}-8e^{-4t} \\ 0 & 1 & -7 & e^{-3t}-e^{-4t} \end{array}\right)$$

$$b_2 = \frac{8e^{-4t}-9e^{-3t}+e^{5t}}{72}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 60 & -5e^{-3t}+5e^{-4t}+e^{5t} \\ 0 & 0 & -72 & 9e^{-3t}-e^{5t}-8e^{-4t} \\ 0 & 1 & -7 & e^{-3t}-e^{-4t} \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 6 & 0 & 360 & -30e^{-3t}+30e^{-4t}+6e^{5t} \\ 0 & 0 & -72 & 9e^{-3t}-e^{5t}-8e^{-4t} \\ 0 & 1 & -7 & e^{-3t}-e^{-4t} \end{array}\right)$$

Según eso

$$b_1 = \frac{9e^{-3t}+7e^{5t}-16e^{-4t}}{72}$$

$$\left(\begin{array}{ccc|c} 6 & 0 & 0 & 15e^{-3t}-10e^{-4t}+e^{5t} \\ 0 & 0 & -72 & 9e^{-3t}-e^{5t}-8e^{-4t} \\ 0 & 1 & -7 & e^{-3t}-e^{-4t} \end{array}\right)$$

$$b_0 = \frac{15e^{-3t}-10e^{-4t}+e^{5t}}{6} = \frac{180e^{-3t}-120e^{-4t}+12e^{5t}}{72}$$

Entonces,

$$e^{At} = A^0 b_0 + A^1 b_1 + A^2 b_2 = \frac{1}{72}\begin{pmatrix} \begin{matrix}180e^{-3t}-120e^{-4t}+12e^{5t}\\ -4\left(9e^{-3t}+7e^{5t}-16e^{-4t}\right)\\ +17\left(9e^{-3t}-e^{5t}-8e^{-4t}\right)\end{matrix} & \begin{matrix}9e^{-3t}+7e^{5t}-16e^{-4t}\\ +2\left(9e^{-3t}-e^{5t}-8e^{-4t}\right)\end{matrix} & \begin{matrix}9e^{-3t}+7e^{5t}-16e^{-4t}\\ -8\left(9e^{-3t}-e^{5t}-8e^{-4t}\right)\end{matrix} \\ \begin{matrix}9e^{-3t}+7e^{5t}-16e^{-4t}\\ +\left(9e^{-3t}-e^{5t}-8e^{-4t}\right)\end{matrix} & \begin{matrix}180e^{-3t}-120e^{-4t}+12e^{5t}\\ +5\left(9e^{-3t}+7e^{5t}-16e^{-4t}\right)\\ +25\left(9e^{-3t}-e^{5t}-8e^{-4t}\right)\end{matrix} & \begin{matrix}-\left(9e^{-3t}+7e^{5t}-16e^{-4t}\right)\\ -\left(9e^{-3t}-e^{5t}-8e^{-4t}\right)\end{matrix} \\ 9e^{-3t}-e^{5t}-8e^{-4t} & \begin{matrix}9e^{-3t}+7e^{5t}-16e^{-4t}\\ +2\left(9e^{-3t}-e^{5t}-8e^{-4t}\right)\end{matrix} & \begin{matrix}180e^{-3t}-120e^{-4t}+12e^{5t}\\ +3\left(9e^{-3t}+7e^{5t}-16e^{-4t}\right)\\ +8\left(9e^{-3t}-e^{5t}-8e^{-4t}\right)\end{matrix} \end{pmatrix}$$

$$A = \begin{pmatrix} -4 & 1 & 1 \\ 1 & 5 & -1 \\ 0 & 1 & -3 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} -4 & 1 & 1 \\ 1 & 5 & -1 \\ 0 & 1 & -3 \end{pmatrix}\begin{pmatrix} -4 & 1 & 1 \\ 1 & 5 & -1 \\ 0 & 1 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 17 & 2 & -8 \\ 1 & 25 & -1 \\ 1 & 2 & 8 \end{pmatrix}$$

$$= \frac{1}{72}\begin{pmatrix} 180e^{-3t}-120e^{-4t}+12e^{5t} -4(9e^{-3t}+7e^{5t}-16e^{-4t}) +17(-9e^{-3t}+e^{5t}+8e^{-4t}) & 9e^{-3t}+7e^{5t}-16e^{-4t} +2(9e^{-3t}+e^{5t}+8e^{-4t}) & 9e^{-3t}+7e^{5t}-16e^{-4t} -8(-9e^{-3t}+e^{5t}+8e^{-4t}) \\ 9e^{-3t}+7e^{5t}-16e^{-4t} +(-9e^{-3t}+e^{5t}+8e^{-4t}) & 180e^{-3t}-120e^{-4t}+12e^{5t} +5(9e^{-3t}+7e^{5t}-16e^{-4t}) +25(-9e^{-3t}+e^{5t}+8e^{-4t}) & -(9e^{-3t}+7e^{5t}-16e^{-4t}) -(-9e^{-3t}+e^{5t}+8e^{-4t}) \\ -9e^{-3t}+e^{5t}+8e^{-4t} & 9e^{-3t}+7e^{5t}-16e^{-4t} +2(-9e^{-3t}+e^{5t}+8e^{-4t}) & 180e^{-3t}-120e^{-4t}+12e^{5t} +3(9e^{-3t}+7e^{5t}-16e^{-4t}) +8(-9e^{-3t}+e^{5t}+8e^{-4t}) \end{pmatrix}$$

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(x)\, e^{-i\omega x}\, dx$$

$$= \int_{-\sqrt{2}}^{0} -x\, e^{-i\omega x}\, dx + \int_{0}^{\sqrt{2}} x\, e^{-i\omega x}\, dx$$

$$= \int_{0}^{\sqrt{2}} y\, e^{i\omega y}\, dy + \int_{0}^{\sqrt{2}} x\, e^{-i\omega x}\, dx$$

$$= \int_{0}^{\sqrt{2}} x\left(e^{i\omega x} + e^{-i\omega x}\right) dx$$

$$= -\int_{0}^{\sqrt{2}} \left(\frac{e^{i\omega x}}{i\omega} + \frac{e^{-i\omega x}}{-i\omega}\right) dx + x\left(\frac{e^{i\omega x}}{i\omega} - \frac{e^{-i\omega x}}{i\omega}\right)\Bigg|_{0}^{\sqrt{2}}$$

$$= -\left(\frac{e^{i\omega x}}{-\omega^2} + \frac{e^{-i\omega x}}{-\omega^2}\right)\Bigg|_0^{\sqrt{2}} + \frac{2x}{\omega}\left(\frac{e^{i\omega x} - e^{-i\omega x}}{2i}\right)\Bigg|_0^{\sqrt{2}}$$

$$= \frac{2}{\omega^2}\cos(\omega x)\Big|_0^{\sqrt{2}} + \frac{2\sqrt{2}}{\omega}\sin(\sqrt{2}\omega)$$

$$= \frac{2}{\omega^2}\left(\cos(\sqrt{2}\omega) - 1\right) + \frac{2\sqrt{2}}{\omega}\sin(\sqrt{2}\omega)$$

$$= \frac{2}{\omega^2}\left[\cos(\sqrt{2}\omega) - 1 + \omega\sqrt{2}\sin(\sqrt{2}\omega)\right]$$

$$= \frac{2}{\omega^2}\left[\cos(\sqrt{2}\omega) + \sqrt{2}\omega\sin(\sqrt{2}\omega) - 1\right]$$

$$f(t) = e^{-bt} u(t)$$

$$\hat{f}(\omega) = \int_0^\infty e^{-bt} e^{-i\omega t} dt = \left. \frac{e^{-(b+i\omega)t}}{-(b+i\omega)} \right|_0^\infty$$

$$= \frac{1-1}{-(b+i\omega)} = \frac{1}{b+i\omega}$$

$$f(x) = e^{-bx} \cos(ax)\, u(x)$$

$$\hat{f}(\omega) = \int_0^\infty e^{-bx} \cos(ax)\, e^{-i\omega x}\, dx$$

$\sin$

$$= \int_0^\infty e^{-(b+i\omega)x} \cos(ax)\, dx$$

$\sin$

$$= -\int_0^\infty \frac{e^{-(b+i\omega)x}}{-(b+i\omega)} (-a)\sin(ax)\, dx + \frac{e^{-(b+i\omega)x}}{-(b+i\omega)} \cos(ax) \Big|_0^\infty$$

$(+a)\cos(ax)$ $\qquad$ $\sin$

$$= \int_0^\infty \frac{e^{-(b+i\omega)x}}{(b+i\omega)^2} (-a^2)\sin(ax)\, dx - \frac{e^{-(b+i\omega)x}}{(b+i\omega)^2} (-a)\sin(ax) \Big|_0^\infty \nearrow 0 + \frac{e^{-(b+i\omega)x}}{-(b+i\omega)} \cos(ax) \Big|_0^\infty \nearrow 0$$

$(+a)\cos(ax)$ $\qquad$ $\sin$

$$= \frac{-a^2}{(b+i\omega)^2} \int_0^\infty e^{-(b+i\omega)x} \sin(ax)\,dx + \frac{1}{b+i\omega}$$

$$+ \frac{a}{(b+i\omega)^2}$$

$$\left(\frac{a^2}{(b+i\omega)^2}+1\right) \int_0^\infty e^{-(b+i\omega)x} \sin(ax)\,dx = \frac{1}{b+i\omega}$$

$$= \frac{a}{(b+i\omega)^2}$$

$$\hat{f}(\omega) = \int_0^\infty e^{-(b+i\omega)x} \sin(ax)\,dx = \frac{(b+i\omega)^2}{a^2+(b+i\omega)^2} \frac{1}{b+i\omega}$$

$$= \frac{b+i\omega}{a^2+(b+i\omega)^2}$$

$$= \frac{a}{a^2+(b+i\omega)^2}$$

$$f x = \begin{cases} 1 & (0, k) \\ \frac{1}{2} & k \\ 0 & (k, \infty) \end{cases}$$

$$B(\omega) = \int_0^\infty f(x) \sin(\omega x)\, dx$$

$$A(\omega) + iB(\omega) = \int_0^\infty f(x)\, e^{i\omega x}\, dx$$

$$= \int_0^k e^{i\omega x}\, dx$$

$$= \frac{1}{i\omega} e^{i\omega x} \Big|_{x=0}^{k}$$

$$= \frac{1}{i\omega}\left(e^{i\omega k} - 1\right) = -i\frac{1}{\omega}\left[\cos(\omega k) - 1 + i\sin(\omega k)\right]$$

$$= \sin(\omega k) - i\left[\frac{1}{\omega}\cos(\omega k) - 1\right]$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\xi)\, e^{-i\tau\omega\xi}\, d\xi\; e^{i\tau\omega t}\, d\omega$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\xi)\, e^{-i\tau\omega(\xi - t)}\, d\xi\, d\omega$$

$v = \xi - t$

$dv = d\xi$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(v-t)\, e^{-i\tau\omega v}\, d\omega\, dv$$

$\xi = v - t$

$$= \int_{-\infty}^{\infty} \frac{f(v-t)}{-i\tau v}\, e^{-i\tau\omega v}\Big|_{\omega=-\infty}^{\infty} dv$$

$$= \int_{-\infty}^{\infty} \frac{f(v-t)}{-i\tau v} \lim_{\omega\to\infty}\left(e^{-i\tau\omega v} - e^{i\tau\omega v}\right) dv$$

$$= \int_{-\infty}^{\infty} 2 \frac{f(v-t)}{\tau v} \lim_{\omega \to \infty} \frac{e^{i\tau\omega v} - e^{-i\tau\omega v}}{2i} \, dv$$

$$= 2 \lim_{\omega \to \infty} \int_{-\infty}^{\infty} \frac{f(v-t)}{\tau v} \sin(\tau\omega v) \, dv$$

$$\int_{-\infty}^{\infty} \frac{f(v-t)}{\tau v} \sin(\tau\omega v) \, dv$$