$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I_{(-\infty, x]}(X_i)$$

$$\begin{aligned}
ECM\left(F_n(x)\right) &= E\left[\left(F_n(x) - F_X(x;\theta)\right)^2\right] \\
&= \underbrace{E\left[F_n(x)^2\right]} - 2F_X(x;\theta)\,\underbrace{E\left[F_n(x)\right]} + F_X(x;\theta)^2
\end{aligned}$$

$$\begin{aligned}
E\left[F_n(x)\right] &= \frac{1}{n}\sum_{i=1}^{n} E\left[I_{(-\infty,x]}(X_i)\right] \\
&= \frac{1}{n}\sum_{i=1}^{n} \int_{-\infty}^{\infty} I_{(-\infty,x]}(t)\, f_{X_i}(t)\, dt \\
&= \int_{-\infty}^{\infty} I_{(-\infty,x]}(t)\, I_{[0,\infty)}(t)\, \frac{1}{\theta} e^{-\frac{t}{\theta}}\, dt \\
&= \begin{cases} 0 & x < 0 \\ \frac{1}{\theta}\int_0^x e^{-\frac{t}{\theta}}\, dt & x \geq 0 \end{cases} \\
&= \begin{cases} 0 & x < 0 \\ \frac{1}{\theta}\int_0^x e^{-\frac{t}{\theta}}\, dt & x \geq 0 \end{cases}
\end{aligned}$$

$$\boxed{\int_0^x e^{-\frac{t}{\theta}}\, dt = \theta\left(1 - e^{-\frac{x}{\theta}}\right)}$$

$$\begin{aligned}
&= \begin{cases} 0 & x < 0 \\ 1 - e^{-\frac{x}{\theta}} & x \geq 0 \end{cases} \\
&= I_{[0,\infty)}(x)\left(1 - e^{-\frac{x}{\theta}}\right) = F_X(x;\theta)
\end{aligned}$$

$$E[F_n(x)^2] = E\left[\left(\frac{1}{n}\sum_{i=1}^{n} I_{(-\infty,x]}(X_i)\right)^2\right]$$

$$= \frac{1}{n^2}\sum_{i,j=1}^{n} E\left[I_{(-\infty,x]}(X_i)\, I_{(-\infty,x]}(X_j)\right]$$

$$= \frac{1}{n^2}\left(\sum_{i=j} E\left[I_{(-\infty,x]}(X_i)\right] + \sum_{i\neq j} E\left[I_{(-\infty,x]}(X_i)\right] E\left[I_{(-\infty,x]}(X_j)\right]\right)$$

$$= \frac{1}{n^2}\left(\sum_{i=j} E\left[I_{(-\infty,x]}(X_i)\right] + \sum_{i\neq j} E\left[I_{(-\infty,x]}(X_i)\right] E\left[I_{(-\infty,x]}(X_j)\right]\right)$$

$$= \frac{1}{n^2}\left(\sum_{i=j} F_X(x;\theta) + \sum_{i\neq j} F_X(x;\theta)^2\right)$$

$$= \frac{1}{n^2}\left(n F_X(x;\theta) + (n^2-n) F_X(x;\theta)^2\right)$$

$$= \frac{1}{n} F_X(x;\theta) + \frac{n-1}{n} F_X(x;\theta)^2$$

$$ECM(F_n(x)) = E\left[(F_n(x) - F_X(x;\theta))^2\right]$$

$$= \underbrace{E[F_n(x)^2]} - 2F_X(x;\theta)\, \underbrace{E[F_n(x)]} + F_X(x;\theta)^2$$

$$= \frac{1}{n} F_X(x;\theta) + (\cancel{1}-\tfrac{1}{n}) F_X(x;\theta)^2 \cancel{- 2F_X(x;\theta)^2} + \cancel{F_X(x;\theta)^2}$$

$$= \frac{1}{n}(1 - F_X(x;\theta))\, F_X(x;\theta)$$

$$E\left[e^{-\frac{xn}{n\hat{\theta}_n}}\right] = E\left[e^{(-xn)H}\right] = M_H(-xn) \qquad H = \frac{1}{n\hat{\theta}_n}$$

$$E\left[F_X\left(x; \tfrac{1}{n}(n\hat{\theta}_n)\right)\right] = E\left[1 - e^{-\frac{x}{\hat{\theta}_n}}\right]$$

$$= 1 - E\left[e^{-\frac{xn}{n\hat{\theta}_n}}\right]$$

$$= 1 - \int_{-\infty}^{\infty} e^{-\frac{xn}{t}} \underline{f_{n\hat{\theta}_n}}(t)\, dt$$

$$= 1 - \frac{1}{\Gamma(n)\theta^n} \int_0^{\infty} e^{-\frac{xn}{t}} t^{n-1} e^{-\frac{t}{\theta}}\, dt$$

$$= 1 - \frac{1}{\Gamma(n)\theta^n} \int_0^{\infty} t^{n-1} e^{-\frac{xn + \theta t^2}{t\theta}}\, dt$$