(a)

$$L(\mu_1,\mu_2,X) = \frac{1}{\sqrt{2\pi}^3}\exp\Big(-\frac{1}{2}\big[x_1^2+x_2^2+x_3^2-2\mu_1(x_1+x_3)-2\mu_2(x_2+x_3)+2\mu_1\mu_2+2\mu_1^2+2\mu_2^2\big]\Big)$$

$$\ell(\mu_1,\mu_2,X) = -\frac{3}{2}\log(2\pi) - \frac{1}{2}\big[x_1^2+x_2^2+x_3^2-2\mu_1(x_1+x_3)-2\mu_2(x_2+x_3)+2\mu_1\mu_2+2\mu_1^2+2\mu_2^2\big]$$

$$\partial_1\ell(\mu_1,\mu_2,X) = -\frac{1}{2}\big[-2(x_1+x_3)+2\mu_2+4\mu_1\big]$$

$$= x_1+x_3-\mu_2-2\mu_1$$

$$\partial_2\ell(\mu_1,\mu_2,X) = -\frac{1}{2}\big[-2(x_2+x_3)+2\mu_1+4\mu_2\big]$$

$$= x_2+x_3-\mu_1-2\mu_2$$

$$\boxed{\begin{aligned} x_1+x_3 &= 2\mu_1+\mu_2 \\ x_2+x_3 &= \mu_1+2\mu_2 \end{aligned}} \Rightarrow \begin{cases} \hat{\mu}_1 = \dfrac{2x_1+(x_3-x_2)}{3} \\ \hat{\mu}_2 = \dfrac{2x_2+(x_3-x_1)}{3} \end{cases}$$

$$2x_1-x_2+x_3 = 3\mu_1$$

(b)

$$R(\mu_1, \mu_2, X) = \frac{L(\mu_1, \mu_2, X)}{L(\hat{\mu}_1, \hat{\mu}_2, X)}$$

$$\frac{1}{\sqrt{2\pi}^3} \exp\left(-\frac{1}{2}\left[x_1^2 + x_2^2 + x_3^2 - 2\mu_1(x_1 + x_3) - 2\mu_2(x_2 + x_3) + 2\mu_1\mu_2 + 2\mu_1^2 + 2\mu_2^2\right]\right)$$

$$= \frac{\frac{1}{\sqrt{2\pi}^3} \exp\left(-\frac{1}{2}\left[x_1^2 + x_2^2 + x_3^2 - 2\mu_1(x_1 + x_3) - 2\mu_2(x_2 + x_3) + 2\mu_1\mu_2 + 2\mu_1^2 + 2\mu_2^2\right]\right)}{\frac{1}{\sqrt{2\pi}^3} \exp\left(-\frac{1}{2}\left[x_1^2 + x_2^2 + x_3^2 - 2\hat{\mu}_1(x_1 + x_3) - 2\hat{\mu}_2(x_2 + x_3) + 2\hat{\mu}_1\hat{\mu}_2 + 2\hat{\mu}_1^2 + 2\hat{\mu}_2^2\right]\right)}$$

$$= \exp\left(-\frac{1}{2}\left[x_1^2 + x_2^2 + x_3^2 - 2\mu_1(x_1 + x_3) - 2\mu_2(x_2 + x_3) + 2\mu_1\mu_2 + 2\mu_1^2 + 2\mu_2^2\right] + \frac{1}{2}\left[x_1^2 + x_2^2 + x_3^2 - 2\hat{\mu}_1(x_1 + x_3) - 2\hat{\mu}_2(x_2 + x_3) + 2\hat{\mu}_1\hat{\mu}_2 + 2\hat{\mu}_1^2 + 2\hat{\mu}_2^2\right]\right)$$

$$= \exp\left((\mu_1 - \hat{\mu}_1)(x_1 + x_3) + (\mu_2 - \hat{\mu}_2)(x_2 + x_3) - \mu_1\mu_2 - \mu_1^2 - \mu_2^2 + \hat{\mu}_1\hat{\mu}_2 + \hat{\mu}_1^2 + \hat{\mu}_2^2\right)$$

$$R(\mu_1, \mu_2, \underline{X}) \equiv \exp\Big((\mu_1 - \mu_1)(X_1 + X_3) + (\mu_2 - \hat{\mu}_2)(X_2 + X_3) - \mu_1\mu_2 - \mu_1^2 - \mu_2^2 + \hat{\mu}_1\hat{\mu}_2 + \hat{\mu}_1^2 + \hat{\mu}_2^2 \Big)$$

$$\hat{\mu}_1 = \frac{2X_1 + (X_3 - X_2)}{3}$$

$$\hat{\mu}_2 = \frac{2X_2 + (X_3 - X_1)}{3}$$

(c)

$$X_1 + X_3 = 2\mu_1 + \mu_2$$

$$X_2 + X_3 = \mu_1 + 2\mu_2$$

$$\hat{\mu}_1(\mu_2) = \frac{X_1 + X_3 - \mu_2}{2}$$

(μ_2 fijo)

$$\hat{\mu}_2(\mu_1) = \frac{X_2 + X_3 - \mu_1}{2}$$

(d)

$$L(\mu_1, \mu_2, X) = \frac{1}{\sqrt{2\pi}^3} \exp\left(-\frac{1}{2}\sum x_i^2 + \left[\mu_1(X_1 + X_3) + \mu_2(X_2 + X_3) - \mu_1\mu_2 - 2\mu_1^2 + 2\mu_2^2\right]\right)$$

$$R_p(\mu_1, \underline{X}) = \frac{L(\mu_1, \hat{\mu}_2(\mu_1), \underline{X})}{L(\hat{\mu}_1, \hat{\mu}_2, \underline{X})}$$

$$= \frac{\frac{1}{\sqrt{2\pi}^3} \exp\left(-\frac{1}{2}\sum x_i^2 + \left[\mu_1(X_1 + X_3) + \hat{\mu}_2(\mu_1)(X_2 + X_3) - \mu_1\hat{\mu}_2(\mu_1) - \mu_1^2 + 2\hat{\mu}_2(\mu_1)\right]\right)}{\frac{1}{\sqrt{2\pi}^3} \exp\left(-\frac{1}{2}\sum x_i^2 + \left[\hat{\mu}_1(X_1 + X_3) + \hat{\mu}_2(X_2 + X_3) - \hat{\mu}_1\hat{\mu}_2 - 2\hat{\mu}_1^2 + 2\hat{\mu}_2^2\right]\right)}$$

$$R_p(\mu_2, \underline{x}) = \frac{L(\hat{\mu}_1(\mu_2), \mu_2, \underline{x})}{L(\hat{\mu}_1, \hat{\mu}_2, \underline{x})}$$