$$T = \{\underbrace{a, b, c}_{\Delta}, \underbrace{d}_{@}\}$$

$$S = \{\Delta, @, \square\}$$

$$\downarrow$$

$$S = \{\Delta, @, \square\}$$

ya es étale

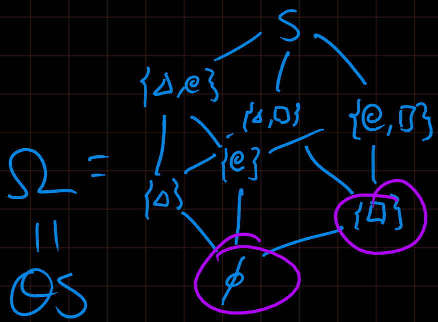
$$\Omega = \mathcal{O}S = \left\{ \begin{tikzcd} & S \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & \\ \{\Delta, @\} \arrow[d, dash] \arrow[dr, dash] & \{\Delta, \square\} \arrow[dl, dash] \arrow[dr, dash] & \{@, \square\} \arrow[dl, dash] \arrow[d, dash] \\ \{\Delta\} \arrow[dr, dash] & \{@\} \arrow[d, dash] & \{\square\} \arrow[dl, dash] \\ & \emptyset & \end{tikzcd} \right\}$$

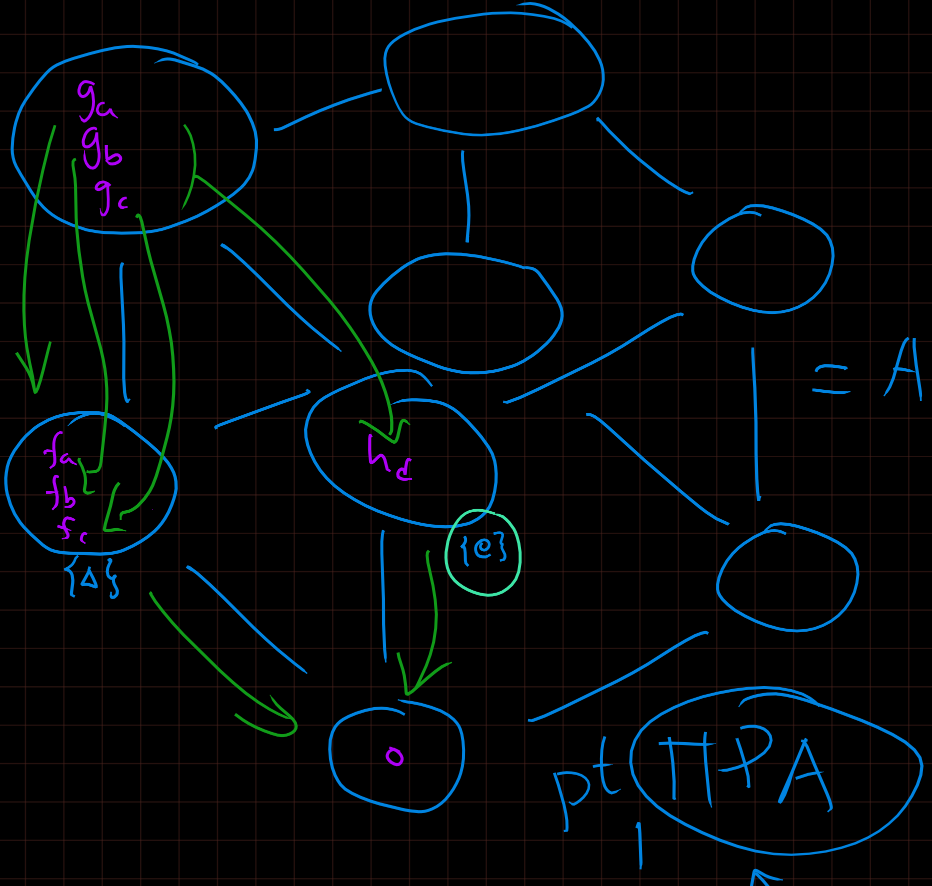
$$Et/S \longrightarrow Stk(\Omega)$$

$$T = \{\underbrace{a, b, c}_{\Delta}, \underbrace{d}_{@}\}$$

$$\pi$$

$$S = \{\Delta, @, \square\}$$





$$\hat{g}_a : \begin{array}{l} g_a \longmapsto \gamma \\ g_b \longmapsto \beta \\ g_c \longmapsto \beta \\ f_a \longmapsto \alpha \\ f_b \longmapsto 0 \\ f_c \longmapsto 0 \end{array}$$

$$\begin{tikzcd}
\gamma \arrow[d, dash] \arrow[dr, dash] & \\
\alpha \arrow[dr, dash] & \beta \arrow[d, dash] \\
& 0
\end{tikzcd}$$

$$[\![a=b]\!] = \bigvee\{u \in \mathcal{O}S \mid a|_u = b|_u\}$$

$$= \{s \in S \mid a(s) = b(s)\}$$

$$[\![g_a]\!] = \{\Delta, @\} = \alpha$$

$$\mathcal{S}A = \left\{ P : A \to \Omega \;\middle|\; \begin{array}{ll} P_a \wedge [\![a=b]\!] \leq P_b & (\text{ext}) \\ P_a \wedge P_b \leq [\![a=b]\!] & (\text{s.f}) \end{array} \right\}$$

$$\begin{array}{c}\psi\\ P\end{array}$$

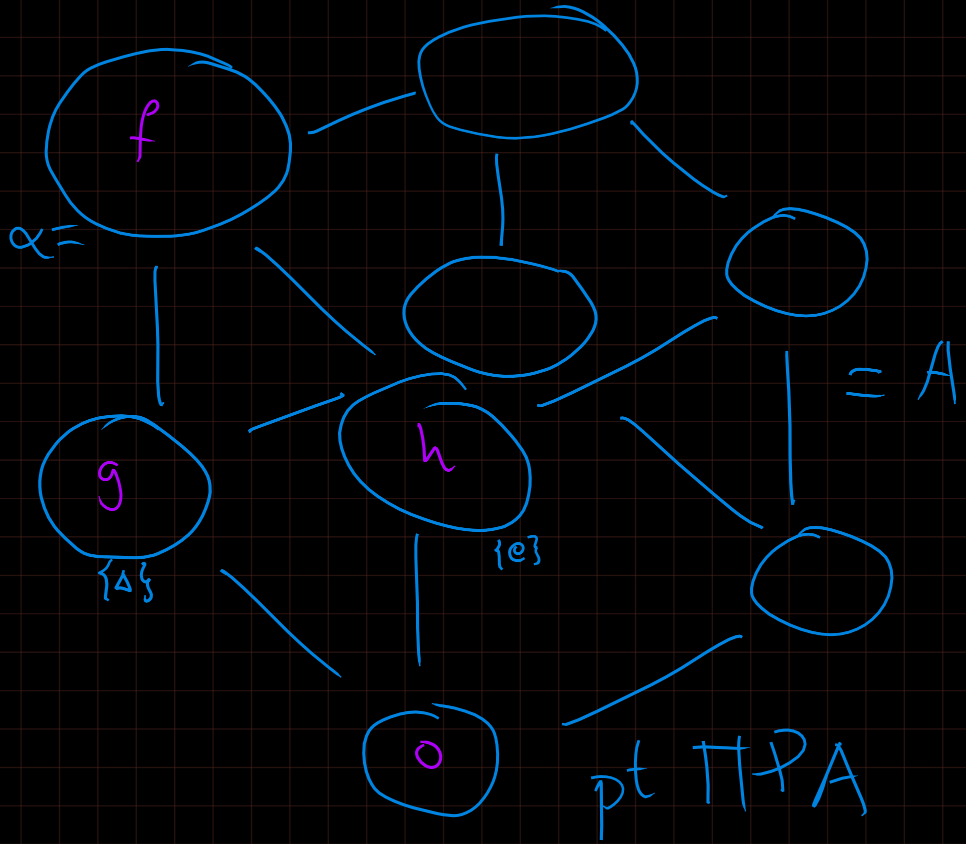
$)_{\sigma}$

$$\mathcal{S}A = \left\{ \langle P, x \rangle \;\middle|\; \begin{array}{ll} P : A \to \Omega & \\ Pa \wedge [\![a=b]\!] \le P_b & (\text{ext}) \\ Pa \le [\![a]\!] \wedge x & (\text{est}) \end{array} \right\} \mathrm{pt}\,\Omega$$

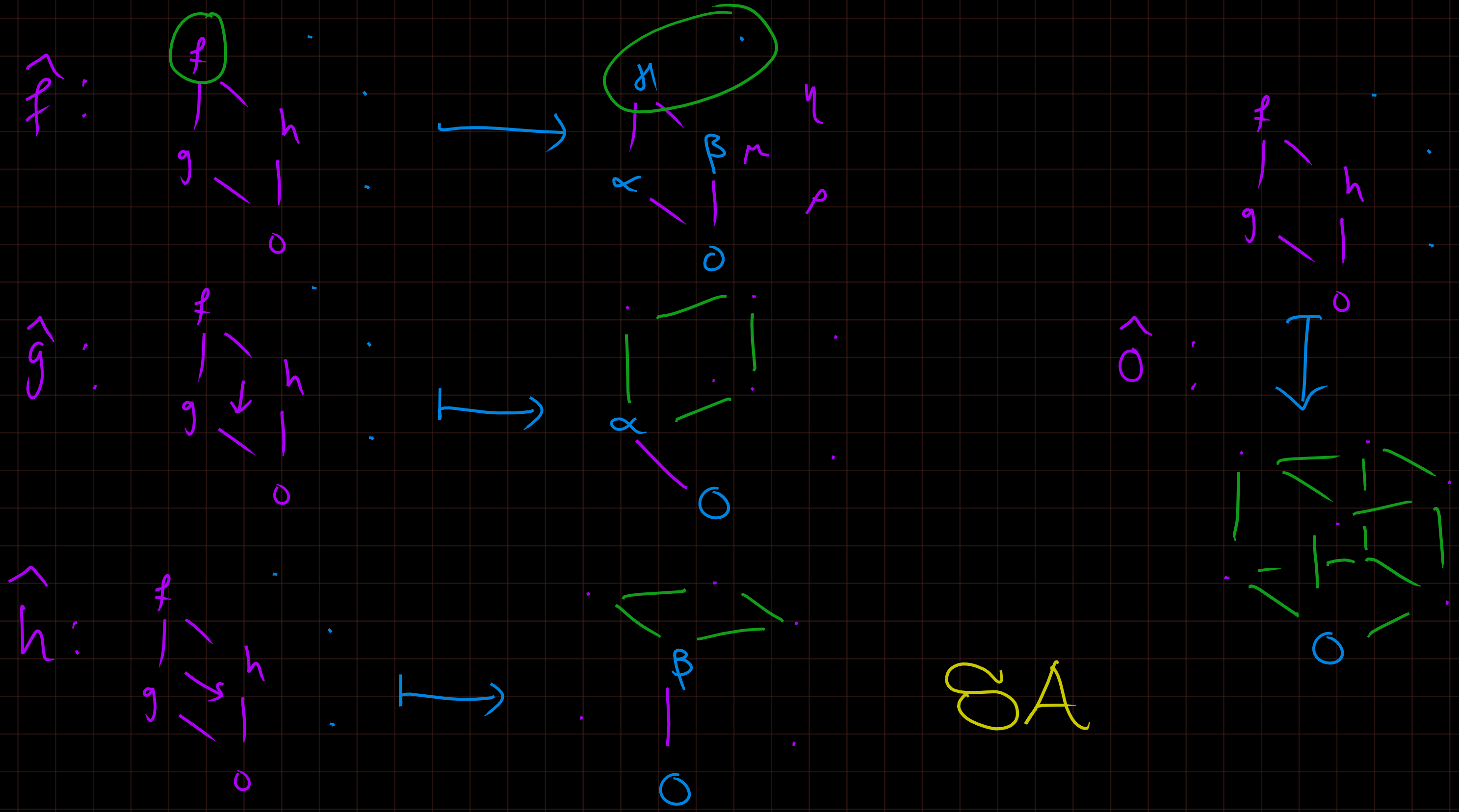
$R = \{a, b\}$

$\downarrow \pi$

$S = \{\Delta, e, \square\}$



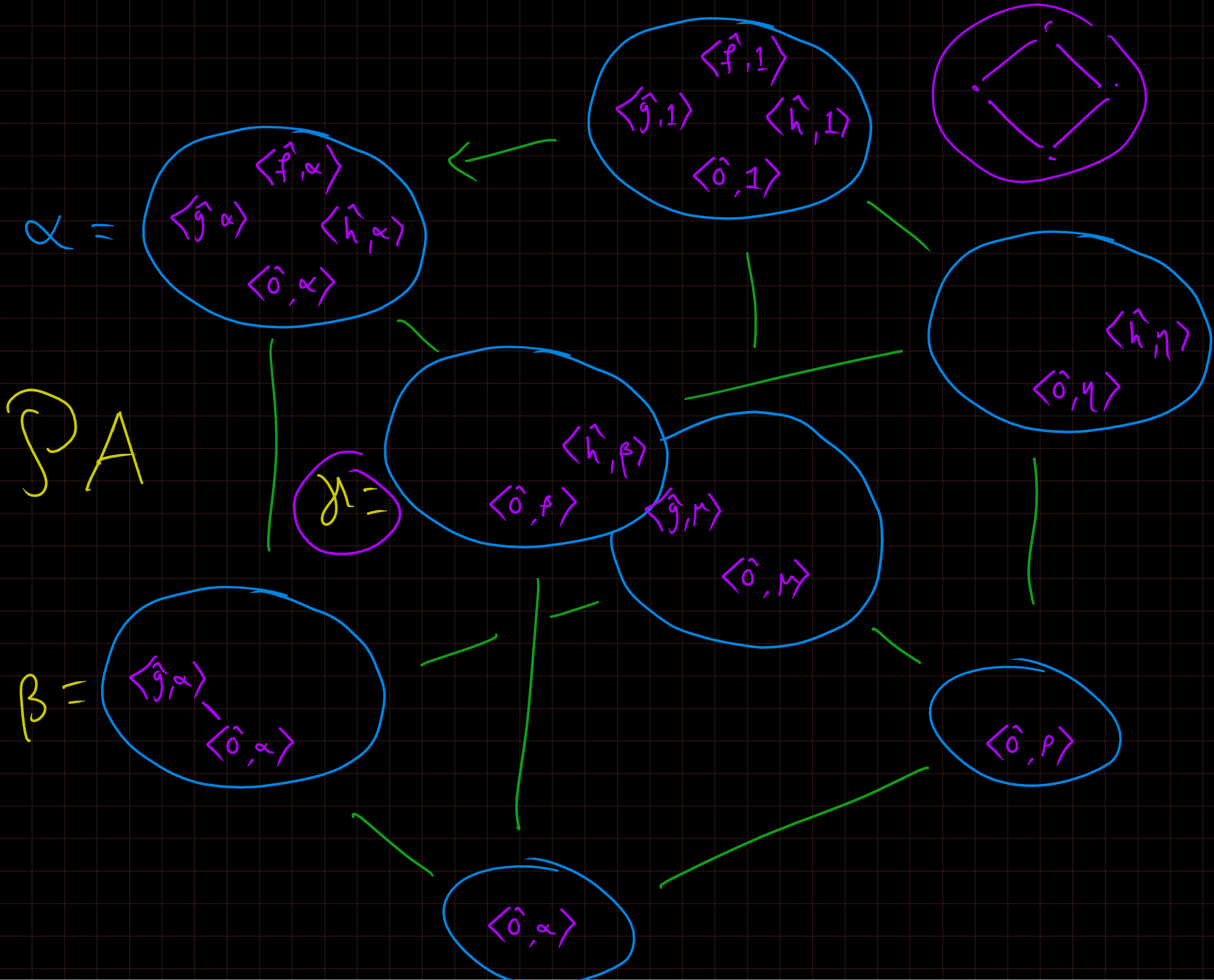
1. estos son todos los unip.


$\hat{f}$:
$\hat{g}$:
$\hat{h}$:
$\hat{0}$:
f
g
h
0
α
β
γ
η
μ
ρ
SA

$$PA = \left\{ \langle p, x \rangle \;\middle|\; \begin{array}{l} Pa \wedge [\![a = b]\!] \leq Pb \\ Pa \leq [\![a]\!] \wedge x \end{array} \right\}$$

$$[\![\langle p, x \rangle]\!] = x$$

2. Basta tomar P unipuntual.



α =
⟨f̂, α⟩
⟨ĝ, α⟩
⟨ĥ, α⟩
⟨ô, α⟩
⟨f̂, 1⟩
⟨ĝ, 1⟩
⟨ĥ, 1⟩
⟨ô, 1⟩
℘A
γ =
⟨ĥ, β⟩
⟨ô, β⟩
⟨ĥ, η⟩
⟨ô, η⟩
⟨ĝ, μ⟩
⟨ô, μ⟩
β =
⟨ĝ, α⟩
⟨ô, α⟩
⟨ô, ρ⟩
⟨ô, α⟩

$$
\begin{tikzcd}
\mathcal{P} : \Omega^{op} \arrow[rr] \arrow[dr] & & \mathrm{Con} \\
& \mathrm{Frm} \arrow[ur] &
\end{tikzcd}
$$

$$\mathcal{P}A \simeq \mathrm{Hom}_{\mathrm{Con}(\Omega)}(A, \Omega)$$

$X : \mathcal{C}$ es "objeto $\underline{\quad \omega \quad}$"

$$
\begin{tikzcd}
\mathrm{Hom}_{\mathcal{C}}(-, X) : \mathcal{C}^{op} \arrow[rr] \arrow[dr] & & Con \\
& \omega \arrow[ur] &
\end{tikzcd}
$$

$$\mathrm{Hom}_{Con(\Omega)}\left[-, \underset{\sim}{\Omega}\right]$$

$\underline{\quad \Omega \qquad [\![a=b]\!] = (a \succ b) \wedge (b \succ a)}$

$$\Omega(-) : \Omega^{op} \longrightarrow \mathrm{Con}$$
$$x \longmapsto \downarrow x$$

$$\left\{ |\Omega| = \{ (x, y) \mid x \leq y \} \right.$$

$$X = \{\Delta, \odot, \square\}$$

$$Et/_X \simeq Sh(X) \simeq Stk(\;)$$

Existe un morfismo $1 \xrightarrow{t} \Omega$ tal que todo monomorfismo $B \xrightarrow{f} A$ encaja en un diagrama "cartesiano"

$$\begin{tikzcd} B \arrow[r] \arrow[d, hook] \arrow[dr, phantom, "\lrcorner", very near start] & 1 \arrow[d, "t"] \\ A \arrow[r] & \Omega \end{tikzcd}$$

$$Sub(A) \simeq Hom_{Stk(\Omega)}(A, \Omega).$$

$$\begin{tikzcd}
\{a,b\} = R \arrow[d, shift right] \arrow[d, shift left] \arrow[d, "\pi"'] \\
\{\triangle, \bigcirc, \square\} = S \arrow[u, bend right] & \arrow[l]
\end{tikzcd}$$

$$\begin{tikzcd}
1 \arrow[r] & A \arrow[d, "{[\![\cdot]\!]}"] & \wp A \arrow[d, "{[\![\cdot]\!]}"] \\
& \mathcal{O}S & \mathcal{O}S
\end{tikzcd}$$

$$\Theta R = \pi(\wp A) = \{ p \in \wp A \mid [\![p]\!] = 1 \}$$

$$\Theta(\pi^{-1}(x)) \longrightarrow \{ p \in \wp A \mid [\![p]\!] = x \}$$

$$
\begin{tikzcd}
1 \arrow[d, tail] & 1 \arrow[d, "t"] \\
1 \arrow[r, "t"] & \underset{\sim}{\Omega}
\end{tikzcd}
$$

Con

$$\{*\} \xrightarrow{x} A$$

Sub(1)

conjunto.

Es un marco de conjuntos

Externo (afuera de Stk(Ω))

$\underset{\sim}{\Omega}$

Ω-conjunto

Es un marco de Ω-conjuntos

Interno en Stk(Ω)

$|\underset{\sim}{\Omega}| = \Omega$, $[\![a=b]\!] = (a \Leftrightarrow b)$ es un Ω-conjunto

Ω-conjunto $[\![a]\!] = 1$

$\mathrm{Con}(\Omega)$

$|\mathcal{S}\underset{\sim}{\Omega}| = \{(x,y) \mid x \leq y\}$ $(x,y)\cdot z = (x \wedge z, y \wedge z)$

$[\![(x,y)]\!] = y$ $\mathrm{Stk}(\Omega)$

$= \coprod_{y \in \Omega} \downarrow y$

$\mathrm{Gav}(\Omega) \longrightarrow \mathcal{S}k(\Omega)$

$\Omega[y] = \downarrow y$

$\underset{\sim}{\Omega} \simeq \mathcal{S}\underset{\sim}{\Omega}$ en $\mathrm{Con}(\Omega)$

$|1| = \Omega \qquad [\![a = b]\!] = a \wedge b \qquad$ en $\mathrm{Stk}(\Omega)$

$a \cdot x = a \wedge x.$

$$\begin{tikzcd}
B \arrow[r] \arrow[d, tail, "f"'] \arrow[dr, phantom, "\lrcorner", very near start] & 1 \arrow[d, "t"] & a \arrow[d, mapsto] \\
A \arrow[r, "\chi_f"'] & \underset{\sim}{\Omega} & (a,a)
\end{tikzcd}$$

$$\mathrm{Hom}(A, \underset{\sim}{\Omega}) \simeq \mathrm{Sub}(A)$$

$$
\begin{tikzcd}
B \arrow[r] \arrow[d, tail, "f"'] \arrow[dr, phantom, "\lrcorner", very near start] & 1 \arrow[d, "t"] & C \arrow[l] \arrow[d, tail, "g"] \arrow[dl, phantom, "\llcorner", very near start] \\
A \arrow[r, "\chi_f"'] & \Omega & A \arrow[l, "\chi_g"] \\
A \arrow[r, "\chi_f"'] & \Omega & A \arrow[l, "\chi_g"]
\end{tikzcd}
$$

$$
\begin{tikzcd}
& A \arrow[dl, "\chi_f"'] \arrow[dr, "\chi_g"] \arrow[d, dashed] \arrow[dd, bend right=60, "\chi_{f\wedge g}"'] & \\
\Omega & \Omega \times \Omega \arrow[l, "\pi_1"] \arrow[r, "\pi_2"'] \arrow[d, "\wedge"] & \Omega \\
& \Omega &
\end{tikzcd}
$$

$$\begin{tikzcd}
B \wedge C \arrow[r] \arrow[d, tail] \arrow[dr, phantom, "\lrcorner", very near start] & 1 \arrow[d, tail, "t"] \\
A \arrow[r, "\chi_{f \wedge g}"'] & \underset{\sim}{\Omega}
\end{tikzcd}$$

$$P\underset{\sim}{\Omega} \longrightarrow \underset{\sim}{\Omega}$$

Con

$B, C \subseteq A$

$B \cap C \subseteq A$

$\mathrm{Stk}(\Omega)$

$B, C \hookrightarrow A$

$B \wedge C \hookrightarrow A$

$\mathcal{G} \in \mathrm{Stk}(\Omega) \simeq \mathrm{Con}(\Omega)$

$$\begin{tikzcd} & \mathcal{G} \times \mathcal{G} \arrow[r, "\cdot"] & \mathcal{G} \\ A \arrow[ur, "\mathcal{G}"] \arrow[r] & \mathcal{G} \quad 1 \arrow[r, "e"'] & \mathcal{G} \end{tikzcd}$$

$$\mathcal{G} \xrightarrow[i]{} \mathcal{G}$$

$g, *$

$$\begin{tikzcd} \mathcal{G} \times 1 \arrow[r] \arrow[dr] & {} \\ & \mathcal{G} \end{tikzcd}$$

$$\begin{tikzcd} g, h, k \arrow[r, mapsto] \arrow[d, mapsto] & g, hk \arrow[d, mapsto] \\ gh, k \arrow[r, mapsto] & (gh)k \end{tikzcd} \qquad g(hk)$$

$$\begin{tikzcd} \mathcal{G}\times\mathcal{G}\times\mathcal{G} \arrow[r, "\mathrm{id}\times \cdot"] \arrow[d, "\cdot \times \mathrm{id}"'] \arrow[dr, phantom, "="] & \mathcal{G} \arrow[d, "\cdot"] \\ \mathcal{G}\times\mathcal{G} \arrow[r, "\cdot"] & \mathcal{G} \end{tikzcd}$$

$A \qquad \mathrm{Hom}_{Stk(\Omega)}(A, \mathcal{G}) \in \mathcal{G}rp$

$$\begin{array}{c} \mathrm{Hom}_{Stk(\Omega)}(A,\mathcal{G}) \times \mathrm{Hom}_{Stk(\Omega)}(A,\mathcal{G}) \\ \downarrow \cdot \\ \mathrm{Hom}_{Stk(\Omega)}(A,\mathcal{G}) \end{array}$$

$$\wp(-): \mathrm{Stk}(\Omega) \longrightarrow \mathrm{Stk}(\Omega)$$

$$\wp(-): \mathrm{Gav}(\Omega) \longrightarrow \mathrm{Gav}(\Omega)$$

$x \in \Omega$