<u>Modelo</u>: Sea $u = u(x,y,t)$ la función que describe las vibraciones, entonces la EDP correspondiente es

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad 0<x<L,\ 0<y<M$$

dado que la membrana está fija $\Rightarrow$

$$\text{CF}: \begin{cases} u(0,y,t) = 0, & u = (L,y,t) = 0, & 0<y<M, & t\geq 0 \\ u(x,0,t) = 0, & u = (x,M,t) = 0, & 0<x<L, & t\geq 0 \end{cases}$$

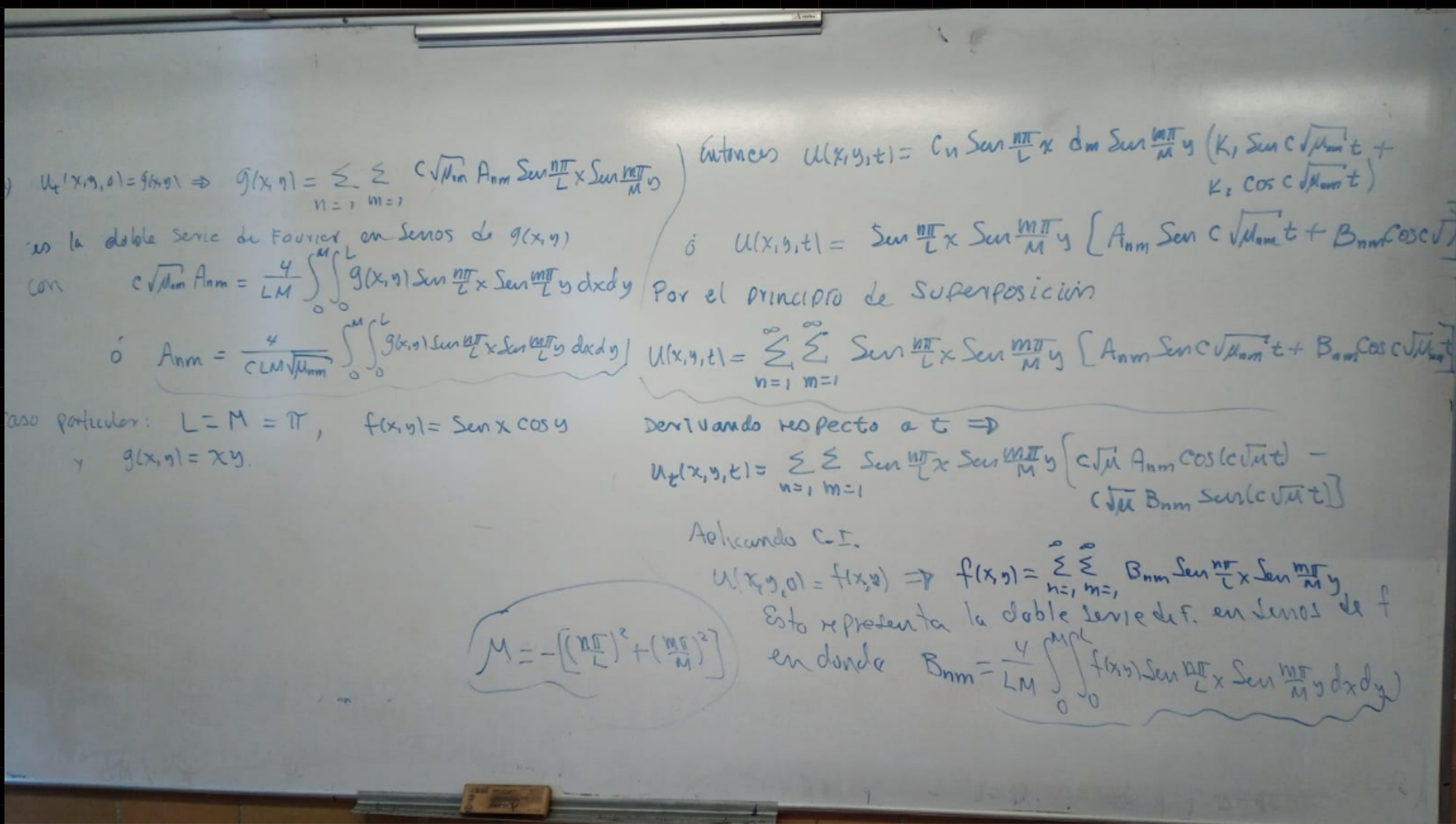
$$\text{CI}: \quad u(x,y,0) = f(x,y), \quad \frac{\partial u}{\partial t}(x,y,0) = g(x,y), \quad \begin{matrix} 0\leq x\leq L \\ 0\leq y\leq M \end{matrix}$$

$$u(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \sin\left(\frac{n\pi}{L}x\right)\sin\left(\frac{m\pi}{M}y\right)\left[A_{nm}\sin\left(c\sqrt{\mu_{nm}}\,t\right) + B_{nm}\cos\left(c\sqrt{\mu_{nm}}\,t\right)\right]$$

$$A_{nm} = \frac{4}{cLM\sqrt{\mu_{nm}}}\int_0^M\int_0^L g(x,y)\sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi y}{M}\right)dx\,dy,$$

$$\mu_{nm} = \left(\frac{n\pi}{L}\right)^2 + \left(\frac{m\pi}{M}\right)^2$$

$$B_{nm} = \frac{4}{LM}\int_0^M\int_0^L f(x,y)\sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi y}{M}\right)dx\,dy,$$



$L = M = \pi$

$f(x,y) = \sin x \cos y$

$g(x,y) = xy$

$$A_{nm} = \frac{4}{CLM\sqrt{\mu_{nm}}} \int_0^M \int_0^L g(x,y) \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi y}{M}\right) dx dy$$

$$= \frac{4}{C\pi^2\sqrt{\mu_{nm}}} \int_0^\pi x \sin(nx)\, dx \int_0^\pi y \sin(my)\, dy$$

$$= \frac{4}{C\pi^2\sqrt{\mu_{nm}}} \frac{\pi \cos(\pi n)}{n} \frac{\pi \cos(\pi m)}{m}$$

$$= \frac{4}{C\sqrt{\mu_{nm}}\, nm} (-1)^n (-1)^m \qquad = \frac{4(-1)^{n+m}}{C\sqrt{\mu_{nm}}\, nm}$$

$$B_{nm} = \frac{4}{LM} \int_0^M \int_0^L f(x,y) \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi y}{M}\right) dx dy$$

$$= \frac{4}{\pi^2} \int_0^\pi \sin x \sin(nx)\, dx \int_0^\pi \cos y \sin(my)\, dy$$

$$= \begin{cases} 0 & m = 1 \\ \dfrac{4}{\pi^2} \displaystyle\int_0^\pi \sin x \sin(nx)\, dx \; \dfrac{m((-1)^m+1)}{m^2-1} & m > 1 \end{cases}$$

$$= \begin{cases} 0 & m = 1 \quad \text{ó} \quad n > 1 \\ \dfrac{2}{\pi^2} \dfrac{m((-1)^m+1)}{m^2-1} & m > 1 \quad \text{y} \quad n = 1 \end{cases}$$

$$x' = -2y + yz - x^3$$
$$y' = x - xz - y^3$$
$$z' = xy - z^3$$

$$V = ax^2 + by^2 + cz^2$$

ent $\quad V' = 2ax(-2y + yz - x^3)$
$$+ 2by(x - xz - y^3)$$
$$+ 2cz(xy - z^3)$$
$$= (-4a + 2b)xy + 2(a - b + c)xyz - 2ax^4 - 2by^4 - 2cz^4$$

$$\left.\begin{aligned} -4a + 2b &= 0 \\ a - b + c &= 0 \end{aligned}\right\} \quad \begin{aligned} b &= 2a \\ a - 2a + c &= 0 \\ c &= a \end{aligned} \qquad \begin{aligned} a &= 1 \\ b &= 2 \\ c &= 1 \end{aligned}$$