Fundamental Algorithms, Section 003
Homework 9, Additional Problems, Fall 22.

1. Let $G = (V, E)$ be an undirected graph, and let $e \in E$ be a designated edge. Give a linear time—$O(|V| + |E|)$ time—algorithm to test if there is a cycle containing edge e. Explain why your algorithm is correct and justify the running time.

2.a. Let $G = (V, E)$ be a dag, a directed acyclic graph, with a designated target vertex $t \in V$. Suppose each edge has an integer length, possible negative. Give a linear time—$O(|V|+|E|)$ time—algorithm to find the length of shortest paths from every vertex $v \in V$ to the target vertex t.

b. Now, let $s \in V$ be a designated source vertex. Give a linear time algorithm to find the length of shortest paths from s to every vertex $v \in V$.

Remember to explain why your algorithms are correct and justify their running times.

3. Let $G = (V, E)$ be an undirected graph. Suppose we perform a DFS traversal of G which produces post-order numbers for each vertex, denoted by $\text{post}(v)$, $v = 1, 2, \ldots, n = |V|$. Let $u, v \in V$. Suppose that $\text{post}(u) < \text{post}(v)$. Must v be an ancestor of u, yes or no? Justify your answer. For a "yes", prove the claim; for a "no" give a graph and a DFS traversal of the graph in which the claim is not true.