Fundamental Algorithms, Section 003
Homework 11, Additional Problems, Fall 22.

1. Let $G = (V, E)$ be a weighted directed graph and let $s \in V$ be a designated vertex. In addition to the weight $w(e)$ for each edge $e \in E$, there is a weight $c(v)$ for each vertex $v \in V$. All weights are non-negative. The length of a path is the sum of the edge and vertex weights on the path. The task is to provide an algorithm for the single source shortest paths problem with this modified definition of path length.
i. Solve the problem by by modifying Dijkstra's algorithm. Argue that your modification is correct. Also, argue that the resulting algorithm has the same running time as Dijkstra's algorithm on a graph of n vertices and m edges, up to constant factors.
ii. Solve the problem by means of a reduction to the standard SSSP problem. Argue that your reduction is correct. Again, argue that the resulting algorithm has the same running time as Dijkstra's algorithm on a graph of n vertices and m edges, up to constant factors. You may assume $n \leq m$.

2. Let $G = (V, E)$ be a directed weighted graph and let $s \in V$ be a designated vertex. Assume that all edges have non-negative weight. An even path is defined to be a path with an even number of edges. By means of a reduction, use the unmodified Dijkstra's algorithm to compute the shortest even path from s to each vertex $v \in V$. Argue that your reduction is correct. Your algorithm should run in the same time as Dijkstra's algorithm on a graph of n vertices and m edges, up to constant factors; argue that it does so.