

Fundamental Algorithms, Section 003  
Homework 10, Additional Problems, Fall 22.

1. Let  $H = (X, F)$  be a dag. Suppose you are given a linear time algorithm to compute for each vertex  $x$  in a dag the sum of the weights of the vertices reachable from  $x$  (which includes vertex  $x$ ).

Let  $G = (V, E)$  be a directed graph. Suppose that each vertex  $v$  has a weight  $W[v]$  stored in array  $W[1 : n]$ . Using the above algorithm as a subroutine, give a linear time algorithm to compute the sum of the weights of the vertices reachable from each vertex in  $G$ , storing the result in array  $\text{ReachWt}[1 : n]$ . You may quote standard procedures. Remember to justify your runtime.

2. Consider the single source low altitude path problem; the input consists of a directed graph  $G = (V, E)$ , and a source vertex  $s \in V$ . Now, each edge has a positive integer height. The height of a path is the maximum of the heights of the edges on the path. The task is to find the minimum height path from  $s$  to each vertex  $v \in V$ . Solve this problem by modifying Dijkstra's algorithm.

Hint. What changes is how you calculate the “length” of a path.