

1. Consider Diffie-Hellman Key Exchange where Alice and Bob have chosen prime value 17 and primitive root 5.

If Alice's secret key is 4 and Bob's secret key is 6, determine the secret key they exchanged. (5 points)

From Alice, $\alpha = 4 \xleftarrow{R} Z_{17}, u \leftarrow g^\alpha = 5^4 = 625$

From Bob, $\beta = 6 \xleftarrow{R} Z_{17}, v \leftarrow g^\beta = 5^6 = 15625$

Secret key from Alice: $w \leftarrow v^\alpha = 15625^4 = 59,604,644,775,390,625$

Secret key from Bob: $w \leftarrow u^\beta = 625^6 = 59,604,644,775,390,625$

2. Using the RSA cryptosystem, encrypt the message $x = 3$, assuming the two primes chosen to generate the keys are $p = 13$ and $q = 7$.

You should choose a value $e < 10$. Show your calculations and assumptions. (5 points)

Assume $e = 5$, here we have $\gcd(e, p-1) = 1, \gcd(e, q-1) = 1$ and $p \nmid q$

$n \leftarrow pq = 91$

$d \leftarrow e^{-1} \bmod (p-1)(q-1) = 5^{-1} \bmod 72 = 29$

$pk \leftarrow (91, 5), sk \leftarrow (91, 29)$

$F(pk, x) := 3^5 \bmod 91 = 61$

Check: $I(sk, y) := 61^{29} \bmod 91 = 3$