Fundamental Algorithms, Section 003
Homework 4, Additional Problems, Fall 22.

1. Let $A[1:m]$ and $B[1:n]$ be two unsorted arrays of integers. Suppose that $m \leq n$. Show how to test if $A$ and $B$ are disjoint in $O(n \log m)$ time. Justify your running time bound and explain why your algorithm is correct.

2. Suppose $n$ distinct items are inserted one by one into an initially empty binary search tree. Suppose further that all $n!$ orderings of the items are equally likely. Averaging over all $n!$ trees, give a recurrence equation for the sum of the depths of the $n$ items, measured in nodes traversed to reach each item.

You have seen a very similar recurrence equation for another problem. What is this other recurrence equation measuring?
Hint. What is the probability that the $i$th smallest item is the first to be inserted? If it is the first, where will it be located in the search tree, and what can you say about where the other items are located?

3. Suppose you are given an array $A[1:n]$ of integers but with only $k$ distinct values. Give an $O(n \log k)$ time algorithm to sort this array. Justify your running time bound and explain why your algorithm is correct.
Hint: modify merge sort to use buckets for equal valued items.