Module 2: Lesson 1

Merton Model Calibration

Outline

- Revisiting Merton (1976) model
- Merton (1976) pricing via Lewis (2001)
- Merton model calibration

Merton (1976)

During the previous course on Derivative Pricing, we looked at the most famous example of jump diffusion models: Merton (1976).

Let's revisit the risk-neutral dynamics (SDE) of an asset following this model:

$$dS_t = (r - r_J)S_t dt + \sigma S_t dZ_t + J_t S_t dN_t$$

where,

r is constant risk-free short rate,

$r_J \equiv \lambda \left(e^{\mu_J + \delta^2/2} - 1 \right)$ is the drift correction for the jump,

σ is constant volatility of S,

Z_t is a standard Brownian motion,

J_t is the jump at date t, with normal distribution $log(1 + J_t) \approx N(log(1 + \mu_J) - \frac{\delta^2}{2}, \delta^2)$,

N_t is a Poisson process with intensity λ.

We have already worked with this model in Derivative Pricing, via Monte-Carlo methods.

Now, we are going to see how we can apply Fourier methods to this model.

Merton (1976) via Lewis (2001)

Remember that the beauty of the Lewis (2001) approach is that we just need the characteristic function ,$\varphi(u, T)$, of the underlying asset process to price an option:

$$C_0 = S_0 - \frac{\sqrt{S_0 K} e^{-rT}}{\pi} \int_0^{\infty} \mathbf{Re}[e^{izk} \varphi(z - i/2)] \frac{dz}{z^2 + 1/4}$$

In the case of Merton (1976), the expression for the characteristic function is:

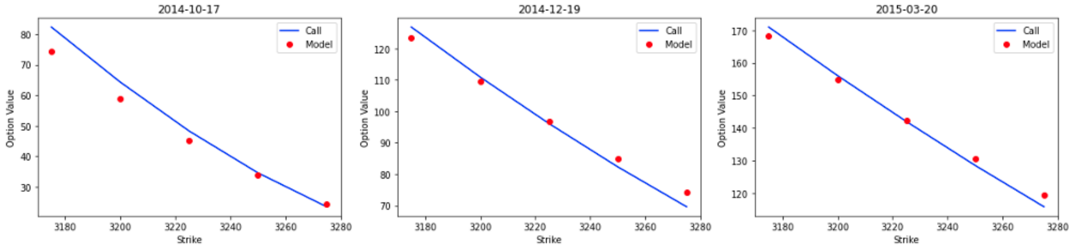
$$\varphi_0^{M76}(u, T) = e^{\left(\left(iu\omega - \frac{u^2\sigma^2}{2} + \lambda(e^{iu\mu_j - u^2\delta^2/2} - 1)\right) T\right)}$$

where,

$$\omega = r - \frac{\sigma^2}{2} - \lambda\left(e^{\mu_j + \delta^2/2} - 1\right)$$

Merton (1976) model calibration

As was the case with Heston model before, now that we have a semi-analytical expression for the price of the option, we can calibrate the model to options market data for different maturities:



Summary of Lesson 1

In Lesson 1, we have:

- Revisited the Merton (1976) model
- Used the Lewis (2001) approach to Merton (1976)
- Calibrated the Merton (1976) model to market prices

⇒ **References for this Lesson:**

Hilpisch, Yves. *Derivatives Analytics with Python: Data Analysis, Models, Simulation, Calibration and Hedging*. John Wiley & Sons, 2015.

⇒ **TO DO NEXT**: In the notebook associated with this lesson, you will find a detailed practical example of pricing and calibration under the Merton (1976) model.

⇒ In the next lesson, we will see how we can combine jump diffusion and stochastic volatility models so as to move closer to real observed prices.