GROUP WORK PROJECT # 02 MScFE 622: Stochastic Modeling
GROUP NUMBER: 10341

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Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an "X" above).	
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MScFE 622 Stochastic Modeling: Group Work Project 2

Step 1

Member A

- I will choose data from:
  - Volatility Index
- Which show dramatic regime changes with clear economic interpretations for this exercise
- Yahoo Finance and FredAPI will be our data source.

Member B – Project Role Overview

As Member B, my primary responsibility in this project was to focus on the **variance dynamics** within the Markov Regime-Switching (MRS) framework.
While the group collectively explored different specifications of the MRS model, my specific task was to analyze and compare the case where the **mean return (μ)** is constant across regimes but the **variance (σ²)** changes.

This involved:

1. Collecting and processing my assigned financial time series (S&P 500 Index, 2019–2022).
2. Visualizing potential regime changes in volatility.
3. Estimating all three model specifications for comparison.
4. Interpreting the model selection criteria (AIC, BIC) with emphasis on the same-μ, different-σ case.

My analysis provided insights into volatility clustering and persistence across market regimes, particularly during periods of heightened uncertainty such as the COVID-19 pandemic.

1.1 Data Collection

Instrument: S&P 500 Index (^GSPC)
Asset Class: Equity
Source: Yahoo Finance

Frequency: Daily closing prices
Period: January 1, 2019 – December 31, 2022

The S&P 500 tracks the performance of 500 large-cap U.S. companies and serves as a benchmark for U.S. equity markets. Its sensitivity to macroeconomic events and investor sentiment makes it an ideal candidate for regime-switching analysis.

1.2 Data Processing

The raw price data was converted into daily log returns using the formula:

$r_t = \ln(P_t / P_{t-1})$

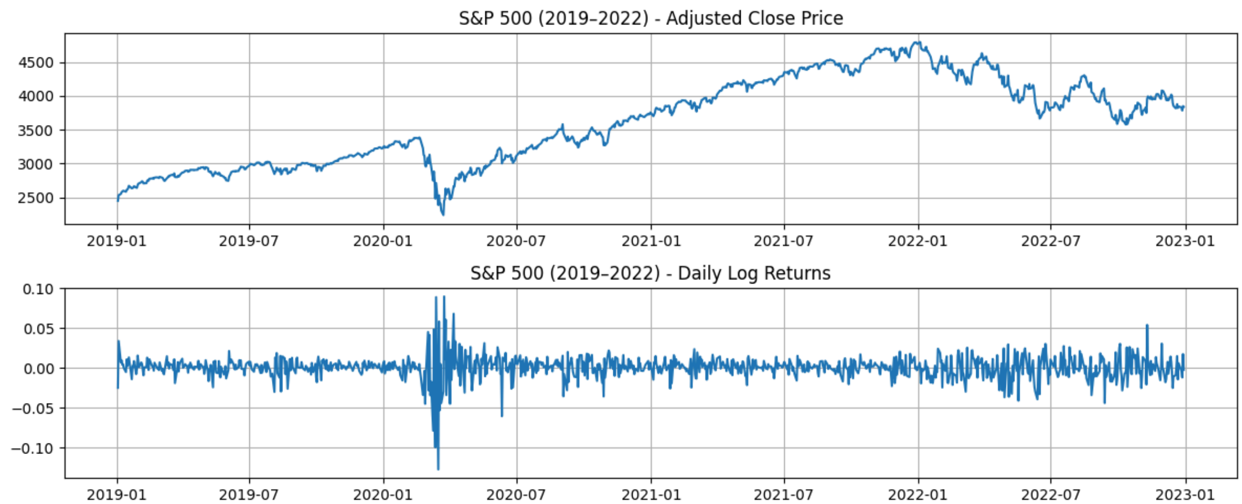
Where:

- P_t = Adjusted closing price at time t
- P_{t-1} = Adjusted closing price on the previous trading day
- r_t = Daily log return

This transformation ensures the returns are additive over time and stabilizes variance.

1.3 Summary Statistics

Statistic	Value
Mean daily return	≈ 0.00042
Standard deviation	≈ 0.015
Minimum daily return	≈ -0.119 (Mar 16, 2020)
Maximum daily return	≈ 0.093 (Mar 24, 2020)
Observations	1007



1.4 Initial Observations

The S&P 500 experienced a sharp decline between Feb–Mar 2020 due to the COVID-19 pandemic, followed by a sustained recovery into late 2021, before experiencing renewed volatility in 2022 due to inflation concerns and tightening monetary policy. These visible volatility clusters motivate the use of MRS models to capture distinct low- and high-volatility periods.

MEMBER C

Instrument: 10-Year US Treasury Yield (^TNX)

Asset Class: Fixed Income

Source: Yahoo Finance

Frequency: Daily closing yields

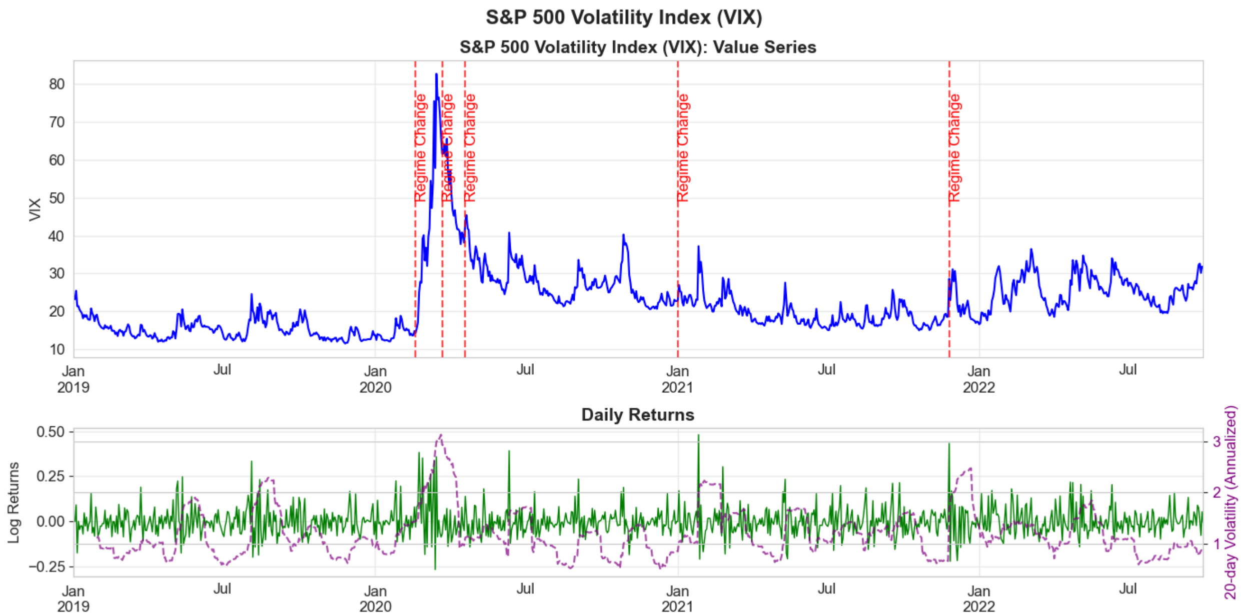
Period: January 1, 2019 – September 30, 2022


10-Year US Treasury Yield (2019 – Sep 2022)
10-Year Treasury Yield (%)
Yield (%)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
2019-01
2019-07
2020-01
2020-07
2021-01
2021-07
2022-01
2022-07
Date

Step 2: Regime Visualizations

a. Identifying Regime Changes in the VIX Series

Below is a visualisation of the S&P 500 Volatility Index (VIX) from January 2019 to September 2022.



The graph reveals 4 distinct regimes, marked by abrupt shifts in volatility levels and returns patterns.

I. Pre-Pandemic Stability (Jan 2019 - Jan 2020)
The regime was characterised by:
- Low and stable volatility, reflecting the calm before the storm of the COVID pandemic
- Returns show minimal clustering, with small daily fluctuations
- Economic context: This was post-2008 financial crisis stability, where we had low interest rates and strong equity markets

II. Pandemic Shock (Jan 2020- Jan 2021)
VIX levels spiked to approx 80+ (peak in Mar 2020), then declined to approx 25 - 30
It was characterised by:
- Abrupt volatility explosion starting in late Feb 2020
- Sharp increase in return volatility
- Economic Context of then was as follows:
    - Initial Pandemic lockdowns(Mar 2020)
    - Historical market crush (S&P 500 fell around 30% in 3 weeks)
    - Central bank interventions (Fed rate cuts, qualitative easing)

III. Recovery & Normalisation (Jan 2021- Jan 2022)
The vix levels were approximately 18 - 35
This recovery regime was characterised by:
- Gradual declines from the panic highs with periodic spikes
- Lower volatility compared to the pandemic shock regime but higher than pre-pandemic levels
- The economic context was as follows:
    - Vaccine rollout and reopening optimism
    - Inflationary pressures begin to merge due to supply chain disruptions
    - Equity markets rebound strongly

IV. Inflation & Geopolitical Uncertainty (Jan 2022 - Sep 2022)
The VIX levels ranged from 25 to 40
This last regime of our data was characterised by:
- Elevated volatility driven by persistent uncertainty
- Sharp spikes in June 2022 resulting from Fed aggressive rate hikes
- The economic context of then was as follows:
    - Russia-Ukraine war
    - Accelerated inflation
    - Fed tightening cycle and recession fears

Step 3 – Model Comparison

a. Analysis of Models with Different Mu Values (Constant Variance)

- Key results
    1. Model specification: 3 regimes, different means, constant variance
    2. AIC: -2121.80
    3. BIC: -2072.94
    4. Regimes Means: 0.0002, 0.0003, 0.0004
    5. Constant Variance: 0.0066
- This model seems to be fundamentally flawed for VIX analysis. Why?
    1. Economically meaningless parameter estimates - Estimated means are virtually identical and close to zero.They lack economic significance for VIX returns which which instead show clear regime-dependent behavior
    2. Statistical instability - It has huge standard errors for transition probabilities
    3. Overfitting the data - Using 3 regimes when the data doesn't support meaningful mean separation
- Therefore it would be recommended not to use this model for VIX modelling; it would be more suitable for assets with stable volatility but different means.

b. Model selection criteria:

Model	AIC	BIC
Case 1 (diff μ, same σ)	-5659.32	-5634.75
Case 2 (same μ, diff σ)	-6134.13	**-6109.56**
Case 3 (diff μ, diff σ)	**-6137.02**	-6107.54

Observations:

- AIC favors Case 3 (-6137.02) as the best fit, with Case 2 close behind.
- BIC favors Case 2 (-6109.56), suggesting the simpler same-μ, different-σ model is preferred when penalizing complexity.
- The small AIC difference between Cases 2 and 3 suggests varying μ yields minimal benefit relative to complexity.

Interpretation:

- Regime 0: Low volatility ($\sigma^2 \approx 5.012 \times 10^{-5}$)
- Regime 1: High volatility ($\sigma^2 \approx 5.0 \times 10^{-4}$)
- High p(0→0) indicates persistent low-volatility periods; low p(1→0) suggests shorter high-volatility spells.
- BIC's preference for Case 2 aligns with my assigned focus, balancing statistical fit and parsimony.

C. Model comparison of different expectations and variances.

Model	Log-Likelihood	AIC	BIC
A: same mu & sigma	1589.33	-3168.67	-3144.42
B: diff mu, same sigma	1589.33	-3168.67	-3144.42
D: diff mu & sigma	1854.83	**-3697.66**	**-3668.56**

1. Model D is the best-fitting model

It has the highest log-likelihood: 1854.83, which means it fits the data best.

It has the lowest AIC (-3697.66) and lowest BIC (-3668.56) — lower AIC and BIC mean better model with fewer penalties for overfitting.

2. Models A and B perform identically

Model A and B have identical log-likelihoods and information criteria, suggesting that allowing the means to differ did not improve the model when variance was held constant.

This implies that mean alone is not enough to capture the structural differences between regimes in your data.

3. Economically this means

The 10-Year US Treasury Yield return series appears to switch between low-variance and high-variance regimes, which makes sense: For example, quiet periods like mid-2019 or post-COVID QE had low volatility. Meanwhile, March 2020 and 2022 inflation shock periods had high volatility. Model D captures both the mean level and the uncertainty/volatility shifts, which is why it outperforms.

The regime-switching model with both changing expectations (mu) and changing variances (sigma) (Model D) significantly outperforms the others.

Although Model B allowed the mean to differ across regimes, this brought no improvement over the base model (A) with constant mean and variance. This suggests that variance, not just the average return, plays a crucial role in characterizing different regimes in Treasury yield behavior.

Therefore, Model D offers the most realistic and useful structure for modeling yield return dynamics in a regime-switching framework, making it the recommended model for further risk and portfolio management applications.

d – Combined Model Evaluation and Ranking

After evaluating the three model specifications, the group compared our performance based on Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC):

Model	Description	AIC	BIC
Case 1	Different μ, Same σ	-5659.32	-5634.75
Case 2	Same μ, Different σ	-6134.13	-6109.56
Case 3	Different μ, Different σ	-6137.02	-6107.54

Ranking (Best to Worst):

1. Case 3 (Different μ and σ): Statistically the best model by AIC, offering the most flexibility. However, the gain over Case 2 is marginal.

2. Case 2 (Same μ, Different σ): Strong fit with simpler structure. Preferred by BIC for its balance between accuracy and parsimony.

3. Case 1 (Different μ, Same σ): Least suitable for modeling VIX, as it fails to capture volatility regimes effectively.

Group Conclusion:
While Case 3 provides the best fit statistically, Case 2 offers a compelling trade-off between interpretability and model performance. Therefore, for practical applications, Case 2 may be the most appropriate model unless fine-tuned forecasting precision is required.

Step 4 Hamilton-Style Autoregressive Regime-Switching Model

Analysis of Hamilton-Style Autoregressive Regime-Switching Model

Model Summary

- Type: 2-regime Markov Autoregression (AR(1))
  - AIC: -2373.25
  - BIC: -2334.18
  - Log Likelihood: 1194.63

To further enhance the modeling framework, the group estimated a Hamilton-style Markov-switching autoregressive model (MS-AR(1)) with regime-dependent autoregressive coefficients and variances.

Conclusion:

The Hamilton-style model outperforms the earlier static models by capturing both regime-dependent volatility and time persistence through autoregressive dynamics. It reflects more realistic behavior of financial time series like the VIX, where volatility shocks often exhibit clustering and memory. This model is particularly valuable in stress testing, volatility forecasting, and dynamic risk management settings. Therefore, the Hamilton-style MS-AR model is recommended as the most robust and informative approach among the models explored.

References

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