Module 2: Lesson 3

Bates (1996) in practice

WorldQuant University

Outline

- Lewis (2001) approach
- Carr and Madan (1999) -FFT- approach

Pricing with Bates (1996)

In the previous lesson, we sketched the main ingredient we need for pricing under the Bates (1996) model using Fourier transform methods: the characteristic function (CF).

But, as we also know, there are at least two methods that we can use for pricing that require Fourier transforms:

- Lewis (2001), namely, solving the following integral using Bates (1996) CF, $\varphi_0^{B96}(u, T)$:

$$C_0 = S_0 - \frac{\sqrt{S_0 K} e^{-rT}}{\pi} \int_0^\infty \mathbf{Re}[e^{izk} \varphi^{B96}(z - i/2)] \frac{dz}{z^2 + 1/4}$$

- Carr and Madan (1999) approach, where the value of the call option is determined by:

$$C_0 = \frac{e^{-\alpha\kappa}}{\pi} \int_0^\infty e^{-i\nu\kappa} \frac{e^{-rT} \varphi^{B96}(\nu - (\alpha + 1)i, T)}{\alpha^2 + \alpha - \nu^2 + i(2\alpha + 1)\nu} d\nu$$

  where the integral can be estimated using Fast-Fourier Transform (FFT).

In this lesson, we will review both methods for the case of Bates (1996).

Summary of Lesson 3

In Lesson 3, we have covered:

- Lewis (2001) approach to the Bates (1996) model
- Carr and Madan (1999) approach to the Bates (1996) model

⇒ **References for this lesson:**

Hilpisch, Yves. *Derivatives Analytics with Python: Data Analysis, Models, Simulation, Calibration and Hedging.* John Wiley & Sons, 2015. (see Chapter 9.)

Bates, David S. "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options." *The Review of Financial Studies,* vol. 9, no. 1, 1996, pp. 69–107.

⇒ **TO DO NEXT**: In the notebook accompanying this lesson, you will see in detail how to implement the Bates (1996) model in practice in Python using both approaches.

⇒ In the next lesson, we will perform a full calibration of the Bates (1996) model.