Module 2: Lesson 2

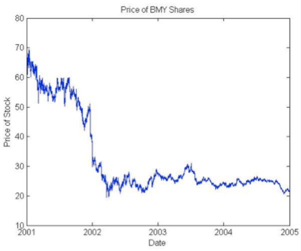
# Combining Heston '93 and Merton '76

# Outline

- Stochastic volatility and Jumps
- Bates (1996) model
- Characteristic function of combined process

# Stochastic Vol and Jump diffusion

- During this and the previous courses, we have covered a bunch of different option pricing models that try to capture different features of the observed distribution of asset prices: heavy tails, excess kurtosis, ...
- It seems that, among all those models, stochastic volatility models such as Heston (1993) are the best to reproduce facts like the volatility surface.
- Still, features like the jump diffusion of Merton (1976) are desirable to model certain asset prices:






**So... why don't we combine in a model stochastic volatility and jump diffusion features?**

# Bates (1996) model

The latter question is exactly what the Bates model (among others) answers.

Bates (1996) is a stochastic volatility jump-diffusion model with the following risk-neutral dynamics:

$$dS_t = (r_t - r_J)S_t dt + \sqrt{\nu_t} S_t dZ_t^1 + J_t S_t dN_t \tag{1}$$

$$d\nu_t = \kappa_\nu(\theta_\nu - \nu_t)dt + \sigma_\nu \sqrt{\nu_t} dZ_t^2 \tag{2}$$

where,

- $r_t$ is the short risk-free rate at date $t$
- $r_J \equiv \lambda(e\mu_J`\delta^2/2 - 1)$ is the drift correction for jump
- $\nu_t$ is the variance at date $t$
- $\kappa_\nu$ is the speed of adjustment of $\nu_t$ to $\theta_\nu$, the long-term variance
- $\sigma_\nu$ volatility of variance
- $Z_t^n$ are standard Brownian motions
- $N_t$ is a Poisson process with intensity $\lambda$
- $J_t$ is a jump at date $t$ with $log(1 + J_t)$ following a normal distribution $N(log(1 + \mu_J) - \frac{\delta^2}{2}, \delta^2)$

# Characteristic function in Bates (1996)

As you already know, we can price and calibrate the Bates (1996) model by using Fourier transform methods. To do so, we just need the characteristic function of the model:

- Bates (1996) combines two basic models: Heston (1993) and Merton (1976).
- It makes sense that its characteristic function is a combination of these two, which we already know.
- We will take the characteristic function of Heston (1993) as is, but we will need to adjust that of Merton (1976) because we just need the part of the jump (not the diffusive part, which is already given by Heston).
  - Thus, the 'jump-part' characteristic function of Merton (1976), $\varphi_0^{M76J}(u, T)$, becomes:

$$\varphi_0^{M76J}(u, T) = e^{\left(\left(iu\omega + \lambda\left(e^{iu\mu_j - u^2\delta^2/2} - 1\right)\right)T\right)}$$

where,

$$\omega = -\lambda\left(e^{\mu_j + \delta^2/2} - 1\right)$$

# Characteristic function in Bates (1996)

Once we have adjusted the Merton (1976) characteristic function, we can recognize that jumps occur independently of the drift process. That is, **correlation between the trend component of the asset price and jump component is zero**.

Hence, we can express the characteristic function of Bates (1996) simply as the product of two characteristic functions:

$$\varphi_0^{B96}(u,T) = \varphi_0^{H93}\varphi_0^{M76J}(u,T)$$

where $\varphi_0^{H93}$ stands for the characteristic function of Heston (1993) that we defined in module 1:

$$\varphi^{H93}(u,T) = e^{H_1(u,T)+H_2(u,T)\nu_0}$$

$$H_1(u,T) \equiv r_0 uiT + \frac{c_1}{\sigma_\nu^2}\left\{(\kappa_\nu - \rho\sigma_\nu ui + c_2)T - 2log\left[\frac{1-c_3e^{c_2T}}{1-c_3}\right]\right\}$$

$$H_2(u,T) \equiv \frac{\kappa_\nu - \rho\sigma_\nu ui + c_2}{\sigma_\nu^2}\left[\frac{1-e^{c_2T}}{1-c_3e^{c_2T}}\right]$$

# Summary of Lesson 2

In Lesson 2, we looked at:

- Bates (1996) stochastic volatility jump-diffusion model
- Characteristic function of Bates (1996) model

⇒ **References for this lesson:**

Hilpisch, Yves. *Derivatives Analytics with Python: Data Analysis, Models, Simulation, Calibration and Hedging*. John Wiley & Sons, 2015. (see Chapter 9)

Bates, David S. "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options." *The Review of Financial Studies*, vol. 9, no. 1, 1996, pp. 69–107.

⇒ **TO DO NEXT**: This lesson does not have an associated notebook. Jump to the next lesson to see how to implement the Bates (1996) model in practice in Python.

⇒ In the next lessons, we will see how to price and calibrate the Bates (1996) model using Fourier transform techniques.