Module 4: Lesson 2

Markov Processes: Further Applications

Outline

- Bond valuation using rating transitions
- The Gambler's ruin
- Markov-chain representation of a first-order autoregressive process

Bond valuation using rating transitions

In the Jupyter notebook for Module 2, we develop an example where we the value of a BB-rated outstanding bond that matures in 5 years and has a 4% coupon.

We combine information from the rating transition matrix, market forward rates, and recovery rates for defaulting bonds.

We develop our computations in two steps:

1. Compute the present value of the bond + coupon in one year's time using the forward rates.

2. Compute the expected value of the bond and the distribution of value changes using the transition matrix.

The methodology we develop underlies the valuation methods of JP Morgan's *CreditMetrics* for a single instrument.

Two absorbing states: The Gambler's ruin

Consider the following situation. A gambler bets on the outcome of a sequence of independent fair coin tosses. After heads, the gambler gains one dollar. After tails, the gambler losses one dollar. The gambler stops betting after reaching a fortune of $\bar{s}$ dollars or after emptying their pockets.

- What are the probabilities of each stopping outcome?
- How long will it take for the gambler, in expectation, to arrive at one of the stopping outcomes?

To answer these questions, we can represent the $(\bar{s}+1) \times (\bar{s}+1)$ transition matrix as:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0.5 & 0 & 0.5 & 0 & 0 & \cdots & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

In the Jupyter notebook for Module 2, we solve, numerically and analytically, the questions above.

Discretization of autoregressive processes

Consider a continuous random variable z_t that follows a first-order autoregressive process (AR(1)):

$$z_t = \rho z_{t-1} + \varepsilon_t \tag{2}$$

where $|\rho| < 1$, and ε_t is white noise with variance σ_ε^2.

For simulation or optimization purposes, it is useful to obtain a Markov-chain that approximates the realizations of the autoregressive random variable.

In the Jupyter notebook for Module 2, we develop two different discretization methods:

- Tauchen method: Intuitive method that fails to match the distribution of the original process.
- Rouwenhorst method: Method that approximates better the target distribution with simple calibration, in particular for processes that are close to a random walk.

Summary of Lesson 2

In Lesson 2, we have looked at:

- Applications of Markov processes with absorbing states
- Discretized, Markov-chain, representations of autoregressive processes

⇒ **References for this lesson:**

Kopecky, K. A., Suen, R. M. (2010). "Finite State Markov-Chain Approximations to Highly Persistent Processes." *Review of Economic Dynamics*, vol. 13, no. 3, pp. 701–714.

Gupton, G. M., et al.*CreditMetrics: Technical Document.* J.P. Morgan & Company, 1997.

Wu, H. "Introduction to Stochastic Processes." Spring 2015. Massachusetts Institute of Technology: MIT OpenCouseWare, https://ocw.mit.edu/. (Lecture 1)

TO DO NEXT: Now, please go to the associated Jupyter notebook for this lesson to practice with the numerical solutions to the examples and methods covered in the slides.

In the next lesson, we will learn the basics of hidden Markov models.