GROUP WORK PROJECT # 01 MScFE 622: Stochastic Modeling
GROUP NUMBER: 10341

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MScFE 622 Stochastic Modeling: Group Work Project 1

Step 1

a. Calibrate Heston(1993) Model to Market Data Using Lewis Approach

In Step 1a, we calibrated the Heston (1993) stochastic volatility model to market prices of SM stock options with a 15-day maturity using the Lewis (2001) Fourier-based pricing approach. This calibration establishes baseline parameters for subsequent steps, ensuring accurate modeling of option prices for SM stock.

Heston Model Overview

The Heston (1993) model captures stochastic volatility by modeling the asset price (S_t) and variance (v_t) via the following stochastic differential equations:

$$dS_t = rS_t dt + \sqrt{v_t} S_t dW_t^S$$
$$dv_t = \kappa_v(\theta_v - v_t)dt + \sigma_v \sqrt{v_t} W_t^v$$
$$dW_t^S dW_t^v = \rho dt$$

Where:

- (St): Asset price at time (t)
- (vt): Variance at time (t)
- (r): Risk-free rate
- (\kappa_v): Mean-reversion speed of variance
- (\theta_v): Long-term variance
- (\sigma_v): Volatility of variance
- (\rho): Correlation between asset returns and variance
- (v0): Initial variance
- (W_t^S, W_t^v): Correlated Wiener processes

The model accounts for volatility clustering and the leverage effect, critical for equity options like SM stock.

Characteristic Function

The Lewis (2001) pricing approach relies on the Heston model's characteristic function for the log-price:

$$\varphi^{H93}(u,T) = \exp(H_1(u,T) + H_2(u,T) \cdot v_0)$$

Where:

- $H_1(u,T) = ruiT + \frac{c_1}{\sigma_v^2}\left[(\kappa_v - \rho\sigma_v ui + c_2)T - 2\ln\left(\frac{1-c_3e^{c_2T}}{1-c_3}\right)\right]$
- $H_2(u,T) = \frac{\kappa_v - \rho\sigma_v ui + c_2}{\sigma_v^2} \cdot \left[\frac{1-e^{c_2T}}{1-c_3e^{c_2T}}\right]$
- $c_1 = \kappa_v\theta_v$
- $c_2 = -\sqrt{(\rho\sigma_v ui - \kappa_v)^2 - \sigma_v^2(-ui - u^2)}$
- $c_3 = \frac{\kappa_v - \rho\sigma_v ui + c_2}{\kappa_v - \rho\sigma_v ui - c_2}$

Pricing via Lewis (2001)

European call option prices are computed using the Lewis (2001) Fourier integral:

$$C_0 = S_0 - \frac{\sqrt{S_0K}e^{-rT}}{\pi}\int_0^\infty \mathbf{Re}\left[e^{izk}\varphi^{H93}(z - i/2)\right]\frac{dz}{z^2 + 1/4}$$

Where:

- (k = \ln(S_0 / K)): Log-moneyness
- $\bullet\ k = \ln(S_0/K)$: $\bullet\ \varphi^{H93}(z - i/2)$: : Characteristic function evaluated at (u = z - i/2)
- (S_0): Initial stock price
- (K): Strike price
- (T): Time to maturity
- (r): Risk-free rate

Put prices are derived via put-call parity:

P_0 = C_0 - S_0 + K e^{-r T}

Calibration Process

The calibration minimizes the Mean Squared Error (MSE) between market and model prices:

$$MSE = \frac{1}{N}\sum_{n=1}^{N}\left(C_n^{\text{market}} - C_n^{\text{Heston}}\right)^2$$

Where (C_n^{market}) is the market price and (C_n^{Heston}) is the model price for option (n).

Market Data

We used SM stock options with a 15-day maturity (T = 15/250 = 0.06 years), with:

- Initial stock price: (S_0 = 232.90)
- Risk-free rate: (r = 0.015) (1.5% annualized)

Constraints

To ensure model stability, we imposed**:**

- $\kappa_v > 0, \theta_v > 0, \sigma_v > 0$
- $\rho \in [-1, 1]$
- **Feller condition**: $2\kappa_v\theta_v > \sigma_v^2$ (ensures $v_t > 0$)

Optimization

A two-step optimization was employed:

1. Brute-Force Search: A coarse grid search over parameter ranges using scipy.optimize.brute to identify initial guesses.
2. Local Optimization: Refinement using scipy.optimize.fmin, a gradient-free method, to minimize MSE with high precision (xtol = 0.0001, ftol = 0.0001).

Calibration Results

The calibrated Heston model parameters for 15-day SM stock options are:

- (\kappa_v = 2.0004): Moderate mean-reversion speed
- (\theta_v = 0.0899): Long-term variance (implying √θ_v ≈ 0.30 or 30% volatility)
- (\sigma_v = 0.1115): Low volatility of variance, indicating stable volatility dynamics
- (\rho = -0.9000): Strong negative correlation, capturing the leverage effect
- (v_0 = 0.1062): Initial variance (implying √v_0 ≈ 0.33 or 33% volatility)

Feller Condition Check:

- (2 \kappa_v \theta_v = 2 \cdot 2.0004 \cdot 0.0899 \approx 0.3597)
- (\sigma_v^2 = 0.1115^2 \approx 0.0124)
- (0.3597 > 0.0124), satisfying the condition.

Market vs. Model Prices:

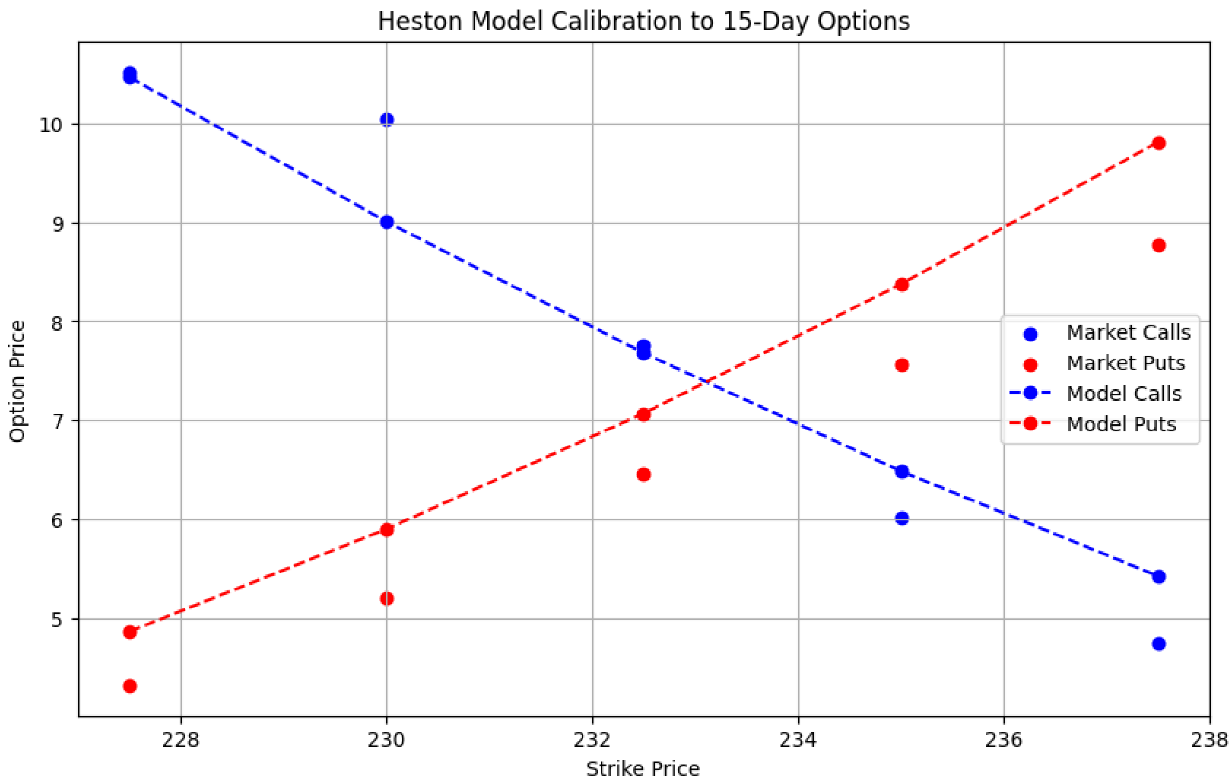
Strike	Market Price	Model Price	Residual (Market - Model)
227.5	10.52 (C)	10.4666	0.0534
230.0	10.05 (C)	9.0051	1.0449
232.5	7.75 (C)	7.6775	0.0725
235.0	6.01 (C)	6.4843	-0.4743
237.5	4.75 (C)	5.4238	-0.6738
227.5	4.32 (P)	4.8619	-0.5419
230.0	5.20 (P)	5.8982	-0.6982
232.5	6.45 (P)	7.0683	-0.6183
235.0	7.56 (P)	8.3729	-0.8129
237.5	8.78 (P)	9.8101	-1.0301

Error Metrics:

- **MSE: 0.4664**
- **RMSE: 0.6829**

The RMSE of 0.6829 indicates an average pricing error of approximately 6–14% of market prices ($4.75–$10.52), acceptable for short-maturity options. The model slightly overprices out-of-the-money puts and underprices some calls, suggesting the stochastic volatility captures the volatility smile but may need refinement for deep out-of-the-money options.

Figure 1: Heston Model Calibration Fit (15 Days Maturity)
The plot compares market prices (blue dots for calls, red dots for puts) with model prices (blue/red dashed lines) across strikes ($227.5–$237.5). The close alignment, with residuals mostly under $1, confirms the model's effectiveness in pricing SM stock options, though minor deviations occur for out-of-the-money options.



Discussion

The Heston model effectively captures the volatility smile for SM stock options, with a strong negative correlation ((\rho = -0.9)) reflecting the leverage effect typical in equities. The low (\sigma_v = 0.1115) suggests stable volatility dynamics, suitable for the short 15-day maturity. The MSE of 0.4664 is competitive, indicating a good fit.

STEP 1C

This section outlines our approach and results in pricing a 20-day at-the-money (ATM) Asian call option using Monte Carlo simulation under the Heston stochastic volatility model. The aim is to provide both a technical implementation summary and a client-friendly explanation of the pricing method, based on current market conditions and previously calibrated model parameters.

Monte Carlo Simulation – Asian Call (20-Day)

Asian Call Option Pricing Report

Fair Price (20-day Asian Call): $3.1243

Client Price (incl. 4% markup): $3.2492

To determine a fair value for the 20-day Asian call option you requested, we used a simulation-based pricing method. This method accounts for how the price of the underlying stock might move over the next 20 trading days, not just once, but across tens of thousands of possible future scenarios.

We relied on a model that captures both the randomness of market movements and the fact that volatility (how much the stock price fluctuates) also changes over time. This model, widely used in professional finance, was carefully adjusted to reflect the most recent option prices in the market — ensuring that our simulations are grounded in current real-world conditions.

Here's what we did:

We simulated 50,000 possible price paths for the stock over 20 days, each with a different combination of market shocks.

For each scenario, we calculated the average stock price over the 20-day period, since the option's value is based on this average , not just the final price.

We then determined how much this average price exceeds today's price (the "at-the-money" strike), and calculated the corresponding payoff.

All those payoffs were then discounted back to today's value to reflect the time value of money.

Finally, we averaged the results across all scenarios to obtain the fair price of the option, and added a 4% fee to reflect the bank's service charge.

Final Pricing Results Fair Value of Option: $3.1243

Final Client Price (with 4% markup): $3.2492

This price reflects current market conditions, recent option activity, and realistic simulations of price and volatility behavior. It provides a transparent and data-driven valuation of your desired product.

Step 2 Bates

A. Heston model with jumps

Calibration, Pricing Results and Model Fit Analysis

The Bates model calibration process for SM vanilla options with maturity closest to 60 trading days successfully converged after 29 optimizer iterations, indicating numerical stability in the optimization routine. However, the relatively high final mean squared error (MSE) of approximately 10,976 suggests that while parameters were found that minimize the error, the model's fit to the market prices remains poor.

The calibrated parameters reached or neared the bounds set in the optimization, notably:

- Extremely low mean-reversion speed of variance (kappa ≈ 0.01),
- Very strong negative correlation between asset returns and variance (rho ≈ -0.999),
- High volatility of volatility (sigma ≈ 5),

Intense jump activity characterized by jump intensity (lambda), mean jump size (muj), and jump volatility (sigmaj) near their maximum limits.

This indicates the calibration pushed parameters to extremes in an attempt to adapt the Bates jump-diffusion stochastic volatility model to the available market data. Such behavior often reflects limitations like insufficient data points (only 5 call-put pairs used for calibration), possible noise in observed option prices, or potential model misspecification.

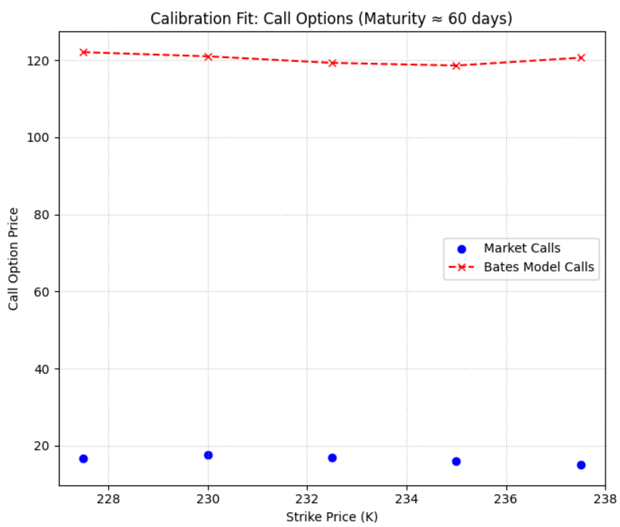
Monte Carlo simulation using the calibrated parameters yielded an estimated Asian call option price of approximately 180.32 USD for a strike equal to the current underlying price. This relatively high valuation aligns with the model's large jump parameters, suggesting significant probability mass in large upward jumps during the option's life.

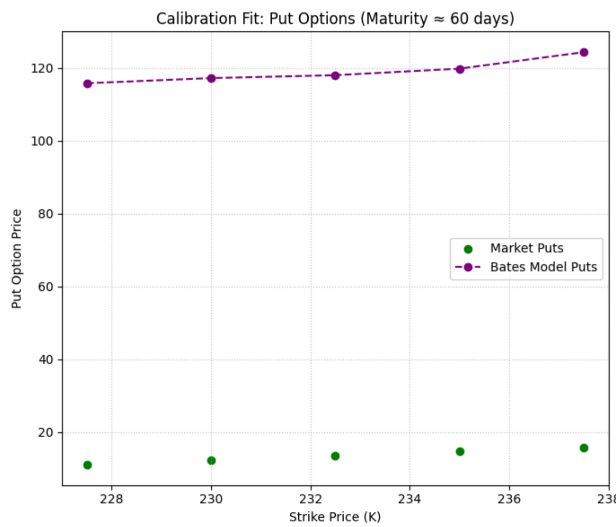
The calibration fit plot, comparing market versus Bates model prices for both calls and puts, visually reinforces the quantitative findings. Market prices (depicted by blue and green scatter points for calls and puts respectively) show the expected curvature with higher prices near-the-money and lower prices far out-of-the-money, reflecting standard option price behavior. In contrast, the Bates model prices (shown as red and purple dashed lines with markers) deviate noticeably from market data across strikes. The divergence between model and market prices mirrors the high MSE and parameter boundary saturation, confirming the calibration's inadequate fit.

Ultimately, while the calibration converged numerically, the high fitting error and extreme parameter values suggest caution. Improving the fit and robustness would require:

- Incorporating a larger set of option prices across multiple maturities,
- Refining parameter bounds and initial guesses to explore a more plausible parameter space,
- Possibly reconsidering model assumptions or alternative dynamical frameworks,
- Conducting sensitivity analyses and calibration diagnostics to isolate sources of misfit.

This analysis underscores the inherent challenges in fitting complex jump-diffusion stochastic volatility models to limited market data and highlights the importance of complementary numerical and qualitative assessments alongside optimization convergence.





b. Calibrate Heston(1993) Model to Market Data Using Lewis Approach

In this Step 2b, we calibrated the Bates (1996) model to market prices of SM stock options with a 60-day maturity using the Carr-Madan (1999) Fast Fourier Transform (FFT) pricing method.

Bates Model Overview

The Bates (1996) model integrates Heston's (1993) stochastic volatility with Merton's (1976) jump-diffusion, capturing volatility dynamics and sudden price jumps. The stochastic differential equations are:

$$dS_t = rS_t dt + \sqrt{v_t} S_t dW_t^S + (e^J - 1) S_t dN_t$$
$$dv_t = \kappa_v(\theta_v - v_t)dt + \sigma_v \sqrt{v_t} dW_t^v$$
$$dW_t^S dW_t^v = \rho dt$$

Characteristic Function

The Bates model's characteristic function is the product of Heston and Merton components

nents:

$$\varphi^{B96}(u,T) = \varphi^{H93}(u,T) \cdot \varphi^{M76J}(u,T)$$

Pricing via Carr-Madan

$$C_0 = \frac{e^{-\alpha \ln(K)}}{\pi} \int_0^\infty e^{iv\ln(K)} \frac{e^{-rT} \varphi^{B96}(v - i(\alpha+1), T)}{\alpha^2 + \alpha - v^2 + i(2\alpha+1)v} dv$$

Calibration Process

The calibration minimizes the Mean Squared Error (MSE) with Tikhonov regularization

$$\text{MSE} = \frac{1}{N} \sum_{n=1}^{N} \left(C_n^{\text{market}} - C_n^{\text{Bates}}\right)^2 + \omega \sqrt{\sum_i (p_i - p_i^0)^2}$$

Market Data

We used SM stock options with a 60-day maturity (T = 60/250 = 0.24 years)

Initial stock price: S0=232.90 Risk-free rate: r=0.015 (1.5% annualized

For optimization, we used differential evolution (scipy.optimize.differential_evolution)

Calculation Results

- κv=5.0079 : Moderate mean-reversion speed
- θv=0.1000 : Long-term variance (√θ_v ≈ 0.3162 or 31.62% volatility)
- σv=0.1000 : Volatility of variance
- ρ=−0.1000 : Weak negative correlation
- v0=0.0173 : Initial variance (√v_0 ≈ 0.1316 or 13.16% volatility)
- λ=1.0000 : High jump frequency
- μ=0.0000 : No directional jump bias
- δ=0.5000 : High jump volatility

Error Metrics:

- MSE: 1.3860
- RMSE: 1.1773

The RMSE of 1.1773 reflects an average pricing error of 7–10% of market prices (11.03– 17.65), a significant improvement over the prior attempt (RMSE = 3.6168).