Module 3: Lesson 2

Calibrating CIR (1985)

Outline

- Calibration process in CIR (1985)
- Forward rates in CIR (1985)
- Interpolation of forward rates
- Error and optimization functions

Calibration process in CIR (1985)

We have done many model calibrations up to this point. Hence, in the calibration of the CIR (1985) model, there are going to be a lot of common features compared to previous instances.

The general process always takes a very similar path:

1. Gather market data
   - In this case, risk-free rates data (e.g., U.S. Treasury Yield curve for U.S., Euribor for Europe,...)
2. Bond pricing functions
   - Build up our valuation functions that aim to reproduce market quotes.
   - In the case of interest rates models, this will require an extra step, since we will focus on forward rates and interpolation of data points.
3. Error function
   - Which kind of error function should we use for our optimization target (e.g., MSE, RMSE, ...)?
4. Optimization function
   - In general, our goal will be to minimize the differences between model results and market data.

Forward rates in CIR (1985)

The calibration process in CIR has a few small peculiarities:

- We calibrate forward rates, instead of ZCB prices.
- We need to interpolate forward rates, due to the absence of many data points.

$\Rightarrow$ As you know, the forward rate is the rate that would be applied between two times t to T, with $t < T$.

Formally, we can define the forward rate between $t = 0$ and T as:

$$f(0,T) = Y(0,T) + \frac{\partial Y(0,T)}{\partial T} T$$

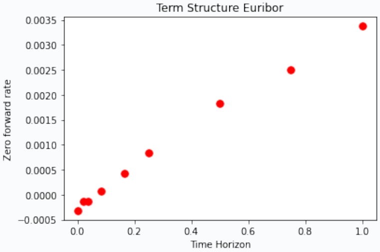
$\Rightarrow$ So, how can we define an expression for forward rates in the CIR (1985) model?

$$f^{CIR}(t,T;\alpha) = \frac{\kappa_r \theta_r (e^{\gamma t} - 1)}{2\gamma + (\kappa_r + \gamma)(e^{\gamma t} - 1)} + r_0 \frac{4\gamma^2 e^{\gamma t}}{(2\gamma + (\kappa_r + \gamma)(e^{\gamma t} - 1))^2}$$

Interpolation of forward rates

In the notebook accompanying this lesson, you have more information on forward rates in case you need a reminder.

Once we have extracted the forward rates from market quotes, we will have a so-called term structure:



The problem is we have few data points (i.e., forward rates) for a very specific set of maturities.

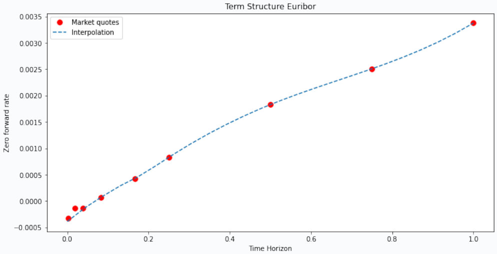
$\Rightarrow$ Can we do something in order to have more points? $\rightarrow$ Interpolation

Interpolation of forward rates

The idea behind interpolation is to fill in the blanks by obtaining a functional expression that can reproduce the observed data points.

There are a bunch of ways to interpolate a term structure of interest rates:

- Linear interpolation
- Fitting a model via OLS (e.g., Nelson-Siegel-Svensson, ...)
- (Cubic) Spline methods $\rightarrow$ This is the one we will use (more on this in the notebook)



Error function and optimization

As in other calibration processes, we need to define two more functions: error and optimization.

$\Rightarrow$ Which error function should we use to measure differences between model and market quotes?

- In general, for interest rates we will opt for the Mean Squared Error (MSE).
- In CIR (1985), model error would be determined by differences in forward rates, given model parameters:

$$\Delta f(0,t) \equiv f(0,t) - f^{CIR}(0,t;\kappa_r,\theta_r,\sigma_r,r_0)$$

$\Rightarrow$ What then do we have to optimize to calibrate our model?

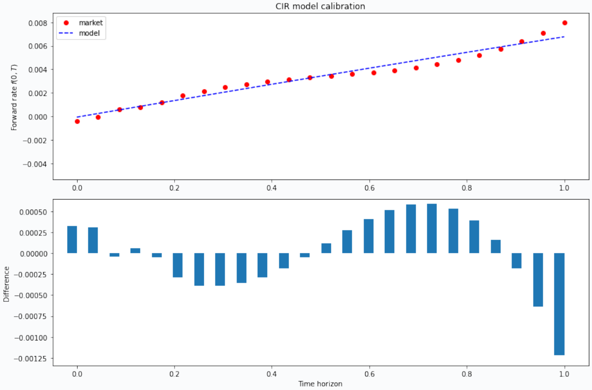
- For $f(0,m\Delta t)$ being the forward rate at each time horizon $m\Delta t \leq T$, we will minimize the MSE for the set of parameters $\alpha = \{\kappa_r,\theta_r,\sigma_r\}$:

$$min \frac{1}{M} \sum_{m=0}^{M} \left(f(0,m\Delta t) - f^{CIR}(0,m\Delta t;\alpha) \right)^2$$

Calibration results

As you will see in the notebook accompanying this lesson, when we calibrate the CIR model to Euribor rates on September 30th, 2014, we obtain the following values for model parameters:

$\Rightarrow \kappa_r = 0.068; \ \theta_r = 0.207; \ \sigma_r = 0.112$



Summary of Lesson 2

In Lesson 2, we have:

- Defined forward rates and capitalization factors in the context of CIR (1985)
- Interpolated market forward rates
- Calibrated the CIR (1985) model to market quotes

⇒ **References:**

Cox, John C., et al. "An Intertemporal General Equilibrium Model of Asset Prices." *Econometrica: Journal of the Econometric Society*, 1985, pp. 363–384.

⇒ **TO DO NEXT**: In the notebook associated with this lesson, we will guide you step by step through the calibration process for the CIR (1985) model.

⇒ In the next lesson, we will see how to incorporate stochastic interest rate models into the Bates (1996) general framework.