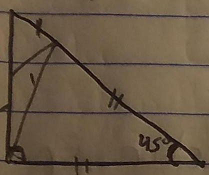
Love: $S \to \{0,1\}$ given 1 if "Love" appears & 0 otherwise.

$$\text{Love} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \qquad \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \overset{?}{=} \alpha\begin{pmatrix}0\\1\\0\\1\end{pmatrix} + \beta\begin{pmatrix}0\\0\\1\\1\end{pmatrix}$$

1st component $\alpha \cdot 0 + \beta \cdot 0 = 0$.

Lecture #2



Puzzle: what does this picture prove?

Where are we headed?
Dantzig algorithm / simplex method
Least squares problem

Last time we were studying the vector space

$\mathbb{Z}_2^S = \{$all ~~functions~~ functions $S \to \mathbb{Z}_2\}$ (set of all tweets)

field

Pair of indicator functions Hillary and Donald deciding whether a tweet contained those words and assigning 1's & 0's according by
unanswered question ii) How many elements does $V = \mathbb{Z}_2^S$ have?

i.e. $|\mathbb{Z}_2^S| = \text{card}(\mathbb{Z}_2^S) = ?$

A. Q: $\log_{10} 2 \approx .30$

Now $|S| = 100$ billion $= 10^{11}$ and there are $2^{|S|}$ possible functions (just go through each element in S and decide whether your f^n assigns $\underbrace{0 \text{ or } 1}$ and

$$2^{10^{11}} \simeq 10^{.3 \times 10^{11}} = 10^{3\times 10^{10}}$$
$$= \underbrace{10 \ldots 0}_{3\times 10^{10} \text{ zeros!}}$$

Self Q: What is the dimension of the vector space $\mathbb{Z}_2^S$?

Q: What is a linear map? (linear transformation)

A: A vector space morphism.

A: A map/function between vector spaces V & W respecting the rules of $+, \cdot$

<u>I.e.</u> $L: V \xrightarrow{\text{linear}} W$ "means"

$$L(\alpha \cdot_V u +_V \beta v) = \alpha L_W(u) +_W \beta_W L(v)$$

scalars

$\Rightarrow$ Shorthand $L(u) = Lu$

$\Rightarrow L(\alpha u + \beta v) = \alpha Lu + \beta Lv$ "distributive"

Example $S \underset{\text{as before}}{=} \{$ I love Bernie, 3 more $\}$

Let $\tilde{T} = \left\{ \begin{matrix} \text{all 2016} \\ \text{political} \\ \text{conventions} \end{matrix} \right\} = \{D, R\}$

Moreover, let $S^T \ni h: T \longrightarrow S$ where

h(R) = GoDonald

h(D) = Hillary Rox

Then we get a linear map $L: \mathbb{Z}_2^S \xrightarrow{\text{linear}} \mathbb{Z}_2^T$

From the diagram

$$\begin{tikzcd}
\underset{\{D,R\}}{\tilde{T}} \arrow[r, "h"] \arrow[dr, "v \circ h = L(v) \in \mathbb{Z}_2^T"'] & S \arrow[d, "v \in \mathbb{Z}_2^S = V"] \\
& \mathbb{Z}_2
\end{tikzcd}$$

Ex. $L(\text{Hillary})(R) = \text{Hillary}(h(R))$ $\in \mathbb{Z}_2^S$
$= \text{Hillary}(\text{Go Donald}) = 0$

$L(\text{Hillary})(D) = \text{Hillary}(h(D)) = \text{Hillary}(\text{Hillary Rox}) = 1$

In the less cucumbersome notation we wrote
$\text{Hillary} = \begin{pmatrix}0\\1\\0\\1\end{pmatrix}$ and lets call the D, R indicator functions $\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}$ then $\begin{pmatrix}0\\1\\0\\1\end{pmatrix} \xrightarrow{L} \begin{pmatrix}1\\0\end{pmatrix}$

$$\begin{pmatrix}* & * & * & *\\ * & * & * & *\end{pmatrix}\begin{pmatrix}0\\1\\0\\1\end{pmatrix} \rightarrow \begin{pmatrix}1\\0\end{pmatrix}$$

Linearity? $\begin{pmatrix}0\\1\\0\\1\end{pmatrix} \xrightarrow{L} \begin{pmatrix}1\\0\end{pmatrix}$ and $\underset{\text{Donald}}{\begin{pmatrix}0\\0\\1\\1\end{pmatrix}} \rightarrow \underset{\text{Republican indicator function}}{\begin{pmatrix}0\\1\end{pmatrix}}$

Then, expect by linearity,
$$L(\text{Hillary} + \text{Donald}) = \begin{pmatrix}0\\1\\0\\1\end{pmatrix} + \begin{pmatrix}0\\0\\1\\1\end{pmatrix} = \begin{pmatrix}0\\1\\1\\0\end{pmatrix}$$
$$\downarrow$$
$$\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}$$

Check. $L(\text{Hillary} + \text{Donald})(D) = (\text{Hillary} + \text{Donald})\, h(D)$
$= (\text{Hillary Rox})(\text{Hillary} + \text{Donald})$
$= 0 + 1 = 1$

Remarks (i) Not all vector spaces can have their elements labeled by column vectors, this is only possible when the dimension of ~~the~~ the vector space is finite.
(ii) Self.Q: Make sure you can define the dimension of a vector space