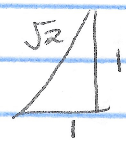
10/02 Lecture #3

- Get computer accnt. or octane
- wednesday 10/04 is a computer session, bring laptop
- puzzle?
  - ↳ Hint:



- det $\mathbb{Z}_2$ is a great trick... if matrix is large calculate det. over "bits" (mod 2)

Last time:

$$S \longrightarrow T$$

$$K^S \xleftarrow[\text{map}]{\text{linear}} K^T \leftarrow \text{numbers}$$

- A map from vectors to vectors

$$\mathbb{R}^{\{1,2\}} = \mathbb{R}^2 = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} \middle| \, a,b \in \mathbb{R} \right\}$$

<u>Matrices</u>

<u>Example:</u> Suppose $L : \mathbb{R}^2 \xrightarrow{\text{linear}} \mathbb{R}^3$ obeys

$$L\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \text{ and } L\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

Q: What is $L\begin{pmatrix} 1 \\ 1 \end{pmatrix}$?

$$L\left(1\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 1\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = 1L\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 1L\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= 1\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + 1\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix}$$

10/02 Because $\begin{pmatrix}1\\2\\3\end{pmatrix}$ and $\begin{pmatrix}3\\2\\1\end{pmatrix}$ give us full information of L, we encode these in a matrix

$$L = \begin{pmatrix}1 & 3\\2 & 2\\3 & 1\end{pmatrix}$$

DIRTY

and the above computation is your old friend

$$\begin{pmatrix}1 & 3\\2 & 2\\3 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}4\\4\\4\end{pmatrix}$$ Matrix · vector

$L\begin{pmatrix}5\\1\end{pmatrix}$, $L\begin{pmatrix}3\\2\end{pmatrix}$ known bases (this is like using different coordinates)

→ Change of base:
call this $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$

Systems of equations & the Dantzig algorithm

Given finite dimensional vectorspaces U and V we can represent elements $x \in U$ by

$$X = \begin{pmatrix}x_1\\x_2\\\vdots\\x_n\end{pmatrix}$$

where $n = \dim U$ and $E_1 = \begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix}, \ldots, E_n = \begin{pmatrix}0\\0\\\vdots\\1\end{pmatrix}$

correspond to some basis $\{e_1, \ldots, e_n\} = U$

* For square & symmetric matrices look for eigenvectors for basis

10/02 Review: Basis - span the vectorspace & are linearly independent

Note: $\{e_1, \dots, e_n\}$ a basis for U means that span $\{e_1, \dots, e_n\} = \{x_1 e_1 + \dots + x_n e_n \mid x_i \in \mathbb{R}\} = U$ (linear combo)
and $e_1, \dots, e_n$ are linearly independent,
i.e. the only solution to $y_1 e_1 + \dots + y_n e_n = 0$
is $y_1 = y_2 = \dots = y_n = 0$

This means $U \ni x = x_1 e_1 + \dots + x_n e_n$

$$\underset{\text{unique}}{=} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

So long as we all agree on a basis

Similarly, we can choose a basis $\{f_1 \dots f_m\}$ = dim V
represented as $f_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$ and $f_m = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$

can express uniquely

$$V \ni v = \begin{pmatrix} v_1 \\ \vdots \\ v_m \end{pmatrix}$$

Having chosen input and output bases for U and V we can represent

$$L : U \xrightarrow{\text{linear}} V$$

as a matrix.

10/02 For example if $L(e_1) = 2f_2 + 3f_5$ the first column of the matrix of L would be $\left.\begin{matrix}0\\2\\0\\0\\3\\0\\\vdots\\0\end{matrix}\right\} m$ and then do this n times over to form an $m \times n$ matrix.

Muemonic:

$$\text{rows}\,(m \times \overset{\text{Columns}}{n})(n \times 1) = (m \times 1)$$

Matrix Column vector Column vector labels vectors in V,

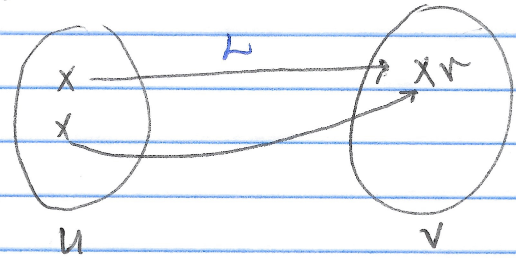
Key Question:

Given $L: U \longrightarrow V$ and $v \in V$. Find x such that $L(x) = v$ or writing (using bases) M for L, X for x and V for v, Solve $MX = V$.

This is a linear system of m equations for the n unknowns.

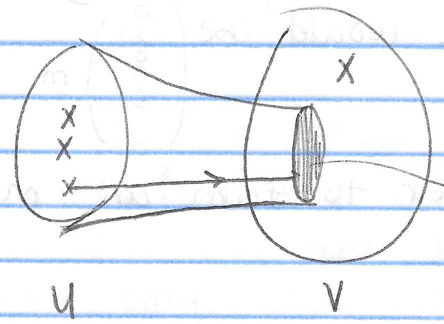
$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \cdots$$ Gaussian Elimination

Blob Diagram of Stuff that can happen:



Might be a unique solution for x.

10/02 OR



$v \notin L(U)$

(image of L / Range)

$L(U) = \mathrm{Im}\, L$

No solution → Least squares Method. Find Closest soln.