STA 137—Applied Time Series Analysis — Practice Midterm II

Please note that this is a practice exam. The actual midterm questions will be different.

[1] Figure 1 displays the time series plot, the ACF and the qq plot of a series of residuals that has been obtained after detrending and deseasonalizing a data set of size $n = 100$.

(a) Based on the ACF alone would you suggest that the residuals are dependent?

(b) Based on the time series plot, would you trust your answer in (a)? Why or why not?

(c) Do the time series plot and the qq plot support the claim of normally distributed residuals?

Give precise arguments for each of your choices.

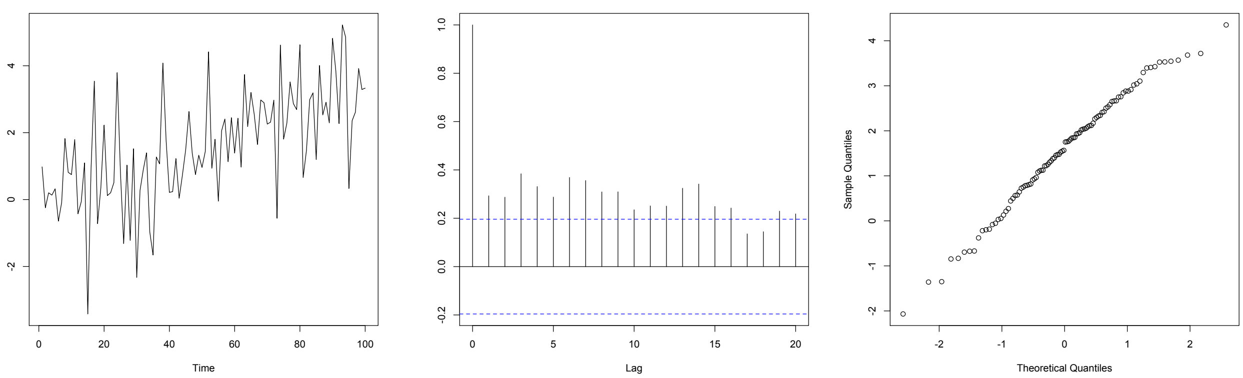

Figure 1: Time series plot, ACF and qq plot of a residual series.

[2] Figure 2 on the next page shows the time series plot and the sample ACF of an AR(1) process $X_t = \phi X_{t-1} + Z_t$. Which parameter ϕ has been used to generate these observations?

(a) $\phi = -0.9$ (b) $\phi = -0.3$ (c) $\phi = -0.1$ (d) $\phi = 0.3$ (e) $\phi = 0.9$

Circle the correct answer.

[3] Give the conditions that make the ARMA(1,1) process

$$X_t = \phi X_{t-1} + Z_t + \theta Z_{t-1}$$

both causal and invertible.

[4] Let $(X_t \colon t \in \mathbb{Z})$ be stationary and represented by the model

$$X_t = \phi X_{t-2} + Z_t,$$

where $(Z_t \colon t \in \mathbb{Z}) \sim \mathrm{WN}(0, \sigma^2)$. Find the variance of X_t.

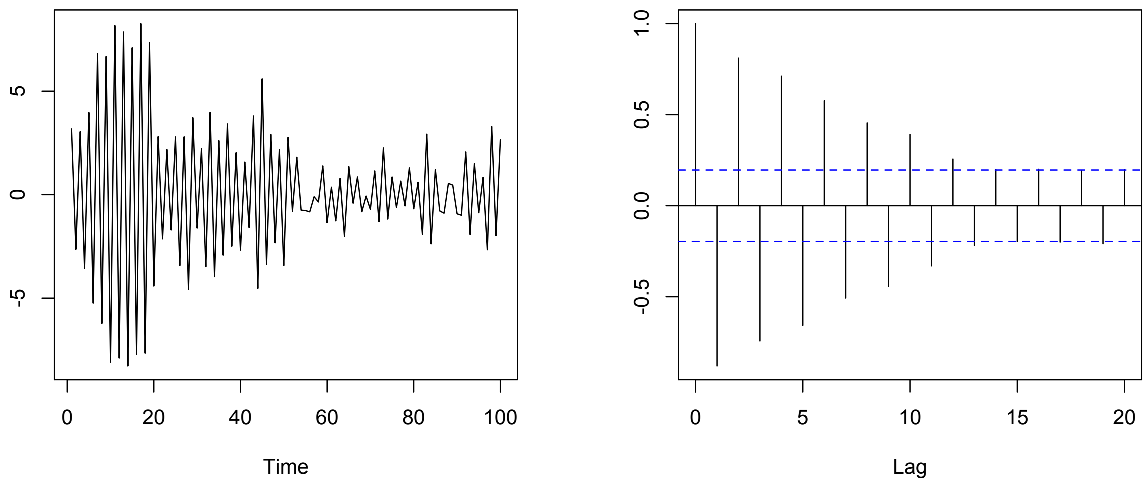

Figure 2: Time series plot and ACF of an AR(1) process.

[5] The process $(X_t \colon t \in \mathbb{Z})$ is expressed as

$$X_t = \beta_1 \sin\left(\frac{2\pi}{n}t\right) + \beta_2 \cos\left(\frac{2\pi}{n}t\right) + Z_t,$$

where $(Y_t \colon t \in \mathbb{Z}) \sim \text{WN}(0, \sigma^2)$. Find the autocovariance function of X_t. Is the process stationary?

[6] Show that for AR(1) model

$$X_t = \phi X_{t-1} + Z_t,$$

where $(Z_t \colon t \in \mathbb{Z}) \sim \text{WN}(0, \sigma^2)$ and $|\phi| > 1$, can be represented as

$$X_t = -\sum_{j=1}^{\infty} \phi^{-j} Z_{t+j}$$

and find the autocovariance function of $(X_t \colon t \in \mathbb{Z})$.

[7] Suppose the roots of the AR polynomial

$$\phi(z) = 1 - \phi_1 z - \phi_2 z^2$$

from some ARMA process are $z_1 = 0.5 + i0.5$ and $z_2 = 0.5 + i0.5$. Is this process causal?

[8] Determine if the following ARMA process is invertible:

$$X_t + 0.2X_{t-1} - 0.48X_{t-2} = Z_t - 0.04Z_{t-1} - 0.4Z_{t-2}.$$

[9] Consider the linear process

$$Y_t = \sum_{j=-\infty}^{\infty} a_j Z_{t-j}$$

where $(Z_t \colon t \in \mathbb{Z}) \sim \text{WN}(0, 4)$, with $a_0 = 1$, $a_2 = -1$ and $a_j = 0$ for all $j \neq 0, 2$. What is the variance of of the output series?

[10] Suppose

$$X_t = \mu + Z_t + \theta Z_{t-1}$$

where $(Z_t \colon t \in \mathbb{Z}) \sim \mathrm{WN}(0, \sigma^2)$.

(a) Find the mean function $\mathbb{E}[X_t]$;

(b) Show that the autocovariance function of $(X_t \colon t \in \mathbb{Z})$ is given by $\gamma(0) = \sigma^2(1+\theta^2)$, $\gamma(\pm 1) = \sigma^2\theta$, and $\gamma(h) = 0$ otherwise;

(c) Show that $(X_t \colon t \in \mathbb{Z})$ is stationary for all values of $\theta \in \mathbb{R}$;

(d) Calculate $\mathrm{Var}(\bar{X})$ for estimating μ when (i) $\theta = 1$, (ii) $\theta = 0$, and (iii) $\theta = -1$.