STA 137—Applied Time Series Analysis **Practice Midterm I**

Please note that this is a practice exam. The actual midterm questions will be different.

[1] Let ∇ be the difference operator and specify the quadratic trend $m_t = b_0 + b_2 t^2$ for some constants b_0 and b_2. What is ∇m_t?

(a) 0 (b) b_0 (c) $2b_2 t - b_2$ (d) b_2 (e) none of the above

[2] Show that the one-sided moving average filter gives exponentially decaying weights to past observations.

[3] Let $(Y_t \colon t \in \mathbb{Z})$ be a weakly stationary process with mean zero. Define the process $(X_t \colon t \in \mathbb{Z})$ by letting

$$X_t = s_t + Y_t, \qquad t \in \mathbb{Z},$$

where $(s_t \colon t \in \mathbb{Z})$ is a seasonal component of period d. Apply ∇_d to the process $(X_t \colon t \in \mathbb{Z})$ and compute $\mathrm{Cov}(\nabla_d X_{t+1}, \nabla_d X_t)$.

[4] In Figure 1, $n = 100$ observations simulated from time series (X_t), (Y_t) and (Z_t) are shown. Which of the three processes is not stationary?

(a) (X_t) (b) (Y_t) (c) (Z_t) (d) none of the processes is stationary

[5] Let $(Z_t \colon t \in \mathbb{Z}) \sim \mathrm{WN}(0, \sigma^2)$ and let $(X_t \colon t \in \mathbb{Z})$ be given by the equations

$$X_t = Z_t - Z_{t-1}, \qquad t \in \mathbb{Z}.$$

What are the values of the ACF $\rho(h) = \mathrm{Corr}(X_t, X_{t+h})$ for $h \in \mathbb{Z}$?

[6] Let $(X_t \colon t \in \mathbb{Z})$ be a stationary process with ACVF $\gamma(h) = \exp(1.5h^2)$ for all $h \in \mathbb{Z}$. Give the expression for the ACF $\rho(h)$ at lag $h \in \mathbb{Z}$.

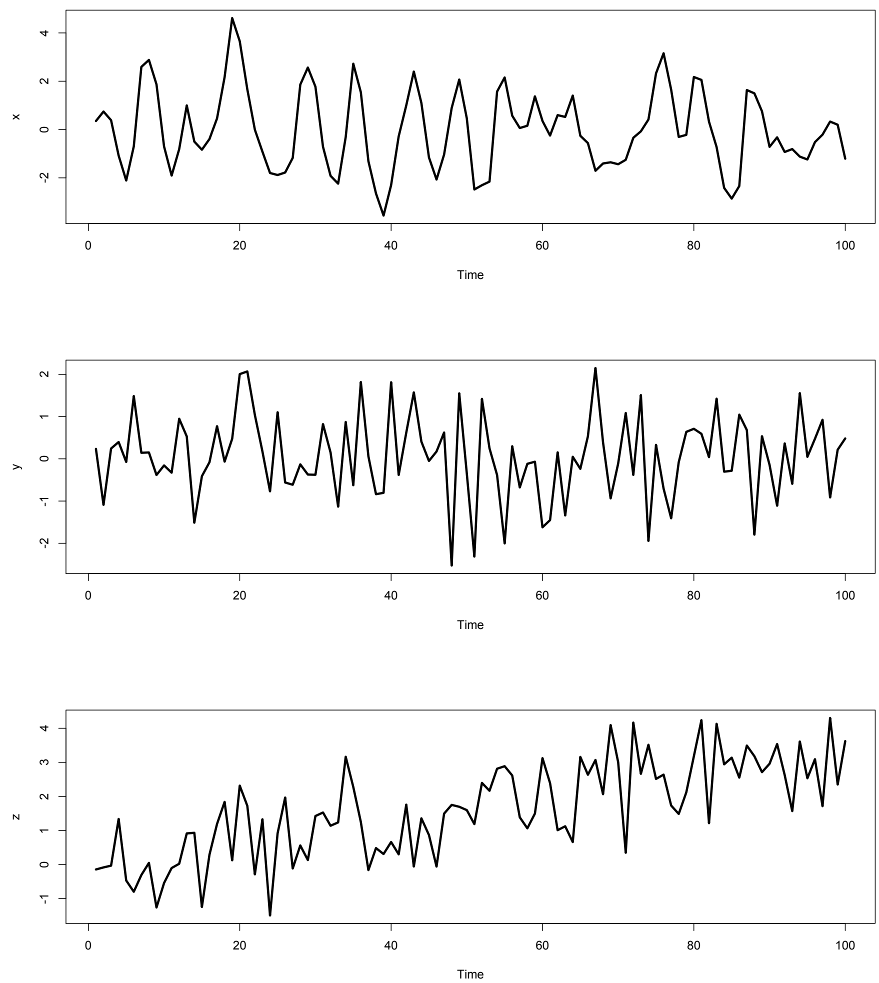

Figure 1: Realizations of (X_t) (top), (Y_t) (middle) and (Z_t) (bottom).