Some simple LR parse tables

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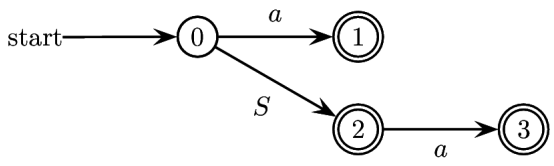
Left recursion: $S \to Sa \mid a$

Stack	Input
0	a a a a $
0 a 1	a a a $
0 S 2	a a a $
0 S 2 a 3	a a $
0 S 2	a a $
0 S 2 a 3	a $
0 S 2	a $
0 S 2 a 3	$
0 S 2	$

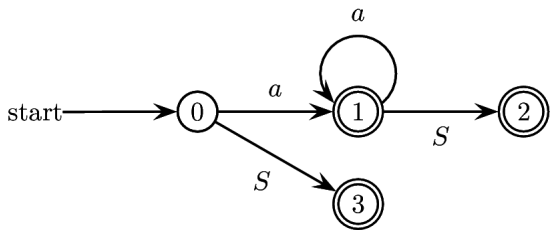
	a	$	S
0	1		2
1	$S \to a$		
2	3	accept	
3	$S \to Sa$	$S \to Sa$	



Right recursion: $S \to aS \mid a$

Stack	Input
0	a a a a $
0 a 1	a a a $
0 a 1 a 1	a a $
0 a 1 a 1 a 1	a $
0 a 1 a 1 a 1 a 1	$
0 a 1 a 1 a 1 S 2	$
0 a 1 a 1 S 2	$
0 a 1 S 2	$
0 S 3	$

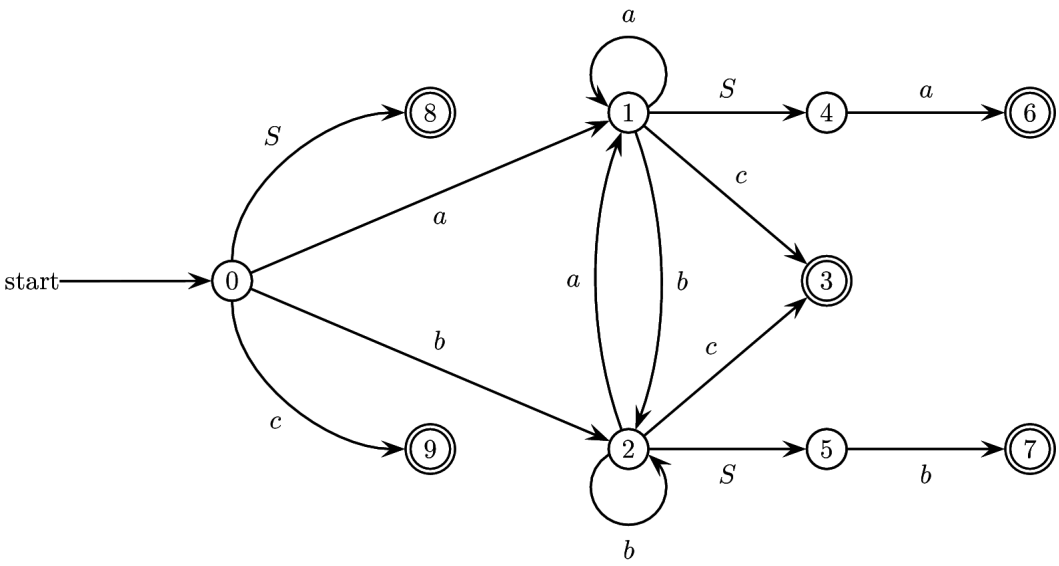
	a	$	S
0	1		3
1	1	$S \to a$	2
2		$S \to aS$	
3		accept	



Middle recursion: $S \to aSa \mid bSb \mid c$

Stack	Input
0	a b c b a $
0 a 1	b c b a $
0 a 1 b 2	c b a $
0 a 1 b 2 c 3	b a $
0 a 1 b 2 S 5	b a $
0 a 1 b 2 S 5 b 7	a $
0 a 1 S 4	a $
0 a 1 S 4 a 6	$
0 S 8	$

	a	b	c	$	S
0	1	2	9		8
1	1	2	3		4
2	1	2	3		5
3	$S \to c$	$S \to c$			
4	6				
5		7			
6	$S \to aSa$	$S \to aSa$		$S \to aSa$	
7	$S \to bSb$	$S \to bSb$		$S \to bSb$	
8				accept	
9				$S \to c$	

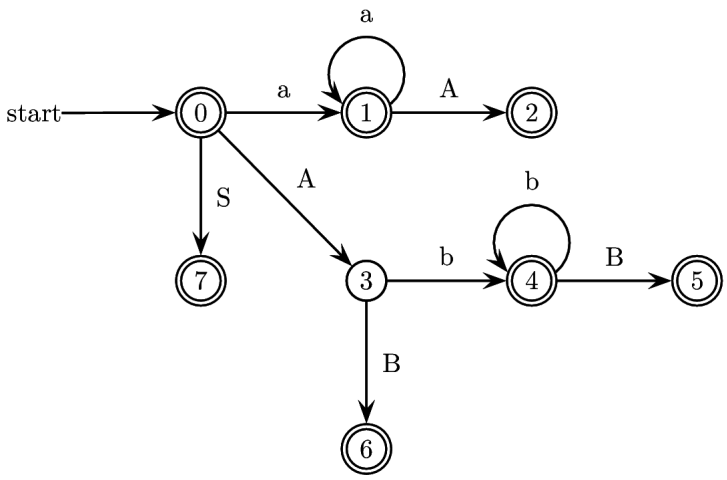


Example: a^*b^*b

$$
\begin{aligned}
S &\rightarrow AB \\
A &\rightarrow aA \mid \epsilon \\
B &\rightarrow bB \mid b
\end{aligned}
$$

Stack	Input
0	a a b b b $
0 a 1	a b b b $
0 a 1 a 1	b b b $
0 a 1 A 2	b b b $
0 A 3	b b b $
0 A 3 b 4	b b $
0 A 3 b 4 b 4	b $
0 A 3 b 4 b 4 b 4	$
0 A 3 b 4 b 4 B 5	$
0 A 3 b 4 B 5	$
0 A 3 B 6	$
0 S 7	$

	a	b	$	S	A	B
0	1	$A \rightarrow \epsilon$		7	3	
1	1	$A \rightarrow \epsilon$			2	
2		$A \rightarrow aA$				
3		4				6
4		4	$B \rightarrow b$			5
5			$B \rightarrow bB$			
6			$S \rightarrow AB$			
7			accept			



Stack	Input
0	b b b $
0 A 3	b b b $
0 A 3 b 4	b b $
0 A 3 b 4 b 4	b $
0 A 3 b 4 b 4 b 4	$
0 A 3 b 4 b 4 B 5	$
0 A 3 b 4 B 5	$
0 A 3 B 6	$
0 S 7	$

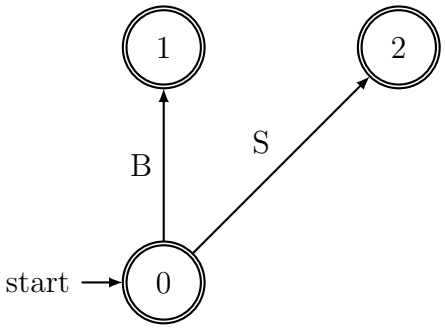
Example: $a^m b^n c^{m+n}$, Part I

$$\begin{aligned} S &\rightarrow aSc \mid B \\ B &\rightarrow bBc \mid \epsilon \end{aligned}$$

$S \overset{S \to B}{\Longrightarrow} B \overset{B \to \epsilon}{\Longrightarrow} \epsilon$

Stack	Input	Rule
0	\$	$B \to \epsilon$
0 B 1	\$	$S \to B$
0 S 2	\$	

	a	b	c	\$	S	B
0				$B \to \epsilon$	2	1
1				$S \to B$		
2				accept		



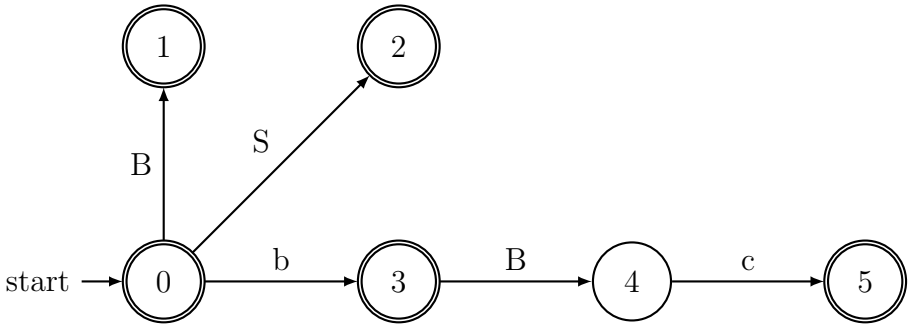
Example: $a^m b^n c^{m+n}$**, Part II**

$$\begin{aligned} S &\rightarrow aSc \mid B \\ B &\rightarrow bBc \mid \epsilon \end{aligned}$$

$S \overset{S \to B}{\Longrightarrow} B \overset{B \to bBc}{\Longrightarrow} bBc \overset{B \to \epsilon}{\Longrightarrow} bc$

Stack	Input	Rule
0	bc$	shift
0 b 3	c$	$B \to \epsilon$
0 b 3 B 4	c$	shift
0 b 3 B 4 c 5	$	$B \to bBc$
0 B 1	$	$S \to B$
0 S 2	$	$S \to B$

	a	b	c	$	S	B
0		3		$B \to \epsilon$	2	1
1				$S \to B$		
2				accept		
3			$B \to \epsilon$			4
4			5			
5				$B \to bBc$		



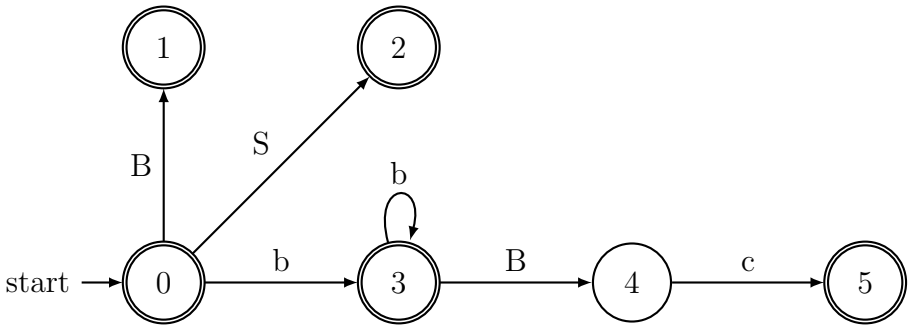
Example: $a^m b^n c^{m+n}$**, Part III**

$$\begin{aligned} S &\rightarrow aSc \mid B \\ B &\rightarrow bBc \mid \epsilon \end{aligned}$$

$S \overset{S \to B}{\Longrightarrow} B \overset{B \to bBc}{\Longrightarrow} bBc \overset{B \to bBc}{\Longrightarrow} bbBcc \overset{B \to \epsilon}{\Longrightarrow} bbcc$

Stack	Input	Rule
0	bbcc$	shift
0 b 3	bcc$	shift
0 b 3 b 3	cc$	$B \to \epsilon$
0 b 3 b 3 B 4	cc$	shift
0 b 3 b 3 B 4 c 5	c$	$B \to bBc$
0 b 3 B 4	c$	shift
0 b 3 B 4 c 5	$	$B \to bBc$
0 B 1	$	$S \to B$
0 S 2	$	accept

	a	b	c	$	S	B
0		3		$B \to \epsilon$	2	1
1				$S \to B$		
2				accept		
3		3	$B \to \epsilon$			4
4			5			
5			$B \to bBc$	$B \to bBc$		



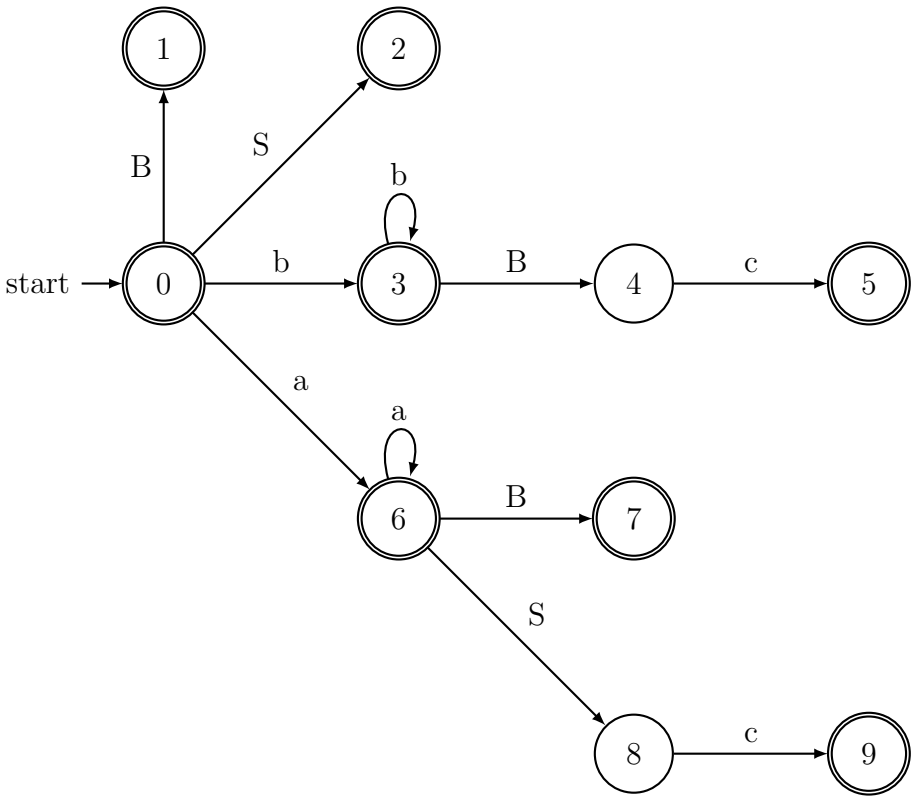
Example: $a^m b^n c^{m+n}$**, Part IV**

$$\begin{aligned} S &\rightarrow aSc \mid B \\ B &\rightarrow bBc \mid \epsilon \end{aligned}$$

$S \overset{S \to aSc}{\Longrightarrow} aSc \overset{S \to aSc}{\Longrightarrow} aaScc \overset{S \to B}{\Longrightarrow} aaBcc \overset{B \to \epsilon}{\Longrightarrow} aacc$

Stack	Input	Rule
0	aacc$	shift
0 a 6	acc$	shift
0 a 6 a 6	cc$	$B \to \epsilon$
0 a 6 a 6 B 7	cc$	$S \to B$
0 a 6 a 6 S 8	cc$	shift
0 a 6 a 6 S 8 c 9	c$	$S \to aSc$
0 a 6 S 8	c$	shift
0 a 6 S 8 c 9	$	$S \to aSc$
0 S 2	$	accept

	a	b	c	$	S	B
0		3		$B \to \epsilon$	2	1
1				$S \to B$		
2				accept		
3		3	$B \to \epsilon$			4
4			5			
5			$B \to bBc$	$B \to bBc$		
6	6		$B \to \epsilon$			7
7			$S \to B$			
8			9			
9			$S \to aSc$	$S \to aSc$		



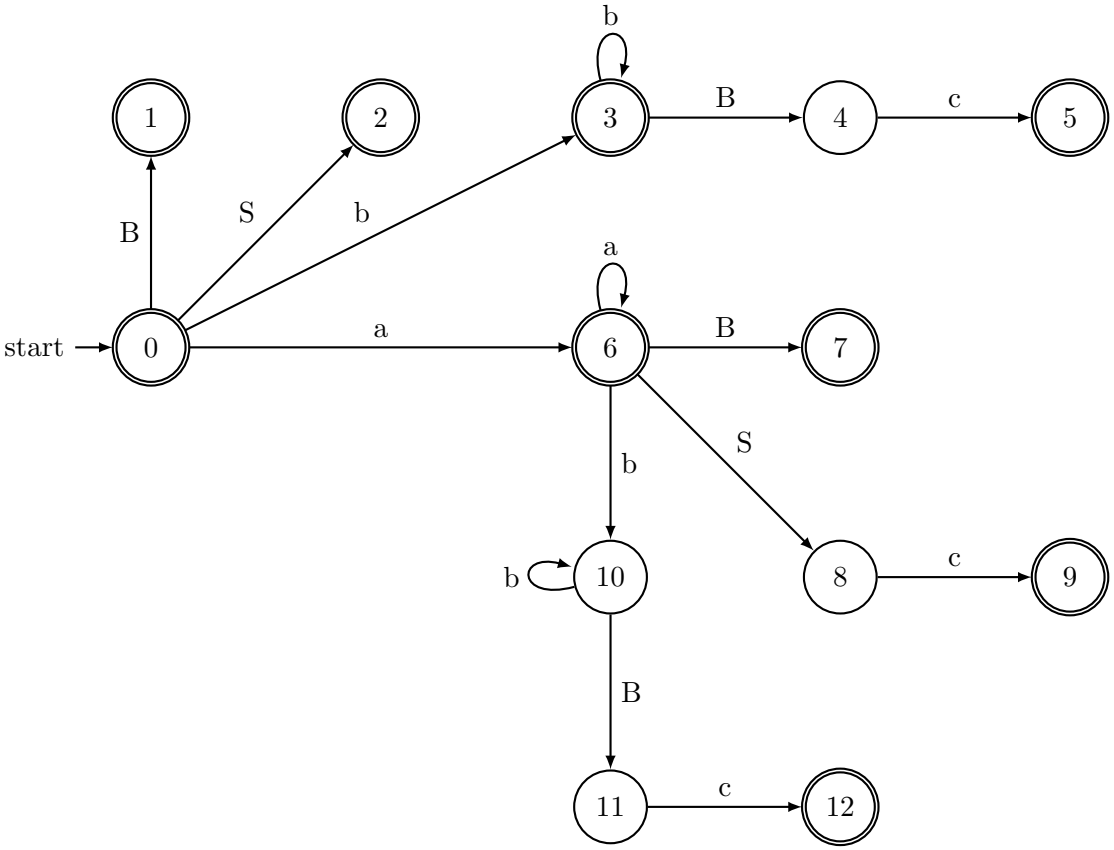
Example: $a^m b^n c^{m+n}$, Part V

$$\begin{aligned} S &\rightarrow aSc \mid B \\ B &\rightarrow bBc \mid \epsilon \end{aligned}$$

$$S \overset{S\to aSc}{\Longrightarrow} aSc \overset{S\to aSc}{\Longrightarrow} aaScc \overset{S\to B}{\Longrightarrow} aaBcc \overset{B\to bBc}{\Longrightarrow} aabBccc \overset{B\to bBc}{\Longrightarrow} aabbBcccc \overset{B\to\epsilon}{\Longrightarrow} aabbcccc$$

Stack	Input	Rule
0	aabbcccc$	shift
0 a 6	abbcccc$	shift
0 a 6 a 6	bbcccc$	shift
0 a 6 a 6 b 10	bcccc$	shift
0 a 6 a 6 b 10 b 10	cccc$	$B \rightarrow \epsilon$
0 a 6 a 6 b 10 b 10 B 11	cccc$	shift
0 a 6 a 6 b 10 b 10 B 11 c 12	ccc$	$B \rightarrow bBc$
0 a 6 a 6 b 10 B 11	ccc$	shift
0 a 6 a 6 b 10 B 11 c 12	cc$	$B \rightarrow bBc$
0 a 6 a 6 B 7	cc$	$S \rightarrow B$
0 a 6 a 6 S 8	cc$	shift
0 a 6 a 6 S 8 c 9	c$	$S \rightarrow aSc$
0 a 6 S 8	c$	shift
0 a 6 S 8 c 9	$	$S \rightarrow aSc$
0 S 2	$	accept

	a	b	c	$	S	B
0		3		$B \rightarrow \epsilon$	2	1
1				$S \rightarrow B$		
2				accept		
3		3	$B \rightarrow \epsilon$			4
4			5			
5			$B \rightarrow bBc$	$B \rightarrow bBc$		
6	6	10	$B \rightarrow \epsilon$			7
7			$S \rightarrow B$			
8			9			
9			$S \rightarrow aSc$	$S \rightarrow aSc$		
10		10				11
11			12			
12			$B \rightarrow bBc$			



Same number of as and bs, Part I

$$
\begin{aligned}
S &\rightarrow \epsilon \mid aB \mid bA \\
A &\rightarrow aS \mid bAA \\
B &\rightarrow bS \mid aBB
\end{aligned}
$$

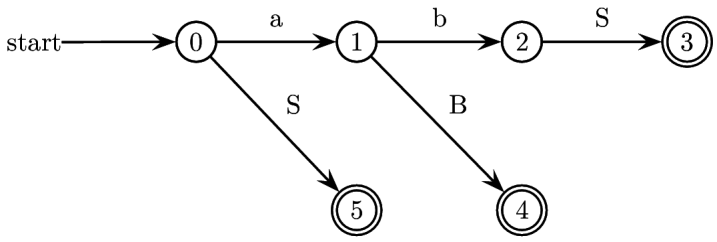
$S \Rightarrow \epsilon$

Stack	Input
0	\$
0 S 5	\$

$S \Rightarrow aB \Rightarrow abS \Rightarrow ab$

Stack	Input
0	a b \$
0 a 1 b 2	\$
0 a 1 b 2 S 3	\$
0 a 1 B 4	\$
0 S 5	\$

	a	b	\$	S	A	B
0	1		$S \rightarrow \epsilon$	5		
1		2				4
2			$S \rightarrow \epsilon$	3		
3			$B \rightarrow bS$			
4			$S \rightarrow aB$			
5			accept			



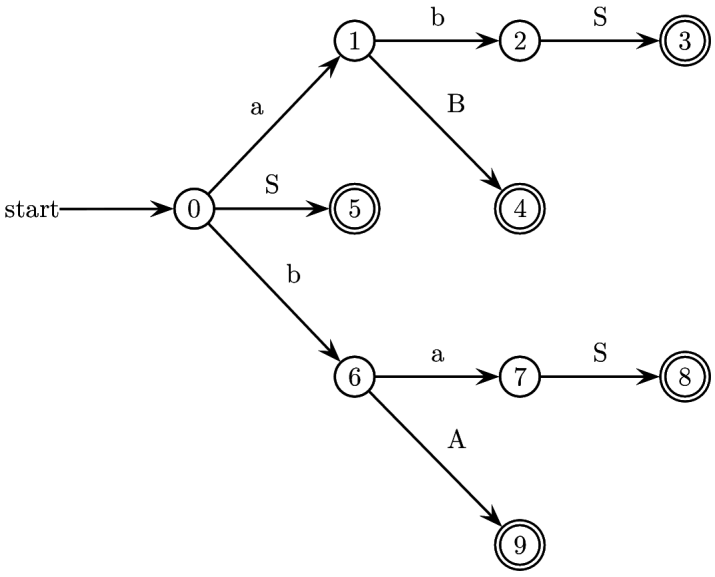
Same number of as and bs, Part II

$$
\begin{aligned}
S &\rightarrow \epsilon \mid aB \mid bA \\
A &\rightarrow aS \mid bAA \\
B &\rightarrow bS \mid aBB
\end{aligned}
$$

$$S \Rightarrow bA \Rightarrow baS \Rightarrow ba$$

Stack	Input
0	b a $
0 b 6	a $
0 b 6 a 7	$
0 b 6 a 7 S 8	$
0 b 6 A 9	$
0 S 5	$

	a	b	$	S	A	B
0	1	6		5		
1		2				4
2			$S \rightarrow \epsilon$	3		
3			$B \rightarrow bS$			
4			$S \rightarrow aB$			
5			accept			
6	7				9	
7			$S \rightarrow \epsilon$	8		
8			$A \rightarrow aS$			
9			$S \rightarrow bA$			



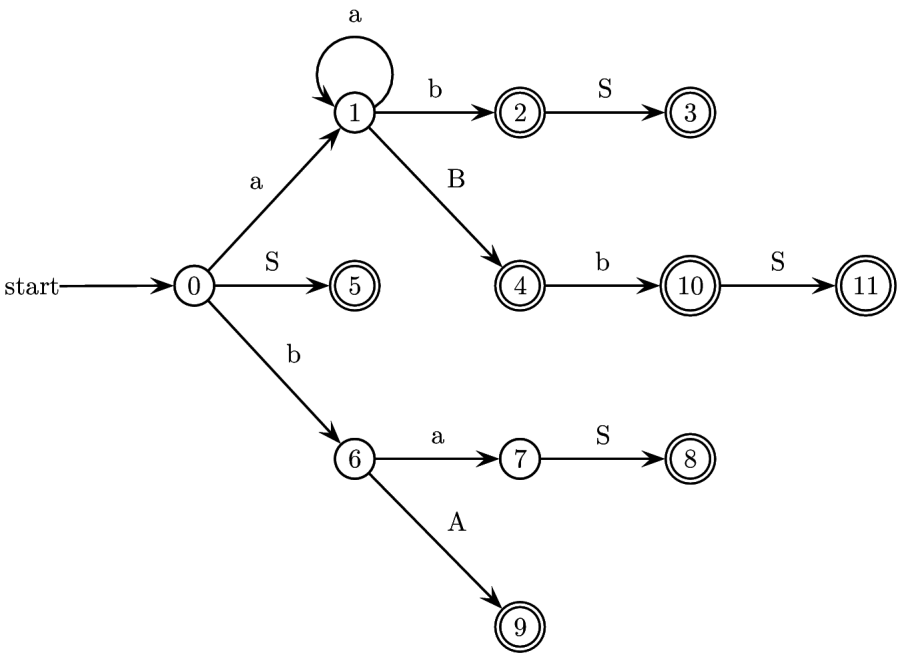
Same number of as and bs, Part III

$$
\begin{aligned}
S &\rightarrow \epsilon \mid aB \mid bA \\
A &\rightarrow aS \mid bAA \\
B &\rightarrow bS \mid aBB
\end{aligned}
$$

$$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aaBbS \Rightarrow aaBb \Rightarrow aabSb \Rightarrow aabb$$

Stack	Input
0	a a b b $
0 a 1	a b b $
0 a 1 a 1	b b $
0 a 1 a 1 b 2	b $
0 a 1 a 1 b 2 S 3	b $
0 a 1 a 1 B 4	b $
0 a 1 a 1 B 4 b 10	$
0 a 1 a 1 B 4 b 10 S 11	$

	a	b	$	S	A	B
0	1	6		5		
1	1	2				4
2		$S \rightarrow \epsilon$	$S \rightarrow \epsilon$	3		
3		$B \rightarrow bS$	$B \rightarrow bS$			
4		10	$S \rightarrow aB$			
5			accept			
6	7				9	
7			$S \rightarrow \epsilon$	8		
8			$A \rightarrow aS$			
9			$S \rightarrow bA$			
10			$S \rightarrow \epsilon$	11		
11			$B \rightarrow bS$			



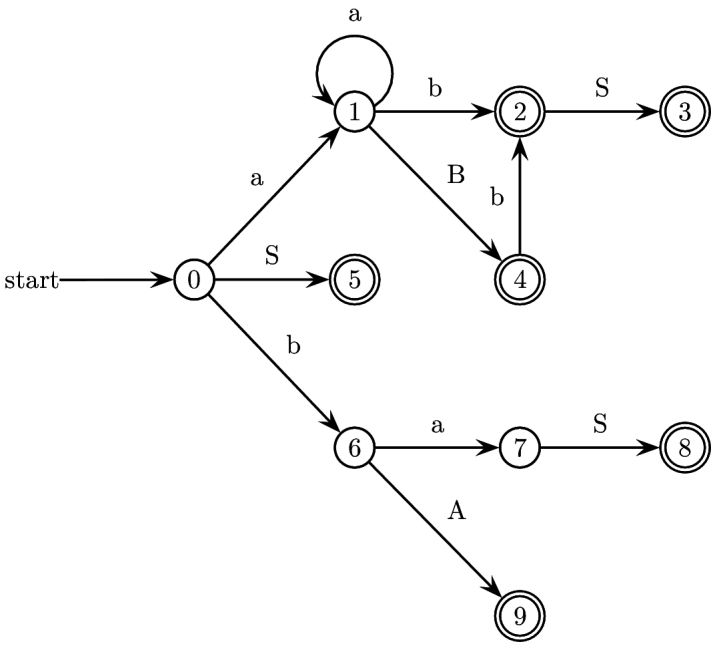
Same number of as and bs, Part IV

$$\begin{aligned} S &\rightarrow \epsilon \mid aB \mid bA \\ A &\rightarrow aS \mid bAA \\ B &\rightarrow bS \mid aBB \end{aligned}$$

$$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aaBbS \Rightarrow aaBb \Rightarrow aabSb \Rightarrow aabb$$

Stack	Input
0	a a b b $
0 a 1	a b b $
0 a 1 a 1	b b $
0 a 1 a 1 b 2	b $
0 a 1 a 1 b 2 S 3	b $
0 a 1 a 1 B 4	b $
0 a 1 a 1 B 4 b 2	$
0 a 1 a 1 B 4 b 2 S 3	$

	a	b	$	S	A	B
0	1	6		5		
1	1	2				4
2		$S \rightarrow \epsilon$	$S \rightarrow \epsilon$	3		
3		$B \rightarrow bS$	$B \rightarrow bS$			
4		2	$S \rightarrow aB$			
5			accept			
6	7				9	
7			$S \rightarrow \epsilon$	8		
8			$A \rightarrow aS$			
9			$S \rightarrow bA$			



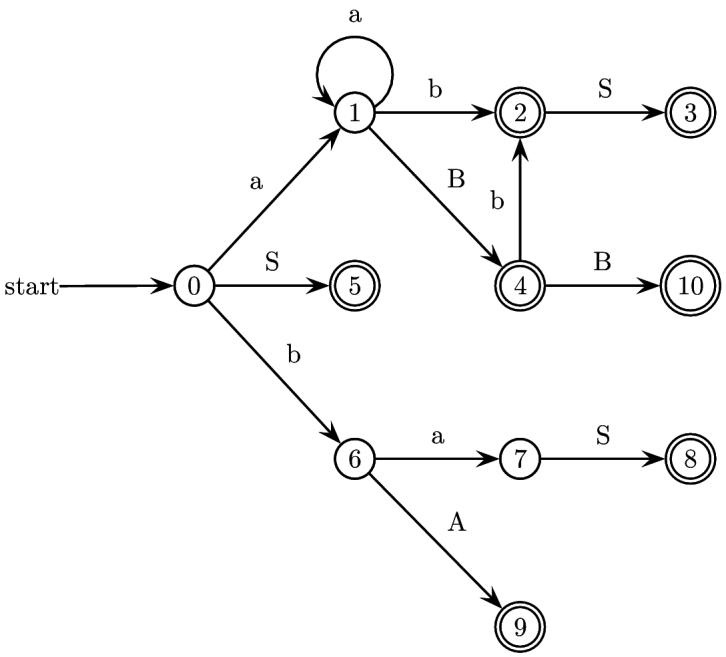
Same number of as and bs, Part V

$$
\begin{aligned}
S &\rightarrow \epsilon \mid aB \mid bA \\
A &\rightarrow aS \mid bAA \\
B &\rightarrow bS \mid aBB
\end{aligned}
$$

$$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aaBbS \Rightarrow aaBb \Rightarrow aabSb \Rightarrow aabb$$

Stack	Input
0	a a b b $
0 a 1	a b b $
0 a 1 a 1	b b $
0 a 1 a 1 b 2	b $
0 a 1 a 1 b 2 S 3	b $
0 a 1 a 1 B 4	b $
0 a 1 a 1 B 4 b 2	$
0 a 1 a 1 B 4 b 2 S 3	$
0 a 1 a 1 B 4 B 10	$

	a	b	$	S	A	B
0	1	6		5		
1	1	2				4
2		$S \rightarrow \epsilon$	$S \rightarrow \epsilon$	3		
3		$B \rightarrow bS$	$B \rightarrow bS$			
4		2	$S \rightarrow aB$			10
5			accept			
6	7				9	
7			$S \rightarrow \epsilon$	8		
8			$A \rightarrow aS$			
9			$S \rightarrow bA$			
10			$B \rightarrow aBB$			



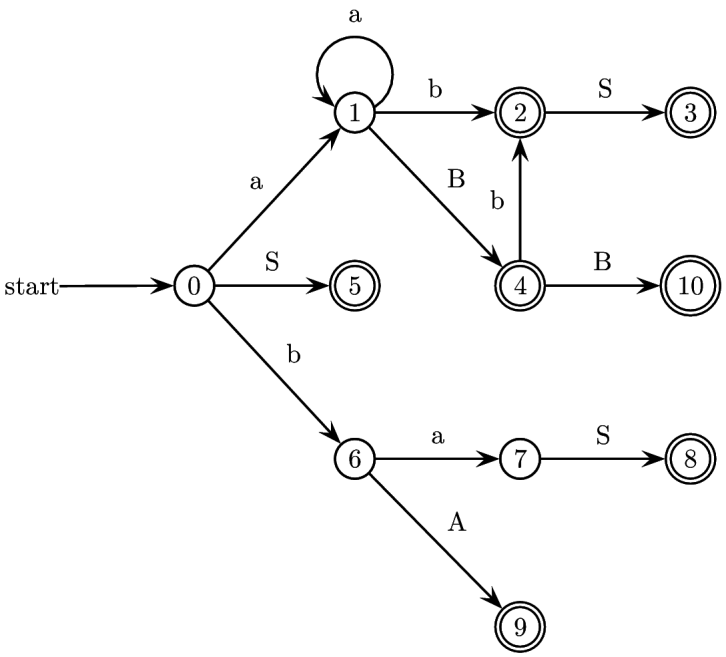
Same number of as and bs, Part VI

$$\begin{aligned} S &\rightarrow \epsilon \mid aB \mid bA \\ A &\rightarrow aS \mid bAA \\ B &\rightarrow bS \mid aBB \end{aligned}$$

$$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aaBbS \Rightarrow aaBb \Rightarrow aabSb \Rightarrow aabb$$

Stack	Input
0	a a b b $
0 a 1	a b b $
0 a 1 a 1	b b $
0 a 1 a 1 b 2	b $
0 a 1 a 1 b 2 S 3	b $
0 a 1 a 1 B 4	b $
0 a 1 a 1 B 4 b 2	$
0 a 1 a 1 B 4 b 2 S 3	$
0 a 1 a 1 B 4 B 10	$
0 a 1 B 4	$
0 S 5	$

	a	b	$	S	A	B
0	1	6		5		
1	1	2				4
2		$S \rightarrow \epsilon$	$S \rightarrow \epsilon$	3		
3		$B \rightarrow bS$	$B \rightarrow bS$			
4		2	$S \rightarrow aB$			10
5			accept			
6	7				9	
7			$S \rightarrow \epsilon$	8		
8			$A \rightarrow aS$			
9			$S \rightarrow bA$			
10			$B \rightarrow aBB$			

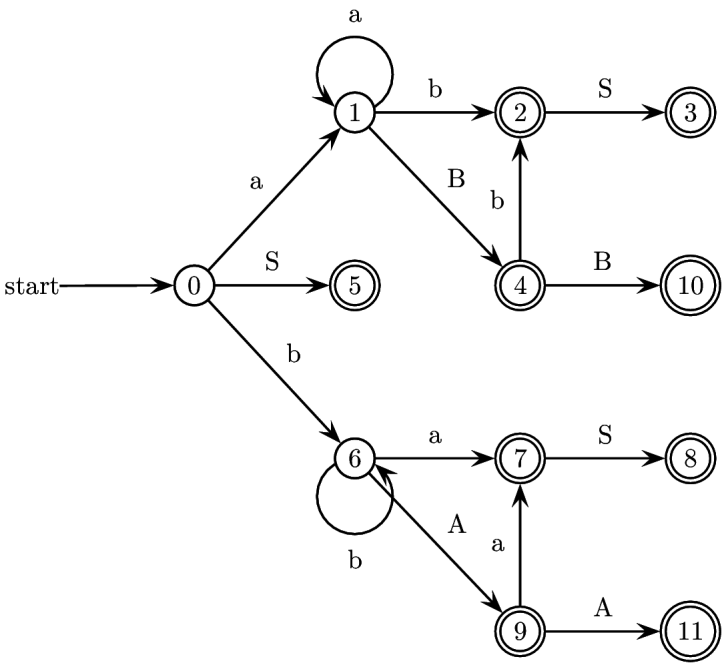


Same number of as and bs, Part VII

$$
\begin{aligned}
S &\rightarrow \epsilon \mid aB \mid bA \\
A &\rightarrow aS \mid bAA \\
B &\rightarrow bS \mid aBB
\end{aligned}
$$

$$S \Rightarrow bA \Rightarrow bbAA \Rightarrow bbAaS \Rightarrow bbAa \Rightarrow bbaSa \Rightarrow bbaa$$

Stack	Input
0	b b a a $
0 b 6	b a a $
0 b 6 b 6	a a $
0 b 6 b 6 a 7	a $
0 b 6 b 6 a 7 S 8	a $
0 b 6 b 6 A 9	a $
0 b 6 b 6 A 9 a 7	$
0 b 6 b 6 A 9 a 7 S 8	$
0 b 6 b 6 A 9 A 11	$
0 b 6 A 9	$
0 S 5	$

	a	b	$	S	A	B
0	1	6		5		
1	1	2				4
2		$S \rightarrow \epsilon$	$S \rightarrow \epsilon$	3		
3		$B \rightarrow bS$	$B \rightarrow bS$			
4		2	$S \rightarrow aB$			10
5			accept			
6	7				9	
7	$S \rightarrow \epsilon$		$S \rightarrow \epsilon$	8		
8	$A \rightarrow aS$		$A \rightarrow aS$			
9	7		$S \rightarrow bA$			
10			$B \rightarrow aBB$			
11			$A \rightarrow bAA$			

By symmetry: