

# Some simple LR parse tables

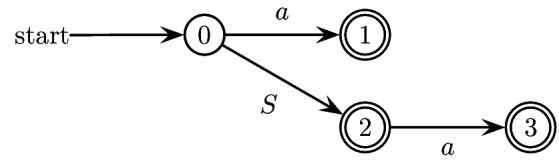
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**Left recursion:**  $S \rightarrow Sa \mid a$

Stack	Input
0	a a a a \$
0 a 1	a a a \$
0 S 2	a a a \$
0 S 2 a 3	a a \$
0 S 2	a a \$
0 S 2 a 3	a \$
0 S 2	a \$
0 S 2 a 3	\$
0 S 2	\$

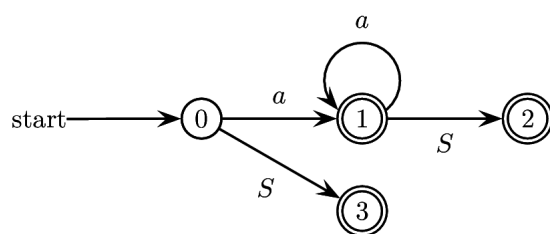
	$a$	$\$$	$S$
0	1		2
1	$S \rightarrow a$		
2	3	accept	
3	$S \rightarrow Sa$	$S \rightarrow Sa$	



**Right recursion:**  $S \rightarrow aS \mid a$

Stack	Input
0	a a a a \$
0 a 1	a a a \$
0 a 1 a 1	a a \$
0 a 1 a 1 a 1	a \$
0 a 1 a 1 a 1 a 1	\$
0 a 1 a 1 a 1 S 2	\$
0 a 1 a 1 S 2	\$
0 a 1 S 2	\$
0 S 3	\$

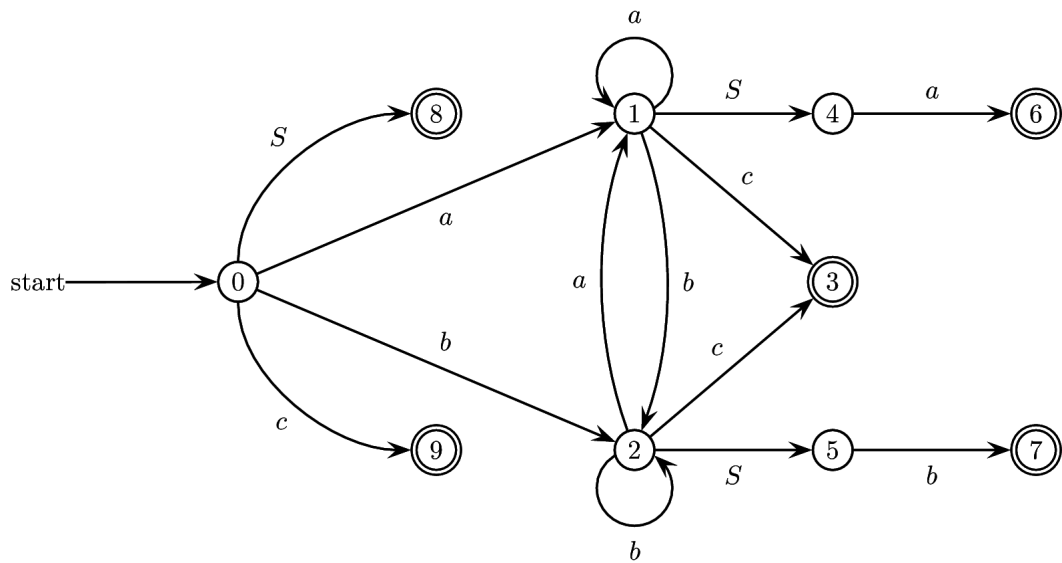
	$a$	$\$$	$S$
0	1		3
1	1	$S \rightarrow a$	2
2		$S \rightarrow aS$	
3		accept	



**Middle recursion:**  $S \rightarrow aSa \mid bSb \mid c$

Stack	Input
0	a b c b a \$
0 a 1	b c b a \$
0 a 1 b 2	c b a \$
0 a 1 b 2 c 3	b a \$
0 a 1 b 2 S 5	b a \$
0 a 1 b 2 S 5 b 7	a \$
0 a 1 S 4	a \$
0 a 1 S 4 a 6	\$
0 S 8	\$

	a	b	c	\$	S
0	1	2	9		8
1	1	2	3		4
2	1	2	3		5
3	$S \rightarrow c$	$S \rightarrow c$			
4	6				
5		7			
6	$S \rightarrow aSa$	$S \rightarrow aSa$		$S \rightarrow aSa$	
7	$S \rightarrow bSb$	$S \rightarrow bSb$		$S \rightarrow bSb$	
8				accept	
9				$S \rightarrow c$	



**Example:**  $a^*b^*b$

$$S \rightarrow AB$$

$$A \rightarrow aA \mid \epsilon$$

$$B \rightarrow bB \mid b$$

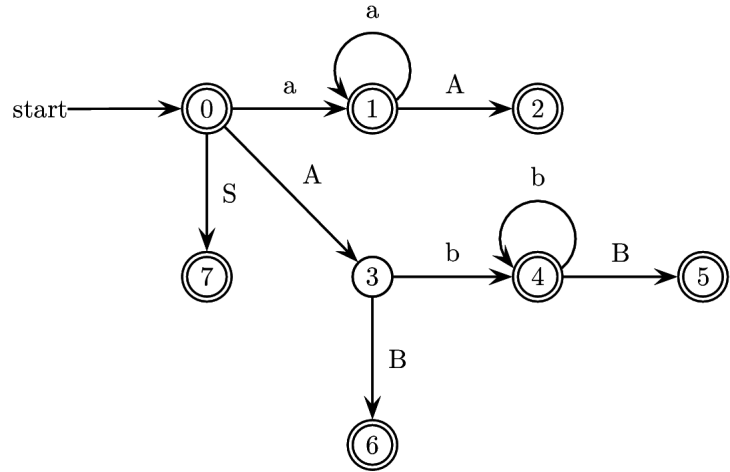
Stack

0  
0 a 1  
0 a 1 a 1  
0 a 1 A 2  
0 A 3  
0 A 3 b 4  
0 A 3 b 4 b 4  
0 A 3 b 4 b 4 b 4  
0 A 3 b 4 b 4 B 5  
0 A 3 b 4 B 5  
0 A 3 B 6  
0 S 7

Input

a a b b b \$  
a b b b \$  
b b b \$  
b b b \$  
b b b \$  
b b \$  
b \$  
\$  
\$  
\$  
\$  
\$

	a	b	\$	S	A	B
0	1	$A \rightarrow \epsilon$		7	3	
1	1	$A \rightarrow \epsilon$			2	
2		$A \rightarrow aA$				
3		4				6
4		4	$B \rightarrow b$			5
5			$B \rightarrow bB$			
6			$S \rightarrow AB$			
7			accept			



Stack

0  
0 A 3  
0 A 3 b 4  
0 A 3 b 4 b 4  
0 A 3 b 4 b 4 b 4  
0 A 3 b 4 b 4 B 5  
0 A 3 b 4 B 5  
0 A 3 B 6  
0 S 7

Input

b b b \$  
b b b \$  
b b \$  
b \$  
\$  
\$  
\$  
\$  
\$

**Example:**  $a^m b^m c^{m+n}$ , **Part I**

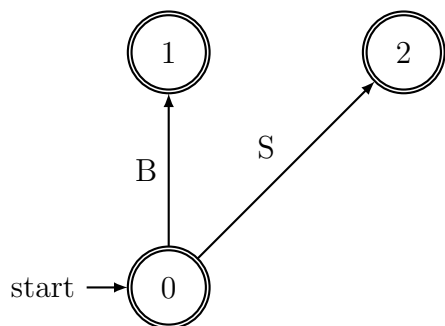
$$S \rightarrow aSc \mid B$$

$$B \rightarrow bBc \mid \epsilon$$

$$S \xRightarrow{S \rightarrow B} B \xRightarrow{B \rightarrow \epsilon} \epsilon$$

Stack	Input	Rule
0	\$	$B \rightarrow \epsilon$
0 B 1	\$	$S \rightarrow B$
0 S 2	\$	

	a	b	c	\$	S	B
0				$B \rightarrow \epsilon$	2	1
1				$S \rightarrow B$		
2				accept		



**Example:  $a^m b^m c^{m+n}$ , Part II**

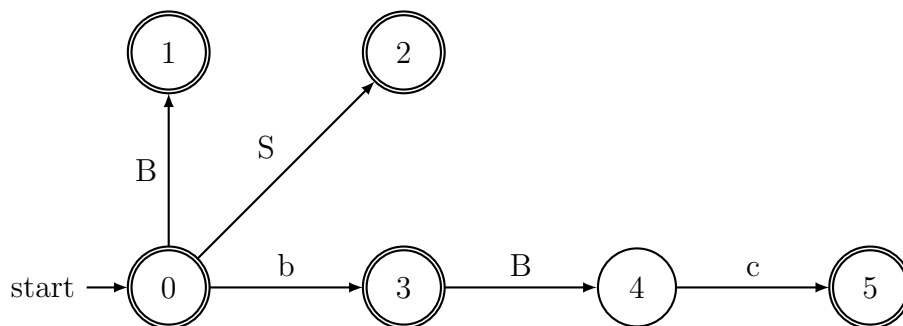
$$S \rightarrow aSc \mid B$$

$$B \rightarrow bBc \mid \epsilon$$

$$S \xRightarrow{S \rightarrow B} B \xRightarrow{B \rightarrow bBc} bBc \xRightarrow{B \rightarrow \epsilon} bc$$

Stack	Input	Rule
0	bc\$	shift
0 b 3	c\$	$B \rightarrow \epsilon$
0 b 3 B 4	c\$	shift
0 b 3 B 4 c 5	\$	$B \rightarrow bBc$
0 B 1	\$	$S \rightarrow B$
0 S 2	\$	$S \rightarrow B$

	a	b	c	\$	S	B
0		3		$B \rightarrow \epsilon$	2	1
1				$S \rightarrow B$		
2				accept		
3			$B \rightarrow \epsilon$			4
4			5			
5				$B \rightarrow bBc$		



**Example:  $a^m b^m c^{m+n}$ , Part III**

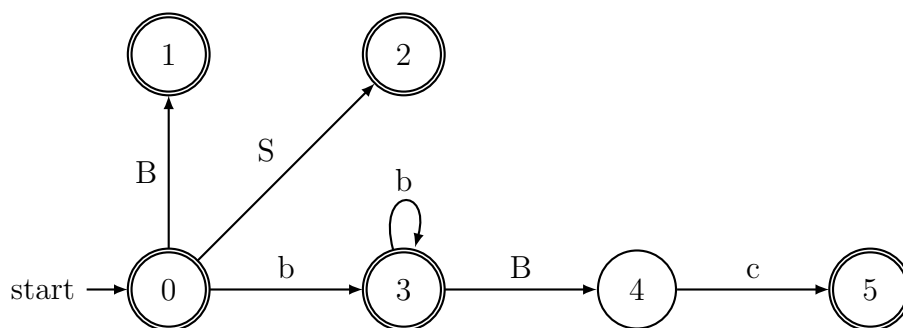
$$S \rightarrow aSc \mid B$$

$$B \rightarrow bBc \mid \epsilon$$

$$S \xRightarrow{S \rightarrow B} B \xRightarrow{B \rightarrow bBc} bBc \xRightarrow{B \rightarrow bBc} bbBcc \xRightarrow{B \rightarrow \epsilon} bbcc$$

Stack	Input	Rule
0	bbcc\$	shift
0 b 3	bcc\$	shift
0 b 3 b 3	cc\$	$B \rightarrow \epsilon$
0 b 3 b 3 B 4	cc\$	shift
0 b 3 b 3 B 4 c 5	c\$	$B \rightarrow bBc$
0 b 3 B 4	c\$	shift
0 b 3 B 4 c 5	\$	$B \rightarrow bBc$
0 B 1	\$	$S \rightarrow B$
0 S 2	\$	accept

	a	b	c	\$	S	B
0		3		$B \rightarrow \epsilon$	2	1
1				$S \rightarrow B$		
2				accept		
3		3	$B \rightarrow \epsilon$			4
4			5			
5			$B \rightarrow bBc$	$B \rightarrow bBc$		



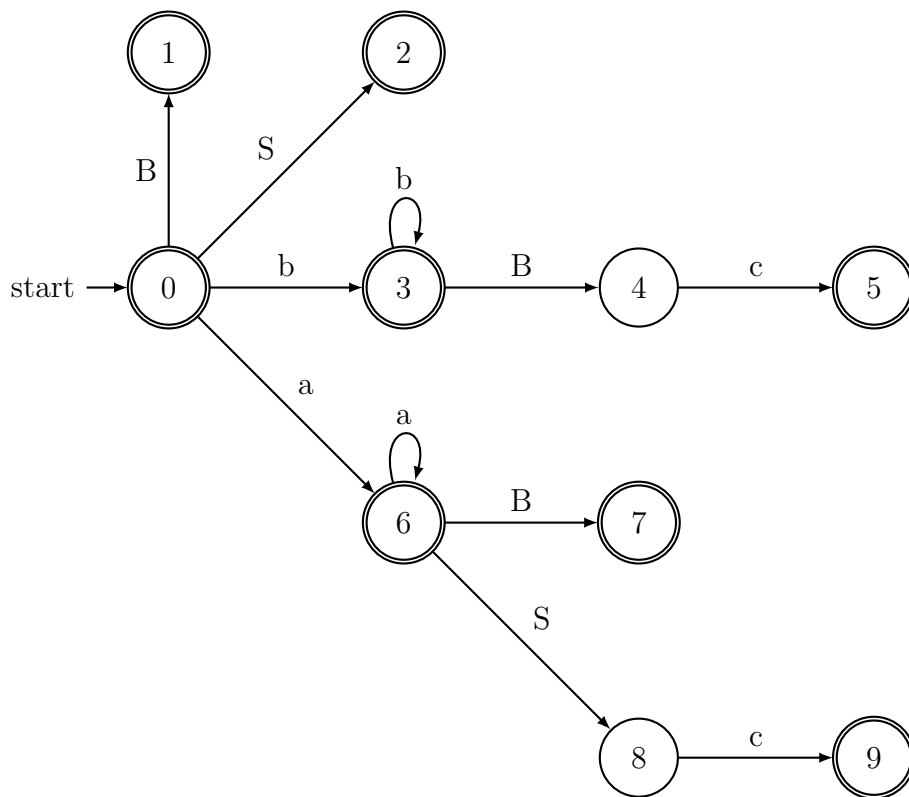
**Example:  $a^m b^m c^{m+n}$ , Part IV**

$$S \rightarrow aSc \mid B$$

$$B \rightarrow bBc \mid \epsilon$$

$$S \xRightarrow{S \rightarrow aSc} aSc \xRightarrow{S \rightarrow aSc} aaSc \xRightarrow{S \rightarrow B} aaBcc \xRightarrow{B \rightarrow \epsilon} aacc$$

Stack	Input	Rule		a	b	c	\$	S	B
0	aacc\$	shift			3		$B \rightarrow \epsilon$	2	1
0 a 6	acc\$	shift					$S \rightarrow B$		
0 a 6 a 6	cc\$	$B \rightarrow \epsilon$					accept		
0 a 6 a 6 B 7	cc\$	$S \rightarrow B$			3	$B \rightarrow \epsilon$			4
0 a 6 a 6 S 8	cc\$	shift				5			
0 a 6 a 6 S 8 c 9	c\$	$S \rightarrow aSc$				$B \rightarrow bBc$	$B \rightarrow bBc$		
0 a 6 S 8	c\$	shift		6		$B \rightarrow \epsilon$			7
0 a 6 S 8 c 9	\$	$S \rightarrow aSc$				$S \rightarrow B$			
0 S 2	\$	accept				9			
						$S \rightarrow aSc$	$S \rightarrow aSc$		





**Example:  $a^m b^m c^{m+n}$ , Part V**

$$S \rightarrow aSc \mid B$$

$$B \rightarrow bBc \mid \epsilon$$

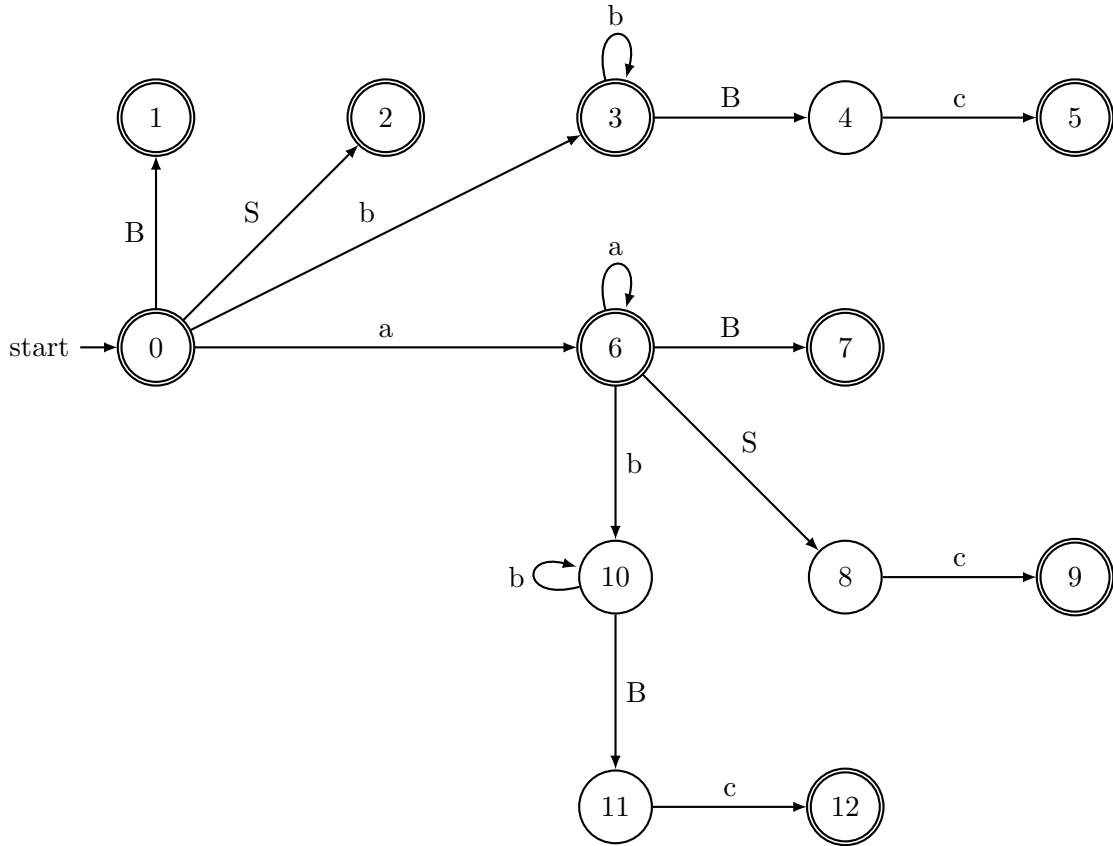
$$S \xRightarrow{S \rightarrow aSc} aSc \xRightarrow{S \rightarrow aSc} aaSc \xRightarrow{S \rightarrow B} aaBcc \xRightarrow{B \rightarrow bBc} aabBccc \xRightarrow{B \rightarrow bBc} aabbBcccc \xRightarrow{B \rightarrow \epsilon} aabbccccc$$

Stack

0  
0 a 6  
0 a 6 a 6  
0 a 6 a 6 b 10  
0 a 6 a 6 b 10 b 10  
0 a 6 a 6 b 10 b 10 B 11  
0 a 6 a 6 b 10 b 10 B 11 c 12  
0 a 6 a 6 b 10 B 11  
0 a 6 a 6 b 10 B 11 c 12  
0 a 6 a 6 B 7  
0 a 6 a 6 S 8  
0 a 6 a 6 S 8 c 9  
0 a 6 S 8  
0 a 6 S 8 c 9  
0 S 2

Input Rule  
aabbccccc\$ shift  
abbccccc\$ shift  
bbccccc\$ shift  
bccccc\$ shift  
cccc\$  $B \rightarrow \epsilon$   
cccc\$ shift  
ccc\$  $B \rightarrow bBc$   
ccc\$ shift  
cc\$  $B \rightarrow bBc$   
cc\$  $S \rightarrow B$   
cc\$ shift  
c\$  $S \rightarrow aSc$   
c\$ shift  
\$  $S \rightarrow aSc$   
\$ accept

	a	b	c	\$	S	B
0		3		$B \rightarrow \epsilon$	2	1
1				$S \rightarrow B$		
2				accept		
3		3	$B \rightarrow \epsilon$			4
4			5			
5			$B \rightarrow bBc$	$B \rightarrow bBc$		
6	6	10	$B \rightarrow \epsilon$			7
7			$S \rightarrow B$			
8			9			
9			$S \rightarrow aSc$	$S \rightarrow aSc$		
10		10				11
11			12			
12			$B \rightarrow bBc$			



# Same number of *as* and *bs*, Part I

$$S \rightarrow \epsilon \mid aB \mid bA$$

$$A \rightarrow aS \mid bAA$$

$$B \rightarrow bS \mid aBB$$

$$S \Rightarrow \epsilon$$

Stack Input

0 \$

0 S 5 \$

$$S \Rightarrow aB \Rightarrow abS \Rightarrow ab$$

Stack Input

0 a b \$

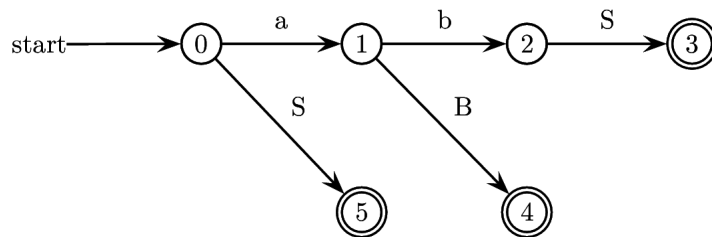
0 a 1 b 2 \$

0 a 1 b 2 S 3 \$

0 a 1 B 4 \$

0 S 5 \$

	a	b	\$	S	A	B
0	1		$S \rightarrow \epsilon$	5		
1		2				4
2			$S \rightarrow \epsilon$	3		
3			$B \rightarrow bS$			
4			$S \rightarrow aB$			
5			accept			



## Same number of *as* and *bs*, Part II

$$S \rightarrow \epsilon \mid aB \mid bA$$

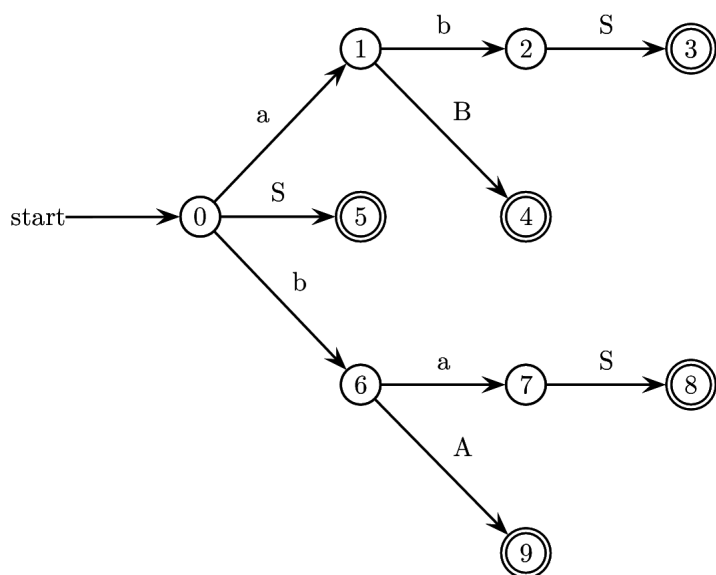
$$A \rightarrow aS \mid bAA$$

$$B \rightarrow bS \mid aBB$$

$$S \Rightarrow bA \Rightarrow baS \Rightarrow ba$$

Stack	Input
0	b a \$
0 b 6	a \$
0 b 6 a 7	\$
0 b 6 a 7 S 8	\$
0 b 6 A 9	\$
0 S 5	\$

	a	b	\$	S	A	B
0	1	6		5		
1		2				4
2			$S \rightarrow \epsilon$	3		
3			$B \rightarrow bS$			
4			$S \rightarrow aB$			
5			accept			
6	7				9	
7			$S \rightarrow \epsilon$	8		
8			$A \rightarrow aS$			
9			$S \rightarrow bA$			



### Same number of *as* and *bs*, Part III

$$S \rightarrow \epsilon \mid aB \mid bA$$

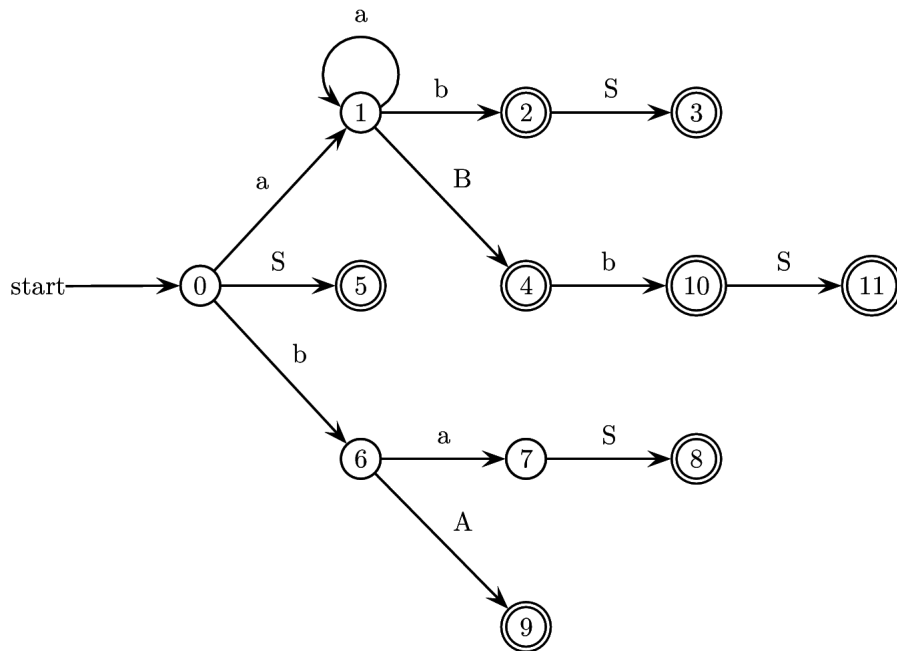
$$A \rightarrow aS \mid bAA$$

$$B \rightarrow bS \mid aBB$$

$$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aaBbS \Rightarrow aaBb \Rightarrow aabSb \Rightarrow aabb$$

Stack	Input
0	a a b b \$
0 a 1	a b b \$
0 a 1 a 1	b b \$
0 a 1 a 1 b 2	b \$
0 a 1 a 1 b 2 S 3	b \$
0 a 1 a 1 B 4	b \$
0 a 1 a 1 B 4 b 10	\$
0 a 1 a 1 B 4 b 10 S 11	\$

	a	b	\$	S	A	B
0	1	6		5		
1	1	2				4
2		$S \rightarrow \epsilon$	$S \rightarrow \epsilon$	3		
3		$B \rightarrow bS$	$B \rightarrow bS$			
4		10	$S \rightarrow aB$			
5			accept			
6	7				9	
7			$S \rightarrow \epsilon$	8		
8			$A \rightarrow aS$			
9			$S \rightarrow bA$			
10			$S \rightarrow \epsilon$	11		
11			$B \rightarrow bS$			



# Same number of *as* and *bs*, Part IV

$$S \rightarrow \epsilon \mid aB \mid bA$$

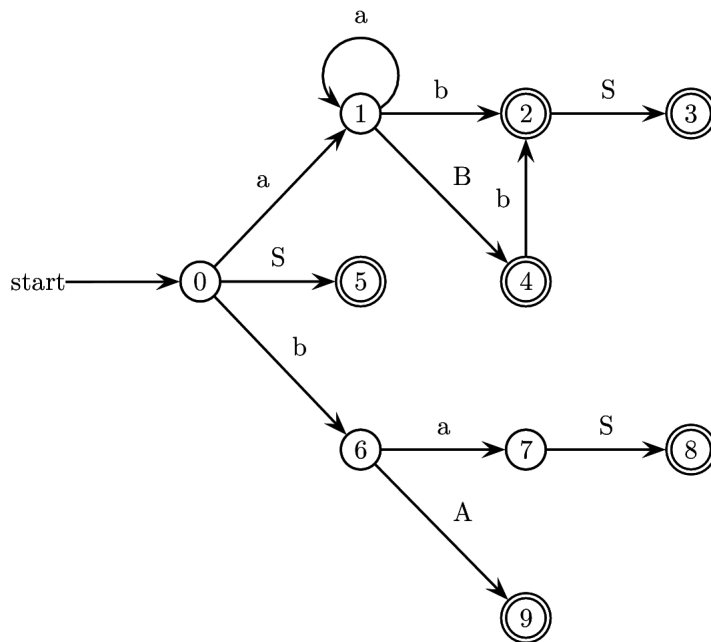
$$A \rightarrow aS \mid bAA$$

$$B \rightarrow bS \mid aBB$$

$$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aaBbS \Rightarrow aaBb \Rightarrow aabSb \Rightarrow aabb$$

Stack	Input
0	a a b b \$
0 a 1	a b b \$
0 a 1 a 1	b b \$
0 a 1 a 1 b 2	b \$
0 a 1 a 1 b 2 S 3	b \$
0 a 1 a 1 B 4	b \$
0 a 1 a 1 B 4 b 2	\$
0 a 1 a 1 B 4 b 2 S 3	\$

	a	b	\$	S	A	B
0	1	6		5		
1	1	2				4
2		$S \rightarrow \epsilon$	$S \rightarrow \epsilon$	3		
3		$B \rightarrow bS$	$B \rightarrow bS$			
4		2	$S \rightarrow aB$			
5			accept			
6	7				9	
7			$S \rightarrow \epsilon$	8		
8			$A \rightarrow aS$			
9			$S \rightarrow bA$			



# Same number of *as* and *bs*, Part V

$$S \rightarrow \epsilon \mid aB \mid bA$$

$$A \rightarrow aS \mid bAA$$

$$B \rightarrow bS \mid aBB$$

$$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aaBbS \Rightarrow aaBb \Rightarrow aabSb \Rightarrow aabb$$

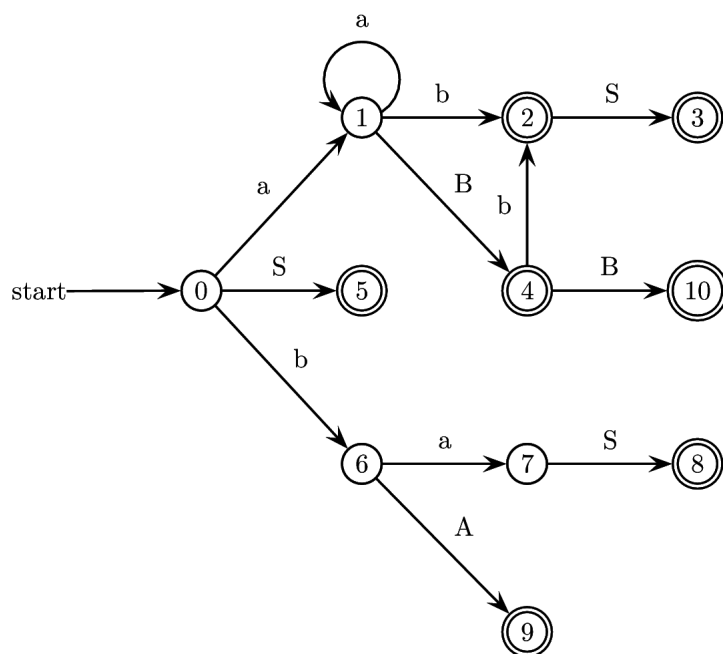
Stack

0  
0 a 1  
0 a 1 a 1  
0 a 1 a 1 b 2  
0 a 1 a 1 b 2 S 3  
0 a 1 a 1 B 4  
0 a 1 a 1 B 4 b 2  
0 a 1 a 1 B 4 b 2 S 3  
0 a 1 a 1 B 4 B 10

Input

a a b b \$  
a b b \$  
b b \$  
b \$  
b \$  
b \$  
\$  
\$  
\$

	a	b	\$	S	A	B
0	1	6		5		
1	1	2				4
2		$S \rightarrow \epsilon$	$S \rightarrow \epsilon$	3		
3		$B \rightarrow bS$	$B \rightarrow bS$			
4		2	$S \rightarrow aB$			10
5			accept			
6	7				9	
7			$S \rightarrow \epsilon$	8		
8			$A \rightarrow aS$			
9			$S \rightarrow bA$			
10			$B \rightarrow aBB$			



# Same number of *as* and *bs*, Part VI

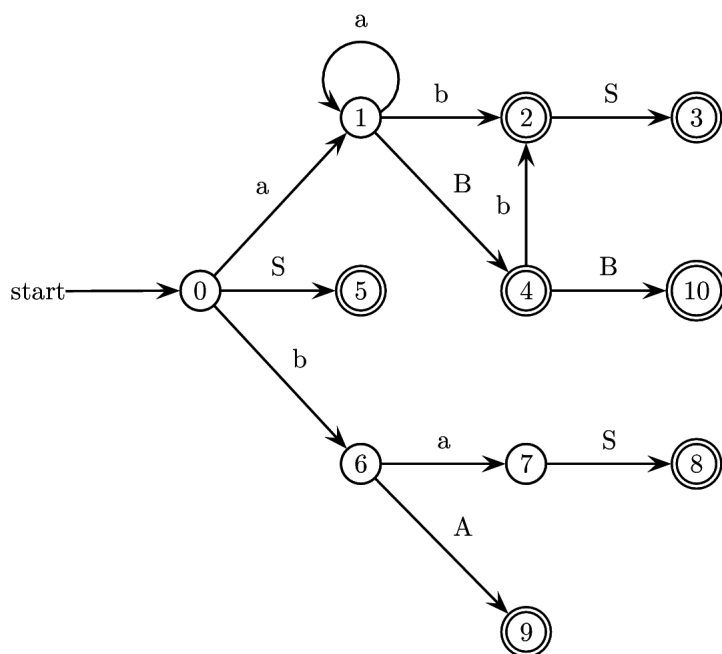
$$S \rightarrow \epsilon \mid aB \mid bA$$

$$A \rightarrow aS \mid bAA$$

$$B \rightarrow bS \mid aBB$$

$$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aaBbS \Rightarrow aaBb \Rightarrow aabSb \Rightarrow aabb$$

Stack	Input		a	b	\$	S	A	B
0	a a b b \$	0	1	6		5		
0 a 1	a b b \$	1	1	2				4
0 a 1 a 1	b b \$	2		$S \rightarrow \epsilon$	$S \rightarrow \epsilon$	3		
0 a 1 a 1 b 2	b \$	3		$B \rightarrow bS$	$B \rightarrow bS$			
0 a 1 a 1 b 2 S 3	b \$	4		2	$S \rightarrow aB$			10
0 a 1 a 1 B 4	b \$	5			accept			
0 a 1 a 1 B 4 b 2	\$	6	7				9	
0 a 1 a 1 B 4 b 2 S 3	\$	7			$S \rightarrow \epsilon$	8		
0 a 1 a 1 B 4 B 10	\$	8			$A \rightarrow aS$			
0 a 1 B 4	\$	9			$S \rightarrow bA$			
0 S 5	\$	10			$B \rightarrow aBB$			



## Same number of *as* and *bs*, Part VII

$$S \rightarrow \epsilon \mid aB \mid bA$$

$$A \rightarrow aS \mid bAA$$

$$B \rightarrow bS \mid aBB$$

$$S \Rightarrow bA \Rightarrow bbAA \Rightarrow bbAaS \Rightarrow bbAa \Rightarrow bbaSa \Rightarrow bbaa$$

Stack	Input	a	b	\$	S	A	B
0	b b a a \$	1	6		5		
0 b 6	b a a \$	1	2				4
0 b 6 b 6	a a \$		$S \rightarrow \epsilon$	$S \rightarrow \epsilon$	3		
0 b 6 b 6 a 7	a \$		$B \rightarrow bS$	$B \rightarrow bS$			
0 b 6 b 6 a 7 S 8	a \$		2	$S \rightarrow aB$			10
0 b 6 b 6 A 9	a \$			accept			
0 b 6 b 6 A 9 a 7	\$	7				9	
0 b 6 b 6 A 9 a 7 S 8	\$	$S \rightarrow \epsilon$		$S \rightarrow \epsilon$	8		
0 b 6 b 6 A 9 A 11	\$	$A \rightarrow aS$		$A \rightarrow aS$			
0 b 6 A 9	\$	7		$S \rightarrow bA$			
0 S 5	\$			$B \rightarrow aBB$			
	\$			$A \rightarrow bAA$			

By symmetry:

