Notes on LR Parsing

Geoffrey Matthews

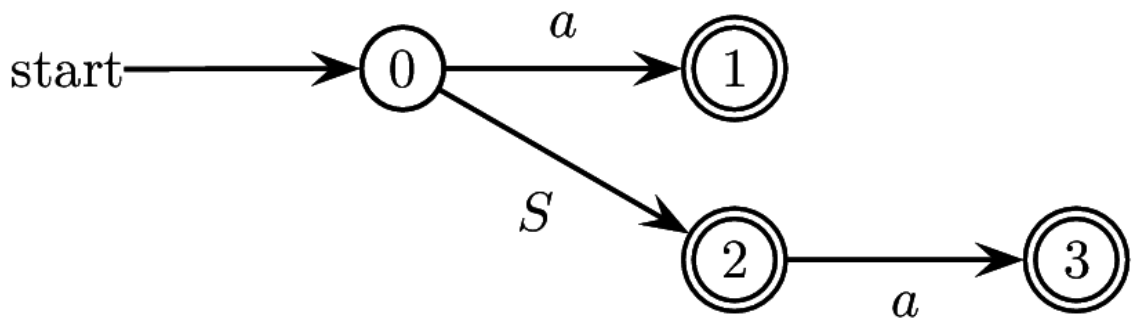
Department of Computer Science
Western Washington University

November 10, 2013

Left recursion: $S \to Sa \mid a$

Stack	Input
0	a a a a $
0 a 1	a a a $
0 S 2	a a a $
0 S 2 a 3	a a $
0 S 2	a a $
0 S 2 a 3	a $
0 S 2	a $
0 S 2 a 3	$
0 S 2	$

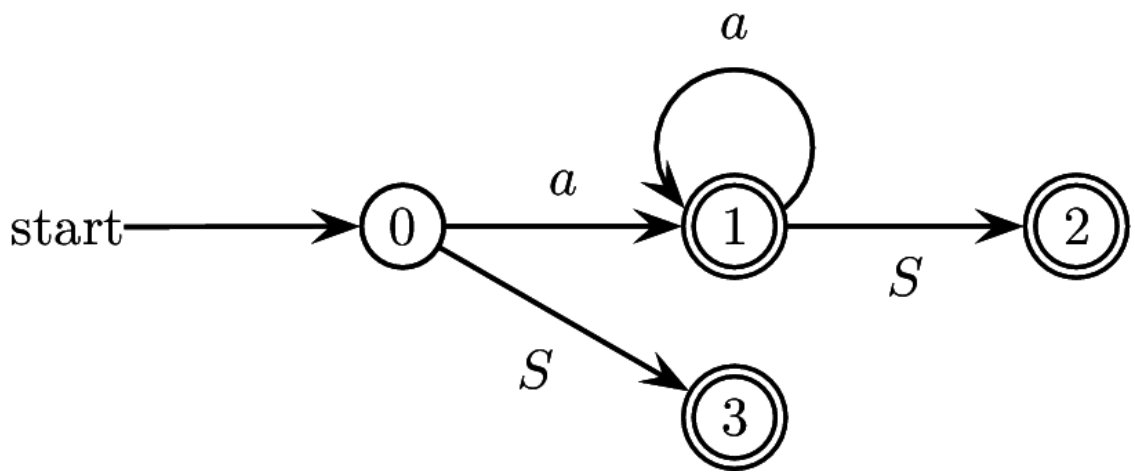
	a	\$	S
0	1		2
1	$S \to a$		
2	3	accept	
3	$S \to Sa$	$S \to Sa$	



Right recursion: $S \rightarrow aS \mid a$

Stack	Input
0	a a a a $
0 a 1	a a a $
0 a 1 a 1	a a $
0 a 1 a 1 a 1	a $
0 a 1 a 1 a 1 a 1	$
0 a 1 a 1 a 1 S 2	$
0 a 1 a 1 S 2	$
0 a 1 S 2	$
0 S 3	$

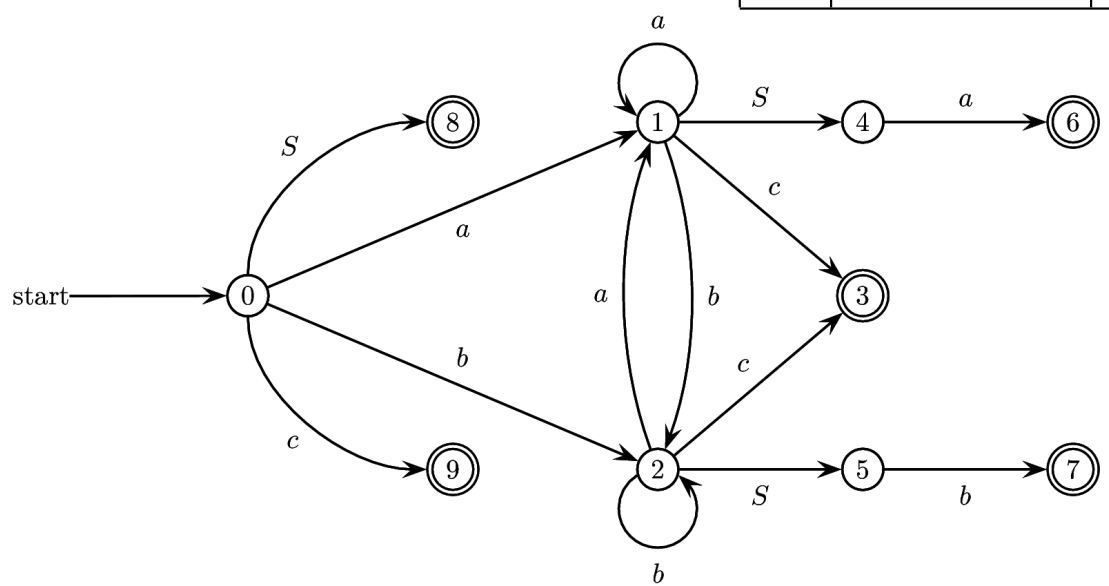
	a	\$	S
0	1		3
1	1	$S \rightarrow a$	2
2		$S \rightarrow aS$	
3		accept	



Middle recursion: $S \to aSa \mid bSb \mid c$

Stack	Input
0	a b c b a $
0 a 1	b c b a $
0 a 1 b 2	c b a $
0 a 1 b 2 c 3	b a $
0 a 1 b 2 S 5	b a $
0 a 1 b 2 S 5 b 7	a $
0 a 1 S 4	a $
0 a 1 S 4 a 6	$
0 S 8	$

	a	b	c	$	S
0	1	2	9		8
1	1	2	3		4
2	1	2	3		5
3	$S \to c$	$S \to c$			
4	6				
5		7			
6	$S \to aSa$	$S \to aSa$		$S \to aSa$	
7	$S \to bSb$	$S \to bSb$		$S \to bSb$	
8				accept	
9				$S \to c$	

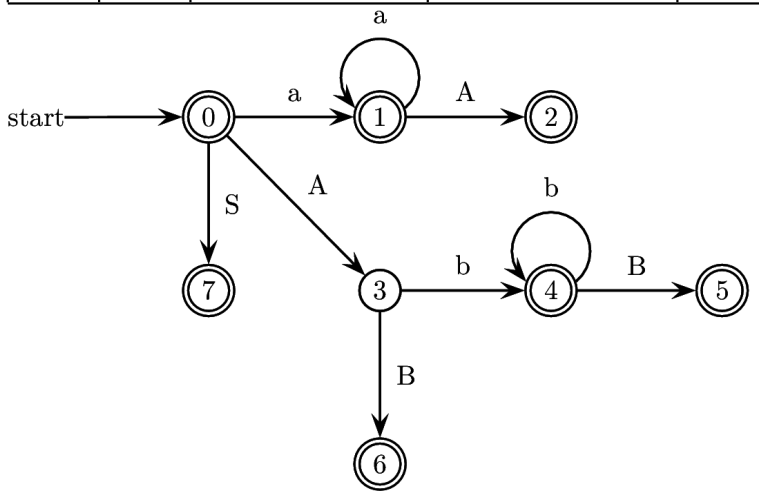


$S \to AB$ $A \to aA \mid \Lambda$ $B \to bB \mid b$

Stack	Input
0	a a b b b $
0 a 1	a b b b $
0 a 1 a 1	b b b $
0 a 1 A 2	b b b $
0 A 3	b b b $
0 A 3 b 4	b b $
0 A 3 b 4 b 4	b $
0 A 3 b 4 b 4 b 4	$
0 A 3 b 4 b 4 B 5	$
0 A 3 b 4 B 5	$
0 A 3 B 6	$
0 S 7	$

Stack	Input
0	b b b $
0 A 3	b b b $
0 A 3 b 4	b b $
0 A 3 b 4 b 4	b $
0 A 3 b 4 b 4 b 4	$
0 A 3 b 4 b 4 B 5	$
0 A 3 b 4 B 5	$
0 A 3 B 6	$
0 S 7	$

	a	b	$	S	A	B
0	1	$A \to \Lambda$		7	3	
1	1	$A \to \Lambda$			2	
2		$A \to aA$				
3		4				6
4		4	$B \to b$			5
5			$B \to bB$			
6			$S \to AB$			
7			accept			

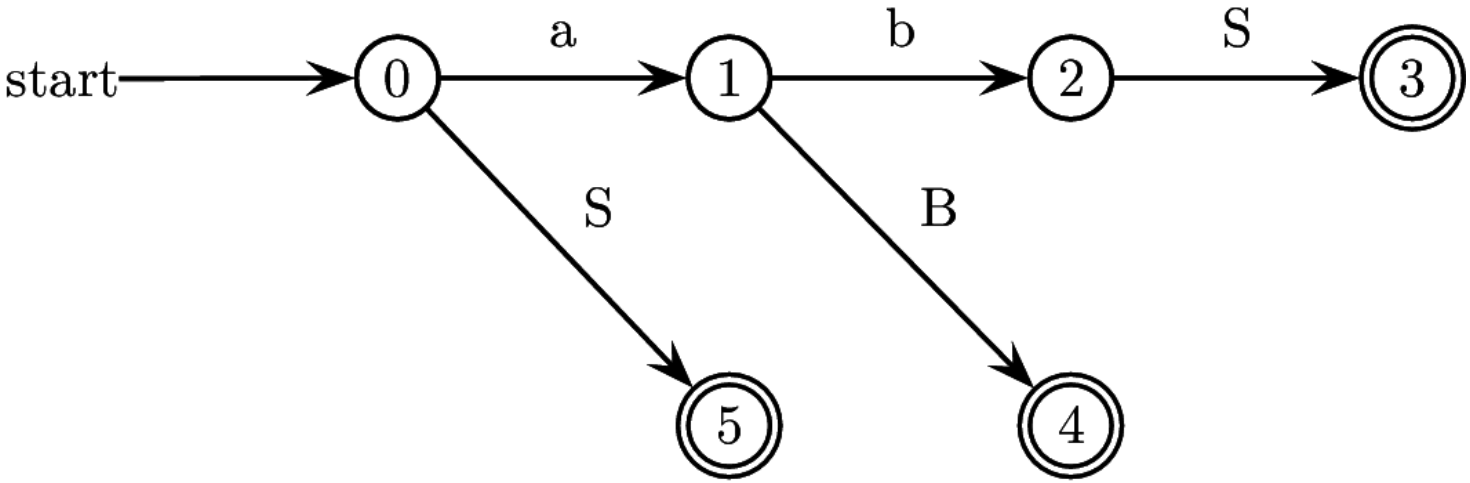


Part I $S \to \Lambda|aB|bA \quad A \to aS|bAA \quad B \to bS|aBB$

$$S \Rightarrow aB \Rightarrow abS \Rightarrow ab$$

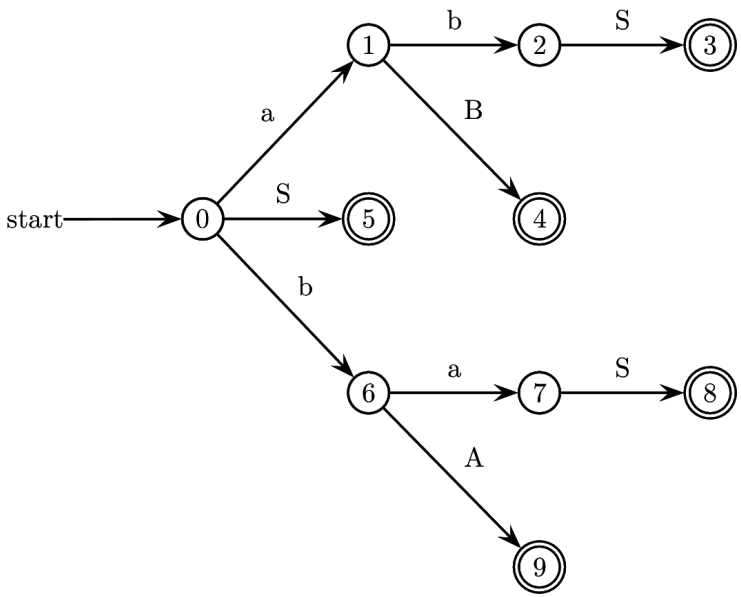
Stack	Input
0	a b $
0 a 1 b 2	$
0 a 1 b 2 S 3	$
0 a 1 B 4	$
0 S 5	$

	a	b	$	S	A	B
0	1			5		
1		2				4
2			$S \to \Lambda$	3		
3			$B \to bS$			
4			$S \to aB$			
5			accept			



Part II $S \to \Lambda|aB|bA \quad A \to aS|bAA \quad B \to bS|aBB$

Stack	Input
0	b a $
0 b 6	a $
0 b 6 a 7	$
0 b 6 a 7 S 8	$
0 b 6 A 9	$
0 S 5	$



$$S \Rightarrow bA \Rightarrow baS \Rightarrow ba$$

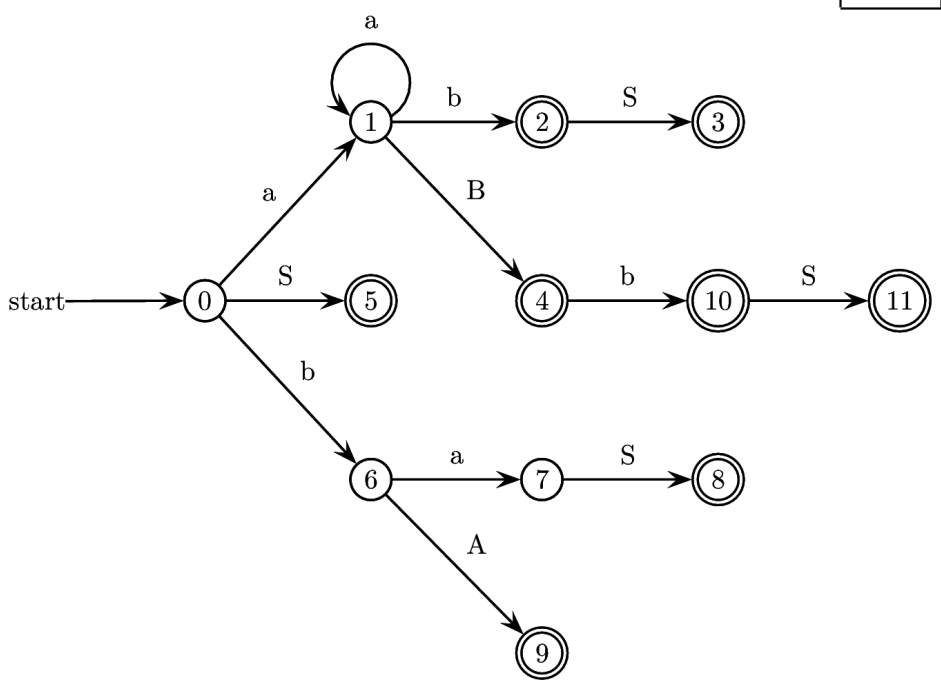
	a	b	$	S	A	B
0	1	6		5		
1		2				4
2			$S \to \Lambda$	3		
3			$B \to bS$			
4			$S \to aB$			
5			accept			
6	7				9	
7			$S \to \Lambda$	8		
8			$A \to aS$			
9			$S \to bA$			

Part III $S \to \Lambda|aB|bA \quad A \to aS|bAA \quad B \to bS|aBB$

$$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aaBbS \Rightarrow aaBb \Rightarrow aabSb \Rightarrow aabb$$

Stack	Input
0	a a b b $
0 a 1	a b b $
0 a 1 a 1	b b $
0 a 1 a 1 b 2	b $
0 a 1 a 1 b 2 S 3	b $
0 a 1 a 1 B 4	b $
0 a 1 a 1 B 4 b 10	$
0 a 1 a 1 B 4 b 10 S 11	$

	a	b	$	S	A	B
0	1	6		5		
1	1	2				4
2		$S \to \Lambda$	$S \to \Lambda$	3		
3		$B \to bS$	$B \to bS$			
4		10	$S \to aB$			
5			accept			
6	7				9	
7			$S \to \Lambda$	8		
8			$A \to aS$			
9			$S \to bA$			
10			$S \to \Lambda$	11		
11			$B \to bS$			

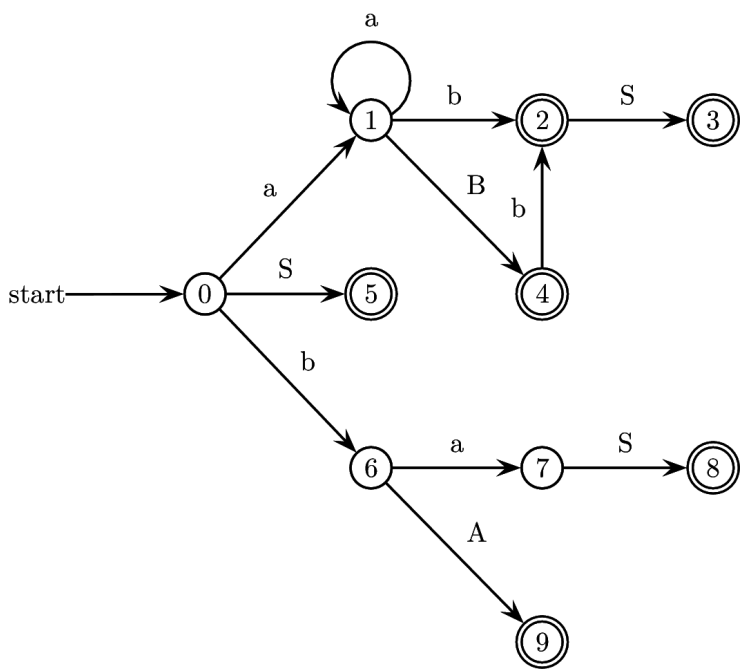


Part IV $S \to \Lambda|aB|bA \quad A \to aS|bAA \quad B \to bS|aBB$

$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aaBbS \Rightarrow aaBb \Rightarrow aabSb \Rightarrow aabb$

Stack	Input
0	a a b b $
0 a 1	a b b $
0 a 1 a 1	b b $
0 a 1 a 1 b 2	b $
0 a 1 a 1 b 2 S 3	b $
0 a 1 a 1 B 4	b $
0 a 1 a 1 B 4 b 2	$
0 a 1 a 1 B 4 b 2 S 3	$

	a	b	$	S	A	B
0	1	6		5		
1	1	2				4
2		$S \to \Lambda$	$S \to \Lambda$	3		
3		$B \to bS$	$B \to bS$			
4		2	$S \to aB$			
5			accept			
6	7				9	
7			$S \to \Lambda$	8		
8			$A \to aS$			
9			$S \to bA$			

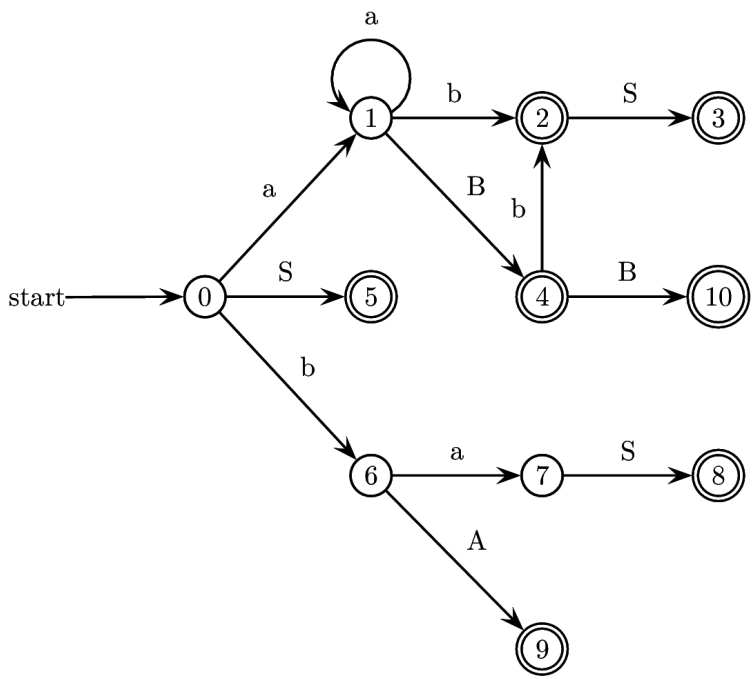


Part V $S \to \Lambda|aB|bA \quad A \to aS|bAA \quad B \to bS|aBB$

$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aaBbS \Rightarrow aaBb \Rightarrow aabSb \Rightarrow aabb$

Stack	Input
0	a a b b $
0 a 1	a b b $
0 a 1 a 1	b b $
0 a 1 a 1 b 2	b $
0 a 1 a 1 b 2 S 3	b $
0 a 1 a 1 B 4	b $
0 a 1 a 1 B 4 b 2	$
0 a 1 a 1 B 4 b 2 S 3	$
0 a 1 a 1 B 4 B 10	$

	a	b	$	S	A	B
0	1	6		5		
1	1	2				4
2		$S \to \Lambda$	$S \to \Lambda$	3		
3		$B \to bS$	$B \to bS$			
4		2	$S \to aB$			10
5			accept			
6	7				9	
7			$S \to \Lambda$	8		
8			$A \to aS$			
9			$S \to bA$			
10			$B \to aBB$			

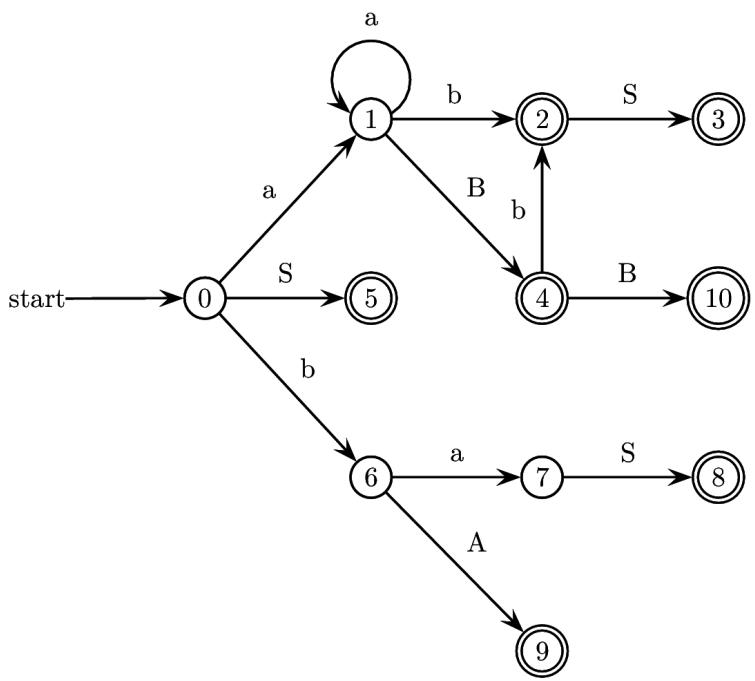


Part VI $S \to \Lambda|aB|bA \quad A \to aS|bAA \quad B \to bS|aBB$

$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aaBbS \Rightarrow aaBb \Rightarrow aabSb \Rightarrow aabb$

Stack	Input
0	a a b b $
0 a 1	a b b $
0 a 1 a 1	b b $
0 a 1 a 1 b 2	b $
0 a 1 a 1 b 2 S 3	b $
0 a 1 a 1 B 4	b $
0 a 1 a 1 B 4 b 2	$
0 a 1 a 1 B 4 b 2 S 3	$
0 a 1 a 1 B 4 B 10	$
0 a 1 B 4	$
0 S 5	$

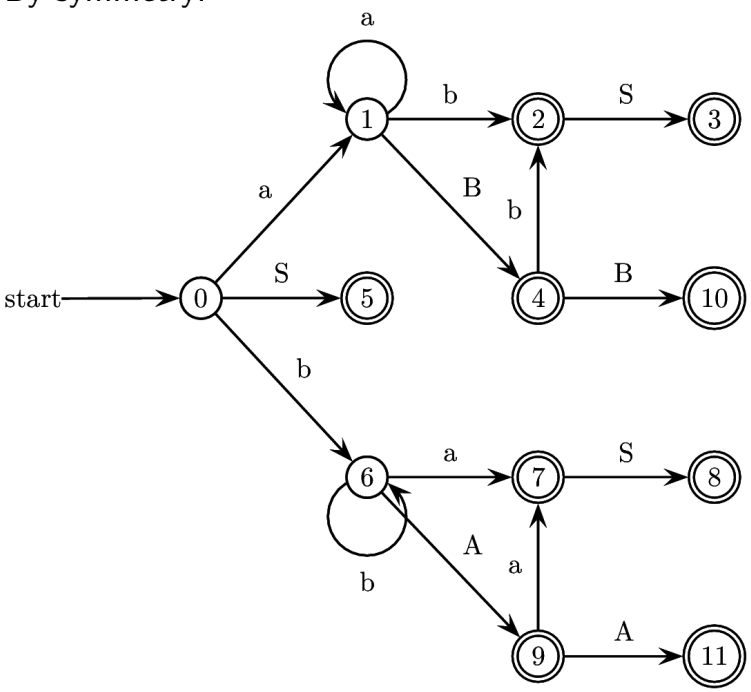
	a	b	$	S	A	B
0	1	6		5		
1	1	2				4
2		$S \to \Lambda$	$S \to \Lambda$	3		
3		$B \to bS$	$B \to bS$			
4		2	$S \to aB$			10
5			accept			
6	7				9	
7			$S \to \Lambda$	8		
8			$A \to aS$			
9			$S \to bA$			
10			$B \to aBB$			



Part VII $S \to \Lambda|aB|bA \quad A \to aS|bAA \quad B \to bS|aBB$

$S \Rightarrow bA \Rightarrow bbAA \Rightarrow bbAaS \Rightarrow bbAa \Rightarrow bbaSa \Rightarrow bbaa$

Stack	Input
0	b b a a $
0 b 6	b a a $
0 b 6 b 6	a a $
0 b 6 b 6 a 7	a $
0 b 6 b 6 a 7 S 8	a $
0 b 6 b 6 A 9	a $
0 b 6 b 6 A 9 a 7	$
0 b 6 b 6 A 9 a 7 S 8	$
0 b 6 b 6 A 9 A 11	$
0 b 6 A 9	$
0 S 5	$

	a	b	$	S	A	B
0	1	6		5		
1	1	2				4
2		$S \to \Lambda$	$S \to \Lambda$	3		
3		$B \to bS$	$B \to bS$			
4		2	$S \to aB$			10
5			accept			
6	7				9	
7	$S \to \Lambda$		$S \to \Lambda$	8		
8	$A \to aS$		$A \to aS$			
9	7		$S \to bA$			
10			$B \to aBB$			
11			$A \to bAA$			

By symmetry: