Sympy

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You may want to familiarize yourself with symbolic algebra packages, such as the **Sympy** package which runs in python.

Example 1. The following session allowed me to simplify two complex expressions and prove they were identical.

I had derived this expression:

$$\frac{k\left(k+1\right)\left(2k+1\right)}{6}+\left(k+1\right)^{2}$$

And another expression I derived:

$$\frac{1\left(k+1\right)\left(k+2\right)\left(2k+3\right)}{6}$$

And I wanted to show that these were equal.

Sympy to the rescue! Sympy even formatted them in LaTeX for me:

```
>>> import sympy
>>> k = sympy.symbols('k')
>>> exp1 = k*(k+1)*(2*k+1)/6 + (k+1)**2
>>> print(sympy.latex(exp1))
\frac{k}{6} \left(k + 1\right) \left(2 k + 1\right) + \left(k + 1\right)^{2}
>>> exp2 = (k+1)*((k+1)+1)*(2*(k+1)+1)/6
>>> print(sympy.latex(exp2))
\frac{1}{6} \left(k + 1\right) \left(k + 2\right) \left(2 k + 3\right)
>>> sympy.expand(exp1)
k**3/3 + 3*k**2/2 + 13*k/6 + 1
>>> print(sympy.latex(sympy.expand(exp1)))
\frac{k^{3}}{3} + \frac{3 k^{2}}{2} + \frac{13 k}{6} + 1
>>> sympy.expand(exp2)
k**3/3 + 3*k**2/2 + 13*k/6 + 1
>>> print(sympy.latex(sympy.expand(exp2)))
\frac{k^{3}}{3} + \frac{3 k^{2}}{2} + \frac{13 k}{6} + 1
>>>
```

Thus I knew that both expressions were equal to

$$\frac{k^{3}}{3}+\frac{3k^{2}}{2}+\frac{13k}{6}+1$$

Example 2. For another example, consider the example in *the Book of Proof*, p. 155. We need to prove that if

$$k^5 - k = 5a$$

for some $a \in \mathbb{Z}$, then

$$(k+1)^5 - (k+1) = 5b$$

for some $b \in \mathbb{Z}$.

We can prove that they are both multiples of 5 if we can prove that their difference is, so let's look at the difference:

```
>> from sympy import *
>>> k = symbols('k')
>>> exp1 = k**5 - k
>>> exp2 = (k+1)**5 - (k+1)
>>> expand(exp2 - exp1)
5*k**4 + 10*k**3 + 10*k**2 + 5*k
```

From which we easily get the proof as in the book.