GEO1003 - Shared Notes

Master Geomatics Students

2024-12-07

Contents

Introduction

This is the introduction to the notes.

Example

Introduction

The goal of this chapter is just to demonstrate how things should be organized. It will be removed from the notes in the end.

Markdown Basics

Resources and Helpers

A nice cheat sheet about Markdown can be found at this link: https://www.markdownguide.org/cheat-sheet/.

On VS Code, there are some nice extensions that can help you write Markdown files:

- Markdown All in One to provide useful shortcuts and commands
- markdownlint to properly format your Markdown files

Feel free to ask me if you have questions about Markdown.

Comments

```
This <!--This is a comment.--> is
<!--
Comments are not rendered.
They can take multiple lines
-->
a
sentence.
```

This is a sentence.

Headers

```
<!-- Comment the fist headers to avoid messing up the outline of this file -->
<!--
# Level 1

## Level 2

### Level 3
-->

#### Level 4

##### Level 5

###### Level 6
```

Level 4

Level 5

Level 6

Bold and Italic

```
- Normal text
- **Bold text**
- _Italic text_
- **_Bold and italic text_**
```

- Normal text
- **Bold text**
- *Italic text*
- ***Bold and italic text***

Lists

```
Unordered list:

- Unordered list item 1
- Unordered list item 2
  - Nested unordered list item

Ordered list:

1. Ordered list item 1
2. Ordered list item 2
   1. Nested ordered list item
```

Unordered list:

- Unordered list item 1
- Unordered list item 2
    - Nested unordered list item

Ordered list:

1. Ordered list item 1
2. Ordered list item 2
    1. Nested ordered list item

Links

```
[Example link](https://www.example.com)
```

Example link

Images

```
![Example image](../../../images/example.jpg){ width="250" }
```

Figure 1: Example image

Blockquotes

```
> This is a blockquote.
```

> This is a blockquote.

Code

```
Inline code: `print("Hello, World!")`

Code block:

```python
def hello_world():
    print("Hello, World!")
```
```

Inline code: `print("Hello, World!")`

Code block:

```python
def hello_world():
    print("Hello, World!")
```

Tables

```
Table: A simple table

| Header 1 | Header 2 |
| -------- | -------- |
| Cell 1   | Cell 2   |
| Cell 3   | Cell 4   |
```

Table 1: A simple table

Header 1	Header 2
Cell 1	Cell 2
Cell 3	Cell 4

Math

```
Inline math: $x^2$ is the square of $x$.

Block math:

$$
\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}
$$
```

Inline math: x^2 is the square of x.

Block math:

$$\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$$

Definition Blocks

```
**Lorem ipsum dolor sit amet**
:   Sed sagittis eleifend rutrum. Donec vitae suscipit est. Nullam tempus
    tellus non sem sollicitudin, quis rutrum leo facilisis.

**Cras arcu libero**
:   Aliquam metus eros, pretium sed nulla venenatis, faucibus auctor ex. Proin
    ut eros sed sapien ullamcorper consequat. Nunc ligula ante.
```

Lorem ipsum dolor sit amet Sed sagittis eleifend rutrum. Donec vitae suscipit est. Nullam tempus tellus non sem sollicitudin, quis rutrum leo facilisis.

Cras arcu libero Aliquam metus eros, pretium sed nulla venenatis, faucibus auctor ex. Proin ut eros sed sapien ullamcorper consequat. Nunc ligula ante.

Definition Blocks + Lists

```
- **Lorem ipsum dolor sit amet**
  :   Sed sagittis eleifend rutrum. Donec vitae suscipit est. Nullam tempus
      tellus non sem sollicitudin, quis rutrum leo facilisis.

- **Cras arcu libero**
  :   Aliquam metus eros, pretium sed nulla venenatis, faucibus auctor ex. Proin
      ut eros sed sapien ullamcorper consequat. Nunc ligula ante.
```

- **Lorem ipsum dolor sit amet** Sed sagittis eleifend rutrum. Donec vitae suscipit est. Nullam tempus tellus non sem sollicitudin, quis rutrum leo facilisis.

- **Cras arcu libero** Aliquam metus eros, pretium sed nulla venenatis, faucibus auctor ex. Proin ut eros sed sapien ullamcorper consequat. Nunc ligula ante.

Empty Section

This section gives more information about the empty section. Or not.

How does GNSS work?

Introduction

The Global Positioning System (**GPS**), also known as the NAVigation Satellite Time And Ranging (**NAVSTAR**) system had its first satellite launched back in February 1978. GPS is a *one-way* radio ranging system which provides realtime knowledge of one's Position and Velocity, and a very accurate Time reference as well (all together referred to as **PVT**).

GPS segments

The GPS system consists of three segments:

1. The **space segment**, consisting of 24 or more satellites, with accurate atomic clocks on board, continuously transmitting ranging signals to Earth.
2. The **control segment**, consisting of a number of ground stations, which monitors the satellites, computes their orbits and clock offsets, and uploads this information to the satellites, which in turn encode this information on the ranging signal (the so-called navigation data).
3. The **user segment**, simply consisting of many GPS receivers, which each track four or more GPS satellites, and compute their own position.

Radio Signal

The GPS radio signal contains:

- the **L-band carrier frequency** between 1 and 2 GHz
- the **Pseudo Random Noise** (PRN, also called the **spreading code**), unique to each satellite, publicly available
- the **navigation message** containing the satellite orbit and clock information

Initialisation

A GPS receiver typically consists of tens to hundreds of so-called **channels**, and will allocate each of these to a specific GPS (GNSS) satellite. When a GPS receiver first starts up, it will begin to *search for a particular GPS satellite on each of its channels*, by trying to scan for the corresponding **spreading codes** at different *Doppler offsets* and *time delays*. This is done by overlaying the received signal with a *local copy or replica of the same code* and then (time) shifting it until correlation shows a maximum (best fit, or match).

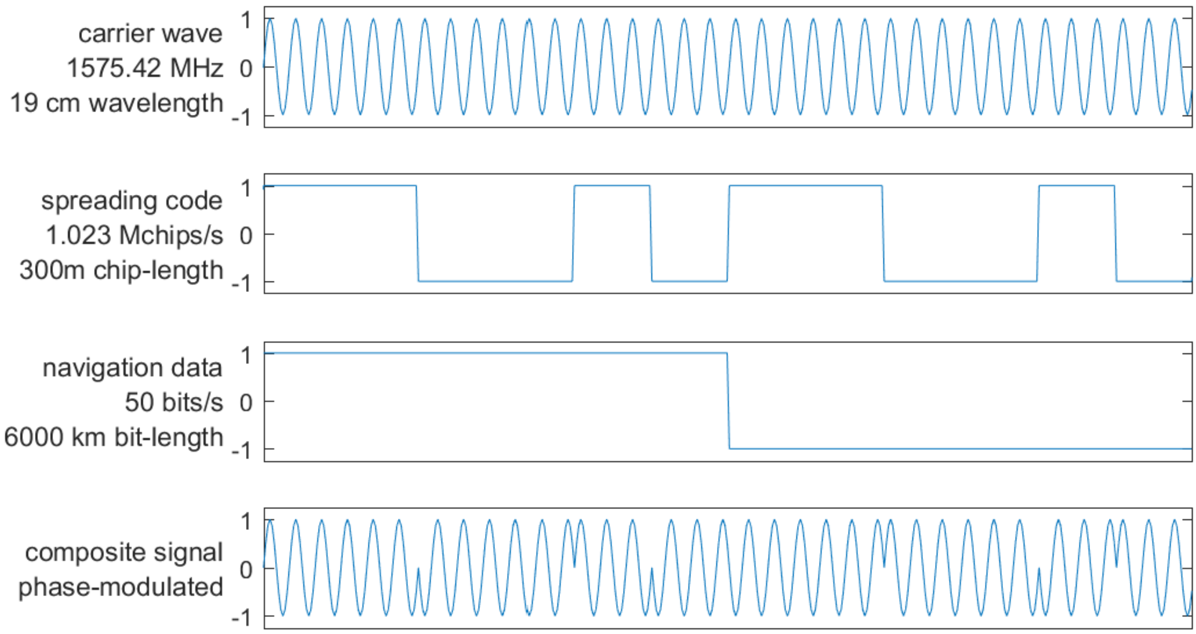


Figure 2: GPS L1 CA-signal (scale is not accurate)

Pseudorange Measurement

Once the receiver is locked on to a satellite's spreading code, it will continue to track it by *continuously adjusting the time delay and Doppler offset* to keep the correlation at a maximum. The time delay is then used to calculate the **pseudorange** $p_{r,s}$ to the satellite, which is (theoretically) the travel time $\tau_{r,s}$ multiplied by the speed of light c:

$$p_{r,s} = c \cdot \tau_{r,s} \text{ where } \tau_{r,s} = t_r - t_s$$

See Error Sources for more information on the errors in the pseudorange measurement.

Carrier Phase Measurement

A GPS receiver may measure the **fractional phase difference** between the received *carrier wave* from the satellite and a locally generated copy (replica). The carrier wave measurement is a **very precise measure** of the distance between the satellite and the receiver, but the initial number of carrier wave cycles is *unknown*, and needs to be estimated before the carrier phase measurements can be effectively used.

The much better precision of the carrier phase measurement with respect to the pseudorange code measurement can be explained by **much smaller period** of the carrier compared to the code chip duration (for the L1 CAcode signal, 1540 periods of the carrier fit in one chip of the Pseudo Random Noise (PRN) spreading code).

Jamming and Spoofing

There are multiple mays a GNSS signal may be threatened, jamming and spoofing being intentional attacks.

Jamming

By the time GNSS signals arrive at the antennas of a GNSS positioning system, the power level of these signals is very low. This low power level makes the signals susceptible to interference from other signals transmitted in the GNSS frequency range.

Jamming is a special case of signal interference where an attacker tries to block the incoming GNSS signal to a specific person/area.

GNSS receivers can use several methods to protect against interference and jamming:

- Signal filtering
- Multiple navigation sensors. For short-term interference, other sensors can help the receiver bridge brief periods of GNSS outage.
- Multi-frequency/multi-constellation GNSS makes it much harder to jam a signal on multiple different frequencies at once.
- Anti-jam antennas use multiple antenna elements to control the amount of signal received from a particular direction. When an anti-jam system senses interference from a direction, it turns down the antenna gain for it.

Spoofing

Unlike interference where GNSS is denied by overpowering the satellite signal, spoofing tricks the receiver into reporting an incorrect position. Spoofing is done by first jamming the GNSS receiver and then providing a false satellite signal that is either created by a signal generator or is a rebroadcast of a pre-recorded GNSS signal. Unlike interference, spoofing is always an intentional attack.

To protect against spoofing the same methods apply as against interference. Additionally, one of the most effective ways to protect against spoofing is to track encrypted signals that are broadcast by several of the GNSS constellations. Access to the encrypted signals is restricted and not available to all users.

Signal blockage

The GNSS signal can be blocked by many objects like trees or buildings, especially in urban areas. The main protection is again using multiple constellations and using additional sensors like an IMU.

Constellation failure

Although it is extremely unlikely that an entire constellation will fail, receivers that can track more than one constellation protect against this unlikely scenario.

GNSS performance

Introduction

Error Sources

There are a few issues with this calculation:

- the **satellite clock** has an **offset**, which is *known* (part of the navigation message)
- the **receiver clock** has an **offset**, which is *unknown*
- the **ionosphere** causes a delay (due to a lower speed of light), which is *unknown*
- there might be other errors, such as multipath, which are unknown

Any of these issues will cause the calculated pseudorange to be **inaccurate**. The calculation is very sensible since $c \approx 3 \times 10^8$ m/s, and a **1 µs** error will cause a **300 m** error in the calculated distance.

Ionosphere Delay

One of the *major error sources* in GPS is due to the **ionosphere**, which contains *free electrons* that cause the speed of light to be lower than in vacuum.

The ionospheric delay may be **highly variable**, as a function of both **time** and **space**. In terms of distance ranging, it can go from *a few meter to hundreds of meters*, and is maximum round the geomagnetic equator around local noon, and during solar maxima.

The ionosphere delay scales, to a very good approximation, with the **inverse of the square of the radio frequency** of the signal, so using two different frequencies allows to create the so-called *ionosphere-free range measurements*. This is why GPS satellites were originally designed to transmit ranging signals on both the **L1** (1575.42 MHz) and **L2** (1227.60 MHz) frequency.

Accuracy and Precision

The receiver can measure the received **signal strength**, through the so-called carrier-to-noise-density ratio C/N_0, which gives an indication of the **quality of the measurement** (larger signal strength yields more precise measurement).

The **pseudorange measurement** precision is typically at the *one or few meter* level for low-cost, mass-market equipment, and can get down to the *few decimeter level* for professional highend equipment.

The **carrier phase measurement** precision ranges from the *few centimeter to the millimeter level*. The carrier phase is an ambiguous measurement of distance, but it is more precise than the pseudorange, typically by **two orders of magnitude**.

Dilution of Precision

Availability, Continuity and Integrity

Availability

Continuity

Integrity

PPP-RTK

PPP

Precise Point Positioning (PPP) is a GNSS signal augmentation technique that offers high accuracy positioning using a single receiver. The errors which PPP corrected:

- satellite errors
  - SV Orbit error
  - SV Clock error
  - SV Bias

Key features of PPP

- **Does not resolve carrier phase ambiguities**
  Instead, it uses an estimation, leading to a longer initialization time and requiring full re-initialization if the signal is lost. This is a key difference from RTK, which uses carrier phase measurements for precise positioning.
- **Eliminates GNSS system errors**
  PPP uses GNSS satellite clock and orbit corrections to achieve high-accuracy positioning without needing a local base station. PPP typically offers a decimetre accuracy (10 cm).
- **Relies on a global network of Continuously Operating Reference Station (CORS)**
  These stations generate the corrections needed to eliminate system errors.
- **Delivers corrections via satellite or internet**
  This allows for global coverage and removes the need for local infrastructure.
- **Provides dm-level or better real-time positioning**
  This accuracy surpasses standalone GNSS capabilities, making it suitable for applications requiring higher precision.
- **Requires a convergence period**
  Typically ranging from 5 to 30 minutes, this time is needed to resolve local biases such as atmospheric conditions, multipath, and satellite geometry.
- **Uses the State Space Representation (SSR) message format**
  This format separates and corrects individual error components, unlike OSR used in RTK.
- **Suitable for applications with no local infrastructure**
  Ex: sparsely populated areas and marine applications where setting up a network of base stations is challenging.

While PPP offers global coverage and good accuracy, its long convergence times can be a drawback for applications that require rapid positioning. Nonetheless, it's a valuable technique for situations where high accuracy is needed without relying on local base stations.

RTK

RTK, which utilises the OSR approach, corrects for location-dependent errors like Ionospheric and Tropospheric delays by providing a localized solution based on a network of base stations (CORS). The errors which RTK corrected:

- satellite errors
  - SV Orbit error
  - SV Clock error
  - SV Bias
- location-dependent errors
  - Ionospheric delay
  - Tropospheric delay

Key features of RTK

- **RTK enables the rover to resolve the ambiguities of the differenced carrier phase data and estimate the coordinates of the rover position.**
- **A CORS transmits its raw measurements or observation corrections to a rover receiver.**
  This is done via a direct (two-way) communication channel. The rover is a potentially moving receiver whose position is being determined.
- **Very high accuracy positioning over a short range (30–50 km).**
  This is due to the degradation of distance-dependent biases, such as orbit error and ionospheric and tropospheric signal refraction.
- **Within close proximities of the base station (10–20 km), RTK provides near-instant high accuracy positioning of up to 1 cm + 1 ppm.**
- **A direct communication channel is required between the rover and the base station.**
  Bandwidth limitations prevent large numbers of users utilising the same base station, making RTK ill-suited to mass-market applications.
- **RTK is the most popular GNSS signal augmentation technology.**
  It is used in industries such as surveying and agriculture, and is especially common in regions with well-developed CORS networks.
- **RTK uses the Observation Space Representation (OSR) approach.**
  This groups the errors together and provides the total correction measurements, rather than for the individual parameters. All parameters are updated at the same frequency regardless of their time sensitivity.

OSR Approach The OSR approach (and thus RTK) has high bandwidth requirements because:

- **It requires a two-way communication channel for each user.**
  Both the base station and the rover need to transmit data back and forth, increasing the amount of data being transferred.
- **OSR groups errors together and provides total correction measurements.**
  Instead of sending corrections for individual parameters separately, OSR sends all the corrections together. This leads to a larger data packet size compared to SSR, which separates individual error components.
- **All parameters are updated at the same frequency.**
  Regardless of the time sensitivity of each parameter, they are all updated at the same rate (the most time-sensitive one), leading to more frequent data transmissions and increased bandwidth usage.

The high bandwidth requirement of OSR is a major limitation, particularly for mass-market applications. If a large number of users were to utilise the OSR approach, current mobile networks would likely be overwhelmed. This is why OSR is not well-suited for applications like smartphones, IoT, and the automotive industry.

PPP-RTK

PPP-RTK is a hybrid GNSS signal augmentation technology that combines the strengths of both Precise Point Positioning (PPP) and Real-Time Kinematic (RTK). Here's a breakdown of its key features:

- **Utilises a network of CORS stations**
  Similar to RTK, it relies on a network of Continuously Operating Reference Stations (CORS) to generate corrections.
- **Provides atmospheric error corrections**
  PPP-RTK utilises a "un-differenced" map of atmospheric errors generated by a network of CORS, specifically for ionospheric and tropospheric delays, which are calculated using the CORS network. Achieving fast ambiguity resolution and high accuracy.
- **Enables fast convergence times**
  Thanks to the atmospheric error corrections, convergence times are significantly reduced, typically in the range of 1-10 minutes and potentially within seconds under ideal conditions.
- **Delivers cm-level accuracy**
  Comparable to traditional RTK techniques, PPP-RTK can achieve centimetre-level accuracy, exceeding the performance of standalone PPP.
- **Employs the State Space Representation (SSR) message format**
  Unlike RTK, which uses OSR, PPP-RTK uses SSR to broadcast corrections. This allows for efficient data transmission and enables an unlimited number of users to connect without overloading the system.
- **Has lower bandwidth requirements than RTK**
  The use of SSR and the efficient transmission of corrections result in significantly lower bandwidth requirements compared to RTK, making it suitable for mass-market applications.

- **Offers global coverage with graceful degradation**
  While it requires a regional CORS network, if a user moves beyond its range, the service seamlessly transitions to standard PPP, ensuring continuous positioning capability.

Overall, PPP-RTK offers a promising solution for mass-market applications by providing high accuracy, fast convergence, global coverage, and efficient bandwidth usage.
It bridges the gap between traditional PPP and RTK, offering a more versatile and scalable approach to high-accuracy positioning.

Solution	Benefits	Drawbacks
PPP	Has no local ground infrastructure requirements Global	Long convergence times Lower accuracy
RTK	High accuracy (2cm) Near-instant convergence times	Highly reliant upon local ground infrastructure Short range of transmissions
PPP-RTK	Fast convergence times High accuracy Lower density CORS network than NRTK Degrades to standard PPP	Reliant upon local ground infrastructure

Figure 3: PPP vs RTK

Exhibit 7: High-Level View of Main Benefits and Drawbacks of PPP-RTK Compared to PPP and RTK Only

Comparing RTK, PPP, and PPP-RTK

Feature	RTK	PPP	PPP-RTK
Accuracy	**cm-level** (up to 1 cm + 1 ppm)	**dm-level or better** (less than 10 cm)	**cm-level**, similar to RTK
Coverage Area	**Limited range** (typically 30-50 km from the base station)	**Global**	**Global** with graceful degradation to standard PPP outside the range of the CORS network
Message Format	**OSR** (Observation Space Representation)	**SSR** (State Space Representation)	**SSR** (State Space Representation)
Transmission Channel	**Two-way communication** between base station and rover	Corrections delivered via **satellite or the internet**	Corrections **broadcast to users**, enabling a large number of users to connect simultaneously
Convergence Time	**Near-instantaneous** (typically less than 5 seconds)	**Relatively long** (typically 5-30 minutes)	**Fast** (typically 1-10 minutes, potentially within seconds under ideal conditions)

Feature	RTK	PPP	PPP-RTK
Errors Solved	Orbit errors, clock errors, bias, **ionospheric delay, tropospheric delay**	Orbit errors, clock errors, bias	Orbit errors, clock errors, bias, **ionospheric delay, tropospheric delay**, enabling **integer ambiguity resolution**
Key Strengths	High accuracy, very fast convergence time	Global coverage, no reliance on local base stations	High accuracy, fast convergence time, global coverage, lower bandwidth requirements compared to RTK, graceful degradation outside CORS range
Key Limitations	Limited range, high bandwidth requirements, reliance on local base stations	Long convergence time, lower accuracy compared to RTK	Still requires a CORS network (though less dense than RTK) and may degrade to standard PPP with increasing distance from CORS station

OSR vs. SSR:

- **OSR:** Groups errors together, requires **two-way communication**, higher bandwidth requirements.
- **SSR:** Separates individual error components, enables efficient data transmission, **one-way communication (broadcast)**, lower bandwidth requirements.

The choice depends on the application's needs and available infrastructure.

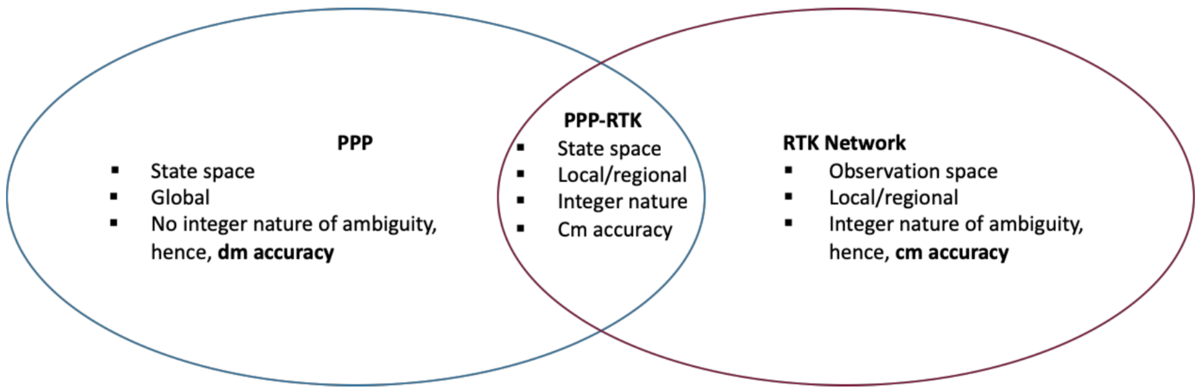


Figure 4: PPP vs RTK vs PPP-RTK

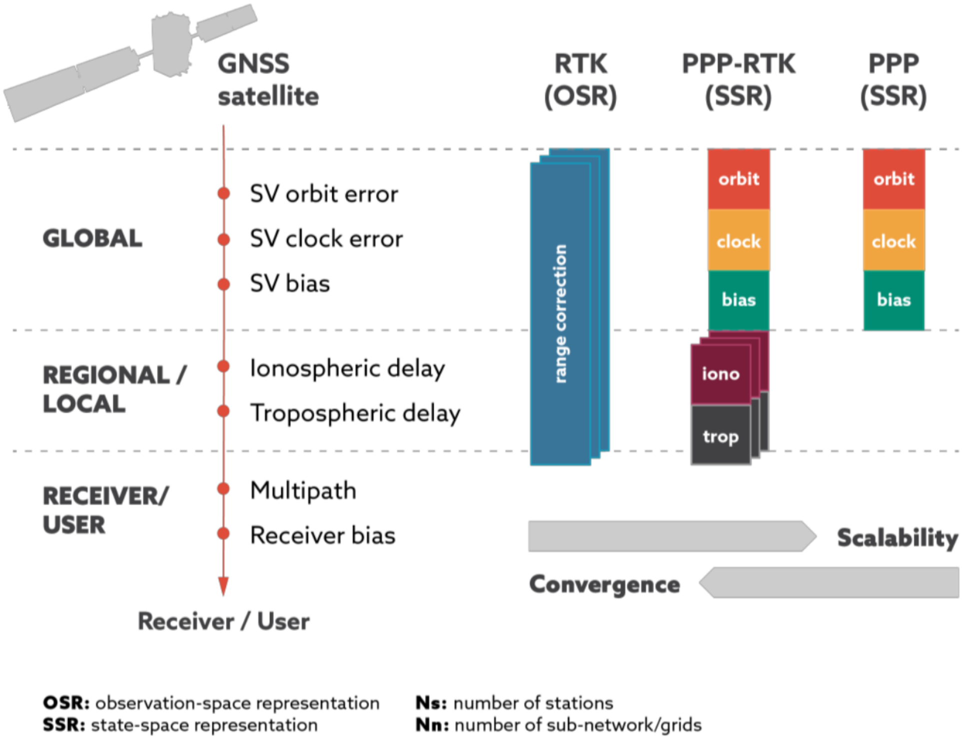


Figure 5: difference in message format and resolved errors

DGNSS

GNSS in the built environment (outdoor, indoor and in between)

Introduction

Multipath

Urban Canyon

Shadow Matching

CRS

Introduction

Coordinate Systems

Coordinate Reference Systems

According to the EPSG and ISO guidelines, four groups of CRSs can be distinguished:

- **Geographic CRSs** give geographic coordinates in degrees (2D latitude and longitude, and for 3D also height) relative to an ellipsoidal model of the Earth's surface. Examples include the European ETRS89, the global ITRF2020, and WGS 84. Geographic CRSs, such as ETRS89 and WGS 84, cannot be visualized on a flat plane without a map projection.
- **Projected CRSs** present geo-information on a flat surface in Cartesian (x and y) coordinates. A projected CRS is a derivative of a geographic CRS, where a map projection is used for the depiction on the flat surface. An example of this is the Dutch RD with the geographic CRS RD-Bessel as its basis.
- **Vertical CRSs** for recording height and depth relative to a reference plane. This reference plane is often based on the direction of gravity and normally does not coincide with the surface of an ellipsoid. With a reference plane based on the direction of gravity, no water flows between two points of equal height, but water usually does flow between two points of equal ellipsoidal height. Examples of vertical CRSs are the NAP and the LAT.
- **Compound CRSs** are composite CRSs, for example: RDNAP which is composed of the projected CRS RD and the vertical CRS NAP.

In this chapter of the summary, we will highlight two examples of Coordinate Reference Systems. Before doing so, here is an overview of some terminology and their examples:

Geodetic terminology

- Terrestrial Reference System (definition) (e.g. ETRS89, WGS84)
- Terrestrial Reference Frame (realisation) (e.g. ETRF2000, ITRF2020)
- Ellipsoid (e.g. GRS80, Bessel 1841)
- 'Coordinate notation' (e.g. axis, order, units)
- Map projection (optional) (e.g. UTM, LCC)
- Epoch (for time-dependent coordinates) (e.g. 2024.91 (27 November 2024)

NB: be aware of the difference between epoch and the year in names!

Geo-spatial terminology

- Datum ensemble != Terrestrial Reference System
- Datum = Terrestrial Reference Frame + ellipsoid
- Coordinate system = Coordinate notation (+ map projection)
- CRS (= projection in some software) = datum + coordinate system

Geographic Coordinate Reference Systems

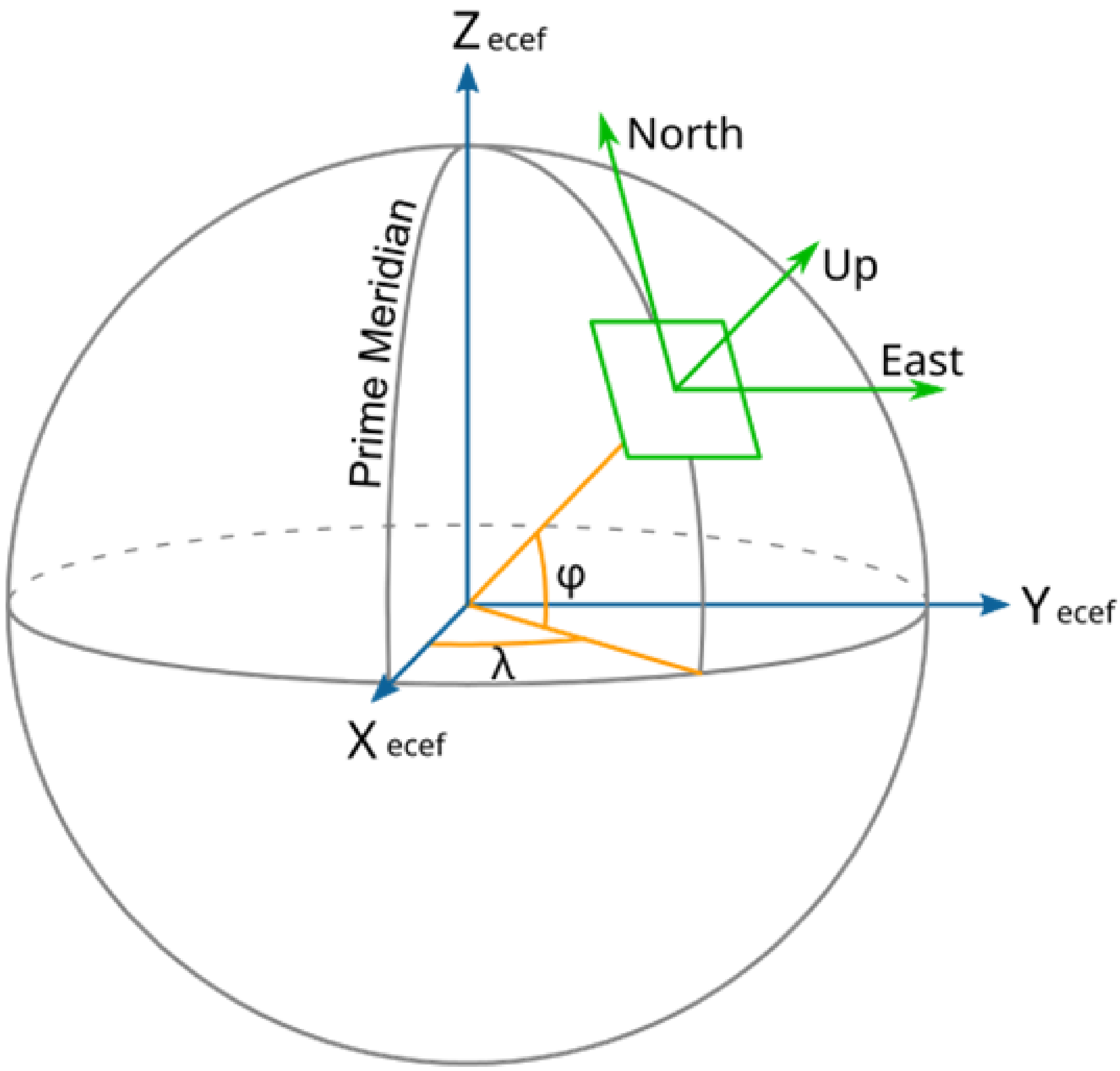
The use of Geographic Coordinate Reference Systems is very common. They use degrees of latitude and longitude and sometimes also a height value to describe a location on the earth's surface.

Lines of latitude run parallel to the equator and divide the earth into 180 equally spaced sections from North to South. The reference line for latitude is the equator and each **hemisphere** is divided into ninety sections, each representing one degree of latitude.

Wherever you are on the earth's surface, the distance between the lines of latitude is the same (60 nautical miles).

Lines of longitude, on the other hand, do not stand up so well to the standard of uniformity. Lines of longitude run perpendicular to the equator and converge at the poles. The reference line for longitude (the prime meridian) runs from the North Pole to the South Pole through Greenwich, England. At the equator, and only at the equator, the distance represented by one line of longitude is equal to the distance represented by one degree of latitude.

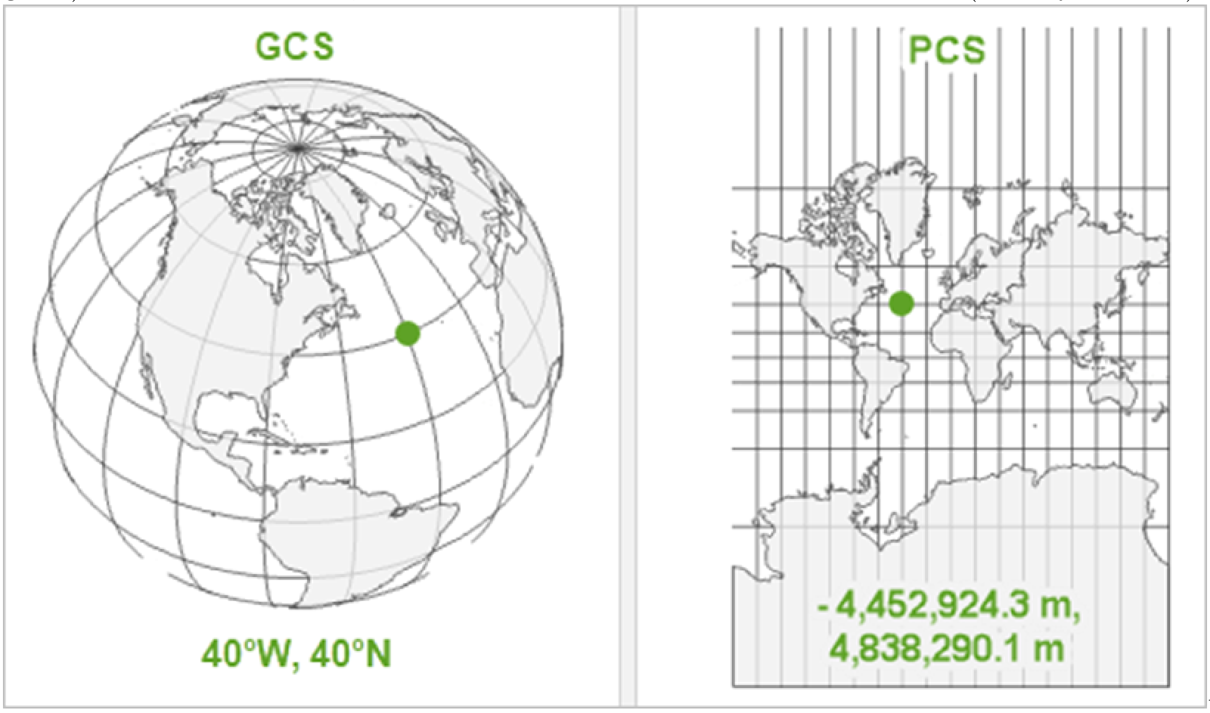
Using the geographic coordinate system, we have a grid of lines dividing the earth into squares that cover approximately 12363.365 square kilometres at the equator — a good start, but not very useful for determining the location of anything within that square. To be truly useful, a map grid must be divided into small enough sections so that they can be used to describe (with an acceptable level of accuracy) the location of a point on the map. To accomplish this, degrees are divided into **minutes** (') and **seconds** ("). There are sixty minutes in a degree, and sixty seconds in a minute (3600 seconds in a degree). So, at the equator, one second of latitude or longitude = 30.87624 meters.



{width:"50%"

Projected Coordinate Reference Systems

There is a difference between a geographic coordinate system (GCS) and a projected coordinate system (PCS). In short, a GCS defines **where** the data is located on the earth's surface; a PCS tells the data **how** to draw on a flat surface. A GCS is round, and so records locations in angular units (usually degrees). A PCS is flat, so it records locations in linear units (usually meters).



{width:"50%"

The GCS is what ties your coordinate values to real locations on the earth. Only knowing the latitude and longitude of a location is thus not good enough, as it only tells you where a location is within a GCS. To draw a graticule, you need a model of the earth that is at least a regular spheroid, if not a perfect sphere. There are many different models of the earth's surface, and therefore many different GCS!

Once you know where to draw something, you need to know how. The earth's surface — and your GCS — are round, but the map is flat. This is where the map projections come in. They tell you how to distort the earth so the parts that are most important to your map get the least distorted and are displayed best on the flat surface of the map. A **projected coordinate system** (PCS) is a GCS that has been flattened using a map projection.

Your data must have a GCS before it knows where it is on Earth. Projecting your data is optional, but projecting your map is not. Maps are flat, so your map must have a PCS to know how to draw.

Coordinates in a PCS are recorded in **a Linear Unit**, often meters. A PCS also contains a Geographic Coordinate System (e.g. WGS 1984)! Remember that a PCS is just a GCS that has been projected.

Coordinates in a GCS are recorded in an Angular Unit, usually degrees. The **Prime Meridian** is an arbitrary line of longitude that is defined as 0°. The **Datum** defines which model is used to represent the earth's surface and where that model is positioned relative to the surface. The **Spheroid** is the regular model of the irregular earth. It's part of the datum. **Semimajor Axis, Semiminor Axis**, and **Inverse Flattening** define the size of the spheroid.

Linear Reference Systems

Linear referencing is the method to store and geographically locate data using relative positions along a measured line feature without the need to explicitly use x,y coordinates or an address. When data is linearly referenced, measure values are used to measure the distance along a line feature, allowing multiple sets of dynamically changing attribute data to be associated with any portion of an existing linear feature, independent of its beginning and end. Linear referencing is used for many reasons. The following are the two primary reasons:

- Many locations are recorded as events along linear features.

  For example, locations of traffic accidents are recorded using a convention such as "27 meters east of reference mile marker 35 along State Highway 287." Many sensors record conditions using measures of distance or time along the lines—along pipelines, along roads, along streams, and so forth.

- Linear referencing is also used to associate multiple sets of attributes to portions of linear features without requiring that underlying lines be segmented (split) each time that attribute values change.

  For example, most road centreline feature classes are segmented where three or more road segments intersect and where the road names change.

{width:"50%"

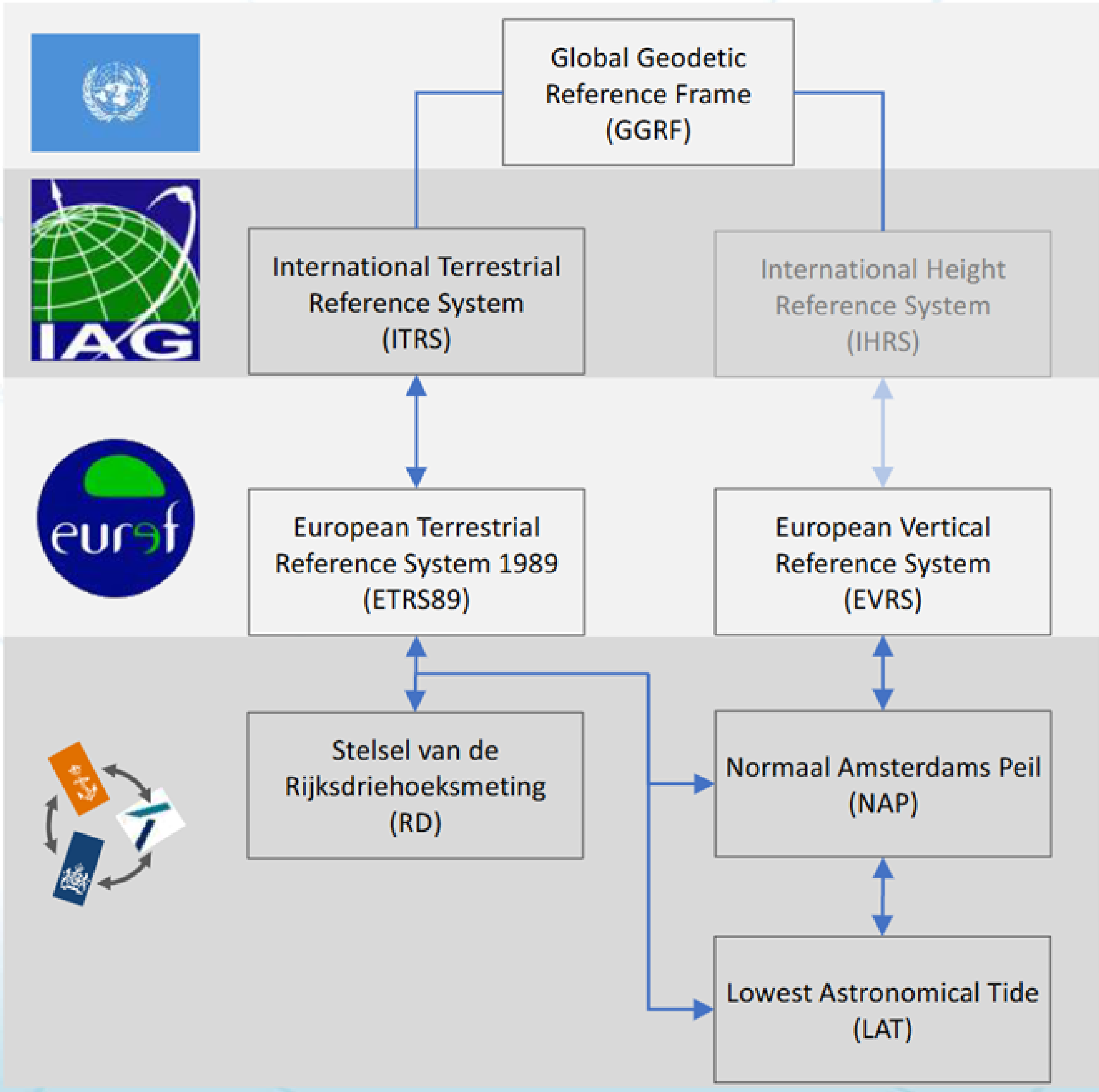
Great examples of Linear Reference Systems are the Dutch **hectometre markers**. As the name suggests, Dutch hectometre markers are spaced at 100-metre intervals. In addition to showing the motorway number and location, they also bear a **carriageway identifier** – Li for Links (Left) and Re for Rechts (Right). The carriageways are identified as being left-hand and right-hand as viewed by somebody looking in the direction of increasing location numbers. By and large, Dutch location numbers increase as one moves away from Amsterdam, or in the case of roads that do not originate in Amsterdam, location numbers increase as one moves eastwards away from the North Sea.

Dynamic segmentation is the process of computing the map locations of events stored and managed in an event table and displaying them on a map using route features. The term dynamic segmentation is derived from the concept that line features need not be split (in other words, segmented) each time an attribute value changes; you can dynamically locate the segment.

Using dynamic segmentation, multiple sets of attributes can be associated with any portion of an existing linear feature independently of where it begins or ends. These attributes can be displayed, queried, edited, and analyzed without affecting the underlying linear feature's geometry.

Terrestrial Reference Systems and Frames

In this chapter of the summary, we will highlight some examples of Coordinate Reference Systems and Frames:

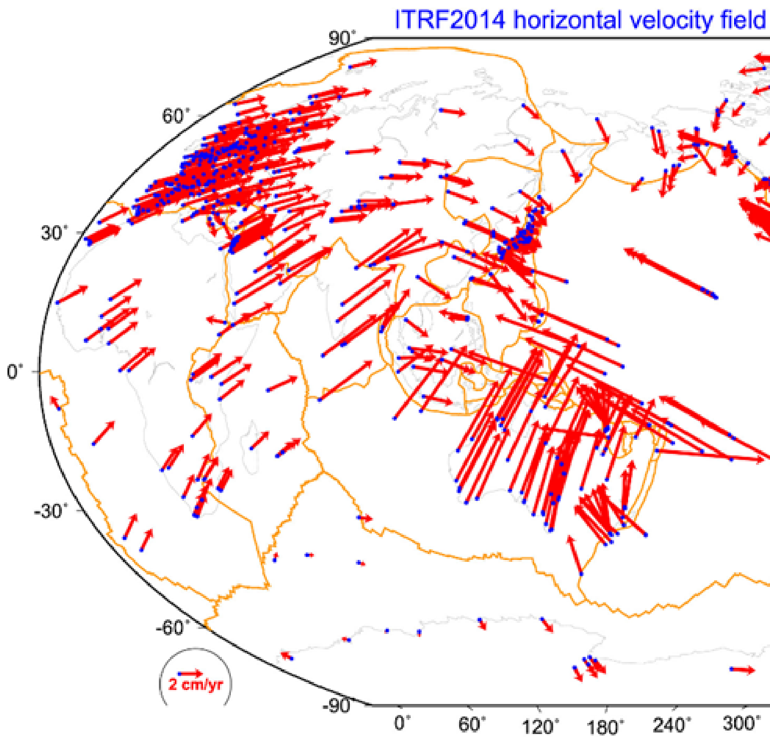


{width:"50%"

ITRS: International Terrestrial Reference System uses the GRS80 ellipsoid, with as its origin the centre of mass of the Earth. Greenwich (UK) is used as a meridian. When using time-dependent coordinates, you always need to specify the epoch (e.g. @2024.91).

ITRF: International Terrestrial Reference Frames are realisations of the ITRS. They publish updates every ~1-6 years. Due to the movement of tectonic plates, the differences between each iteration can be multiple centimetres to decimetres. Two examples are ITRF2014 and ITRF2020 (NB: Frame != epoch, e.g. ITRF2014@2022.90). These updates reflect:

- Improved precision of the station positions r(\$t\$0) and velocities r due to the availability of a longer period of observations, which is particularly important for the velocities,
- Improved datum definition due to the availability of more observations and better models,
- Discontinuities in the time series due to earthquakes and other geophysical events,
- Newly added and discontinued stations,



- Occasionally a new reference epoch t0.

WGS 84 is aligned with ITRS (WGS 84-G2296 (2024) = ITRF2020). For time-dependent coordinates, however, it has limited precision. The realisations (frames) often have differences between m – cm. The ensemble code (+- 2m): EPSG:4326 for 2D (often used as unknown latlon).

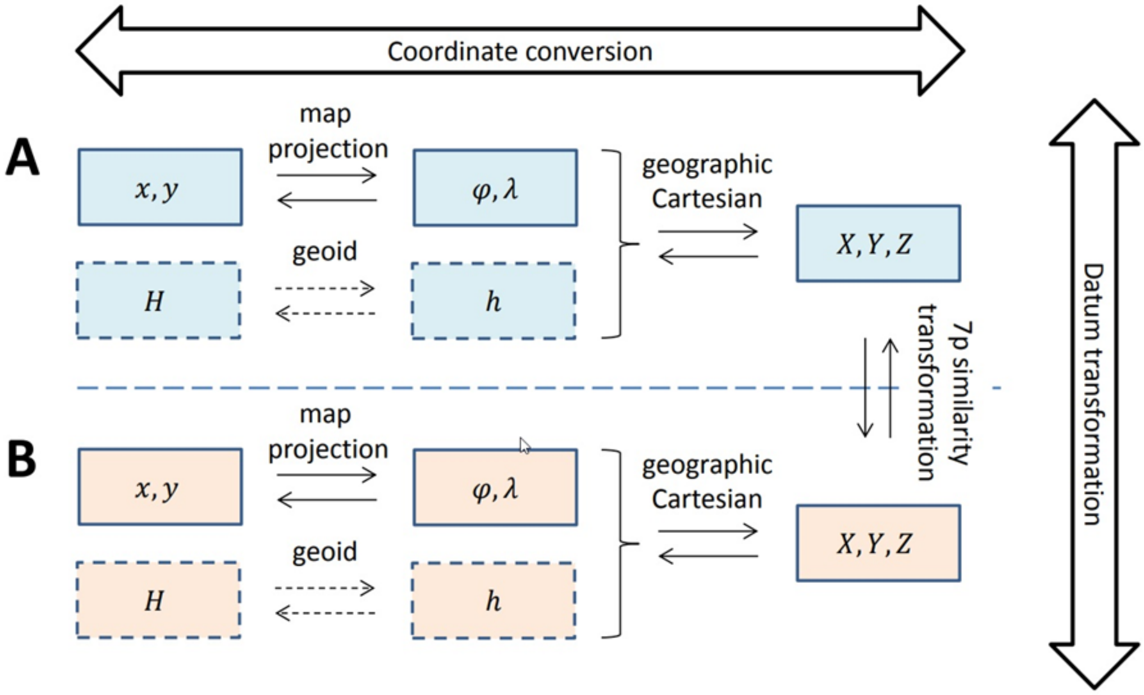
ETRS89: European Terrestrial Reference System 1989 is ITRF89 at epoch 1989.00. It moves with a stable part of Europe, "no" time-dependency. A new realisation (frame) comes with every ITRF. ETRF2000 is recommended for mapping and the ensemble code has an accuracy of +- 0.1m.

A transformation from ETRS89 to ITRS (and WGS84) is an example of a **Time-dependent transformation** (2.4 cm/year), but a specification of realisation and epoch is needed and most software does not include this transformation. A **Null transformation** is possible, simple and the current practice, but it is not future-proof, since the difference is growing (now 0.9m).

Datum and Transformations

Transformations and conversions

The International Association of Oil and Gas Producers (EPSG) used a *de facto* standard instead of an ISO standard. The EPSG collects all the different reference systems and their transformations. There are two different steps when working with 3D data. First, the coordinates need to be converted to a new coordinate system, after which the height values are transformed.



{width:“50%”

Note, that a geoid is a stochastic height transformation and not a conversion. It uses in total 14 parameters: * 7 parameters;

3 Translations [m];

3 Rotations [“];

1 Scale factor [ppm];

- 7 rates and reference epoch (t0).

These parameters can be given (by an official source), user-estimated (empirical) and/or conventional (by definition). A correction grid can be used as an alternative transformation or an additional conversion.

A big difference between datum transformations and coordinate conversions is that the parameters for the datum transformation are often empirically determined and thus subject to measurement errors, whereas coordinate conversions are fully deterministic. More specific, three possibilities need to be distinguished for the datum transformation parameters:

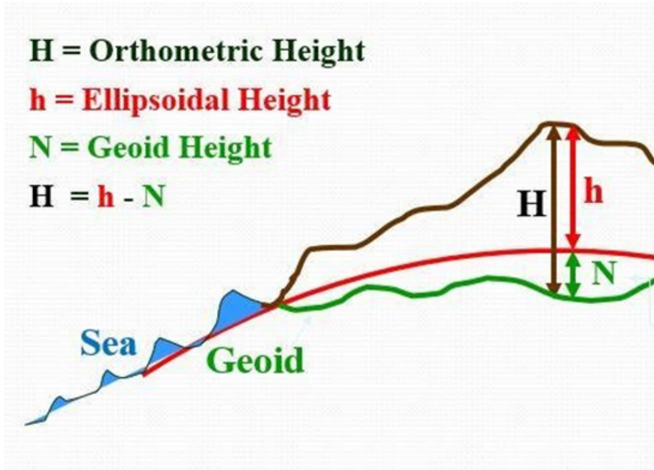
1. **The datum transformation parameters are conventional**. This means they are chosen and therefore not stochastic. The datum transformation is then just some sort of coordinate conversion (which is also not stochastic).
2. **The datum transformation parameters are given but have been derived by a third party through measurements**. This third party often does new measurements and updates the transformation parameters occasionally or at regular intervals. This is also related to the concepts of reference systems and reference frames. Reference frames are considered (different) realisations of the same reference system, with different numerical values assigned to the coordinates of the points in the reference frame, and often with different realisations of the transformation parameters. The station coordinates and transformation parameters are stochastic, so new measurements, mean new estimates that are different from previous estimates.

3. **There is no third party that has determined the transformation parameters, and you as a user, have to estimate them using at least three common points in both systems**. In this case you will need coordinates from the other reference system. Keep in mind that the coordinates from the external reference system should all come from the same realization, or, reference frame.

Datums

When people in the field of Geomatics are talking about height, they can reference multiple different definitions of height:

- **Geometric**: Ellipsoidal height (max. MSL deviation ±150m)
- **Physical**: Height above the geoid ≈ Mean Sea Level (MSL)
- **Relative**: Height above ground level (DTM)



- **Water depth**: Lowest Astronomical Tide (LAT)
  There are multiple physical height standards depending on where you want to know the height:
- **International Height Reference Systems** (IHRS) – No realisation yet
- **European Vertical Reference System** (EVRS) – Realisations available, but not widely used yet (dm – cm)
- **Earth Gravitation Model** (EGM84, 96, 2008, 2020) – By USA like WGS84 (m – dm)
- National Height Systems – Based on local MSL

Map Projections

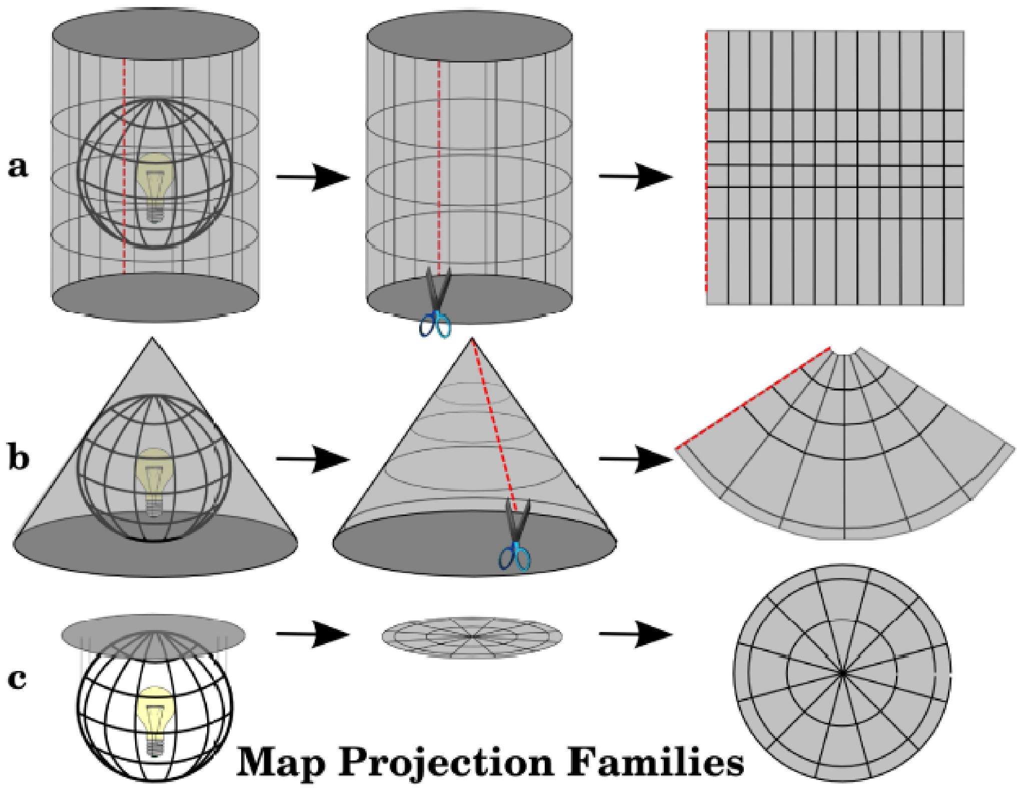
Map projections try to portray the surface of the earth or a portion of the earth, on a flat piece of paper or computer screen.They try to transform the earth from its spherical shape (3D) to a planar shape (2D).

A coordinate reference system (CRS) then defines how the two-dimensional, projected map relates to real places on Earth. The decision of which map projection and CRS to use depends on the regional extent of the area you want to work in, on the analysis you want to do, and often on the availability of data.

When viewed at close range the earth appears to be relatively flat. Maps are representations of reality: they are designed to not only represent features but also their shape and spatial arrangement. Each map projection has **advantages** and **disadvantages**.

The best projection for a map depends on the **scale** of the map, and on the purposes for which it will be used. For example, a projection may have unacceptable distortions if used to map the entire African continent but may be an excellent choice for a **large-scale (detailed) map** of your country.

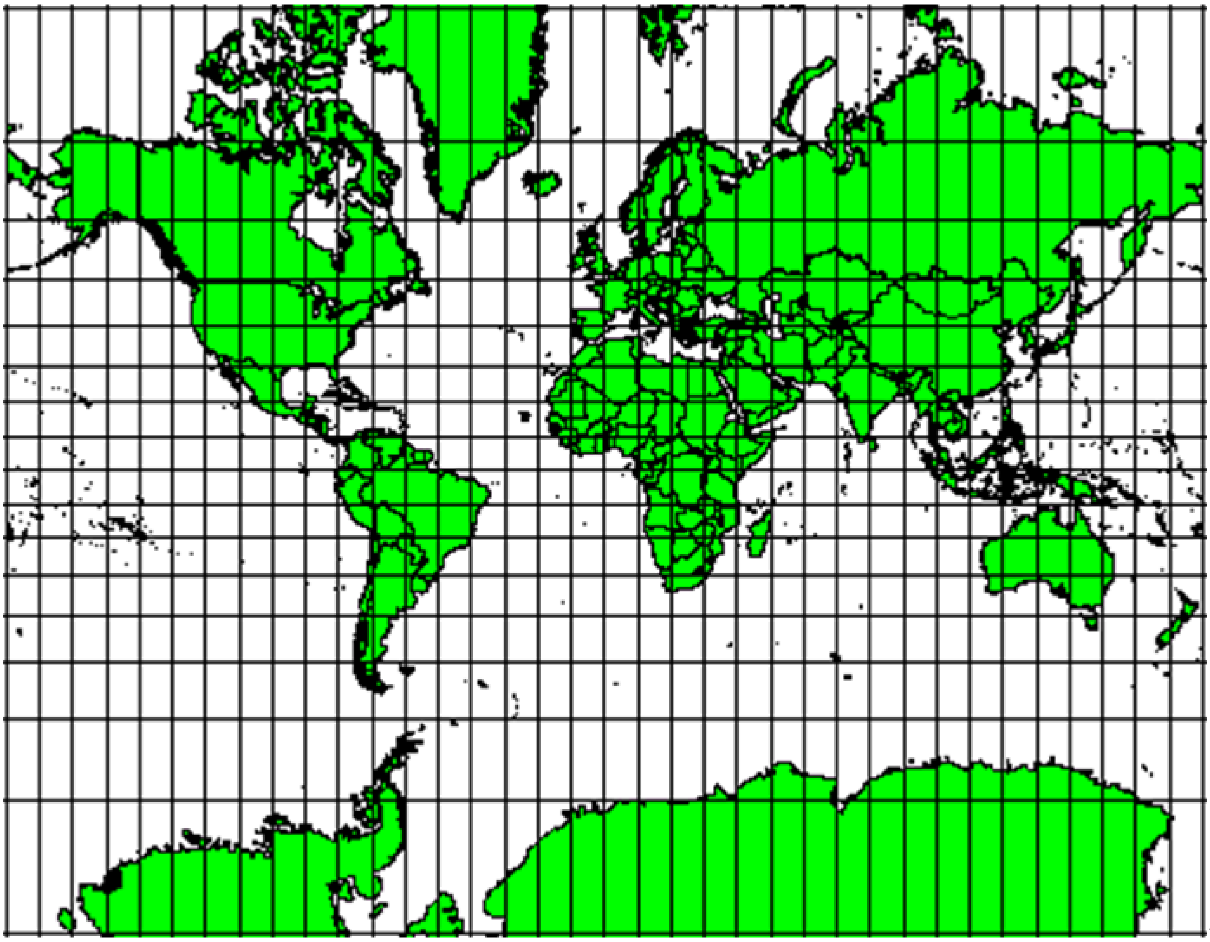
Different projection methods can be produced by surrounding the globe in a **cylindrical** fashion, as a **cone**, or even as a **flat surface**. Each of these methods produces what is called a **map projection family**. Therefore, there is a family of **planar/azimuthal projections (c)**, a family of **cylindrical projections (a)**, and another called **conical projections (b).**



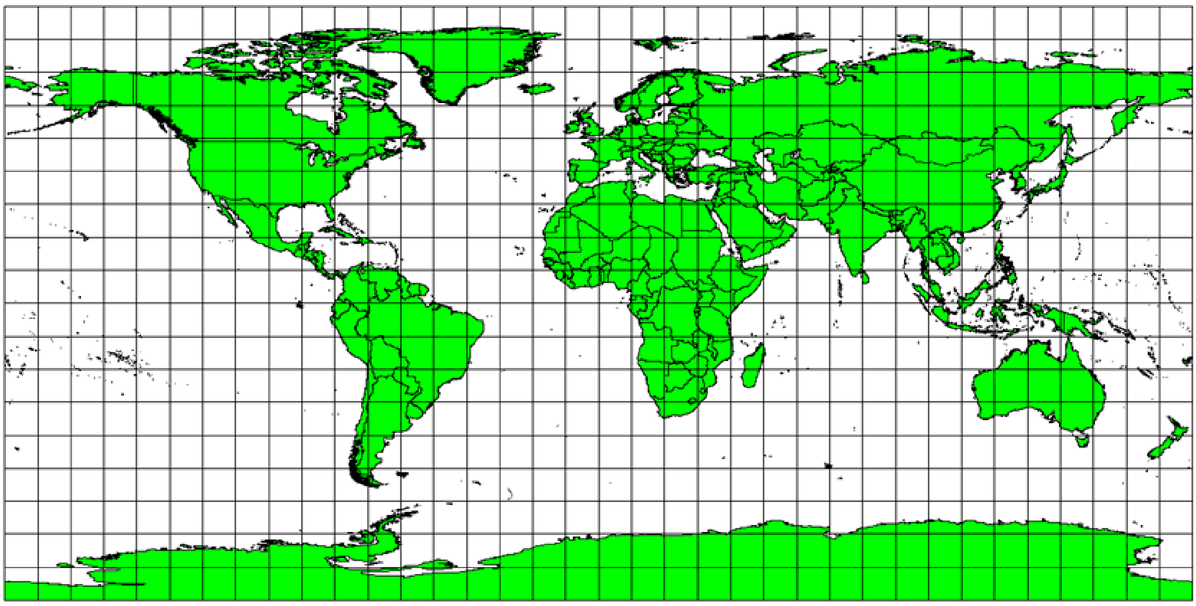
{width:"50%"

Map projections are never absolutely accurate representations of the spherical Earth. As a result of the map projection process, every map shows **distortions of angular conformity, distance and/or area**. It is usually impossible to preserve all characteristics at the same time in a map projection. This means that when you want to carry out accurate analytical operations, you need to use a map projection that provides the best characteristics for your analyses.

When working with a globe, the main directions of the compass (North, East, South and West) will always occur at 90 degrees to one another. In other words, the East will always occur at a 90-degree angle to the North. Maintaining correct **angular properties** can be preserved on a map projection as well. A map projection that retains this property of angular conformity is called a **conformal** or **orthomorphic projection**. These projections are used when the **preservation of angular relationships** is important. They are commonly used for navigational or meteorological tasks.

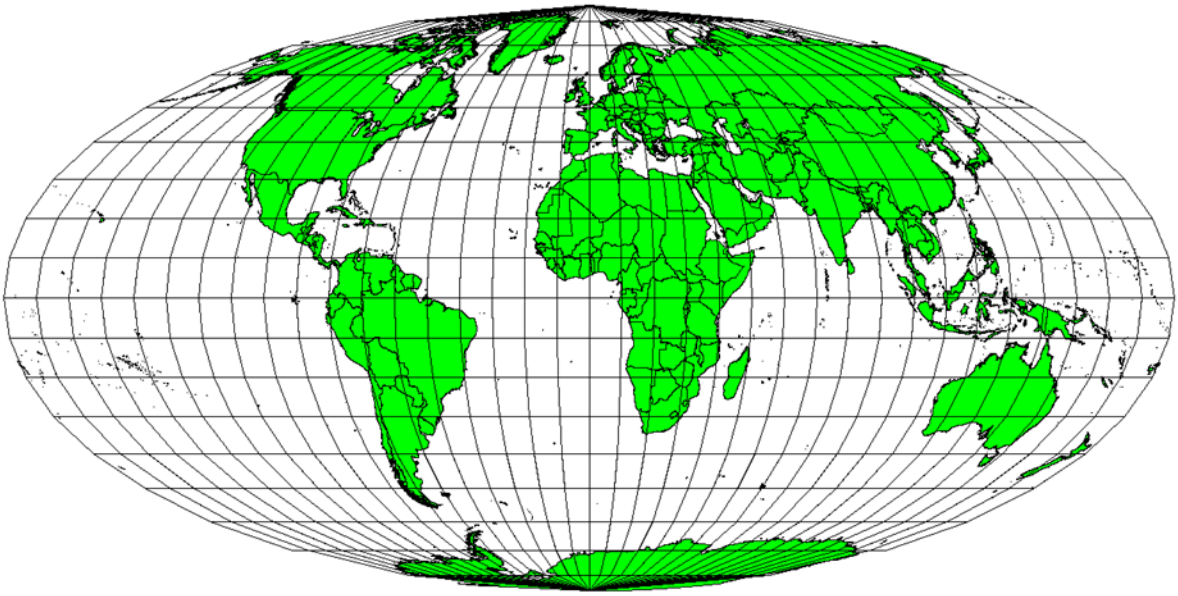

{width:“50%”

If your goal in projecting a map is to accurately measure distances, you should select a projection that is designed to preserve distances well. Such projections, called **equidistant projections**, require that the **scale** of the map is **kept constant**. A map is equidistant when it correctly represents distances from the centre of the projection to any other place on the map.**Equidistant projections** maintain accurate distances from the centre of the projection or along given lines. These projections are used for radio and seismic mapping, and navigation.


{width:“50%”

When a map portrays areas over the entire map, so that all mapped areas have the same proportional relationship to the areas on the Earth that they represent, the map is an

equal area map. As the name implies, these maps are best used when calculations of area are the dominant calculations you will perform. If, for example, you are trying to analyse a particular area in your town to find out whether it is large enough for a new shopping mall, **equal area projections** are the best choice. On the one hand, the larger the area you are analysing, the more precise your area measures will be, if you use an equal area projection rather than another type. On the other hand, an equal area projection results in **distortions of angular conformity** when dealing with large areas.



{width:"50%"

RDNAP

0. Coordinate Systems

Official 3D coordinate system of the Netherlands and Europe: European Terrestrial Reference System 1989 (ETRS89). ETRS89 is linked to the International Terrestrial Reference System (ITRS) by a time-dependant coordinate transformation.
National coordinate systems in Europe are linked to ETRS89.

Rijksdriehoeksmeting (RD) Coordinates in the Dutch **Stelsel van de Rijksdriehoeksmeting (RD)** are the most-frequently used 2D coordinates on land and internal waters. RD coordinates are defined by the official transformation from ETRS89 coordinates. Maintaining reference points for ETRS89 and the transformation to RD coordinates are legal responsibilities of Kadaster.

The **Stelsel van de Rijksdriehoeksmeting (RD)** was created using triangulation from church spires and stone markers (historical). It has no time dependence; the differences are <1cm since 2000. It uses one EPSG-code (EPSG:28992 (RD New) for 2D and EPSG:7415 for compound CRS with NAP). The RD projection has its origin in Amersfoort in the middle of the Netherlands, it uses conformal stereographic projection (angles are preserved) and the north of the map is not equal to the true north. Unlike what you might think, the highest accuracy is not in Amersfoort itself, but rather in a circle ~100km around it.

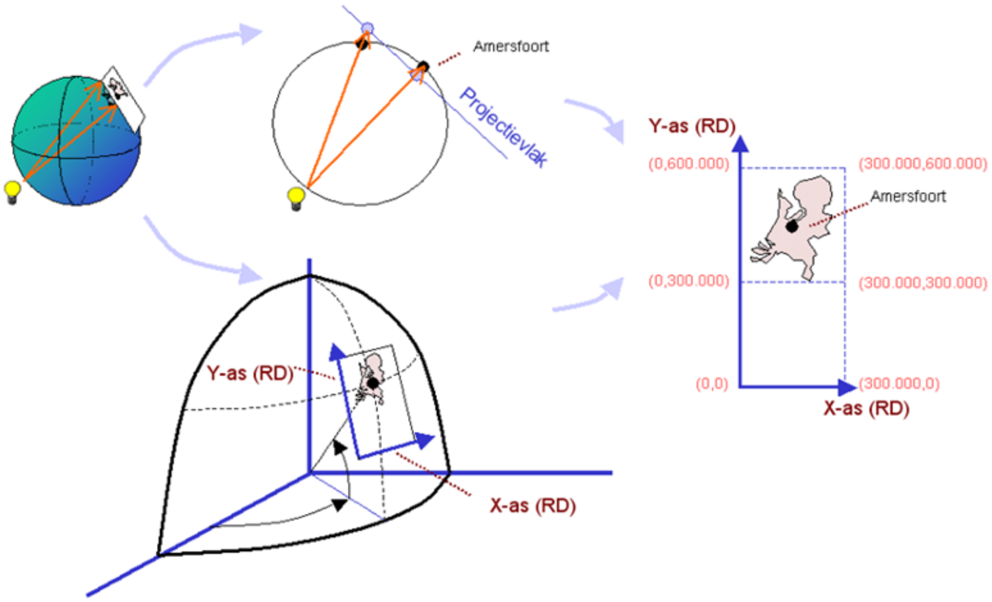


Figure 6: img.png

To prevent confusion between the x-coordinates and y-coordinates, and to obtain always positive coordinates, the origin of the coordinates was shifted 155 km to the West and 463 km to the South (**False Easting and Northing**). This resulted in only positive x- and y-coordinates, where the y-coordinates are always larger than the x-coordinates.

Normaal Amsterdams Peil (NAP) The national height system on land and internal waters in the Netherlands is called **Normaal Amsterdams Peil (NAP)** and is based on the average summer flood in 1683-1684. Maintenance is based on point stability and it uses the NLGEO2018 geoid for GNSS measurements. Ellipsoidal heights in ETRS89 can be transformed with the quasi-geoid model to NAP with a precision higher than ETRS89 coordinates obtained with most GNSS measurements. Values range between 39.1 – 48.7m. The NAP is a legal responsibility of Rijkswaterstaat

For parts of the Noordzee however, the NAP cannot be used. Instead, they use the **Lowest Astronomical Tide** (LAT), which is the water depth in Worst-case astronomical conditions and average meteorological conditions.

1. Coordinate transformation

The official coordinate transformation between European ETRS89 coordinates and Dutch coordinates in RD and NAP is called RDNAPTRANS™. It uses a Datum transformation and a correction grid (see fig.) in combination with the map projection to transform the values. The height of the transformation is determined by a quasi-geoid.

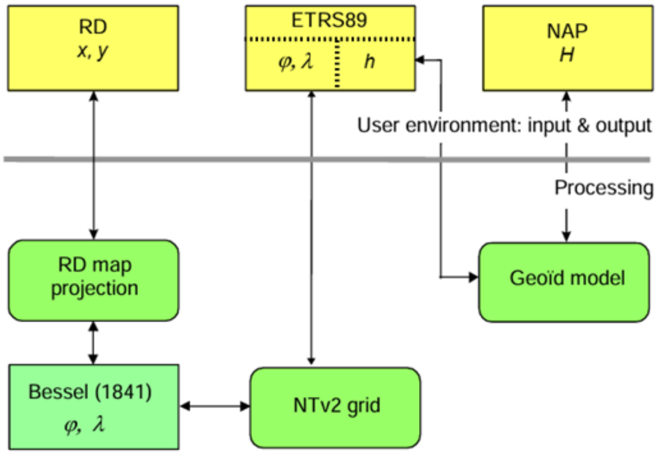
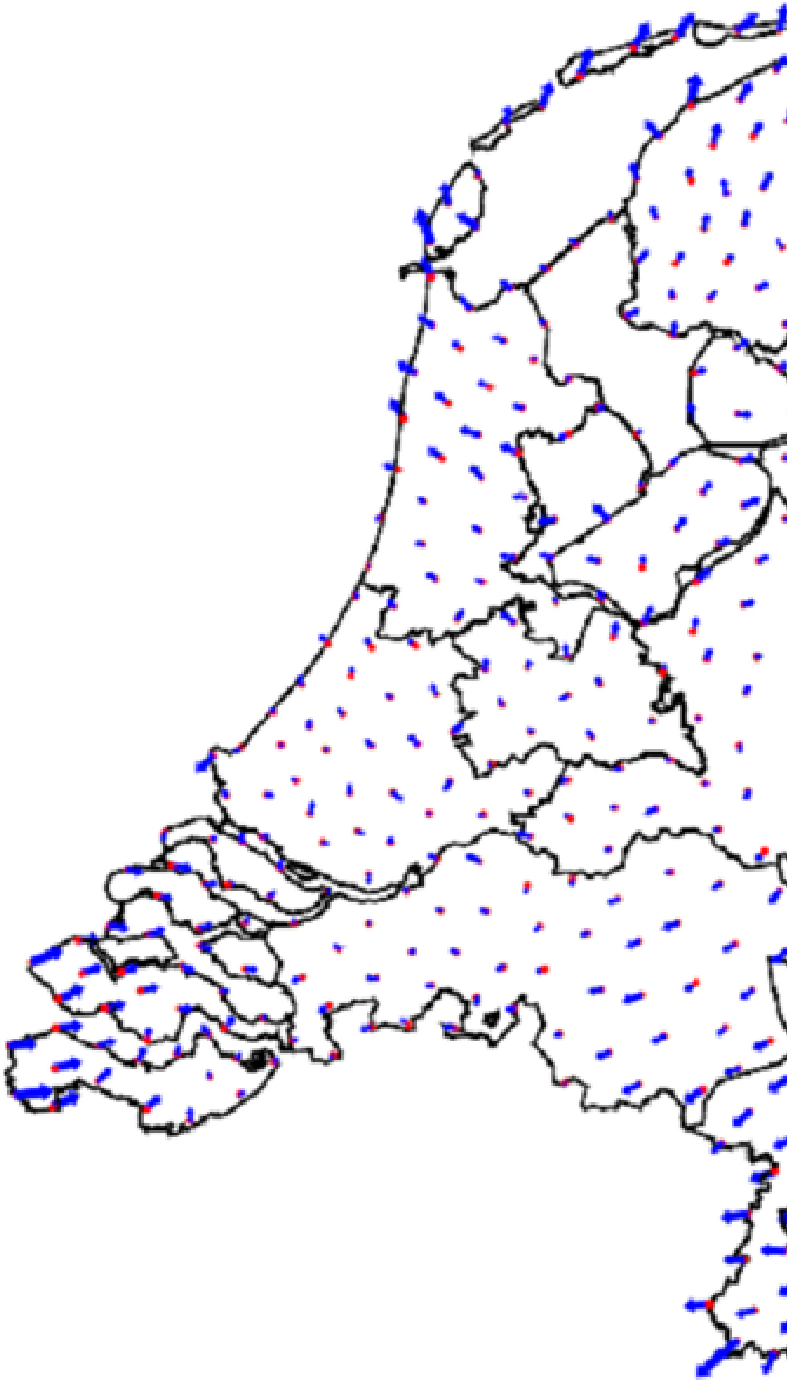


Figure 10.5: NTv2 transformation procedure used by RDNAPTRANS™2018. The figure outlines the relationships and transformations between ETRS89, RD2000 and NAP using the proposed NTv2 procedure, in variant 2 of RDNAPTRANS™2018 where the datum transformation is included in the correction grid. The coordinates below the line are used only for computational purposes and should never be published or distributed to other users.



Below are the errors of the RDNAPTRANS:

The recommended ETRS89 realisation is ETRF2000 at epoch 2010.50 (AGRS2010). When using RDNAPTRANS™2018 it is important to use this realisation and epoch, especially for the height. For applications demanding high accuracy, it is recommended to obtain the NAP height of the point of interest by levelling to nearby NAP benchmarks.

There are two variants for the implementation of the horizontal component of RDNAPTRANS™2018 and two variants for the vertical component (Figure 1.2.2).

Implementation variant 1 applies the datum transformation as a separate step using a 3D similarity transformation.
The advantage of implementation variant 1 is that it has no strict bounds for the area where horizontal coordinates can be transformed correctly. The disadvantage is that

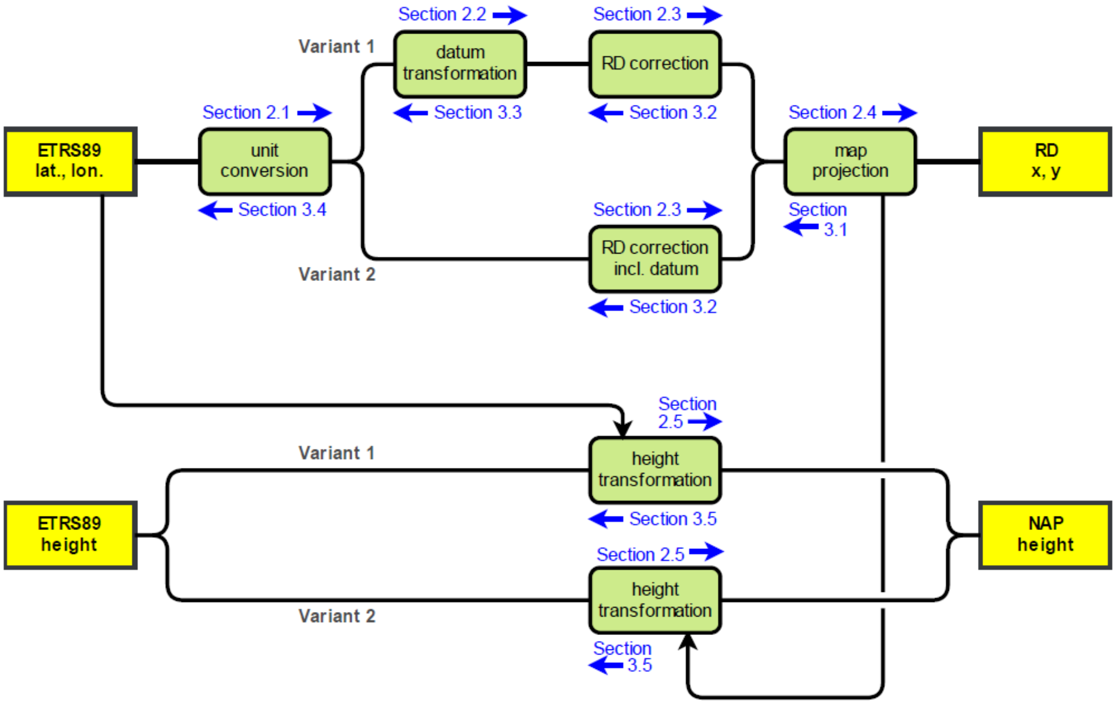


Figure 1.2.2. Guide to the sections (blue) of this document for the different steps of the transformation procedure.

Figure 7: Figure 1.2.2

many software packages do not support implementation variant 1 for the horizontal component.

Implementation variant 2 includes the datum transformation in the correction grid and uses a different quasi-geoid grid for the height transformation. Implementation variant 2 for the horizontal component is supported by more software but can only be used within the bounds of the correction grid (Figure 1.1.1). The difference in the resulting coordinates between the two variants is well below 0.0010 m within the bounds of the RDNAPTRANS™2018 grids.

2. Transformation from ETRS89 to RD and NAP

2.1 Notation in degrees, minutes and seconds ETRS89 coordinates are commonly expressed in ellipsoidal geographic coordinates latitude, longitude and ellipsoidal height.

2.2 Datum transformation

2.2.1 Conversion to geocentric Cartesian coordinates Variant 1 - The ellipsoidal geographic ETRS89 coordinates of a point of interest must be converted to geocentric Cartesian ETRS89 coordinates to be able to apply a 3D similarity transformation. (Formula 2.2.1) Variant 2 - the datum transformation (Section 2.2.1, 2.2.2 and 2.2.3) is included in the correction grid (Section 2.3).

A fixed ellipsoidal height is used instead of the actual height of the point of interest. As a result, points with the same latitude and longitude in ETRS89 that differ in height get exactly the same RD coordinates.

This enables 2D transformation between ETRS89 and RD and straightforward implementation in software like GIS packages.
However, it introduces small differences between back and forth transformation.

2.2.2 3D similarity transformation The formula for a 3D similarity transformation must be applied to the geocentric Cartesian ETRS89 coordinates of the point of interest (Formula 2.2.2). The obtained geocentric Cartesian coordinates are in the geodetic datum of RD. Since the name RD is often used for projected coordinates only, the geodetic datum is often referred to as RD Bessel or just Bessel.

2.2.3 Conversion from geocentric Cartesian coordinates After the 3D similarity transformation, the geocentric Cartesian Bessel coordinates of the point of interest must be converted back to ellipsoidal geographic Bessel coordinates (Formula 2.2.3).

2.3 RD correction

2.3.1 Bilinear correction grid interpolation The ellipsoidal geographic coordinates of a point of interest obtained by datum transformation of implementation variant 1 are pseudo Bessel coordinates. Due to the error propagation of measurement noise of the original (1888–1928) measurements of RD, the pseudo Bessel coordinates must be corrected up to 0.25 m to obtain real Bessel coordinates.
For implementation variant 2, the datum transformation is included in the correction grid (Section 2.3.4). The corrections are obtained from a regular grid of values for latitude correction and a regular grid of values for longitude correction, using bilinear interpolation (Formula 2.3.1).

2.3.2 Determine nearest grid points To transform the point of interest, the nearest NW, NE, SW and SE grid values are required.
Grid values can be read one by one from the binary grid file by direct access or the entire grid of the binary or ASCII text file can be assigned to an array variable first.

2.3.3 Iterative correction The horizontal ellipsoidal geographic pseudo Bessel coordinates of the point of interest must be corrected to real Bessel coordinates (Formula 2.3.3) using the interpolated correction grid value of the point of interest.
The horizontal ellipsoidal geographic coordinates of the correction grid points are in real Bessel. Therefore, also the coordinates of the point of interest are needed in real Bessel to determine the right correction.
To solve this, the real Bessel coordinates are computed iteratively, until the difference between subsequent iterations becomes smaller than the precision threshold.

2.3.4 Datum transformation in the correction grid It is possible to include the datum transformation in the correction grid.
The alternative grid for this implementation variant 2 contains the latitude and longitude corrections up to 0.25 m, but also the datum difference (about 0.1 km in the central part of the Netherlands).
In that way the 3D similarity transformation (Section 2.2) is not needed.

2.4 Map projection

2.4.1 Projection from ellipsoid to sphere The corrected ellipsoidal geographic Bessel coordinates of a point of interest must be projected to obtain RD coordinates.
The used RD map projection is a double projection.
The first step is a Gauss conformal projection from the ellipsoid to a sphere (Formula 2.4.1).

2.4.2 Projection from sphere to plane The second step of the RD map projection of the point of interest is an oblique stereographic conformal projection from sphere to a plane to obtain RD coordinates (Formula 2.4.2).

2.5 Height transformation

2.5.1 Bilinear quasi-geoid grid interpolation The ellipsoidal height is not used with RD coordinates as it is purely geometrical and has no physical meaning. The height transformation from ellipsoidal ETRS89 height of a point of interest to NAP height is based on the quasi-geoid model NLGEO2018.
The quasi-geoid height at the point of interest is obtained by bilinear interpolation of a regular grid of quasi-geoid height values (Formula 2.5.1).

To transform the point of interest, the nearest NW, NE, SW and SE grid values are required.

The horizontal coordinates of the grid points for which the quasi-geoid height is given are in ETRS89 (variant 1) or in Bessel (variant 2), but the quasi-geoid height is relative to the ETRS89 ellipsoid in both cases.

Implementation variant 1 uses the ETRS89 grid for transformation in both transformation directions, for ETRS89 to RD and NAP as well as RD and NAP to ETRS89.
Using a different grid for the transformation back is not recommended, as it can result in too large differences after repeatedly transforming back and forth.

The quasi-geoid heights are in metres, spacing and coordinates of grid bounds are given in decimal degrees with conventional sign, thus east of the Greenwich meridian is positive.

2.5.2 Transformation to NAP The ellipsoidal ETRS89 height of the point of interest must be transformed to NAP height (Formula 2.5.2) using the interpolated quasi-geoid height of the point of interest.

3. Transformation from RD and NAP to ETRS89

3.1 Inverse map projection

3.1.1 Projection from plane to sphere RD coordinates of a point of interest must be converted to Bessel coordinates before the other steps of the transformation can be performed.
The RD map projection is a double projection. The first step of the inverse map projection is an inverse oblique stereographic conformal projection from the RD projection plane to a sphere (Formula 3.1.1).

3.1.2 Projection from sphere to ellipsoid The second step of the inverse RD map projection is an inverse Gauss conformal projection from the sphere to the Bessel ellipsoid to obtain Bessel coordinates of the point of interest (Formula 3.1.2).

3.2 RD correction

3.2.1 Direct correction The ellipsoidal geographic coordinates of a point of interest obtained by the inverse map projection, are real Bessel coordinates. Due to the error propagation of measurement noise of the original (1888–1928) measurements of RD, the real Bessel coordinates must be corrected up to 0.25 m to obtain pseudo Bessel coordinates. For implementation variant 2, the datum transformation is included in the correction grid (Section 3.2.2). The corrections are obtained from a regular grid of values for latitude correction and a regular grid of values for longitude correction, using bilinear interpolation (Formula 2.3.1).

To transform the point of interest, the nearest NW, NE, SW and SE grid values are required The horizontal ellipsoidal geographic real Bessel coordinates of the point of interest must be corrected to pseudo Bessel coordinates (Formula 3.2.1) using the interpolated correction grid value of the point of interest.
No iteration is needed for the transformation from RD to ETRS89 coordinates as the grid is given in real Bessel coordinates.

3.2.2 Datum transformation in the correction grid It is possible to include the datum transformation in the correction grid.
In that way the 3D similarity transformation (Section 3.3) is not needed.
With this alternative grid a bilinear interpolation of the latitude and longitude corrections (Formula 2.3.1) at the nearest grid points (Formula 2.3.2) and correction of real Bessel coordinates (Formula 3.2.1) can be applied as for a correction grid without the datum transformation, but in this case the output are ETRS89 coordinates of the point of interest instead of pseudo Bessel coordinates.

3.3 Datum transformation The corrected ellipsoidal geographic Bessel coordinates of a point of interest must be transformed to ellipsoidal geographic ETRS89 coordinates.
This is only needed for implementation variant 1, for variant 2 the datum transformation (Section 3.3) is included in the correction grid (Section 3.2).
First, the ellipsoidal geographic Bessel coordinates of a point of interest must be converted to geocentric Cartesian Bessel coordinates (Formula 2.2.1) to be able to apply a 3D similarity transformation.

Unlike conventional use in transformations, including RDNAPTRANS™2008 and earlier versions of RDNAPTRANS™, a fixed ellipsoidal height is used instead of the actual

height of the point of interest.
Points with the same latitude and longitude in RD that differ in height get exactly the same horizontal ETRS89 coordinates. This enables 2D transformation between RD and ETRS89 and straightforward implementation in software like GIS packages.
However, it introduces small differences between back and forth transformation. These differences are below 0.0010 m up to 500 km outside the bounds of the RDNAP-TRANS™2018 grids.

The 3D similarity transformation must be applied to the geocentric Cartesian Bessel coordinates of the point of interest to obtain geocentric Cartesian ETRS89 coordinates. The datum transformation uses the same formulas for the 3D similarity transformation from RD to ETRS89 as from ETRS89 to RD (Formula 2.2.2). So the order is first rotate and then translate, instead of inversing the order to translating and rotating. As a result, the inverse transformation parameters do not only have opposite sign, but the parameters also have slightly different numbers (except for the scale difference parameter). Sometimes the differences are only apparent in the last digit. The parameters could be computed from the inverse parameters too (Section 2.2.2).

After the 3D similarity transformation, the geocentric Cartesian ETRS89 coordinates of the point of interest must be converted back to ellipsoidal geographic ETRS89 coordinates (Formula 2.2.3). The latitude is computed iteratively. The parameters of the GRS80 ellipsoid (Section 2.2.1) are needed for the conversion to ellipsoidal geographic coordinates for the transformation from RD to ETRS89.

3.4 Notation in degrees, minutes and seconds The ETRS89 coordinates of a point of interest obtained after transformation are in radians or decimal degrees, depending on the type of goniometry functions used. These coordinates must be converted to decimal degrees and optionally to degrees, minutes and seconds (Formula 3.4).

3.5 Height transformation The physical NAP height of a point of interest can be transformed to the purely geometrical ellipsoidal ETRS89 height. The height transformation from NAP to ETRS89 is based on the quasi-geoid model NLGEO2018. The quasi-geoid height at the point of interest is obtained by bilinear interpolation of a regular grid of quasi-geoid height values (Formula 2.5.1). A unit conversion (Formula 2.1) is only needed when working with radians.

To transform the point of interest, the nearest NW, NE, SW and SE grid values are required. Grid values can be read one by one from the binary grid file by direct access or the entire grid of the binary or ASCII text file can be assigned to an array variable first. In both cases, the indices of the required grid values need to be determined.
The horizontal coordinates of the grid points for which the quasi-geoid height is given are in ETRS89 (variant 1) or in Bessel (variant 2), but the quasi-geoid height is relative to the ETRS89 ellipsoid in both cases. Implementation variant 1 uses the ETRS89 grid for transformation in both transformation directions, for RD and NAP to ETRS89 as well as ETRS89 to RD and NAP. Using a different grid for the transformation back is not recommended, as it can result in too large differences after repeatedly transforming back and forth.

The NAP height of the point of interest must be transformed to ellipsoidal ETRS89 height (Formula 3.5) using the interpolated quasi-geoid height of the point of interest.

Wi-Fi-monitoring / Fingerprinting

Introduction

Wi-Fi-Based Approaches

Wi-Fi Monitoring

Wi-Fi Fingerprinting

Radio Signal Based Techniques

Received Signal Strength (RSS)

Time of Arrival (ToA)

Time Difference of Arrival (TDoA)

Angle of Arrival (AOA)

Path-Loss

Fine Timing Measurement (FTM)

Radio Frequency Identification (RFID)

Hybrid and Other Techniques

Trilateration

Inertial Navigation Systems (INS)

Visual Based Indoor Localisation

Isovists

Performance Metrics

Position

Location

Yield

Consistency

Overhead

Latency

Power Consumption