Final Exam for: Jackson Bilello
MCEN 3030, Computational Methods - Fall 2020

Prof. Robert MacCurdy

Noon December 11, 2020 - to - Noon December 13, 2020

Guidance:

- Spend some time looking over the exam and ask any clarifying questions asap using the course discussion board. See: **Getting Help** below.
- Do your best, you got this!
- Include your work for partial credit!
- For numerical solutions, show symbolic work for partial credit.
- **Show all work and assumptions.**

Honor Pledge, Please type the following at the top of your submission:
"I have neither given nor received unauthorized assistance on this exam."

Exam Rules: Open-Book. Open-notes. Open-YOUR-Homework. You may NOT use posted Homework Solutions. NO COLLABORATION. This is an INDIVIDUAL EFFORT TEST. Open-MATLAB You may use a computer running MATLAB. You may use the built-in MATLAB help functionality. You may use web searches for clarifications and debugging, but NOT for algorithms or code. THE WORK YOU SUBMIT ON THIS EXAM MUST BE YOUR OWN. Note: by making small changes to each question, each exam is unique to each student.

Submission Guidelines: The exam submission format will be like the homeworks: You will upload a SINGLE .ZIP file with your submission. This ZIP file must contain:

1. A SINGLE PDF document that is reasonably well formatted. This PDF document will contain all your solutions (for example: images generated via MATLAB, any scanned or photographed work-on-paper, a listing/copy-paste printout of all MATLAB code used to solve a problem and any console output)
2. You must also provide ALL code files used to solve the problems. This code MUST be executed by a single "runme.m" script that takes no arguments. WE MUST BE ABLE TO RUN YOUR CODE VIA THIS FILE.

Please follow any specific instructions provided in each question carefully.

Getting Help: You should post questions to the course discussion board. Students should NOT answer any exam questions via the discussion board. To ensure a uniform and fair process, Prof. MacCurdy and the TAs will not answer any exam questions submitted via email or via the Canvas private messaging/chat apps. ONLY questions posted to the course public discussion board will be answered. We will monitor this board frequently during the exam period.

NOTES:

- This exam is out of 230 points
- Some of the problems include numerical data that you will need to load into MATLAB. It may be easiest/fastest to copy and paste the text from the PDF into MATLAB. Be sure to check for any copy/paste errors if you do this.

Question 1 - 1D Root Finding (20 points)

Given the function $f(x) = -5.74x^3 + -4.33x^2 + 7.26x + -1.05$

1. (4 points) Use MATLAB to plot the function $f(x)$, showing the approximate locations (values) of the roots. You may find the MATLAB DataTips tool in the plot window useful to find these approximate values. Include the plot in your solution's PDF.

2. (8 points) Demonstrate the use of the Bisection Method as it would be used to find the UPPER (highest) root, but only compute the first 3 iterations (i.e. you dont actually need to find the root). Generate and show a table with 5 columns and 3 rows (including the initial value). Your table should have these columns: iteration number (i), x_L, x_m, x_U, $f(x_m)$. You are NOT required to write a MATLAB function to do this, unless you think it will save you time, but your solution table MUST be digital (no handwritten tables).

3. (8 points) Demonstrate the use of the Newton-Raphson Method as it would be used to find the LOWER (lowest) root, but only compute the first 3 iterations (i.e. you dont actually need to find the root). Generate and show a table with 2 columns and 3 rows (including the initial value). Your table should have these columns: iteration number (i), x_i. You are NOT required to write a MATLAB function to do this, unless you think it will save you time, but your solution table MUST be digital (no handwritten tables).

Question 2 - 1D Optimization (15 Points)

Given the function $f(x) = -5.74x^3 + -4.33x^2 + 7.26x + -1.05$

1. (5 points) If you were using an iterative method to find an optimal value, what stopping criteria would you use for the approximation to be accurate to 7 significant figures?

2. (10 points) In MATLAB use the Golden Section Search algorithm to find the local MINIMUM, accurate to 7 significant figures. Generate and show a table with 4 columns and 3 rows (including the initial value). Your table should have these columns: iteration number (i); current x value of the optimum (x_i), current value of the optimum ($f(x_i)$), current percent relative error Ea_i. Also, plot the iterations overlaid atop the function. Meaning: plot the function $f(x)$ and then plot the $(x_i, f(x_i)$ locations of each of the 3 guesses. Include this plot, as well as the table in your solution's PDF.

Question 3 - Multivariate Unconstrained Optimization (20 points)

Given the function $f(x) = f(x, y) = -x^2 - y^2 + 2x - 4y - 5$

1. (8 points) If performing a search for the local optimum using the *Method of Steepest Ascent* and the starting (x, y) position $(2.09, -1.93)$, in which direction will you take your first step? Express your answer as a vector with unit length. If you work this problem by hand, show your work; if you use MATLAB functions then include your code.

2. (12 points) If you continue from the starting (x, y) position $(2.09, -1.93)$, in the direction your found in part 1) until the objective function f stops increasing, how far will you travel? Express your answer as a scalar. If you work this problem by hand, show your work; if you use MATLAB functions then include your code.

Question 4 - Solving Linear Systems (25 points)

Given the matrix $A = \begin{pmatrix} 2.5 & 1.1 & 0.92 \\ 0.54 & 0.83 & 0.91 \\ 0.16 & 1.6 & 0.34 \end{pmatrix}$, and the vector $b = \begin{pmatrix} 0.75 \\ 2.8 \\ 0.79 \end{pmatrix}$

1. (10 points) Use LU decomposition to generate the L and U matrices for A. Do this "by hand" (not with a built-in MATLAB function) and show your work.

2. (5 points) Write down the MATLAB line(s) you would use to solve the previous problem (using built-in functions) and verify your correct answer

3. (10 points) Perform two iterations of the Gauss-Seidel method on the linear system in the previous question.

   Use an initial guess of zero for all unknowns, and perform two iterations *after* the initial guess. Report the values of x_1,x_2, and x_3 for each of these three (counting the initial guess) iterations in a table. You may use Matlab or work this problem by hand.

Question 5 - Norm Measures (10 points)

Given the matrix $A = \begin{pmatrix} 2.5 & 1.1 & 0.92 \\ 0.54 & 0.83 & 0.91 \\ 0.16 & 1.6 & 0.34 \end{pmatrix}$, and the vector $b = \begin{pmatrix} 0.75 \\ 2.8 \\ 0.79 \end{pmatrix}$

(5 points) Compute the following for the A Matrix. You may use MATLAB.

1. The L1 Norm
2. The L2 Norm
3. The Frobenius Norm
4. Infinity Norm

(5 points) Compute the following for the b Matrix. You may use MATLAB.

1. The L1 Norm
2. The L2 Norm
3. Infinity Norm

Question 6 - 1D Regression (20 points)

Prof. Rick has been researching a new material and wants to find its electrical resistance. He entrusts his assistant Morty to help him out with it. Morty being an amateur does not make perfect measurements, but has managed to take a lot of them.

Morty applied various voltages (V) across this material, and measured the current (I) flowing through it using an ammeter.

I = [0.1, 1.3, 2.5, 3.7, 4.9, 6.1, 7.3, 8.5, 9.7, 11, 12]
V = [8.4, 3.6, 14, 23, 28, 43, 47, 56, 64, 70, 87]

1. (15 points) Using Morty's data, estimate the resistance of this material by fitting a 1D regression line to this data using a least-squares approach. Recall that resistance R(ohms) = V/I. Note: despite this equation (R=V/I), for the purposes of this exam problem, your fit line does not need to pass through (0,0).

2. (5 points) Compute and show the standard error of the estimate $S_{y/x}$ for the data collected by Morty.

Question 7 - Linear Programming (20 Points)

Consider the linear programming problem:
Maximize $f(x, y) = 2.2x + 3.3y$
Subject to:
$5.5x + 3.0y \leq 9$
$1.1x + 2y \leq 6$
$1.0x + 2.5y \leq 4$
$x \geq 0$
$y \geq 0$

1. (10 points) Plot the given constraint equations and identify the region over which the input variables satisfy these constraints. You may use MATLAB to do this if you wish, or you may sketch out the plot.

2. (10 points) Find the maximum value of the objective function $f(x, y)$, and the (x, y) pair that maximize it using MATLAB's *linprog* function. Mark the position of the maximum value on your plot above.

Question 8 - Numerical Integration (20 points)

An entomologist who is trying to estimate the average flight velocity of a dragonfly in a straight line, collects the following data at various times for the velocity. Using this data, estimate the total distance flown during the experiment, and the average velocity of this dragonfly by employing a method or combination of methods **offering highest accuracy**.

Remember: Average Velocity is defined as "Total Distance/Total Time".

$T(s)$	0.0	3.0	6.0	6.5	7.0	7.5	8.5
$V\ (m/s)$	13.698	12.358	12.109	12.723	12.551	13.06	12.612

Question 9 - Spring mass damper (30 Points)

A 1 kg mass rests on a frictionless horizontal surface. It is connected by a damped spring (a spring in parallel with a dashpot) to a fixed plane. The mass is given an initial velocity of -0.4 m/s at initial position 0.1 m. The spring's stiffness coefficient is 100 and the damper's damping coefficient is 0.2. (Note: do not worry about the spring's resting length, it can be 0. Yes, this is not physically plausible but it's fine for this problem.)

1. (10 points) Draw the free-body diagram of the system showing all forces acting on it and write the second order differential equations of motion of the system along the x direction.

2. (20 points) Using the ode23 function of MATLAB plot the position and velocity vs time in a single figure. Use an appropriate time span to show the system behaviour.

   A code fragment is provided here (note that you will need to modify this fragment):

   ```
   t=[0,1];
   x0=[0;0];
   [t,x]=ode23(@myODE,t,x0);
   plot(t,x(:,1));
   ```

Question 10 - Numbers (20 Points)

1. (10 points) Convert the following base-2 numbers to base-10:
   (a) 101101
   (b) 101.011
   (c) 0.01101

2. (5 points) State the difference between fixed and floating point numbers. Use an equation to explain this difference.

3. (5 points) What floating point numbers are represented by the following mantissa, base, and exponent? (the mantissa, base, and exponent are stated as decimal base-10 numbers):
   (a) mantissa = 1.5, base = 2, and exponent = 6
   (b) mantissa = 1.5, base = 3.7, and exponent = 2.5

Question 11 - PDE Heat Equations (30 points)

You are given an Aluminum square plate of size 0.5meter * 0.5meter that is insulated on the left and bottom edges, and is placed in contact with a heating source at 100C on the top and a heat-sink at 20C on the right edge.

Use the following values for material properties of Aluminum:

Thermal conductivity = $204.3 (W/m.k)$
Specific heat = $910 (J/kg.K)$
Density = $2700.0 (kg/m^3)$

Use 25 nodes each in X and Y directions.

1. (20 points) Suppose you start at $t = 0$ seconds with all nodes (except fixed temp. nodes) at $0°C$, obtain the time-varying solution for the temperature of the plate until it reaches steady-state. Show color plots of the temperature map of this plate at $t = 50$s.

2. (10 points) For the node located at (X=19,Y=6), plot the temperature vs time from t=0 until steady-state.