2004

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4. Two antibiotics are available as treatment for a common ear infection in children.

   - Antibiotic A is known to effectively cure the infection 60 percent of the time. Treatment with antibiotic A costs \$50.
   - Antibiotic B is known to effectively cure the infection 90 percent of the time. Treatment with antibiotic B costs \$80.

The antibiotics work independently of one another. Both antibiotics can be safely administered to children. A health insurance company intends to recommend one of the following two plans of treatment for children with this ear infection.

- Plan I: Treat with antibiotic A first. If it is not effective, then treat with antibiotic B.
- Plan II: Treat with antibiotic B first. If it is not effective, then treat with antibiotic A.

(a) If a doctor treats a child with an ear infection using plan I, what is the probability that the child will be cured?

If a doctor treats a child with an ear infection using plan II, what is the probability that the child will be cured?

(b) Compute the expected cost per child when plan I is used for treatment.

Compute the expected cost per child when plan II is used for treatment.

(c) Based on the results in parts (a) and (b), which plan would you recommend?

Explain your recommendation.

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3. A local arcade is hosting a tournament in which contestants play an arcade game with possible scores ranging from 0 to 20. The arcade has set up multiple game tables so that all contestants can play the game at the same time; thus contestant scores are independent. Each contestant's score will be recorded as he or she finishes, and the contestant with the highest score is the winner.

After practicing the game many times, Josephine, one of the contestants, has established the probability distribution of her scores, shown in the table below.

Josephine's Distribution				
Score	16	17	18	19
Probability	0.10	0.30	0.40	0.20

Crystal, another contestant, has also practiced many times. The probability distribution for her scores is shown in the table below.

Crystal's Distribution			
Score	17	18	19
Probability	0.45	0.40	0.15

(a) Calculate the expected score for each player.

(b) Suppose that Josephine scores 16 and Crystal scores 17. The difference (Josephine minus Crystal) of their scores is -1. List all combinations of possible scores for Josephine and Crystal that will produce a difference (Josephine minus Crystal) of -1, and calculate the probability for each combination.

(c) Find the probability that the difference (Josephine minus Crystal) in their scores is -1.

(d) The table below lists all the possible differences in the scores between Josephine and Crystal and some associated probabilities.

Distribution (Josephine minus Crystal)						
Difference	-3	-2	-1	0	1	2
Probability	0.015			0.325	0.260	0.090

Complete the table and calculate the probability that Crystal's score will be higher than Josephine's score.

3. A test consisting of 25 multiple-choice questions with 5 answer choices for each question is administered. For each question, there is only 1 correct answer.

   (a) Let X be the number of correct answers if a student guesses randomly from the 5 choices for each of the 25 questions. What is the probability distribution of X?

   This test, like many multiple-choice tests, is scored using a penalty for guessing. The test score is determined by awarding 1 point for each question answered correctly, deducting 0.25 point for each question answered incorrectly, and ignoring any question that is omitted. That is, the test score is calculated using the following formula.

   Score = (1 × number of correct answers) – (0.25 × number of incorrect answers) + (0 × number of omits)

   For example, the score for a student who answers 17 questions correctly, answers 3 questions incorrectly, and omits 5 questions is

   $$\text{Score} = (1 \times 17) - (0.25 \times 3) + (0 \times 5) = 16.25.$$

   (b) Suppose a student knows the correct answers for 18 questions, answers those 18 questions correctly, and chooses randomly from the 5 choices for each of the other 7 questions. Show that the expected value of the student's score is 18 when using the scoring formula above.

   (c) A score of at least 20 is needed to pass the test. Suppose a student knows the correct answers for 18 questions, answers those 18 questions correctly, and chooses randomly from the 5 choices for each of the other 7 questions. What is the probability that the student will pass the test?

2004 AP® STATISTICS FREE-RESPONSE QUESTIONS

3. At an archaeological site that was an ancient swamp, the bones from 20 brontosaur skeletons have been unearthed. The bones do not show any sign of disease or malformation. It is thought that these animals wandered into a deep area of the swamp and became trapped in the swamp bottom. The 20 left femur bones (thigh bones) were located and 4 of these left femurs are to be randomly selected without replacement for DNA testing to determine gender.

   (a) Let X be the number out of the 4 selected left femurs that are from males. Based on how these bones were sampled, explain why the probability distribution of X is not binomial.

   (b) Suppose that the group of 20 brontosaurs whose remains were found in the swamp had been made up of 10 males and 10 females. What is the probability that all 4 in the sample to be tested are male?

   (c) The DNA testing revealed that all 4 femurs tested were from males. Based on this result and your answer from part (b), do you think that males and females were equally represented in the group of 20 brontosaurs stuck in the swamp? Explain.

   (d) Is it reasonable to generalize your conclusion in part (c) pertaining to the group of 20 brontosaurs to the population of all brontosaurs? Explain why or why not.

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4. An automobile company wants to learn about customer satisfaction among the owners of five specific car models. Large sales volumes have been recorded for three of the models, but the other two models were recently introduced so their sales volumes are smaller. The number of new cars sold in the last six months for each of the models is shown in the table below.

Car Model	A	B	C	D	E	Total
Number of new cars sold in the last six months	112,338	96,174	83,241	3,278	2,323	297,354

The company can obtain a list of all individuals who purchased new cars in the last six months for each of the five models shown in the table. The company wants to sample 2,000 of these owners.

(a) For simple random samples of 2,000 new car owners, what is the expected number of owners of model E and the standard deviation of the number of owners of model E?

(b) When selecting a simple random sample of 2,000 new car owners, how likely is it that fewer than 12 owners of model E would be included in the sample? Justify your answer.

(c) The company is concerned that a simple random sample of 2,000 owners would include fewer than 12 owners of model D or fewer than 12 owners of model E. Briefly describe a sampling method for randomly selecting 2,000 owners that will ensure at least 12 owners will be selected for each of the 5 car models.

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3. An airline claims that there is a 0.10 probability that a coach-class ticket holder who flies frequently will be upgraded to first class on any flight. This outcome is independent from flight to flight. Sam is a frequent flier who always purchases coach-class tickets.

   (a) What is the probability that Sam's first upgrade will occur after the third flight?

   (b) What is the probability that Sam will be upgraded exactly 2 times in his next 20 flights?

   (c) Sam will take 104 flights next year. Would you be surprised if Sam receives more than 20 upgrades to first class during the year? Justify your answer.

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3. Each full carton of Grade A eggs consists of 1 randomly selected empty cardboard container and 12 randomly selected eggs. The weights of such full cartons are approximately normally distributed with a mean of 840 grams and a standard deviation of 7.9 grams.

   (a) What is the probability that a randomly selected full carton of Grade A eggs will weigh more than 850 grams?

   (b) The weights of the empty cardboard containers have a mean of 20 grams and a standard deviation of 1.7 grams. It is reasonable to assume independence between the weights of the empty cardboard containers and the weights of the eggs. It is also reasonable to assume independence among the weights of the 12 eggs that are randomly selected for a full carton.

   Let the random variable X be the weight of a single randomly selected Grade A egg.

   i) What is the mean of X ?

   ii) What is the standard deviation of X ?

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3. Schools in a certain state receive funding based on the number of students who attend the school. To determine the number of students who attend a school, one school day is selected at random and the number of students in attendance that day is counted and used for funding purposes. The daily number of absences at High School A in the state is approximately normally distributed with mean of 120 students and standard deviation of 10.5 students.

   (a) If more than 140 students are absent on the day the attendance count is taken for funding purposes, the school will lose some of its state funding in the subsequent year. Approximately what is the probability that High School A will lose some state funding?

   (b) The principals' association in the state suggests that instead of choosing one day at random, the state should choose 3 days at random. With the suggested plan, High School A would lose some of its state funding in the subsequent year if the mean number of students absent for the 3 days is greater than 140. Would High School A be more likely, less likely, or equally likely to lose funding using the suggested plan compared to the plan described in part (a)? Justify your choice.

   (c) A typical school week consists of the days Monday, Tuesday, Wednesday, Thursday, and Friday. The principal at High School A believes that the number of absences tends to be greater on Mondays and Fridays, and there is concern that the school will lose state funding if the attendance count occurs on a Monday or Friday. If one school day is chosen at random from each of 3 typical school weeks, what is the probability that none of the 3 days chosen is a Tuesday, Wednesday, or Thursday?

2. For an upcoming concert, each customer may purchase up to 3 child tickets and 3 adult tickets. Let C be the number of child tickets purchased by a single customer. The probability distribution of the number of child tickets purchased by a single customer is given in the table below.

c	0	1	2	3
$p(c)$	0.4	0.3	0.2	0.1

(a) Compute the mean and the standard deviation of C.

(b) Suppose the mean and the standard deviation for the number of adult tickets purchased by a single customer are 2 and 1.2, respectively. Assume that the numbers of child tickets and adult tickets purchased are independent random variables. Compute the mean and the standard deviation of the total number of adult and child tickets purchased by a single customer.

(c) Suppose each child ticket costs \$15 and each adult ticket costs \$25. Compute the mean and the standard deviation of the total amount spent per purchase.

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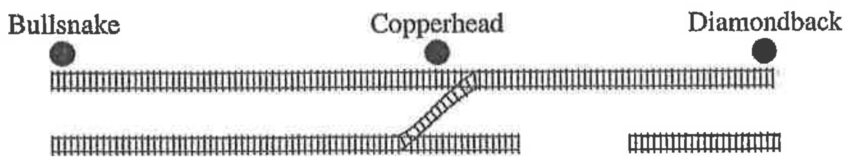
5. Flooding has washed out one of the tracks of the Snake Gulch Railroad. The railroad has two parallel tracks from Bullsnake to Copperhead, but only one usable track from Copperhead to Diamondback, as shown in the figure below. Having only one usable track disrupts the usual schedule. Until it is repaired, the washed-out track will remain unusable. If the train leaving Bullsnake arrives at Copperhead first, it has to wait until the train leaving Diamondback arrives at Copperhead.



Every day at noon a train leaves Bullsnake heading for Diamondback and another leaves Diamondback heading for Bullsnake.

Assume that the length of time, X, it takes the train leaving Bullsnake to get to Copperhead is normally distributed with a mean of 170 minutes and a standard deviation of 20 minutes.

Assume that the length of time, Y, it takes the train leaving Diamondback to get to Copperhead is normally distributed with a mean of 200 minutes and a standard deviation of 10 minutes.

These two travel times are independent.

(a) What is the distribution of $Y - X$?

(b) Over the long run, what proportion of the days will the train from Bullsnake have to wait at Copperhead for the train from Diamondback to arrive?

(c) How long should the Snake Gulch Railroad delay the departure of the train from Bullsnake so that the probability that it has to wait is only 0.01 ?

AP Stats

2002, Question 3

Suppose the times it takes Jack to run the 100m are normally distributed with a mean of 12.3 seconds and a standard deviation of 0.26 seconds.

What is the probability that he will run his 100m in less than 12 seconds?

Now suppose he is part of a 400m relay team. The other members are Tim, Carl, and Ben. Their means and standard deviations are listed in the table. Assume the individual times are all normally distributed.

Person	Mean	Standard Dev
Tim	11.1	0.2
Carl	10.9	0.22
Ben	11.7	0.31

Find the distribution (type, mean, standard deviation) of the **total** time it takes the team to complete their race. (They each run one leg of the 400m). Assume that their times are independent of each other.

What is the probability that the team will run their race in less than 45 seconds?

Suppose the team runs this race 7 times. Assume that each run is independent of the others. What is the probability that they will be under 45 seconds at least twice?

What is the probability that they will have to run the race more than 5 times before they get a time under 45 seconds?