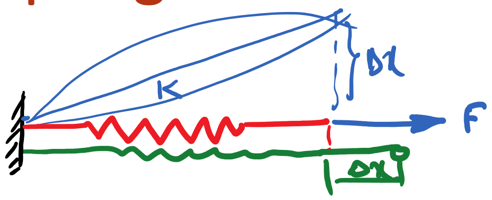
Springs

①



$F \propto \Delta x$

$F = K\Delta x$

$K = 1000\ kN/m$

$F = 5000\ N$

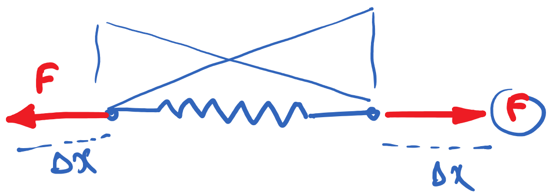
$5000 = 1000\times10^3 \times \Delta x$

$\therefore \Delta x = \dfrac{5000}{1000\times10^3}$

$\therefore \Delta x = 0.005\ m$

FBD:

②



$F = K\Delta x$

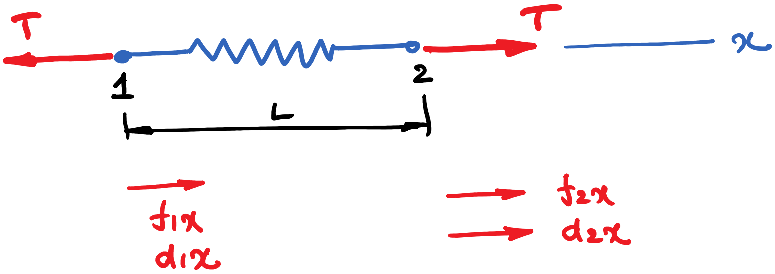
$\Delta x = \left(\dfrac{F}{K}\right)$

$-\Delta x$

Total deflection $= \Delta x - (-\Delta x)$

$\boxed{= 2\Delta x}$

Springs



$$u = a_1 + a_2 x$$

at $x = 0$; $\quad d_{1x} = a_1 + 0$

$$\therefore a_1 = d_{1x}$$

at $x = L$; $\quad d_{2x} = a_1 + a_2 L$

$$\therefore d_{2x} = d_{1x} + a_2 L$$

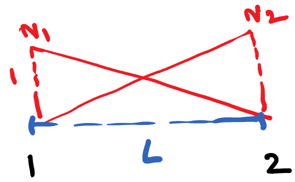
$$\therefore a_2 = \frac{d_{2x} - d_{1x}}{L}$$

$$u = a_1 + a_2 x$$

$$= d_{1x} + \frac{d_{2x} - d_{1x}}{L} x$$

$$\therefore u = \left[1 - \frac{x}{L}\right] d_{1x} + \left[\frac{x}{L}\right] d_{2x}$$

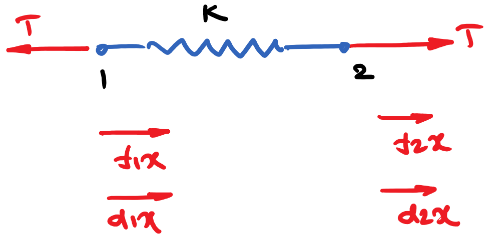
$$\therefore u = \begin{bmatrix} \left(1 - \frac{x}{L}\right) & \frac{x}{L} \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \end{Bmatrix}$$



$$\therefore u = \begin{bmatrix} N_1 & N_2 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \end{Bmatrix}$$

where $\left.\begin{aligned} N_1 &= 1 - \frac{x}{L} \\ N_2 &= \frac{x}{L} \end{aligned}\right\}$ shape functions

Springs



$$T = K(d_{2x} - d_{1x})$$

$$f_{1x} = -T$$

$$\therefore f_{1x} = -K(d_{2x} - d_{1x})$$

$$\therefore f_{1x} = K d_{1x} - K d_{2x} \rightarrow (1)$$

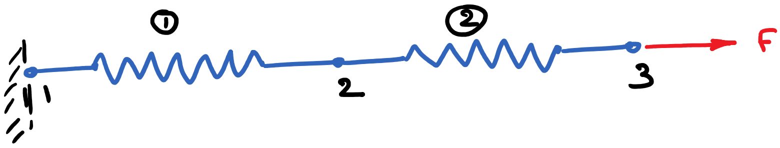
$$f_{2x} = T$$

$$\therefore f_{2x} = K(d_{2x} - d_{1x})$$

$$= -K d_{1x} + K d_{2x} \rightarrow (2)$$

$$\begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = \begin{bmatrix} K & -K \\ -K & K \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \end{Bmatrix}$$

Springs

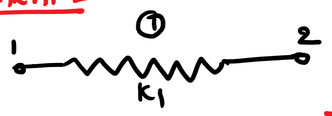


Node1 $d_{1x}^1 = d_{1x}$

Node2 $d_{2x}^1 = d_{2x}^2 = d_{2x}$

Node3 $d_{3x}^2 = d_{3x}$

Element 1

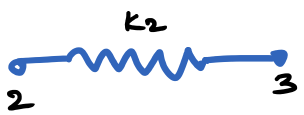


$\overrightarrow{f_{1x}^1}$ d_{1x}^1 $\qquad$ $\overrightarrow{f_{2x}^1}$ d_{2x}^1

$$\begin{Bmatrix} f_{1x}^1 \\ f_{2x}^1 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \end{Bmatrix}$$

$$\begin{Bmatrix} f_{1x}^1 \\ f_{2x}^1 \\ f_{3x}^1 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \end{Bmatrix}$$

Element 2



$\overrightarrow{f_{2x}^2}$ d_{2x} $\qquad$ $\overrightarrow{f_{3x}^2}$ d_{3x}

$$\begin{Bmatrix} f_{2x}^2 \\ f_{3x}^2 \end{Bmatrix} = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} d_{2x} \\ d_{3x} \end{Bmatrix}$$

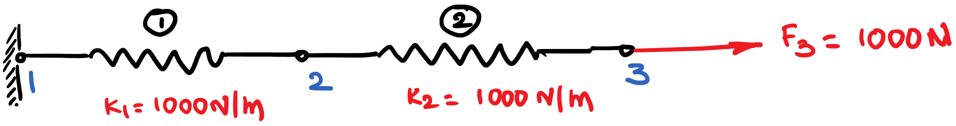
$$\begin{Bmatrix} f_{1x}^2 \\ f_{2x}^2 \\ f_{3x}^2 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \end{Bmatrix}$$

overall force displacement equation:

$$\underbrace{\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \end{Bmatrix}}_{\text{overall Force vector}} = \begin{Bmatrix} f_{1x}^1 \\ f_{2x}^1 \\ f_{3x}^1 \end{Bmatrix} + \begin{Bmatrix} f_{1x}^2 \\ f_{2x}^2 \\ f_{3x}^2 \end{Bmatrix} = \underbrace{\begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix}}_{\text{overall Stiffness matrix}} \underbrace{\begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \end{Bmatrix}}_{\text{overall displacement vector}}$$

Springs

Example 1



Calculate the unknown forces and displacements at nodes

Boundary conditions: $d_{1x} = 0$; $F_{2x} = 0$; $F_{3x} = 1000\,N$

[$d_{2x} = ?$ $F_{1x} = ?$ $d_{3x} = ?$]

Element 1

$$\begin{Bmatrix} f^1_{1x} \\ f^1_{2x} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \end{Bmatrix}; \quad \begin{Bmatrix} f^1_{1x} \\ f^1_{2x} \\ f^1_{3x} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 & 0 \\ -1000 & 1000 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \end{Bmatrix}$$

Element 2

$$\begin{Bmatrix} f^2_{2x} \\ f^2_{3x} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{Bmatrix} d_{2x} \\ d_{3x} \end{Bmatrix}; \quad \begin{Bmatrix} f^2_{1x} \\ f^2_{2x} \\ f^2_{3x} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1000 & -1000 \\ 0 & -1000 & 1000 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \end{Bmatrix}$$

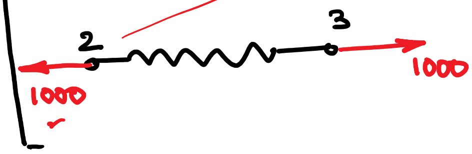
$$\therefore \begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \end{Bmatrix} = \begin{Bmatrix} f^1_{1x} \\ f^1_{2x} \\ f^1_{3x} \end{Bmatrix} + \begin{Bmatrix} f^2_{1x} \\ f^2_{2x} \\ f^2_{3x} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 & 0 \\ -1000 & 2000 & -1000 \\ 0 & -1000 & 1000 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \end{Bmatrix}$$

Substitute the boundary conditions:

$$\begin{bmatrix} 1000 & -1000 & 0 \\ -1000 & 2000 & -1000 \\ 0 & -1000 & 1000 \end{bmatrix} \begin{Bmatrix} 0 \\ d_{2x} \\ d_{3x} \end{Bmatrix} = \begin{Bmatrix} F_{1x} \\ 0 \\ 1000 \end{Bmatrix}$$

Element 1

Element 2



$$\begin{bmatrix} 2000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{Bmatrix} d_{2x} \\ d_{3x} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1000 \end{Bmatrix}$$

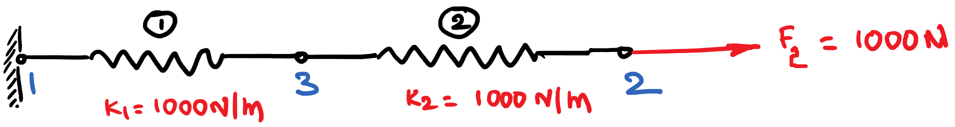
$$\therefore \begin{Bmatrix} d_{2x} \\ d_{3x} \end{Bmatrix} = \frac{1}{(2000 \times 1000 - 1000 \times 1000)} \begin{bmatrix} 1000 & 1000 \\ 1000 & 2000 \end{bmatrix} \times \begin{Bmatrix} 0 \\ 1000 \end{Bmatrix}$$

$$\therefore \begin{Bmatrix} d_{2x} \\ d_{3x} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix} m$$

$$\therefore \begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 & 0 \\ -1000 & 2000 & -1000 \\ 0 & -1000 & 1000 \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \\ 2 \end{Bmatrix}; \quad \therefore \begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \end{Bmatrix} = \begin{Bmatrix} -1000 \\ 0 \\ 1000 \end{Bmatrix}$$

Springs

Example 2



calculate the unknown forces and displacements at nodes

Boundary conditions: $d_{1x} = 0$; $F_{3x} = 0$; $F_{2x} = 1000N$

$[d_{2x} = ? \quad F_{1x} = ? \quad d_{3x} = ?]$

$$\begin{Bmatrix} f_{1x}^{(1)} \\ f_{3x}^{(1)} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{3x} \end{Bmatrix}$$

$$\begin{Bmatrix} f_{1x}^{(1)} \\ f_{2x}^{(1)} \\ f_{3x}^{(1)} \end{Bmatrix} = \begin{bmatrix} 1000 & 0 & -1000 \\ 0 & 0 & 0 \\ -1000 & 0 & 1000 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \end{Bmatrix}$$

$$\begin{Bmatrix} f_{3x}^{(2)} \\ f_{2x}^{(2)} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{Bmatrix} d_{3x} \\ d_{2x} \end{Bmatrix}$$

$$\begin{Bmatrix} f_{2x}^{(2)} \\ f_{3x}^{(2)} \end{Bmatrix} = \begin{bmatrix} -1000 & 1000 \\ 1000 & -1000 \end{bmatrix} \begin{Bmatrix} d_{3x} \\ d_{2x} \end{Bmatrix}$$

$$\begin{Bmatrix} f_{2x}^{(2)} \\ f_{3x}^{(2)} \end{Bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{Bmatrix} d_{2x} \\ d_{3x} \end{Bmatrix}$$

$$\begin{Bmatrix} f_{1x}^{(2)} \\ f_{2x}^{(2)} \\ f_{3x}^{(2)} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1000 & -1000 \\ 0 & -1000 & 1000 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \end{Bmatrix}$$

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \end{Bmatrix} = \begin{bmatrix} 1000 & 0 & -1000 \\ 0 & 1000 & -1000 \\ -1000 & -1000 & 2000 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \end{Bmatrix}$$

$$\begin{bmatrix} 1000 & -1000 \\ -1000 & 2000 \end{bmatrix} \begin{Bmatrix} d_{2x} \\ d_{3x} \end{Bmatrix} = \begin{Bmatrix} 1000 \\ 0 \end{Bmatrix}$$

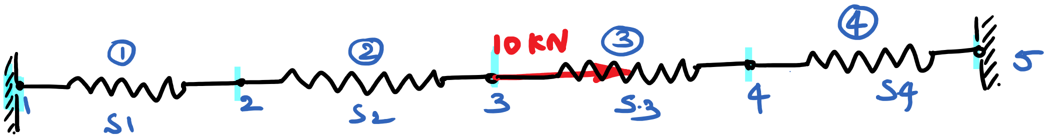
$$\therefore \begin{Bmatrix} d_{2x} \\ d_{3x} \end{Bmatrix} = \begin{Bmatrix} 2 \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \end{Bmatrix} = \begin{Bmatrix} -1000 \\ 1000 \\ 0 \end{Bmatrix}$$

Springs

Springs

Example 3



$S_1 = S_2 = S_3 = S_4 = 20\ \text{kN/m}$

Calculate: F_{1x}, F_{5x}, d_{2x}, d_{3x}, and d_{4x}

Element 1

$$\begin{Bmatrix} f_{1x}^1 \\ f_{2x}^1 \end{Bmatrix} = \begin{bmatrix} 20 & -20 \\ -20 & 20 \end{bmatrix}^{(1)} \begin{Bmatrix} d_{1x} \\ d_{2x} \end{Bmatrix};$$

$$\begin{Bmatrix} f_{1x}^1 \\ f_{2x}^1 \\ f_{3x}^1 \\ f_{4x}^1 \\ f_{5x}^1 \end{Bmatrix} = \begin{bmatrix} 20 & -20 & 0 & 0 & 0 \\ -20 & 20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \\ d_{4x} \\ d_{5x} \end{Bmatrix}$$

Elemental Stiffness matrix $[K_1]$

Element 2

$$\begin{Bmatrix} f_{2x}^2 \\ f_{3x}^2 \end{Bmatrix} = \begin{bmatrix} 20 & -20 \\ -20 & 20 \end{bmatrix} \begin{Bmatrix} d_{2x} \\ d_{3x} \end{Bmatrix};$$

$$\begin{Bmatrix} f_{1x}^2 \\ f_{2x}^2 \\ f_{3x}^2 \\ f_{4x}^2 \\ f_{5x}^2 \end{Bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 20 & -20 & 0 & 0 \\ 0 & -20 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{[K_2]} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \\ d_{4x} \\ d_{5x} \end{Bmatrix}$$

Working is similar lines with elements ③ & ④:

$$K_3 = \begin{array}{c|l} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & -20 & 0 \\ 0 & 0 & -20 & 20 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} d_{1x} \\ d_{2x} \\ d_{3x} \\ d_{4x} \\ d_{5x} \end{matrix} \end{array};
\qquad
K_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & -20 \\ 0 & 0 & 0 & -20 & 20 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \\ d_{4x} \\ d_{5x} \end{Bmatrix}$$

$$F(x) = [K]\{d\} = \underbrace{[K_1 + K_2 + K_3 + K_4]}_{[K]}\{d\}$$

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \\ F_{5x} \end{Bmatrix} = \begin{bmatrix} 20 & -20 & 0 & 0 & 0 \\ -20 & 40 & -20 & 0 & 0 \\ 0 & -20 & 40 & -20 & 0 \\ 0 & 0 & -20 & 40 & -20 \\ 0 & 0 & 0 & -20 & 20 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \\ d_{4x} \\ d_{5x} \end{Bmatrix}$$

Boundary conditions:

$d_{1x} = 0$; $d_{5x} = 0$; $F_{2x} = 0$; $F_{3x} = 10000\text{N}$; $F_{4x} = 0$

$$\begin{bmatrix} 20 & -20 & 0 & 0 & 0 \\ -20 & 40 & -20 & 0 & 0 \\ 0 & -20 & 40 & -20 & 0 \\ 0 & 0 & -20 & 40 & -20 \\ 0 & 0 & 0 & -20 & 20 \end{bmatrix} \begin{Bmatrix} 0 \\ d_{2x} \\ d_{3x} \\ d_{4x} \\ 0 \end{Bmatrix} = \begin{Bmatrix} F_{1x} \\ 0 \\ 10000 \\ 0 \\ F_{5x} \end{Bmatrix}$$

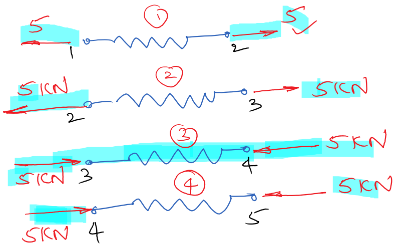
$$\underbrace{\begin{bmatrix} 40 & -20 & 0 \\ -20 & 40 & -20 \\ 0 & -20 & 40 \end{bmatrix}}_{[KI]} \underbrace{\begin{Bmatrix} d_{2x} \\ d_{3x} \\ d_{4x} \end{Bmatrix}}_{\{DI\}} = \underbrace{\begin{Bmatrix} 0 \\ 10000 \\ 0 \end{Bmatrix}}_{\{FI\}}$$

$$\{F\} = [K]\{d\}$$

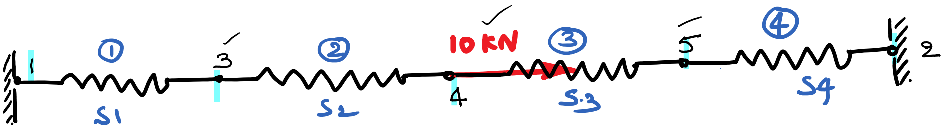
$$\{d\} = \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \\ d_{4x} \\ d_{5x} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0.25 \\ 0.50 \\ 0.25 \\ 0 \end{Bmatrix}$$

Springs

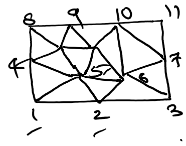
$$\therefore \begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \\ F_{5x} \end{Bmatrix} = \begin{Bmatrix} -5 \\ 0 \\ 10 \\ 0 \\ -5 \end{Bmatrix} \text{KN}$$



Springs



$S_1 = S_2 = S_3 = S_4 = 20$ kN/m



$$K_1 = \begin{bmatrix} 20 & -20 \\ -20 & 20 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{3x} \end{Bmatrix}$$

$$K_2 = \begin{bmatrix} 20 & -20 \\ -20 & 20 \end{bmatrix} \begin{Bmatrix} d_{3x} \\ d_{4x} \end{Bmatrix}$$

$$K_3 = \begin{bmatrix} 20 & -20 \\ -20 & 20 \end{bmatrix} \begin{Bmatrix} d_{4x} \\ d_{5x} \end{Bmatrix}$$

$$K_4 = \begin{bmatrix} 20 & -20 \\ -20 & 20 \end{bmatrix} \begin{Bmatrix} d_{5x} \\ d_{2x} \end{Bmatrix}$$

$$K_1 = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{matrix} d_{1x} \\ d_{2x} \\ d_{3x} \\ d_{4x} \\ d_{5x} \end{matrix}$$

$$K_2 = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{matrix} d_{1x} \\ d_{2x} \\ d_{3x} \\ d_{4x} \\ d_{5x} \end{matrix}$$

$$K_3 = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{matrix} d_{1x} \\ d_{2x} \\ d_{3x} \\ d_{4x} \\ d_{5x} \end{matrix}$$

$$K_4 = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & \cdot & \cdot & 0 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \\ d_{4x} \\ d_{5x} \end{Bmatrix}$$

$$\begin{Bmatrix} f_{5x}^{4} \\ f_{2x}^{4} \end{Bmatrix} = \begin{bmatrix} 20 & -20 \\ -20 & 20 \end{bmatrix} \begin{Bmatrix} d_{5x} \\ d_{2x} \end{Bmatrix}$$

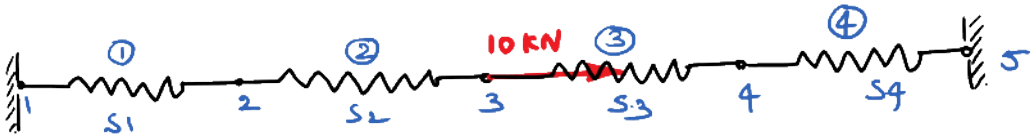
$$\begin{Bmatrix} f_{2x}^{4} \\ f_{5x}^{4} \end{Bmatrix} = \begin{bmatrix} -20 & 20 \\ 20 & -20 \end{bmatrix} \begin{Bmatrix} d_{5x} \\ d_{2x} \end{Bmatrix}$$

$$\therefore \begin{Bmatrix} f_{2x}^{4} \\ f_{5x}^{4} \end{Bmatrix} = \begin{bmatrix} 20 & -20 \\ -20 & 20 \end{bmatrix} \begin{Bmatrix} d_{2x} \\ d_{5x} \end{Bmatrix}$$

$$\begin{bmatrix} 20 & 0 & -20 & 0 & 0 \\ 0 & 20 & 0 & 0 & -20 \\ -20 & 0 & 40 & -20 & 0 \\ 0 & 0 & -20 & 40 & -20 \\ 0 & -20 & 0 & -20 & 40 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ d_{3x} \\ d_{4x} \\ d_{5x} \end{Bmatrix} = \begin{Bmatrix} F_{1x} \\ F_{2x} \\ 0 \\ 10 \\ 0 \end{Bmatrix}$$

$K_I \quad D_I \quad F_I$

Springs

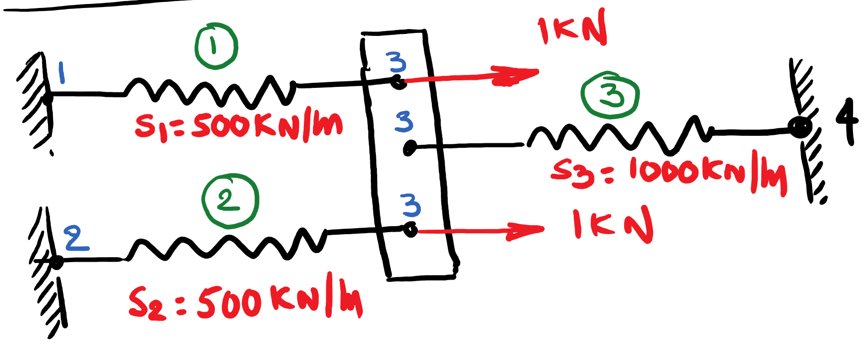


$$
\begin{array}{cc}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\
\begin{bmatrix}
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot
\end{bmatrix} &
\begin{array}{c} d_{1x} \\ d_{2x} \\ d_{3x} \\ d_{4x} \\ d_{5x} \end{array}
\end{array}
$$

$$
\begin{array}{cc}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\
\begin{bmatrix}
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot
\end{bmatrix} &
\begin{array}{c} d_{1x} \\ d_{2x} \\ d_{3x} \\ d_{4x} \\ d_{5x} \end{array}
\end{array}
$$

Springs

Example 3



Calculate : F_{1x}, F_{2x}, F_{4x}, and d_{3x}

Boundary Conditions: $d_{1x} = d_{2x} = d_{4x} = 0$

$F_{3x} = 2$ KN

Element 1

$$\begin{Bmatrix} f'_{1x} \\ f'_{3x} \end{Bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{3x} \end{Bmatrix}; \quad \begin{Bmatrix} f'_{1x} \\ f'_{2x} \\ f'_{3x} \\ f'_{4x} \end{Bmatrix} = \begin{bmatrix} 500 & 0 & -500 & 0 \\ 0 & 0 & 0 & 0 \\ -500 & 0 & 500 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \\ d_{4x} \end{Bmatrix}$$

$$K_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 500 & -500 & 0 \\ 0 & -500 & 500 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \quad K_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1000 & -1000 \\ 0 & 0 & -1000 & 1000 \end{bmatrix}$$

$\therefore K = K_1 + K_2 + K_3$

$$= \begin{bmatrix} 500 & 0 & -500 & 0 \\ 0 & 500 & -500 & 0 \\ -500 & -500 & 2000 & -1000 \\ 0 & 0 & -1000 & 1000 \end{bmatrix}$$

$$\{F\} = [K]\{X\}$$

$$\therefore \begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{bmatrix} 500 & 0 & -500 & 0 \\ 0 & 500 & -500 & 0 \\ -500 & -500 & 2000 & -1000 \\ 0 & 0 & -1000 & 1000 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \\ d_{4x} \end{Bmatrix} \begin{matrix} - 0 \\ - 0 \\ \\ - 0 \end{matrix}$$

$$\therefore F_{3x} = -500 \times \cancel{d_{1x}}^{0} - 500 \times \cancel{d_{2x}}^{0} + 2000 \times d_{3x} - 1000 \times \cancel{d_{4x}}^{0}$$

$$\therefore 2 = 2000\, d_{3x}; \qquad d_{3x} = 0.001 \text{ m}$$

Springs

$$\therefore \begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = \begin{bmatrix} K \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \\ d_{4x} \end{Bmatrix} \qquad X = \begin{Bmatrix} 0 \\ 0 \\ 0.001 \\ 0 \end{Bmatrix} x$$

$$F \qquad\qquad X$$

$$\therefore F = K * X$$

$$\therefore F = \begin{Bmatrix} -0.5 \\ -0.5 \\ 2.0 \\ -1.0 \end{Bmatrix} \text{KN}$$

Springs