Tuesdays &
Thursdays 12:00-
13:30 ET
GGBL 2147

Lecture 10

ME599-004: Data-Driven Methods for Control Systems

Winter 2024

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MECHANICAL
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Dynamical System and Control

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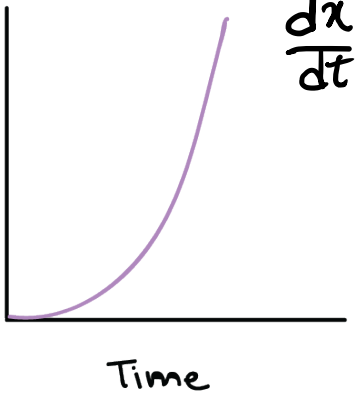
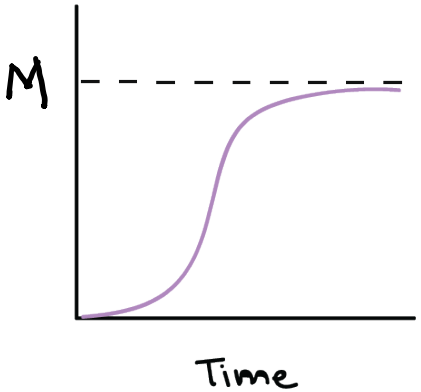
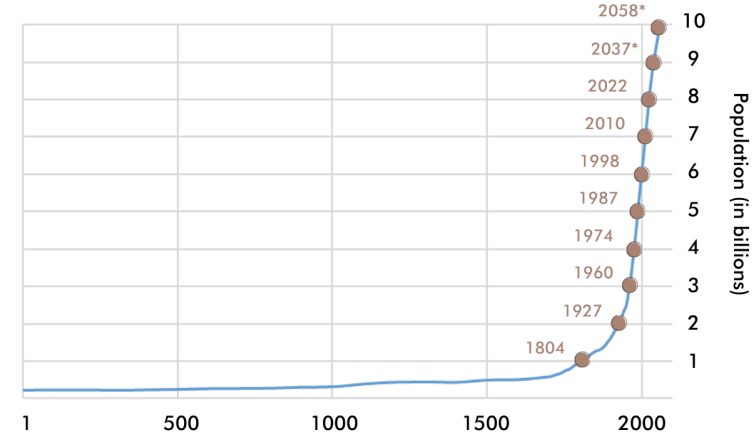
Exponential Growth



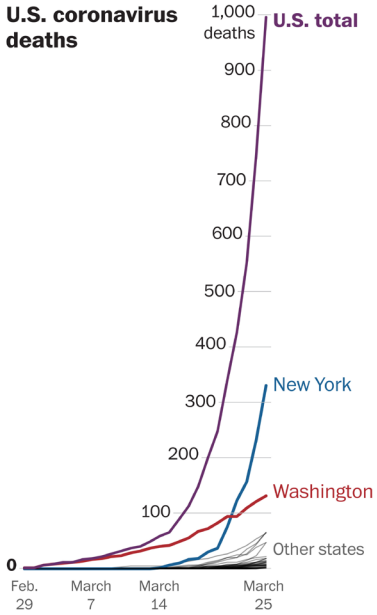
$$\frac{dx}{dt} = kx$$



$$\frac{dx}{dt} = \frac{k}{M}(M-x)x$$

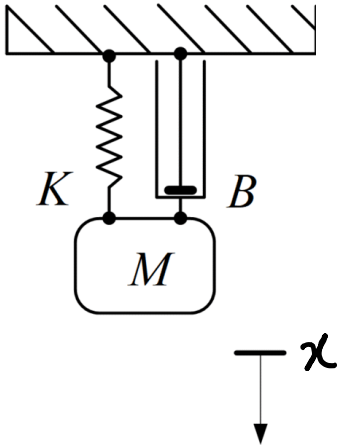


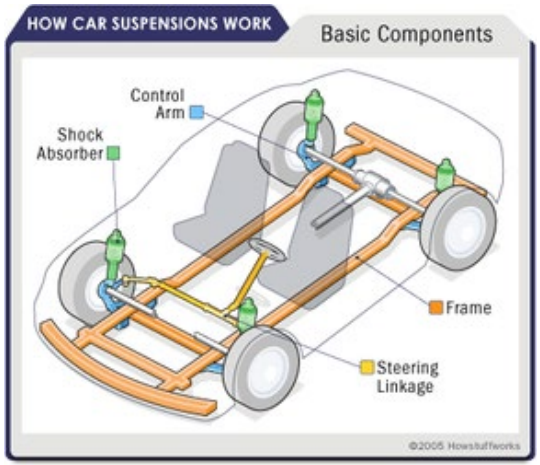
Source: UN Population Division (2022)

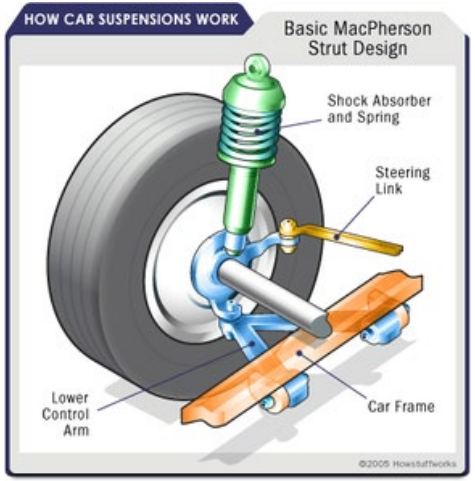


Source: Post reporting from state health department data

Mechanical Vibration



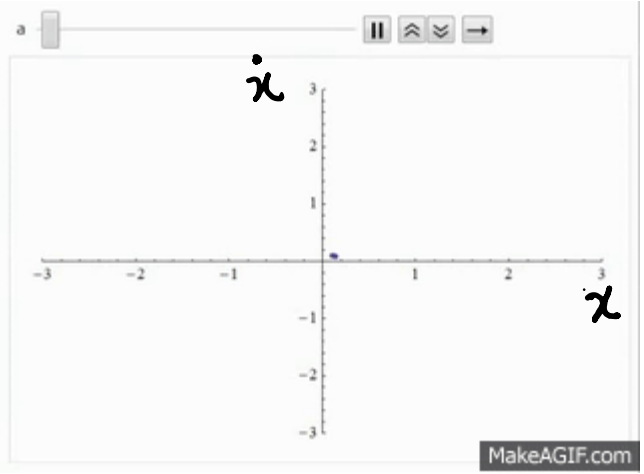




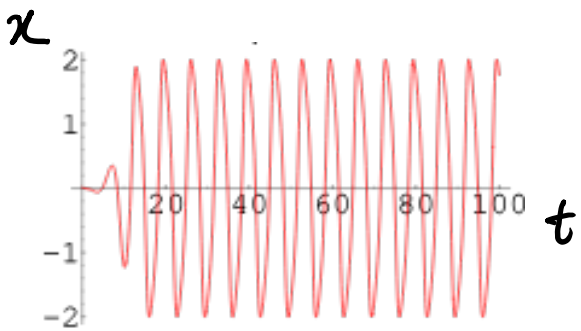
$$M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + kx = g(t)$$

Van der pol Oscillator

$$\frac{d^2x}{dt^2} - \mu(1-x^2)\frac{dx}{dt} + x = 0$$

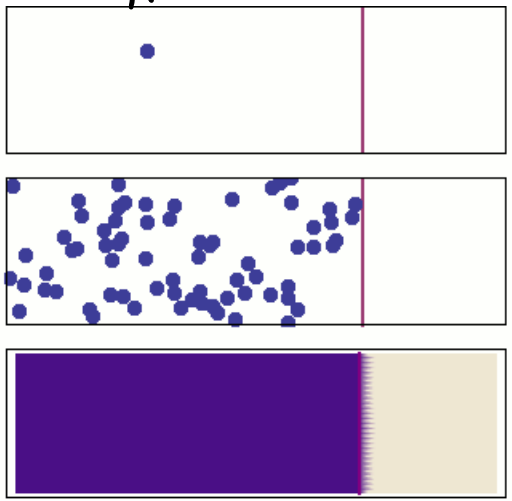


https://i.makeagif.com/media/7-04-2016/mitx1D.mp4



PDE's

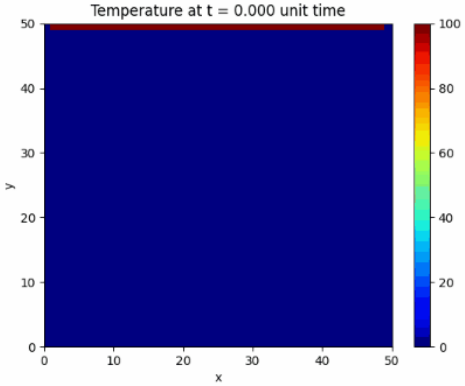
Diffusion



https://en.wikipedia.org/wiki/File:DiffusionMicroMacro.gif

$$\frac{\partial u}{\partial t} = -c\frac{\partial u}{\partial x}$$

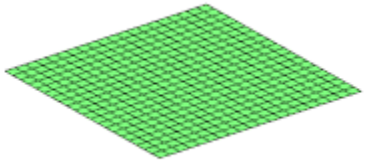
Heat Equation



SOURCE

$$\frac{1}{\alpha}\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

Wave Equation



https://en.wikipedia.org/wiki/File:2D_Wave_Function_resize.gif

https://upload.wikimedia.org/wikipedia/commons/1/1f/Wave_equation_1D_fixed_endpoints.gif

$$\frac{\partial^2 u}{\partial t} = c\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

System of DE's

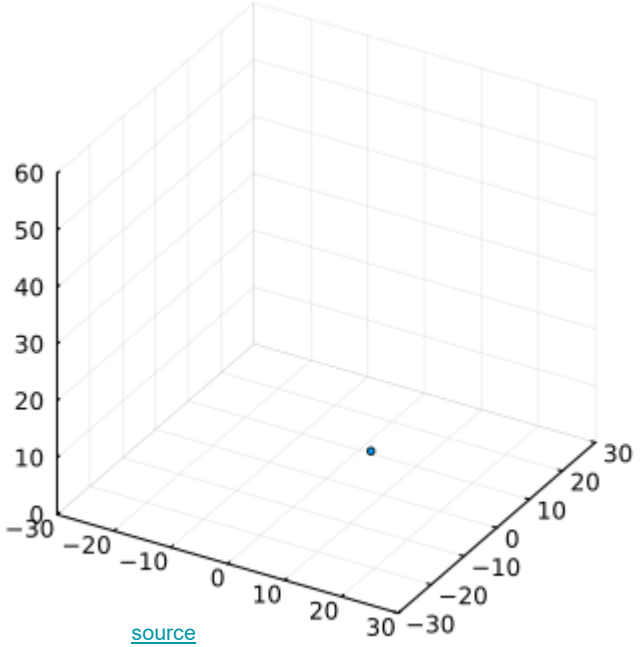
Lorenz Equations

$$\dot{x} = \sigma(y - x)$$

$$\dot{y} = x(\rho - z) - y$$

$$\dot{z} = xy - \beta z$$

SOURCE



source

PDE 2 ODE

MAKE THE WORLD WORK BETTER

Heat Equation: Conduction in a rod

$$u_t = \alpha^2 u_{xx}$$

$$u_t = -\alpha\omega^2 \hat{u}$$