Tuesdays & Thursdays 12:00-13:30 ET
GGBL 2147

Lecture 4

ME599-004: Data-Driven Methods for Control Systems

Winter 2024

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MECHANICAL ENGINEERING
UNIVERSITY OF MICHIGAN

Dimensional Analysis

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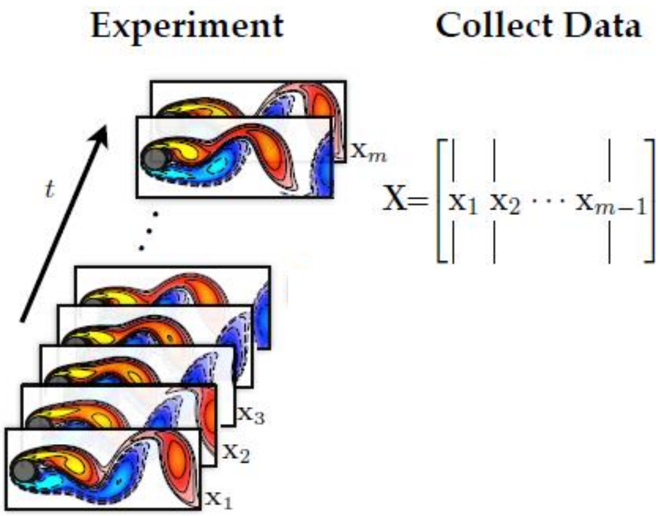
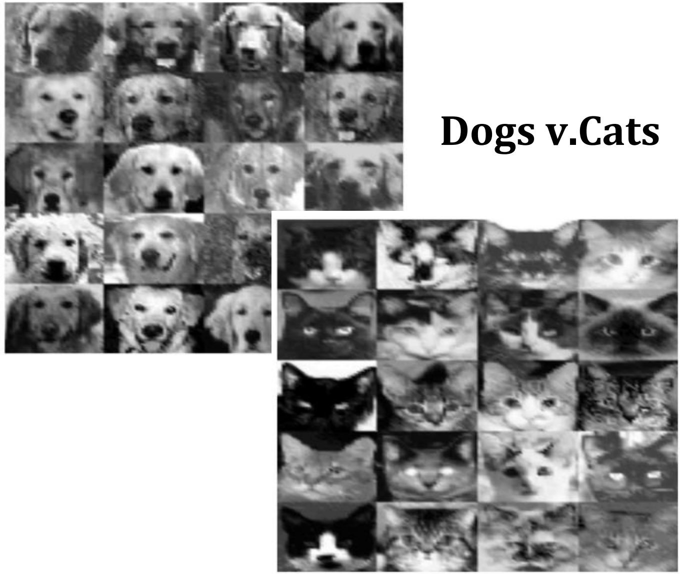
Motivation

High dimensionality is a challenge in processing data.

- Dynamic System Identification
- Image compression
- Classification





Dogs v.Cats

Dimensionality Reduction

- Dimensionality reduction is the process of **reducing the number of attributes** in a data set while **keeping as much of the variation in the original data** set as possible
- Linear methods e.g. Principal component analysis (PCA)
- Nonlinear methods e.g. Autoencoders

Random Variables

A random variable X models a random phenomena

- One in which many different outcomes are possible
- Some outcomes may be more likely than others

A random variable represents two things: all possible outcomes & their respective likelihoods

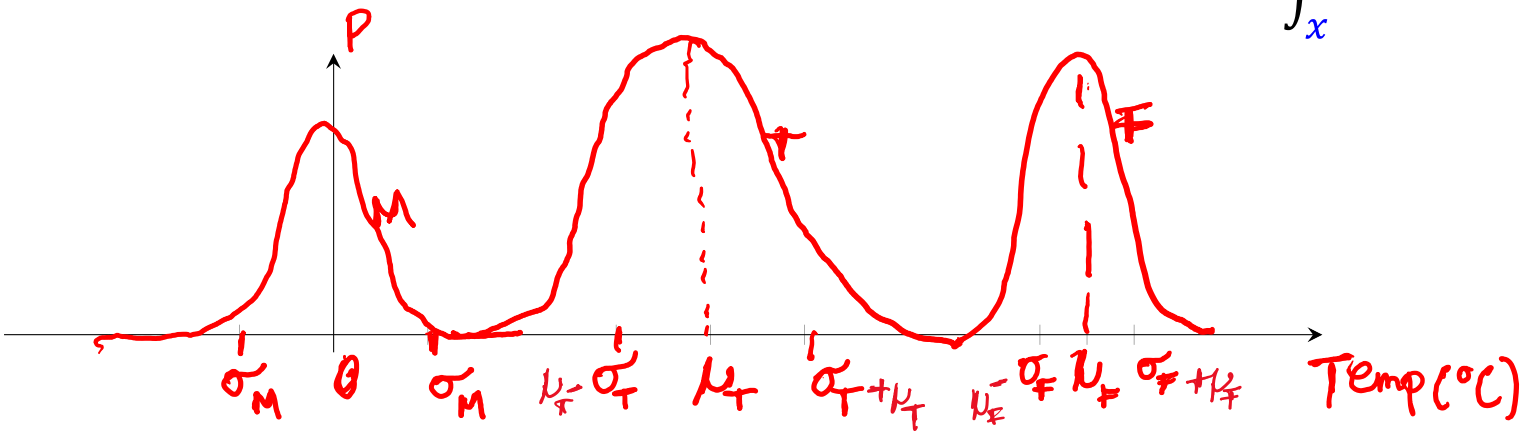
Could be continuous eg average temp. in a state
or discrete eg no. of pizza shops open in a state

Probabilities

- Random variables represented by uppercase e.g., X
- Values that it can take represented by lowercase e.g., x *(realization of X)*
- Multivariate random variable if $\boldsymbol{x}$ is a vector

- $p_X(x)$ $\approx$ How likely random variable X is to take a value around x

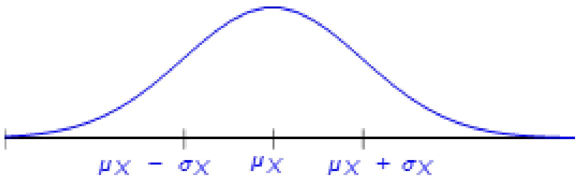
$$P\,(x < X \leq x') = \int_{x}^{x'} p_X(u)\,du$$



Gaussian (or Normal) random variable

- Gaussian (or Normal) random variable pdf if

$$p_x(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{(x-\mu)^2/\sigma^2}$$



Expectation

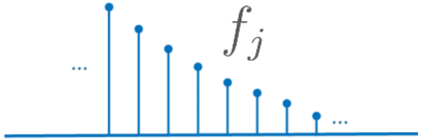
Variance

Discrete Random Variable

Eigenvector and Eigenvalue of Covariance Matrix

MAKE THE WORLD WORK BETTER

Recall Discrete Fourier Transforms



$$\omega_N = e^{-i\frac{2\pi}{N}}$$

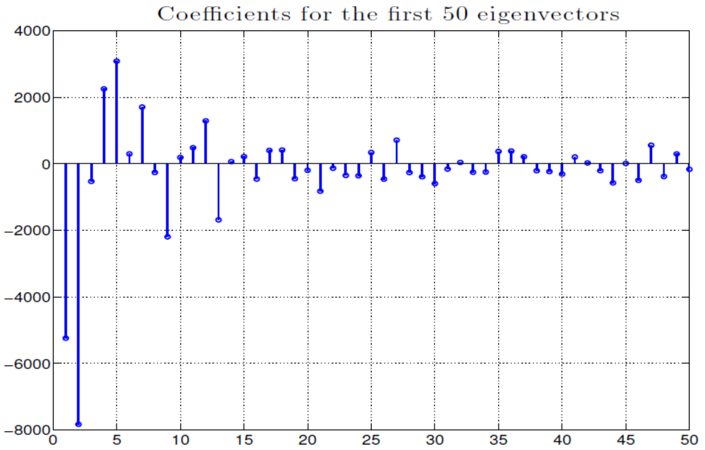
$$\hat{f}_k = \sum_{j=0}^{N-1} f_j e^{-ik\frac{2\pi}{N}j}$$

$$f_j = \frac{1}{N}\sum_{k=0}^{N-1} \hat{f}_k e^{ij\frac{2\pi}{N}k}$$

$$\begin{array}{cccccc}
 & j & & \begin{array}{ccccc} 0 & 1 & 2 & & N-1 \end{array} & \\
k & & & & & \\
\begin{array}{c} 0 \\ 1 \\ 2 \\ \vdots \\ N-1 \end{array} & \begin{bmatrix} \hat{f}_0 \\ \hat{f}_1 \\ \hat{f}_2 \\ \vdots \\ \hat{f}_{N-1} \end{bmatrix} & = & \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega_N^1 & \omega_N^2 & & \omega_N^{N-1} \\ 1 & \omega_N^2 & \omega_N^4 & \cdots & \omega_N^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega_N^{N-1} & \omega_N^{2(N-1)} & \cdots & \omega_N^{(N-1)^2} \end{bmatrix} & \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ \vdots \\ f_{N-1} \end{bmatrix} &
\end{array}$$

Illustration: Coefficients of a Projected Face

- PCA transform coefficients for given face with 10, 304 pixels



Slide courtesy of Alejandro Ribeiro of UPenn

Illustration: Reconstructing Face Image

- Reconstructed image for increasing number of PCA coefficients

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Illustration: Coefficients for same & different persons

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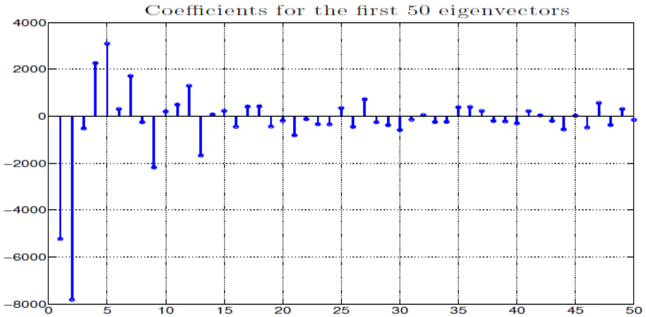
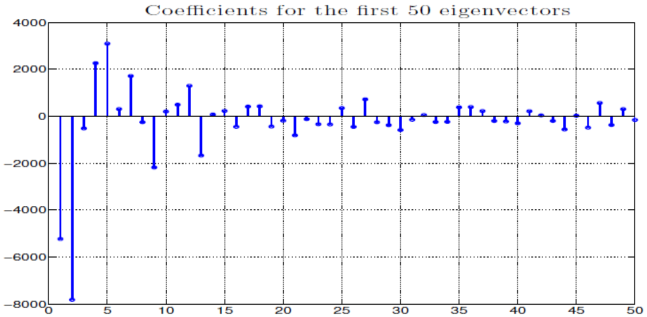
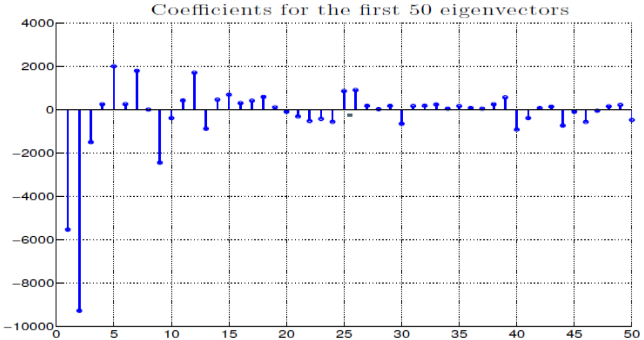
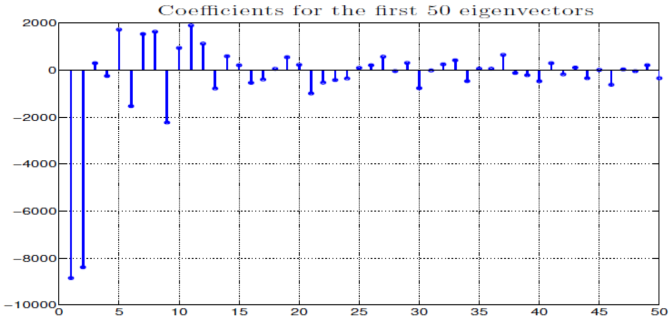
- PCA transform coefficients for the pictures









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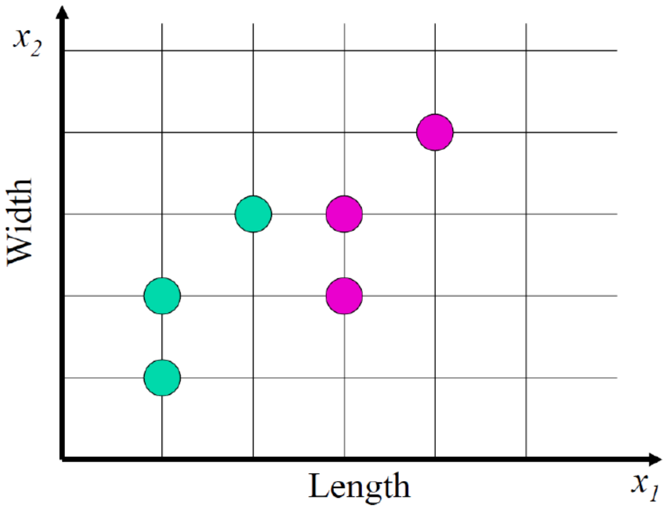
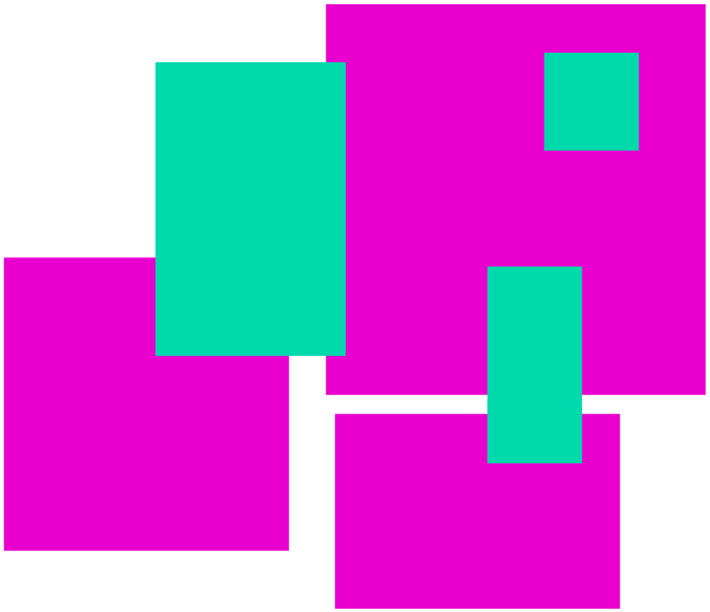
Dimensionality Reduction Example

- $x \in R^2$

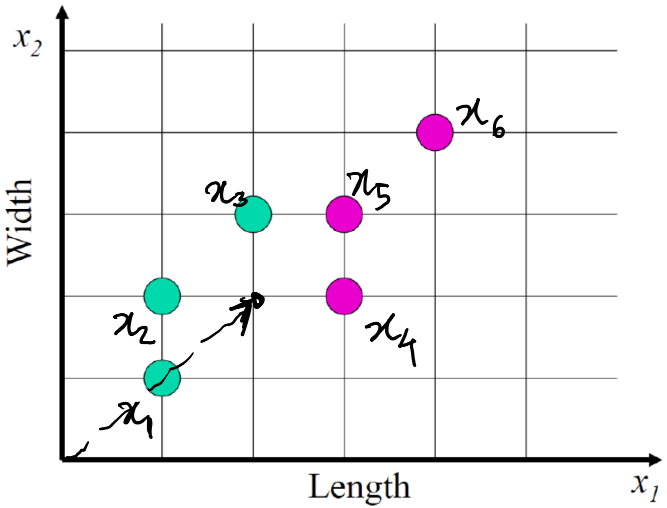




Dimensionality Reduction Example

- $x \in R^2$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



$x_1 = [1, 1]$

$x_2 = [1, 2]$

$x_3 = [2, 3]$

$x_4 = [3, 2]$

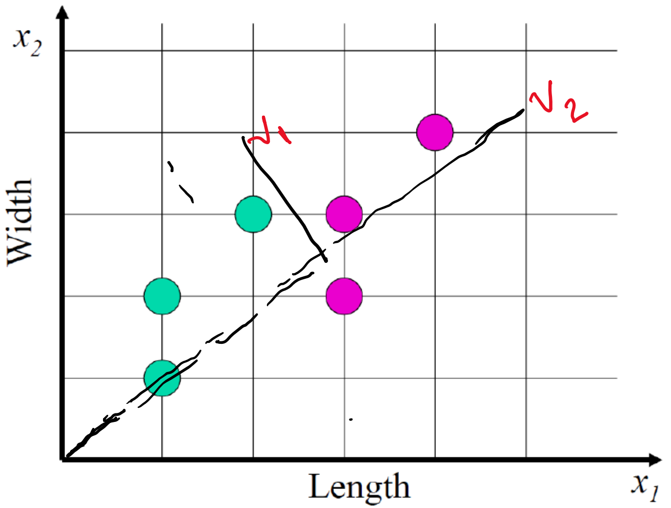
$x_5 = [3, 3]$

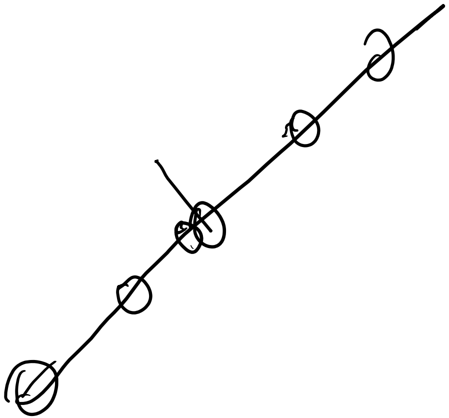
$x_6 = [4, 4]$

Dimensionality Reduction Example

- $x \in R^2$

$$y_k = v_2^T (x_k - \mu)$$





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