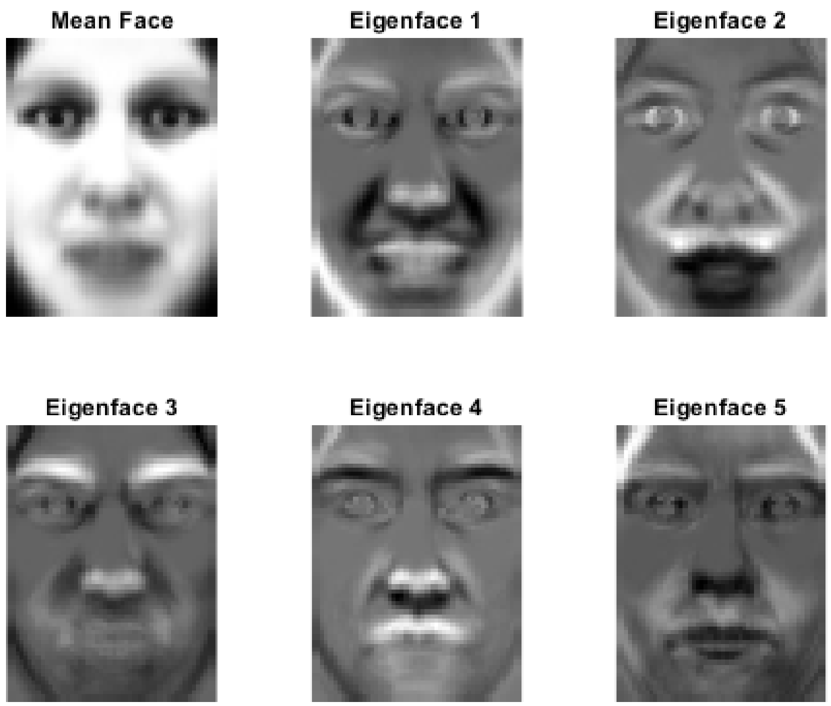
Contents

Part (a.1) Perform PCA for the mean faces and the first five eigen-faces (with pca function)

```
% First we try the pca function from Statistics and Machine Learning add-on
% We have m = 66x50 = 3300 pixel per picture
% We have n = 40x7x0.75 = 210 number of training pictures
% Correspondingly, the train_data is a 3300x210 double, matching m and n

% Perform PCA on the training data
% Note: feed each data as a row vector to the function
[coeff, score, latent, tsquared, explained, mu] = pca(train_data');


% OUTPUTS: (from documentation)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% coeff: A matrix of principal component coefficients (also known as loadings or eigenvectors),
% where each column represents a principal component, and rows correspond to the original variables.
% The columns are in order of decreasing component variance.

% score: The representation of 'X'(train_data) in the principal component space.
% Rows of 'score' correspond to observations, and columns to components.

% latent: A vector containing the eigenvalues of the covariance matrix of 'X',
% which represent the variance explained by the corresponding principal components.

% tsquared: Hotellingts T-squared statistic for each observation.

% explained: A vector containing the percentage of the total variance
% explained by each principal component.

% mu: A vector of the mean of each variable in 'X', used to center the data during the PCA process.
% i.e. the mean value of each data, AKA the mean_face
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Since all the data have been falltened (1xn) undergoing PCA we need to
% Reshape the mean_face and the first five eigenfaces back to 66x50 to plot
Mean_FaceImg = reshape(mu, [66, 50]);
EigenfacesImg = reshape(coeff(:,1:5), [66, 50, 5]);

% Plot the six figures
% Mean_face
figure;
subplot(2,3,1); % 2x3 layout (better than 1x6)
imshow(Mean_FaceImg); % Visualize the matrix
colormap gray; % In gray
title('Mean Face');
axis image; % fix ratio of images

% First five eigen_faces, similar process
for i = 1:5
```

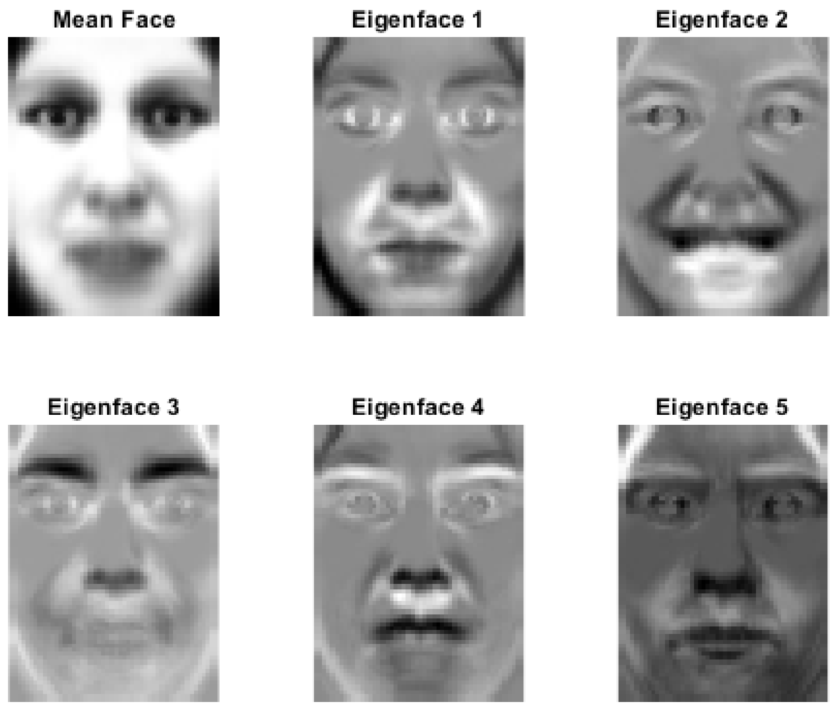
```
    subplot(2,3,i+1);
    imagesc(EigenfacesImg(:,:,i)); % imagesc since eigenfaces are stored in a 3D array
    colormap gray;
    title(['Eigenface ', num2str(i)]);
    axis image; % Ensure the aspect ratio is not distorted
    axis off;
end
```



Part (a.2) Perform PCA for the mean faces and the first five eigen-faces (without pca function)

```
% Here we manually calculate the eigenvectors and mean of each data

% Normalize the data by substracting the mean
X = train_data;
meanval = mean(X, 2); % Mean of each row (mean(X, 1) for column) [3300x1]
X = X - meanval; % Subtract mean [3300x210 - 3300x1]

% Carry out SVD
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% What are we doing here?
% 1. To do Dimensionality Reduction
% Why SVD?
% 1. SVD is numerically stable
% 2. SVD is the most efficient method to find the most representative bases
% for the data. Namely, it is used to find the pricipal components
% containing the most variance of the data with the fewest number of
% components.

[U, S, V] = svd(X);

% U contains the eigenvectors
% S contains the singular values
```

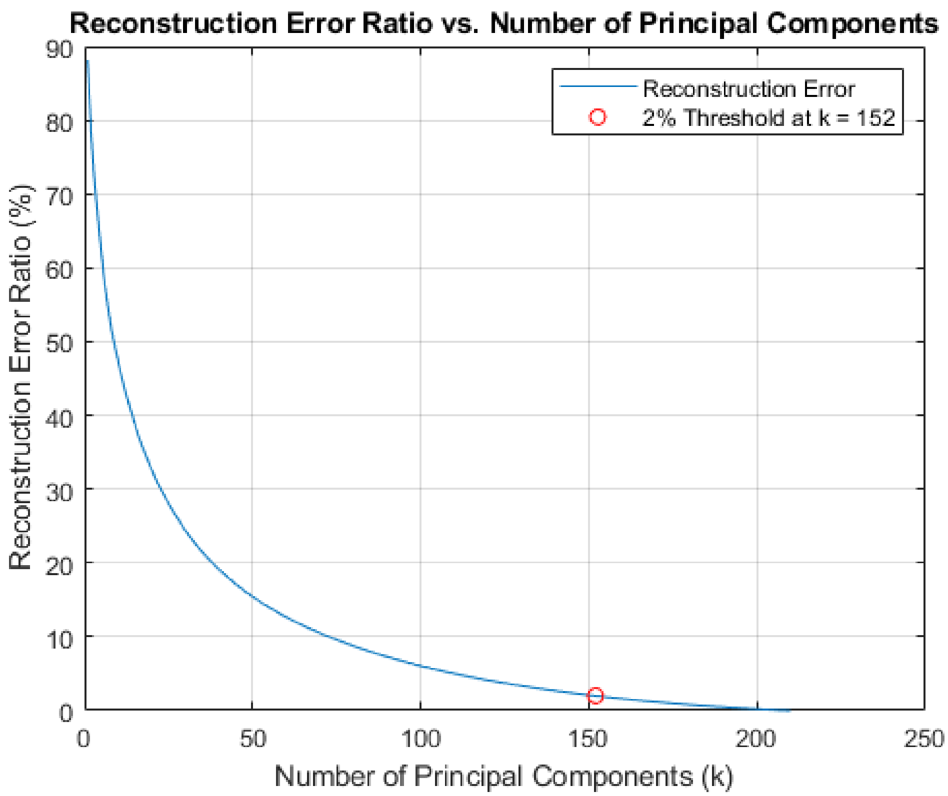
```
% V's columns are the principal components of the covariance matrix of X
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Reshape for visualization
Mean_FaceImg = reshape(meanval, 66, 50);
EigenfacesImg = reshape(U(:, 1:5), 66, 50, 5);

% Plot
figure;
subplot(2,3,1);
imshow(Mean_FaceImg);
colormap gray;
title('Mean Face');
axis image;

for i = 1:5
    subplot(2,3,i+1);
    imagesc(EigenfacesImg(:,:,i));
    colormap gray;
    title(['Eigenface ', num2str(i)]);
    axis image;
    axis off;
end

% Comments for manual pca and pca function from add-on:
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% The outputs of manual pca and those from pca function are different.
% The reason they are different is that we are getting bases with opposite
% directions i.e. getting eigenvectors with different signs.
% Thus we can draw the conclusion that we are getting equivalent results
% from manul pca and from the pca function dispite the different
% presentations of eigenfaces.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```



Part (b) Plot the reconstruction error against the number of principal components and determine how many principal components c are needed to achieve a reconstruction error of 2% or less.

```
% Part b.1 Find c for 2% error
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% We have X from Part (a), the centered data matrix
% And we have the U, S, V too
% Now we calculate the eigenvalues of X, we have the covariance matrix C as
% C = XX^T / (n - 1)
% Plug in X = USV^T
% We have C = US^2U^T / (n - 1), which is diagonalizable
% The eigenvalue matrix Lambda = S^2 / (n - 1)
X = test_data;
X = X - meanval;
eigenvalues = diag(S).^2 / (size(X, 2) - 1);

% Cumulative sum of eigenvalues for total variance
total_variance = sum(eigenvalues);

% Reconstruction error ratio rk for different k
numComponents = length(eigenvalues); % Number of principal components
rk = zeros(numComponents, 1); % Initialize reconstruction error ratio array

for k = 1:numComponents
    rk(k) = sum(eigenvalues(k+1:end)) / total_variance;
end

% Find k where rk is 2% or less
cMax = find(rk <= 0.02, 1, 'first');

% Plot rk vs. k
figure;
plot(1:numComponents, rk * 100); % Convert to percentage
```

```
xlabel('Number of Principal Components (k)');
ylabel('Reconstruction Error Ratio (%)');
title('Reconstruction Error Ratio vs. Number of Principal Components');
grid on;
hold on;
if ~isempty(cMax)
    plot(cMax, rk(cMax) * 100, 'ro');
    legend('Reconstruction Error', ['2% Threshold at k = ', num2str(cMax)]);
else
    legend('Reconstruction Error');
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Part b.2 Reconstruction of Images with c = 1, 5, 10, and cMax
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Number of components to use for reconstruction
components = [1, 5, 10, cMax];

% Select a random image from your dataset
randIndex = randi(size(X, 2));
originalImage = X(:, randIndex) + meanval; % Adding mean back to visualize

% Reshape and display the original image
figure;
subplot(1, 5, 1);
imagesc(reshape(originalImage, 66, 50)); % Original image
colormap gray;
title('Original Image');
axis image;
axis off;

for i = 1:4
    c = components(i);
    % Project the image onto the first 'c' principal components
    y = U(:, 1:c)' * X(:, randIndex);

    % Reconstruct the image from its reduced representation
    reconstructed = U(:, 1:c) * y;  % Correct: Use matrix multiplication '*'
    reconstructed = reconstructed + meanval;  % Adding the mean back

    % Ensure that 'reconstructed' is properly reshaped to match the original image size
    % Using '[]' as one of the arguments to 'reshape' lets MATLAB automatically calculate the correct dimension
    subplot(1, 5, i+1);
    imagesc(reshape(reconstructed, 66, []));  % Adjust dimensions as per your data
    colormap gray;  % Apply gray colormap for visual consistency
    title(['Reconstructed with ', num2str(c), ' PCs']);
    axis image;  % Ensure aspect ratio matches that of an image
    axis off;  % Hide axis for clarity
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```



Part (c)

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