Course: MECH 567: Robot Kinematics and Dynamics **Assigned Date:** 02/24/2023
Instructor: Robert Gregg, PhD **Due:** 11:59 PM on Sunday, 03/19/2023
We are using Gradescope.

Homework 4

Problem 1

Euler angles can be used to represent rotations via the product of exponentials formula. If we think of (α, β, γ) as joint angles of a robot manipulator, then we can find the singularities of an Euler angle parameterization by calculating the Jacobian of the "forward kinematics," where we are concerned only with the rotation portion of the forward kinematics map. Use this point of view to find singularities for the following class of Euler angles: i) ZYZ sequence, and ii) ZXY sequence.

Problem 2

Recall for a particle with kinetic energy $K = \frac{1}{2}m\dot{x}^2$, the **momentum** is defined as

$$p = m\dot{x} = \frac{dK}{d\dot{x}}$$

Therefore, for a mechanical system with generalized coordinates $q_1, ..., q_n$, we define the **generalized momentum** p_k as

$$p_k = \frac{\partial L}{\partial \dot{q}_k}$$

where L is the Lagrangian of the system. With $K = \frac{1}{2}\dot{q}^\top D(q)\,\dot{q}$ and $L = K - V$ prove that

$$\sum_{k=1}^{n} \dot{q}_k\, p_k = 2K$$

Problem 3

There is another formulation of the equations of motion of a mechanical system that is useful, the so-called **Hamiltonian** formulation. Define the Hamiltonian function H by

$$H = \sum_{k=1}^{n} \dot{q}_k\, p_k - L$$

(a) Show that $H = K + V$.

(b) Using the Euler-Lagrange equations, derive Hamilton's equations

$$\dot{q}_k = \frac{\partial H}{\partial p_k}$$
$$\dot{p}_k = -\frac{\partial H}{\partial q_k} + \tau_k$$

where τ_k is the input generalized force.

Problem 4

For two-link manipulator of Figure 1 compute Hamiltonian equations in vector form. Note that Hamilton's equations are a system of first order differential equations as opposed to a second order system given by Lagrange's equations.

As derived in SHV, the inertia matrix $D(q)$ is given as $D(q) = \begin{bmatrix} d_{11}(q) & d_{12}(q) \\ d_{12}(q) & d_{22} \end{bmatrix}$. The total potential energy is $V(q) = V_1(q) + V_2(q)$.

Note: You do not have to plug in the actual expressions for $d_{ij}(q)$, $V_1(q)$, or $V_2(q)$, but feel free to use Mathematica to see how it turns out.

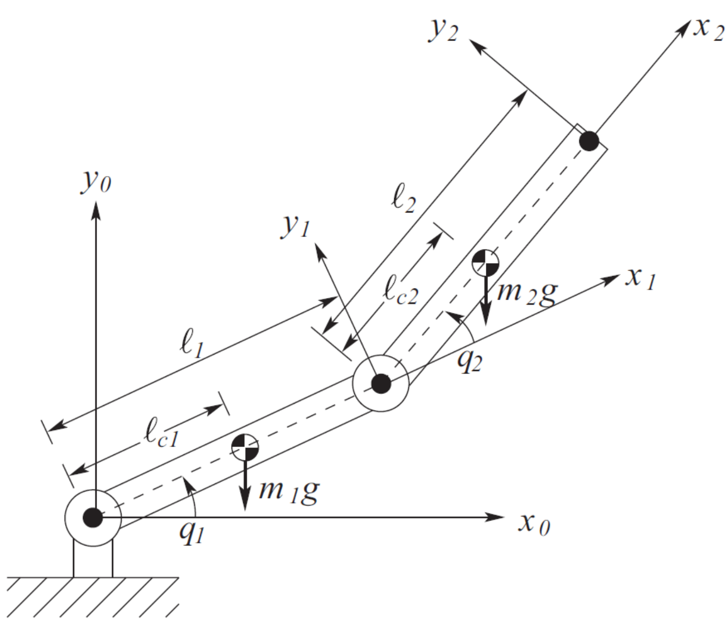


Figure 1: Two-link revolute joint arm. The rotational joint motion introduces dynamic coupling between the joints. (Figure 6.9 in SHV)

Problem 5

In this problem, use Mathematica with the screw theory package. Copy and paste your code in the homework.

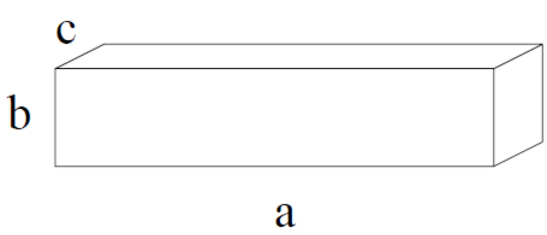


Figure 2: Block

Consider a uniform rectangular solid of mass m and dimensions $a \times b \times c$ as shown in Figure 2. The inertia tensor with respect to a coordinate frame located at the geometric center of the solid can be computed in terms of a, b, c and m .

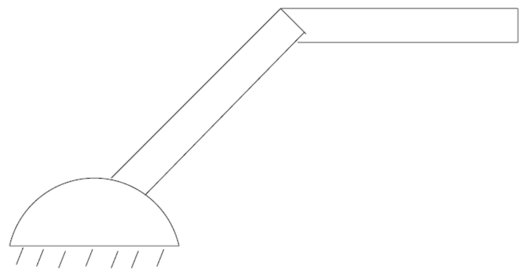
For each of the two-link planar robots shown on the next page, assume that the links are uniform rectangular solids as above. Let $a = 1$, $b = 0.2$, and $m = 2$ in each case. The value of c does not matter since the motion is planar. You may take $c = 0$ for simplicity. Assume that the gravity vector is in the direction of the negative $y-$axis. **Define the positive x-axis pointing towards right, positive y-axis pointing upwards, and positive z-axis pointing out of the page.** Robots (i), (iii), and (iv) are shown with angles to help understand where the links are. For the zero configuration of each of these robots assume that the links that are shown at 45 degrees are only along the x-direction. This means that the two links for (i) and (iii) are both along x *only*.

(a) Compute the inertia tensor and center of mass vector for each link.

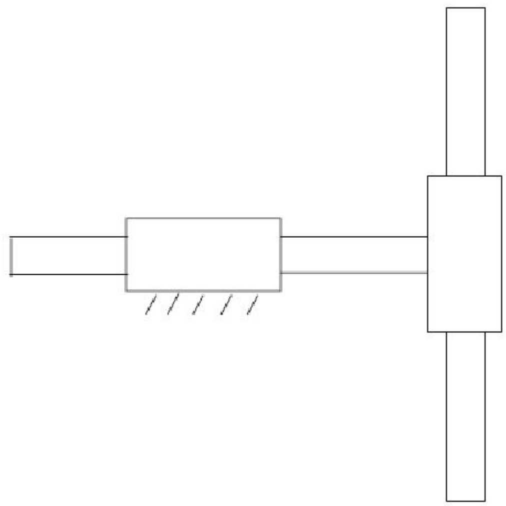
(b) Compute the Euler-Lagrange dynamic equations using the Lagrangian function L.

(c) Output the inertia matrix $D(q)$, the Coriolis/centrifugal matrix $C(q, \dot{q})$, and the gravitational torques/forces vector $g(q)$ for each robot.

(d) Compute the expression $\dot{D}(q) - 2C(q, \dot{q})$ for each robot and verify that it is a skew symmetric matrix.



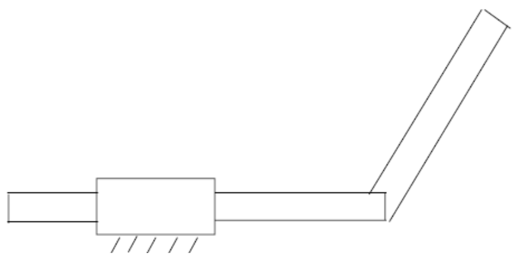
(i) A two-link RR manipulator



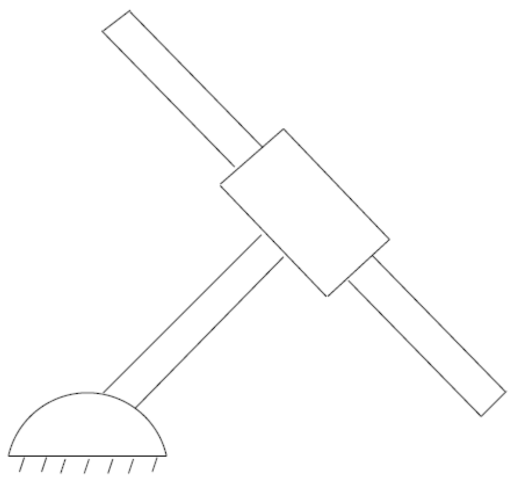
(ii) A two-link PP manipulator.

Problem 6

Following the same steps in Problem 5 (a) to (d), work with (iii) and (iv) planar robots shown below.



(iii) A two-link PR manipulator.



(iv) A two-link RP manipulator.