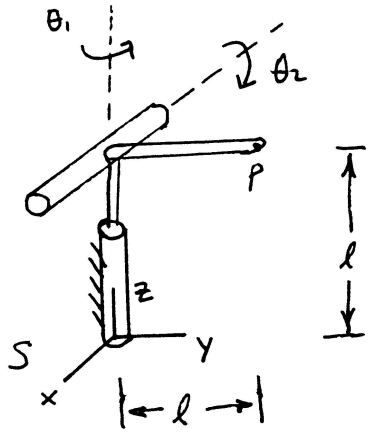
Course: Mech 567: Robot Kinematics & Dynamics
Instructor: Robert Gregg, PhD

Assigned Date: 02/10/2023
Due: 11:59 PM on Thursday, 02/23/2023
We are using Gradescope.

Homework 3

Problem 1

Use Paden-Kahan subproblem 2 to find θ_1 and θ_2 necessary to rotate an initial point $\mathrm{p} = \begin{bmatrix} 0 \\ l \\ l \end{bmatrix}$ to a final position $\mathrm{q} = \begin{bmatrix} -l/\sqrt{2} \\ 0 \\ l + l/\sqrt{2} \end{bmatrix}$.



Problem 2

Show how you would solve for the inverse kinematics of the 6 DOF manipulator shown in Figure 3.24 (ii) of MLS book, given a desired g_d. Show which subproblems you would solve and which points you would use to solve them (or how you would use geometric reasoning instead of one or more of the subproblems). How many solutions are possible?

Problem 3

(a) A body B is initially located with origin (1,2,0) relative to a fixed frame A, and coordinate axes parallel to A. What is the spatial velocity V_{ab}^s if the body rotates at speed $\frac{\pi}{2}$ radians per second about its z axis?

(b) What is the initial velocity, relative to the A frame (i.e., v_{q_a}), of a point with coordinates $q_b = (0, 1, 0)$ in the body frame? Show your math.

Problem 4

(a) Continued with Problem 3, what is the location of the point q after 1 second, in A coordinates? Show your math.

(b) Suppose that B is translating with a velocity of 2π units per second along the z axis, in addition to the rotation described in Problem 3 (a). Now what is V_{ab}^s? What would the location of the point q be after 1 second, in A coordinates? Again, show your math.

Problem 5

For manipulator (i) in Figure 3.23 (MLS book), find the spatial Jacobian. Please find this by hand; do not use Mathematica or the equivalent. Use a spatial frame with its origin at the intersection of ξ_1 and ξ_2 and principal axes parallel to those used in the book; (i.e., z vertical, y to the right, x out of the page).

Problem 6

Suppose that each of the four manipulators in Figure 3.24 of MLS experiences the following wrench on their end-effectors (Given in **body** coordinates): $F_B = \left[f_x, f_y, f_z, \tau_x, \tau_y, \tau_z\right]^T$. Use the values of $q, \omega, g_{st}(0)$ and the resulting twists ξ **as shown in the solutions of homework 2, problem 2**, and $\theta = \left[0, \frac{\pi}{2}, 0, 0, 0, 0\right]^T$ for each manipulator.

(a) Find the joint torque array τ that would be necessary to resist the wrench F_B for each manipulator. Note: in this context, "resist the wrench" means to exert the appropriate joint torque to keep the manipulator in equilibrium, *i.e.* sum of forces = 0.

(b) If not all of six terms of F_B appear in the solution of τ, explain why (your explanation should be general, not for each manipulator individually).

Note: Problem 6 allows the use of Mathematica. You are also permitted to use the "Screws" and "RobotLinks" packages, but specifically **not** the BodyJacobian and SpatialJacobian functions (or any other function that finds the Jacobian directly). Copy and paste your code in the homework.