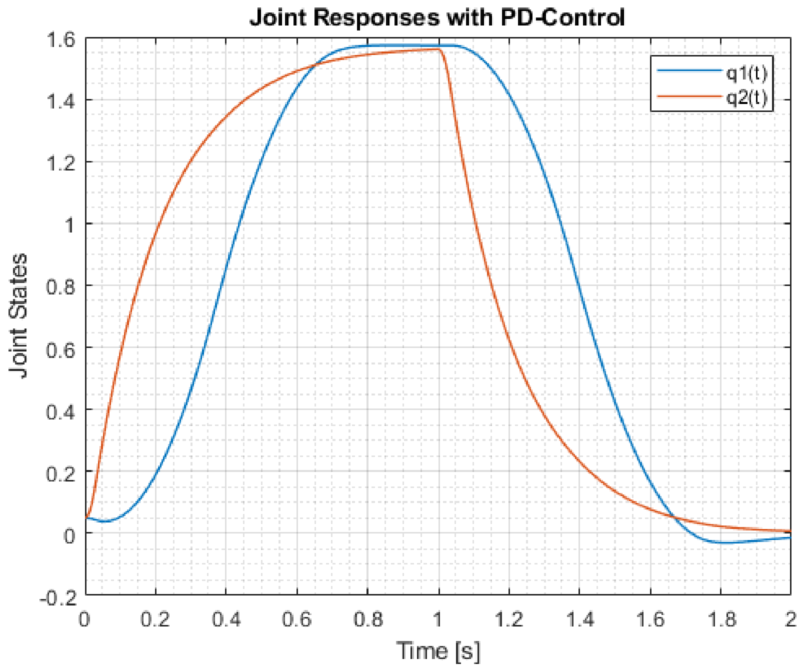
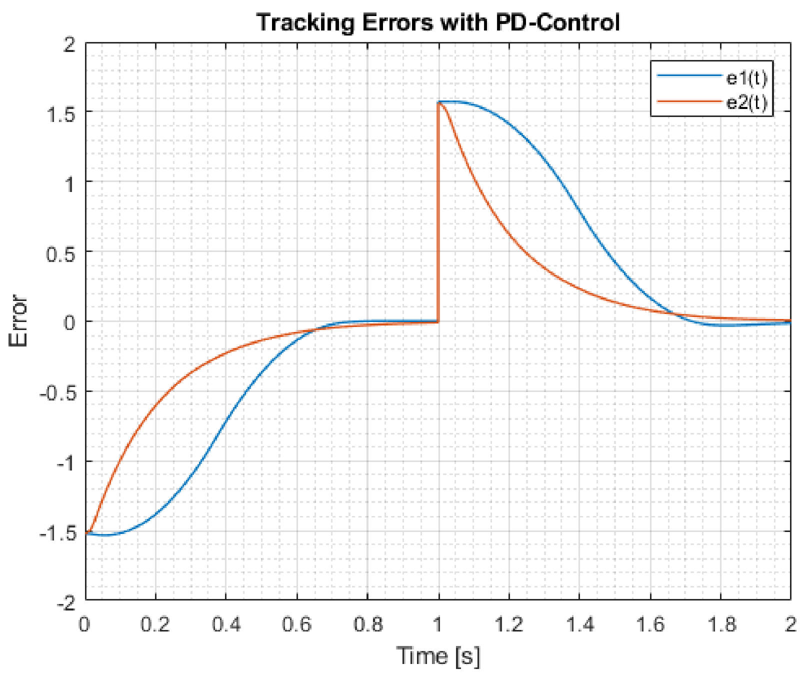
Contents

Problem1 (a)

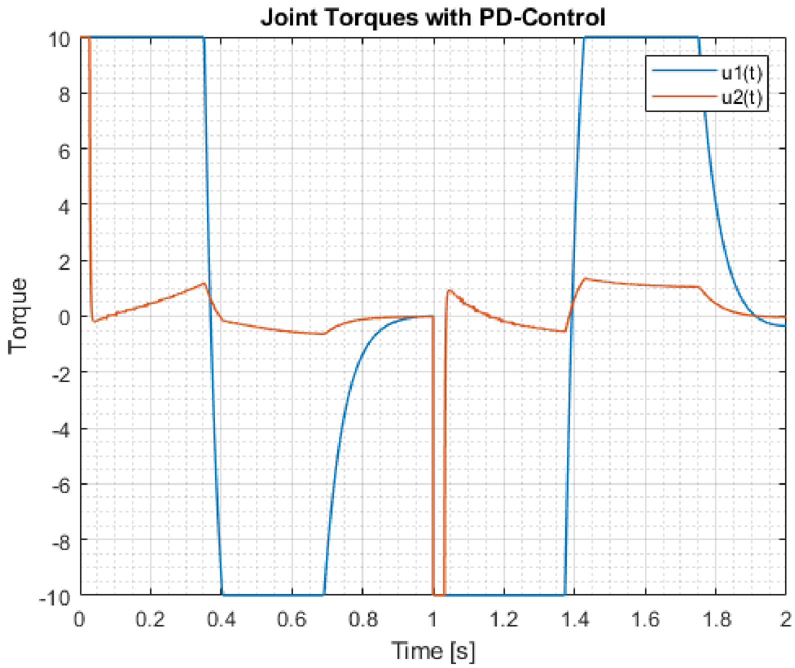
```
% Initial conditions
x0 = [0.05; 0; 0.05; 0];

% Simulate the system using ode45 with PD control
[T, X] = ode45(@(t, x) EoM(t, x, @PD), [0, 2], x0);

% Plot joint responses
figure(1)
plot(T, X(:, 1), T, X(:, 3), LineWidth = 1)
legend('q1(t)', 'q2(t)')
title('Joint Responses with PD-Control')
ylabel('Joint States')
xlabel('Time [s]')
grid on
grid minor

% Calculate torques and tracking errors
tau = zeros(2, length(T));
e = zeros(2, length(T));
for i = 1:length(T)
    [tau(:, i), e(:, i)] = PD(T(i), X(i, :));
end

% Plot tracking errors
figure(2)
plot(T, e(1, :), T, e(2, :), LineWidth = 1)
legend('e1(t)', 'e2(t)')
title('Tracking Errors with PD-Control')
xlabel('Time [s]')
ylabel('Error')
grid on
grid minor

% Plot joint torques
figure(3)
plot(T, tau(1, :), T, tau(2, :), LineWidth = 1)
legend('u1(t)', 'u2(t)')
title('Joint Torques with PD-Control')
xlabel('Time [s]')
ylabel('Torque')
grid on
grid minor
```



Joint Responses with PD-Control
q1(t)
q2(t)
Joint States
Time [s]



Tracking Errors with PD-Control
e1(t)
e2(t)
Error
Time [s]



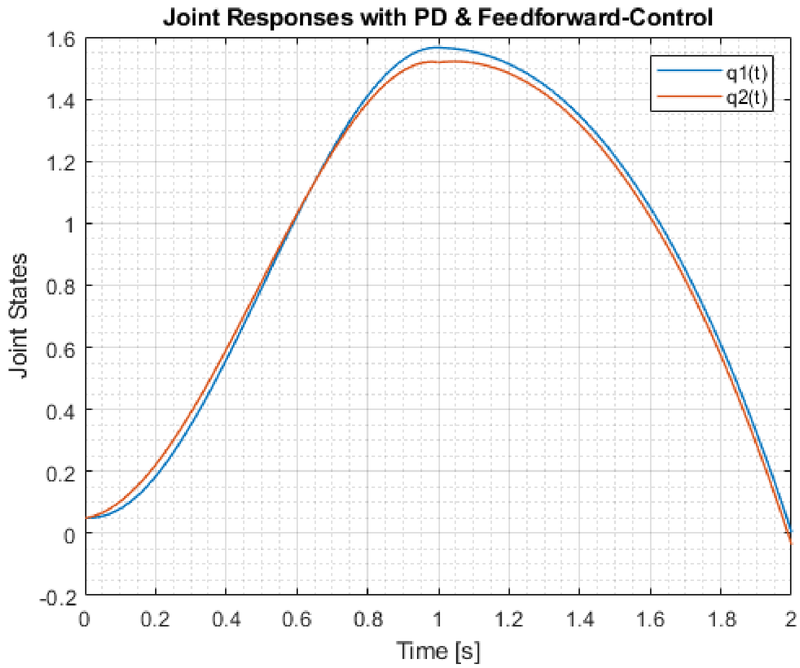
Problem1 (b)

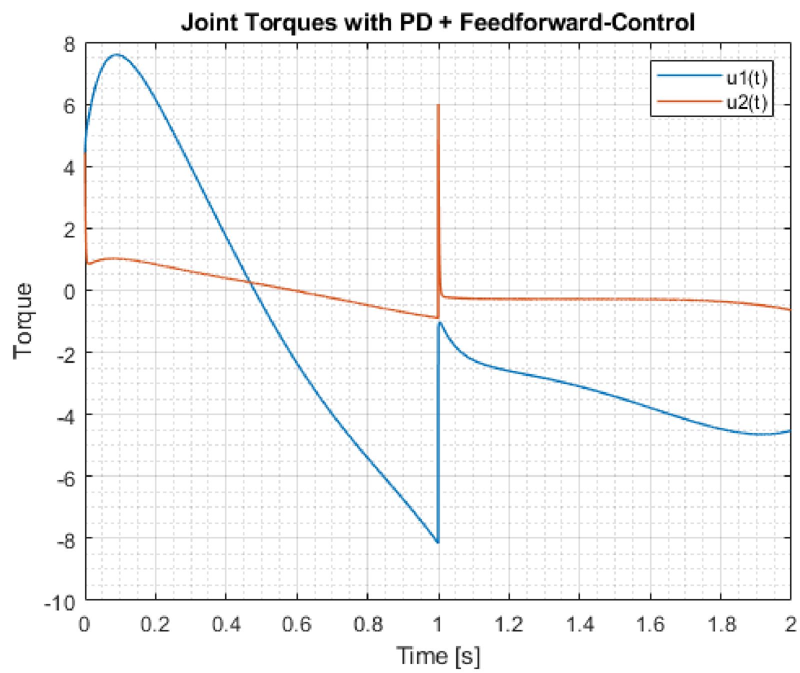
```
% error1(t=1) = 0.0051; Actual e(1) is around 1.5 on the plot
% error2(t=1) = -0.0101;
% error1(t=2) = -0.0134;
% error2(t=2) = 0.0080.

% Torque1 saturates in the intervals [0, 0.360), (0.41, 0.71], [1, 1.37], and [1.42, 1.75]
% Torque2 almost does not saturate. It does for around 0.03s at the beginning and near 1s

% Large tracking errors might be due to the initialization of the system, where the joint angles and velocities are not desired ones.
% Large initial torques might be due to the high control gain trying to eliminate the tracking error rapidly.
```

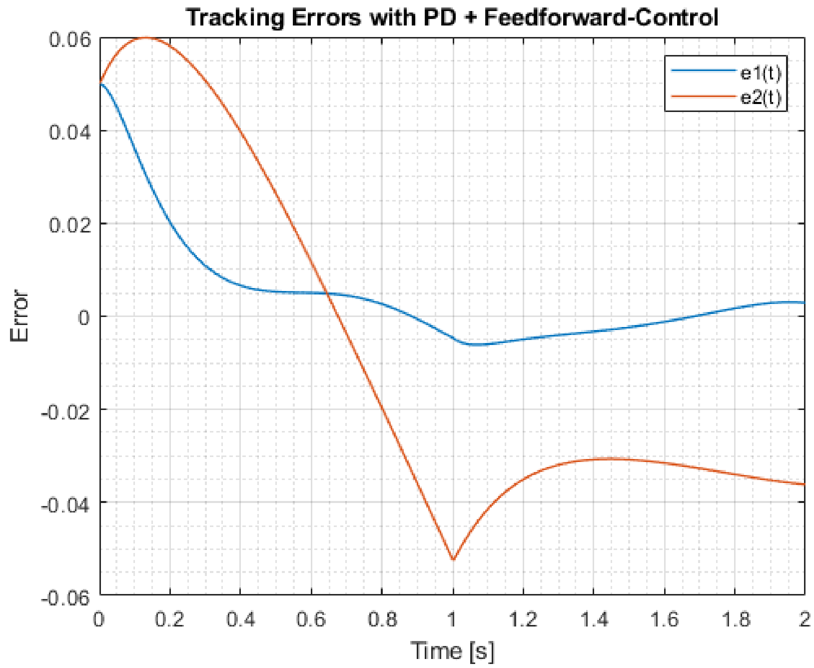
Problem2 (a)

```
% Generate cubic polynomial reference trajectories
t = 0:0.01:2;
qd = zeros(2, length(t));
vd = zeros(2, length(t));
ad = zeros(2, length(t));
for i = 1:length(t)
    [qd(:, i), vd(:, i), ad(:, i)] = cubicpoly(t(i));
end

% Initial conditions
x0 = [0.05; 0; 0.05; 0];

% Simulate the system using ode45 with PD + Feedforward control
[T, X] = ode45(@(t, x) EoM(t, x, @PD_FeedForward), [0, 2], x0);

% Plot joint responses
figure(1)
plot(T, X(:, 1), T, X(:, 3), LineWidth=1)
legend('q1(t)', 'q2(t)')
title('Joint Responses with PD & Feedforward-Control')
ylabel('Joint States')
xlabel('Time [s]')
grid on
grid minor

% Calculate torques and tracking errors
tau = zeros(2, length(T));
e = zeros(2, length(T));
```

```
for i = 1:length(T)
    [tau(:, i), e(:, i)] = PD_FeedForward(T(i), X(i, :));
end

% Plot joint torques
figure(2)
plot(T, tau(1, :), T, tau(2, :), LineWidth=1)
legend('u1(t)', 'u2(t)')
title('Joint Torques with PD + Feedforward-Control')
xlabel('Time [s]')
ylabel('Torque')
grid on
grid minor

% Plot tracking errors
figure(3)
plot(T, e(1, :), T, e(2, :), LineWidth=1)
legend('e1(t)', 'e2(t)')
title('Tracking Errors with PD + Feedforward-Control')
xlabel('Time [s]')
ylabel('Error')
grid on
grid minor
```


Joint Responses with PD & Feedforward-Control
q1(t)
q2(t)
Joint States
Time [s]
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


Joint Torques with PD + Feedforward-Control
u1(t)
u2(t)
Torque
Time [s]
8
6
4
2
0
-2
-4
-6
-8
-10
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2



Problem2 (b)

error1(t=1) = -0.0048; error2(t=1) = -0.0523; error1(t=2) = 0.0029; error2(t=2) = -0.0361.

```
% The errors and input torques are much smaller than those of pure PD
% control. Joint responses and part of the torque inputs get smoother with
% the feedforward added on.
```

Problem3 Drawing

```
clear; clc; close all

% Initial conditions
x0 = [0.0; 0; 0.0; 0];

% Simulate the system using ode45 with Inverse Dynamics control
[T, X] = ode45(@(t, x) EoM(t, x, @InvDynamics), [0, 2], x0);

% Plot joint responses
figure(1)
plot(T, X(:, 1), T, X(:, 3), LineWidth=1)
legend('q1(t)', 'q2(t)')
title('Joint Responses with Inverse Dynamics Control')
xlabel('Time [s]')
grid on
grid minor

% Calculate torques and tracking errors
tau = zeros(2, length(T));
e = zeros(2, length(T));
for i = 1:length(T)
    [tau(:, i), e(:, i)] = InvDynamics(T(i), X(i, :));
end

% Plot joint torques
figure(2)
plot(T, tau(1, :), T, tau(2, :), LineWidth=1)
legend('u1(t)', 'u2(t)')
title('Joint Torques with Inverse Dynamics Control')
xlabel('Time [s]')
ylabel('Torque')
grid on
grid minor
```

```
% Plot tracking errors
figure(3)
plot(T, e(1, :), T, e(2, :), LineWidth=1)
legend('e1(t)', 'e2(t)')
title('Tracking Errors with Inverse Dynamics Control')
xlabel('Time [s]')
ylabel('Error')
grid on
grid minor
```

Problem3 (b)

```
% error1(t=1) = -0.0048;
% error2(t=1) = -0.0523;
% error1(t=2) = 0.0029;
% error2(t=2) = -0.0361.

% Appreantly the figures are not what they supposed to be. But
% theoretically, with Inverse Dynamics Control, the errors and the torques should be similar
% to thoses in PD + FeedForward ones.
```

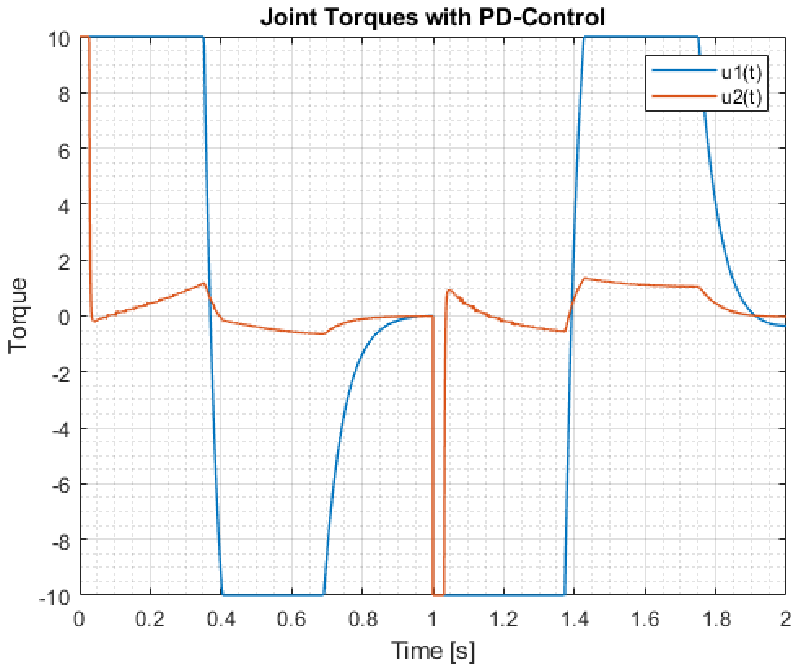
Problem 4

```
% PD controller achieves good tracking with some initial errors presented.
% PD + FeedForward Controller improves by reducing initial errors with
% added feedforward terms, but may still exhibit slight overshoot. Inverse
% Dynamics Control, somehow accurate, but requires very precious dynamics
% parameters.
```

Problem1 PD Control

```
function dx_dt = EoM(t, x, controller)

    % Given Dynamic Parameters
    m1 = 7.848; m2 = 4.49;
    l1 = 0.3; lc1 = 0.1554; lc2 = 0.0341;
    I1 = 0.176; I2 = 0.0411;

    % Retrive qi from State State Representation [X]
    q1 = x(1); q1dot = x(2);
    q2 = x(3); q2dot = x(4);

    % Inertia matrix D(q)
    D = zeros(2, 2);
    D(1, 1) = m1 * lc1^2 + m2 * (l1^2 + lc2^2 + 2 * l1 * lc2 * cos(q2)) + I1 + I2;
    D(1, 2) = m2 * (lc2^2 + l1 * lc2 * cos(q2)) + I2;
    D(2, 1) = D(1, 2);
    D(2, 2) = m2 * lc2^2 + I2;

    % Compute Christoffel symbols
    C121 = -m2 * l1 * lc2 * sin(q2);
    C211 = C121;
    C221 = C121;
    C112 = -C121;

    % Compute control inputs (torques) using the provided controller function
    [tau, ~] = controller(t, x);

    % Compute accelerations
    a1 = tau(1) - C121 * q1dot * q2dot - C211 * q2dot * q1dot - C221 * q2dot^2;
    a2 = tau(2) - C112 * q1dot^2;

    % Compute state derivatives
    dx_dt = [x(2); 1 / (D(1, 1) * D(2, 2) - D(2, 1) * D(1, 2)) * (D(2, 2) * a1 - D(1, 2) * a2);
            x(4); 1 / (D(1, 1) * D(2, 2) - D(2, 1) * D(1, 2)) * (-D(2, 1) * a1 + D(1, 1) * a2)];
end
```

```matlab
% Function generates cubic polynomial reference trajectories
% [qd (positions), vd (velocities), ad (accelerations)]
function [qd, vd, ad] = cubicpoly(t)
    if t <= 1 && t >= 0
        a = [1 0 0 0;
             0 1 0 0;
             1 1 1 1;
             0 1 2 3] \ [0; 0; pi/2; 0];
    elseif t > 1
        a = [1 1 1 1;
             0 1 2 3;
             1 2 4 8;
             0 1 4 1] \ [pi/2; 0; 0; 0];
    end

    qd = [a' * [1; t; t.^2; t.^3];
          a' * [1; t; t.^2; t.^3]];

    vd = [a' * [0; 1; 2*t; 3*t.^2];
          a' * [0; 1; 2*t; 3*t.^2]];

    ad = [a' * [0; 0; 2; 6*t];
          a' * [0; 0; 2; 6*t]];
end


function [tau, e] = PD(t, x)
    % Extract states
    q1 = x(1); q1dot = x(2);
    q2 = x(3); q2dot = x(4);

    % Define gains
    kp1 = 100; kp2 = 100;
    kd1 = 20; kd2 = 20;

    % Calculate desired positions based on time
    if t <= 1 && t >= 0
        q1d = pi/2;
        q2d = pi/2;
    elseif t > 1
        q1d = 0;
        q2d = 0;
    end

    % Compute control inputs (torques)
    tau1 = min(max(-10, kp1 * (q1d - q1) - kd1 * q1dot), 10);
    tau2 = min(max(-10, kp2 * (q2d - q2) - kd2 * q2dot), 10);

    tau = [tau1; tau2];

    % Calculate tracking errors
    e = [q1 - q1d;
         q2 - q2d];
end
```



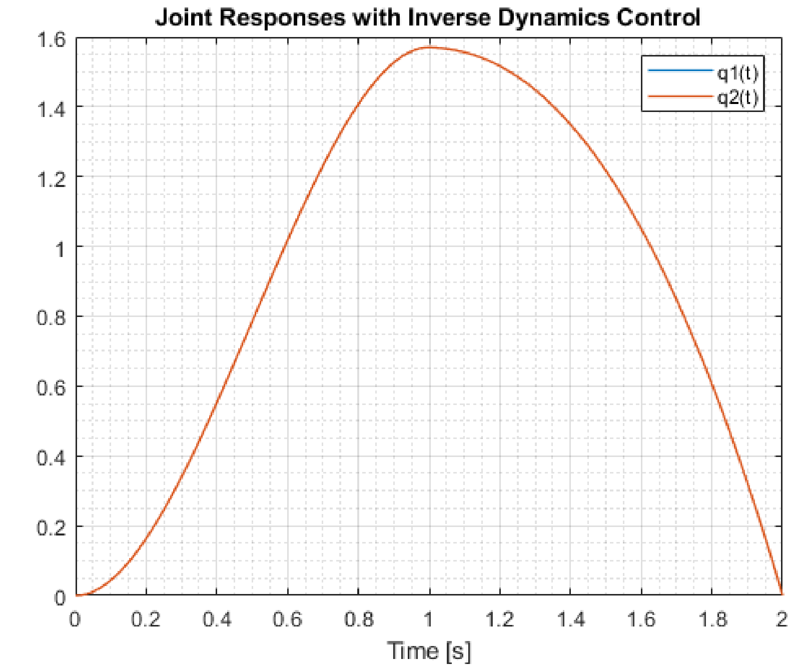
Problem2 PD + Feedforward Control

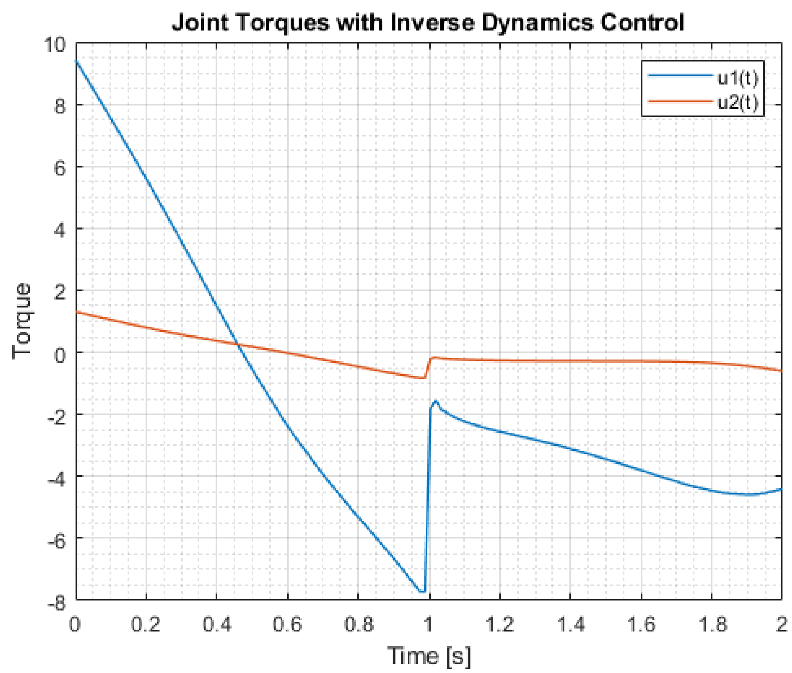
```
function [tau, e] = PD_FeedForward(t, x)
    % Extract states
    q1 = x(1); q1dot = x(2);
    q2 = x(3); q2dot = x(4);

    % Define gains
    kp1 = 100; kp2 = 100;
    kd1 = 20; kd2 = 20;

    % Compute desired positions, velocities, and accelerations using cubic polynomial function
    [qd, vd, ad] = cubicpoly(t);

    % Compute control inputs (torques) with feedforward
    tau1 = min(max(-10, ad(1) + kp1 * (qd(1) - q1) + kd1 * (vd(1) - q1dot)), 10);
    tau2 = min(max(-10, ad(2) + kp2 * (qd(2) - q2) + kd2 * (vd(2) - q2dot)), 10);

    tau = [tau1; tau2];

    % Calculate tracking errors
    e = [q1 - qd(1);
         q2 - qd(2)];
end
```

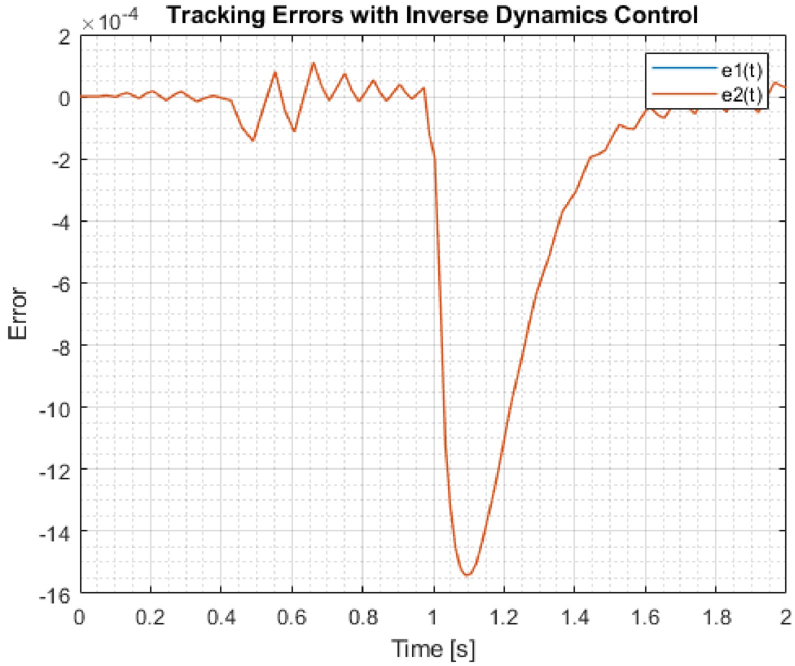
Problem3 Inverse Dynamics Control

```
function [tau, e] = InvDynamics(t, x)
    % Extract states
    q1 = x(1); q1dot = x(2);
    q2 = x(3); q2dot = x(4);

    % Define gains
    kp1 = 100; kp2 = 100;
    kd1 = 20; kd2 = 20;

    % Define parameters
    m1 = 7.848; m2 = 4.49;
    l1 = 0.3; lc1 = 0.1554; lc2 = 0.0341;
    I1 = 0.176; I2 = 0.0411;

    % Compute inertia matrix D
    D = zeros(2, 2);
```

```
    D(1, 1) = m1 * lc1^2 + m2 * (l1^2 + lc2^2 + 2 * l1 * lc2 * cos(q2)) + I1 + I2;
    D(1, 2) = m2 * (lc2^2 + l1 * lc2 * cos(q2)) + I2;
    D(2, 1) = D(1, 2);
    D(2, 2) = m2 * lc2^2 + I2;

    % Compute Christoffel symbols
    C121 = -m2 * l1 * lc2 * sin(q2);
    C211 = C121;
    C221 = C121;
    C112 = -C121;

    % Compute desired positions, velocities, and accelerations using cubic polynomial function
    [qd, vd, ad] = cubicpoly(t);

    % Compute desired accelerations with PD control
    aq = ad + [kp1; kp2] * (qd(1) - q1) + [kd1; kd2] * (vd(1) - q1dot);

    % Compute torques (control inputs) using inverse dynamics
    tau = D * aq + [C121 * q1dot * q2dot + C211 * q2dot * q1dot + C221 * q2dot^2;
                    C112 * q1dot^2];

    % Ensure torque limits are respected
    tau = [min(max(-10, tau(1)), 10);
           min(max(-10, tau(2)), 10)];

    % Calculate tracking errors
    e = [q1 - qd(1);
         q2 - qd(2)];
end
```


Joint Responses with Inverse Dynamics Control
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Time [s]
q1(t)
q2(t)


Joint Torques with Inverse Dynamics Control
10
8
6
4
2
0
-2
-4
-6
-8
Torque
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Time [s]
u1(t)
u2(t)



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