# ROB510 Robot Kinematics and Dynamics
# Homework 2

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| T | O | B | O | R |
| H | C | I | M | U |

# Problem I

Verify that for $\omega \in \mathrm{R}^3, \|\omega\| \neq 1$

$$e^{\widehat{\omega}\theta} = I + \frac{\widehat{\omega}}{\|\omega\|}\sin(\|\omega\|\theta) + \frac{\widehat{\omega}^2}{\|\omega\|^2}(1 - \cos(\|\omega\|\theta))$$

**Proof:**

Let $\omega = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3$, then the skew-symmetric matrix $\widehat{\omega}$ will be defined as:

$$\widehat{\omega} = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}$$

Since $\widehat{\omega}$ is skew-symmetric, indicating $\widehat{\omega}^T = -\widehat{\omega}$, which yields

$$\widehat{\omega}^2 = \omega\omega^T - \|\omega\|^2 I \tag{2.12}$$

$$\widehat{\omega}^3 = -\|\omega\|^2\widehat{\omega} \tag{2.13}$$

Since the Taylor Expansion of matrix exponential is:

$$e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \frac{A^4}{4!} + \cdots$$

Then,

$$\begin{aligned} e^{\widehat{\omega}\theta} &= I + \widehat{\omega}\theta + \frac{\widehat{\omega}^2\theta^2}{2!} + \frac{\widehat{\omega}^3\theta^3}{3!} + \frac{\widehat{\omega}^4\theta^4}{4!} + \cdots \\ &= I + (\widehat{\omega}\theta + \frac{\widehat{\omega}^3\theta^3}{3!} + \cdots) + (\frac{\widehat{\omega}^2\theta^2}{2!} + \frac{\widehat{\omega}^4\theta^4}{4!} + \cdots) \\ &= I + (\widehat{\omega}\theta - \frac{\widehat{\omega}(\|\omega\|\theta)^3}{\|\omega\|3!} + \cdots) + (\frac{\widehat{\omega}^2(\|\omega\|\theta)^2}{\|\omega\|^2 2!} - \frac{\widehat{\omega}^2(\|\omega\|\theta)^4}{\|\omega\|^2 4!} + \cdots) \\ &= I + \frac{\widehat{\omega}}{\|\omega\|}(\|\omega\|\theta - \frac{(\|\omega\|\theta)^3}{3!} + \frac{(\|\omega\|\theta)^5}{5!} - \cdots) + \frac{\widehat{\omega}^2}{\|\omega\|^2}(\frac{(\|\omega\|\theta)^2}{2!} - \frac{(\|\omega\|\theta)^4}{4!} + \frac{(\|\omega\|\theta)^6}{6!} - \cdots) \end{aligned}$$

Since

$$\sin(\|\omega\|\theta) = \|\omega\|\theta - \frac{(\|\omega\|\theta)^3}{3!} + \frac{(\|\omega\|\theta)^5}{5!} - \frac{(\|\omega\|\theta)^7}{7!} + \cdots,$$

and

$$\cos(\|\omega\|\theta) = 1 - \frac{(\|\omega\|\theta)^2}{2!} + \frac{(\|\omega\|\theta)^4}{4!} - \frac{(\|\omega\|\theta)^6}{6!} + \cdots,$$

Thus,

$$e^{\widehat{\omega}\theta} = I + \frac{\widehat{\omega}}{\|\omega\|}\sin(\|\omega\|\theta) + \frac{\widehat{\omega}^2}{\|\omega\|^2}(1 - \cos(\|\omega\|\theta)).$$

***Q.E.D***

## Problem II

A robot threads a nut onto a bolt with a pitch of 10 threads per inch parallel to the z axis and intersecting the $x-y$ plane at $[1'', 1'', 0]$. Find the twist coordinates $\xi$ that describe the motion of the nut.

Since the pitch is 10 threads per inch parallel to the z axis, we can have the twist axis as

$$\omega = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Since it's intersecting the $x - y$ plane at $[1'', 1'', 0]$, we can have

$$q = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Now, we have the pitch

$$h = d/\theta = \frac{1}{10}/2\pi = \frac{1}{20\pi}$$

Now, we can calculate

$$v = -\omega \times q + h\omega, \ \textit{h finite case}$$

$$v = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \frac{1}{20\pi} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ \frac{1}{20\pi} \end{bmatrix}.$$

Thus, we have

$$\xi = \begin{bmatrix} 1 \\ -1 \\ \frac{1}{20\pi} \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

## Problem III

Find the twist coordinates $\xi$ and $g_{st}(0)$ ($g_{st}(0)$ represents the Rigid Body Transformation between the frame attached to the last link of the manipulator and the base frame ) for all 4 manipulators shown below (show your chosen base and tool frames, but please keep them parallel to those used in the book; i.e., $z$ vertical, $y$ to the right, $x$ out of the page). In this problem, we assume there are no offsets between joints in $x$ and $z$. Please label the offsets in $y$ as $l_1, l_2, \ldots, l_n$ accordingly.

For the Elbow Manipulator, since there is no rotation between the base frame and the tool frame,

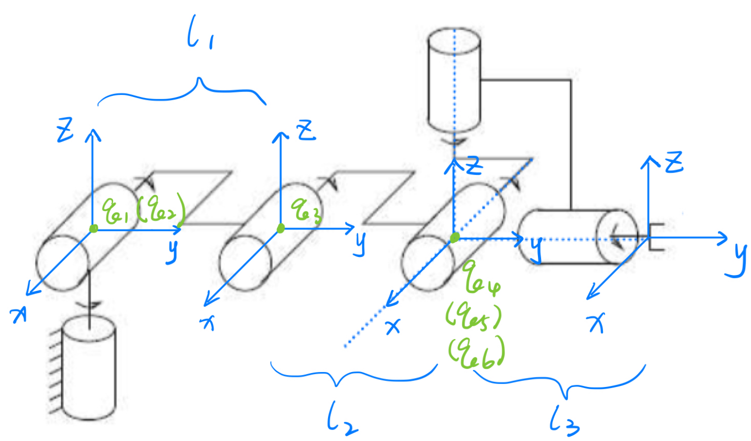


Figure 1: Elbow Manipulator

we can have

$$g_{st}(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_1 + l_2 + l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Then we can write out each twist coordinate as

$$\omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\ q_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ \xi_1 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

$$\omega_2 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix},\ q_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ \xi_2 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}.$$

$$\omega_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \; q_3 = \begin{bmatrix} 0 \\ l_1 \\ 0 \end{bmatrix}, \; \xi_3 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ l_1 \\ -1 \\ 0 \\ 0 \end{bmatrix}.$$

$$\omega_4 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \; q_4 = \begin{bmatrix} 0 \\ l_1 + l_2 \\ 0 \end{bmatrix}, \; \xi_4 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ l_1 + l_2 \\ -1 \\ 0 \\ 0 \end{bmatrix}.$$

$$\omega_5 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \; q_5 = \begin{bmatrix} 0 \\ l_1 + l_2 \\ 0 \end{bmatrix}, \; \xi_5 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} l_1 + l_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

$$\omega_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \; q_6 = \begin{bmatrix} 0 \\ l_1 + l_2 \\ 0 \end{bmatrix}, \; \xi_6 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

For the Inverse Elbow Manipulator, we can have

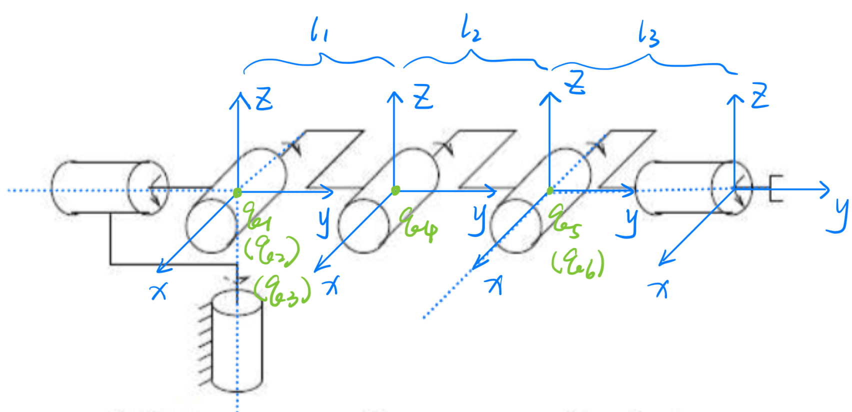


Figure 2: Inverse Elbow Manipulator

$$g_{st}(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_1 + l_2 + l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Then we can write out each twist coordinate as

$$\omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\ q_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ \xi_1 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

$$\omega_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ q_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ \xi_2 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

$$\omega_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix},\ q_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ \xi_3 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}.$$

$$\omega_4 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix},\ q_4 = \begin{bmatrix} 0 \\ l_1 \\ 0 \end{bmatrix},\ \xi_4 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ l_1 \\ -1 \\ 0 \\ 0 \end{bmatrix}.$$

$$\omega_5 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix},\ q_5 = \begin{bmatrix} 0 \\ l_1 + l_2 \\ 0 \end{bmatrix},\ \xi_5 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ l_1 + l_2 \\ -1 \\ 0 \\ 0 \end{bmatrix}.$$

$$\omega_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ q_6 = \begin{bmatrix} 0 \\ l_1 + l_2 \\ 0 \end{bmatrix},\ \xi_6 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

For the Stanford Manipulator, we can have

$$g_{st}(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_1 + l_2 + l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

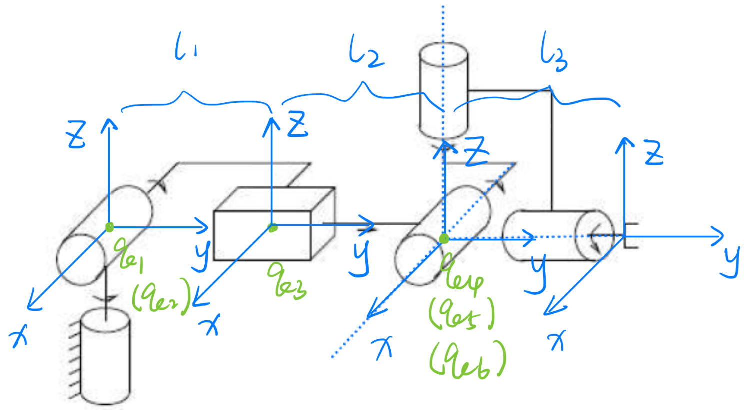


Figure 3: Stanford Manipulator

Then we can write out each twist coordinate as (Note that no rotation at $q_3$)

$$\omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \; q_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \; \xi_1 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

$$\omega_2 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \; q_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \; \xi_2 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}.$$

$$v_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \; q_3 = \begin{bmatrix} 0 \\ l_1 \\ 0 \end{bmatrix}, \; \xi_3 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\omega_4 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \; q_4 = \begin{bmatrix} 0 \\ l_1 + l_2 \\ 0 \end{bmatrix}, \; \xi_4 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ l_1 + l_2 \\ -1 \\ 0 \\ 0 \end{bmatrix}.$$

$$\omega_5 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\ q_5 = \begin{bmatrix} 0 \\ l_1 + l_2 \\ 0 \end{bmatrix},\ \xi_5 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} l_1 + l_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

$$\omega_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ q_6 = \begin{bmatrix} 0 \\ l_1 + l_2 \\ 0 \end{bmatrix},\ \xi_6 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

For the Rhino Robot, we can have

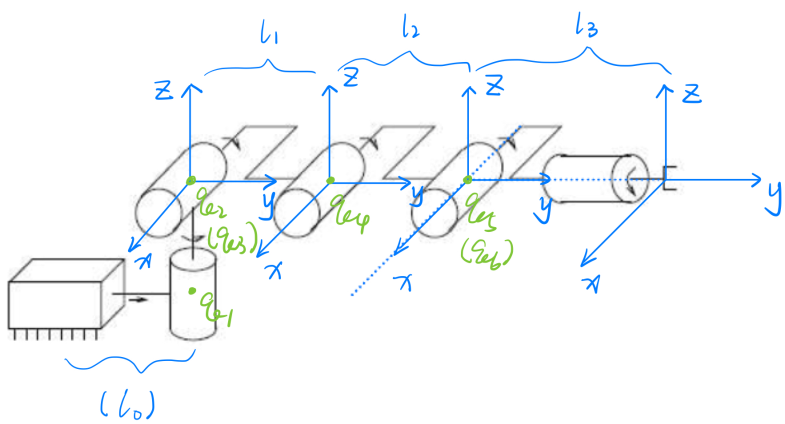


Figure 4: Rhino Robot

$$g_{st}(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_1 + l_2 + l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Then we can write out each twist coordinate as (Note that no rotation at $q_1$)

$$v_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ q_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ \xi_1 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\omega_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\ q_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ \xi_2 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

$$\omega_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix},\ q_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ \xi_3 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\omega_4 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix},\ q_4 = \begin{bmatrix} 0 \\ l_1 \\ 0 \end{bmatrix},\ \xi_4 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ l_1 \\ -1 \\ 0 \\ 0 \end{bmatrix}.$$

$$\omega_5 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix},\ q_5 = \begin{bmatrix} 0 \\ l_1 + l_2 \\ 0 \end{bmatrix},\ \xi_5 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ l_1 + l_2 \\ -1 \\ 0 \\ 0 \end{bmatrix}.$$

$$\omega_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ q_6 = \begin{bmatrix} 0 \\ l_1 + l_2 \\ 0 \end{bmatrix},\ \xi_6 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

# Problem IV

Find the matrix exponential $e^{\widehat{\xi}\theta}$ for each of the six twists for the Stanford manipulator (Figure 3.24 iii) that you found in problem 3. Please calculate these by hand; **do not use** Mathematica or equivalent.

$$e^{\widehat{\xi}\theta} = \begin{bmatrix} e^{\widehat{\omega}\theta} & \left(I - e^{\widehat{\omega}\theta}\right)(\omega \times v) + \omega\omega^T v\theta \\ 0 & 1 \end{bmatrix}.$$

1.
$$\widehat{w}_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \widehat{w}_1^2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

$$\begin{aligned} e^{\hat{\omega}\theta} &= I + \frac{\hat{\omega}}{\|\omega\|}\sin(\|\omega\|\theta) + \frac{\hat{\omega}^2}{\|\omega\|^2}(1 - \cos(\|\omega\|\theta)) \\ &= I + \begin{bmatrix} 0 & -\sin\theta & 0 \\ \sin\theta & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} -1+\cos\theta & 0 & 0 \\ 0 & -1+\cos\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \end{aligned}$$

Thus, we have

$$e^{\widehat{\xi}\theta} = \begin{bmatrix} e^{\widehat{\omega}\theta} & \left(I - e^{\widehat{\omega}\theta}\right)(\omega \times v) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

2. Similarly, we have

$$\widehat{w}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix},\ \widehat{w}_2^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ v_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

and

$$e^{\widehat{\xi}\theta} = \begin{bmatrix} e^{\widehat{\omega}\theta} & \left(I - e^{\widehat{\omega}\theta}\right)(\omega \times v) + \omega\omega^T v\theta \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

3. Here, we have a prismatic joint, which yields

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ v_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

and

$$e^{\widehat{\xi}\theta} = \begin{bmatrix} e^{\widehat{\omega}\theta} & \left(I - e^{\widehat{\omega}\theta}\right)(\omega \times v) + \omega\omega^T v\theta \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \theta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

4. Similarly, we have

$$\widehat{w}_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix},\ \widehat{w}_4^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ v_4 = \begin{bmatrix} 0 \\ 0 \\ -l_1 - l_2 \end{bmatrix},$$

and

$$e^{\widehat{\xi}\theta} = \begin{bmatrix} e^{\widehat{\omega}\theta} & \left(I - e^{\widehat{\omega}\theta}\right)(\omega \times v) + \omega\omega^T v\theta \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & (l_1 + l_2)(1 - \cos\theta) \\ 0 & -\sin\theta & \cos\theta & (l_1 + l_2)\sin\theta \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

5. Similarly, we have

$$\widehat{w}_5 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \widehat{w}_5^2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ v_5 = \begin{bmatrix} l_1 + l_2 \\ 0 \\ 0 \end{bmatrix},$$

and

$$e^{\widehat{\xi}\theta} = \begin{bmatrix} e^{\widehat{\omega}\theta} & \left(I - e^{\widehat{\omega}\theta}\right)(\omega \times v) + \omega\omega^T v\theta \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & (l_1 + l_2)\sin\theta \\ \sin\theta & \cos\theta & 0 & (l_1 + l_2)(1 - \cos\theta) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

6. Similarly, we have

$$\widehat{w}_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix},\ \widehat{w}_6^2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix},\ v_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

and

$$e^{\widehat{\xi}\theta} = \begin{bmatrix} e^{\widehat{\omega}\theta} & \left(I - e^{\widehat{\omega}\theta}\right)(\omega \times v) + \omega\omega^T v\theta \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

# Problem V

Using **Mathematica**, derive the complete transform $g_{st}(\theta)$ via the product of exponential for all four manipulators in problem 3 (Figure 3.24). Copy and paste your code in the homework. The problem will be graded based on the setup of the twists and syntax of commands rather than the final output. **Hint:** Read Appendix B in MLS for a brief description of Mathematica.

```
(* Mathematica Code *)
ROB510 Homework2
Needs["Screws`", "E:\\Screws.m"]

Elbow Manipulator
g0 = {{1, 0, 0, 0}, {0, 1, 0, l1 + l2 + l3}, {0, 0, 1, 0}, {0, 0, 0, 1}};
s1 = {0, 0, 0, 0, 0, 1};
s2 = {0, 0, 0, \[Minus]1, 0, 0};
s3 = {0, 0, l1, \[Minus]1, 0, 0};
s4 = {0, 0, l1 + l2, \[Minus]1, 0, 0};
s5 = {l1 + l2, 0, 0, 0, 0, 1};
s6 = {0, 0, 0, 0, 1, 0};
q1 = {0, 0, 0};
q2 = q1;
q3 = q2;
q4 = {0, l1 , 0};
q5 = {0, l1 + l2, 0};
q6 = q5;
e1 = TwistExp[s1, q1];
e2 = TwistExp[s2, q2];
e3 = TwistExp[s3, q3];
e4 = TwistExp[s4, q4];
e5 = TwistExp[s5, q5];
e6 = TwistExp[s6, q6];
MatrixForm[gst = Simplify[e1 . e2 . e3 . e4 . e5 . e6 . g0]]

Inverse Elbow Manipulator
gst0 = {{1, 0, 0, 0}, {0, 1, 0, l1 + l2 + l3}, {0, 0, 1, 0}, {0, 0, 0, 1}};
s1 = {0, 0, 0, 0, 0, 1};
s2 = {0, 0, 0, -1, 0, 0};
s3 = {0, 1, 0, 0, 0, 0};
s4 = {0, 0, l1 + l2, \[Minus]1, 0, 0};
s5 = {l1 + l2, 0, 0, 0, 0, 1};
s6 = {0, 0, 0, 0, 1, 0};
q1 = {0, 0, 0};
q2 = q1;
q3 = {0, l1, 0};
q4 = {0, l1 + l2, 0};
q5 = q4;
q6 = q5;
e1 = TwistExp[s1, q1];
e2 = TwistExp[s2, q2];
e3 = TwistExp[s3, q3];
e4 = TwistExp[s4, q4];
e5 = TwistExp[s5, q5];
e6 = TwistExp[s6, q6];
MatrixForm[gst = Simplify[e1 . e2 . e3 . e4 . e5 . e6 . gst0]]
```

```
Stanford  Manipulator
g0 = {{1, 0, 0, 0}, {0, 1, 0, l1 + l2 + l3}, {0, 0, 1, 0}, {0, 0, 0,
    1}};
s1 = {0, 0, 0, 0, 0, 1};
s2 = {0, 0, 0, \[Minus]1, 0, 0};
s3 = {0, 1, 0, 0, 0, 0};
s4 = {0, 0, l1 + l2, \[Minus]1, 0, 0};
s5 = {l1 + l2, 0, 0, 0, 0, 1};
s6 = {0, 0, 0, 0, 1, 0};
q1 = {0, 0, 0};
q2 = q1;
q3 = {0, l1, 0};
q4 = {0, l1 + l2, 0};
q5 = q4;
q6 = q5;
e1 = TwistExp[s1, q1];
e2 = TwistExp[s2, q2];
e3 = TwistExp[s3, q3];
e4 = TwistExp[s4, q4];
e5 = TwistExp[s5, q5];
e6 = TwistExp[s6, q6];
MatrixForm[gst = Simplify[e1 . e2 . e3 . e4 . e5 . e6 . g0]]

Rhino  Robot
g0 = {{1, 0, 0, 0}, {0, 1, 0, l1 + l2 + l3}, {0, 0, 1, 0}, {0, 0, 0,
    1}};
s1 = {0, 1, 0, 0, 0, 0};
s2 = {0, 0, 0, 0, 0, 1};
s3 = {0, 0, -1, 0, 0, 0};
s4 = {0, 0, l1, \[Minus]1, 0, 0};
s5 = {0, 0, l1 + l2, -1, 0, 0};
s6 = {0, 0, 0, 0, 1, 0};
q1 = {0, 0, 0};
q2 = q1;
q3 = q2;
q4 = {0, l1, 0};
q5 = {0, l1 + l2, 0};
q6 = q5;
e1 = TwistExp[s1, q1];
e2 = TwistExp[s2, q2];
e3 = TwistExp[s3, q3];
e4 = TwistExp[s4, q4];
e5 = TwistExp[s5, q5];
e6 = TwistExp[s6, q6];
MatrixForm[gst = Simplify[e1 . e2 . e3 . e4 . e5 . e6 . g0]]
```