

The given solutions assume that the  $a=1$  dimension is always in the x-direction. The following solutions assume that the  $a=1$  dimension is always along the long axis of the link as shown in the homework. This will end up changing the inertia tensors, which trickles down.

```

In[757]:= ClearAll["Global`*"]
Needs["Screws`", "C:\\Mathematica\\Screws.m"]
Needs["RobotLinks`", "C:\\Mathematica\\RobotLinks.m"]
Needs["VariationalMethods`"]

Problem 1: Two-link RR manipulator

In[761]:= m = 2;
a = 1;
b = 0.2;
c = 0;

 $\theta = \{\theta_1[t], \theta_2[t]\};$ 
 $\dot{\theta} = D[\theta, t];$ 

xi1 = {0, 0, 0, 0, 0, 1};
xi2 = {0, -a, 0, 0, 0, 1};

gst0b1 = {{1, 0, 0, a/2}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};
gst0b2 = {{1, 0, 0, a + a/2}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};

e1 = TwistExp[xi1,  $\theta[[1]]$ ];
e2 = TwistExp[xi2,  $\theta[[2]]$ ];

gstb1 = TwistExp[xi1,  $\theta[[1]]$ ].gst0b1;
gstb2 = TwistExp[xi1,  $\theta[[1]]$ ].TwistExp[xi2,  $\theta[[2]]$ ].gst0b2 // FullSimplify;

Labeled[MatrixForm[COM1 = RigidPosition[gstb1]], "COM1"]
Labeled[MatrixForm[COM2 = RigidPosition[gstb2]], "COM2"]
Labeled[
  MatrixForm[I1 = {{m/12 * (b^2 + c^2), 0, 0}, {0, m/12 * (a^2 + c^2), 0}, {0, 0, m/12 * (a^2 + b^2)}}], Inertia Tensor]
I2 = I1;

GenM1 = {m * IdentityMatrix[3], 0}, {0, I1} // ArrayFlatten;

```

```

GenM2 = {{m * IdentityMatrix[3], 0}, {0, I2}} // ArrayFlatten;

xi1cross = Inverse[RigidAdjoint[e1.gst0b1]].xi1 // Simplify;
Jbs11 = {xi1cross, {0, 0, 0, 0, 0, 0}} // Transpose;

xi1cross = Inverse[RigidAdjoint[e1.e2.gst0b2]].xi1 // Simplify;
xi2cross = Inverse[RigidAdjoint[e2.gst0b2]].xi2 // Simplify;
Jbs12 = {xi1cross, xi2cross} // Transpose;

M = Transpose[Jbs11].GenM1.Jbs11 + Transpose[Jbs12].GenM2.Jbs12 // Simplify;

P = m * g * COM1[[2]] + m * g * COM2[[2]];
G = {0, 0};
G[[1]] = D[P,  $\theta$ [[1]]];
G[[2]] = D[P,  $\theta$ [[2]]];
christoffel = ConstantArray[0, {2, 2, 2}];

For[i = 1, i ≤ 2, i++,
  For[j = 1, j ≤ 2, j++,
    For[k = 1, k ≤ 2, k++,
      christoffel[[i, j, k]] = 1/2 * (D[M[[i, j]],  $\theta$ [[k]]] + D[M[[i, k]],  $\theta$ [[j]]] - D[M[[k, j]],  $\theta$ [[i]]]);
    ]
  ]
]

MatrixForm[christoffel];

Coriolis = ConstantArray[0, {2, 2}];
For[i = 1, i ≤ 2, i++,
  For[j = 1, j ≤ 2, j++,
    Coriolis[[i, j]] = christoffel[[i, j, All]]. $\dot{\theta}$ ;
  ]
]

```

```
K = 1 / 2 * Transpose[θdot].M.θdot;
```

```
L = K - P // FullSimplify;
```

```
EL = {0, 0};
```

```
For [i = 1, i ≤ 2, i++,
```

```
  EL[[i]] = D[D[L, θdot[[i]]], t] - D[L, θ[[i]]];
```

```
]
```

```
Labeled[MatrixForm[EL // FullSimplify], "Euler-Lagrange Equations"]
```

```
Labeled[MatrixForm[M], "M(θ)"]
```

```
Labeled[MatrixForm[Coriolis // FullSimplify], "C(θ,θ̇)"]
```

```
MatrixForm[InertiaToCoriolis[M, θ, θdot] // FullSimplify];
```

```
Labeled[MatrixForm[G // FullSimplify], "G(θ)"]
```

```
Labeled[MatrixForm[D[M, t] - 2 * Coriolis // Simplify], "(Ḣ - 2C)"]
```

```
MatrixForm[D[M, t] - 2 * Coriolis // Simplify] == MatrixForm[-1 * Transpose[D[M, t] - 2 * Coriolis // Simplify]]
```

$$\text{Out[775]} = \begin{pmatrix} \frac{1}{2} \cos[\theta_1[t]] \\ \frac{1}{2} \sin[\theta_1[t]] \\ 0 \end{pmatrix}$$

COM1

$$\text{Out[776]} = \begin{pmatrix} \cos[\theta_1[t]] + \frac{1}{2} \cos[\theta_1[t] + \theta_2[t]] \\ \sin[\theta_1[t]] + \frac{1}{2} \sin[\theta_1[t] + \theta_2[t]] \\ 0 \end{pmatrix}$$

COM2

$$\text{Out[777]} = \begin{pmatrix} 0.00666667 & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0.173333 \end{pmatrix}$$

Inertia Tensor

$$\text{Out[800]} = \left( \begin{array}{c} g (3 \cos[\theta_1[t]] + \cos[\theta_1[t] + \theta_2[t]]) - \sin[\theta_2[t]] \theta_2'[t] (2 \theta_1'[t] + \theta_2'[t]) + 2 (1.673 + \cos[\theta_2[t]]) \theta_1''[t] + (0.673 + \cos[\theta_2[t]]) \theta_2''[t] \\ g \cos[\theta_1[t] + \theta_2[t]] + \sin[\theta_2[t]] \theta_1'[t]^2 + (0.673333 + \cos[\theta_2[t]]) \theta_1''[t] + 0.673333 \theta_2''[t] \end{array} \right)$$

Euler-Lagrange Equations

$$\text{Out[801]} = \left( \begin{array}{cc} 2 (1.67333 + \cos[\theta_2[t]]) & 0.673333 + \cos[\theta_2[t]] \\ 0.673333 + \cos[\theta_2[t]] & 0.673333 \end{array} \right)$$

$M(\theta)$

$$\text{Out[802]} = \left( \begin{array}{cc} -1 \sin[\theta_2[t]] \theta_2'[t] & -1 \sin[\theta_2[t]] (\theta_1'[t] + \theta_2'[t]) \\ \sin[\theta_2[t]] \theta_1'[t] & 0 \end{array} \right)$$

$C(\theta, \dot{\theta})$

$$\text{Out[804]} = \left( \begin{array}{c} g (3 \cos[\theta_1[t]] + \cos[\theta_1[t] + \theta_2[t]]) \\ g \cos[\theta_1[t] + \theta_2[t]] \end{array} \right)$$

$G(\theta)$

$$\text{Out[805]} = \left( \begin{array}{c} 0 \sin[\theta_2[t]] (2 \theta_1'[t] + \theta_2'[t]) \\ -\sin[\theta_2[t]] (2 \theta_1'[t] + \theta_2'[t]) \end{array} \right)$$

$(\dot{M} - 2C)$

Out[806]= True

## Problem 2: Two-link PP manipulator

In[807]:=

```
m = 2;
a = 1;
b = 0.2;
c = 0;
```

```
theta = {theta1[t], theta2[t]};
thetadot = D[theta, t];
```

```
xi1 = {1, 0, 0, 0, 0, 0};
xi2 = {0, 1, 0, 0, 0, 0};
```

```
gst0b1 = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};
```

```

gst0b2 = {{0, -1, 0, a/2}, {1, 0, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};

e1 = TwistExp[xi1,  $\theta[1]$ ];
e2 = TwistExp[xi2,  $\theta[2]$ ];

gstb1 = TwistExp[xi1,  $\theta[1]$ ].gst0b1;
gstb2 = TwistExp[xi1,  $\theta[1]$ ].TwistExp[xi2,  $\theta[2]$ ].gst0b2 // FullSimplify;

Labeled[MatrixForm[COM1 = RigidPosition[gstb1]], "COM1"]
Labeled[MatrixForm[COM2 = RigidPosition[gstb2]], "COM2"]
Labeled[
  MatrixForm[I1 = {{m/12*(b^2 + c^2), 0, 0}, {0, m/12*(a^2 + c^2), 0}, {0, 0, m/12*(a^2 + b^2)}}], Inertia Tensor1]
Labeled[
  MatrixForm[I2 = {{m/12*(a^2 + c^2), 0, 0}, {0, m/12*(b^2 + c^2), 0}, {0, 0, m/12*(a^2 + b^2)}}], Inertia Tensor2]

GenM1 = {{m*IdentityMatrix[3], 0}, {0, I1}} // ArrayFlatten;
GenM2 = {{m*IdentityMatrix[3], 0}, {0, I2}} // ArrayFlatten;

xi1cross = Inverse[RigidAdjoint[e1.gst0b1]].xi1 // Simplify;
Jbsl1 = {xi1cross, {0, 0, 0, 0, 0, 0}} // Transpose;

xi1cross = Inverse[RigidAdjoint[e1.e2.gst0b2]].xi1 // Simplify;
xi2cross = Inverse[RigidAdjoint[e2.gst0b2]].xi2 // Simplify;
Jbsl2 = {xi1cross, xi2cross} // Transpose;

M = Transpose[Jbsl1].GenM1.Jbsl1 + Transpose[Jbsl2].GenM2.Jbsl2 // Simplify;

P = m*g*COM1[[2]] + m*g*COM2[[2]];
G = {0, 0};
G[[1]] = D[P,  $\theta[1]$ ];
G[[2]] = D[P,  $\theta[2]$ ];
christoffel = ConstantArray[0, {2, 2, 2}];

For[i = 1, i ≤ 2, i++,

```

```

For [j = 1, j ≤ 2, j++,
  For[k = 1, k ≤ 2, k++,
    christoffel[[i, j, k]] = 1/2 * (D[M[[i, j]], θ[[k]]] + D[M[[i, k]], θ[[j]]] - D[M[[k, j]], θ[[i]]]);
  ]
]

MatrixForm[christoffel];

Coriolis = ConstantArray[0, {2, 2}];
For[i = 1, i ≤ 2, i++,
  For[j = 1, j ≤ 2, j++,
    Coriolis[[i, j]] = christoffel[[i, j, All]].θdot;
  ]
]

K = 1/2 * Transpose[θdot].M.θdot;
L = K - P // FullSimplify;

EL = {0, 0};
For[i = 1, i ≤ 2, i++,
  EL[[i]] = D[D[L, θdot[[i]]], t] - D[L, θ[[i]]];
]

Labeled[MatrixForm[EL // FullSimplify], "Euler-Lagrange Equations"]

Labeled[MatrixForm[M], "M(θ)"]
Labeled[MatrixForm[Coriolis // FullSimplify], "C(θ, θ̇)"]
MatrixForm[InertiaToCoriolis[M, θ, θdot] // FullSimplify];
Labeled[MatrixForm[G // FullSimplify], "G(θ)"]

Labeled[MatrixForm[D[M, t] - 2 * Coriolis // Simplify], "(Ḣ - 2C)"]
MatrixForm[D[M, t] - 2 * Coriolis // Simplify] == MatrixForm[-1 * Transpose[D[M, t] - 2 * Coriolis // Simplify]]

```

$$\text{Out}[821]= \begin{pmatrix} \theta 1[t] \\ 0 \\ 0 \end{pmatrix}$$

**COM1**

$$\text{Out}[822]= \begin{pmatrix} \frac{1}{2} + \theta 1[t] \\ \theta 2[t] \\ 0 \end{pmatrix}$$

**COM2**

$$\text{Out}[823]= \begin{pmatrix} 0.00666667 & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0.173333 \end{pmatrix}$$

**Inertia Tensor1**

$$\text{Out}[824]= \begin{pmatrix} \frac{1}{6} & 0 & 0 \\ 0 & 0.00666667 & 0 \\ 0 & 0 & 0.173333 \end{pmatrix}$$

**Inertia Tensor2**

$$\text{Out}[846]= \begin{pmatrix} 4. \theta 1''[t] \\ 2 (g + \theta 2''[t]) \end{pmatrix}$$

**Euler-Lagrange Equations**

$$\text{Out}[847]= \begin{pmatrix} 4. & 0. \\ 0. & 2. \end{pmatrix}$$

**M( $\theta$ )**

$$\text{Out}[848]= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

**C( $\theta, \dot{\theta}$ )**

$$\text{Out}[850]= \begin{pmatrix} 0 \\ 2 g \end{pmatrix}$$

**G( $\theta$ )**



Out[851]= 
$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$
  

$$(\dot{M} - 2C)$$

Out[852]= True

### Problem 3: Two-link PR manipulator

In[853]:=

```

m = 2;
a = 1;
b = 0.2;
c = 0;

θ = {θ1[t], θ2[t]};
θdot = D[θ, t];

xi1 = {1, 0, 0, 0, 0, 0};
xi2 = {0, a/2, 0, 0, 0, 1};

gst0b1 = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};
gst0b2 = {{1, 0, 0, a}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};

e1 = TwistExp[xi1, θ[[1]]];
e2 = TwistExp[xi2, θ[[2]]];

gstb1 = TwistExp[xi1, θ[[1]]].gst0b1;
gstb2 = TwistExp[xi1, θ[[1]]].TwistExp[xi2, θ[[2]]].gst0b2 // FullSimplify;

Labeled[MatrixForm[COM1 = RigidPosition[gstb1]], "COM1"]
Labeled[MatrixForm[COM2 = RigidPosition[gstb2]], "COM2"]
Labeled[
  MatrixForm[I1 = {{m/12*(b^2 + c^2), 0, 0}, {0, m/12*(a^2 + c^2), 0}, {0, 0, m/12*(a^2 + b^2)}}], Inertia Tensor]
I2 = I1;

```

```

GenM1 = {{m * IdentityMatrix[3], 0}, {0, I1}} // ArrayFlatten;
GenM2 = {{m * IdentityMatrix[3], 0}, {0, I2}} // ArrayFlatten;

xi1cross = Inverse[RigidAdjoint[e1.gst0b1]].xi1 // Simplify;
Jbs11 = {xi1cross, {0, 0, 0, 0, 0, 0}} // Transpose;

xi1cross = Inverse[RigidAdjoint[e1.e2.gst0b2]].xi1 // Simplify;
xi2cross = Inverse[RigidAdjoint[e2.gst0b2]].xi2 // Simplify;
Jbs12 = {xi1cross, xi2cross} // Transpose;

M = Transpose[Jbs11].GenM1.Jbs11 + Transpose[Jbs12].GenM2.Jbs12 // Simplify;

P = m * g * COM1[[2]] + m * g * COM2[[2]];
G = {0, 0};
G[[1]] = D[P,  $\theta$ [[1]]];
G[[2]] = D[P,  $\theta$ [[2]]];
christoffel = ConstantArray[0, {2, 2, 2}];

For[i = 1, i ≤ 2, i++,
  For[j = 1, j ≤ 2, j++,
    For[k = 1, k ≤ 2, k++,
      christoffel[[i, j, k]] = 1/2 * (D[M[[i, j]],  $\theta$ [[k]]] + D[M[[i, k]],  $\theta$ [[j]]] - D[M[[k, j]],  $\theta$ [[i]]]);
    ]
  ]
]

MatrixForm[christoffel];

Coriolis = ConstantArray[0, {2, 2}];
For[i = 1, i ≤ 2, i++,
  For[j = 1, j ≤ 2, j++,
    Coriolis[[i, j]] = christoffel[[i, j, All]]. $\dot{\theta}$ ;
  ]
]

```

```
K = 1 / 2 * Transpose[θdot].M.θdot;
L = K - P // FullSimplify;
```

```
EL = {0, 0};
For[i = 1, i ≤ 2, i++,
  EL[[i]] = D[D[L, θdot[[i]], t] - D[L, θ[[i]]];
]
```

```
Labeled[MatrixForm[EL // FullSimplify], "Euler-Lagrange Equations"]
```

```
Labeled[MatrixForm[M], "M(θ)"]
```

```
Labeled[MatrixForm[Coriolis // FullSimplify], "C(θ,θ̇)"]
```

```
MatrixForm[InertiaToCoriolis[M, θ, θdot] // FullSimplify];
```

```
Labeled[MatrixForm[G // FullSimplify], "G(θ)"]
```

```
Labeled[MatrixForm[D[M, t] - 2 * Coriolis // Simplify], "(Ḣ - 2C)"]
```

```
MatrixForm[D[M, t] - 2 * Coriolis // Simplify] == MatrixForm[-1 * Transpose[D[M, t] - 2 * Coriolis // Simplify]]
```

$$\text{Out[867]=} \begin{pmatrix} \theta 1[t] \\ 0 \\ 0 \end{pmatrix}$$

COM1

$$\text{Out[868]=} \begin{pmatrix} -\frac{1}{2} + \frac{3}{2} \cos[\theta 2[t]] + \theta 1[t] \\ \frac{3}{2} \sin[\theta 2[t]] \\ 0 \end{pmatrix}$$

COM2

$$\text{Out[869]=} \begin{pmatrix} 0.00666667 & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0.173333 \end{pmatrix}$$

Inertia Tensor

$$\text{Out[892]} = \begin{pmatrix} -3. (\cos[\theta_2[t]] \theta_2'[t]^2 - 1.33333 \theta_1''[t] + \sin[\theta_2[t]] \theta_2''[t]) \\ 3. g \cos[\theta_2[t]] - 3. \sin[\theta_2[t]] \theta_1''[t] + 4.67333 \theta_2''[t] \end{pmatrix}$$

Euler-Lagrange Equations

$$\text{Out[893]} = \begin{pmatrix} 4. & -3 \sin[\theta_2[t]] \\ -3 \sin[\theta_2[t]] & 4.67333 \end{pmatrix}$$

$M(\theta)$

$$\text{Out[894]} = \begin{pmatrix} 0 & -3 \cos[\theta_2[t]] \theta_2'[t] \\ 0 & 0 \end{pmatrix}$$

$C(\theta, \dot{\theta})$

$$\text{Out[896]} = \begin{pmatrix} 0 \\ 3 g \cos[\theta_2[t]] \end{pmatrix}$$

$G(\theta)$

$$\text{Out[897]} = \begin{pmatrix} 0 & 3 \cos[\theta_2[t]] \theta_2'[t] \\ -3 \cos[\theta_2[t]] \theta_2'[t] & 0 \end{pmatrix}$$

$(\ddot{M} - 2C)$

Out[898] = True

#### Problem 4: Two-link RP manipulator

In[899] :=

```
m = 2;
a = 1;
b = 0.2;
c = 0;
```

```
theta = {theta1[t], theta2[t]};
thetadot = D[theta, t];
```

```
xi1 = {0, 0, 0, 0, 0, 1};
xi2 = {0, 1, 0, 0, 0, 0};
```

```
gst0b1 = {{1, 0, 0, a/2}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};
```

```

gst0b2 = {{0, -1, 0, a}, {1, 0, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};

e1 = TwistExp[xi1,  $\theta[1]$ ];
e2 = TwistExp[xi2,  $\theta[2]$ ];

gstb1 = TwistExp[xi1,  $\theta[1]$ ].gst0b1;
gstb2 = TwistExp[xi1,  $\theta[1]$ ].TwistExp[xi2,  $\theta[2]$ ].gst0b2 // FullSimplify;

Labeled[MatrixForm[COM1 = RigidPosition[gstb1]], "COM1"]
Labeled[MatrixForm[COM2 = RigidPosition[gstb2]], "COM2"]
Labeled[
  MatrixForm[I1 = {{m/12*(b^2 + c^2), 0, 0}, {0, m/12*(a^2 + c^2), 0}, {0, 0, m/12*(a^2 + b^2)}}], Inertia Tensor1]
Labeled[
  MatrixForm[I1 = {{m/12*(a^2 + c^2), 0, 0}, {0, m/12*(b^2 + c^2), 0}, {0, 0, m/12*(a^2 + b^2)}}], Inertia Tensor2]

GenM1 = {{m*IdentityMatrix[3], 0}, {0, I1}} // ArrayFlatten;
GenM2 = {{m*IdentityMatrix[3], 0}, {0, I2}} // ArrayFlatten;

xi1cross = Inverse[RigidAdjoint[e1.gst0b1]].xi1 // Simplify;
Jbsl1 = {xi1cross, {0, 0, 0, 0, 0, 0}} // Transpose;

xi1cross = Inverse[RigidAdjoint[e1.e2.gst0b2]].xi1 // Simplify;
xi2cross = Inverse[RigidAdjoint[e2.gst0b2]].xi2 // Simplify;
Jbsl2 = {xi1cross, xi2cross} // Transpose;

M = Transpose[Jbsl1].GenM1.Jbsl1 + Transpose[Jbsl2].GenM2.Jbsl2 // Simplify;

P = m*g*COM1[[2]] + m*g*COM2[[2]];
G = {0, 0};
G[[1]] = D[P,  $\theta[1]$ ];
G[[2]] = D[P,  $\theta[2]$ ];
christoffel = ConstantArray[0, {2, 2, 2}];

For[i = 1, i ≤ 2, i++,

```

```

For [j = 1, j ≤ 2, j++,
  For[k = 1, k ≤ 2, k++,
    christoffel[[i, j, k]] = 1/2 * (D[M[[i, j]], θ[[k]]] + D[M[[i, k]], θ[[j]]] - D[M[[k, j]], θ[[i]]]);
  ]
]

MatrixForm[christoffel];

Coriolis = ConstantArray[0, {2, 2}];
For[i = 1, i ≤ 2, i++,
  For[j = 1, j ≤ 2, j++,
    Coriolis[[i, j]] = christoffel[[i, j, All]].θdot;
  ]
]

K = 1/2 * Transpose[θdot].M.θdot;
L = K - P // FullSimplify;

EL = {0, 0};
For[i = 1, i ≤ 2, i++,
  EL[[i]] = D[D[L, θdot[[i]]], t] - D[L, θ[[i]]];
]

Labeled[MatrixForm[EL // FullSimplify], "Euler-Lagrange Equations"]

Labeled[MatrixForm[M], "M(θ)"]
Labeled[MatrixForm[Coriolis // FullSimplify], "C(θ, θ̇)"]
MatrixForm[InertiaToCoriolis[M, θ, θdot] // FullSimplify];
Labeled[MatrixForm[G // FullSimplify], "G(θ)"]

Labeled[MatrixForm[D[M, t] - 2 * Coriolis // Simplify], "(Ḣ - 2C)"]
MatrixForm[D[M, t] - 2 * Coriolis // Simplify] == MatrixForm[-1 * Transpose[D[M, t] - 2 * Coriolis // Simplify]]

```

$$\text{Out[913]} = \begin{pmatrix} \frac{1}{2} \cos[\theta_1[t]] \\ \frac{1}{2} \sin[\theta_1[t]] \\ 0 \end{pmatrix}$$

COM1

$$\text{Out[914]} = \begin{pmatrix} \cos[\theta_1[t]] - \sin[\theta_1[t]] \theta_2[t] \\ \sin[\theta_1[t]] + \cos[\theta_1[t]] \theta_2[t] \\ 0 \end{pmatrix}$$

COM2

$$\text{Out[915]} = \begin{pmatrix} 0.00666667 & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0.173333 \end{pmatrix}$$

Inertia Tensor1

$$\text{Out[916]} = \begin{pmatrix} \frac{1}{6} & 0 & 0 \\ 0 & 0.00666667 & 0 \\ 0 & 0 & 0.173333 \end{pmatrix}$$

Inertia Tensor2

$$\text{Out[938]} = \begin{pmatrix} 3. g \cos[\theta_1[t]] + 2.84667 \theta_1''[t] + \theta_2[t] (-2. g \sin[\theta_1[t]] + 4 \theta_1'[t] \theta_2'[t] + 2 \theta_2[t] \theta_1''[t]) + 2. \theta_2''[t] \\ 2. (g \cos[\theta_1[t]] - 1. \theta_2[t] \theta_1'[t]^2 + \theta_1''[t] + \theta_2''[t]) \end{pmatrix}$$

Euler-Lagrange Equations

$$\text{Out[939]} = \begin{pmatrix} 2.84667 + 2 \theta_2[t]^2 & 2. \\ 2. & 2. \end{pmatrix}$$

$M(\theta)$

$$\text{Out[940]} = \begin{pmatrix} 2 \theta_2[t] \theta_2'[t] & 2 \theta_2[t] \theta_1'[t] \\ -2 \theta_2[t] \theta_1'[t] & 0 \end{pmatrix}$$

$C(\theta, \dot{\theta})$

$$\text{Out[942]} = \begin{pmatrix} g (3 \cos[\theta_1[t]] - 2 \sin[\theta_1[t]] \theta_2[t]) \\ 2 g \cos[\theta_1[t]] \end{pmatrix}$$

$G(\theta)$

$$\text{Out}[943]= \begin{pmatrix} 0 & -4 \theta 2[t] \theta 1'[t] \\ 4 \theta 2[t] \theta 1'[t] & 0 \\ (\dot{M} - 2C) \end{pmatrix}$$

Out[944]= **True**

In[945]:=