Course: MECH 567: Robot Kinematics and Dynamics **Assigned Date:** 03/17/2023
Instructor: Robert Gregg, PhD **Due:** 11:59 PM on Thursday, 03/30/2023
We are using Gradescope.

Homework 5

Problem 1

Consider the nonlinear system

$$\dot{x}_1 = x_1 - x_1 x_2$$
$$\dot{x}_2 = 2x_1^2 - 2x_2.$$

Find the equilibrium points and investigate local stability around each equilibrium point.

Problem 2

Consider the nonlinear system

$$\dot{x}_1 = -x_1 - x_1 x_2^2$$
$$\dot{x}_2 = -x_2 - x_2 x_1^2$$

Show that (0,0) is the unique equilibrium point and investigate local stability. Investigate global stability using the Lyapunov function candidate

$$V = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2.$$

Problem 3

SHV book, Problem 9-3, page 342.

9-3 Complete the proof of stability of PD control for the flexible joint robot <u>without</u> gravity terms using the Lyapunov function candidate (9.20) and LaSalle's theorem. Show that $q1 = q2$ in the steady state.
The system is given as

$$D(q_1)\ddot{q}_1 + C(q_1, \dot{q}_1)\dot{q}_1 + g(q_1) + K(q_1 - q_2) = 0 \quad (9.18)$$
$$J\ddot{q}_2 + K(q_2 - q_1) = u$$

with the set point tracking PD control $u = -K_P\tilde{q}_2 - K_D\dot{q}_2$ and $\tilde{q}_2 = q_2 - q^d$. The Lyapunov function candidate is given as

$$V = \frac{1}{2}\dot{q}_1^T D(q_1)\dot{q}_1 + \frac{1}{2}\dot{q}_2^T J\dot{q}_2 + \frac{1}{2}(q_1 - q_2)^T K(q_1 - q_2) + \frac{1}{2}\tilde{q}_2^T K_P\tilde{q}_2 \quad (9.20)$$

Problem 4

SHV book, Problem 9-13, page 343.

9-13 Consider the coupled nonlinear system

$$\begin{aligned} \ddot{y}_1 + 3y_1y_2 + y_2^2 &= u_1 + y_2u_2 \\ \ddot{y}_2 + \cos(y_1)\dot{y}_2 + 3(y_1 - y_2) &= u_2 - 3(\cos(y_1))^2 y_2u_1 \end{aligned}$$

where u_1, u_2 are the inputs and y_1, y_2 are the outputs.

(a) What is the dimension of the state space?

(b) Choose state variables and write the system as a system of first order differential equations in state space.

(c) Find an inverse dynamics control so that the close-loop system is linear and decoupled, with each subsystem having natural frequency 10 radians and damping ratio $\frac{1}{2}$