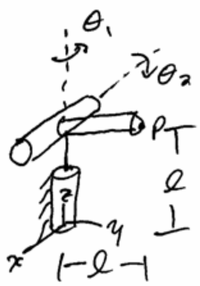
Course: Mech 567: Robot Kinematics and Dynamics
Instructor: Robert Gregg, PhD

Solution 3

Problem 1



$$p = \begin{bmatrix} 0 \\ l \\ l \end{bmatrix} \quad q = \begin{bmatrix} -l/\sqrt{2} \\ 0 \\ l + l/\sqrt{2} \end{bmatrix}$$

SP2: $e^1 e^2 p = q$

$$r = \begin{bmatrix} 0 \\ 0 \\ l \end{bmatrix}$$

$$u = p - r = \begin{bmatrix} 0 \\ l \\ 0 \end{bmatrix} \quad v = \begin{bmatrix} -l/\sqrt{2} \\ 0 \\ l/\sqrt{2} \end{bmatrix}$$

$$\omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \omega_2 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$$\alpha = \frac{\underbrace{(\omega_1 \cdot \omega_2)}_{0}(\omega_2 \cdot u) - \omega_1 \cdot v}{\underbrace{(\omega_1 \cdot \omega_2)^2}_{0} - 1}$$

$$= + l/\sqrt{2}$$

$$\beta = \frac{\underbrace{(\omega_1 \cdot \omega_2)}_{0}(\omega_1 \cdot v) - \underbrace{\omega_2 \cdot u}_{0}}{(\omega_1 \cdot \omega_2)^2 - 1}$$

$$= 0$$

$$\gamma^2 = \frac{\|u\|^2 - \alpha^2 - \beta^2 - 2\alpha\beta\underbrace{\omega_1 \cdot \omega_2}_{0}}{\underbrace{\|\omega_1 \times \omega_2\|}_{1}}$$

$$= l^2 - l^2/2$$

$$= l^2/2$$

$$\gamma = \pm l/\sqrt{2}$$

$$z = \alpha\omega_1 + \beta\omega_2 + \gamma(\omega_1 \times \omega_2)$$

$$z = \begin{bmatrix} 0 \\ 0 \\ l/\sqrt{2} \end{bmatrix} + \pm l/\sqrt{2} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \pm l/\sqrt{2} \\ l/\sqrt{2} \end{bmatrix}$$

$$c = z + r = \begin{bmatrix} 0 \\ \pm l/\sqrt{2} \\ l + l/\sqrt{2} \end{bmatrix}$$

Use SP1 to solve $e^2 p = c$ and $e^{-1} q = c$

$$\theta_1 : \quad e' c = q \qquad r = \begin{bmatrix} 0 \\ 0 \\ \ell \end{bmatrix} \qquad \omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$u_1 = c - r = \begin{bmatrix} 0 \\ \pm \ell/\sqrt{2} \\ \ell/\sqrt{2} \end{bmatrix}$$

$$v_1 = q - r = \begin{bmatrix} -\ell/\sqrt{2} \\ 0 \\ \ell/\sqrt{2} \end{bmatrix}$$

$$u_1' = u_1 - \omega_1 \omega_1^T u_1$$

$$= \begin{bmatrix} 0 \\ \pm \ell/\sqrt{2} \\ \ell/\sqrt{2} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \pm \ell/\sqrt{2} \\ \ell/\sqrt{2} \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 \\ \pm \ell/\sqrt{2} \\ 0 \end{bmatrix}$$

$$v_1' = v_1 - \omega_1 \omega_1^T v_1$$

$$= \begin{bmatrix} -\ell/\sqrt{2} \\ 0 \\ \ell/\sqrt{2} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -\ell/\sqrt{2} \\ 0 \\ \ell/\sqrt{2} \end{bmatrix} \right)$$

$$= \begin{bmatrix} -\ell/\sqrt{2} \\ 0 \\ 0 \end{bmatrix}$$

$$\theta_1 = \operatorname{atan2}\left(\omega_1 \cdot (u_1' \times v_1'),\ \overset{0}{u_1' \cdot v_1'}\right)$$

$$= \operatorname{atan2}\left(\pm \ell/\sqrt{2},\ 0\right)$$

$$= \pm \pi/2$$

θ_2: $e^{2} p = c$ $\quad r = \begin{bmatrix} 0 \\ 0 \\ \ell \end{bmatrix}$ $\quad \omega_2 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$

$u_2 = p - r = \begin{bmatrix} 0 \\ \ell \\ 0 \end{bmatrix}$ $\quad v_2 = \begin{bmatrix} 0 \\ \pm \ell/\sqrt{2} \\ \ell/\sqrt{2} \end{bmatrix}$

$$u_2' = u_2 - \omega_2(\omega_2 \cdot u_2)$$
$$= \begin{bmatrix} 0 \\ \ell \\ 0 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \left(\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \ell \\ 0 \end{bmatrix} \right)$$
$$= \begin{bmatrix} 0 \\ \ell \\ 0 \end{bmatrix}$$

$$v_2' = v_2 - \omega_2(\omega_2 \cdot v_2)$$
$$= \begin{bmatrix} 0 \\ \pm \ell/\sqrt{2} \\ \ell/\sqrt{2} \end{bmatrix}$$

$$\theta_2 = \operatorname{atan2}\left(\omega_2 \cdot (u_2' \times v_2'),\ u_2' \cdot v_2'\right)$$
$$= \operatorname{atan2}\left(\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \ell^2/\sqrt{2} \\ 0 \\ 0 \end{bmatrix},\ \pm \ell^2/\sqrt{2} \right)$$
$$= \operatorname{atan2}\left(-\ell^2/\sqrt{2},\ \pm \ell^2/\sqrt{2}\right)$$
$$= 3\pi/4 \quad \& \quad -\pi/4$$

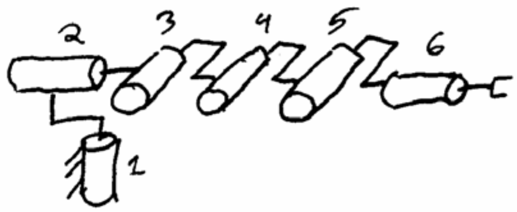
Total soln:

1 $\quad \theta_1 = -\pi/2$; $\theta_2 = -3\pi/4$

2 $\quad \theta_1 = \pi/2$; $\theta_2 = -\pi/4$

Problem 2

. ii



$e^{1} e^{2} e^{3} e^{4} e^{5} e^{6} g_{st}(0) = g_d$

$g_{st}\, g_d^{-1} = e^{-6} e^{-5} e^{-4} e^{-3} e^{-2} e^{-1}$

choose p_{123} which lies ~~on~~ ω_1, ω_2, and ω_3

$g_{st}\, g_d^{-1} p_{123} = e^{-6} e^{-5} e^{-4} p_{123}$

choose p_{56} on ω_5, ω_6

$g_{st}\, g_d^{-1} p_{123} - p_{56} = e^{-6} e^{-5} e^{-4} p_{123} - p_{56}$

$\| g_{st}\, g_d^{-1} p_{123} - p_{56} \| = \| e^{-4} p_{123} - p_{56} \|$

Use SP 3 to solve for $-\theta_4$

$g_{st}\, g_d^{-1} p_{123} = e^{-6} e^{-5} e^{-4} p_{123}$

$g_{st}\, g_d^{-1} p_{123} = e^{-6} e^{-5} p_w$

Use SP 2 to solve for $-\theta_5, -\theta_6$

$e^{-3} e^{-2} e^{-1} = e^{4} e^{5} e^{6} g_{st}\, g_d^{-1}$

choose p_1 on ω_1 and not ω_3 or ω_2

$e^{-3} e^{-2} p_1 = e^{4} e^{5} e^{6} g_{st}\, g_d^{-1} p_1$

use SP 2 to solve for $-\theta_3, -\theta_2$

$e^{-1} = e^{2} e^{3} e^{4} e^{5} e^{6} g_{st}\, g_d^{-1}$

choose p not on ω_1

$e^{-1} p = e^{2} e^{3} e^{4} e^{5} e^{6} g_{st}\, g_d^{-1} p$

Use SP 1 to solve for $-\theta_1$

Max # solutions

		Total
θ_4	SP 3 : 2 solns	2
θ_5, θ_6	SP 2 : 2 solns	4
θ_3, θ_2	SP 2 : 2 solns	8
θ_1	SP 1 : 1 soln	8

Problem 3

(a)

$$V_{ab}^s = \begin{bmatrix} v_{ab}^s \\ \omega_{ab}^s \end{bmatrix} = \begin{bmatrix} -\dot{R}_{ab}\, R_{ab}^\top\, p_{ab} + \dot{p}_{ab} \\ \omega_{ab}^s \end{bmatrix} = \begin{bmatrix} -\hat{\omega}_{ab}^s\, p_{ab} + \dot{p}_{ab} \\ \omega_{ab}^s \end{bmatrix}$$

$$V_{ab}^s = \begin{bmatrix} -\begin{bmatrix} 0 \\ 0 \\ \frac{\pi}{2} \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\ 0 \\ 0 \\ \frac{\pi}{2} \end{bmatrix} = \begin{bmatrix} \pi \\ -\frac{\pi}{2} \\ 0 \\ 0 \\ 0 \\ \frac{\pi}{2} \end{bmatrix}$$

Or

$$\omega = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad q = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \qquad \dot{\theta} = \frac{\pi}{2} \qquad h = 0$$

$$V_{ab}^s = \xi\dot{\theta} = \begin{bmatrix} -\omega \times q + h\omega \\ \omega \end{bmatrix} \dot{\theta} = \begin{bmatrix} \pi \\ -\frac{\pi}{2} \\ 0 \\ 0 \\ 0 \\ \frac{\pi}{2} \end{bmatrix}$$

(b)

$$q_a(0) = g_{ab}(0)\, q_b = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 1 \end{bmatrix}$$

$$v_{q_a} = \hat{V}_{ab}^s\, q_a = \begin{bmatrix} \hat{\omega}_{ab}^s & v_{ab}^s \\ 0 & 0 \end{bmatrix} q_a$$

$$v_{q_a}(0) = \begin{bmatrix} 0 & -\frac{\pi}{2} & 0 & \pi \\ \frac{\pi}{2} & 0 & 0 & -\frac{\pi}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{\pi}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Problem 4

(a)

$$\dot{\theta} = \frac{\pi}{2} \qquad \Rightarrow \qquad \theta = \frac{\pi}{2} \quad \text{at t=1}$$

Note that q_b is constant; point is fixed in body coordinates.

$$q_a(\frac{\pi}{2}) = g_{ab}(\frac{\pi}{2})\, q_b = e^{\hat{\xi}\,\frac{\pi}{2}} g_{ab}(0) q_b(0) = e^{\hat{\xi}\,\frac{\pi}{2}} q_a(0)$$

$$e^{\hat{\xi}\,\frac{\pi}{2}} = \begin{bmatrix} e^{\hat{z}\,\frac{\pi}{2}} & (I - e^{\hat{z}\,\frac{\pi}{2}})\omega \times v \\ 0 & 0 \end{bmatrix} \qquad \omega = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad v = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

$$q_a(\frac{\pi}{2}) = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

(b) Like Problem 3 part (a), but now

$$\dot{p}_{ab} = \begin{bmatrix} 0 \\ 0 \\ 2\,\pi \end{bmatrix} \qquad \text{or} \qquad h = \frac{\text{translational velocity parallel to } \omega}{\text{rotational velocity about } \omega} = \frac{2\pi}{\frac{\pi}{2}} = 4$$

$$\xi = \begin{bmatrix} 2 \\ -1 \\ 4 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad V_{ab}^{s} = \begin{bmatrix} \pi \\ -\frac{\pi}{2} \\ 2\pi \\ 0 \\ 0 \\ \frac{\pi}{2} \end{bmatrix}$$

$$q_a(t) = g_{ab}(t)\; q_b$$

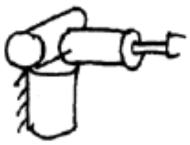
$$q_a(t) = \begin{bmatrix} cos(\frac{\pi}{2}t) & -sin(\frac{\pi}{2}t) & 0 & 1 \\ sin(\frac{\pi}{2}t) & cos(\frac{\pi}{2}t) & 0 & 2 \\ 0 & 0 & 1 & 2\pi t \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

for t=1

$$q_a(1) = \begin{bmatrix} 0 \\ 2 \\ 2\pi \\ 1 \end{bmatrix}$$

Problem 5

(i)



$$\xi_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \xi_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \xi_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

$$e^{\hat{\xi}_1\theta_1} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad e^{\hat{\xi}_2\theta_2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\theta_2 & s\theta_2 & 0 \\ 0 & -s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$e^{\hat{\xi}_3\theta_3} = \begin{bmatrix} c\theta_3 & 0 & s\theta_3 & 0 \\ 0 & 1 & 0 & 0 \\ -s\theta_3 & 0 & c\theta_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\xi_1' = \xi_1$$

$$\xi_2' = Ad_{(e^1)}\,\xi_2 \qquad Ad_{(e^1)} = \begin{bmatrix} R_z(\theta_1) & 0 \\ 0 & R_z(\theta_1) \end{bmatrix}$$

$$\xi_2' = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -c\theta_1 \\ -s\theta_1 \\ 0 \end{bmatrix}$$

$$\xi_3' = Ad_{(e^1e^2)}\,\xi_3 \qquad Ad_{(e^1e^2)} = \begin{bmatrix} R_z(\theta_1)R_x(\theta_2) & 0 \\ 0 & R_z(\theta_1)R_{-x}(\theta_2) \end{bmatrix}$$

$$\xi_3' = \begin{bmatrix} 0 \\ 0 \\ 0 \\ R_z(\theta_1)\begin{bmatrix} 0 \\ c\theta_2 \\ -s\theta_2 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -s\theta_1 c\theta_2 \\ c\theta_1 c\theta_2 \\ -s\theta_2 \end{bmatrix}$$

$$J^s = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -c\theta_1 & -s\theta_1 c\theta_2 \\ 0 & -s\theta_1 & c\theta_1 c\theta_2 \\ 1 & 0 & -s\theta_2 \end{bmatrix}$$

Problem 6

(a) See next pages!

(b) Some components of the force F_B do not appear in τ because they map to the nullspace of $(J^B)^T$. In other words, these forces do not induce torque on the joints because they are resisted by the structure of the links directly.

i) Elbow

```
In[650]:= ClearAll["Global`*"]
Needs["Screws`", "C://Users/            /Desktop//Screws.m"]
xi1 = {0, 0, 0, 0, 0, 1};
xi2 = {0, 0, 0, -1, 0, 0};
xi3 = {0, 0, l1, -1, 0, 0};
xi4 = {0, 0, l1 + l2, -1, 0, 0};
xi5 = {l1 + l2, 0, 0, 0, 0, 1};
xi6 = {0, 0, 0, 0, 1, 0};


MatrixForm[e1 = TwistExp[xi1, (0)]];
MatrixForm[e2 = TwistExp[xi2, (Pi / 2)]];
MatrixForm[e3 = TwistExp[xi3, (0)]];
MatrixForm[e4 = TwistExp[xi4, (0)]];
MatrixForm[e5 = TwistExp[xi5, (0)]];
MatrixForm[e6 = TwistExp[xi6, (0)]];
MatrixForm[gst0 = {{1, 0, 0, 0}, {0, 1, 0, l1 + l2 + l3}, {0, 0, 1, 0}, {0, 0, 0, 1}}];

MatrixForm[gst = e1.e2.e3.e4.e5.e6.gst0 // Simplify];

g1 = e1.e2.e3.e4.e5.e6.gst0;
g2 = e2.e3.e4.e5.e6.gst0;
g3 = e3.e4.e5.e6.gst0;
g4 = e4.e5.e6.gst0;
g5 = e5.e6.gst0;
g6 = e6.gst0;
Ad1 = RigidAdjoint[g1];
Ad2 = RigidAdjoint[g2];
Ad3 = RigidAdjoint[g3];
Ad4 = RigidAdjoint[g4];
Ad5 = RigidAdjoint[g5];
Ad6 = RigidAdjoint[g6];

Ad1 = Inverse[Ad1];
Ad2 = Inverse[Ad2];
Ad3 = Inverse[Ad3];
Ad4 = Inverse[Ad4];
Ad5 = Inverse[Ad5];
Ad6 = Inverse[Ad6];

xi1t = Ad1.xi1;
xi2t = Ad2.xi2;
xi3t = Ad3.xi3;
xi4t = Ad4.xi4;
xi5t = Ad5.xi5;
xi6t = Ad6.xi6;
Jb = {xi1t, xi2t, xi3t, xi4t, xi5t, xi6t};
Fb = {fx, fy, fz, taux, tauy, tauz};
MatrixForm[Jb = Transpose[Jb]]
Tau = MatrixForm[-(Transpose[Jb]).Fb]
```

Out[692]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & -l3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -l1-l2-l3 & -l2-l3 & -l3 & 0 & 0 \\ 0 & -1 & -1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Out[693]//MatrixForm=

$$\begin{pmatrix} tauy \\ fz\,(l1+l2+l3)+taux \\ fz\,(l2+l3)+taux \\ fz\,l3+taux \\ fx\,l3-tauz \\ -tauy \end{pmatrix}$$

ii) Inverse Elbow

In[694]:=
```
ClearAll["Global`*"]
Needs["Screws`", "C://Users            //Desktop//Screws.m"]
xi1 = {0, 0, 0, 0, 0, 1};
xi2 = {0, 0, 0, 0, 1, 0};
xi3 = {0, 0, 0, -1, 0, 0};
xi4 = {0, 0, l1, -1, 0, 0};
xi5 = {0, 0, l1 + l2, -1, 0, 0};
xi6 = {0, 0, 0, 0, 1, 0};


MatrixForm[e1 = TwistExp[xi1, (0)]];
MatrixForm[e2 = TwistExp[xi2, (Pi / 2)]];
MatrixForm[e3 = TwistExp[xi3, (0)]];
MatrixForm[e4 = TwistExp[xi4, (0)]];
MatrixForm[e5 = TwistExp[xi5, (0)]];
MatrixForm[e6 = TwistExp[xi6, (0)]];
MatrixForm[gst0 = {{1, 0, 0, 0}, {0, 1, 0, l1 + l2 + l3}, {0, 0, 1, 0}, {0, 0, 0, 1}}];

MatrixForm[gst = e1.e2.e3.e4.e5.e6.gst0 // Simplify];



g1 = e1.e2.e3.e4.e5.e6.gst0;
g2 = e2.e3.e4.e5.e6.gst0;
g3 = e3.e4.e5.e6.gst0;
g4 = e4.e5.e6.gst0;
g5 = e5.e6.gst0;
g6 = e6.gst0;
Ad1 = RigidAdjoint[g1];
Ad2 = RigidAdjoint[g2];
Ad3 = RigidAdjoint[g3];
Ad4 = RigidAdjoint[g4];
Ad5 = RigidAdjoint[g5];
Ad6 = RigidAdjoint[g6];

Ad1 = Inverse[Ad1];
Ad2 = Inverse[Ad2];
Ad3 = Inverse[Ad3];
Ad4 = Inverse[Ad4];
Ad5 = Inverse[Ad5];
Ad6 = Inverse[Ad6];

xi1t = Ad1.xi1;
xi2t = Ad2.xi2;
xi3t = Ad3.xi3;
xi4t = Ad4.xi4;
xi5t = Ad5.xi5;
xi6t = Ad6.xi6;
Jb = {xi1t, xi2t, xi3t, xi4t, xi5t, xi6t};
Fb = {fx, fy, fz, taux, tauy, tauz};
```

```
MatrixForm[Jb = Transpose[Jb]]
Tau = MatrixForm[-(Transpose[Jb]).Fb]
```

Out[736]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -l1-l2-l3 & 0 & -l1-l2-l3 & -l2-l3 & -l3 & 0 \\ -1 & 0 & -1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Out[737]//MatrixForm=

$$\begin{pmatrix} fz\,(l1+l2+l3)+taux \\ -tauy \\ fz\,(l1+l2+l3)+taux \\ fz\,(l2+l3)+taux \\ fz\,l3+taux \\ -tauy \end{pmatrix}$$

iii) Stanford

In[738]:=
```
ClearAll["Global`*"]
Needs["Screws`", "C://Users//          //Desktop//Screws.m"]
xi1 = {0, 0, 0, 0, 0, 1};
xi2 = {0, 0, 0, -1, 0, 0};
xi3 = {0, 1, 0, 0, 0, 0};
xi4 = {0, 0, l1, -1, 0, 0};
xi5 = {l1, 0, 0, 0, 0, 1};
xi6 = {0, 0, 0, 0, 1, 0};


MatrixForm[e1 = TwistExp[xi1, (0)]];
MatrixForm[e2 = TwistExp[xi2, (Pi/2)]];
MatrixForm[e3 = TwistExp[xi3, (0)]];
MatrixForm[e4 = TwistExp[xi4, (0)]];
MatrixForm[e5 = TwistExp[xi5, (0)]];
MatrixForm[e6 = TwistExp[xi6, (0)]];
MatrixForm[gst0 = {{1, 0, 0, 0}, {0, 1, 0, l1 + l2}, {0, 0, 1, 0}, {0, 0, 0, 1}}];

MatrixForm[gst = e1.e2.e3.e4.e5.e6.gst0 // Simplify];



g1 = e1.e2.e3.e4.e5.e6.gst0;
g2 = e2.e3.e4.e5.e6.gst0;
g3 = e3.e4.e5.e6.gst0;
g4 = e4.e5.e6.gst0;
g5 = e5.e6.gst0;
g6 = e6.gst0;
Ad1 = RigidAdjoint[g1];
Ad2 = RigidAdjoint[g2];
Ad3 = RigidAdjoint[g3];
Ad4 = RigidAdjoint[g4];
Ad5 = RigidAdjoint[g5];
Ad6 = RigidAdjoint[g6];

Ad1 = Inverse[Ad1];
Ad2 = Inverse[Ad2];
Ad3 = Inverse[Ad3];
Ad4 = Inverse[Ad4];
Ad5 = Inverse[Ad5];
Ad6 = Inverse[Ad6];

xi1t = Ad1.xi1;
xi2t = Ad2.xi2;
xi3t = Ad3.xi3;
xi4t = Ad4.xi4;
xi5t = Ad5.xi5;
xi6t = Ad6.xi6;
MatrixForm[Jb = {xi1t, xi2t, xi3t, xi4t, xi5t, xi6t}];
Fb = {fx, fy, fz, taux, tauy, tauz};
```

```
MatrixForm[Jb = Transpose[Jb]]
Tau = MatrixForm[-(Transpose[Jb]).Fb]
```

Out[780]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & -l2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -l1-l2 & 0 & -l2 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Out[781]//MatrixForm=

$$\begin{pmatrix} tauy \\ fz\,(l1+l2)+taux \\ -fy \\ fz\,l2+taux \\ fx\,l2-tauz \\ -tauy \end{pmatrix}$$

iv) Rhino

```
In[870]:= ClearAll["Global`*"]
Needs["Screws`", "C://Users            //Desktop//Screws.m"]
xi1 = {0, 1, 0, 0, 0, 0};
xi2 = {0, 0, 0, 0, 0, 1};
xi3 = {0, 0, 0, -1, 0, 0};
xi4 = {0, 0, l1, -1, 0, 0};
xi5 = {0, 0, l1 + l2, -1, 0, 0};
xi6 = {0, 0, 0, 0, 1, 0};


MatrixForm[e1 = TwistExp[xi1, (0)]];
MatrixForm[e2 = TwistExp[xi2, (Pi / 2)]];
MatrixForm[e3 = TwistExp[xi3, (0)]];
MatrixForm[e4 = TwistExp[xi4, (0)]];
MatrixForm[e5 = TwistExp[xi5, (0)]];
MatrixForm[e6 = TwistExp[xi6, (0)]];
MatrixForm[gst0 = {{1, 0, 0, 0}, {0, 1, 0, l1 + l2 + l3}, {0, 0, 1, 0}, {0, 0, 0, 1}}];

MatrixForm[gst = e1.e2.e3.e4.e5.e6.gst0 // Simplify];



g1 = e1.e2.e3.e4.e5.e6.gst0;
g2 = e2.e3.e4.e5.e6.gst0;
g3 = e3.e4.e5.e6.gst0;
g4 = e4.e5.e6.gst0;
g5 = e5.e6.gst0;
g6 = e6.gst0;
Ad1 = RigidAdjoint[g1];
Ad2 = RigidAdjoint[g2];
Ad3 = RigidAdjoint[g3];
Ad4 = RigidAdjoint[g4];
Ad5 = RigidAdjoint[g5];
Ad6 = RigidAdjoint[g6];

Ad1 = Inverse[Ad1];
Ad2 = Inverse[Ad2];
Ad3 = Inverse[Ad3];
Ad4 = Inverse[Ad4];
Ad5 = Inverse[Ad5];
Ad6 = Inverse[Ad6];

xi1t = Ad1.xi1;
xi2t = Ad2.xi2;
xi3t = Ad3.xi3;
xi4t = Ad4.xi4;
xi5t = Ad5.xi5;
xi6t = Ad6.xi6;
Jb = {xi1t, xi2t, xi3t, xi4t, xi5t, xi6t};
Fb = {fx, fy, fz, taux, tauy, tauz};
```

```
MatrixForm[Jb = Transpose[Jb]]
Tau = MatrixForm[-(Transpose[Jb]).Fb]
```

Out[912]//MatrixForm=

$$\begin{pmatrix} 1 & -l1-l2-l3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -l1-l2-l3 & -l2-l3 & -l3 & 0 \\ 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Out[913]//MatrixForm=

$$\begin{pmatrix} -fx \\ -fx\,(-l1-l2-l3) - tauz \\ -fz\,(-l1-l2-l3) + taux \\ -fz\,(-l2-l3) + taux \\ fz\,l3 + taux \\ -tauy \end{pmatrix}$$