Course: Mech 567: Robot Kinematics & Dynamics
Instructor: Robert Gregg, PhD

Solution 2

Problem 1

$$\omega = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad q = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

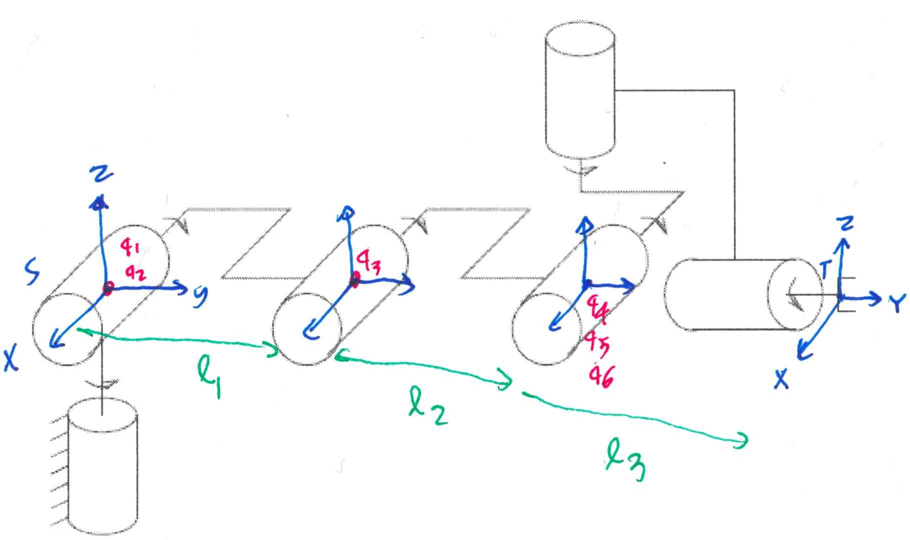
$$h = \frac{1}{10 \cdot 2\pi} = \frac{1}{20\,\pi} \qquad \text{(translation per revolution)}$$

$$v = -\omega \times q + h\,w$$

$$\xi = \begin{bmatrix} 1 \\ -1 \\ \frac{1}{20\,\pi} \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Problem 2

(i)



(i) Elbow manipulator

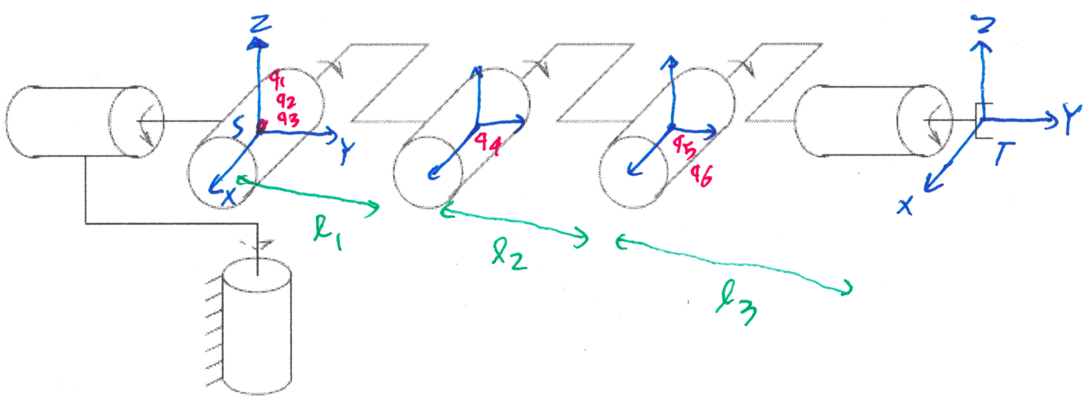
$$\omega_1 = \begin{bmatrix}0\\0\\1\end{bmatrix} \qquad \omega_2 = \omega_3 = \omega_4 = \begin{bmatrix}-1\\0\\0\end{bmatrix} \qquad \omega_5 = \begin{bmatrix}0\\0\\1\end{bmatrix} \qquad \omega_6 = \begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$q_1 = q_2 = \begin{bmatrix}0\\0\\0\end{bmatrix} \qquad q_3 = \begin{bmatrix}0\\L_1\\0\end{bmatrix} \qquad q_4 = q_5 = q_6 = \begin{bmatrix}0\\L_1+L_2\\0\end{bmatrix}$$

$$g_{st}(0) = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & L_1+L_2+L_3\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix} \qquad \xi = \begin{bmatrix}-\omega\times q\\ \omega\end{bmatrix}$$

$$\xi_1 = \begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix} \quad \xi_2 = \begin{bmatrix}0\\0\\0\\-1\\0\\0\end{bmatrix} \quad \xi_3 = \begin{bmatrix}0\\0\\L_1\\-1\\0\\0\end{bmatrix} \quad \xi_4 = \begin{bmatrix}0\\0\\L_1+L_2\\-1\\0\\0\end{bmatrix} \quad \xi_5 = \begin{bmatrix}L_1+L_2\\0\\0\\0\\0\\1\end{bmatrix} \quad \xi_6 = \begin{bmatrix}0\\0\\0\\0\\1\\0\end{bmatrix}$$

(ii)



(ii) Inverse elbow manipulator

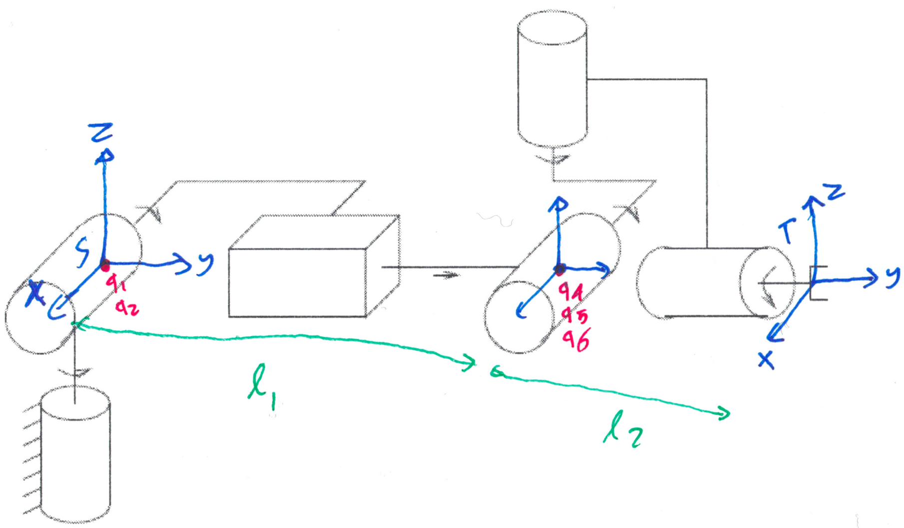
$$\omega_1 = \begin{bmatrix}0\\0\\1\end{bmatrix} \qquad \omega_2 = \begin{bmatrix}0\\1\\0\end{bmatrix} \qquad \omega_3 = \omega_4 = \omega_5 = \begin{bmatrix}-1\\0\\0\end{bmatrix} \qquad \omega_6 = \begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$q_1 = q_2 = q_3 = \begin{bmatrix}0\\0\\0\end{bmatrix} \qquad q_4 = \begin{bmatrix}0\\L_1\\0\end{bmatrix} \qquad q_5 = q_6 = \begin{bmatrix}0\\L_1+L_2\\0\end{bmatrix}$$

$$g_{st}(0) = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & L_1+L_2+L_3\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix} \qquad \xi = \begin{bmatrix}-\omega\times q\\ \omega\end{bmatrix}$$

$$\xi_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \xi_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad \xi_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \xi_4 = \begin{bmatrix} 0 \\ 0 \\ L_1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \xi_5 = \begin{bmatrix} 0 \\ 0 \\ L_1 + L_2 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \xi_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

(iii)



(iii) Stanford manipulator

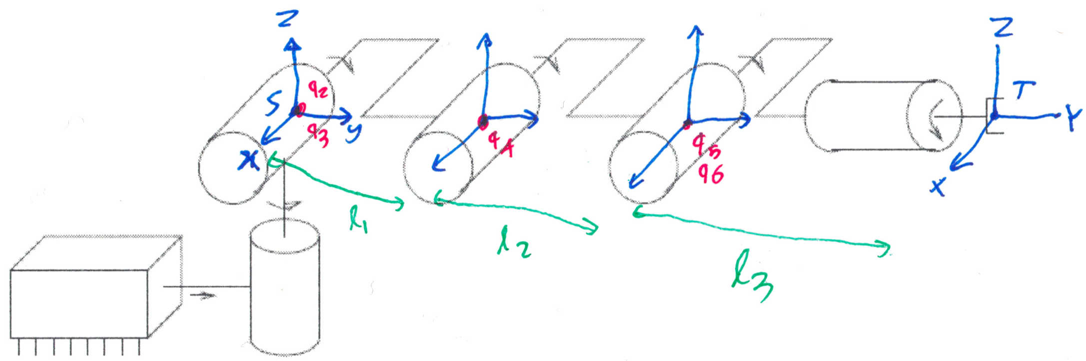
$$\omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \omega_2 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad v_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \omega_4 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad \omega_5 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \omega_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$q_1 = q_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad q_4 = q_5 = q_6 = \begin{bmatrix} 0 \\ L_1 \\ 0 \end{bmatrix}$$

$$g_{st}(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L_1 + L_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \xi = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix}$$

$$\xi_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \xi_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \xi_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \xi_4 = \begin{bmatrix} 0 \\ 0 \\ L_1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \xi_5 = \begin{bmatrix} L_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \xi_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

(iv)



(iv) Rhino robot

$$v_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \omega_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \omega_3 = \omega_4 = \omega_5 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \qquad \omega_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$q_2 = q_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad q_4 = \begin{bmatrix} 0 \\ L_1 \\ 0 \end{bmatrix} \qquad q_5 = q_6 = \begin{bmatrix} 0 \\ L_1 + L_2 \\ 0 \end{bmatrix}$$

$$g_{st}(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L_1 + L_2 + L_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \xi = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix}$$

$$\xi_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \xi_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad \xi_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} \qquad \xi_4 = \begin{bmatrix} 0 \\ 0 \\ L_1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \qquad \xi_5 = \begin{bmatrix} 0 \\ 0 \\ L_1 + L_2 \\ -1 \\ 0 \\ 0 \end{bmatrix} \qquad \xi_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Problem 3

$$e^{\hat{\xi}_1 \theta_1} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad e^{\hat{\xi}_2 \theta_2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_2 & \sin\theta_2 & 0 \\ 0 & -\sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$e^{\hat{\xi}_3 \theta_3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad e^{\hat{\xi}_4 \theta_4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_4 & \sin\theta_4 & L_1\,(1-\cos\theta_4) \\ 0 & -\sin\theta_4 & \cos\theta_4 & L_1\,\sin\theta_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$e^{\hat{\xi}_5 \theta_5} = \begin{bmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & L_1 \sin\theta_5 \\ \sin\theta_5 & \cos\theta_5 & 0 & L_1 (1-\cos\theta_5) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad e^{\hat{\xi}_6 \theta_6} = \begin{bmatrix} \cos\theta_6 & 0 & \sin\theta_6 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_6 & 0 & \cos\theta_6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 4

(i)

```
ClearAll["Global`*"]
Needs["Screws`" , "C://Mathematica//Screws.m"]
xi1 = { 0 , 0 , 0 , 0 , 0 , 1 };
xi2 = { 0 , 0 , 0 , -1 , 0 , 0 };
xi3 = { 0 , 0 , L1 , -1 , 0 , 0 };
xi4 = { 0 , 0 , L1 + L2 , -1 , 0 , 0 };
xi5 = { L1 + L2 , 0 , 0 , 0 , 0 , 1 };
xi6 = { 0 , 0 , 0 , 0 , 1 , 0 };
MatrixForm[ e1 = TwistExp[ xi1 , q1[t] ] ];
MatrixForm[ e2 = TwistExp[ xi2 , q2[t] ] ];
MatrixForm[ e3 = TwistExp[ xi3 , q3[t] ] ];
MatrixForm[ e4 = TwistExp[ xi4 , q4[t] ] ];
MatrixForm[ e5 = TwistExp[ xi5 , q5[t] ] ];
MatrixForm[ e6 = TwistExp[ xi6 , q6[t] ] ];
MatrixForm[ gst0 = { { 1 , 0 , 0 , 0 } , { 0 , 1 , 0 , L1 + L2 + L3 } ,
{ 0 , 0 , 1 , 0 } , { 0 , 0 , 0 , 1 } } ];
MatrixForm[ gst = Simplify[e1.e2.e3.e4.e5.e6.gst0] ]
```

(ii)

```
ClearAll["Global`*"]
Needs["Screws`" , "C://Mathematica//Screws.m"]
xi1 = { 0 , 0 , 0 , 0 , 0 , 1 };
xi2 = { 0 , 0 , 0 , 0 , 1 , 0 };
xi3 = { 0 , 0 , 0 , -1 , 0 , 0 };
xi4 = { 0 ,0 , L1 , -1 , 0 , 0 };
xi5 = { 0 , 0 , L1 + L2 , -1 , 0 , 0 };
xi6 = { 0 , 0 , 0 , 0 , 1 , 0 };
MatrixForm[ e1 = TwistExp[ xi1 , q1[t] ] ];
MatrixForm[ e2 = TwistExp[ xi2 , q2[t] ] ];
MatrixForm[ e3 = TwistExp[ xi3 , q3[t] ] ];
MatrixForm[ e4 = TwistExp[ xi4 , q4[t] ] ];
MatrixForm[ e5 = TwistExp[ xi5 , q5[t] ] ];
MatrixForm[ e6 = TwistExp[ xi6 , q6[t] ] ];
MatrixForm[ gst0 = { { 1 , 0 , 0 , 0 } , { 0 , 1 , 0 , L1 + L2 + L3 } ,
{ 0 , 0 , 1 , 0 } , { 0 , 0 , 0 , 1 } } ];
MatrixForm[ gst = Simplify[e1.e2.e3.e4.e5.e6.gst0] ]
```

(iii)

```
ClearAll["Global`*"]
Needs["Screws`" , "C://Mathematica//Screws.m"]
xi1 = { 0 , 0 , 0 , 0 , 0 , 1 };
xi2 = { 0 , 0 , 0 , −1 , 0 , 0 };
xi3 = { 0 , 1 , 0 , 0 , 0 , 0 };
xi4 = { 0 , 0 , L1 , −1 , 0 , 0 };
xi5 = { L1 , 0 , 0 , 0 , 0 , 1 };
xi6 = { 0 , 0 , 0 , 0 , 1 , 0 };
MatrixForm[ e1 = TwistExp[ xi1 , q1[t] ] ];
MatrixForm[ e2 = TwistExp[ xi2 , q2[t] ] ];
MatrixForm[ e3 = TwistExp[ xi3 , q3[t] ] ];
MatrixForm[ e4 = TwistExp[ xi4 , q4[t] ] ];
MatrixForm[ e5 = TwistExp[ xi5 , q5[t] ] ];
MatrixForm[ e6 = TwistExp[ xi6 , q6[t] ] ];
MatrixForm[ gst0 = { { 1 , 0 , 0 , 0 } , { 0 , 1 , 0 , L1 + L2 } , { 0 , 0 ,
{ 0 , 0 , 0 , 1 } } ];
MatrixForm[ gst = Simplify[e1.e2.e3.e4.e5.e6.gst0] ]
```

(iv)

```
ClearAll["Global`*"]
Needs["Screws`" , "C://Mathematica//Screws.m"]
xi1 = { 0 , 1 , 0 , 0 , 0 , 0 };
xi2 = { 0 , 0 , 0 , 0 , 0 , 1 };
xi3 = { 0 , 0 , 0 , −1 , 0 , 0 };
xi4 = { 0 , 0 , L1 , −1 , 0 , 0 };
xi5 = { 0 , 0 , L1 + L2 , −1 , 0 , 0 };
xi6 = { 0 , 0 , 0 , 0 , 1 , 0 };
MatrixForm[ e1 = TwistExp[ xi1 , q1[t] ] ];
MatrixForm[ e2 = TwistExp[ xi2 , q2[t] ] ];
MatrixForm[ e3 = TwistExp[ xi3 , q3[t] ] ];
MatrixForm[ e4 = TwistExp[ xi4 , q4[t] ] ];
MatrixForm[ e5 = TwistExp[ xi5 , q5[t] ] ];
MatrixForm[ e6 = TwistExp[ xi6 , q6[t] ] ];
MatrixForm[ gst0 = { { 1 , 0 , 0 , 0 } , { 0 , 1 , 0 , L1 + L2 + L3 } ,
{ 0 , 0 , 1 , 0 } , { 0 , 0 , 0 , 1 } } ];
MatrixForm[ gst = Simplify[e1.e2.e3.e4.e5.e6.gst0] ]
```