

NA 568 - Winter 2024

Localization

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Slides: Courtesy of Ryan Eustice

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Localization

“Using sensory information to locate the robot in its environment is the most fundamental problem to providing a mobile robot with autonomous capabilities.” [Cox '91]

- **Given**

- Map of the environment.
 - Sequence of sensor measurements.

- **Wanted**

- Estimate of the robot’s position.

- **Problem classes**

- Position tracking
 - Global localization
 - Kidnapped robot problem (recovery)

Landmark-based Localization



EKF_localization (μ_{t-1} , Σ_{t-1} , \mathbf{u}_t , \mathbf{z}_t , \mathbf{m}):

Prediction:

$$G_t = \frac{\partial g(\mathbf{u}_t, \mu_{t-1})}{\partial \mathbf{x}_{t-1}} = \begin{pmatrix} \frac{\partial x'}{\partial \mu_{t-1,x}} & \frac{\partial x'}{\partial \mu_{t-1,y}} & \frac{\partial x'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial y'}{\partial \mu_{t-1,x}} & \frac{\partial y'}{\partial \mu_{t-1,y}} & \frac{\partial y'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial \theta'}{\partial \mu_{t-1,x}} & \frac{\partial \theta'}{\partial \mu_{t-1,y}} & \frac{\partial \theta'}{\partial \mu_{t-1,\theta}} \end{pmatrix} \xrightarrow{\text{Jacobian of } g \text{ w.r.t location}}$$

$$V_t = \frac{\partial g(\mathbf{u}_t, \mu_{t-1})}{\partial \mathbf{u}_t} = \begin{pmatrix} \frac{\partial x'}{\partial v_t} & \frac{\partial x'}{\partial \omega_t} \\ \frac{\partial y'}{\partial v_t} & \frac{\partial y'}{\partial \omega_t} \\ \frac{\partial \theta'}{\partial v_t} & \frac{\partial \theta'}{\partial \omega_t} \end{pmatrix} \xrightarrow{\text{Jacobian of } g \text{ w.r.t control}}$$

$$M_t = \begin{pmatrix} \alpha_1 v_t^2 + \alpha_2 \omega_t^2 & 0 \\ 0 & \alpha_3 v_t^2 + \alpha_4 \omega_t^2 \end{pmatrix} \xrightarrow{\text{Motion noise}}$$

$$\bar{\mu}_t = g(\mathbf{u}_t, \mu_{t-1}) \xrightarrow{\text{Predicted mean}}$$

$$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + V_t M_t V_t^T \xrightarrow{\text{Predicted covariance}}$$

EKF_localization (μ_{t-1} , Σ_{t-1} , \mathbf{u}_v , \mathbf{z}_v , \mathbf{m}):

Correction:

$$\hat{\mathbf{z}}_t = \begin{pmatrix} \sqrt{(m_x - \bar{\mu}_{t,x})^2 + (m_y - \bar{\mu}_{t,y})^2} \\ \text{atan} 2(m_y - \bar{\mu}_{t,y}, m_x - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{pmatrix} \xrightarrow{\hspace{1cm}} \text{Predicted measurement mean}$$

$$H_t = \frac{\partial h(\bar{\mu}_t, \mathbf{m})}{\partial \mathbf{x}_t} = \begin{pmatrix} \frac{\partial r_t}{\partial \bar{\mu}_{t,x}} & \frac{\partial r_t}{\partial \bar{\mu}_{t,y}} & \frac{\partial r_t}{\partial \bar{\mu}_{t,\theta}} \\ \frac{\partial \phi_t}{\partial \bar{\mu}_{t,x}} & \frac{\partial \phi_t}{\partial \bar{\mu}_{t,y}} & \frac{\partial \phi_t}{\partial \bar{\mu}_{t,\theta}} \end{pmatrix} \xrightarrow{\hspace{1cm}} \text{Jacobian of } h \text{ w.r.t location}$$

$$Q_t = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\phi^2 \end{pmatrix} \xrightarrow{\hspace{1cm}} \text{Measurement covariance}$$

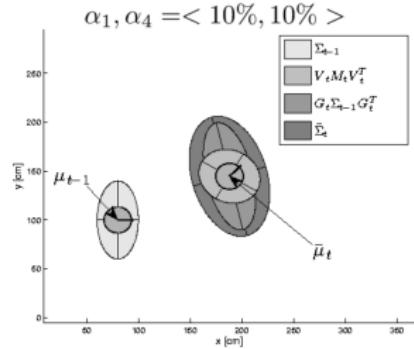
$$S_t = H_t \bar{\Sigma}_t H_t^T + Q_t \xrightarrow{\hspace{1cm}} \text{Innovation covariance}$$

$$K_t = \bar{\Sigma}_t H_t^T S_t^{-1} \xrightarrow{\hspace{1cm}} \text{Kalman gain}$$

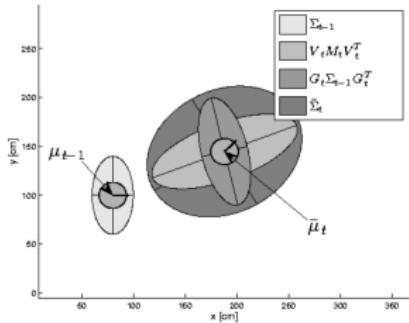
$$\mu_t = \bar{\mu}_t + K_t (\mathbf{z}_t - \hat{\mathbf{z}}_t) \xrightarrow{\hspace{1cm}} \text{Updated mean}$$

$$\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t \xrightarrow{\hspace{1cm}} \text{Updated covariance}$$

EKF Motion Prediction

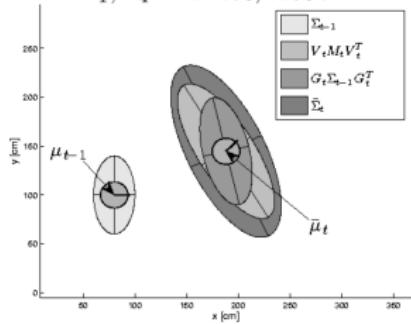


$\alpha_1, \alpha_4 = < 30\%, 10\% >$

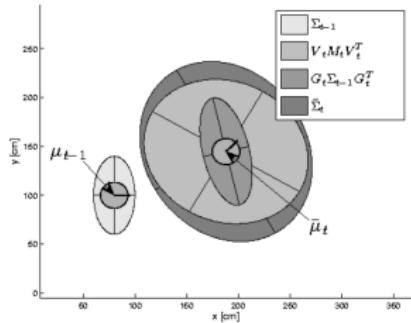


$$M_t = \begin{bmatrix} \alpha_1 v_t^2 + \alpha_2 w_t^2 & 0 \\ 0 & \alpha_3 v_t^2 + \alpha_4 w_t^2 \end{bmatrix}$$

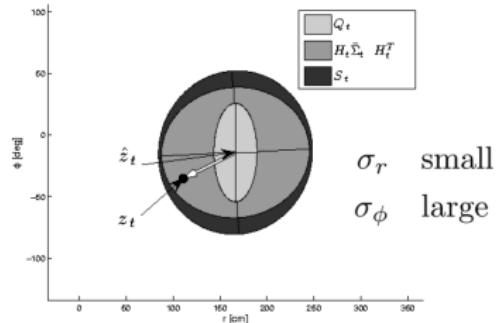
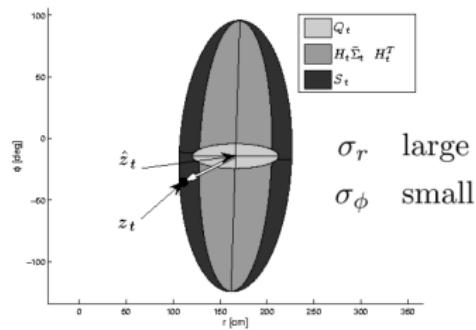
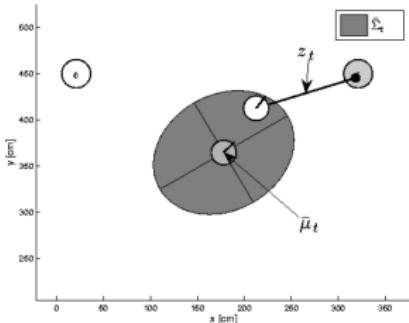
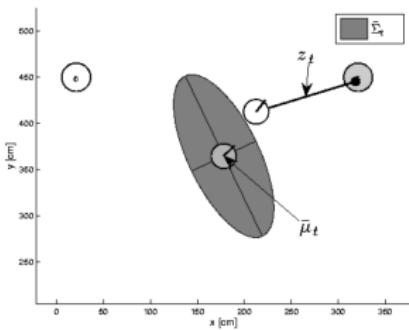
$\alpha_1, \alpha_4 = < 10\%, 30\% >$



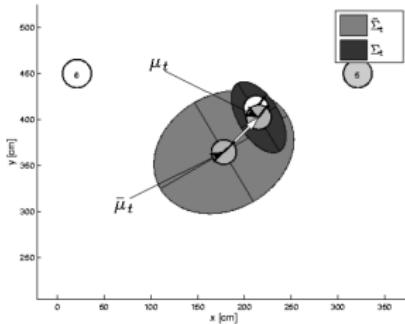
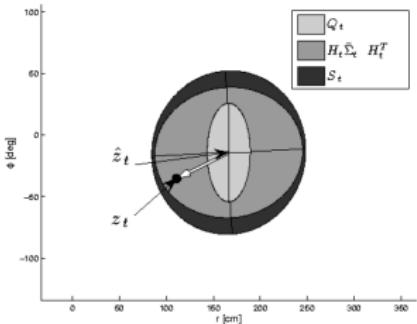
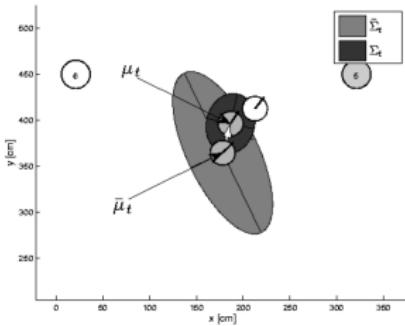
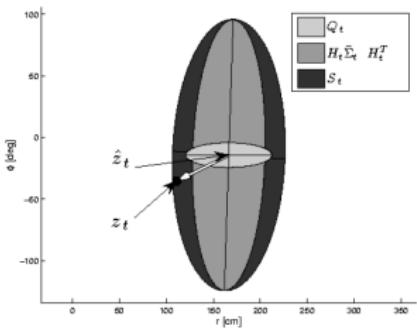
$\alpha_1, \alpha_4 = < 30\%, 30\% >$



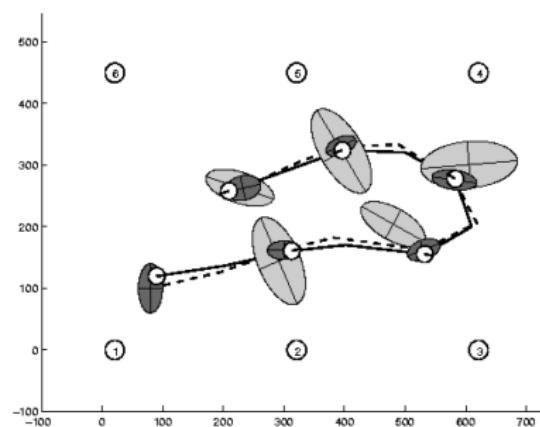
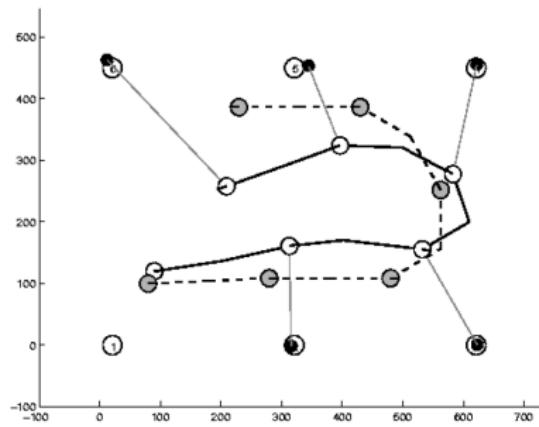
EKF Observation Prediction



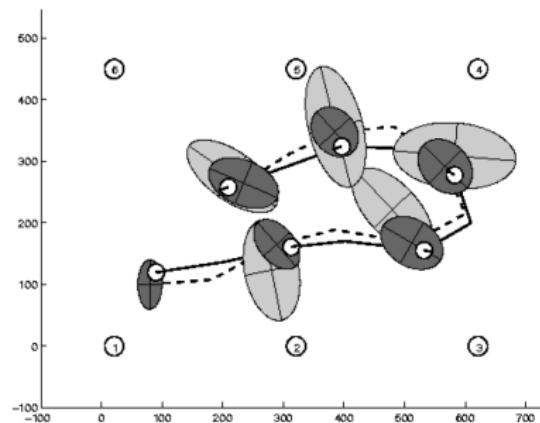
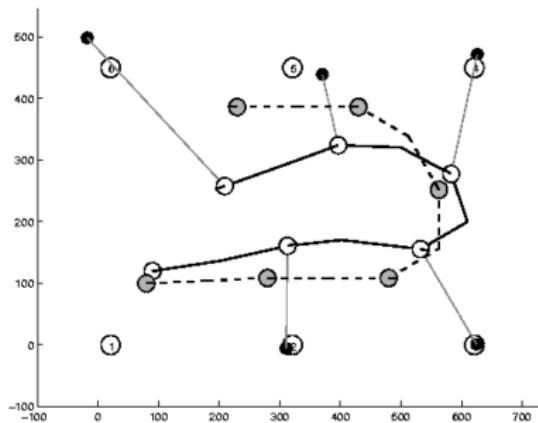
EKF Correction Step



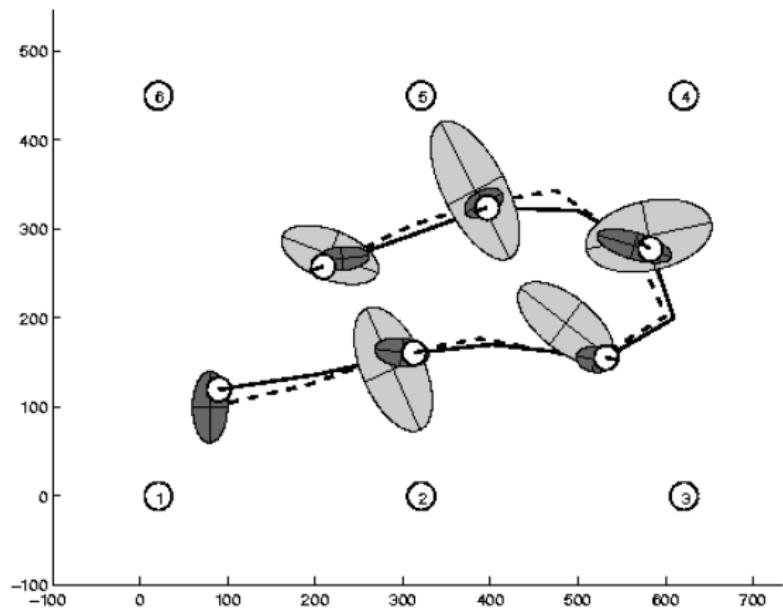
Estimation Sequence (1): Accurate Landmark Sensor



Estimation Sequence (2): Less Accurate Landmark Sensor



Comparison to GroundTruth



UKF_localization ($\mu_{t,1}, \Sigma_{t,1}, \mathbf{u}_t, \mathbf{z}_t, \mathbf{m}$):

Prediction:

$$M_t = \begin{pmatrix} \alpha_1 v_t^2 + \alpha_2 \omega_t^2 & 0 \\ 0 & \alpha_3 v_t^2 + \alpha_4 \omega_t^2 \end{pmatrix} \quad \text{Motion noise}$$

$$Q_t = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_r^2 \end{pmatrix} \quad \text{Measurement noise}$$

$$\mu_{t-1}^a = \left(\mu_{t-1}^T \quad (0\ 0)^T \quad (0\ 0)^T \right) \quad \text{Augmented state mean}$$

$$\Sigma_{t-1}^a = \begin{pmatrix} \Sigma_{t-1} & 0 & 0 \\ 0 & M_t & 0 \\ 0 & 0 & Q_t \end{pmatrix} \quad \text{Augmented covariance}$$

$$\chi_{t-1}^a = \left(\mu_{t-1}^a \quad \mu_{t-1}^a + \gamma \sqrt{\Sigma_{t-1}^a} \quad \mu_{t-1}^a - \gamma \sqrt{\Sigma_{t-1}^a} \right) \quad \text{Sigma points}$$

$$\bar{\chi}_t^x = g(\mathbf{u}_t + \chi_t^u, \chi_{t-1}^x) \quad \text{Prediction of sigma points}$$

$$\bar{\mu}_t = \sum_{i=0}^{2L} w_m^i \bar{\chi}_{i,t}^x \quad \text{Predicted mean}$$

$$\bar{\Sigma}_t = \sum_{i=0}^{2L} w_c^i (\bar{\chi}_{i,t}^x - \bar{\mu}_t) (\bar{\chi}_{i,t}^x - \bar{\mu}_t)^T \quad \text{Predicted covariance}$$

UKF_localization ($\mu_{t,1}, \Sigma_{t,1}, \mathbf{u}_t, \mathbf{z}_t, \mathbf{m}$):

Correction:

$$\bar{\mathbf{Z}}_t = h(\bar{\chi}_t^x) + \chi_t^z \quad \text{Measurement sigma points}$$

$$\hat{\mathbf{z}}_t = \sum_{i=0}^{2L} w_m^i \bar{\mathbf{Z}}_{i,t} \quad \text{Predicted measurement mean}$$

$$S_t = \sum_{i=0}^{2L} w_c^i (\bar{\mathbf{Z}}_{i,t} - \hat{\mathbf{z}}_t)(\bar{\mathbf{Z}}_{i,t} - \hat{\mathbf{z}}_t)^T \quad \text{Pred. measurement covariance}$$

$$\Sigma_t^{x,z} = \sum_{i=0}^{2L} w_c^i (\bar{\chi}_{i,t}^x - \bar{\mu}_t)(\bar{\mathbf{Z}}_{i,t} - \hat{\mathbf{z}}_t)^T \quad \text{Cross-covariance}$$

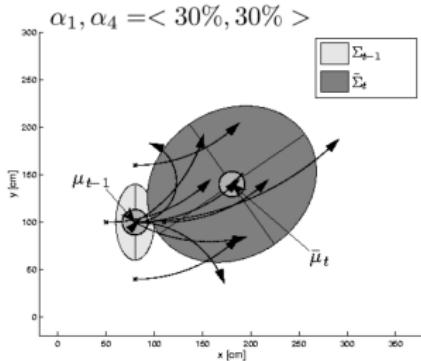
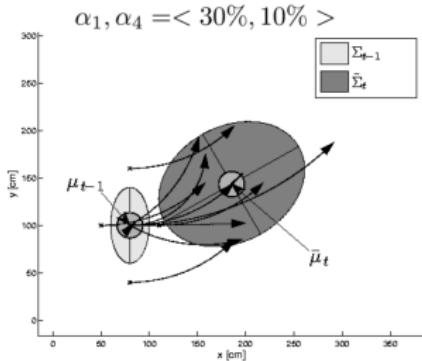
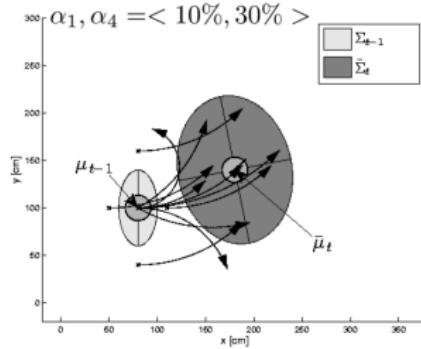
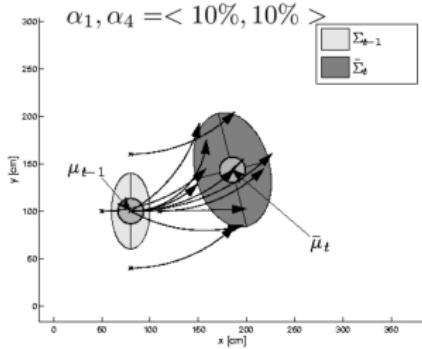
$$K_t = \Sigma_t^{x,z} S_t^{-1} \quad \text{Kalman gain}$$

$$\mu_t = \bar{\mu}_t + K_t (\mathbf{z}_t - \hat{\mathbf{z}}_t) \quad \text{Updated mean}$$

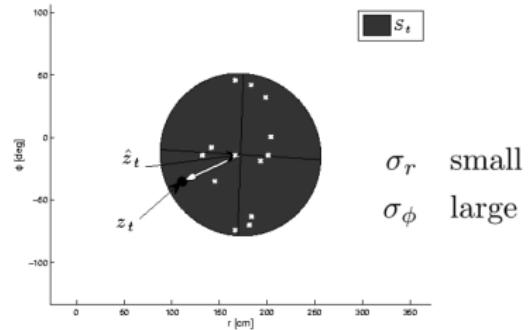
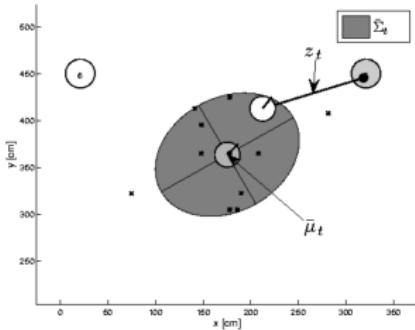
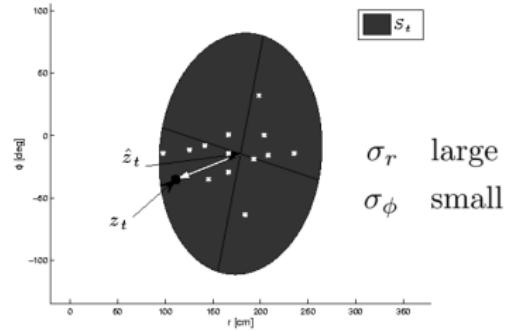
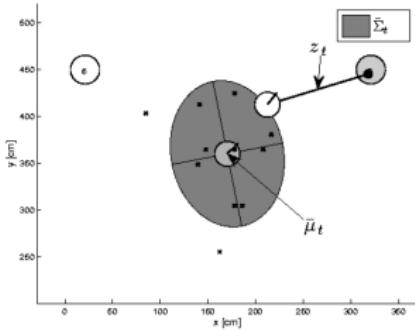
$$\Sigma_t = \bar{\Sigma}_t - K_t S_t K_t^T \quad \text{Updated covariance}$$

UKF Motion Prediction

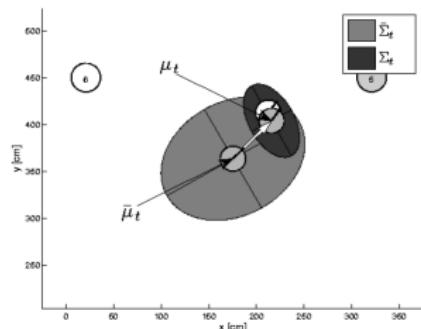
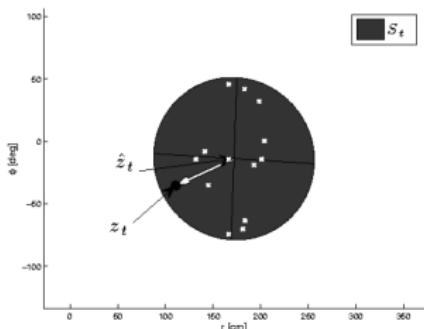
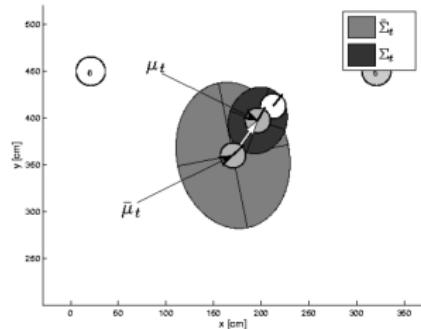
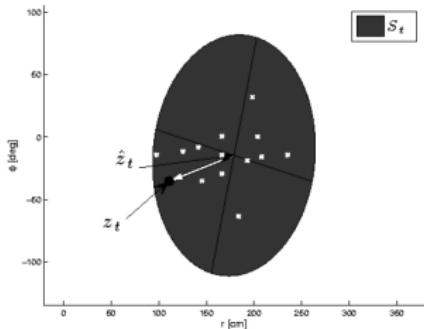
$$M_t = \begin{bmatrix} \alpha_1 v_t^2 + \alpha_2 w_t^2 & 0 \\ 0 & \alpha_3 v_t^2 + \alpha_4 w_t^2 \end{bmatrix}$$



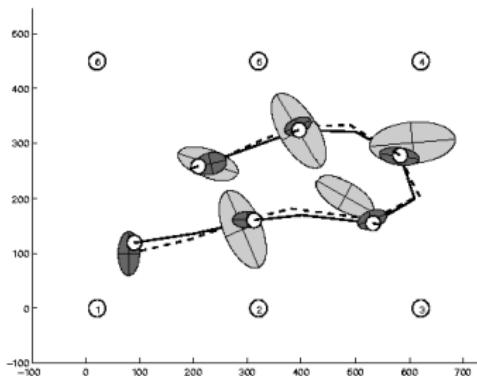
UKF Observation Prediction



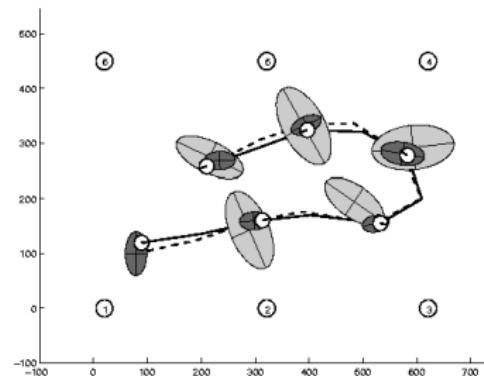
UKF Correction Step



Estimation Sequence

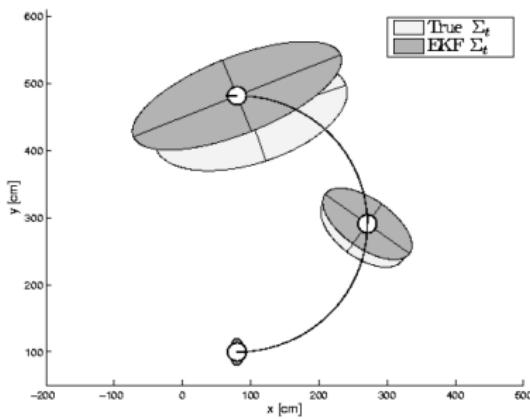


EKF

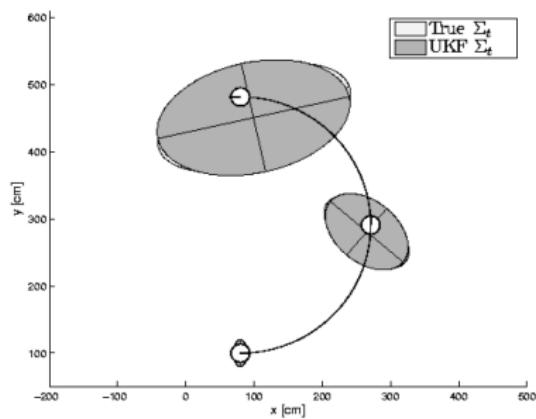


UKF

Prediction Quality

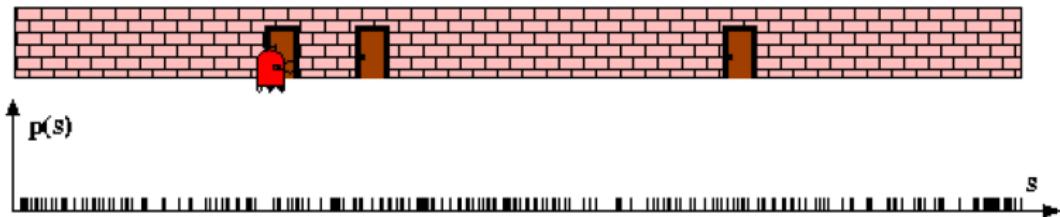


EKF



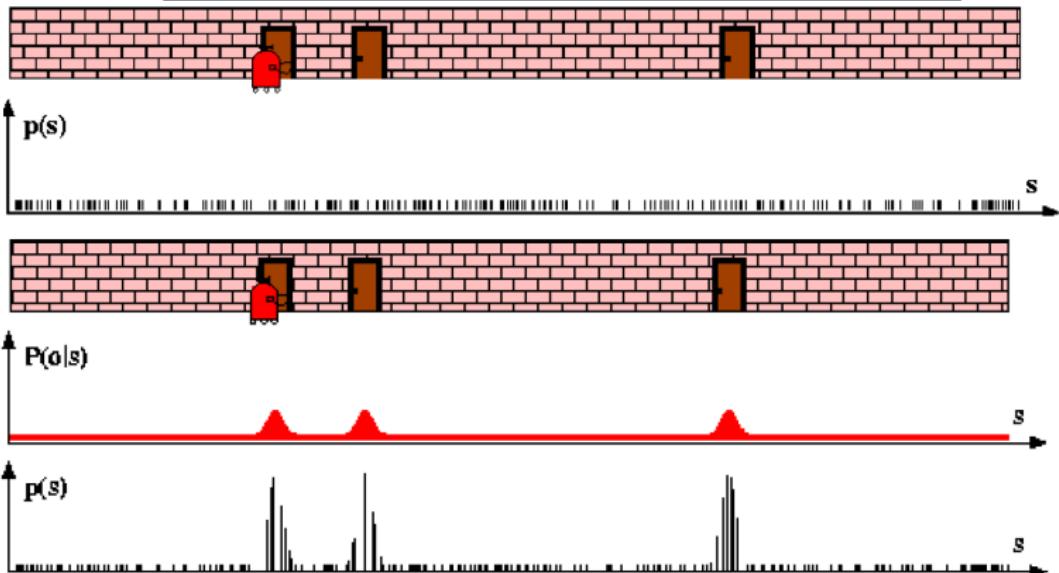
UKF

Particle Filters



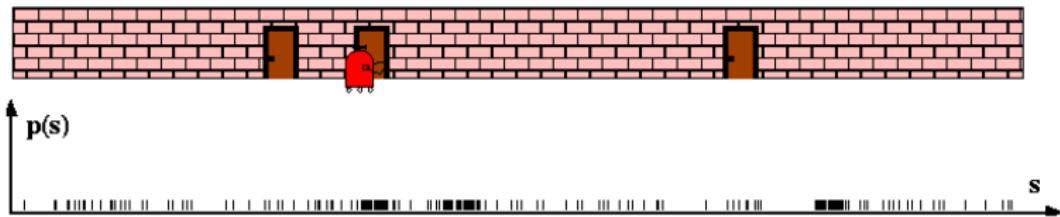
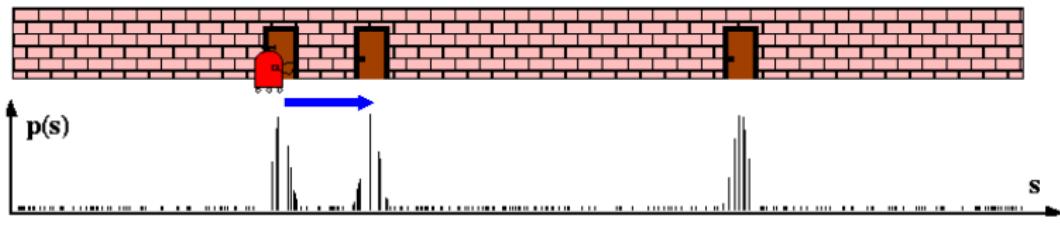
Sensor Information: Importance Sampling

$$Bel(\mathbf{x}) \leftarrow \alpha p(\mathbf{z} | \mathbf{x}) Bel^-(\mathbf{x})$$
$$w \quad \leftarrow \frac{\alpha p(\mathbf{z} | \mathbf{x}) Bel^-(\mathbf{x})}{Bel^-(\mathbf{x})} = \alpha p(\mathbf{z} | \mathbf{x})$$



Robot Motion

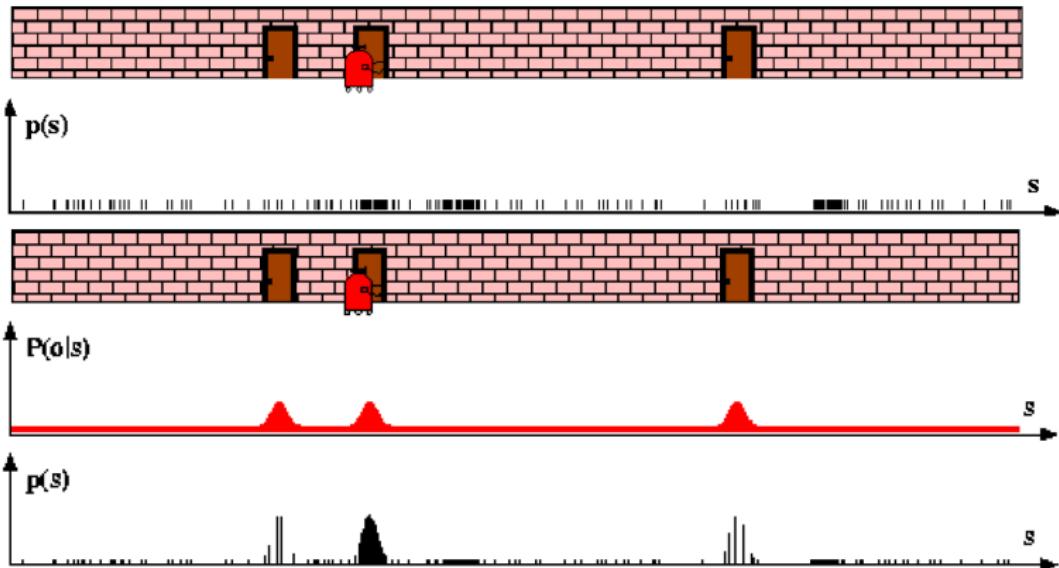
$$Bel^-(\mathbf{x}) \leftarrow \int p(\mathbf{x} | \mathbf{u}, \mathbf{x}') Bel(\mathbf{x}') d\mathbf{x}'$$



Sensor Information: Importance Sampling

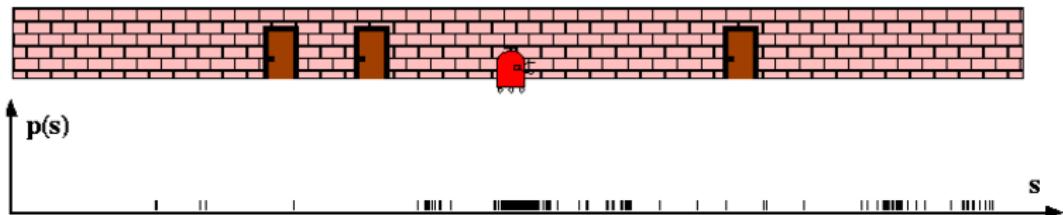
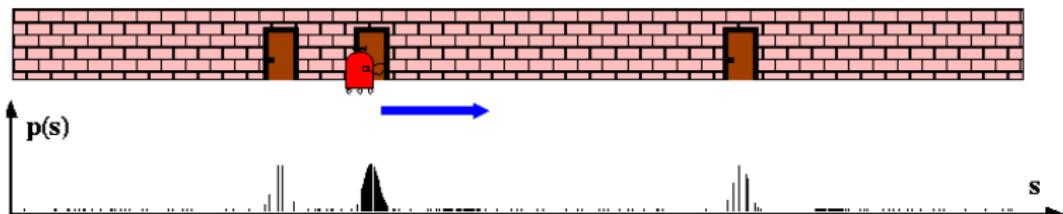
$$Bel(\mathbf{x}) \leftarrow \alpha p(\mathbf{z} | \mathbf{x}) Bel^-(\mathbf{x})$$

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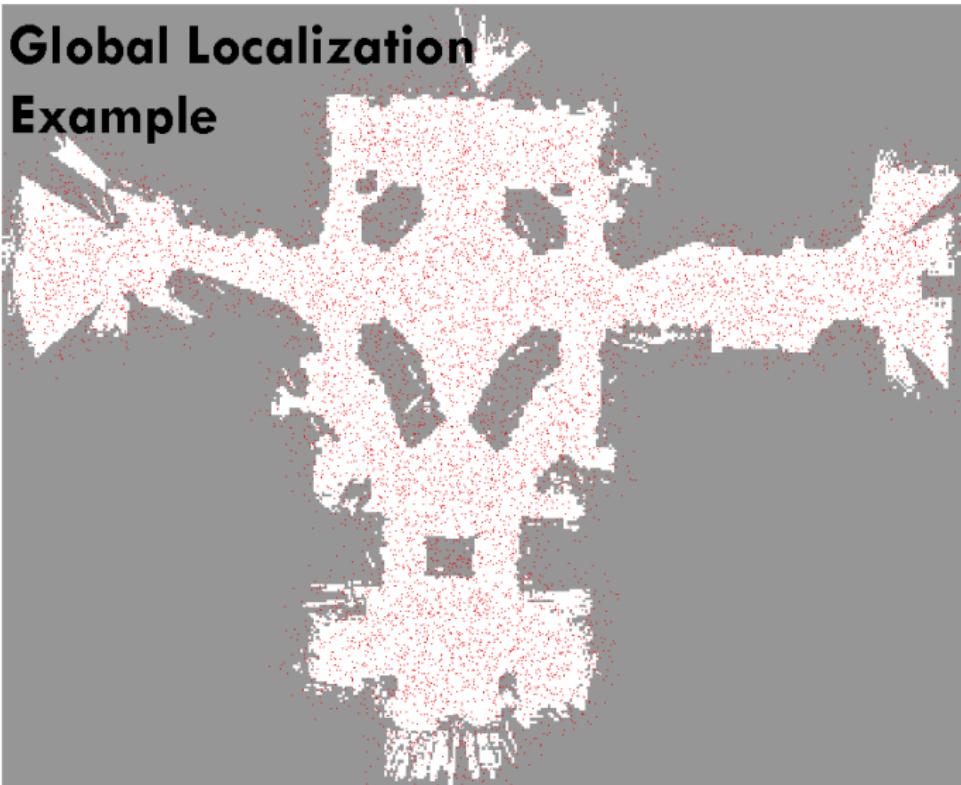


Robot Motion

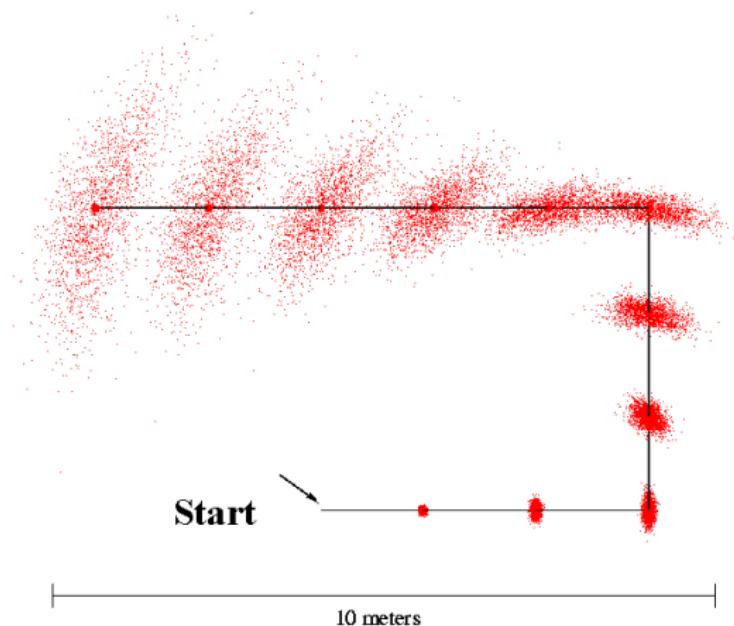
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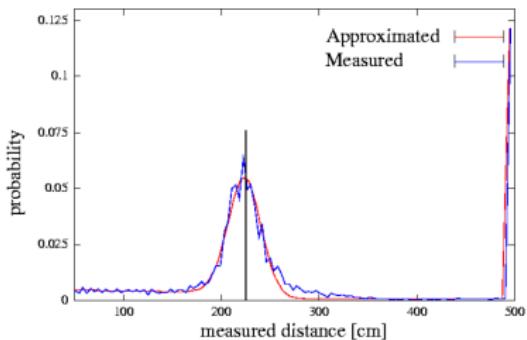
Global Localization Example



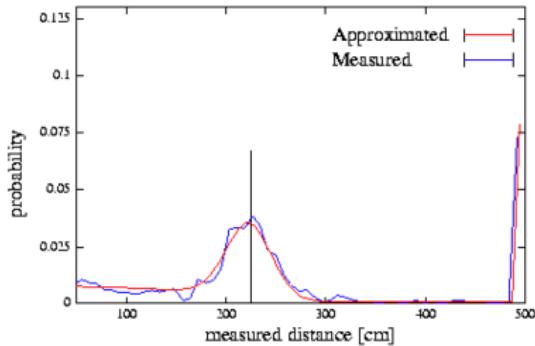
Motion Model Reminder



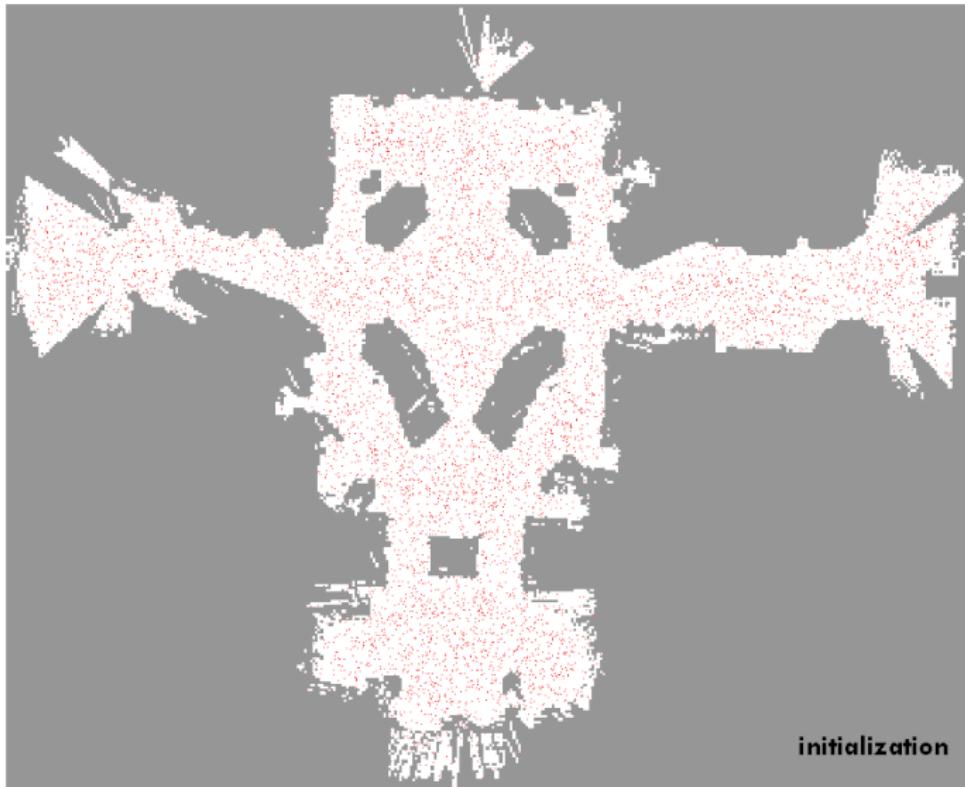
Proximity Sensor Model Reminder



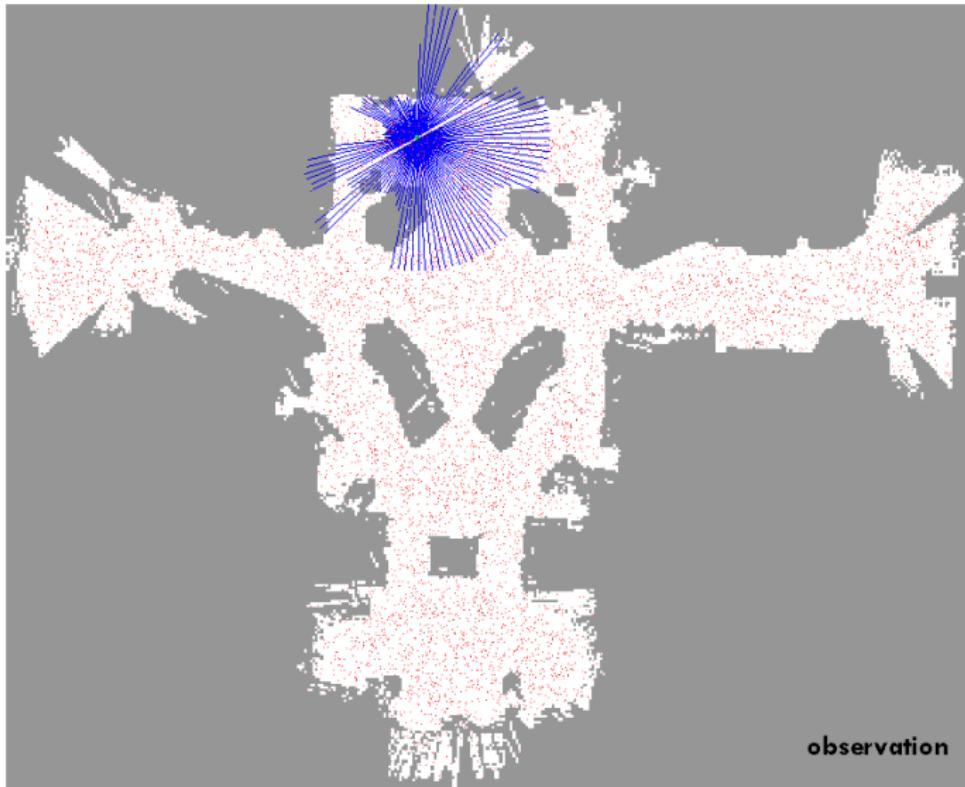
Laser sensor



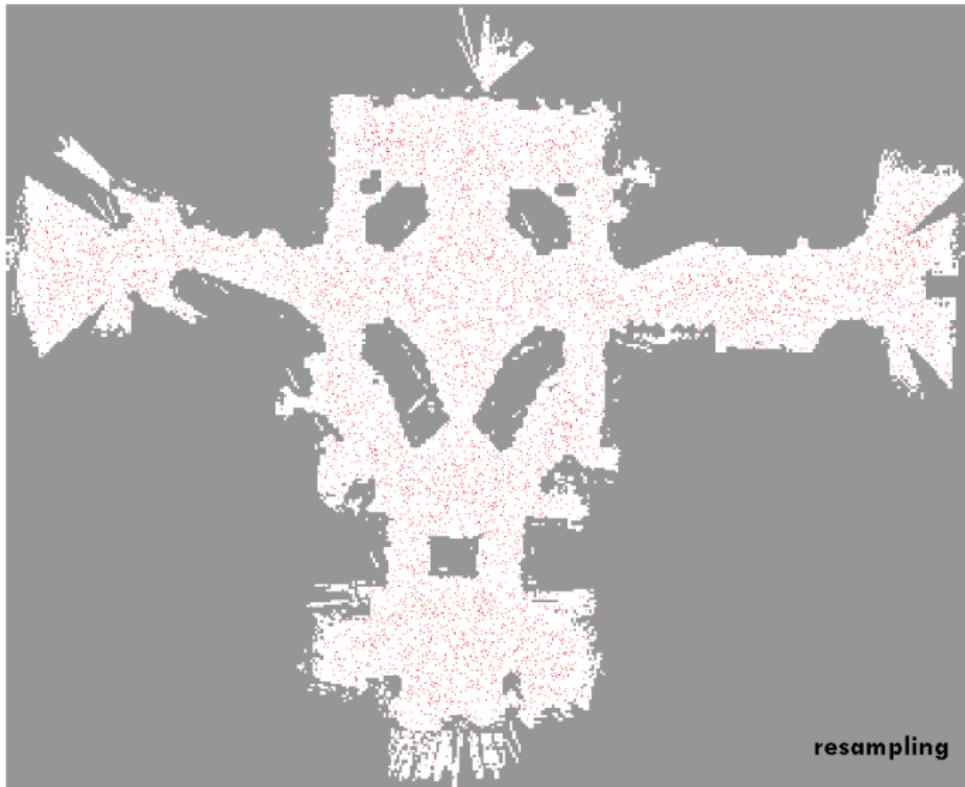
Sonar sensor



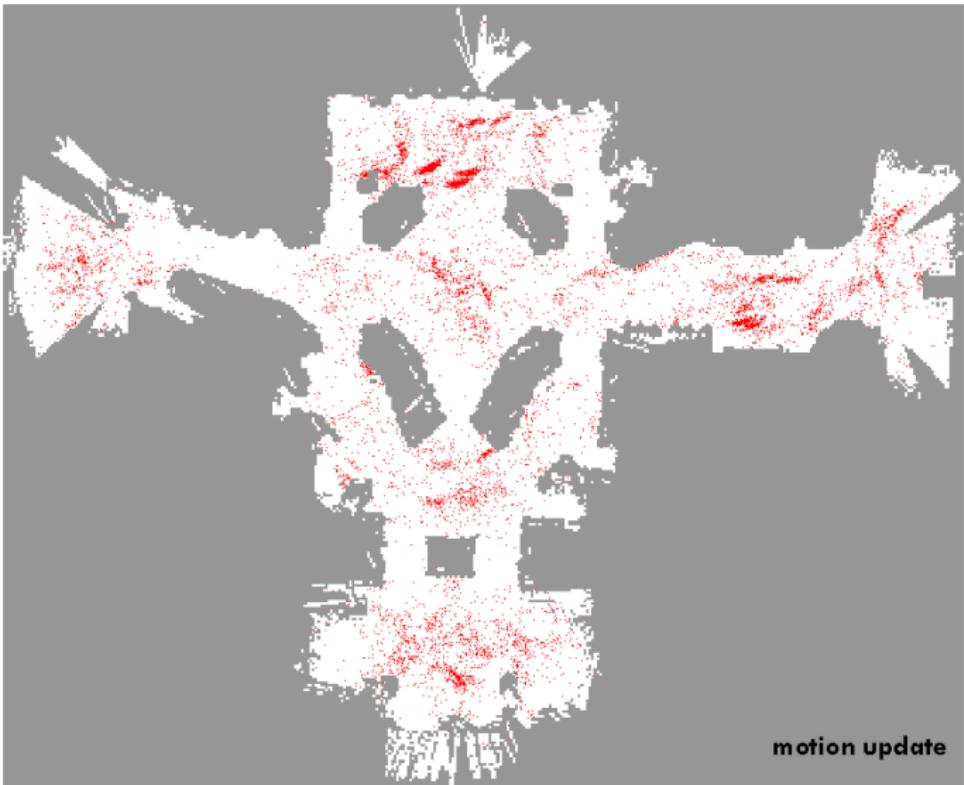
Courtesy: Thrun, Burgard, Fox



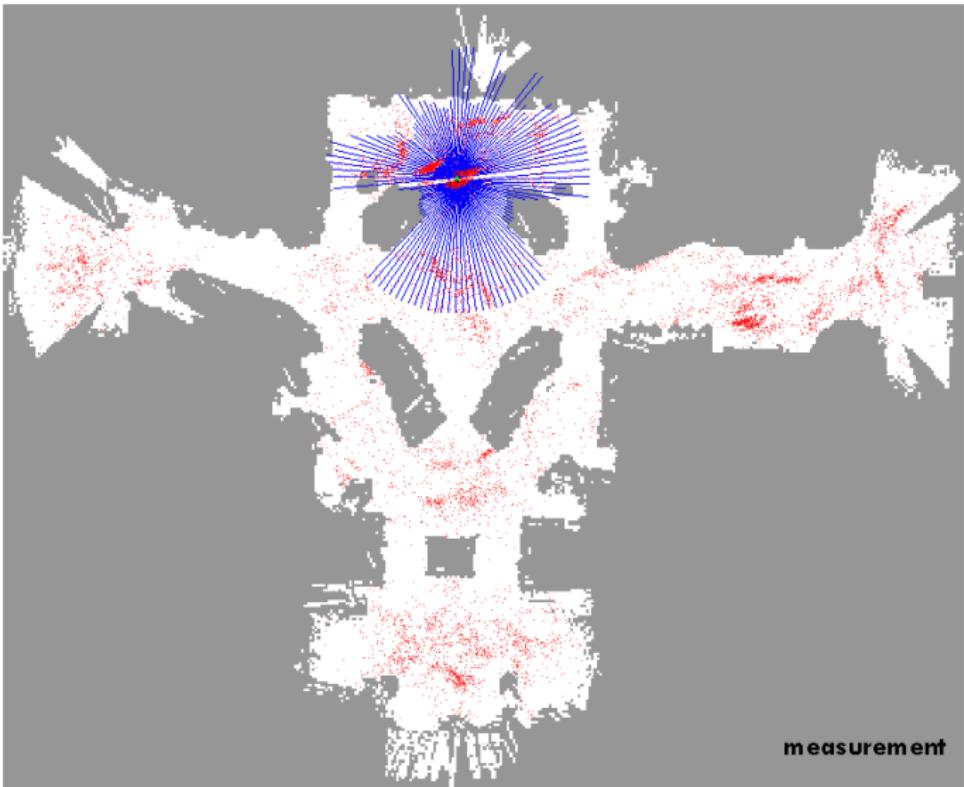
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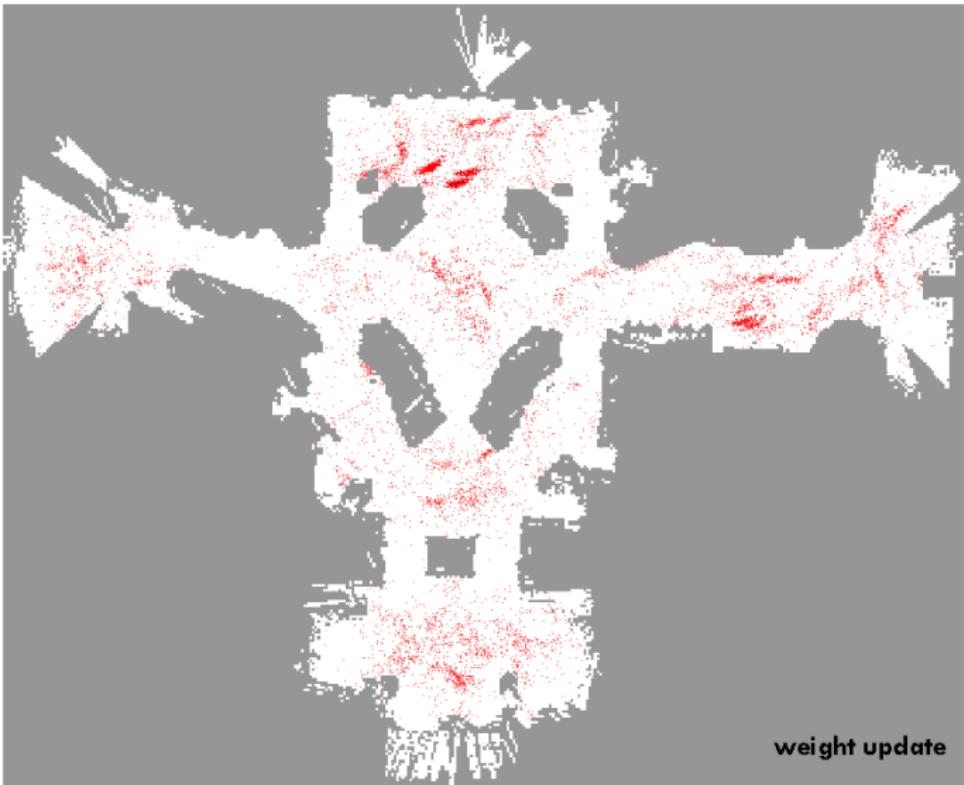
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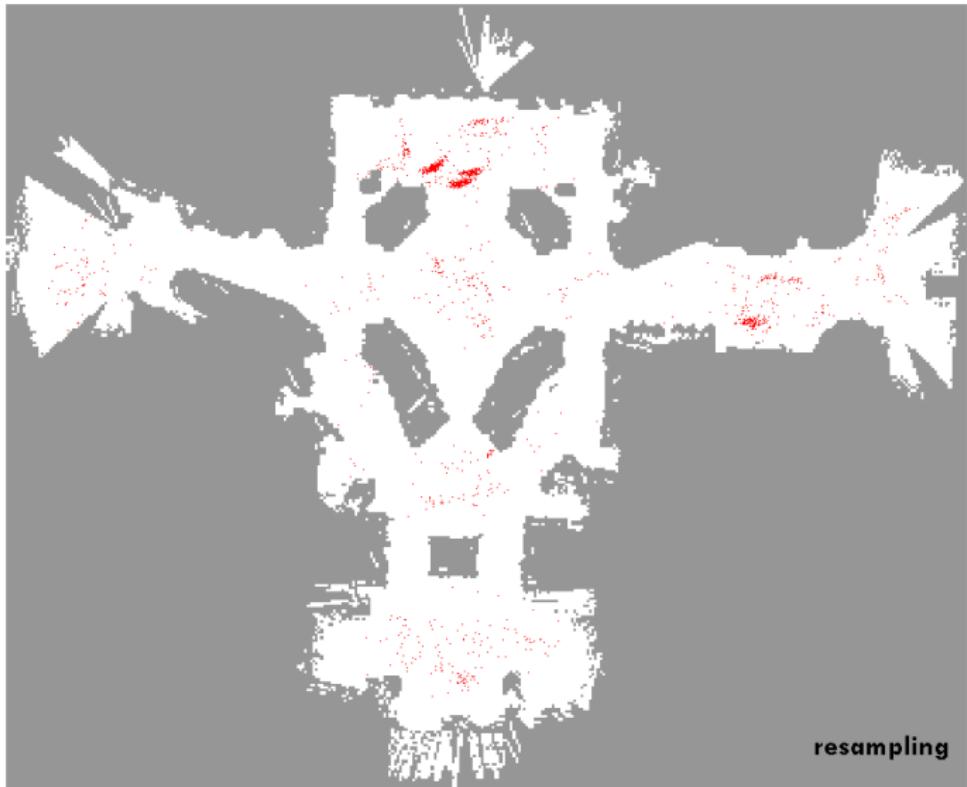
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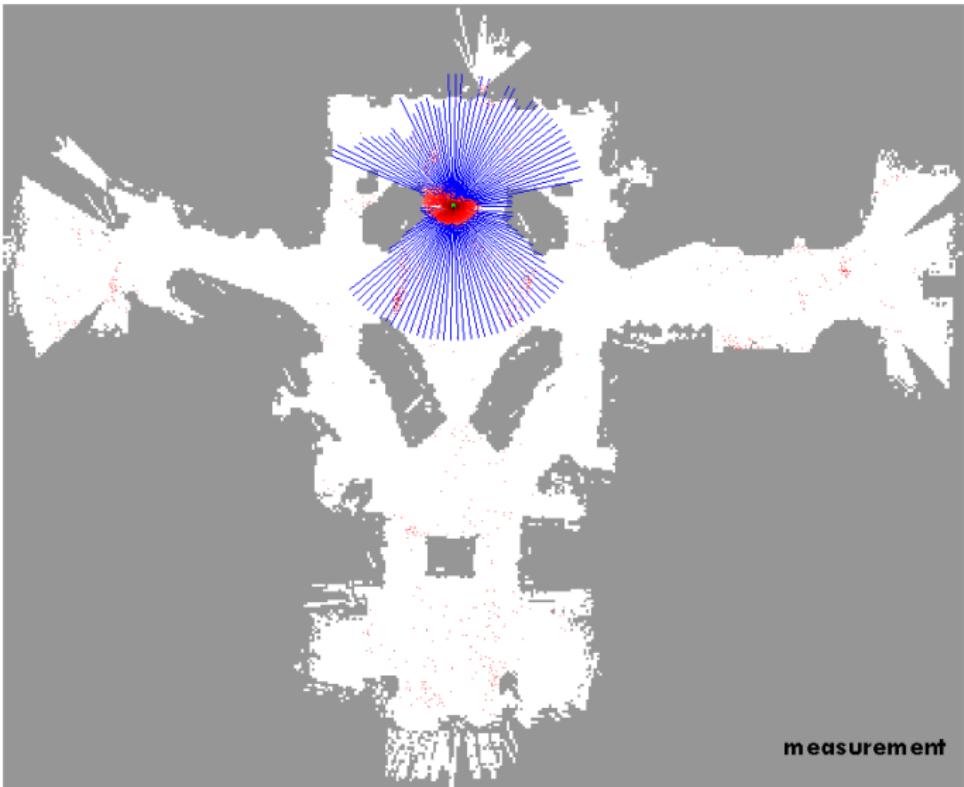
Courtesy: Thrun, Burgard, Fox



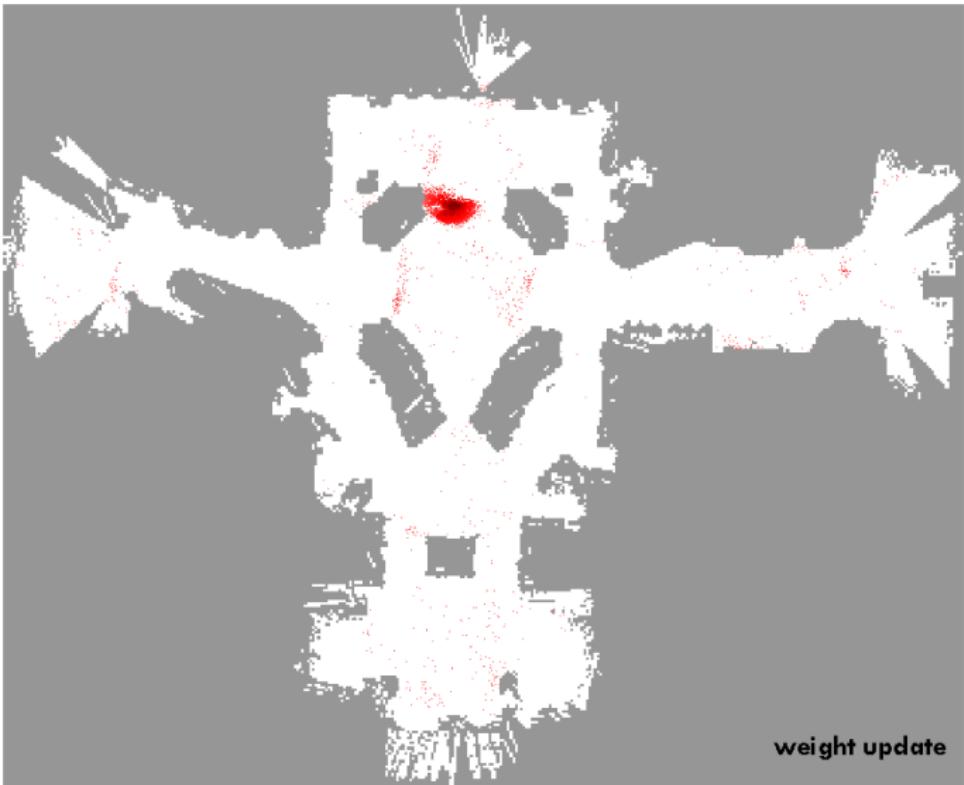
Courtesy: Thrun, Burgard, Fox



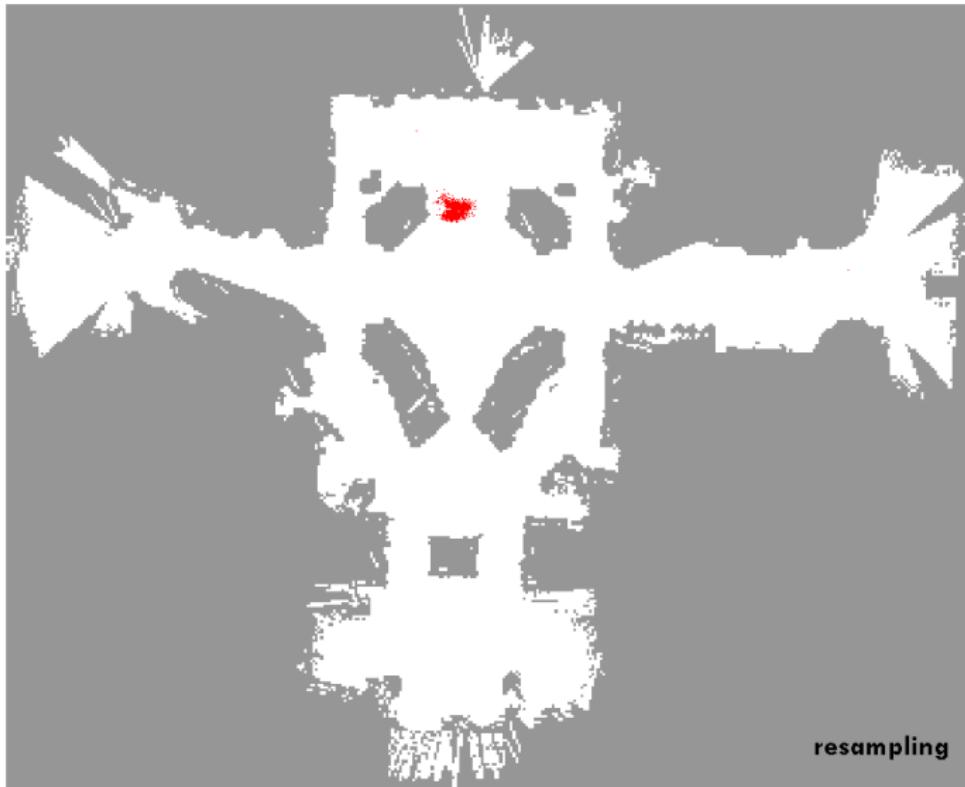
Courtesy: Thrun, Burgard, Fox



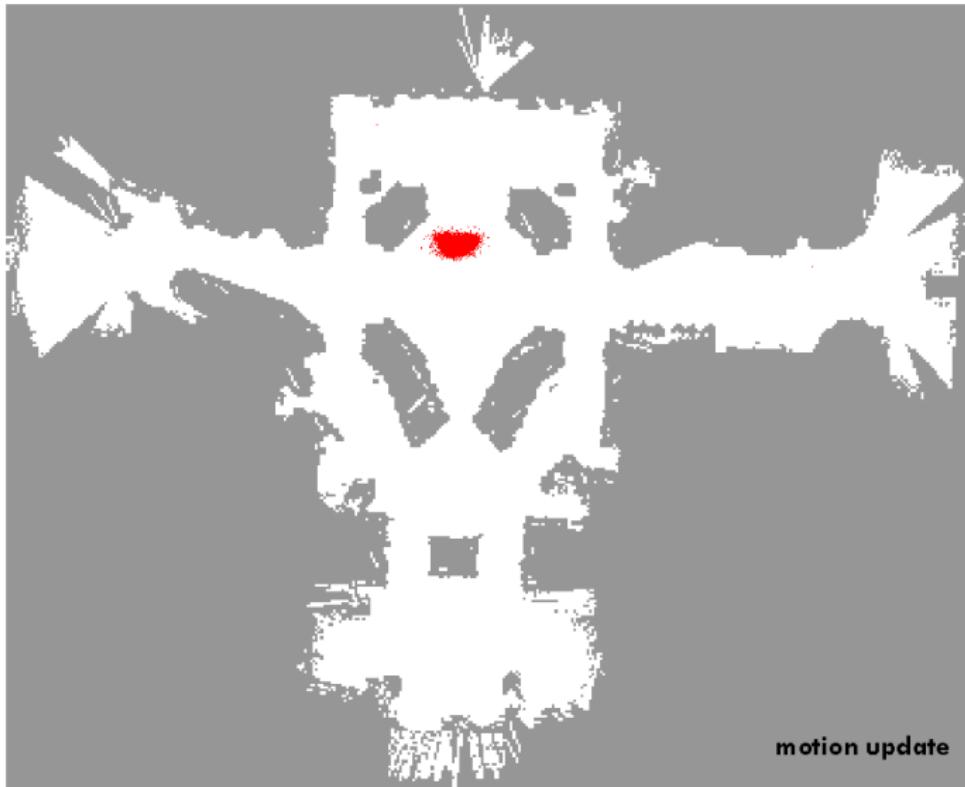
Courtesy: Thrun, Burgard, Fox



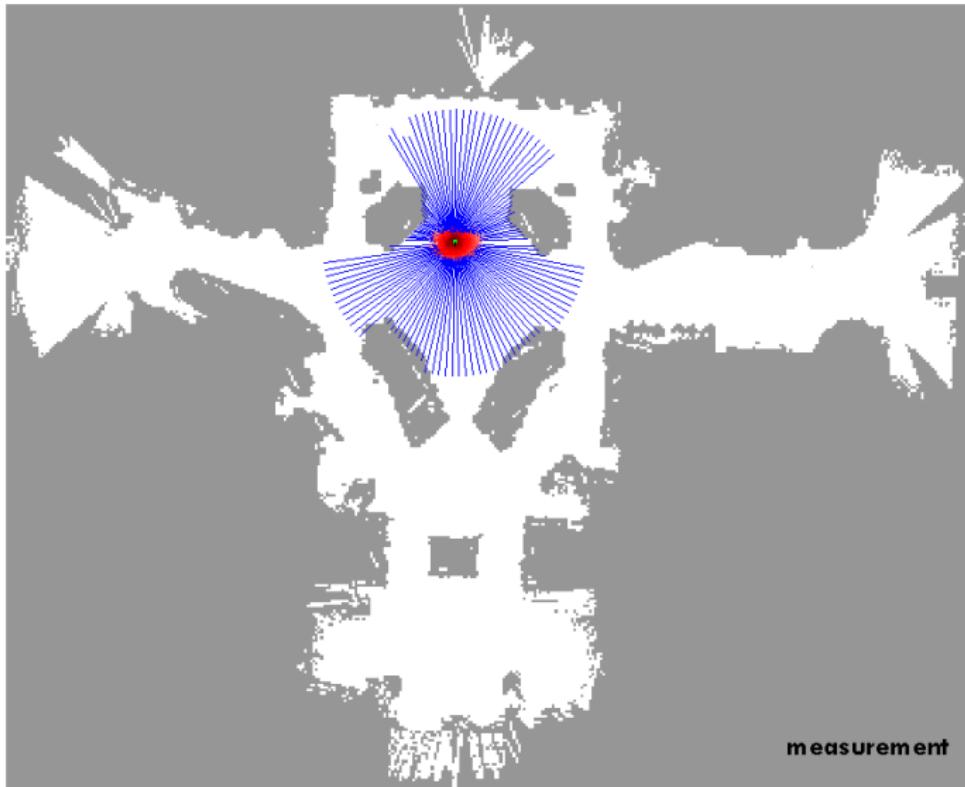
Courtesy: Thrun, Burgard, Fox



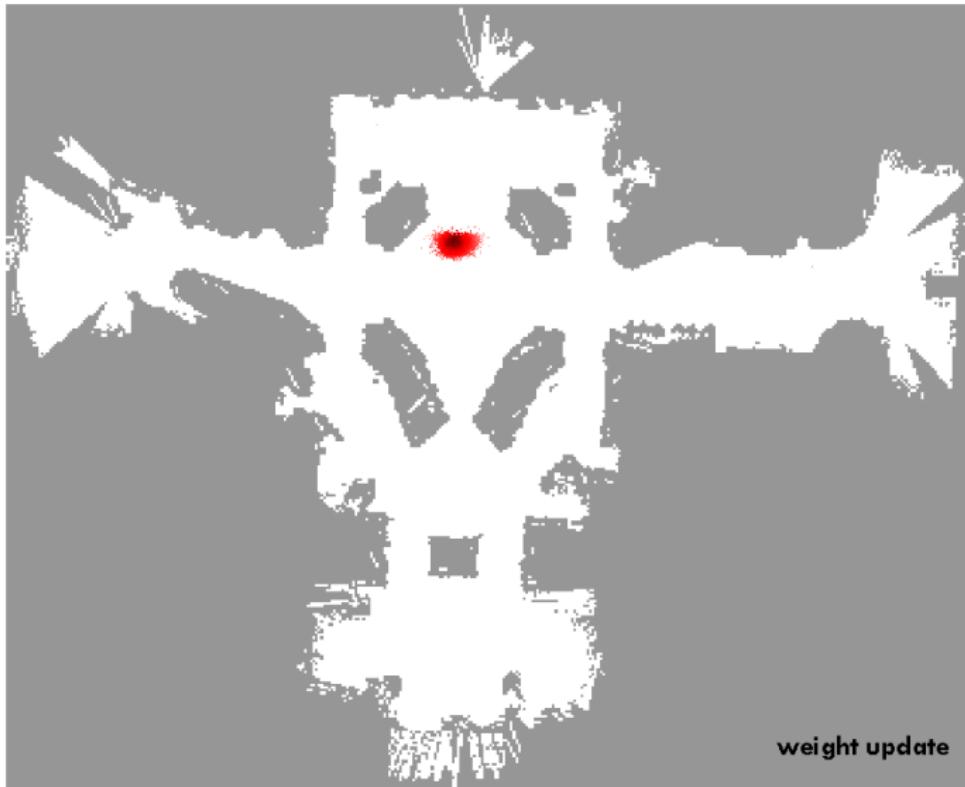
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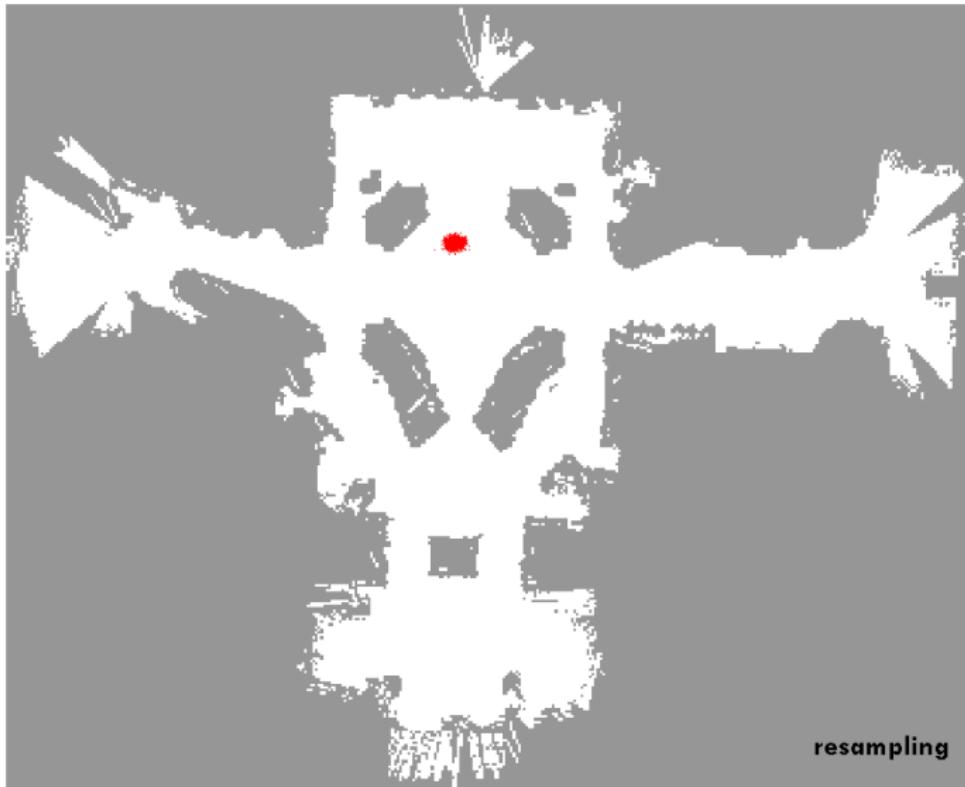
Courtesy: Thrun, Burgard, Fox



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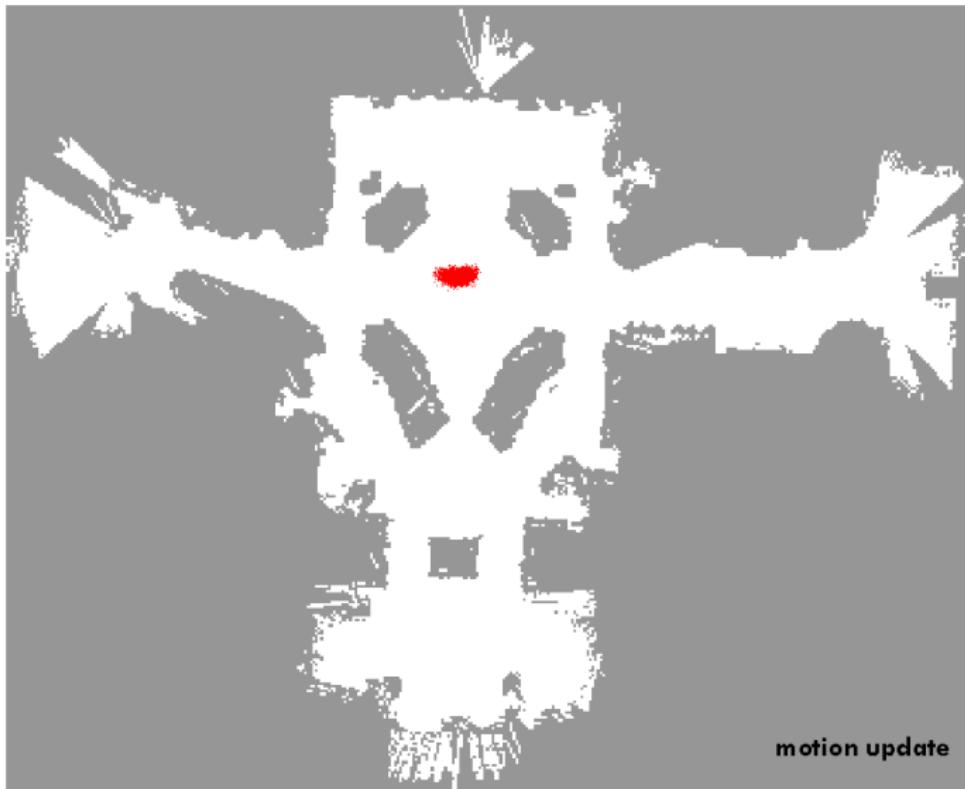


Courtesy: Thrun, Burgard, Fox

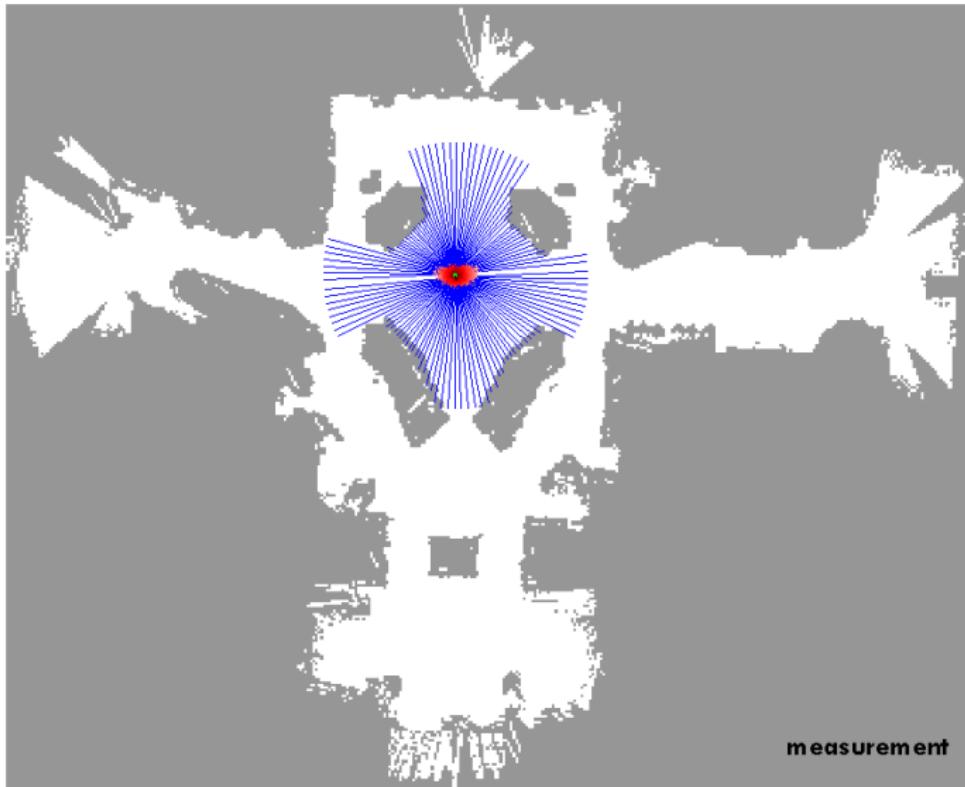


resampling

Courtesy: Thrun, Burgard, Fox



Courtesy: Thrun, Burgard, Fox



measurement

Courtesy: Thrun, Burgard, Fox

Importance Sampling with Resampling: Landmark Detection Example



Summary – Particle Filters

- Particle filters are non-parametric, recursive Bayes filters
- Posterior is represented by a set of weighted samples
- Proposal to draw the samples for $t+1$
- Weight to account for the differences between the proposal and the target
- Work well in low-dimensional spaces

Summary – PF Localization

- Particles are propagated according to the motion model
- They are weighted according to the likelihood of the observation
- Called: Monte-Carlo localization (MCL)
- MCL is the gold standard for mobile robot localization today

- ▶ Probabilistic Robotics: Ch. 7 and 8