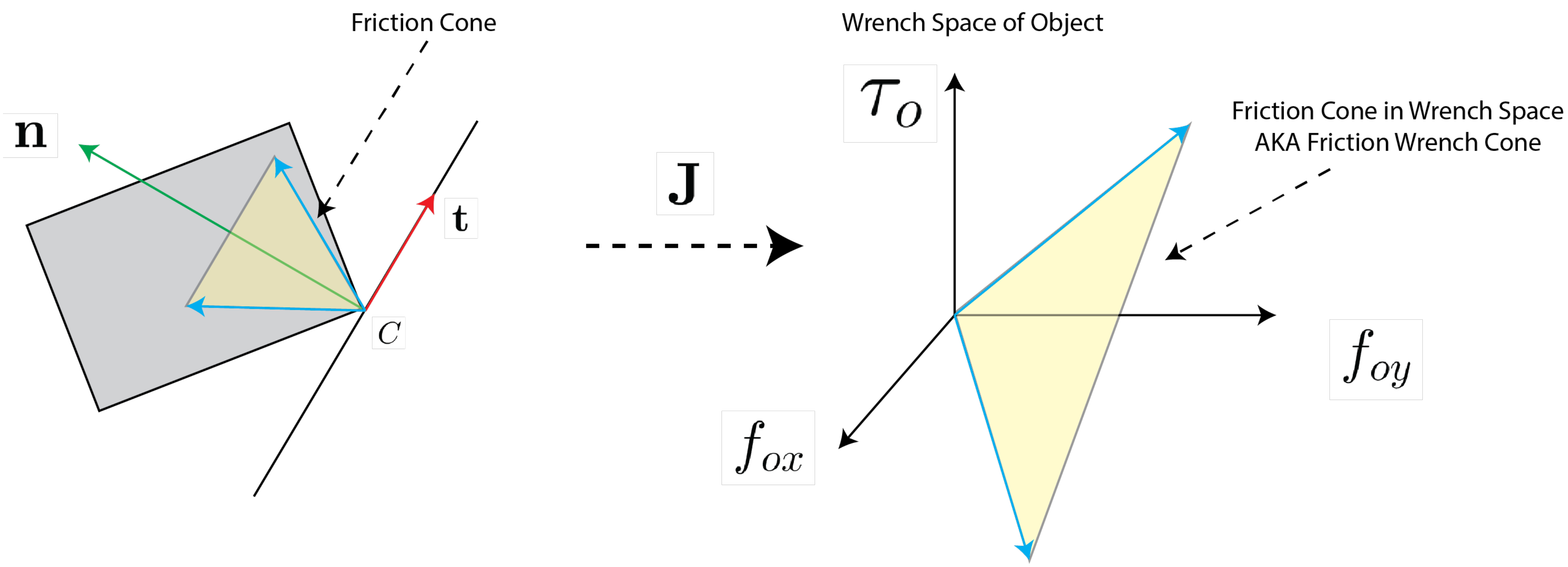
Introduction to Robotic Manipulation

Session 3

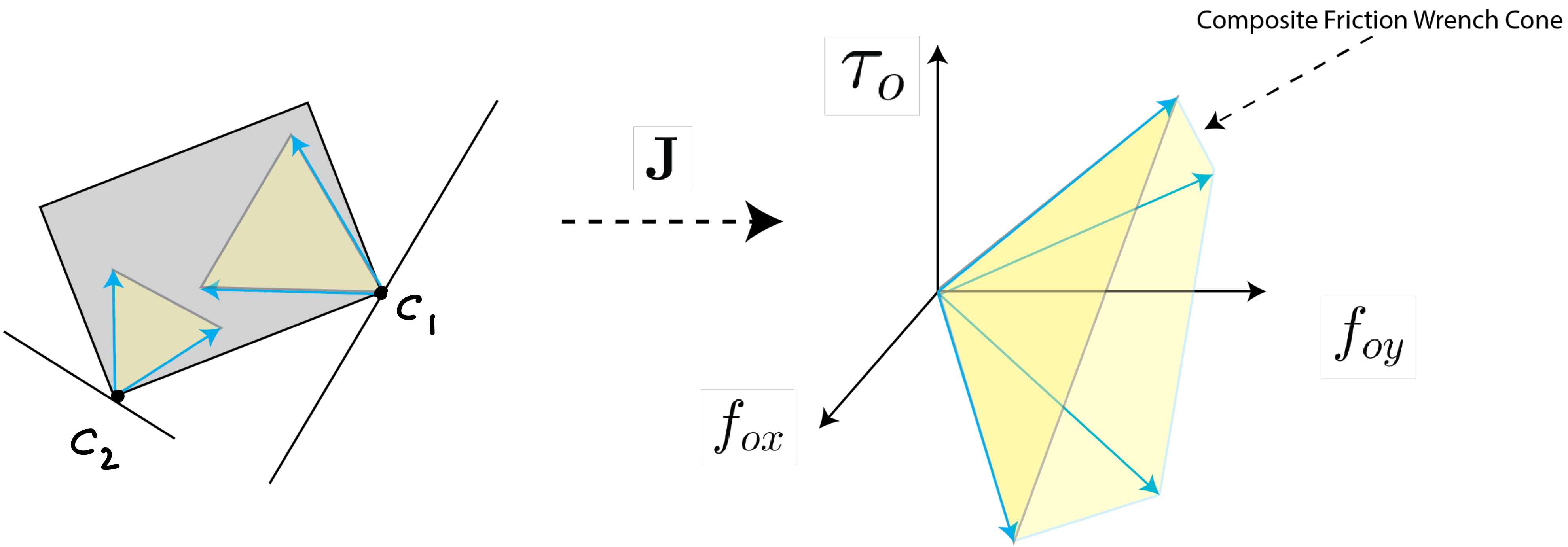
Todays Agenda

- Review of last session
- Finger Jacobians and Forward Kinematics
- Grasping and the Grasp Matrix

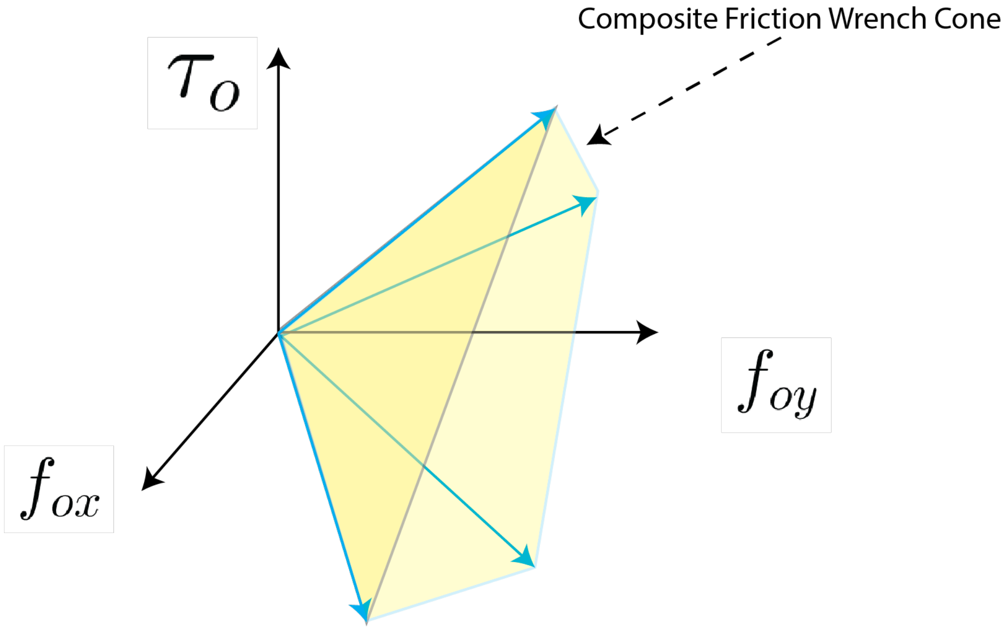
Review - Friction Wrench Cone



Review - Composite Friction Wrench Cone



Review - Composite Friction Wrench Cone

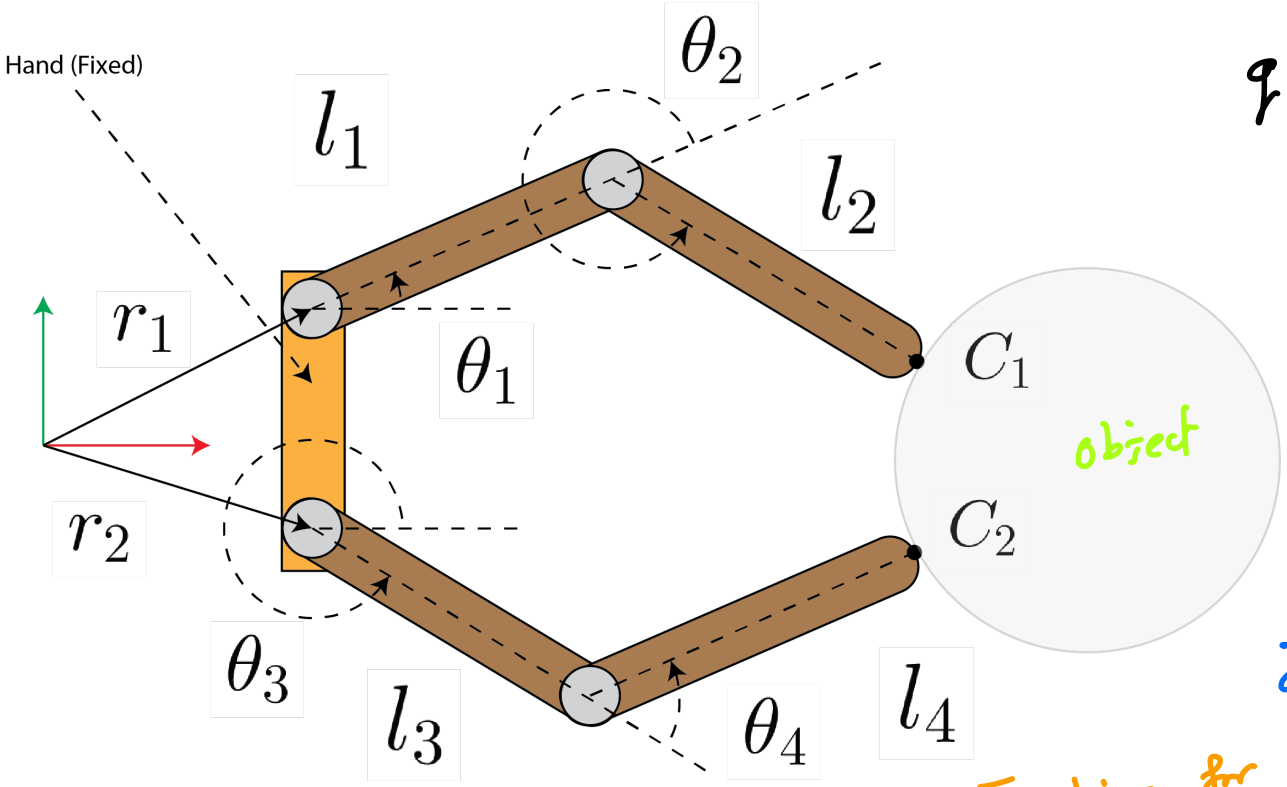


Properties:

- It's still a cone
- Characterizes the set of all possible forces the frictional interaction can apply to the object
- On the boundary = contact is sliding

Finger Jacobians

Goal: Relate the joint torques of my fingers to the forces applied to the object at (c_1, c_2)



$$q = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}$$ ← config space of my fingers

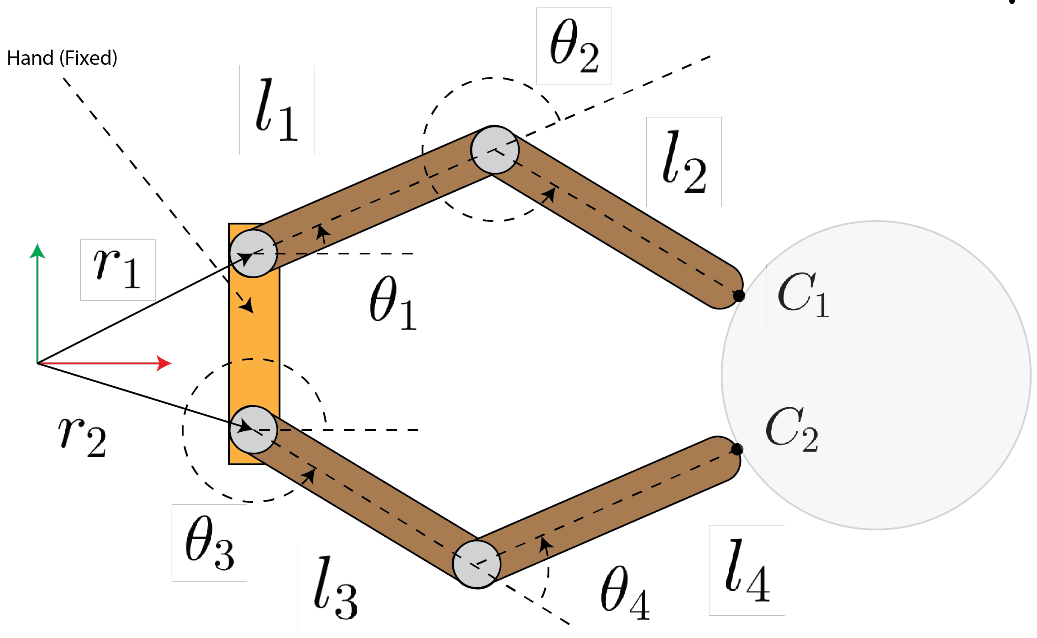
$r_{c_1}(\theta_1, \theta_2)$ ← where is the contact pt c_1 w.r.t. palm frame

$$\frac{d}{dt} r_{c_1} = \frac{\partial r_{c_1}}{\partial \theta} \frac{d\theta}{dt}$$

$$= \underbrace{\frac{\partial r_{c_1}}{\partial \theta}}_{2\times 2} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Jacobian for finger 1

Finger Jacobians – Upper Finger

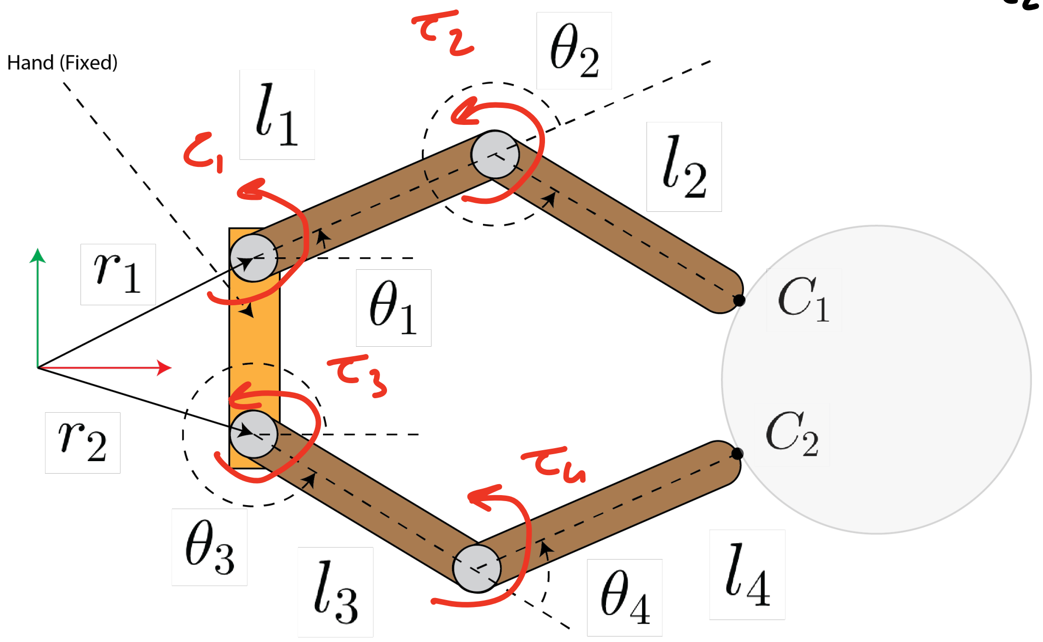


$$r_{c_1} = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) + r_{1x} \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) + r_{1y} \end{bmatrix}$$

$$\frac{\partial r_{c_1}}{\partial \theta}$$

$$= \underbrace{\begin{bmatrix} -l_1 \sin\theta_1 - l_2 \sin(\theta_1+\theta_2) & -l_2 \sin(\theta_1+\theta_2) \\ l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) & l_2 \cos(\theta_1+\theta_2) \end{bmatrix}}_{\text{finger 1 Jacobian}} \quad 2\times2$$

Finger Jacobians – Lower Finger



$$r_{c_2} = \begin{bmatrix} l_3 \cos\theta_3 + l_4 \cos(\theta_3+\theta_4) + r_{2x} \\ l_3 \sin\theta_3 + l_4 \sin(\theta_3+\theta_4) + r_{2y} \end{bmatrix}$$

$$\frac{\partial r_{c_2}}{\partial \theta} = J_{f_2}$$

$$= \begin{bmatrix} -l_3 \sin\theta_3 - l_4 \sin(\theta_3+\theta_4) & -l_4 \sin(\theta_3+\theta_4) \\ l_3 \cos\theta_3 + l_4 \cos(\theta_3+\theta_4) & l_4 \cos(\theta_3+\theta_4) \end{bmatrix}$$

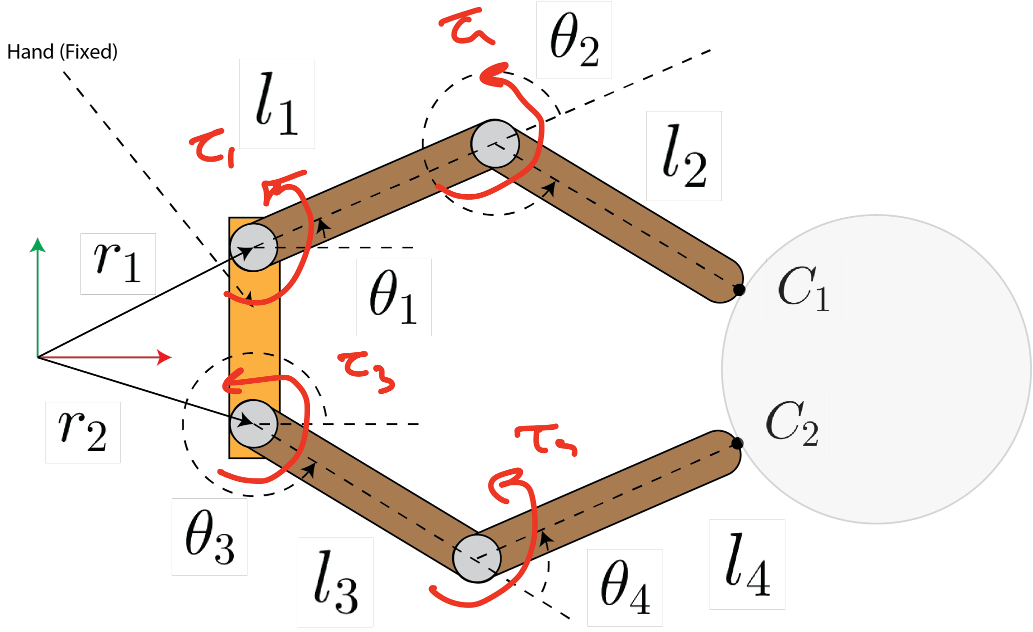
2×2

$\dot{\theta} \rightarrow v_c$ & $\tau \rightarrow f_c$

$r(c_1, c_2) \rightarrow \frac{d}{dt} \rightarrow v_c = J\dot{\theta}$

$f_{c_1} = J_{c_1}^T \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$ $\quad f_{c_2} = J_{c_2}^T \begin{bmatrix} \tau_3 \\ \tau_4 \end{bmatrix}$

Finger Jacobians – Total



$$f_c = \overbrace{\begin{bmatrix} J_{c_1}^T & \underline{\underline{0}} \\ \underline{\underline{0}} & J_{c_2}^T \end{bmatrix}}^{J_g}_{4\times 4} \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{bmatrix}$$

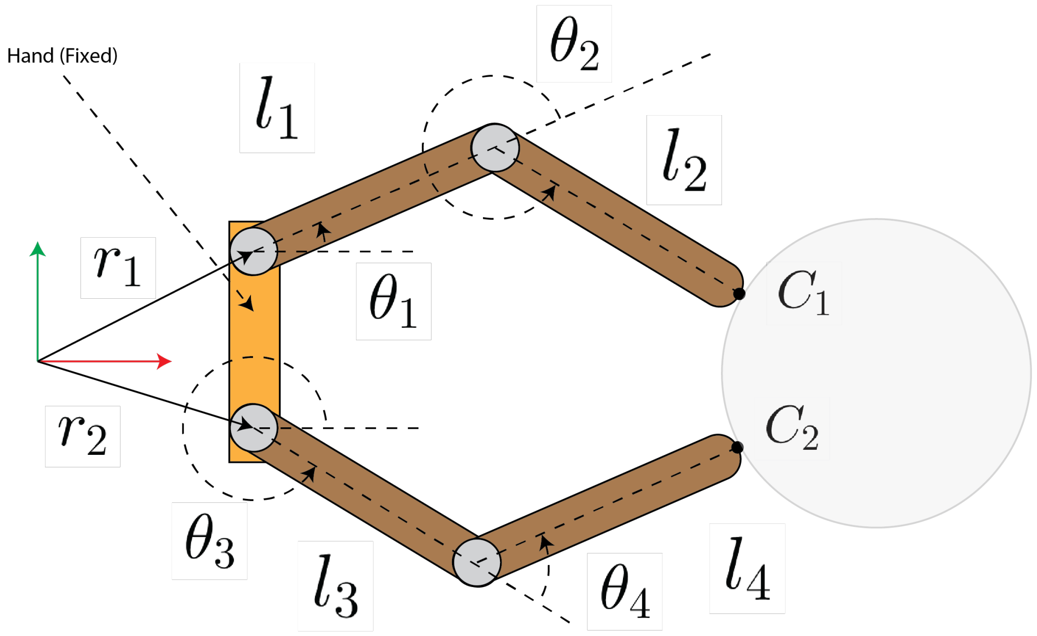
$$2\times 2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$f_c = J_g \, \tau$$

what the object feels

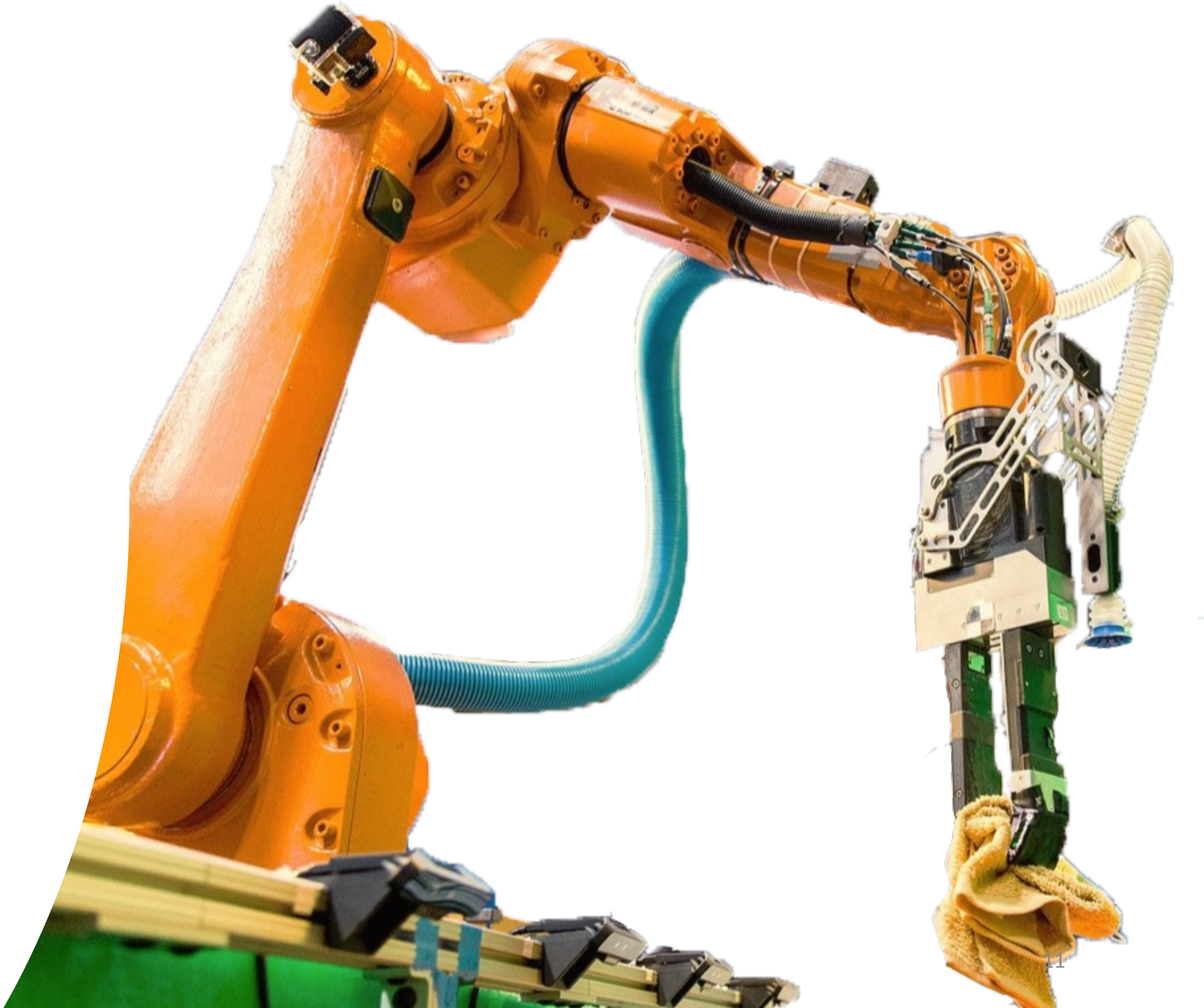
what I control

Finger Jacobians – Total (Intuition)



Grasping

Grasp Analysis



WSG 050-110-B
ID 0306120
SNo. 0047897
www.schunk.com
SCHUNK

6-DOF GraspNet: Variational Grasp Generation for Object Manipulation

https://arxiv.org/pdf/1905.10520.pdf

Our Objectives for Grasp Mechanics

- Grasp Analysis – quantify the quality of a grasp

① stability → It should resist external efforts to move object out of grasp

② Efficiency

③ Downstream tasks

④ observability ...

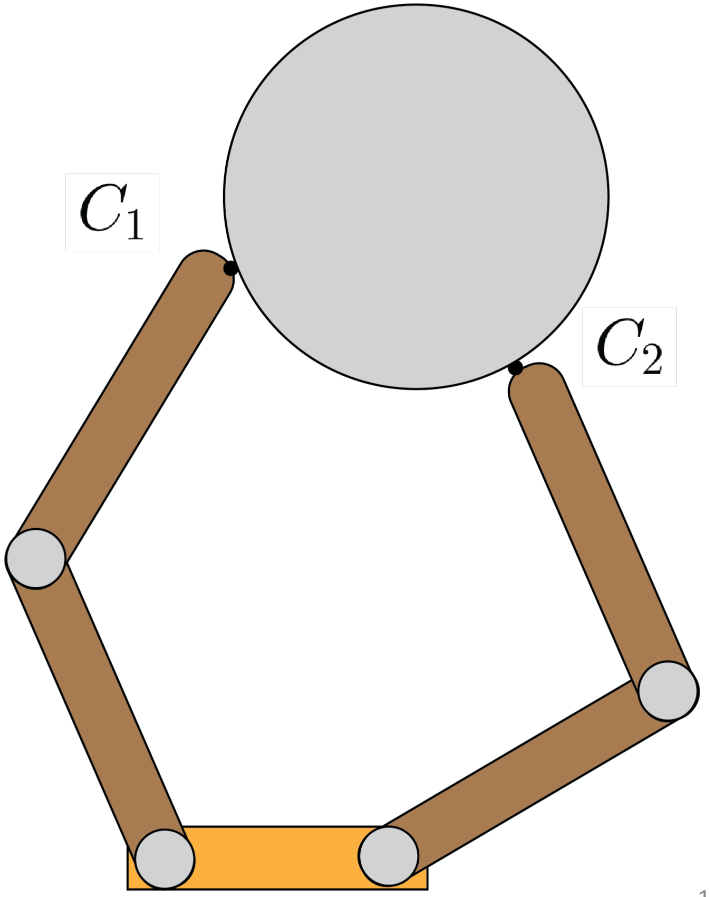
- Identifying grasp restraints

① form closure → geometric analysis → does not need friction

② force closure → friction

A grasp under our assumptions

- Rigid-bodies
- Static
- Coulomb friction
- Known geometries
- Infinite squeeze force



Definition of a Grasp – Grasp Matrix

$$\omega_1 = J_1 f_{c_1}$$

$$\omega_2 = J_2 f_{c_2}$$

$$\underset{3\times 1}{\omega} = \omega_1 + \omega_2 = \underset{3\times 2}{J_1}\,\underset{2\times 1}{f_{c_1}} + \underset{3\times 2}{J_2}\,\underset{2\times 1}{f_{c_2}}$$

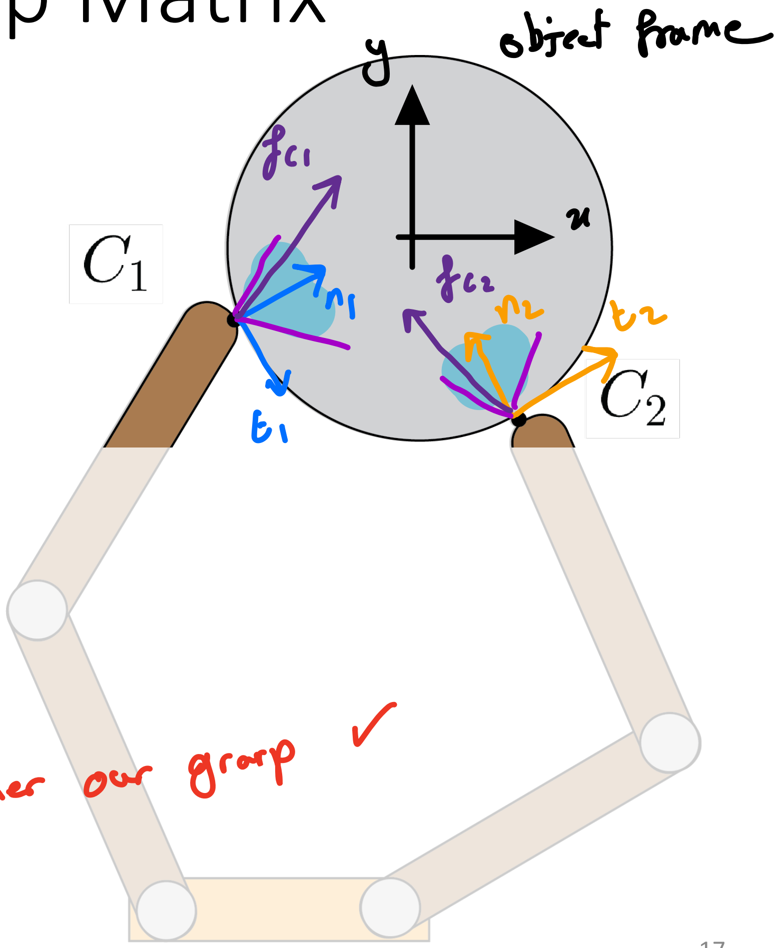
Composite wrench

$$\underline{\omega} = \underbrace{\begin{bmatrix} J_1 & J_2 \end{bmatrix}}_{G_{3\times 4}} \begin{bmatrix} f_{c_1} \\ f_{c_1} \end{bmatrix}_{4\times 1}$$

Contact forces

$$\omega = G f_c$$

Grasp Matrix ← defines our grasp ✓



Definition of a Grasp – Grasp Matrix

w_e := some external wrench applied to the object

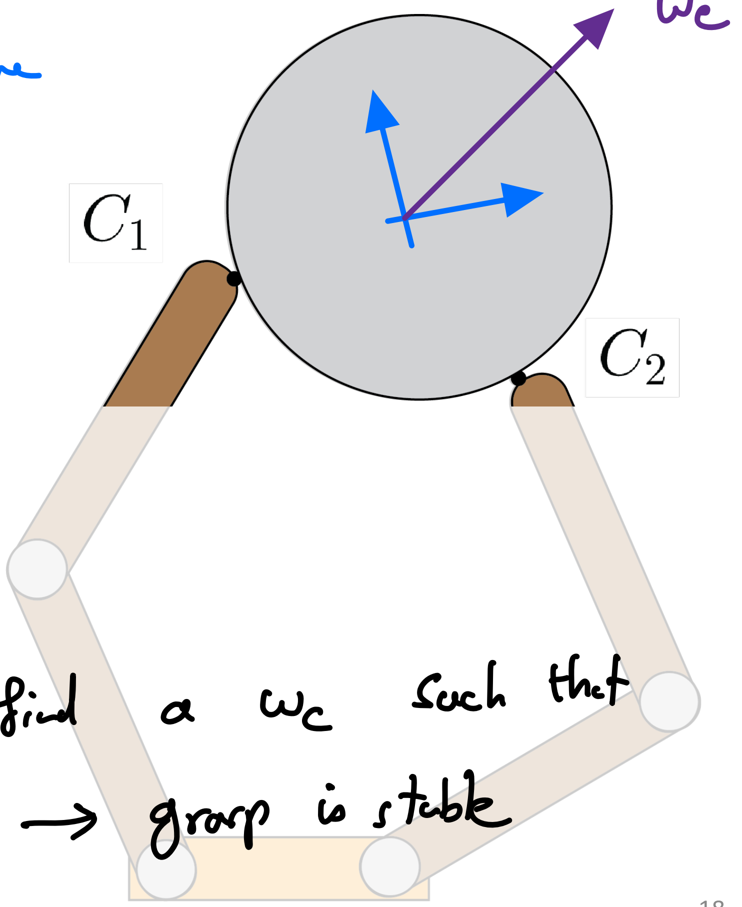
$w_e + \underbrace{w_c}_{\text{wrench provided by contact}} = 0$

Force balance ✓ → stable grasp

fingers can resist the external wrench

If for any choice of w_e, I can find a w_c such that

$w_e + w_c = 0$ / static equilibrium → grasp is stable

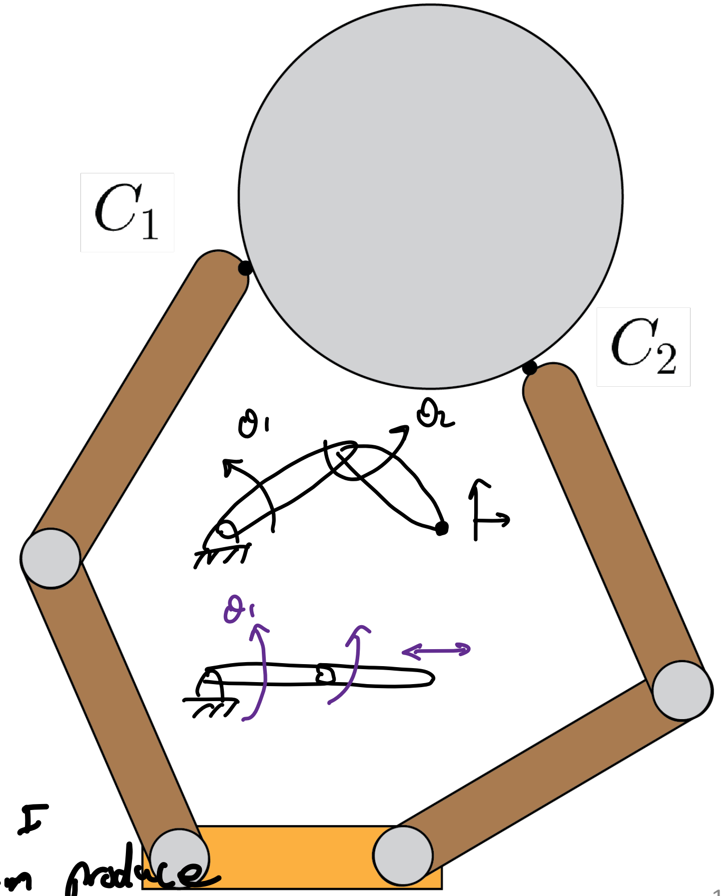


Definition of a Grasp - Augmented

$$\omega = G f_c$$

$$f_c = J_f \tau$$

$$\{ \underbrace{G}_{\text{object centric}}, \underbrace{J_f}_{\text{finger centric}} \} := \text{augmented grasp definition}$$



G: what are the set of forces I can resist

J_f: what are the set of forces I can produce

Grasp Analysis – Form Closure

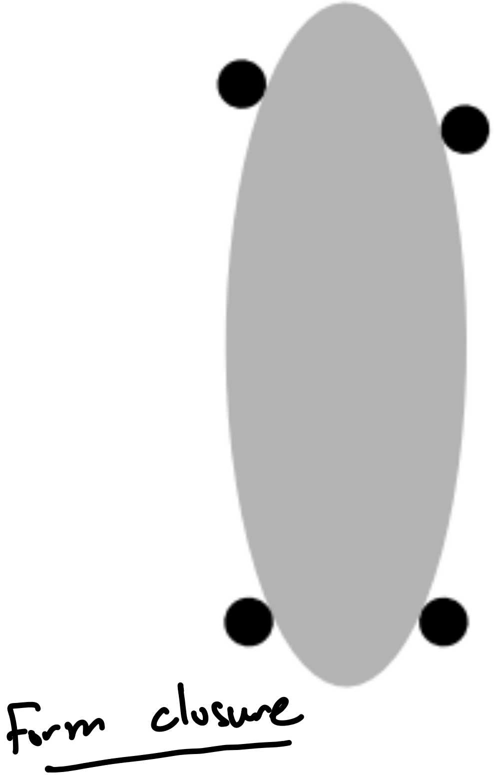
① Geometric Concept

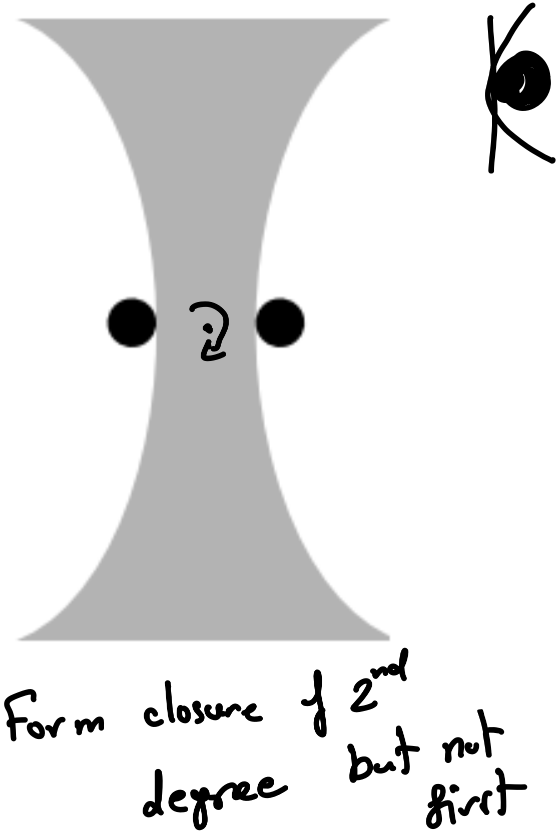
② Only considers contact normals

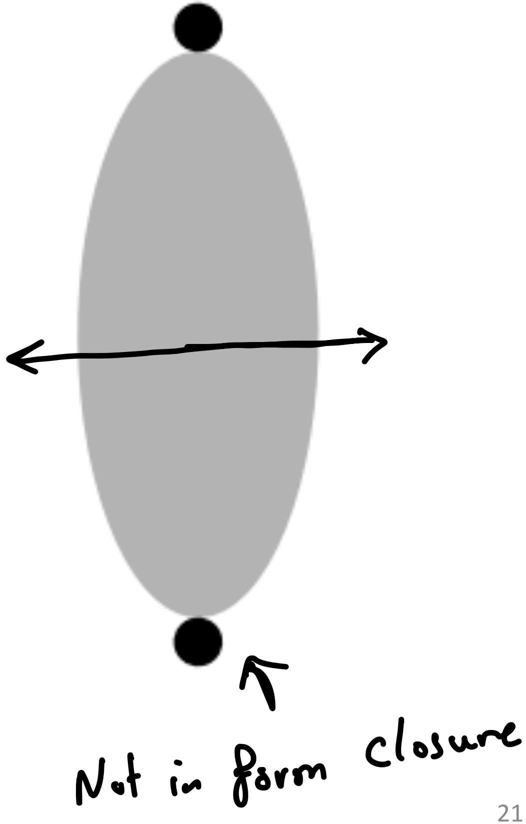
★ does not consider friction

$\equiv$ zero coefficient of friction

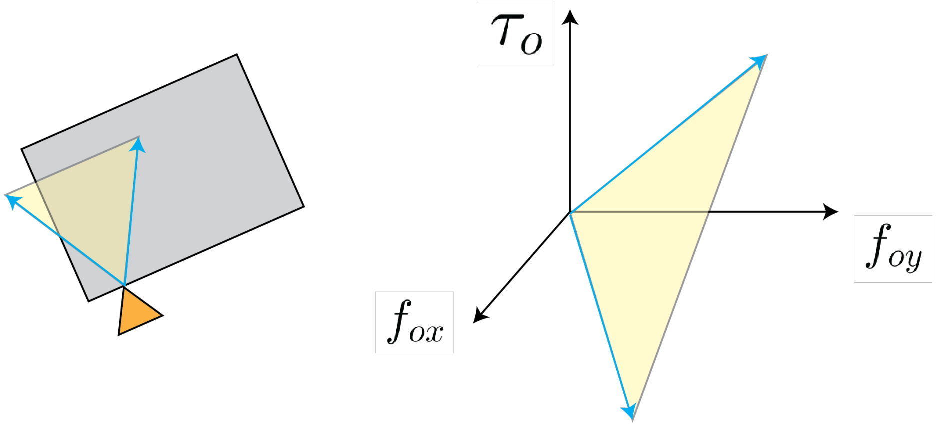
Exercise: Which is a Form Closure?

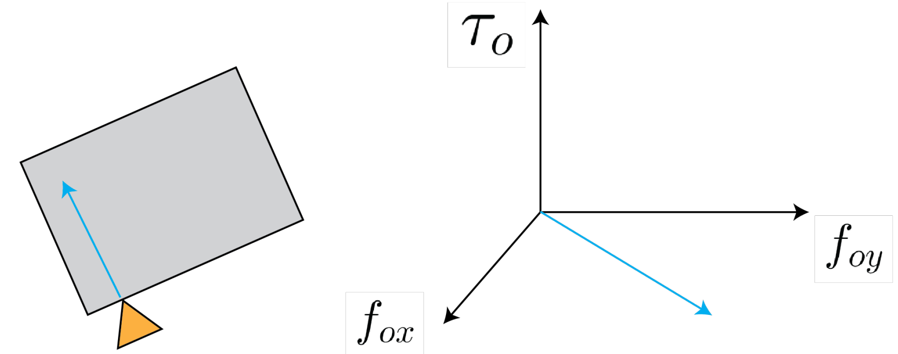




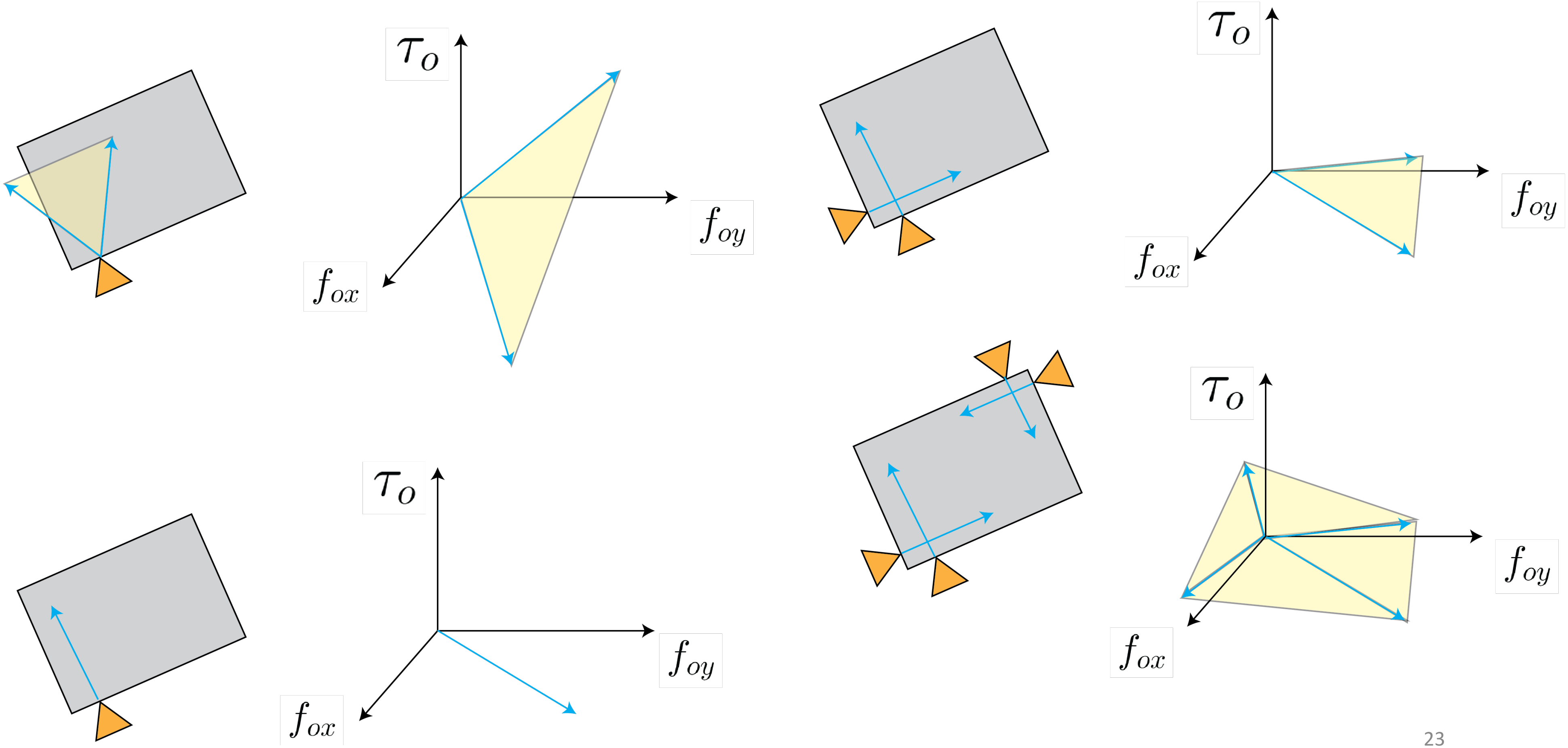


Form Closure – Geometric Intuition

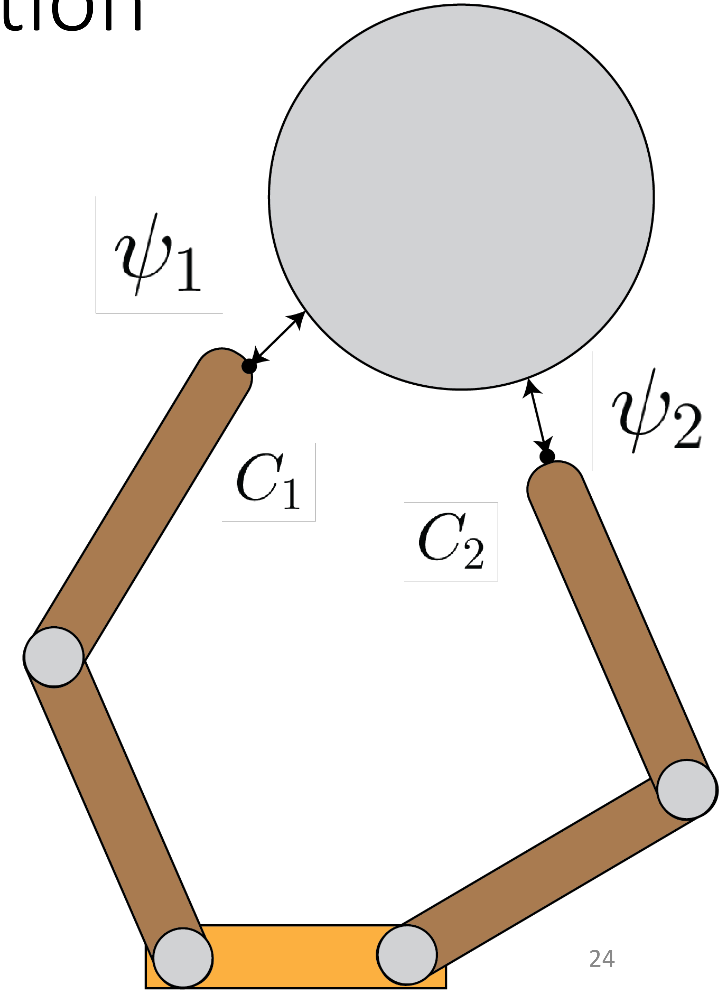




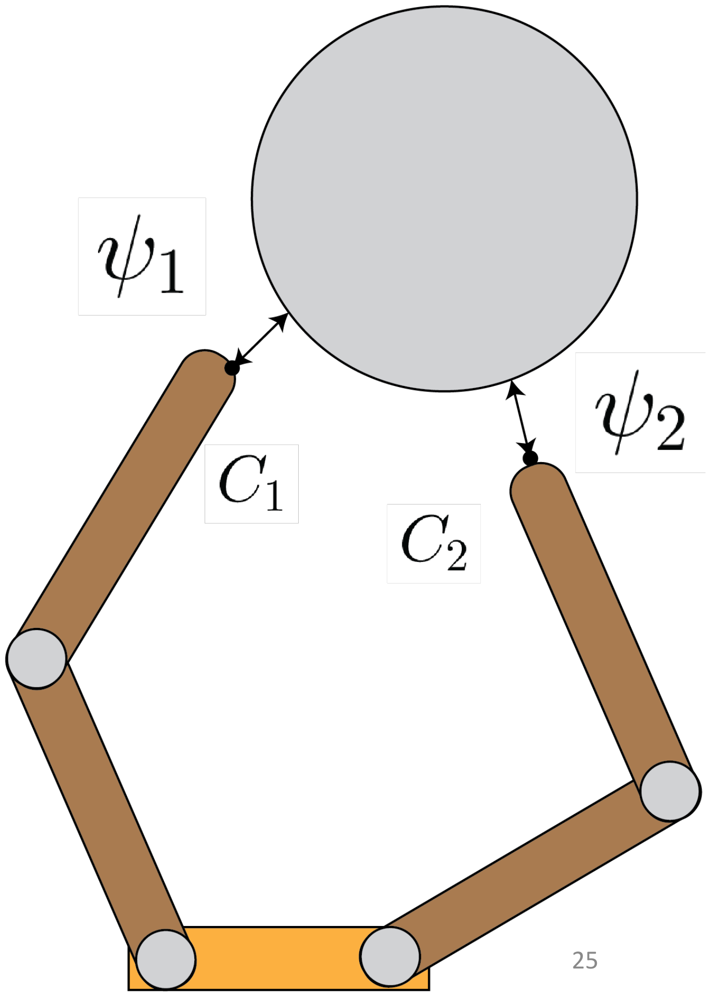
Form Closure – Geometric Intuition



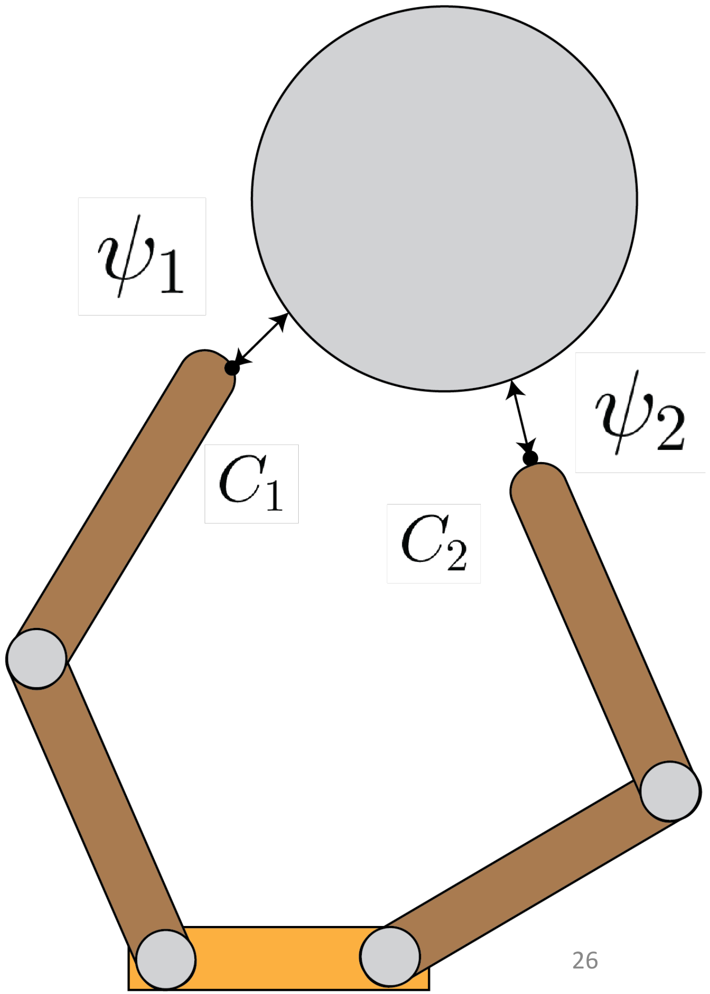
Form Closure – Analytic Definition



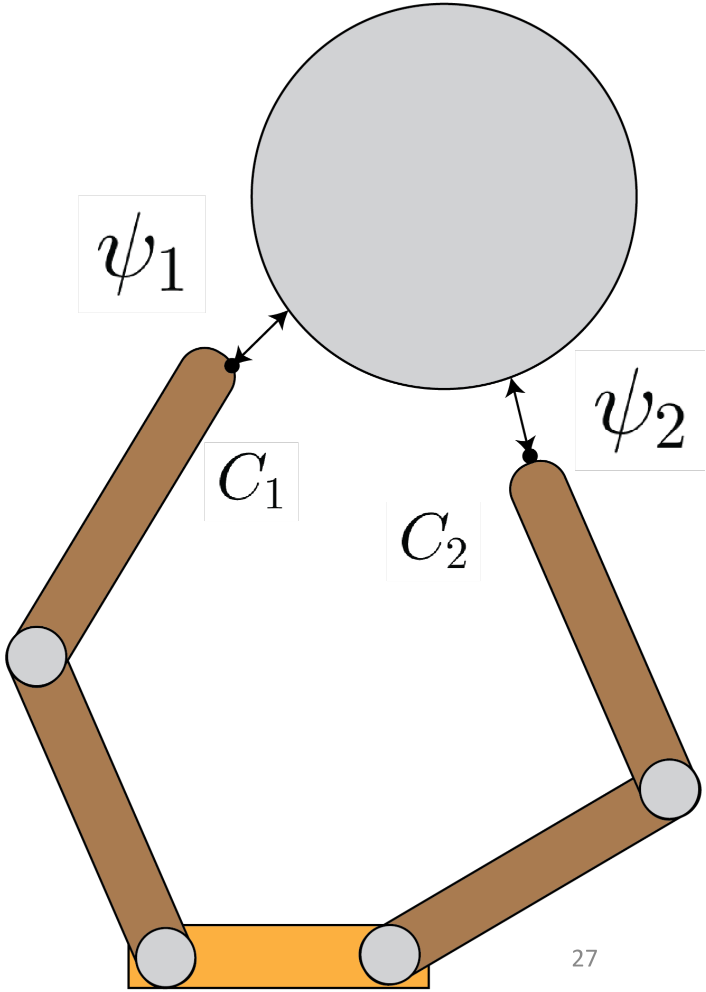
1st Order Form Closure



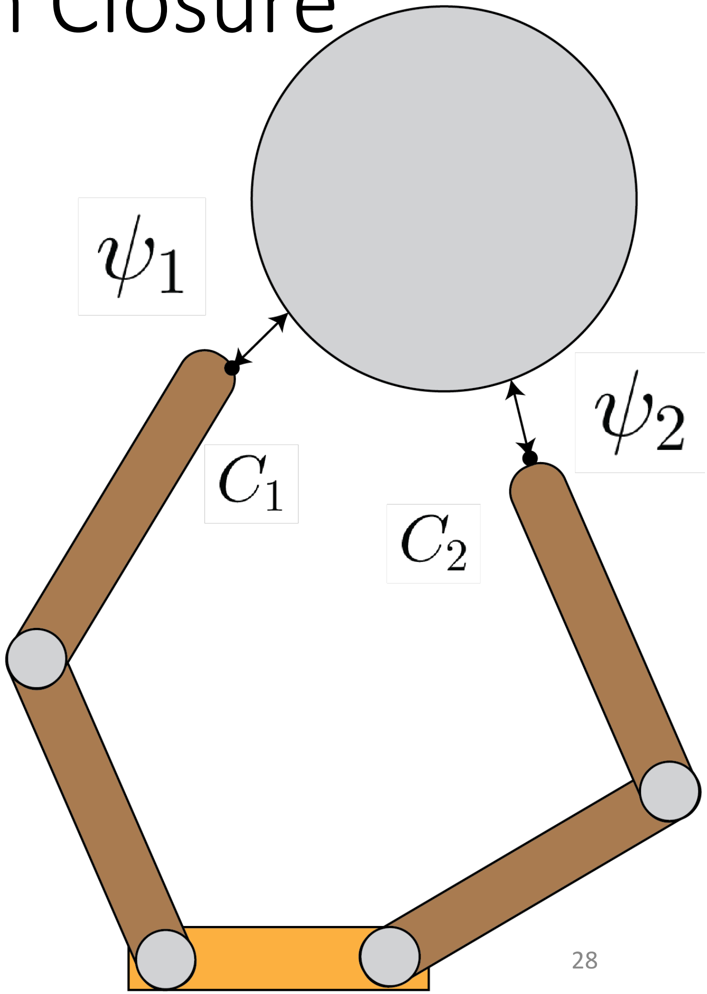
1st Order Form Closure



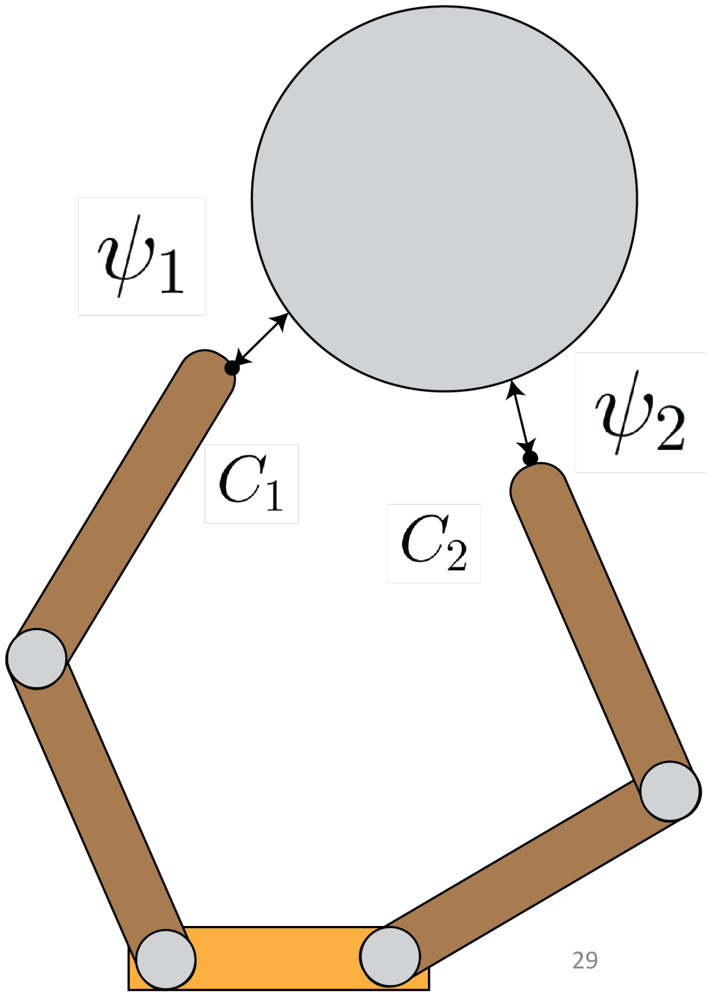
1st Order Form Closure



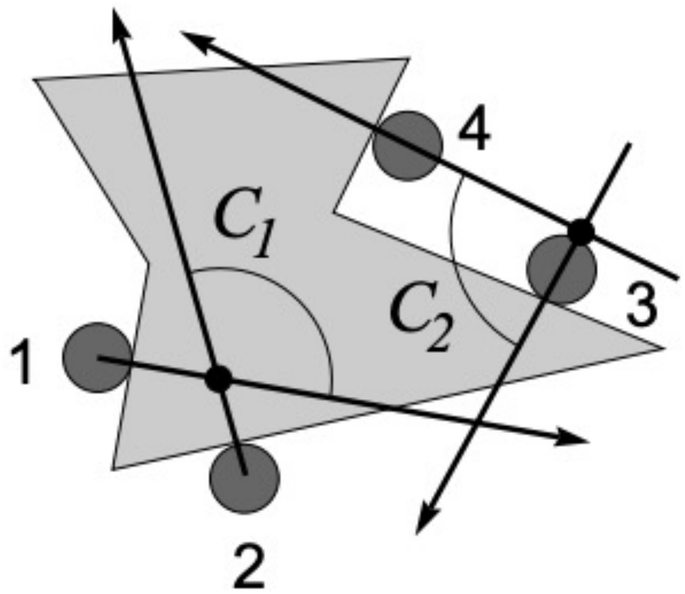
Grasp Analysis – 1st Order Form Closure

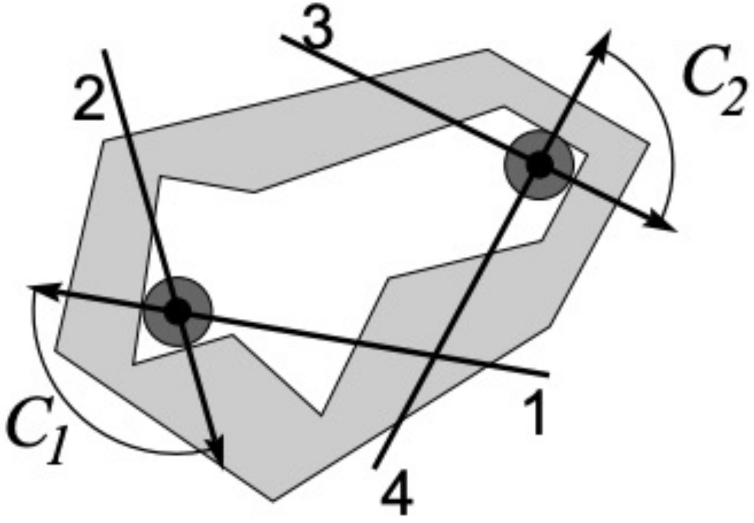


1st Order Form Closure



Grasp Analysis – 1st Order Form Closure





Grasp Analysis – 1st Order Form Closure