Introduction to Robotic Manipulation

Session 2

Todays Agenda

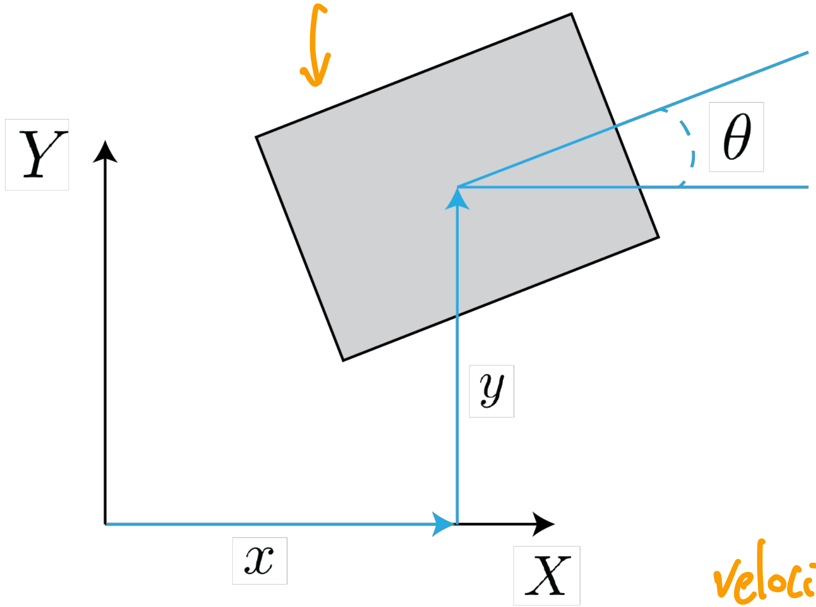
- Announcements
- Review of last session
- Deriving the Contact Jacobian
- Coulomb Friction and Friction Cone
- Finger Jacobians and Forward Kinematics
- Grasping and the Grasp Matrix

Announcements

- Assignment 1 will be out this Friday
- Lecture are actively being uploaded – see website
- Video lectures – find zoom recordings on Canvas!

Review of last session – Config and States

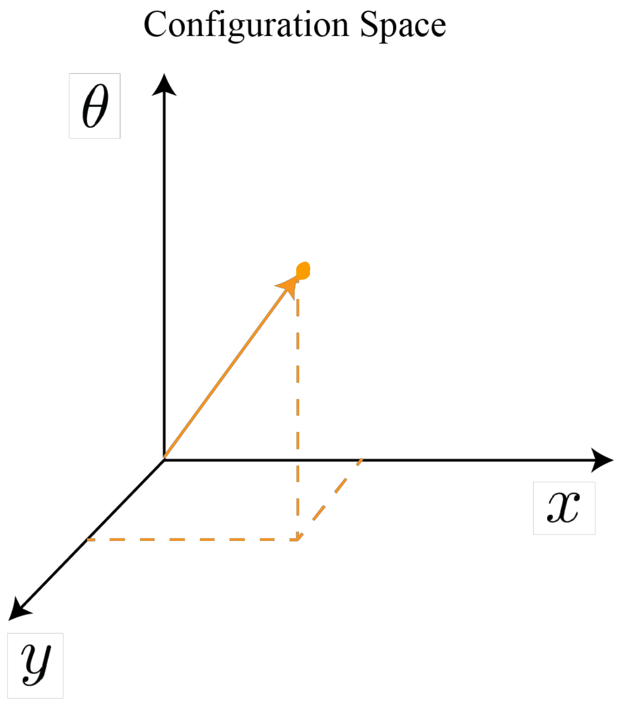
Rigid-body in the plane



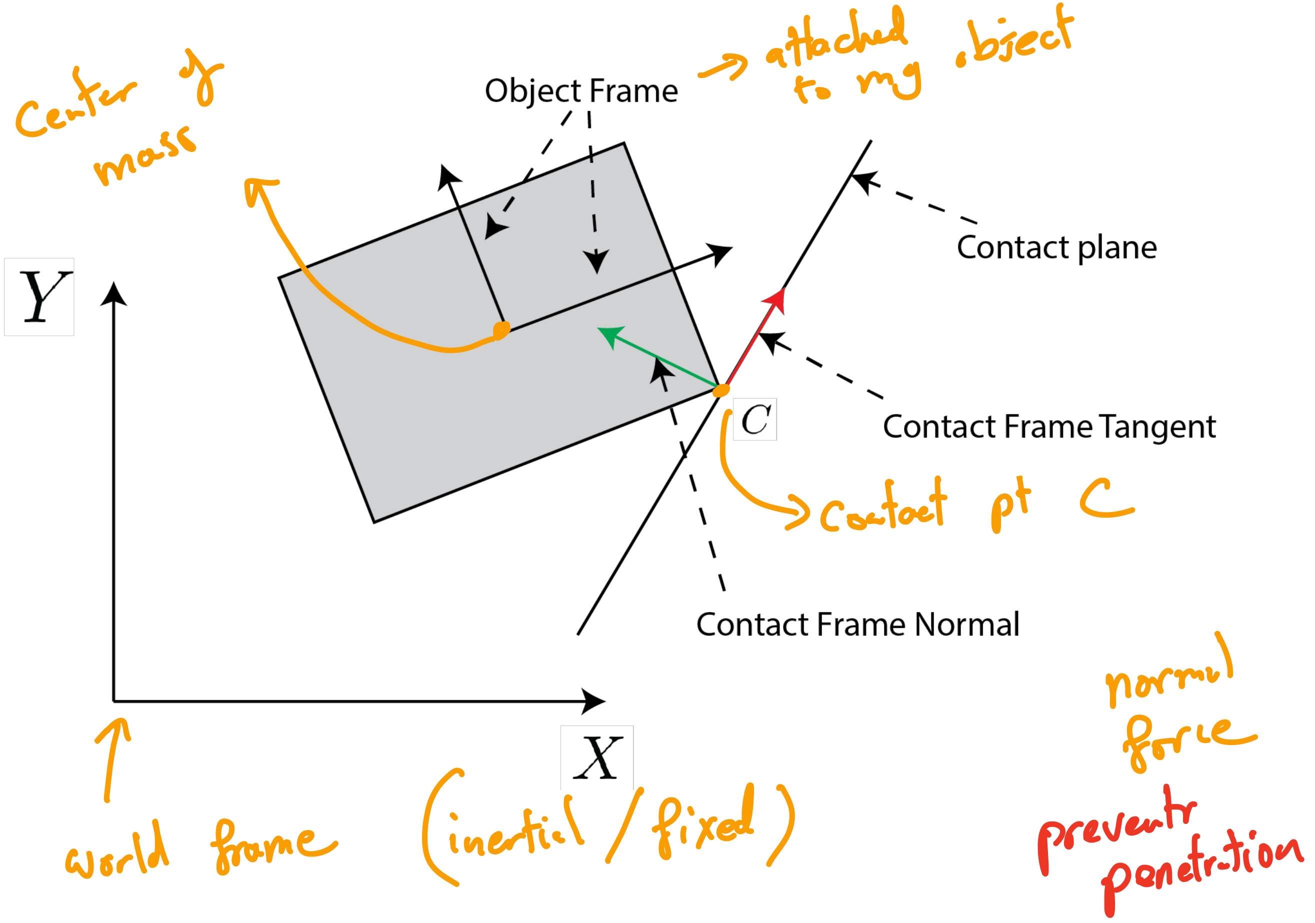
Configurations

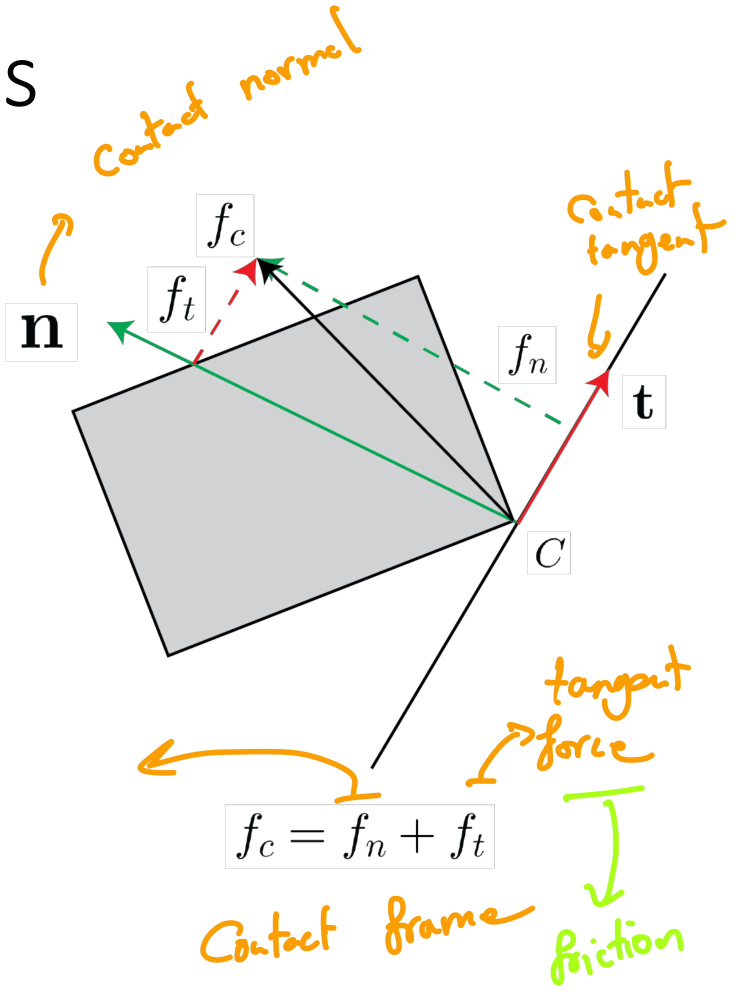
$$s_t = \begin{bmatrix} x \\ y \\ \theta \\ \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

velocities

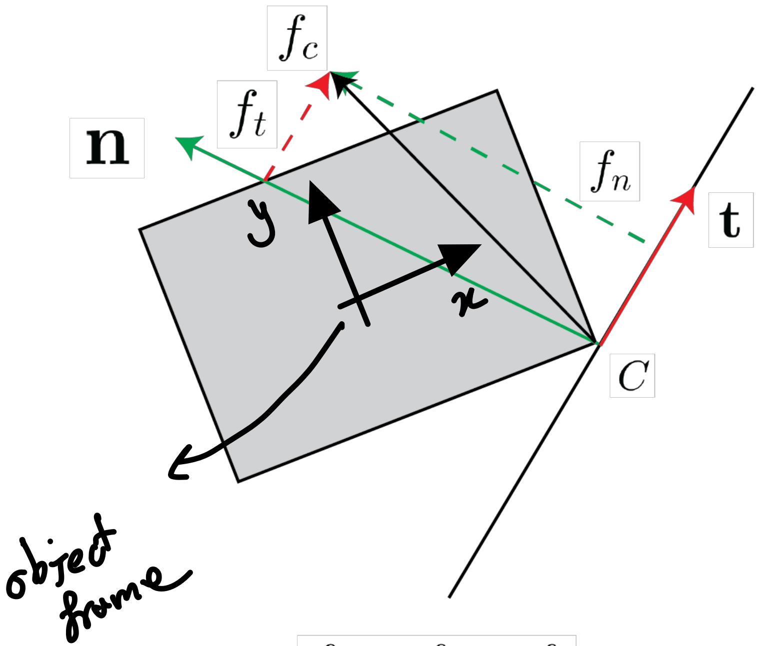


Review of last session -- Frames





Review of last session – Forces and Wrenches

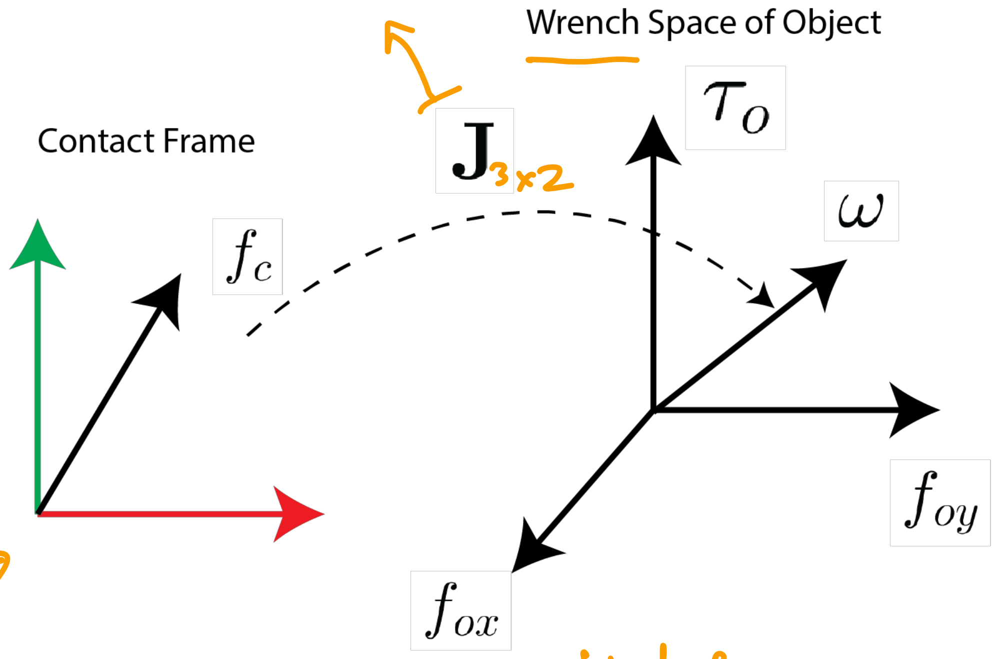


$f_c = f_n + f_t$

Contact frame

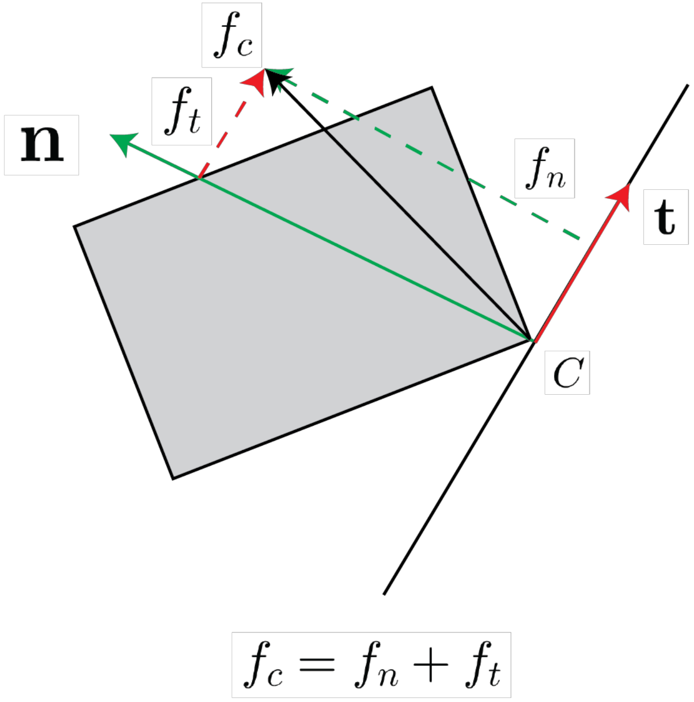
Contact Jacobian

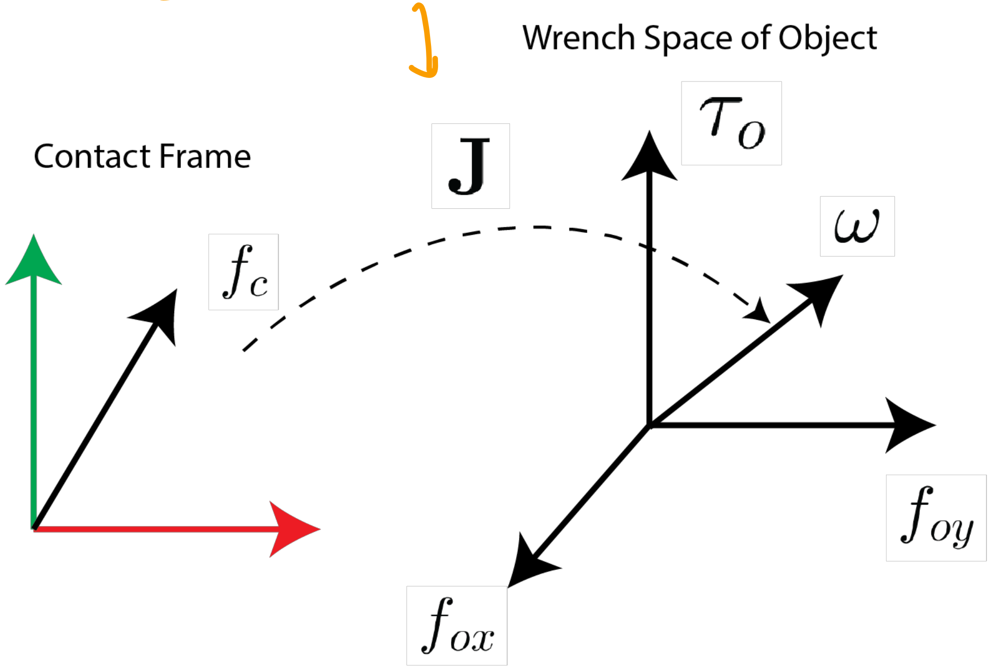
$\omega = \begin{bmatrix} f_x \\ f_y \\ \tau \end{bmatrix}$



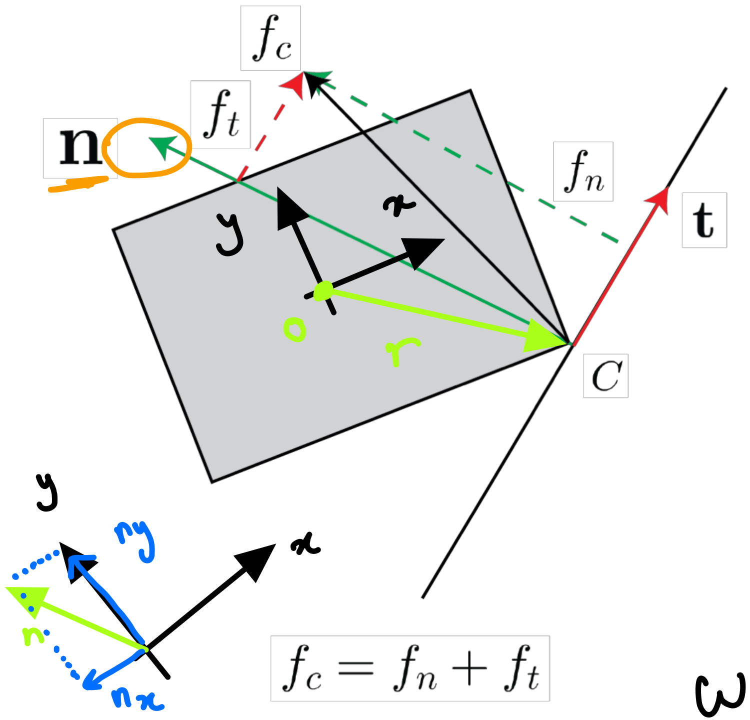
Deriving the Contact Jacobian

$J(q)$: is a function of the config of the object





Deriving the Contact Jacobian – Normal



r: vector connecting O to C

$$n : \begin{bmatrix} n_x \\ n_y \end{bmatrix}$$: unit normal of the contact frame in the object frame

$$w = \begin{bmatrix} f_{x_o} \\ f_{y_o} \\ \tau_o \end{bmatrix} \qquad \underset{3\times1}{w_n} = \underset{3\times1}{J_n} \underset{1\times1}{f_n}$$

f_n: magnitude of normal force $\equiv$ projection of f_c on $\vec{n}$

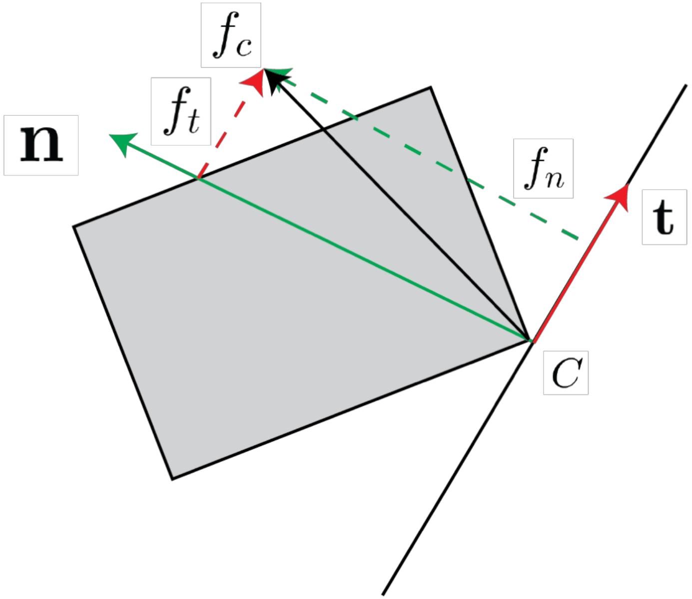
$$w_n = \begin{bmatrix} n_x \\ n_y \\ \boxed{r \times n} \end{bmatrix} f_n$$

n_x, n_y: linear terms

$r \times n$: Torque term

Cross product

Deriving the Contact Jacobian - Tangent



$f_c = f_n + f_t$

$w_t = J_t f_t$ $\quad t \perp n$

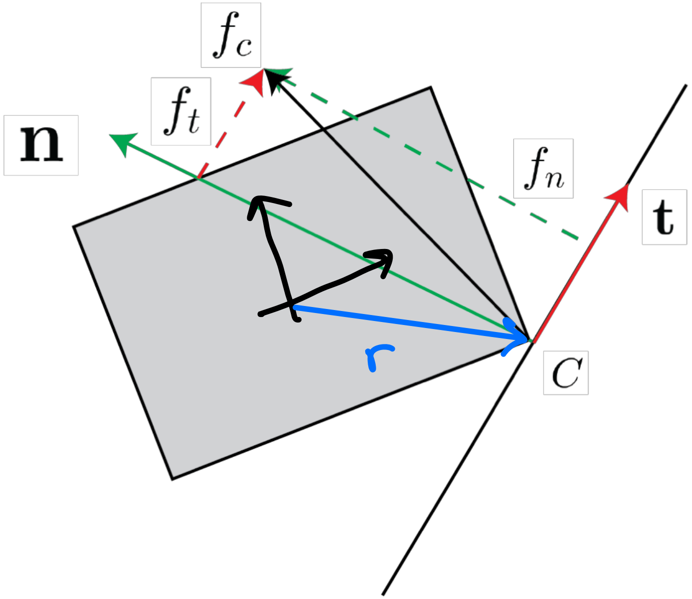
$$n = \begin{bmatrix} n_x \\ n_y \end{bmatrix} \rightarrow t = \begin{bmatrix} n_y \\ -n_x \end{bmatrix}$$

$$w_t = \begin{bmatrix} n_y \\ -n_x \\ r \times t \end{bmatrix}$$

$\rightarrow$ account for linear

$\rightarrow$ account for ~~rotational~~ torque

Deriving the Contact Jacobian

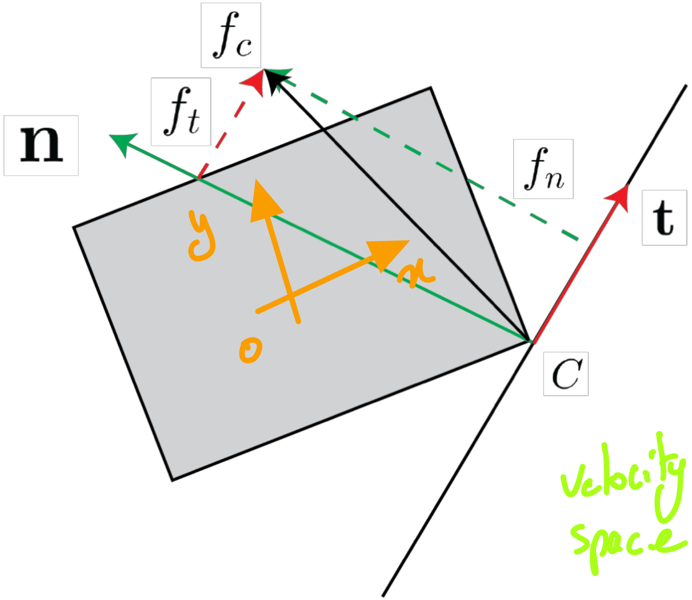


$f_c = f_n + f_t$

$$\underset{3\times 1}{w} = \underbrace{\begin{bmatrix} n_x & n_y \\ n_y & -n_x \\ r \times n & r \times t \end{bmatrix}_{3\times 2}}_{\text{Contact Jacobian}} \begin{bmatrix} f_n \\ f_t \end{bmatrix}_{2\times 1}$$

Map the forces in the contact frame into the wrench space of the object

Force/Velocity Dual of Jacobian



v_o := velocity of the object $\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix}_{3\times1}$

v_c := velocity of the contact location C $_{2\times 1}$

Velocity mapping

velocity space $\left\{ v_c = \underline{J_c}^T v_o \right.$

transpose of Contact Jacobian

&

$\underline{w = J_c f_c}$

contact Jac.

force/wrench space

$v_c = \begin{bmatrix} v_{c,n} \\ v_{c,t} \end{bmatrix} \rightarrow$ velocity of pt C

$\cancel{v_{c,n} < 0}$ penetration

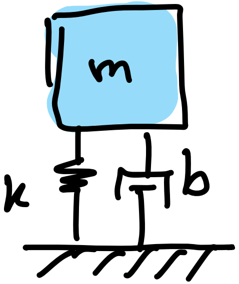
Coulomb Friction Definition

- Approximation $\rightarrow$ to what ?

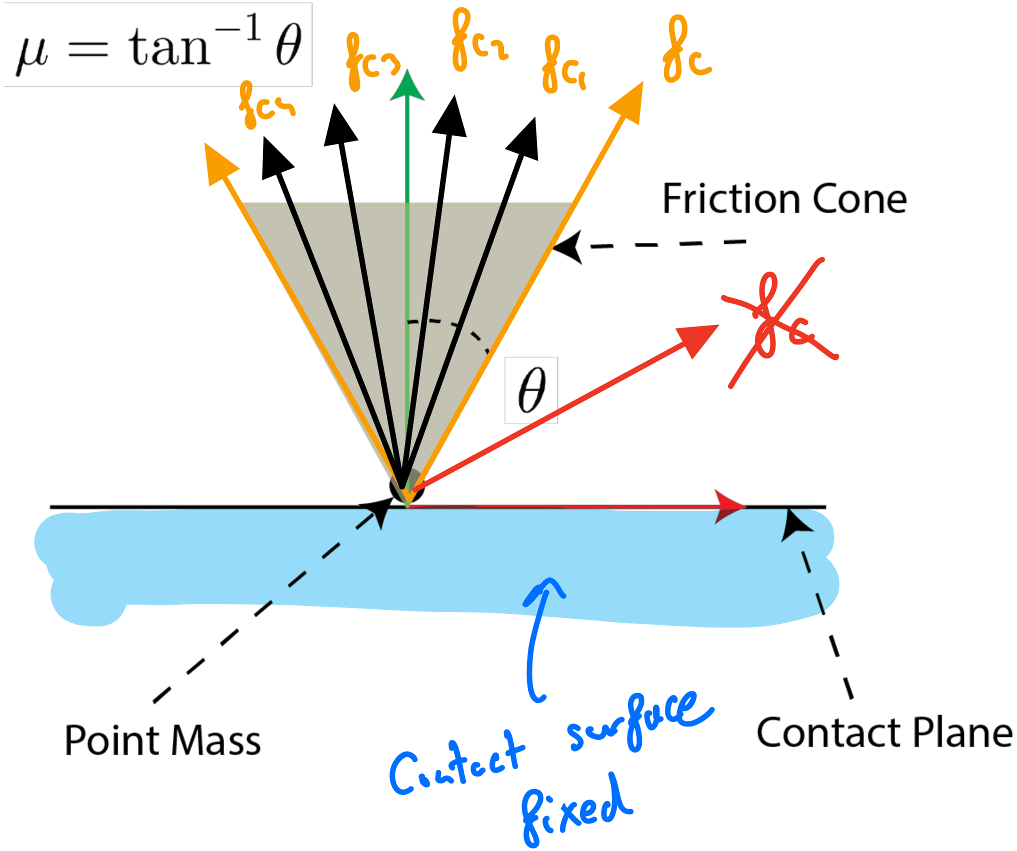
  $\hookrightarrow$ electro magnetic; electro dynamic / static $\rightarrow$ adhesion

- Contrasts viscous laws

$$m\ddot{x} + \underbrace{b\dot{x}}_{\text{viscous dissipation term}} + kx = u$$



Coulomb Friction for Point Mass

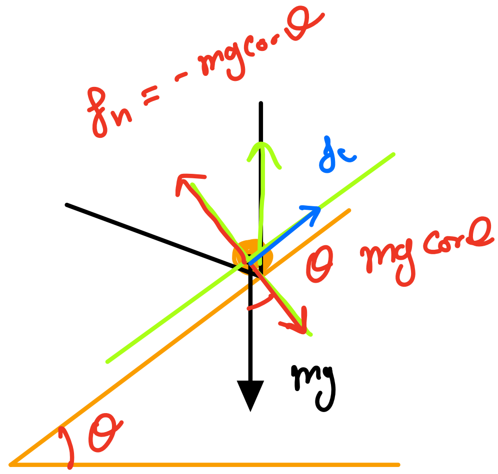


- The frictional force transmitted between the object and the surface must lie with the friction cone

- $|f_t| \leq \mu f_n$

  μ → coefficient of friction

  $|\cdot|$ → mathematical constraint on the magnitude of tangential force

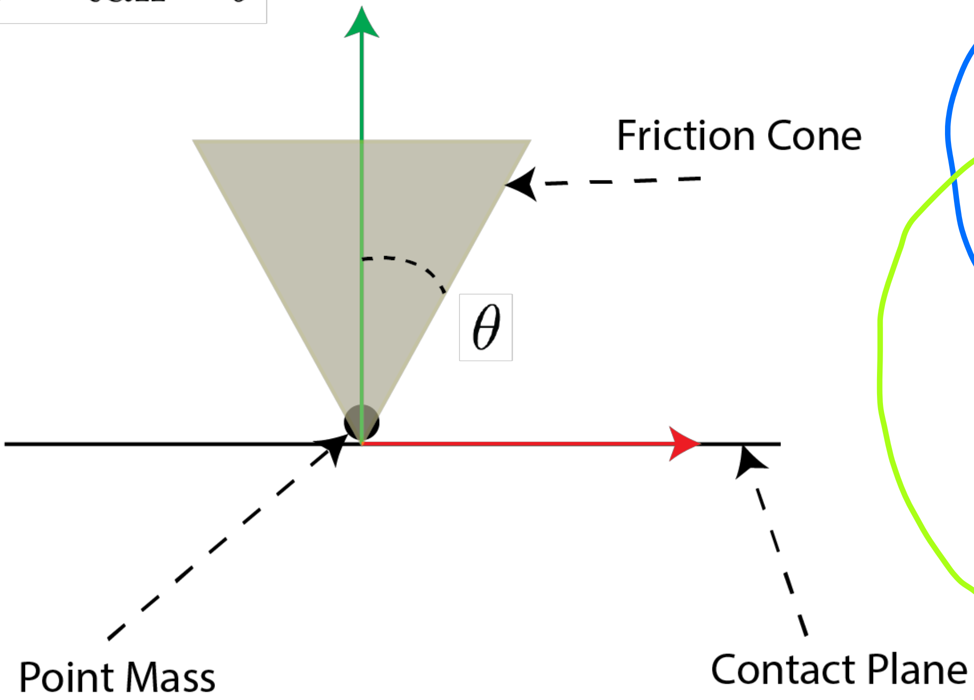


$$|f_t| \leq \mu f_n$$

$$f_t = \mu f_n$$

Coulomb Friction for Point Mass

$$\mu = \tan^{-1}\theta$$

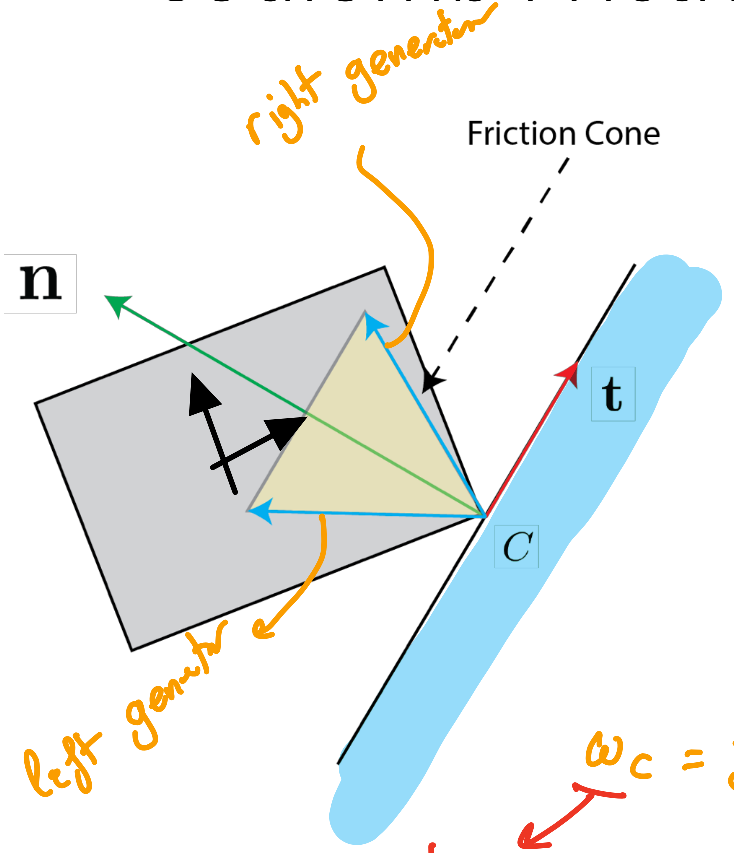


- sticking contact → no motion
- sliding contact → motion

reaction force is always in the interior of the cone $|f_t| < \mu f_n$

$f_t = \mu f_n$ and the object moves

Coulomb Friction for Extended Body



- All contact forces transmitted to the obj. through contact pt C must live inside the friction cone

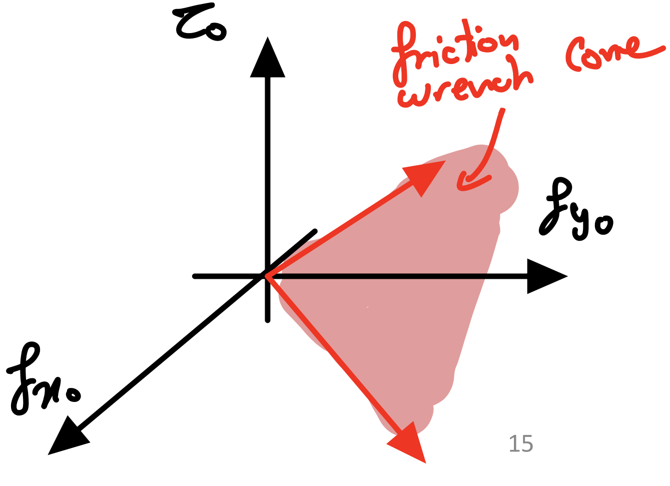
★ Friction cone is always described in the contact frame.

J_c : Contact Jacobian

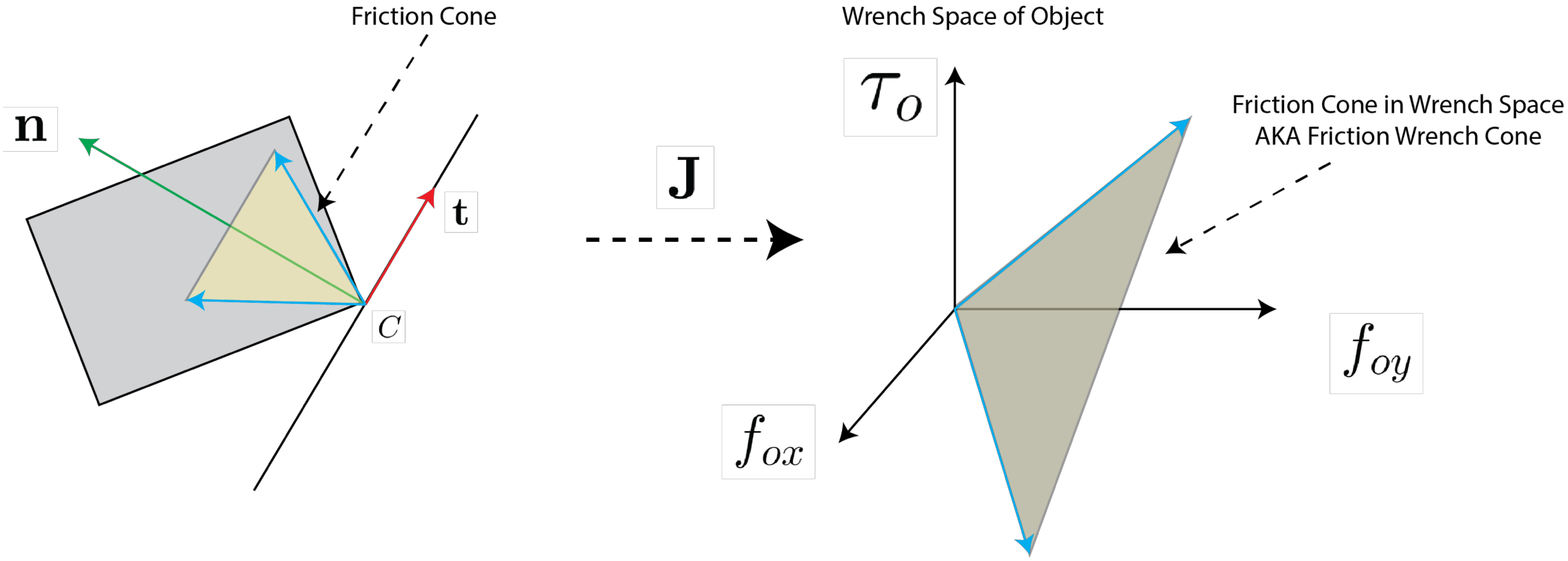
$w_c = J_c \, \underline{FC}$

friction wrench cone

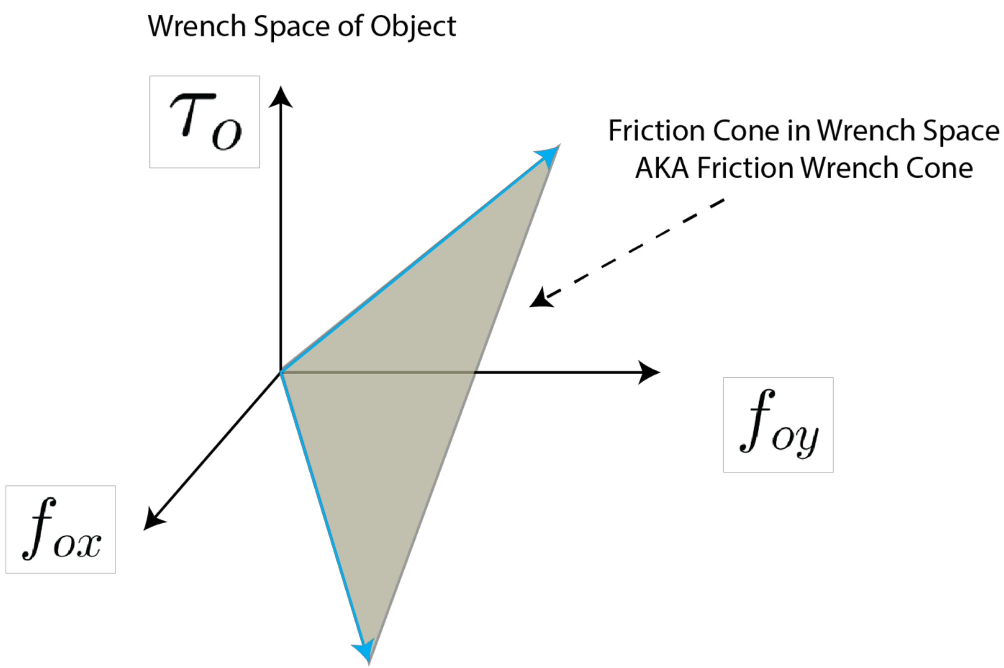
the set of all possible friction reaction forces



Friction Wrench Cone



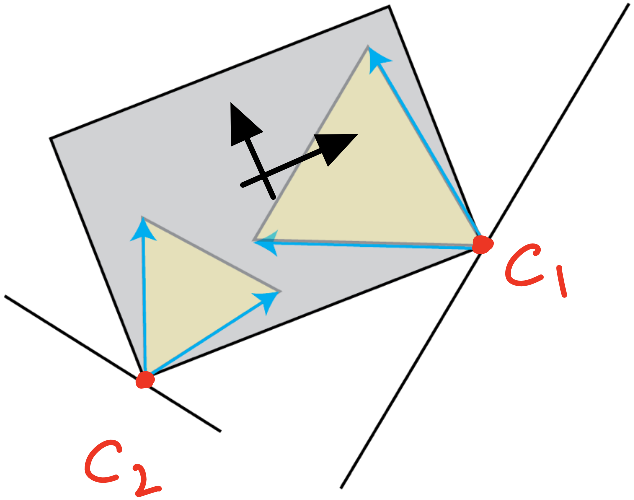
Friction Wrench Cone



Properties:

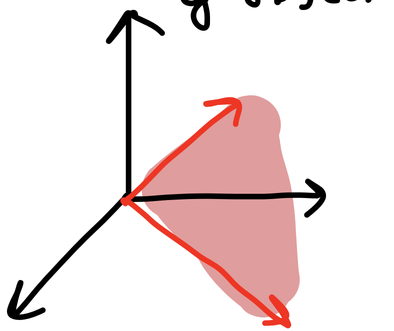
- It's still a cone
- It's not symmetric w.r.t. linear force plane
- Characterizes the set of all possible forces the frictional interaction can apply to the object
- On the boundary = contact is sliding

Composite Friction Wrench Cone



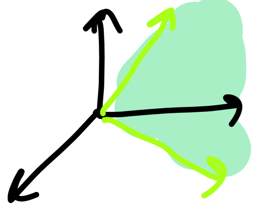
- First ignore c_2

  $J_{c_1} \rightarrow$ apply it to friction cone at c_1

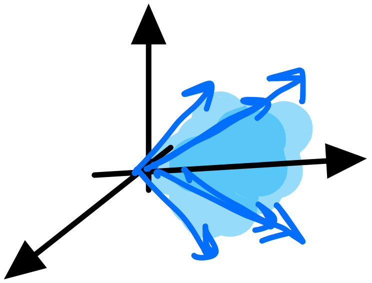


- Next, ignore c_1

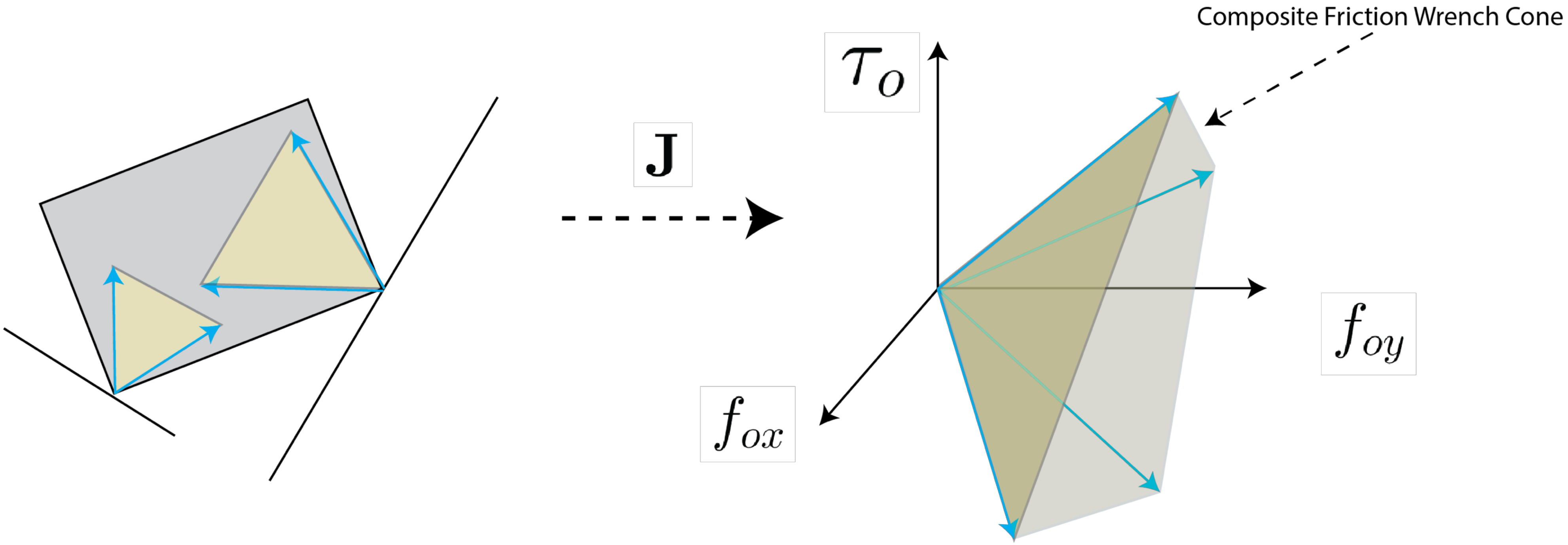
  $J_{c_2} \rightarrow$ apply to FC at c_2



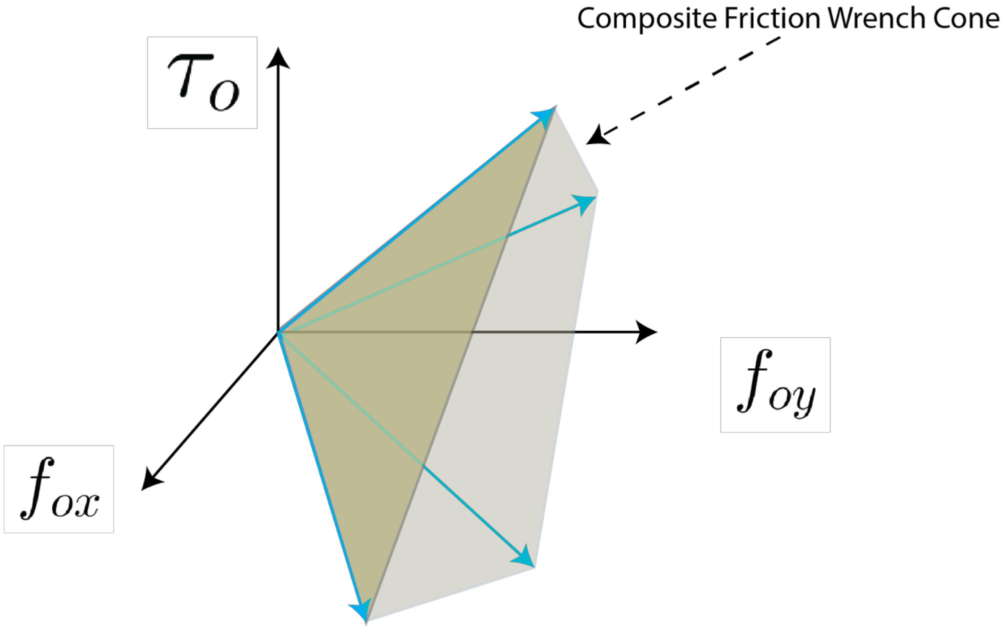
☆ add the contribution from each cone



Composite Friction Wrench Cone



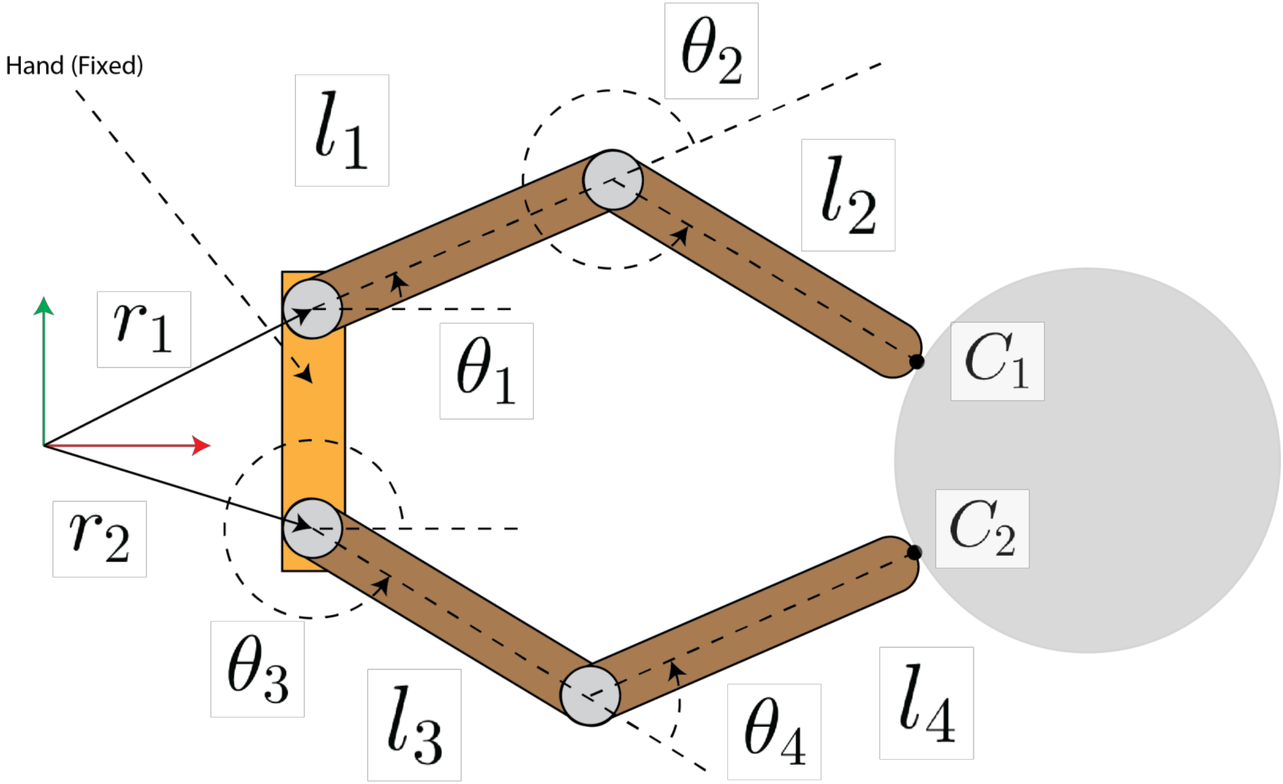
Composite Friction Wrench Cone



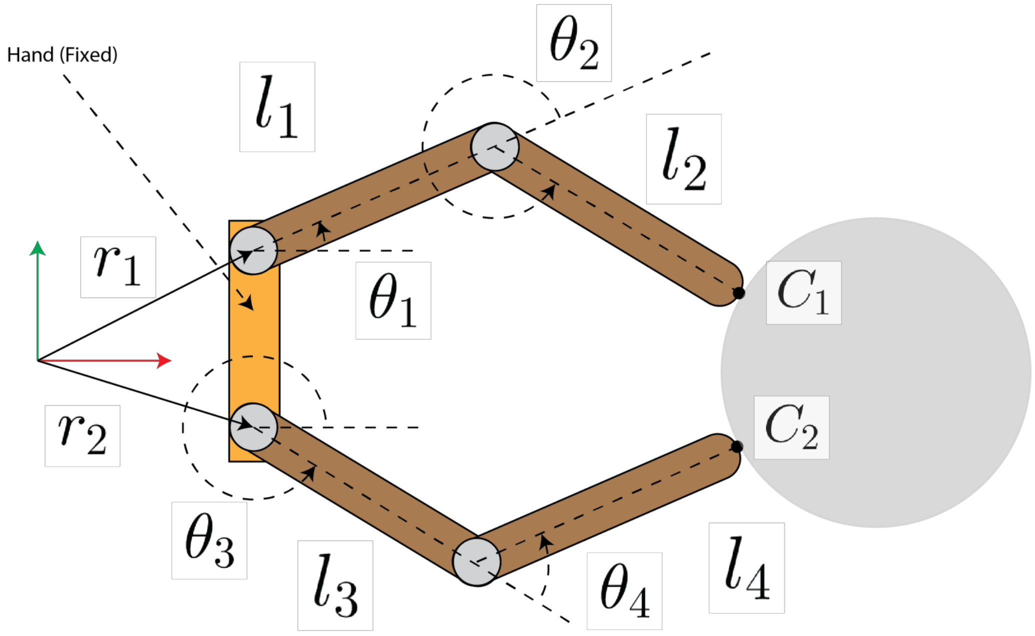
Properties:

- It's still a cone
- Characterizes the set of all possible forces the frictional interaction can apply to the object
- On the boundary = contact is sliding

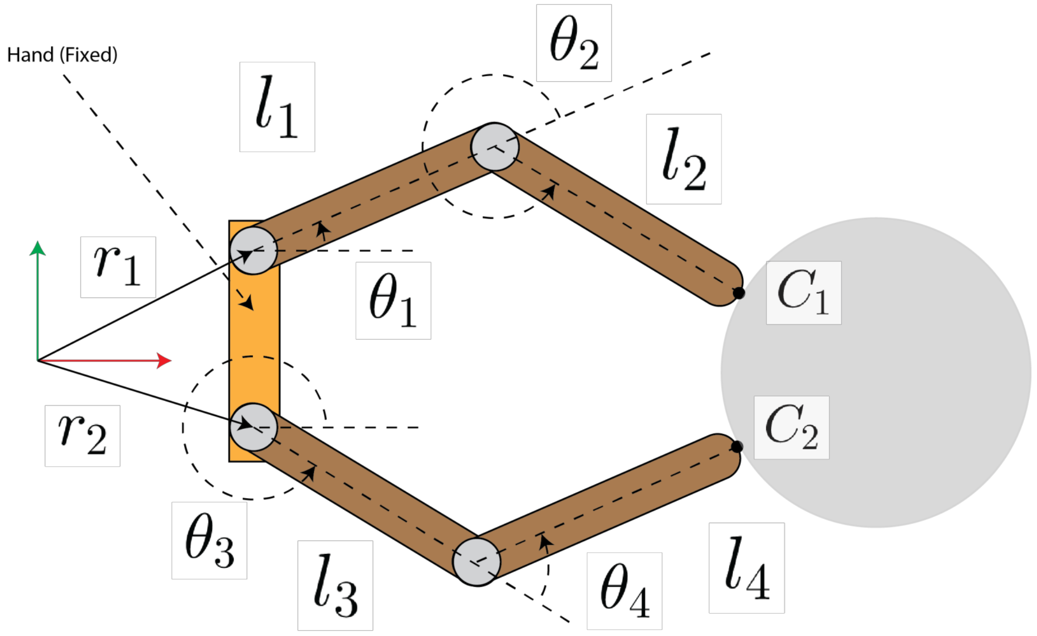
Finger Jacobians



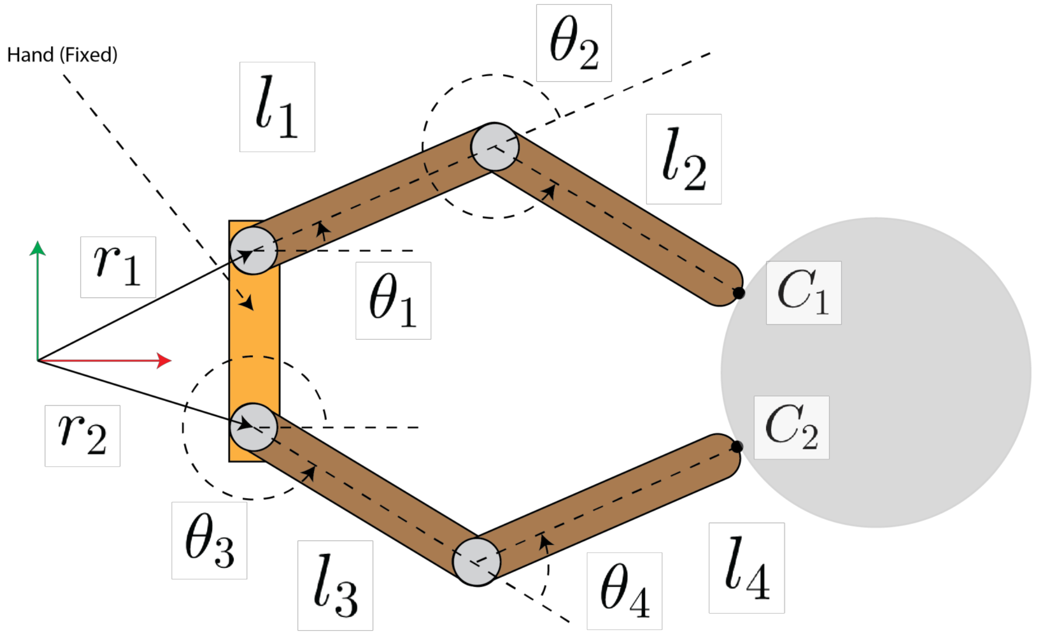
Finger Jacobians – Upper Finger



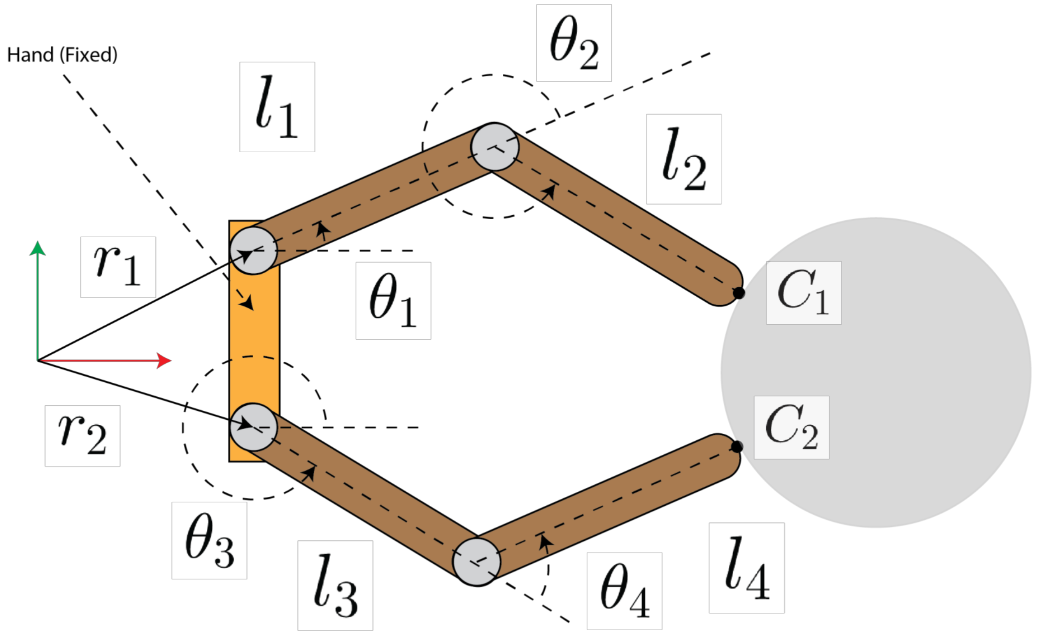
Finger Jacobians – Lower Finger



Finger Jacobians – Total



Finger Jacobians – Total (Intuition)



Finger Jacobians – Dual Force/Velocity