Lab 1 Written 1

Wednesday, August 31, 2022 3:24 PM

1. 1. Consider two random variables X,Y that are not independent. Their probabilities of are given by the following table:

	X=0	X=1
Y=0	1/4	1/4
Y=1	1/6	1/3

(a) What is the probability that $X = 1$?
(b) What is the probability that $X = 1$ conditioned on $Y = 1$?
(c) What is the variance of the random variable X?
(d) What is the variance of the random variable X conditioned that $Y = 1$?
(e) What is $E[X^3 + X^2 + 3Y^7 | Y = 1]$?

a) $P(X=1) =$

$= P(X=1, Y=0) + P(X=1, Y=1)$

$= \frac{1}{4} + \frac{1}{3}$

$= \frac{3}{12} + \frac{4}{12} = \frac{7}{12}$

$\boxed{P(X=1) = \frac{7}{12}}$

b) $P(X=1 \mid Y=1) =$

$= \frac{P(Y=1, X=1)}{P(Y=1)}$ → $\frac{1}{6} + \frac{1}{3}$, $\frac{1}{6} + \frac{2}{6} = \frac{3}{6}$

$= \frac{\frac{1}{3}}{\frac{3}{6}}$ → $\frac{2}{6}$

$= \frac{2}{3}$

$\boxed{P(X=1 \mid Y=1) = \frac{2}{3}}$

c) $Var(X) =$

Marginal PMF:

$$P_X(x) = \begin{cases} \frac{5}{12} & x = 0 \\ \frac{7}{12} & x = 1 \\ 0 & \text{otherwise} \end{cases}$$

$E(X) = \sum_{x=0}^{1} x \cdot P_X(x)$

$= (0)\left(\frac{5}{12}\right) + (1)\left(\frac{7}{12}\right)$

$E(X) = \frac{7}{12}$

$Var(X) = \sigma^2 = E(X - E[X])^2$

$= E[X^2] - (E[X])^2$

$= \left[0^2 \cdot \frac{5}{12} + 1^2\left(\frac{7}{12}\right)\right] - \left(\frac{7}{12}\right)^2$

$= \frac{7}{12} - \left(\frac{7}{12}\right)^2$

$= 0.24$

$\boxed{Var(X) = 0.24}$

d) $Var(X \mid Y=1) =$

$$P_X(x) = \begin{cases} \frac{5}{12} & x = 0 \\ \frac{7}{12} & x = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$P_X(x \mid Y=1) = \begin{cases} \frac{1}{3} & x = 0 \\ \frac{2}{3} & x = 1 \\ 0 & \text{otherwise} \end{cases}$$

$Var(X \mid Y=1) =$

$E[X^2 \mid Y=1] - (E[X \mid Y=1])^2$

$0^2 \cdot \frac{1}{3} + 1^2 \cdot \frac{2}{3}$ → $\frac{2}{3}$

$(0 \cdot \frac{1}{3} + 1 \cdot \frac{2}{3})^2$ → $(\frac{2}{3})^2$

$Var(X \mid Y=1) = \frac{2}{3} - \left(\frac{2}{3}\right)^2 =$

$\boxed{Var(X \mid Y=1) = \frac{2}{9}}$

e) $E[X^3 + X^2 + 3Y^7 \mid Y=1] =$

- $E[X^3] = 0^3 \cdot \frac{5}{12} + 1^3 \cdot \frac{7}{12} = \frac{7}{12}$
- $E[X^2] = 0^2 \cdot \frac{5}{12} + 1^2 \cdot \frac{7}{12} = \frac{7}{12}$
- $E[Y^7] = 0^7 \cdot \frac{1}{2} + 1^7 \cdot \frac{1}{2} = \frac{1}{2}$

$$P_Y(y) = \begin{cases} \frac{1}{2} & Y = 0 \\ \frac{1}{2} & Y = 1 \\ 0 & \text{otherwise} \end{cases}$$

- $|Y=1| = \frac{1}{2}$?

→ $\frac{1}{2} \cdot \frac{1}{2} \cdot 3 = \frac{3}{4}$

$= \frac{7}{12} \cdot \frac{7}{12} \cdot \frac{3}{4} = \frac{49}{192} = 0.26$

$\boxed{E[X^3 + X^2 + 3Y^7 \mid Y=1] = 0.26}$