Lab 1 Written Problems

Wednesday, August 31, 2022 4:05 PM

3. Consider a coin such that probability of heads is 2/3. Suppose you toss the coin 100 times. Estimate the probability of getting 50 or fewer heads. You can do this in a variety of ways. One way is to use the Central Limit Theorem. Be explicit in your calculations and tell us what tools you are using in these.

$X :=$ the number of heads flipped

$X = \sum_i^{100} x_i \qquad x_i \sim \text{Bern}(2/3)$

$E[x_i] = 2/3$

$\text{Var}[x_i] = 2/9$

$E[X] = \sum E[x_i]$

$= \sum 2/3$

$= 100(2/3)$

$E[X] = 66.67$

$\text{Var}[X] = \text{Var}[\sum x_i] = 100\ \text{Var}[x_i] = 200/9$

$\Rightarrow \sigma_X = \frac{\sqrt{200}}{3}$

Applying Central Limit Theorem

$\bar{X} \sim N\left(\frac{200}{3}, \frac{\sqrt{200}}{3\sqrt{100}}\right) = N\left(\frac{200}{3}, \frac{\sqrt{2}}{3}\right)$

$z = \frac{50 - \frac{200}{3}}{\frac{\sqrt{2}}{3}} = \frac{150 - 200}{\sqrt{2}} = \frac{-50}{\sqrt{2}}$

$z \approx -35$

$P\{X \leq 50\} = Q(-35) \approx 0$

Manual check

$P\{X \leq 50\} = \sum_{k=0}^{50} (2/3)^k (1/3)^{100-k} \binom{100}{k}$ $\qquad X \sim \text{Binom}(100, 2/3)$

$= 4.19 E^{-4}$

$= 0.000419 \approx 0$ ✓