7.9 Problems: 1,2,3,4,8,9,19

1. $\arg(1+i) = \pi/4$
   $\arg(1/2 + \sqrt(3)/2) = 1.047$
   $\arg((1+i)^3) = 3\arg(1+i) = 3\pi/4$
   $\arg((1/2+\sqrt(3)/2)^2 43) = 243\arg(1/2+\sqrt(3)/2) = 254.42$
   $\arg((1+i)^2(1/2+\sqrt(3)/2)^3) = 2\arg(1+i)3\arg(1/2+\sqrt(3)/2) = 2*\pi/4*3*1.047 = 4.934$

2. (i) If we can make the argument continuous in this range we can simplify the expression. We can write the limit instead as $\lim_{y\to 0^+}\arg(x+iy)$. We have $y>0$ so we can work in the complex plane $\mathbb{C}_{y>0}$ with this restriction. The function $\cos^{-1}:(-1,1)\to(0,\pi)$ is continuous on this domain.

$$\begin{aligned}\lim_{y\to 0^+}\cos^{-1}\left(\frac{x}{\sqrt{x^2+y^2}}\right) &= \lim_{y\to 0^+}\cos^{-1}\left(\frac{x}{\sqrt{x^2}}\right)\\ &= \lim_{y\to 0^+}\cos^{-1}\left(\frac{x}{|x|}\right)\\ &= \lim_{y\to 0^+}\cos^{-1}(-1)\end{aligned}$$

   As $x/|x|\to -1$ the function $\cos^{-1}(x/|x|)\to\pi$ because it is continous.

   (ii) We can write the limit as $\lim_{y\to 0^+}\arg(x-iy)$. With $y<0$ we can work in the domain $\mathbb{C}_{y<0}$. In this plane this function is continous:

$$\cos^{-1}:(-1,1)\to(-\pi,0)$$

   So we can find the limit:

$$\begin{aligned}\lim_{y\to 0^+}\cos^{-1}\left(\frac{x}{\sqrt{x^2+y^2}}\right) &= \lim_{y\to 0^+}\cos^{-1}\left(\frac{x}{\sqrt{x^2}}\right)\\ &= \lim_{y\to 0^+}\cos^{-1}\left(\frac{x}{|x|}\right)\\ &= \lim_{y\to 0^+}\cos^{-1}(-1)\end{aligned}$$

   As $x/|x|\to -1$ the function $\cos^{-1}(x/|x|)\to -\pi$ because it is continous.

3.

$$
\begin{aligned}
&\log(3i) = \log(3) + i\pi/2 \\
&\log(-2i) = \log(2) + i3\pi/2 \\
&\log(1+i) = \log(\sqrt(2)) + i\pi/4 \\
&\log(-1) = i\pi \\
&\log((e^{i\pi/3})^{10}) = 10(\log|e^{i\pi/3}| + i\pi/3) = 10i\pi/3 \\
&\log(x) = \begin{cases} x, & \text{if } 0 < x \\ \log(-x) + i\pi, & \text{if } 0 > x \end{cases}
\end{aligned}
$$

4.

$$\mathrm{Log}()$$