

CH1 Time Complexity

- recursive

- Factorial

- Fibonacci Number

$$\begin{aligned} \bullet \quad F_n = & \begin{cases} 0, & \text{if } n = 0 \\ 1, & \text{if } n = 1 \\ F_{n-1} + F_{n-2}, & \text{if } n \geq 2 \end{cases} \end{aligned}$$

- the number of recursive calls grows exponentially with n is $1.41^n < F_n < 2^n$
- use DP skill need $O(n)$

- Binomial Coefficient

$$\begin{aligned} \bullet \quad \binom{n}{m} = & \begin{cases} 1, & \text{if } n = m \text{ or } m = 0 \\ \binom{n-1}{m} + \binom{n-1}{m-1}, & \text{otherwise} \end{cases} \\ \bullet \quad \text{use DP skill need } & O(nk) \end{aligned}$$

- GCD

$$\begin{aligned} \bullet \quad \text{GCD}(A, B) = & \begin{cases} A, & \text{if } A \bmod B = 0 \\ \text{GCD}(B, A \bmod B), & \text{otherwise} \end{cases} \end{aligned}$$

- Ackerman function

$$\begin{aligned} \bullet \quad A(m, n) = & \begin{cases} n + 1, & \text{if } m = 0 \\ A(m - 1, 1), & \text{if } n = 0 \\ A(m - 1, A(m, n - 1)), & \text{otherwise} \end{cases} \end{aligned}$$

- Tower of Hanoi $O(2^n)$

$$\begin{aligned} \bullet \quad T(n) = & \begin{cases} 1, & \text{if } n = 1 \\ 2T(n - 1) + 1, & \text{if } n \geq 2 \end{cases} \end{aligned}$$

- permutation: $O(n! * n)$

```

void swap(char *a, char *b){
    char temp=*a;
    *a=*b;
    *b=temp;
}
void perm(char *list, int i, int n){
    int j, temp;
    if(i==n){
        for(j=0;j<n;j++)
            printf("%c", list[j]);
        printf("\n");
    }
    else{
        for(j=i;j<n;j++){
            swap(&list[i], &list[j]); //list[j]當head
            perm(list, i+1, n); //後面(i+1)~n permutation
            swap(&list[i], &list[j]); //還原
        }
    }
}
int main(){
    char list[3]={"abc"};
    perm(list,0,3);
    return 0;
}
output:
abc
acb
bac
bca
cba
cab

```

- basic math
 - $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$
 - $\sum_{i=1}^n i^d \approx n^{d+1}, d \geq 0$
 - $\sum_{i=1}^n \frac{1}{i} = \log n$
 - $n! \leq n^n$
 - $\lg(n!) = \Theta(n \log n)$
 - $\frac{n^{\frac{n}{2}}}{2} \leq n!$
 - Stirling's Formula
 - $n! = \sqrt{2\pi n} \left(\frac{n}{e}\right)^n \left(1 + \Theta\left(\frac{1}{n}\right)\right) \approx n^{(n+\frac{1}{2})} \times e^{-n}$
 - $(\log n)^b = o(n^a), a > 0$
 - e.g. $(\log n)^{100} < n^{0.0001}$
 - $\log^*(\log n) = \log^* n - 1$
- Master Theorem
 - $T(n) = aT\left(\frac{n}{b}\right) + f(n)$
- extended Master Theorem
 - $T(n) = aT\left(\frac{n}{b}\right) + nlgn$

CH2 CH4 Array & Linked Lisd

- linked list
 - invert linked list

```

typedef struct Node {
    int value;
    struct Node *next;
} Node;

Node* invert(Node* head) {
    Node* previous=NULL;
    Node* current=NULL;
    Node* nextNode=head;

    while(nextNode!=NULL) {
        previous=current; // previous 前進
        current=nextNode; // current 前進
        nextNode=nextNode->next; // 暫存下一個節點
        current->next=previous; // 反轉指標

    }
    head=current;
    return head; // 新的頭節點
}

int main()
{
    Node *head = (Node*)malloc(sizeof(Node));
    Node *dummy= (Node*)malloc(sizeof(Node));
    head->value=1;
    dummy=head;
    for(int i=2;i<4;i++){
        Node *temp=(Node*)malloc(sizeof(Node));
        temp->value=i;
        dummy->next=temp;
        dummy=temp;
    }
    dummy=head;
    printf("before invert: ");
    while(dummy!=NULL){
        printf("%d", dummy->value);
        dummy=dummy->next;
        if(dummy!=NULL)
            printf("->");
    }
}

```

```

}

printf("\n");
dummy=head;
head=invert(dummy);
dummy=head;
printf("after invert: ");
while(dummy!=NULL){
    printf("%d", dummy->value);
    dummy=dummy->next;
    if(dummy!=NULL)
        printf("->");
}
return 0;
}

output:
before invert: 1->2->3
after invert: 3->2->1

```

- Insert a node t after x node
 - doubly linked list(change 4 pointers)

```

node* insert(node *head, node *x, node *t){
    t->prev=x;
    t->next=x->next;
    x->next->prev=t;
    x->next=t;
    return head;
}

```

- singly linked list(change 2 pointers)

```

node* insert(node *head, node *x, node *t){
    t->next=x->next;
    x->next=t;
    return head;
}

```

- Delete a node t
 - doubly linked list(change 2 pointers)

```
node* delete(node *head, node *t){
    t->prev->next=t->next;
    t->next->prev=t->prev;
    free(t);
    return head;
}
```

- singly linked list delete node t after node x(change 1 pointers)

```
node* delete(node *head, node *x){
    node *t=x->next;
    x->next=t->next;
    free(t);
    return head;
}
```

- Generalize list
 - 像集合論
 - e.g.
 - $A = (a, (b, c)) \Rightarrow |A| = 2$
 - $B = (A, A, ()) \Rightarrow |B| = 3$ 表達出sharing
 - $C = (a, C) \Rightarrow |C| = 2$ 表達出recursion
- 多項式之表示
 - array
 - linked list
- spare matrix之表示
 - array
 - doubly linked list

CH3 Stack & Queue

- stack
 - stack application
 - parsing context-free languages
 - evaluating arithmetic expressions(infix, postfix, prefix)
 - function call management
 - recursion removal/recursive call
 - traversing tree(preorder, inorder, postorder)
 - DFS graph traversal
 - eight queen problem
 - maze problem
 - reverse output
 - 客人取盤子行為
 - stack implementation
 - array
 - linked list
 - two queues
 - stack permutations
 - $\frac{1}{n+1} \binom{2n}{n}$
 - 與下列問題同義
 - the number of binary tree structures with n nodes
 - the number of valid parentheses with n "("and")"
 - the number of matrix multiply chain with n+1 matrix(∴ 有n個*)
 - the number of train output order with n trains in the gateway

- Infix to Postfix

```
InfixtoPostfix(Infix){  
    while(Infix has not been scanned over){  
        x=NextToken(Infix);  
        if(x is operand)//x是operand  
            print(x);  
        else{//x是operator  
            if(x=='') {  
                while(stack.top() != '('){  
                    y=stack.top();  
                    stack.pop();  
                    print(y);  
                }  
            }  
            else{  
                if(precedence(x) > precedence(stack.top()))  
                    stack.push(x);  
                else{  
                    while(precedence(x) <= precedence(stack.top())){  
                        y=stack.top();  
                        stack.pop();  
                        print(y);  
                    }  
                    stack.push(x);  
                }  
            }  
        }  
    }  
    while(!stack.empty()){//清空stack  
        y=stack.top();  
        stack.pop();  
        print(y);  
    }  
}
```

- Postfix求值

```
Evaluate(Postfix){  
    whlie(Postfix has not been scanned over){  
        x=NextToken(Postfix);  
        if(x is oeprand)  
            stack.push(x);  
        else{//x is operator  
            right_operand=stack.pop();  
            left_operand=stack.pop();  
            stack.push(left_operand opeartor right_operand);//依operator作運算,放入stack  
        }  
    }  
    result=stack.top();  
    stack.pop();  
    return result;  
}
```

- check for balanced brackets(){}[]

```

bool judge(s:string){
    while(s has not been scanned over){
        x=NextToken(s);
        if(x=='(' || x=='[' || x=='{ ')
            stack.push(x);
        else{
            if(stack.isempty())
                return false;
            else{
                if(x==')'){
                    if(stack.top()!='(')
                        return false;
                }
                if(x==']'){
                    if(stack.top()!='[')
                        return false;
                }
                if(x=='}'){
                    if(stack.top()!='{ ')
                        return false;
                }
                stack.pop();
            }
        }
    }
    if(stack.isempty())
        return true;
    return false;
}

```

- queue
 - queue implementaion
 - circular array with no tag -> n-1
 - circular array with tag -> n
 - single linked list
 - circular linked list
 - two stacks

CH5 Tree & Binary Tree

- Tree
 - ancestor=predecessor
 - descendent=successor
 - tree化成binary tree, binary tree化成tree
 - tree化成binary
 - Leftmost-child-Next-Right-sibling
 - Forest化成binary tree, binary tree化成Forest
 - 皆針對Root做操作
- Binary Tree
 - ith level max node= 2^{i-1}
 - height h max node= $2^h - 1$
 - leaf num= n_0 , degree-2= n_2 , $n_0 = n_2 + 1$
 - 不可決定唯一binary tree
 - a. preorder+postorder
 - b. level-order+preorder
 - c. level-order+postorder
 - d. BST+inorder
 - the number of different binary trees with n nodes
 - Catalan number
 - $\frac{1}{n+1} \binom{2n}{n}$

- Binary Search Tree
 - In a BST find i-th smallest data

```

struct Node {
    Node* Lchild;
    int data;
    int Lsize;
    Node* Rchild;
};

search(T:BST, i:int){//在T中找出i-th小之data
    if(T!=Nil){
        k=(T->Lsize)+1;//代表root是kth小的数据
        if(i==k)
            return T->Data;
        else if(i<k)
            return serach(T->Lchild,i);//去左子樹找i-th小
        else
            return search(T->Rchild,i-k);//去右子樹找(i-k)th小
    }
}

```

- Heap
 - build a heap with n nodes
 - Top-Down
 - $O(n \log n)$
 - Bottom-Up
 - $O(n)$
 - Heapify[adjust(tree,i,n)]

```

void adjust(int tree[], int i, int n){
    //調整以i node no.為root之子樹成為Heap
    int j=2*i;//目前j是i之左子點No.
    int x=tree[i];
    while(j<=n){//尚有兒子
        if(j<n && tree[j]<tree[j+1])
            j=j+1;
        if(x>=tree[j])
            break;
        else{
            tree[j/2]=tree[j];//上移至父點
            j=2*j;//新的左子點位置
        }
    }
    tree[j/2]=x;//x置入正確格子中
}
void buildheap(int tree[], int n){
    for(int i=n/2;i>=1;i--)
        adjust(tree, i, n);
}

```

- Disjoin Sets
 - Union
 - Find
- Thread Binary Tree

CH9 Advanced Tree

- Double-Ended Priority Queue
 - Min-Max Heap
 - Deap
 - SMMH
- Extended Binary Tree
 - $E=I+2N$
 - Huffman Algorithm
- AVL Tree
- M-way search tree
 - B Tree of order m
 - B^+ Tree of order m
- Red-Blcak tree
- Optimal Binay Search Tree(OBST)
- Splay Tree
- Leftist Heap
- Binomail Heap
- Fibonacci Heap

CH7 Sort

- Search
 - Linear Search
 - Binary Search
- Sort
 - Elementary/Simple Sorts
 - Insertion sort
 - Selection sort
 - Bubble sort
 - Shell sort
 - Advanced/Efficient Sorts
 - Quick sort
 - Merge sort
 - Heap sort
 - Linear-Time sorting methods
 - LSD Radix sort=Radix sort
 - MSD Radix sort=Bucket sort
 - Counting sort

CH8 Hashing

- Collision
- Overflow
- Identifier Density
 - $\frac{n}{T}$, n:存入hash table之資料總數,T:變數總數
- Loading Density(α)
 - $\frac{n}{b*s}$, n:存入hash table之資料總數,b:Bucket數,s:Slot數
- Hashing 優點
- hashing function design
 - 3 disgn criteria
 - 計算簡單
 - 碰撞少
 - perfect hashing function
 - 不要造成hash table局部偏重儲存的情形
 - uniform hashing function
 - 常見hashing function design methods
 - Middle Square
 - Mod(Division)
 - Folding Addition
 - Digits Analysis
- Overflow Handling
 - Linear Probing
 - Quadratic Probing
 - Double Hashing
 - Chaining
 - Rehashing

CH6 Graph

- DFS
 - adjacency matrix: $O(V^2)$
 - adjacency lists: $O(V + E)$
- BFS
 - adjacency matrix: $O(V^2)$
 - adjacency lists: $O(V + E)$
- Topological sort
 - adjacency lists: $O(V + E)$
- Minimum Spanning Tree
 - Kruskal's algorithm
 - adjacency matrix: $O(E \log E)$
 - adjacency lists : $O(E \log E)$
 - compare to prim's: $\because E << V^2 \therefore \log E = O(\log V), \therefore O(E \log V)$
 - Prim's algorithm
 - adjacency matrix: $O(V^2)$
 - binary heap+adjacency lists: $O(E \log V)$
 - Fibonacci heap+adjacency lists: $O(E + V \log V)$
 - Sollin's algorithm
- Shortest Path Length
 - single source to other destinations
 - Directed Acyclic Graph(DAG)
 - adjacency lists: $O(V + E)$
 - Dijkstra algorithm
 - adjacency matrix: $O(V^2)$
 - binary heap+adjacency lists: $O(E \log V)$
 - Fibonacci heap+adjacency lists: $O(E + V \log V)$
 - Bellman-Ford Algorithm
 - adjacency matrix: $O(V^3)$
 - adjacency lists: $O(VE)$
 - all pairs of vertex
 - Floyd-Warshall algorithm
 - adjacency matrix: $O(V^3)$

- Johnson's algorithm
 - adjacency matrix: $O(V^2 \log V + VE)$
- AOE network
- Articulation Point
- Biconnected Graph
 - a connected undirected graph with no AP
- Biconnected component
 - G' is a subgraph of G, and G' is a biconnected graph
 - G' is Maximum Component