

winning strategy-subtraction game

8. (10%) Suppose that two people play a game taking turns removing, 1, 2, 3 or 4 stones at a time from a pile that begins with 22 stones. The person who removes the last stone wins the game. Show that the first player can win the game no matter what the second player does.

8. 【解】

【106 成大電機類題】

先手先拿 2 個,

接著若後手拿 x 個, 則先手接著拿 $5-x$ 個,

並一直以這種策略進行, 則最後先手必可以拿到最後一個.

假設第二人拿了 k 顆 ($k = 1, 2, 3, 4$)

剩下 :

$$20 - k$$

第一人立刻拿 :

$$5 - k$$

總共一輪拿了 :

$$k + (5 - k) = 5$$

👉 每一輪都把石頭數拉回到 :

$$20 \rightarrow 15 \rightarrow 10 \rightarrow 5 \rightarrow 0$$

最後 :

- 對手面對 5
- 對手拿完後, 你一定能拿到最後一顆