winning strategy-subtraction game

8. (10%) Suppose that two people play a game taking turns removing, 1, 2, 3 or 4 stones at a time from a pile that begins with 22 stones. The person who removes the last stone wins the game. Show that the first player can win the game no matter what the second player does.

8.【解】 【106 成大電機類題】

先手先拿 2 個,

接著若後手拿 x 個, 則先手接著拿 5-x 個,

並一直以這種策略進行, 則最後先手必可以拿到最後一個.

假設第二人拿了 k 顆（$k = 1, 2, 3, 4$）

剩下：

$$20 - k$$

第一人立刻拿：

$$5 - k$$

總共一輪拿了：

$$k + (5 - k) = 5$$

👉 每一輪都把石頭數拉回到：

$$20 \to 15 \to 10 \to 5 \to 0$$

最後：

- 對手面對 5
- 對手拿完後，你一定能拿到最後一顆

difference constraints

A.B,E

32.Consider the following system of difference constraints..

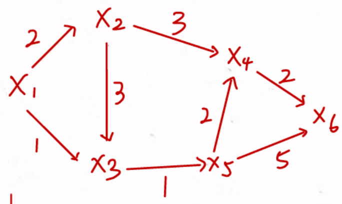
$x_2 - x_1 \le 2 \qquad x_5 - x_3 \le 1$

$x_3 - x_1 \le 1 \qquad x_6 - x_5 \le 5$

$x_4 - x_2 \le 3 \qquad x_6 - x_4 \le 2$

$x_3 - x_2 \le 3 \qquad x_4 - x_5 \le 2$

→



↳ 若 system 有負 cycle：無解

Which of the following statement(s) is (are) correct.

(A) There are infinitely many solutions for x_i 's , $i = 1$ *to* 6 .

(B) The maximum value of $x_6 - x_1$ is 6

(C) The maximum value of $x_6 - x_1$ is 7

) $x_1 \to x_6$ sp=6

(D) The maximum value of $x_4 - x_1$ is 5

(E) The maximum value of $x_4 - x_1$ is 4

) $x_1 \to x_4$ sp = 4

$x_2 = x_1 + 2 \qquad x_5 = x_3 + 1$

$x_3 = x_1 + 1 \qquad x_6 = x_5 + 5$

$x_4 = x_2 + 3 \qquad x_6 = x_4 + 2$

$x_3 = x_2 + 3 \qquad x_4 = x_5 + 2$