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1. Gambarlah sketsa grafik fungsi berikut : $f(x) = \begin{cases} -3x-2 & \text{jika } x \leq 0 \\ 3x+2 & \text{jika } 0 < x < 1 \\ 7-2x & \text{jika } x \geq 1 \end{cases}$

Jawab : ◦ Untuk menjawab pertanyaan di atas, dapat dilakukan dengan mencari grafik dari setiap fungsi di atas.

◦ $f(x) = -3x-2$ jika $x \leq 0$

↳ $x = 0$, $f(x) = -3x-2$

$f(0) = -3 \cdot 0 - 2$

$= 0-2$

$= -2 \rightarrow (0,-2)$

↳ $x = -5$, $f(x) = -3x-2$

$f(-5) = -3(-5)-2$

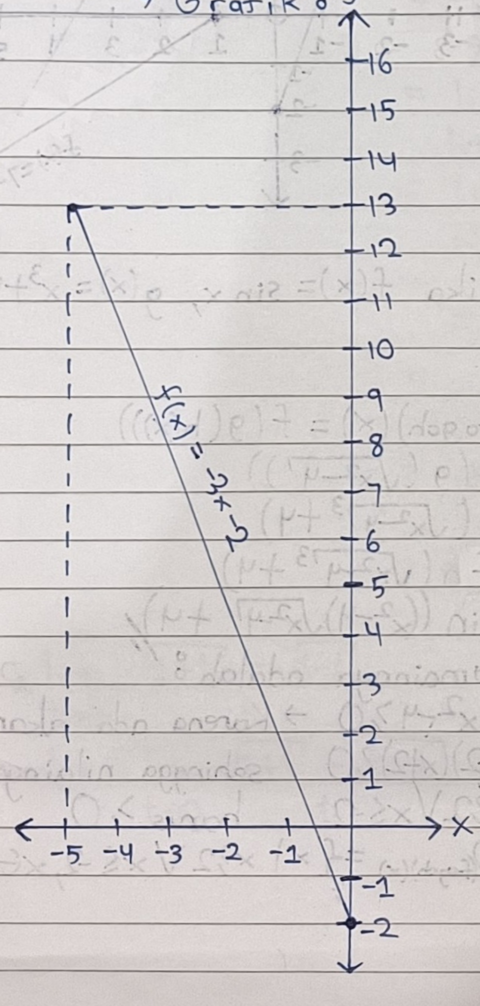
$= 15-2$

$= 13 \rightarrow (-5,13)$

↳ $D_f = \{x \mid x \leq 0, x \in R\}$

↳ $R_f = \{f(x) \mid f(x) \geq -2, f(x) \in R\}$

↳ Grafik :



◦ $f(x) = 3x+2$, jika $0 < x < 1$

↳ $x = 0$, $f(x) = 3x+2$

$f(0) = 3 \cdot 0 + 2$

$= 0+2$

$= 2 \rightarrow (0,2)$

↳ $x = 1$, $f(x) = 3x+2$

$f(1) = 3 \cdot 1 + 2$

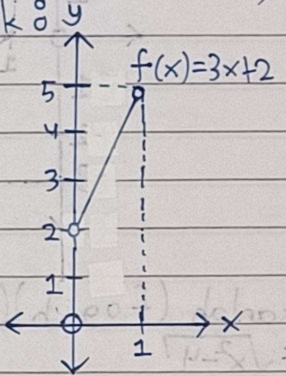
$= 3+2$

$= 5$ $(1,5)$

↳ $D_f = \{x \mid 0 < x < 1, x \in R\}$

↳ $R_f = \{f(x) \mid 2 < f(x) < 5, f(x) \in R\}$

↳ Grafik :



∘) $f(x) = 7-2x$ jika $x \geq 1$

↳ $x = 1$, $f(x) = 7-2x$

$f(1) = 7-2\cdot 1$

$= 7-2$

$= 5 \rightarrow (1,5)$

↳ $x = 5$, $f(x) = 7-2x$

$f(5) = 7-2\cdot 5$

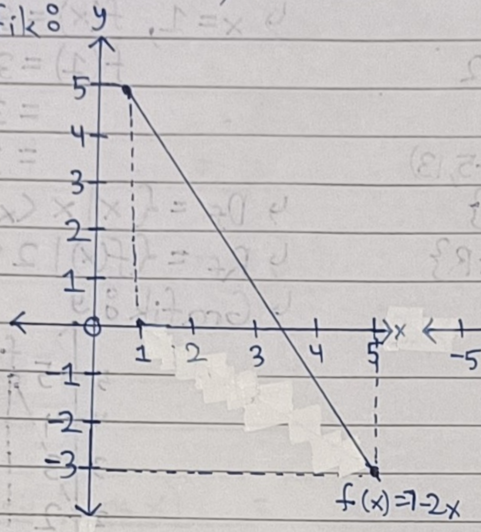
$= 7-10$

$= -3$ $(5,-3)$

↳ $D_f = \{x \mid x \geq 1, x \in R\}$

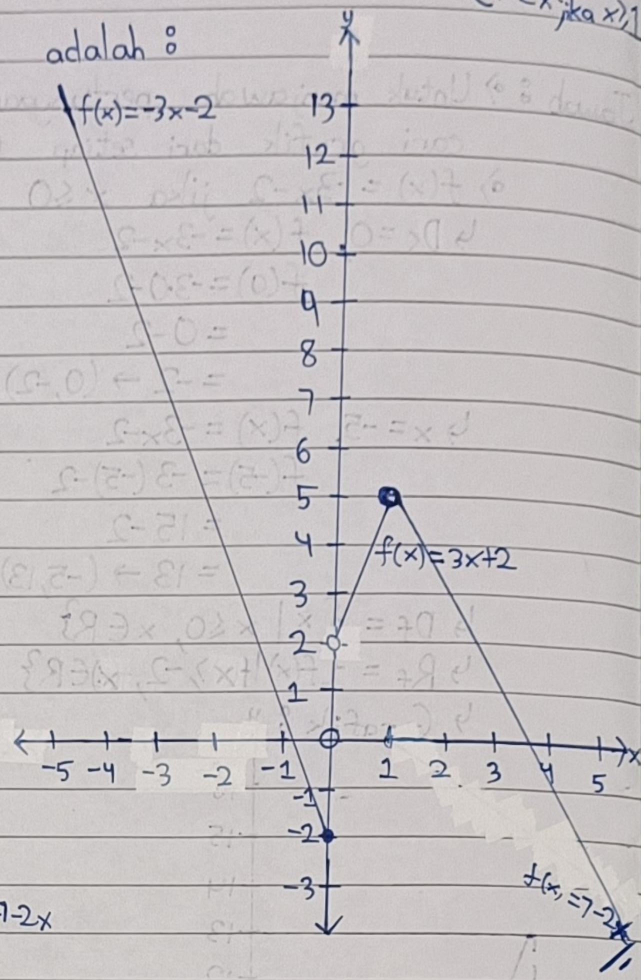
↳ $R_f = \{f(x) \mid f(x) \geq -3, f(x) \in R\}$

↳ Grafik :



∘) Maka, grafik fungsi $f(x) = \begin{cases} -3x-2 & \text{jika } x \\ 3x+2 & \text{jika } 0 \\ 7-2x & \text{jika } x \geq 1 \end{cases}$

adalah :



2. Tentukanlah $(f \circ g \circ h)(x)$ dan domainnya jika $f(x) = \sin x$, $g(x) = x^3+4$, $h(x) = \sqrt{x^2-4}$

Jawab : ∘) $f(x) = \sin x$

↳ $D_f = \{x \mid x \in R\}$

∘) $g(x) = x^3+4$

↳ $D_g = \{x \mid x \in R\}$

∘) $h(x) = \sqrt{x^2-4}$

↳ $x^2-4 \geq 0$

$(x-2)(x+2) \geq 0$

$x \geq 2 \vee x \leq -2$

↳ $D_h = \{x \mid x \geq 2 \vee x \leq -2, x \in R\}$

∘) $(f \circ g \circ h)(x) = f(g(h(x)))$

$= f(g(\sqrt{x^2-4}))$

$= f(\sqrt{x^2-4}^{\,3}+4)$

$= \sin(\sqrt{x^2-4}^{\,3}+4)$

$= \sin((x^2-4)\sqrt{x^2-4}+4)$ //

∘) Domainnya adalah :

↳ $x^2-4 \geq 0$ → Karena ada akar

$(x-2)(x+2) \geq 0$ sehingga nilainya

$x \geq 2 \vee x \leq -2$ harus ≥ 0

↳ $D_{(f \circ g \circ h)(x)} = \{x \mid x \geq 2 \vee x \leq -2, x \in R\}$ //

KIKY

3. Hitunglah nilai limit berikut ini :

a. $\lim\limits_{x\to5} \frac{x^2-6x+5}{x-5}$

Jawab : ∘) $\lim\limits_{x\to5} \frac{x^2-6x+5}{x-5} = \frac{5^2-6\cdot5+5}{5-5} = \frac{25-30+5}{0} = \frac{0}{0}$ Tak terdefinisi

∘∘) $\lim\limits_{x\to5} \frac{x^2-6x+5}{x-5} = \lim\limits_{x\to5} \frac{\cancel{(x-5)}(x-1)}{\cancel{x-5}} = \lim\limits_{x\to5} (x-1) = 5-1 = 4$ //

b. $\lim\limits_{h\to0} \frac{\sqrt{9+h}-3}{h}$

Jawab : ∘) $\lim\limits_{h\to0} \frac{\sqrt{9+h}-3}{h} = \frac{\sqrt{9+0}-3}{0} = \frac{\sqrt{9}-3}{0} = \frac{3-3}{0} = \frac{0}{0}$ Tak terdefinisi

∘∘) $\lim\limits_{h\to0} \frac{\sqrt{9+h}-3}{h} = \lim\limits_{h\to0} \frac{\sqrt{9+h}-3}{h} \cdot \frac{(\sqrt{9+h}+3)}{(\sqrt{9+h}+3)}$ $a^2-b^2 = (a+b)(a-b)$

$$= \lim_{h\to0} \frac{(\sqrt{9+h})^2-3^2}{h\cdot(\sqrt{9+h}+3)}$$

$$= \lim_{h\to0} \frac{9+h-9}{h(\sqrt{9+h}+3)}$$

$$= \lim_{h\to0} \frac{\cancel{h}}{\cancel{h}\cdot(\sqrt{9+h}+3)}$$

$$= \lim_{h\to0} \frac{1}{\sqrt{9+h}+3}$$

$$= \frac{1}{\sqrt{9+0}+3}$$

$$= \frac{1}{\sqrt{9}+3}$$

$$= \frac{1}{3+3}$$

$$= \frac{1}{6}$$ //

c. $\lim\limits_{t\to0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t}$

Jawab : ∘) $\lim\limits_{t\to0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t} = \frac{\sqrt{1+0}-\sqrt{1-0}}{0} = \frac{\sqrt{1}-\sqrt{1}}{0} = \frac{0}{0}$ Tak terdefinisi

$$\circ/\circ\ \lim_{t\to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t} = \lim_{t\to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t} \cdot \frac{(\sqrt{1+t}+\sqrt{1-t})}{(\sqrt{1+t}+\sqrt{1-t})}$$

$a^2-b^2 = (a-b)(a+b)$

$$= \lim_{t\to 0} \frac{(\sqrt{1+t})^2-(\sqrt{1-t})^2}{t\cdot(\sqrt{1+t}+\sqrt{1-t})}$$

$$= \lim_{t\to 0} \frac{1+t-1+t}{t\cdot(\sqrt{1+t}+\sqrt{1-t})}$$

$$= \lim_{t\to 0} \frac{2\cancel{t}}{\cancel{t}\cdot(\sqrt{1+t}+\sqrt{1-t})}$$

$$= \lim_{t\to 0} \frac{2}{\sqrt{1+t}+\sqrt{1-t}}$$

$$= \frac{2}{\sqrt{1+0}+\sqrt{1-0}}$$

$$= \frac{2}{\sqrt{1}+\sqrt{1}}$$

$$= \frac{2}{1+1}$$

$$= 1 //$$

d. $\lim_{x\to\frac{\pi}{2}} \frac{\sin(\cos x)}{x-\frac{\pi}{2}}$

Jawab : $\circ$) $\lim_{x\to\frac{\pi}{2}} \frac{\sin(\cos x)}{x-\frac{\pi}{2}} = \frac{\sin(\cos\frac{\pi}{2})}{\frac{\pi}{2}-\frac{\pi}{2}} = \frac{\sin(0)}{0} = \frac{0}{0}$ Tak terdefinisi

$\circ$) Menggunakan aturan L'Hopital

$$\circ/\circ\ \lim_{x\to\frac{\pi}{2}} \frac{\sin(\cos x)}{x-\frac{\pi}{2}} = \lim_{x\to\frac{\pi}{2}} \frac{\frac{d}{dx}\sin(\cos x)}{\frac{d}{dx}(x-\frac{\pi}{2})}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\cos(\cos x)\cdot -\sin x}{1}$$

$$= -\cos(\cos x)\cdot\sin x$$

$$= -\cos(\cos\tfrac{\pi}{2})\cdot\sin\tfrac{\pi}{2}$$

$$= -\cos(0)\cdot 1$$

$$= -1\cdot 1$$

$$= -1 //$$

KIKY

4. Jika diketahui bahwa $x = 3 + 3\cos\theta$, $y = 3 + 3\sin\theta$, dimana $0 \leq \theta \leq 2\pi$, hitunglah $\frac{dy}{dx}$ dan $\frac{d^2y}{dx^2}$

Jawab : o) $x = 3 + 3\cos\theta$

$$x \cdot \frac{d}{d\theta} = (3 + 3\cos\theta) \cdot \frac{d}{d\theta}$$

$$\frac{d(x)}{dx} \cdot \frac{dx}{d\theta} = 0 + 3 \cdot -\sin\theta$$

$$1 \cdot \frac{dx}{d\theta} = -3\sin\theta$$

$$\frac{d\theta}{dx} = \frac{1}{-3\sin\theta}$$

o) $y = 3 + 3\sin\theta$

$$y \cdot \frac{d}{d\theta} = (3 + 3\sin\theta) \cdot \frac{d}{d\theta}$$

$$\frac{d(y)}{dy} \cdot \frac{dy}{d\theta} = 0 + 3 \cdot \cos\theta$$

$$1 \cdot \frac{dy}{d\theta} = 3\cos\theta$$

$$\frac{dy}{d\theta} = 3\cos\theta$$

o)oo $\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx} = 3\cos\theta \cdot \frac{1}{-3\sin\theta} = -\frac{\cos\theta}{\sin\theta} = -\cot\theta = -\mathrm{ctg}\,\theta = \frac{-1}{\tan\theta}$ //

o) $\frac{d^2y}{dx^2}$ → Turunan kedua o) $\frac{dy}{dx} = -\cot\theta$

o)oo $\frac{d^2y}{dx^2} = -(-\csc^2\theta) = \csc^2\theta = \frac{1}{\sin^2\theta}$ //

ATAU

o) $x = 3 + 3\cos\theta$

$$x - 3 = 3\cos\theta$$

$$\cos\theta = \frac{x-3}{3}$$

o) $y = 3 + 3\sin\theta$

$$y - 3 = 3\sin\theta$$

$$\sin\theta = \frac{y-3}{3}$$

o)oo $\sin^2\theta + \cos^2\theta = 1$

$$\left(\frac{y-3}{3}\right)^2 + \left(\frac{x-3}{3}\right)^2 = 1$$

$$\frac{y^2 - 6y + 9}{9} + \frac{x^2 - 6x + 9}{9} = 1$$

$$y^2 - 6y + x^2 - 6x + 18 = 9$$

$$y^2 - 6y + x^2 - 6x = -9$$

$$\frac{d}{dx}(y^2 - 6y + x^2 - 6x) = \frac{d}{dx}(-9)$$

$$\frac{d(y^2)}{dy} \cdot \frac{dy}{dx} + \frac{d(-6y)}{dy} \cdot \frac{dy}{dx} + 2x - 6 = 0$$

$$2y \cdot \frac{dy}{dx} - 6 \cdot \frac{dy}{dx} + 2x - 6 = 0$$

$$\frac{dy}{dx}(2y - 6) = 6 - 2x$$

$$\frac{dy}{dx} = \frac{6 - 2x}{2y - 6}$$

$$\frac{dy}{dx} = \frac{3 - x}{y - 3} \text{ //}$$

o)oo $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$$= \frac{d}{dx}\left(\frac{3 - x}{y - 3}\right)$$

$$= \frac{-1 \cdot (y-3) - (3-x)\frac{d}{dx}(y-3)}{(y-3)^2}$$

$$= \frac{3 - y - (3-x)\frac{d(y)}{dy} \cdot \frac{dy}{dx}}{(y-3)^2}$$

$$= \frac{3 - y - (3-x) \cdot \left(\frac{3-x}{y-3}\right)}{(y-3)^2}$$

$$= \frac{(3-y)(y-3) - (3-x)(3-x)}{(y-3)^3}$$

$$= \frac{3y - 9 - y^2 + 3y - 9 + 6x - x^2}{(y-3)(y^2 - 6y + 9)}$$

$$= \frac{-x^2 - y^2 + 6x + 6y - 18}{y^3 - 9y^2 + 27y - 27} \text{ //}$$

5. Jika diketahui hubungan implisit $x^4(x+y) = y^2(3x-y)$, hitunglah $\frac{dy}{dx}$

Jawab : ∘) $x^4(x+y) = y^2(3x-y)$

$$x^5 + x^4y = 3xy^2 - y^3$$

$$\frac{d}{dx}(x^5 + x^4y) = \frac{d}{dx}(3xy^2 - y^3)$$

$$5x^4 + (4x^3 \cdot y + x^4 \cdot \frac{d(y)}{dy} \frac{dy}{dx}) = (3 \cdot y^2 + 3x \cdot \frac{d(y^2)}{dy} \cdot \frac{dy}{dx}) - 3y^2 \cdot \frac{dy}{dx}$$

$$5x^4 + 4x^3y + x^4 \cdot 1 \cdot \frac{dy}{dx} = 3y^2 + 3x \cdot 2y \cdot \frac{dy}{dx} - 3y^2 \frac{dy}{dx}$$

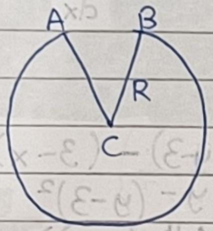
$$3y^2 \cdot \frac{dy}{dx} + x^4 \cdot \frac{dy}{dx} - 6xy \cdot \frac{dy}{dx} = -5x^4 - 4x^3y + 3y^2$$

$$\frac{dy}{dx}(3y^2 + x^4 - 6xy) = -5x^4 - 4x^3y + 3y^2$$

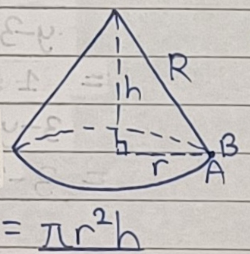
$$\frac{dy}{dx} = \frac{-5x^4 - 4x^3y + 3y^2}{x^4 - 6xy + 3y^2}$$

$$\frac{dy}{dx} = \frac{5x^3 + 4x^2y - 3y^2}{-x^4 + 6xy - 3y^2} //$$

6. Sebuah cangkir berbentuk kerucut hendak dibuat dari selembar kertas berbentuk lingkaran berjari-jari R dengan menggunting sebuah sektornya dan melekatkan sisi CA dan CB (lihat gambar). Tentukan volume maksimum cangkir yang bisa dibuat dengan cara demikian.



Jawab : ∘) Ilustrasi di atas jika dibuat menjadi kerucut akan menjadi seperti :



→ Misalkan :
- Tinggi kerucut = h
- Jari-jari kerucut = r

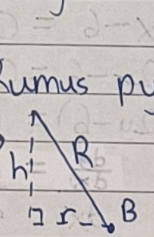
∘) $V_{kerucut} = \frac{\pi r^2 h}{3}$

$$V = \frac{\pi(R^2 - h^2) \cdot h}{3}$$

$$= \frac{\pi(R^2h - h^3)}{3}$$

∘) Rumus pythagoras : $R^2 = r^2 + h^2$

$$r^2 = R^2 - h^2$$



o) Untuk mencari volume maksimum dapat dilakukan dengan turunan V dengan h, bukan V dengan R karena R bernilai tetap dan h tidak. Istilahnya h mau diadjust sedemikian rupa hingga dapat membuat V maksimum. Tentu saja nilai turunan tersebut harus sama dengan 0 untuk mendapatkan nilai maksimumnya.

o) $V = \frac{\pi (R^2 h - h^3)}{3}$

o) $\frac{dV}{dh} = \frac{\pi}{3} (R^2 - 3h^2) = 0$

$$R^2 - 3h^2 = 0$$
$$R^2 = 3h^2$$
$$h^2 = \frac{R^2}{3}$$
$$h = \pm\sqrt{\frac{R^2}{3}}$$

$h = \frac{R}{\sqrt{3}}$ → Bernilai positif karena panjang tidak bernilai negatif

$= \frac{R\sqrt{3}}{3}$ → Berarti volume maksimum dicapai pada saat $h = \frac{R\sqrt{3}}{3}$

o) V maksimum $= \frac{\pi r^2 h}{3}$

$= \frac{\pi}{3} \cdot \frac{2R^2}{3} \cdot \frac{R\sqrt{3}}{3}$

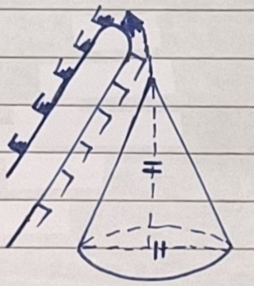
$= \frac{2\pi R^3 \sqrt{3}}{27}$ //

o) $r^2 = R^2 - h^2$

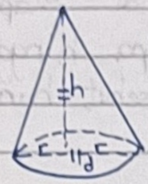
$= \frac{R^2 \cdot 3}{1 \cdot 3} - \frac{R^2}{3}$

$= \frac{2R^2}{3}$

7. Batu kerikil tertumpuk dari sebuah conveyor belt dengan kelajuan 30 ft^3/min, dan kekasarannya sedemikian sehingga ia membentuk tumpukan berbentuk kerucut dengan diameter alas dan tinggi selalu sama. Seberapa cepatkah tinggi tumpukan bertambah ketika tinggi tumpukan tersebut adalah 10 ft?



Jawab : ◦> Ilustrasi kerucut di soal tersebut adalah :



→ Misalkan :
- Tinggi kerucut = h
- Diameter alas kerucut = d
- Jari-jari alas kerucut = r

◦> $d = h = 10$ ft

◦> $V = \frac{\pi r^2 h}{3}$

$= \frac{\pi \left(\frac{d}{2}\right)^2 \cdot h}{3}$

$= \frac{\pi d^2 \cdot h}{4 \cdot 3}$

$= \frac{\pi h^3}{12}$

◦> $\frac{dV}{dt} = 30 \rightarrow 30\ \frac{ft^3}{min}$

ft³ → V = Volume

min → t = Waktu

$\frac{dV}{dh} \cdot \frac{dh}{dt} = 30$

$\frac{\pi h^2}{4} \cdot \frac{dh}{dt} = 30$

$\frac{dh}{dt} = \frac{30 \cdot 4}{\pi \cdot h^2}$

$\frac{dh}{dt} = \frac{12\cancel{0}}{\pi \cdot 10^{\cancel{2}}}$

$\frac{dh}{dt} = \frac{6}{5\pi}$ ft/min //

◦> $d = 2r$

$r = \frac{d}{2}$

◦> Seiring bertambahnya tumpukan batu kerikil tersebut berarti volumenya bertambah juga dengan alas diameternya dan tinggi selalu sama.

◦> $V = \frac{\pi h^3}{12}$

$\frac{dV}{dh} = \frac{3\pi h^2}{12}$

$\frac{dV}{dh} = \frac{\pi h^2}{4}$

◦> Ditanya seberapa cepat berarti melihat satuannya nanti adalah ft/min. Maka, dengan kecepatan $\frac{6}{5\pi}$ ft/min tinggi tumpukan bertambah ketika tinggi tumpukan tersebut adalah 10 ft.

8. Diketahui fungsi : $f(x) = x^4 - 8x^2 + 8$. Dengan menggunakan turunan pertama dan kedua dari fungsi ini, tentukan :

a. Interval di mana fungsi turun dan naik

Jawab : ◦> $f(x) = x^4 - 8x^2 + 8$

$f'(x) = 4x^3 - 16x$

◦> $f'(x) = 4x^3 - 16x = 0$

$4x(x^2 - 4) = 0$

$4x(x-2)(x+2) = 0$

$x_1 = 0 \vee x_2 = 2 \vee x_3 = -2$

∘)

$f'(x)$ $-$ | $+$ | $-$ | $+$
-2 0 2

∘)

Interval	$4x$	$x-2$	$x+2$	$f'(x)$	$f(x)$
$x<-2$	$-$	$-$	$-$	$-$	Turun
$-2<x<0$	$-$	$-$	$+$	$+$	Naik
$0<x<2$	$+$	$-$	$+$	$-$	Turun
$x>2$	$+$	$+$	$+$	$+$	Naik

∘)∘ Interval fungsi turun
$= (-\infty, -2) \cup (0, 2)$//

∘)∘ Interval fungsi naik
$= (-2, 0) \cup (2, \infty)$//

b. Interval dimana fungsi cekung (Concave up) dan cembung (Concave down)

Jawab: ∘) $f'(x) = 4x^3 - 16x$ ∘) $f''(x) = 12x^2 - 16$ ∘)

∘) $f''(x) = 12x^2 - 16 = 0$
$3x^2 - 4 = 0$
$(x\sqrt{3}-2)(x\sqrt{3}+2) = 0$
$x\sqrt{3}-2 = 0 \lor x\sqrt{3}+2 = 0$
$x\sqrt{3} = 2$ $x\sqrt{3} = -2$
$x = \frac{2}{\sqrt{3}}$ $x = \frac{-2}{\sqrt{3}}$
$x = \frac{2\sqrt{3}}{3}$ $x = \frac{-2\sqrt{3}}{3}$

$f''(x)$ $+$ ∘ $-$ ∘ $+$
$\frac{-2\sqrt{3}}{3}$ $\frac{2\sqrt{3}}{3}$

∘)

Interval	$x\sqrt{3}-2$	$x\sqrt{3}+2$	$f''(x)$	$f(x)$
$x<-\frac{2\sqrt{3}}{3}$	$-$	$-$	$+$	Naik
$-\frac{2\sqrt{3}}{3}<x<\frac{2\sqrt{3}}{3}$	$-$	$+$	$-$	Turun
$x>\frac{2\sqrt{3}}{3}$	$+$	$+$	$+$	Naik

∘)∘ Interval fungsi cekung (Concave up) $= \left(-\infty, \frac{-2\sqrt{3}}{3}\right) \cup \left(\frac{2\sqrt{3}}{3}, \infty\right)$//

∘)∘ Interval fungsi cembung (Concave down) $= \left(\frac{-2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}\right)$//

c. Koordinat ekstrema lokal serta titik belok

Jawab: ∘) Koordinat ekstrema lokal dicapai pada saat $f'(x) = 0$, maka:
↳ $x_1 = 0 \lor x_2 = 2 \lor x_3 = -2$ → Dari bagian soal 8a.
↳ $f(x_1) = x_1^4 - 8x_1^2 + 8$
$f(0) = 0 - 8 \cdot 0 + 8$
$= 8 \rightarrow (0, 8)$
↳ $f(x_2) = x_2^4 - 8x_2^2 + 8$
$f(2) = 2^4 - 8 \cdot 2^2 + 8$
$= 16 - 32 + 8$
$= -8 \rightarrow (2, -8)$
↳ $f(x_3) = x_3^4 - 8x_3^2 + 8$
$f(-2) = (-2)^4 - 8(-2)^2 + 8$
$= 16 - 8 \cdot 4 + 8$
$= 16 - 32 + 8 = -8 \rightarrow (-2, -8)$

∘)∘ Maka, koordinat ekstrema lokal adalah
$(0, 8)$; $(2, -8)$; $(-2, -8)$//

KIKY

◦) Koordinat titik belok dicapai pada saat $f''(x)=0$

↳ $x_1 = \frac{2\sqrt{3}}{3} \vee x_2 = \frac{-2\sqrt{3}}{3}$ → Dari bagian soal 8b.

↳ $f(x_1) = x_1^4 - 8x_1^2 + 8$

$$f\left(\frac{2\sqrt{3}}{3}\right) = \left(\frac{2\sqrt{3}}{3}\right)^4 - 8\left(\frac{2\sqrt{3}}{3}\right)^2 + 8$$
$$= \frac{16}{9} - 8 \cdot \frac{4}{3} + 8$$
$$= \frac{16}{9} - \frac{32 \cdot 3}{3 \cdot 3} + \frac{8 \cdot 9}{1 \cdot 9}$$
$$= \frac{16 - 96 + 72}{9}$$
$$= \frac{-8}{9} \rightarrow \left(\frac{2\sqrt{3}}{3}, \frac{-8}{9}\right)$$

↳ $f(x_2) = x_2^4 - 8x_2^2 + 8$

$$f\left(\frac{-2\sqrt{3}}{3}\right) = \left(\frac{-2\sqrt{3}}{3}\right)^4 - 8\left(\frac{-2\sqrt{3}}{3}\right)^2 + 8$$
$$= \frac{16}{9} - 8 \cdot \frac{4}{3} + 8$$
$$= \frac{16}{9} - \frac{32 \cdot 3}{3 \cdot 3} + \frac{8 \cdot 9}{1 \cdot 9}$$
$$= \frac{16 - 96 + 72}{9}$$
$$= \frac{-8}{9} \rightarrow \left(\frac{-2\sqrt{3}}{3}, \frac{-8}{9}\right)$$

◦) Maka, koordinat titik belok adalah $\left(\frac{2\sqrt{3}}{3}, \frac{-8}{9}\right)$; $\left(\frac{-2\sqrt{3}}{3}, \frac{-8}{9}\right)$ //

d. Sketsa grafiknya

Jawab: ◦) Untuk sketsa grafiknya diperlukan beberapa titik/koordinat, yaitu:

↳ Titik ekstrema lokal : $(0,8)$; $(2,-8)$; $(-2,-8)$

↳ Titik balik : $\left(\frac{2\sqrt{3}}{3}, \frac{-8}{9}\right)$; $\left(\frac{-2\sqrt{3}}{3}, \frac{-8}{9}\right) \approx (1{,}1547 ; -0{,}888)$; $(-1{,}1547 ; -0{,}888)$

↳ Titik potong sumbu-x : $y = 0 = f(x)$

↳◦) $f(x) = x^4 - 8x^2 + 8$

$f(x) = x^4 - 8x^2 + 8 = 0$

◦) Misalkan $x^2 = d$

↳ $f(x) = x^4 - 8x^2 + 8 = 0$

$d^2 - 8d + 8 = 0 \leftarrow$ $a = 1$, $b = -8$, $c = 8$

$$↳ d_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 8}}{2 \cdot 1}$$
$$= \frac{8 \pm \sqrt{64 - 32}}{2}$$
$$= \frac{8 \pm \sqrt{32}}{2}$$
$$= \frac{8 \pm 4\sqrt{2}}{2}$$
$$= 4 \pm 2\sqrt{2}$$

$d_1 = 4 + 2\sqrt{2} \vee d_2 = 4 - 2\sqrt{2}$

◦) $d_1 = 4 + 2\sqrt{2}$

$x_{1,2}^2 = 4 + 2\sqrt{2}$

$x_{1,2} = \pm\sqrt{4 + 2\sqrt{2}}$

$x_1 = \sqrt{4 + 2\sqrt{2}}$

$\vee$

$x_2 = -\sqrt{4 + 2\sqrt{2}}$

◦) $d_2 = 4 - 2\sqrt{2}$

$x_{3,4}^2 = \sqrt{4 - 2\sqrt{2}}$

$x_{3,4} = \pm\sqrt{4 - 2\sqrt{2}}$

$x_3 = \sqrt{4 - 2\sqrt{2}}$

$\vee$

$x_4 = -\sqrt{4 - 2\sqrt{2}}$

↳ Maka, titik potong sumbu-x adalah :

- ∘ $(\sqrt{4+2\sqrt{2}}, 0) \approx (2{,}61313 ; 0)$
- ∘ $(-\sqrt{4+2\sqrt{2}}, 0) \approx (-2{,}61313 ; 0)$
- ∘ $(\sqrt{4-2\sqrt{2}}, 0) \approx (1{,}0824 ; 0)$
- ∘ $(-\sqrt{4-2\sqrt{2}}, 0) \approx (-1{,}0824 ; 0)$

↳ Titik potong sumbu $y = f(x) \rightarrow x = 0$

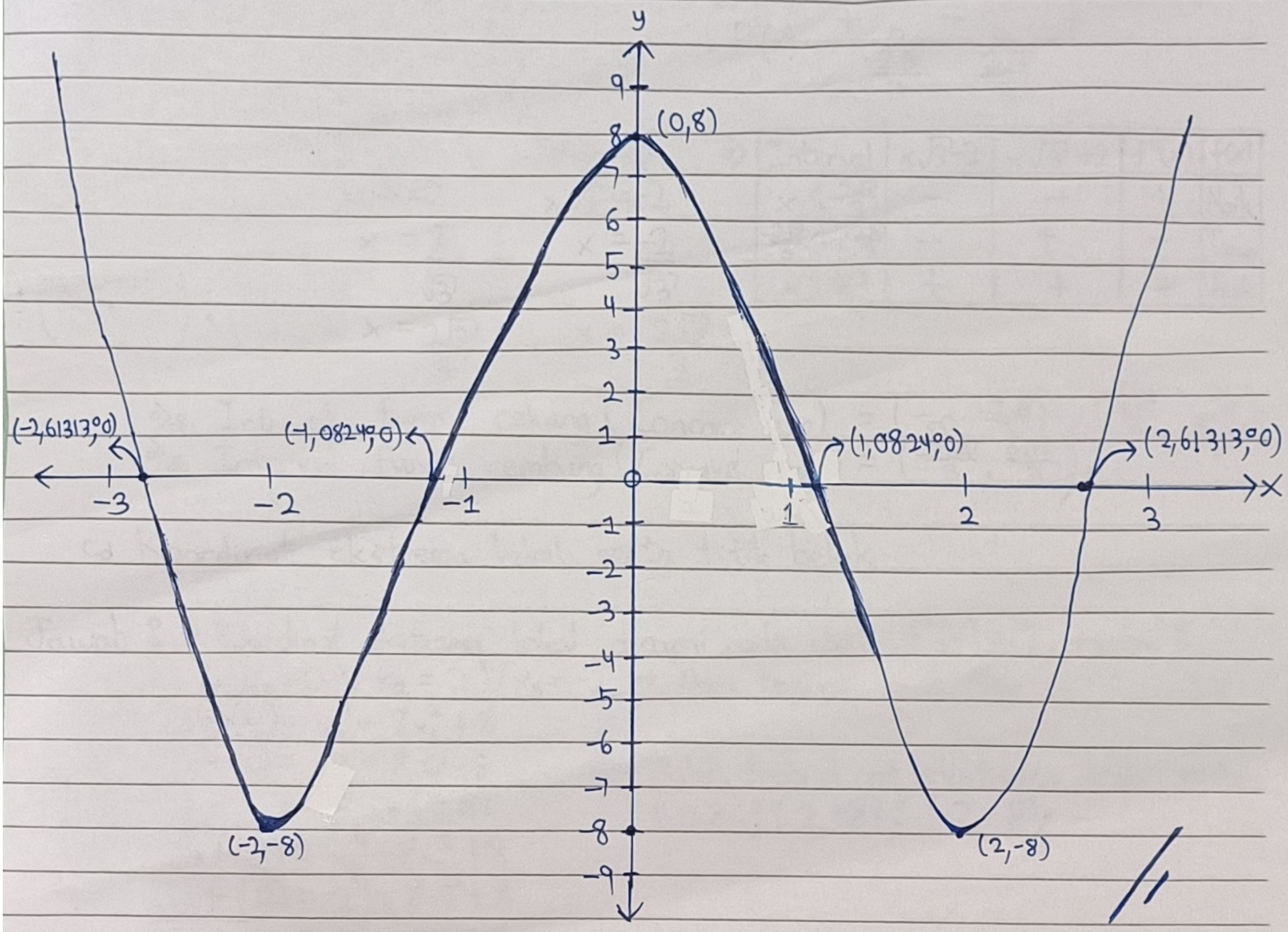
↳ $f(x) = x^4 - 8x^2 + 8$

$= 0 - 8 \cdot 0 + 8$

$= 8$

↳ Maka, titik potong sumbu-y adalah (0,8)

∘) ∴ Oleh karena itu, grafik yang dapat diperoleh adalah :



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