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Warshall's Algorithm's Implementation in Directed Acyclic Graph (DAG)-based Blockchain Network Systems

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ABSTRACT

Blockchain, a key component of digital transformation across many industries, has its limitations in terms of performance because it depends on Proof-of-Work (PoW) consensus processes. Due to these restrictions, there is a significant increase in energy usage, transaction validation delays, and fees. A blockchain based on Directed Acyclic Graphs (DAGs) is suggested as a way to overcome these obstacles, improving speed, efficiency, and scalability by enabling the processing of several transactions at once. The purpose of this study is to examine how Warshall's Algorithm is implemented and performed in DAG-based blockchain networks in order to maximize pathfinding and transitive closure operations, which are essential for network dependability and transaction verification. Because of its high computational efficiency, Warshall's Algorithm is especially well-suited for guaranteeing transaction verification and connection in the DAG-based blockchain. The study shows that combining DAGs with Warshall's Algorithm is a feasible substitute for conventional blockchain systems since it greatly increases transaction processing capacity, lowers energy usage, and improves security with its ability to find all of the possible paths of the graph or transitive closure.

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1. INTRODUCTION

Technology has been one of the most crucial aspects when it comes to online transactions, including utilizing one of cutting-edge technologies approach by blockchain on revolutionizing the process. For instance, banking systems or payment systems have become almost entirely digitized and blockchain itself can handle smart contracts, which increases the efficiency of doing transactions or payments in stock markets or cryptocurrency [1]. However, such efficiency has certain limitations which makes blockchain technology unable to perform at its best. The performance restriction on the transaction processing actually originated from a solid solution of blockchain to cope with the Byzantine Generals Problem [3] by Proof-of-Work (PoW) [4]. Therefore, this results in massive power consumption of Proof-of-Work (PoW), longer delay of validation than traditional card transaction (approximately 45,000 TPS), and high consumption of fee [5], [6]. Consequently, addressing these challenges requires the implementation of various algorithmic approaches to enhance and sustain blockchain's efficiency.

Blockchain technology is a component of a group of innovative technologies that hold the potential to completely transform and alter the course of our sector. These technologies are designed to enable intelligent, digitally connected factories to carry out more autonomous, effective, quick, and secure processes without the need for a third party to manage operations [7]. It is also classified as a distributed database of records or shared public / private ledgers of all conducted digital events that is shared by blockchain agents

[9]. Moreover, the term blockchain is related to Bitcoin because it is a technology underlying Bitcoin and other cryptocurrencies like Ethereum, Manta, and so on [1]. In other words, it is a cryptocurrency essentially, where its creation does not depend on any central authority like the stock market [8]. In addition, blockchain is regarded as a unique data structure where blocks are organized sequentially in chronological order, forming a chain. The data within these blocks is immutable and tamper-proof, making it impossible to forge [8]. It efficiently stores simple, sequential data that can be easily validated within the system, however the blockchain itself does not allow the deletion and modification operations on the transactions and other information stored on its ledger [10] even though it is able to record in detail all transactions according to a mathematical set of rules to prevent illegal inference [1].

Originally, there is a person who proposed a protocol for securing computer systems that can be trusted by mutually suspicious groups in which its performance is similar to blockchain in his Ph.D. thesis namely Chaum in 1982 [11]. Afterwards, Haber and his colleagues, Stornetta, introduced the concept of a cryptographically secured chain of blocks [12]. By 1993, Bayer and his team had incorporated Merkle trees into this design making it more effective by allowing several documents to be gathered into one block [13]. In 1998, Szabo created "bit gold," a decentralized digital currency mechanism [14]. In 2008, Satoshi Nakamoto unveiled Bitcoin, an electronic cash system based on a purely peer-to-peer network [15]. In the same year, the term "blockchain" was coined to describe the distributed ledger technology behind bitcoin transactions [16]. When Satoshi Nakamoto distributed a paper entitled "Bitcoin: A Peer-To-Peer Electronic Money System" in 2008, he also concluded the design of a decentralized electronic cash transaction system in order to solve the problem of double payment and improve the security of information verification [17]. As a result, these historical blockchain technology is quickly implemented and able to be directed in the financial field.

Building on the previous discussion, the development of blockchain technology has emerged as a significant innovation, now applied across various domains / fields such as finance, IoT (Internet of Things), energy, and healthcare [18]. Apart from that, Cyber-Physical Systems (CPS), cloud computing, or even edge computing are also able to be integrated by blockchain technology, particularly within the context of Industry 4.0. [7]. Consequently, understanding the diverse applications of blockchain technology is crucial. In fact, the underlying blockchain technology mechanism is complicated, so it is necessary to provide a concise overview through the lens of transactions. When a transaction between two parties is about to take place in a blockchain network, the transaction is going to record it as block data that consists of the event that happened along with both parties. Such data include basic information such as date / time, sender's information, receiver's information, asset type, and quantity. Afterwards, the transaction is first converted into a hashed transaction proposal, which acts as a chain that connects blocks together, and stored as a candidate to be printed on the ledger. A distinct cryptographic signature is applied to the proposed transaction, guaranteeing the record's integrity and validity. It is then sent to a distributed computer network for processing and validation, but could not alter it. These computers process and authenticate the transaction to be added to the digital ledger, which completes the asset transfer between two parties. Each new transaction is linked to those recorded previously in which every added block strengthens the block verification and eventually strengthens all blockchain, providing a complete, irreversible, and verifiable history of all transactions ever made on this blockchain [19]. Therefore, there is why transaction validation presumably on blockchain technology can be referred to as a network system with connecting nodes as the participants that validate and propagate the transaction across the blockchain.

Blockchain technology was proposed to provide transactional databases to various resources like computers and each resource is represented as a node along the connection as the vertices. On the other hand, it is able to enable a database to be directly shared without a central administrator. Besides that, this brings as a pros of blockchain technology including empowered users, high quality of blockchain data that is complete, consistent, timely, accurate, and widely available, able to withstand malicious attack, transparent to all parties for public blockchains, immutable (Cannot be altered or deleted), faster transaction, decentration, tamper-resistant information [21] lower transaction costs, and so on [20]. Nevertheless, such benefits also face several challenges in adapting new cutting-edge technologies. For instance, must follow the government's financial regulations, bugs in the source code [22], lack of performance, large energy consumption, cost a high amount of money and time, integration concerns, and many more [20]. In addition, blockchain technology is more concerned with the scalability problem, where in Bitcoin block size is limited to 1 MB and a block is mined about every ten minutes. As a result, the Bitcoin network is limited to 7 transactions per second, making it unable to handle high frequency trading. Larger blocks, however, need more storage space and propagate over the network more slowly. This will eventually cause centralization since fewer people would want to keep up with such a big blockchain [23].

Other issues including selfish mining strategies and privacy leakage are also being concerned. Even though there is a current consensus algorithm – an algorithm to build trust and properly stores the transactions on the block – [10] that used to encounter such problems by Proof-of-Work (PoW) or Proof-of-Stake (PoS) approaches, it also face some serious by utilizing that approaches like wasting too much electrical energy and time [23]. Research carried out by Yu and his colleagues concluded that throughput of traditional blockchain protocols is relatively low and solutions based on DAG allow multiple blocks to append the tail of the graph concurrently for balancing between asynchronization and speed of the problems [27]. All of those problems like require a lot of time to search all blocks [24], huge space especially on Bitcoin, energy consumption, etc can be fixed with other unique consensus algorithms that are different from others, namely DAG or Directed Acyclic Graph approach [10], [24]. Hence, the problems in terms of transaction are considered to be more efficient by using DAG approach to address the throughput problem, which allows multiple blocks to join the network concurrently and could perform transactions based on the network system [6], [29], [30]. Specifically, the blocks are generated by distributed nodes through a consensus mechanism and spread and verified across the whole network [33].

DAG-based blockchain is one of the alternative forms of the distributed ledger that store and validate the transactions in a form of DAG [2]. These transactions are often referred to as a network that in terms of graph, the node represents the base network and link communication between the nodes is called vertices. Moreover, DAGs (Directed Acyclic Graphs) are utilized to overcome significant limitations of traditional blockchain networks, particularly in terms of scalability, speed, and efficiency. It is also one of the essential concepts of data structure theory in computer science subject, due to its unique topological structure and its ability to deal with dynamic programming problems [8]. By enabling the simultaneous addition of multiple transactions and linking them to several existing transactions, DAGs significantly boost the transaction processing capacity of blockchain networks. This enhancement is especially beneficial for high transaction throughput requirements in sectors like IoT and financial services. In addition, integrating DAGs into blockchain networks provides several advantages. It reduces dependence on energy-intensive proof-of-work consensus mechanisms, thereby decreasing energy consumption and increasing transaction speed. Additionally, DAGs offer superior scalability compared to traditional blockchains by allowing parallel processing of transactions, which greatly increases the number of transactions that can be processed in a given timeframe. Moreover, the intricate interconnectivity of transaction histories in DAGs inherently enhances security and provides strong resistance to certain types of cyber-attacks [31].

The DAG-based blockchain adopts transactions as vertices of a graph without packing transactions to blocks in order to achieve more efficient data processing inside transactions [34]. In the new ledger, let G = (V, E) be a DAG, where V is the set of nodes or blocks, and E is the set of directed edges [27]. These graph implementations can be put as blockchain networking in routing or transaction systems. The all-pair routing table formed is then included/mined into the blockchain by miner to make available to all nodes publicly by using Floyd-Warshall algorithm [35]. Moreover, most miners or researchers use the Floyd-Warshall algorithm as basis for the computation of all-pairs shortest path and is an example of dynamic programming that is essential to know which path or route that is efficient to take place for the blockchain network systems and privacy-preserving dynamic programming specifically for two parties while considering the multi-party setting [36]. In other words, this algorithm can maximize the speed of doing the action and not consume lots of time but costs a high amount of money. Despite the Floyd-Warshall algorithm being used to find the shortest-path in blockchain technology, it is essential to know the adjacency nodes so the blockchain can recognize each block with its details. One of the adjacency nodes or transitive closure of a directed graph can be determined by utilizing Warshall algorithm to find all paths in a directed graph by using the adjacency matrix [37]. As a result, the blockchain transaction is able to know all of the connection with the details in the networking system on the application of blockchain technology. By far this research is going to find the connection between Warshall algorithms in blockchain network systems by using the adjacency matrix to discover all of the graphs on providing examples.

2. METHOD

2.1 Research Design

The objective of this research is to investigate the implementation and performance of Warshall's Algorithm within Directed Acyclic Graph (DAG)-based blockchain networks. This study is designed to explore the algorithm's effectiveness in optimizing pathfinding and transitive closure operations, which are critical for transaction verification and network reliability in blockchain systems. The research involves several phases: theoretical modeling, algorithm implementation, testing, and result analysis.

A Directed Acyclic Graph (DAG) is a directed graph devoid of directed cycles [40]. It is represented as G = (V, E), where V denotes vertices containing collections of transaction hashes, and E represents the set of edges [42]. Each vertex in this model features two outgoing edges, serving as hash-references to preceding vertices. The terminal vertices of the DAG correspond to vertices lacking incoming edges. This structure imbues the DAG with a partial ordering, wherein the descendants of a vertex vi, comprising the vertices directly and indirectly referenced by it, unequivocally precede it in temporal sequence.

The concept of a Directed Acyclic Graph (DAG)-based blockchain has been proposed to augment blockchain storage capacity and enhance scalability. As illustrated in Figure 1, DAG-type blockchain systems present an alternative to linear blockchain architectures. In traditional linear blockchains, transactions from blocks not belonging to the longest chain are disregarded, leading to inefficiencies. However, in DAG-type blockchains, fewer transactions are discarded, thereby promising greater efficiency.

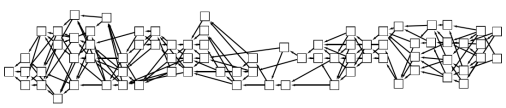

Figure 1. Example of a DAG-type blockchain

The Warshall algorithm, proposed by Stephen Warshall in 1962, is a classical method used to compute the transitive closure of a binary relation. This algorithm is widely utilized in various fields such as computer science, particularly in graph theory for finding the reachability of vertices [39]. Conversely, the Floyd-Warshall algorithm, a more recent development, is widely used to compute the shortest paths between all pairs of vertices in an edge weighted directed graph [41]. The primary distinction between these two algorithms is that Warshall's algorithm identifies reachable vertices, while the Floyd-Warshall algorithm finds the shortest paths.

DAG-based blockchain networks represent an evolution from traditional linear blockchain structures, different from the linear-chain topology in traditional blockchain, DAG-based blockchain removes the limitation of blocks, expanding the network through the directed acyclic graph [38]. Therefore, offering enhanced scalability and efficiency by allowing multiple branches and paths. These characteristics necessitate sophisticated algorithms for managing the increased complexity in verifying transactions and maintaining network integrity. Thus, Warshall's Algorithm, a well-established method for computing transitive closures in graphs, is a better choice than Floyd-Warshall for ensuring connectivity and verifying transactions within the network, as it does not require finding the shortest path. Its computational efficiency and straightforward implementation is particularly suited for this purpose.

2.2 Research's Algorithms

2.2.1 Warshall's Algorithm

This study involves a detailed analysis of a DAG-based blockchain using Warshall's algorithm to assess the reachability of transactions within the network. This analysis is critical for understanding the connectivity and verifying the integrity of transactions in the blockchain, ultimately contributing to network reliability and efficiency. Let D become the directed graph / digraph with V as vertices and E as edge on $D = (V, E)$ equation, then the steps for implementing Warshall's algorithm are as follows [39].

Algorithm 1 Warshall's algorithm

Require: Adjacency matrix M_{nxn} of directed graph D with V as vertices / vertex and E as edge
Ensure: D^t the transitive closure of D

```
for k ← 1 until n do
    for i ← 1 until n do
        for j ← 1 until n do
            M[i, j] ← M[i, j] or (M[i, k] and M[k, j])
        endfor
    endfor
endfor
```

The reachable matrix only considers whether there is a road between nodes. Moreover, based on the algorithm, it performs a time complexity of

$O(n^3)$ and space complexity of $O(n^2)$ because it works on *nxn* matrices. A value of 1 signifies the existence of a path, while 0 indicates no path. This matrix is crucial for understanding the overall connectivity of the DAG-based blockchain.

This procedure aims to leverage the efficiency of Warshall's Algorithm in analyzing the connectivity and transaction verification within a DAG-based blockchain. By utilizing this algorithm, the study provides a robust method for ensuring network reliability, as it efficiently determines the reachability of transactions, thereby verifying the integrity and robustness of the blockchain. This comprehensive analysis is essential for the practical implementation and scalability of DAG-based blockchain systems, offering insights into their operational efficacy and potential improvements.

2.2.2 Floyd-Warshall's Algorithm

Besides the Warshall algorithm on its implementation to find all paths, there is also a similar approach that is called the Floyd-Warshall algorithm. Although powerful and efficient for solving all-pairs shortest path problems in a weighted graph, it is not suitable for this research. This algorithm specifically focuses on determining the shortest path distances between all pairs of nodes, which is not required for our research objectives. Therefore, implementing the Floyd-Warshall algorithm would be unnecessary and irrelevant to our specific needs in blockchain technology. The steps for implementing Floyd-Warshall's algorithm are as follows.

Algorithm 2 Floyd-Warshall's algorithm

Require: Adjacency matrix M_{nxn} of directed graph D with V as vertices / vertex and E as edge
Ensure: D^t the transitive closure of D
1: **for** k ← 1 **until** n **do**
2: **for** i ← 1 **until** n **do**
3: **for** j ← 1 **until** n **do**
4: M[i, j] ← M[i, j] **or** (M[i, k] **and** M[k, j])
5: **endfor**
6: **endfor**
7: **endfor**

Overall, the method adopted in this research ensures a thorough examination of the reachability within a DAG-based blockchain, contributing to the broader field of blockchain technology by offering a detailed and practical approach to transaction verification and network analysis on Warshall algorithm.

2.3 Research Procedure and Visualization of Methodological Framework

This research involves several steps for a thorough analysis of DAG-based blockchain networks using Warshall's Algorithm. These steps are essential for achieving the research objectives and contributing valuable insights to the field of blockchain technology. Moreover, the steps are covered in the SLR method which is provided on the bottom of the research. On the other hand, the research steps are as follows.

1. An extensive literature review will be conducted, encompassing 25-50 recent research papers and publications related to DAG-based blockchains, graph theory, and Warshall's Algorithm. This review will help identify current methodologies, algorithms, and relevant data sources. The data sources will be extracted from these reviewed papers, focusing on examples of DAG-based blockchains, transaction datasets, and case studies. Using validated and peer-reviewed sources ensures the reliability of the data.
2. Data extraction will be conducted from the reviewed papers as it is a critical component of this research, particularly focusing on examples of DAG-based blockchains, transaction datasets, and case studies. Ensure that the data sources are validated and peer-reviewed to maintain reliability.
3. The extracted data will then be formatted into a structured representation, such as adjacency matrices or adjacency lists, which can be used as input for Warshall's Algorithm. This step is crucial as it organizes the transaction data into a graph representation where nodes represent transactions and edges represent the connections between them.
4. The Warshall's Algorithm will be applied to the structured graph representations, such as adjacency matrices or adjacency lists of the DAG-based blockchain networks. The implementation will focus on ensuring the algorithm correctly processes the adjacency matrices or lists, effectively identifying all possible paths within the network.
5. The results will be thoroughly analyzed. This analysis will involve examining the connectivity and pathways identified within the DAG-based blockchain networks, assessing the accuracy and efficiency of the algorithm in this context. The findings will be interpreted

to understand the performance of DAG-based blockchains.

To provide a clearer understanding of the research methodology, the flowchart presented in Figure 2 illustrates the research steps. This flowchart visually represents each phase of the research process, highlighting the sequential and interconnected nature of the approach.

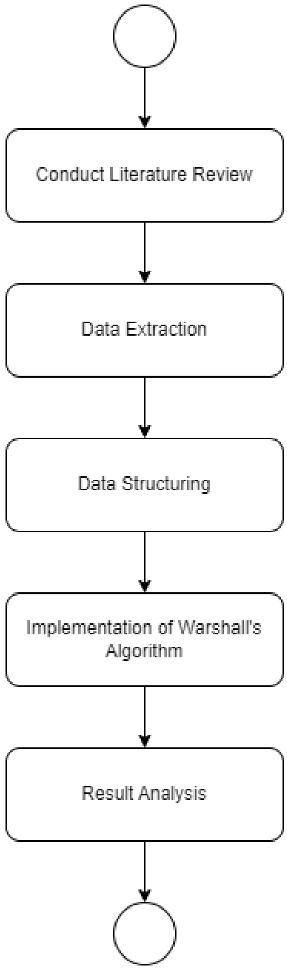

Figure 2. Research steps flowchart

2.4 Data Collection

The DAG-based blockchain that researchers will analyze through the implementation of the Warshall algorithm is illustrated in Figure 3 and 4. These figures provide a detailed visualization of the blockchain's structure, highlighting its vertices and edges.

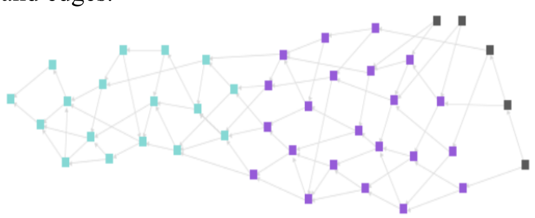

Figure 3. DAG-based blockchain of Tangle network

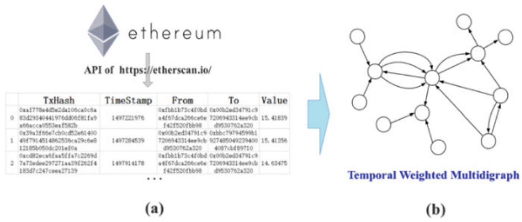

Figure 4. DAG-based blockchain of Ethereum transaction network

In Figure 3, a Tangle network may perform three different kinds of transactions: fully confirmed transactions, not confirmed transactions, and tips, which are determined by a confirmation level. To put it another way, the categories are determined by the quantity of freshly created tips that have been approved directly or indirectly on prior transactions [43]. Basically, the Tangle network is one of the best updated approaches from blockchain based with Directed Acyclic Graph (DAG) to overcome inefficiencies of sequential blockchain design [44]. Meanwhile, Figure 4 provides a model of Ethereum based on blockchain transaction network by graph [45]. Both figures will later be the exemplar of using Warshall algorithm on Section 3.1.

2.5 Data Analysis

In the previous discussion on Section 2.4, a graph was provided for analysis using Warshall's Algorithm. This algorithm is a fundamental tool in graph theory, and the following explanation will present its application, including its generalization and relevant applications. Moreover, it is also another form similar to transitive closure on graphs. By consolidating these elements, the information will be more accessible for further research. In addition to implementing Warshall's algorithm on the graph shown in Figure 3, a simpler example will be provided to illustrate the algorithm's application more clearly.

Let R be a binary on the set $S = \{s_1, s_2, \ldots, s_n\}$, that is written as s_iRs_j if s_i is in relation s_j. The relation R can be represented by a relation matrix, which is as follows.

$$A = (a_{ij})_{\substack{i=\overline{1,n} \\ j=\overline{1,n}}}, \quad \text{where} \quad a_{ij} = \begin{cases} 1, & \text{if } s_iRs_j, \\ 0, & \text{otherwise.} \end{cases}$$

The transitive closure of the relation R is the binary relation R* defined as: $s_iR^*\, s_j$ if and only if there exists s_{p1} , s_{p2} , . . ., s_{pr} , $r \geq 2$ such that $s_i =$

s_{p1} , $s_{p1}Rs_{p2}$, $s_{p2}Rs_{p3}$, . . . , $s_{pr-1}Rs_{pr}$, $s_{pr} = s_{pj}$. The relation matrix of R^* is $A^* = (a^*_{ij})$. In addition, A binary relation can be represented by a directed graph (i.e. digraph) too. The relation matrix is equal to the adjacency matrix of the corresponding graph. Adjacency matrix is a means of representing which vertices of a graph are adjacent to which other vertices [25]. Given a directed graph $G = (V, E)$ with vertex set $V = \{1, 2,..., n\}$, we might wish to determine whether G contains a path from i to j for all vertex pairs $i, j \in V$. We define the transitive closure of G as the graph $G^* = (V, E^*)$, where $E^* = \{(i, j)$: there is a path from vertex i to vertex j in G$\}$. See Fig. 5 for an example of a directed graph and Fig. 6 for its corresponding transitive closure relation.

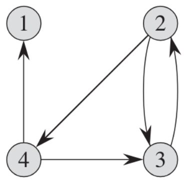

Figure 5. Example of directed graph

$$T^{(0)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix} \quad T^{(1)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix} \quad T^{(2)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$

$$T^{(3)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \quad T^{(4)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Figure 6. Corresponding transitive closure relation from Fig 4

Figure 5 illustrates the matrices computed through the application of Warshall's Algorithm on a sample graph. This figure provides a detailed visual representation of the algorithm's process and outcomes. Initially, the adjacency matrix of the sample graph is presented, showcasing the direct connections between nodes. As Warshall's Algorithm is executed, intermediate matrices at various stages of the computation are displayed, highlighting the progressive identification of indirect paths.

The final matrix, resulting from the complete execution of the algorithm, reveals the comprehensive reachability of nodes within the graph. This final matrix effectively demonstrates all possible paths between nodes, capturing both direct and indirect connections. By examining these matrices, one can observe how Warshall's Algorithm systematically updates the reachability information, transforming the initial adjacency matrix into a matrix that encompasses the full connectivity of the graph [47].

3. RESULTS AND DISCUSSION

Based on the previous discussion on Section 2.4, the Warshall's algorithm to be implemented. To be more precise, the following figure is a clearer picture of a graph with its direction (digraph).

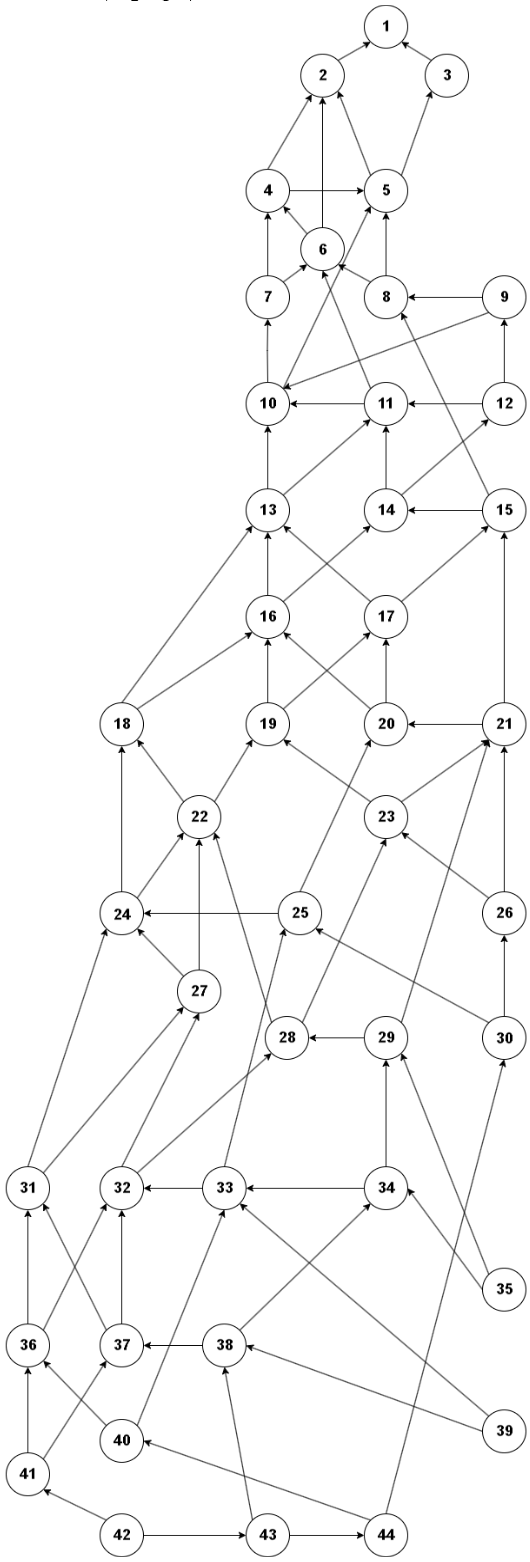

Figure 7. Clearer DAG-based blockchain of Tangle transaction network

Indeed, the digraph above with 3 different colours have the direction to one another. This shows there is a relation between them and can result in making an adjacency matrix to search all the transitive closure of all possible paths or find all possible Tangle network transactions that are connected to each other. As a result, Warshall's algorithm converts the digraph into an adjacency matrix as follows.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0

Figure 8. Adjacency Matrix of DAG-based blockchain of Tangle transaction network

The simulation of Warshall's algorithm on a directed graph, as detailed in Section 2.5, involves an iterative process applied to a *nxn* adjacency matrix. This process can be visualized as a series of matrix transformations. Specifically, each iteration examines the cross product of corresponding column and row indices to update the matrix. Starting with the initial adjacency matrix, the algorithm systematically updates the matrix entries to reflect the transitive closure of the graph. This involves checking and incorporating the reachability information by considering the presence of intermediate nodes (indicated by yellow color), thereby transforming the matrix step-by-step until it accurately represents the transitive closure then adding the matrix again. In other words, firstly it sees column 1 and row 1, but there is no value 1 that indicates there is a path on row 1. Even though there is a path on column 1, which is on {2} and {3} then when it does cross product it results as column 1 x row 1 = {2, 3} x ∅ = ∅. Then there is no addition to the matrix. Row 2 and column 2 cross product become column 2 x row 2 = {4,5,6} x {1} = {(4,1),(5,1),(6,1)} (indicated by purple color) then it adds to the matrix by marking 1 from 0 on the nodes. Moreover, column 3 x row 3 cross products become column 3 x row 3 = {5} x {3} = {5,3}. Column 4 and row 4 cross product become column 4 x row 4 = {6,7} x {2,5} = {(6,2), (6,5),(7,2),(7,5)} (indicated by blue color). Column 5 x row 5 cross product become column 5 x row 5 = {4,8,10} x {2,3} = {(4,2),(4,3),(8,2),(8,3),(10,2),(10,3)} (indicated by green color), and so on.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	1	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0

Figure 8. First Step of Warshall's algorithm on DAG-based blockchain of Tangle transaction network

Thus, the steps involved in implementing Warshall's algorithm for all 44 nodes are executed by updating the adjacency matrix. In this matrix, a value of 1 indicates the existence of a path from the node represented by the row to the node represented by the column, while a value of 0 indicates the absence of such a path. The end results are as follows.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	1	1	1	1	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
34	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
35	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
36	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
37	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
38	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	0	0	1	0	0	0	0	0	0	0
39	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	0	0	1	1	0	0	0	0	0	0
40	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0	1	1	1	0	0	1	0	0	0	0	0	0	0	0
41	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	1	0	0	0	1	1	0	0	0	0	0	0	0
42	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	0	1	0
43	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	0	0	0	1
44	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	1	0	0	0	1	0	0	0	0

Figure 8. Final result of Warshall's algorithm on DAG-based blockchain of Tangle transaction network

Another instance of the algorithm's implementation on the graph can be seen in figure 4. More specifically, a sharper representation of a graph with its direction (digraph) may be seen in the following picture.

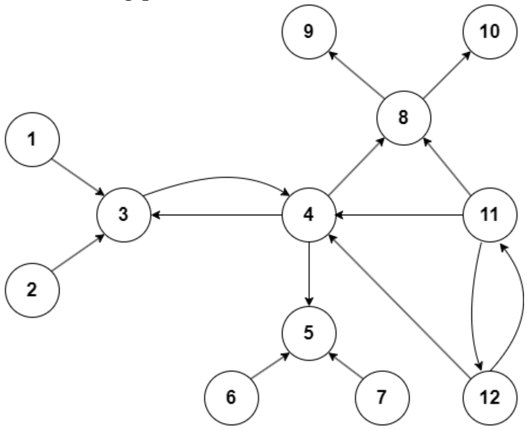

Figure 9. Clearer DAG-based blockchain of Ethereum transaction network

Indeed, the digraph has the direction to one another. This shows there is a relation between them and can result in making an adjacency matrix to search all the transitive closure of all possible paths or find all possible transactions networks of Ethereum that are connected to each other. As a result, Warshall's algorithm converts the digraph into an adjacency matrix as follows.

$$\begin{array}{cc} & \begin{array}{cccccccccccc}1&2&3&4&5&6&7&8&9&10&11&12\end{array} \\ \begin{array}{c}1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\end{array} & \begin{pmatrix}0&0&1&0&0&0&0&0&0&0&0&0\\0&0&1&0&0&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0&0&0&0&0\\0&0&1&0&1&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&1&1&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&1&0&0&0&1\\0&0&0&1&0&0&0&0&0&0&1&0\end{pmatrix}\end{array}$$

Figure 9. Adjacency Matrix of DAG-based blockchain of Ethereum transaction network

The simulation of Warshall's algorithm on a directed graph, as detailed in Section 2.5, involves an iterative process applied to a *nxn* adjacency matrix. This process can be visualized as a series of matrix transformations. Specifically, each iteration examines the cross product of corresponding column and row indices to update the matrix. Starting with the initial adjacency matrix, the algorithm systematically updates the matrix entries to reflect the transitive closure of the graph. This involves checking and incorporating the reachability information by considering the presence of intermediate nodes, thereby transforming the matrix step-by-step until it accurately represents the transitive closure then adding the matrix again. In other words, firstly it sees column 1 and row 1, but there is no value 1 that indicates there is a path on column 1. Even though there is a path on row 1, which is on {3} then when it does cross product it results as column 1 x row 1 = ∅ x {3} = ∅. Then there is no addition to the matrix. Row 2 and column 2 also do the same thing. However, in column 3 x row 3 results as {1,2,4}x{4} = {(1,4),(2,4),(4,4)} . Then there is an addition on the matrix as follows.

$$\begin{array}{cc} & \begin{array}{cccccccccccc}1&2&3&4&5&6&7&8&9&10&11&12\end{array} \\ \begin{array}{c}1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\end{array} & \begin{pmatrix}0&0&1&1&0&0&0&0&0&0&0&0\\0&0&1&1&0&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&0&0&0&0&0\\0&0&1&1&1&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&1&1&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&1&0&0&0&1&0&0&0&1\\0&0&0&1&0&0&0&0&0&0&1&0\end{pmatrix}\end{array}$$

Figure 8. First Step of Warshall's Algorithm on DAG-based Blockchain of Ethereum transaction network

Thus, the steps involved in implementing Warshall's algorithm for all 12 nodes are executed by updating the adjacency matrix. In this matrix, a value of 1 indicates the existence of a path from the node represented by the row to the node represented by the column, while a value of 0 indicates the absence of such a path. The end results are as follows.

$$\begin{array}{cc} & \begin{array}{cccccccccccc}1&2&3&4&5&6&7&8&9&10&11&12\end{array} \\ \begin{array}{c}1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\end{array} & \begin{pmatrix}0&0&1&1&1&0&0&1&1&1&0&0\\0&0&1&1&1&0&0&1&1&1&0&0\\0&0&1&1&1&0&0&1&1&1&0&0\\0&0&1&1&1&0&0&1&1&1&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&0&0&0&0\\0&0&0&0&1&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&1&1&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&1&1&1&0&0&1&1&1&0&1\\0&0&1&1&1&0&0&1&1&1&1&0\end{pmatrix}\end{array}$$

Figure 8. Final result of Warshall's algorithm on DAG-based blockchain of Ethereum transaction network

In conclusion, the matrix depicted in Figure 9 provides a comprehensive representation of the graph's connectivity. For instance, consider the entry at column 10 and row 3, or (3, 10). This indicates the presence of a path from node 3 to node 10, specifically following the route 3 → 4

→ 8 → 10. Similar interpretations apply to the other entries in the matrix.

4. CONCLUSION

As previously said, the investigation of Warshall's Algorithm in DAG-based blockchain networks seeks to resolve the innate scalability and efficiency problems in conventional blockchain systems. We want to show that the use of Warshall's Algorithm efficiently optimizes pathfinding and transitive closure operations, improving transaction verification and overall network reliability, in the "Results and Discussion" section. This connection guarantees the coherence and integration of our study aims, methodology, and outcomes, thereby offering strong support for our hypothesis.

The research findings underscore the algorithm's effectiveness in determining the reachability of transactions, thus ensuring comprehensive connectivity within the blockchain network. This capability is critical for verifying the integrity and robustness of transactions, which in turn enhances the overall reliability of DAG-based blockchain systems. The improved scalability and transaction processing capacity achieved through this method are particularly beneficial for applications requiring high transaction throughput, such as IoT and financial services. Moreover, the reduction in energy consumption and increased transaction speed due to the decreased reliance on proof-of-work mechanisms further highlight the practical benefits of implementing Warshall's Algorithm in these systems.

Additionally, the effective use of Warshall's Algorithm in DAG-based blockchains creates new opportunities for both theoretical and practical study. Future directions for the algorithm's development include enhancing its efficiency even further, investigating its potential for use in other distributed ledger technologies, and fusing it with cutting-edge innovations like edge computing and the Internet of Things. This may result in blockchain systems that are more secure, scalable, and effective, encouraging a wider range of industries to use them, such as supply chain management, healthcare, and banking. The ongoing development and use of these discoveries will greatly advance blockchain technology and open the door to creative solutions to challenging digital transaction problems.

By leveraging Warshall's Algorithm, DAG-based blockchain networks can overcome significant limitations of traditional blockchain systems, paving the way for more efficient, scalable, and secure digital transaction platforms. This research contributes to the broader field of blockchain technology, offering insights and practical approaches that can drive future innovations and applications across various sectors. In other words, Warshall's algorithms are able to make the graph to find all of its transitive closure or all possible paths in the provided graphs as the research's data.

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REFERENCES

[1] Q. K. Nguyen, "Blockchain-A Financial Technology for Future Sustainable Development," in *Proceedings - 3rd International Conference on Green Technology and Sustainable Development, GTSD 2016*, Institute of Electrical and Electronics Engineers Inc., Dec. 2016, pp. 51–54. doi: 10.1109/GTSD.2016.22.

[2] Institute of Electrical and Electronics Engineers, *2019 IEEE International Conference on Communications Workshops (ICC Workshops) : proceedings : Shanghai, China, 22-24 May 2019.*

[3] L. Lamport, R. Shostak, and M. Pease, “The Byzantine Generals Problem,” 1982.
[4] J. A. Garay, A. Kiayias, and N. Leonardos, “The Bitcoin Backbone Protocol: Analysis and Applications,” *Journal of the ACM*, Apr. 2024, doi: 10.1145/3653445.
[5] Y. L. Yonatan Sompolinsky and A. Zohar, “Serialization of proof-ofwork events: Confirming transactions via recursive elections,” IACR Cryptology ePrint Archive. [Online]. Available: https://eprint.iacr.org/ 2016/1159.pdf
[6] S. Popov, “The Tangle,” 2017.
[7] S. B. Elmamy, H. Mrabet, H. Gharbi, A. Jemai, and D. Trentesaux, “A survey on the usage of blockchain technology for cyber-threats in the context of industry 4.0,” *Sustainability (Switzerland)*, vol. 12, no. 21. MDPI, pp. 1–19, Nov. 01, 2020. doi: 10.3390/su12219179.
[8] C. Bai, “State-of-the-Art and Future Trends of Blockchain Based on DAG Structure,” in *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, Springer Verlag, 2019, pp. 183–196. doi: 10.1007/978-3-030-13651-2_11.
[9] S. Saberi, M. Kouhizadeh, J. Sarkis, and L. Shen, “Blockchain technology and its relationships to sustainable supply chain management,” Int. J. Prod. Res., vol. 57, no. 7, pp. 2117–2135, Apr. 2019. [Online]. Available: doi: 10.1080/00207543.2018.1533261.
[10] H. Guo and X. Yu, “A survey on blockchain technology and its security,” *Blockchain: Research and Applications*, vol. 3, no. 2, Jun. 2022, doi: 10.1016/j.bcra.2022.100067.
[11] D. L. Chaum and T. P., "Computer systems established, maintained, and trusted by mutually suspicious groups," 1979.
[12] S. Haber and W. S. Stornetta, "How to Time-Stamp a Digital Document," 1991.
[13] S. H. Bellcore, D. Bayer, and W. S. Stornetta, “Improving the Efficiency and Reliability of Digital Time-Stamping,” 1992.
[14] R. Sharma, Bit gold, Investopedia, 2021. Available online: https://www.investop edia.com/terms/b/bit-gold.asp. (Accessed 24 October 2021).
[15] S. Nakamoto, "Bitcoin: A Peer-to-Peer Electronic Cash System," Bitcoin.org, 2008. [Online]. Available: www.bitcoin.org
[16] R. Sheldon, A timeline and history of blockchain technology. https://whatis.techt arget.com/feature/A-timeline-and-history-of-blockchain-technology, 2021.
[17] S. Underwood, “Blockchain beyond bitcoin,” *Commun ACM*, vol. 59, no. 11, pp. 15–17, Oct. 2016, doi: 10.1145/2994581.
[18] J. Abou Jaoude and R. George Saade, “Blockchain applications - Usage in different domains,” *IEEE Access*, vol. 7, pp. 45360–45381, 2019, doi: 10.1109/ACCESS.2019.2902501.
[19] V. J. Morkunas, J. Paschen, and E. Boon, "How blockchain technologies impact your business model," Bus. Horiz., vol. 62, no. 3, pp. 295–306, May 2019, doi: 10.1016/j.bushor.2019.01.009.
[20] M. Niranjanamurthy, B. N. Nithya, and S. Jagannatha, "Analysis of Blockchain technology: pros, cons and SWOT," Cluster Comput., vol. 22, pp. 14743–14757, Nov. 2019, doi: 10.1007/s10586-018-2387-5.
[21] B. Wu and T. Duan, “The Application of Blockchain Technology in Financial markets,” in *Journal of Physics: Conference Series*, Institute of Physics Publishing, Mar. 2019. doi: 10.1088/1742-6596/1176/4/042094.
[22] S. Gupta, S. Sinha, and B. Bhushan, “International Conference on Innovative Computing and Communication (ICICC 2020) Emergence of Blockchain Technology: Fundamentals, Working and its Various Implementations,” [Online]. Available: https://ssrn.com/abstract=3569577
[23] Z. Zheng, S. Xie, H. Dai, X. Chen, and H. Wang, “An Overview of Blockchain Technology: Architecture, Consensus, and Future Trends,” in *Proceedings - 2017 IEEE 6th International Congress on Big Data, BigData Congress 2017*, Institute of Electrical and Electronics Engineers Inc., Sep. 2017, pp. 557–564. doi: 10.1109/BigDataCongress.2017.85.
[24] H. Watanabe *et al.*, “Enhancing blockchain traceability with DAG-based tokens,” in *Proceedings - 2019 2nd IEEE International Conference on Blockchain, Blockchain 2019*, Institute of Electrical and Electronics Engineers Inc., Jul. 2019, pp. 220–227. doi: 10.1109/Blockchain.2019.00036.
[25] H. Singh and R. Sharma, “Role of Adjacency Matrix & Adjacency List in Graph Theory,” *INTERNATIONAL JOURNAL OF COMPUTERS & TECHNOLOGY*, vol. 3, no. 1, pp. 179–183, Aug. 2012, doi: 10.24297/ijct.v3i1c.2775.
[26] F. R. Yu, J. Liu, Y. He, P. Si, and Y. Zhang, “Virtualization for Distributed Ledger Technology (vDLT),” *IEEE Access*, vol. 6, pp. 25019–25028, Apr. 2018, doi: 10.1109/ACCESS.2018.
[27] S. Yang, Z. Chen, L. Cui, M. Xu, Z. Ming, and K. Xu, “CoDAG: An efficient and compacted DAG-Based blockchain protocol,” in *Proceedings - 2019 2nd IEEE International Conference on Blockchain, Blockchain 2019*, Institute of Electrical and Electronics Engineers Inc., Jul. 2019, pp. 314–318. doi: 10.1109/Blockchain.2019.00049.
[28] T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein, “Introduction to Algorithms, Third Edition.”
[29] Y. Sompolinsky, S. Wyborski, and A. Zohar, “PHANTOM GHOSTDAG: A scalable generalization of Nakamoto consensus: September 2, 2021,” in *AFT 2021 - Proceedings of the 2021 3rd ACM Conference on Advances in Financial Technologies*, Association for Computing Machinery, Inc, Sep. 2021, pp. 57–70. doi: 10.1145/3479722.3480990.
[30] Y. Sompolinsky, Y. Lewenberg, and A. Zohar, “SPECTRE: Serialization of Proof-of-work Events: Confirming Transactions via Recursive Elections.”
[31] A. Tokhmetov, V. Lee, and L. Tanchenko, “DEVELOPMENT OF DAG BLOCKCHAIN MODEL,” *Scientific Journal of Astana IT University*, Jan. 2024, doi: 10.37943/16cgoy7609.
[32] Z. Kása, “Warshall’s algorithm—survey and applications,” *Annales Mathematicae et Informaticae*, vol. 54, pp. 17–31, 2021, doi: 10.33039/ami.2021.08.001.
[33] T. T. Kuo, H. E. Kim, and L. Ohno-Machado, “Blockchain distributed ledger technologies for biomedical and health care applications,” *Journal of the American Medical Informatics Association*, vol. 24, no. 6. Oxford University Press, pp. 1211–1220, Nov. 01, 2017. doi: 10.1093/jamia/ocx068.
[34] W. Yang, X. Dai, J. Xiao, and H. Jin, “LDV: A Lightweight DAG-Based Blockchain for Vehicular Social Networks,” *IEEE Trans Veh Technol*, vol. 69, no. 6, pp. 5749–5759, Jun. 2020, doi: 10.1109/TVT.2020.2963906.
[35] A. K. Biswas and M. Dasgupta, “Modification of DSDV and Secure Routing using Blockchain Technology,” in *2020 4th International Conference on Electronics, Materials Engineering and Nano-Technology, IEMENTech 2020*, Institute of Electrical and Electronics Engineers Inc., Oct. 2020. doi: 10.1109/IEMENTech51367.2020.9270125.
[36] ACM Digital Library. and A. Association for Computing Machinery. Special Interest Group on Security, *Proceedings of the 7th ACM workshop on Privacy in the*

electronic society. ACM, 2008.
[37] H. S. Warren, “A Modification of Warshall’s Algorithm for the Transitive Closure of Binary Relations,” 1975.
[38] Q. Wang, “Improving the scalability of blockchain through DAG,” in *Middleware 2019 - Proceedings of the 2019 20th International Middleware Conference Doctoral Symposium, Part of Middleware 2019*, Association for Computing Machinery, Inc, Dec. 2019, pp. 34–35. doi: 10.1145/3366624.3368165.
[39] C. E. R. Alves, E. N. Cáceres, A. A. de Castro, S. W. Song, and J. L. Szwarcfiter, “Parallel transitive closure algorithm,” *Journal of the Brazilian Computer Society*, vol. 19, no. 2, pp. 161–166, 2013, doi: 10.1007/s13173-012-0089-z.
[40] J. Kawahara, “Graph Optimization Problems and Algorithms for DAG-Type Blockchains,” in *Advanced Mathematical Science for Mobility Society*, Springer Nature Singapore, 2024, pp. 109–124. doi: 10.1007/978-981-99-9772-5_7.
[41] A. Brodnik, M. Grgurovič, and R. Požar, “Modifications of the Floyd-Warshall algorithm with nearly quadratic expected-time,” *Ars Mathematica Contemporanea*, vol. 22, no. 1, 2022, doi: 10.26493/1855-3974.2467.497.
[42] M. Abram, D. Galindo, D. Honerkamp, J. Ward, and J.-M. Wong, “Democratising blockchain: A minimal agency consensus model,” Jun. 2020, [Online]. Available: http://arxiv.org/abs/2006.05390
[43] H.-K. Yang, H.-J. Cha, and Y.-J. Song, “A Study on the Application of Blockchain Technology based on Direct Acyclic Graph in IoT environment,” 2021.
[44] M. N. Halgamuge, “Optimization framework for Best Approver Selection Method (BASM) and Best Tip Selection Method (BTSM) for IOTA tangle network: Blockchain-enabled next generation Industrial IoT,” *Computer Networks*, vol. 199, Nov. 2021, doi: 10.1016/j.comnet.2021.108418.
[45] Z. Zheng, S. Xie, H. Dai, X. Chen, and H. Wang, “An Overview of Blockchain Technology: Architecture, Consensus, and Future Trends,” in *Proceedings - 2017 IEEE 6th International Congress on Big Data, BigData Congress 2017*, Institute of Electrical and Electronics Engineers Inc., Sep. 2017, pp. 557–564. doi: 10.1109/BigDataCongress.2017.85.

ID	Publication Author [Reference]	Year	Title	Total of Page	Citation	Peer-review Journal
1	Quoc Khanh Nguyen [1]	2016	Blockchain - A Financial Technology for Future Sustainable Development	4	144	Conference Papers
2	Seongjoon Park [2]	2019	Performance Analysis of DAG-Based Cryptocurrency	6	12	Conference Papers
3	Chong Bai [8]	2019	State-of-the-Art and Future Trends of Blockchain Based on DAG Structure	14	10	Book Chapters
4	Sara Saberi [9]	2018	Blockchain technology and its relationships to sustainable supply chain management	20	1831	Journal Articles
5	M. Niranjanamurthy [20]	2018	Analysis of Blockchain technology: pros, cons and SWOT	15	262	Journal Article
6	Hiroki Watanabe [24]	2019	Enhancing Blockchain Traceability with DAG-Based Tokens	8	15	Conference Papers
7	Shu Yang [27]	2019	CoDAG: An Efficient and Compacted DAG-Based Blockchain Protocol	5	17	Conference Papers
8	Wenhui Yang [34]	2020	LDV: A Lightweight DAG-based Blockchain for Vehicular Social Networks	11	57	Journal Article

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