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- $|x|$ = Cardinality of X, if X is a finite set.
- Z = Integers
- Q = Rational numbers
- R = Real numbers
- Himpunankosong = $\emptyset$ = { } = Empty set = Null set = Void set
- If x is in the set X, then $x \in X$
- If x isn't inthe set X, then $x \notin X$
- X is a subset of Y, then $X \subseteq Y$
- X is a proper subset of Y, then $X \subset Y$
- Power set of X = P(x) = The set of all subjects of a set X
- The power set of a set with n elements has 2^n elements
- If A = {a, b, c}, the members of P(A) are $\emptyset$, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}.
- $X \cup Y$ = Union of X and Y
- $X \cap Y$ = Intersection of X and Y
- $X - Y$ = Difference or relative complement = Consits of all elements in X that are not in Y

↳ e.g.: A = {1, 3, 5} B = {4, 5, 6}

A − B = {1, 3}

B − A = {4, 6}

- X and Y are disjoint / tdk berhubungan if $X \cap Y = \emptyset$.
- A collection of sets S is said to be pairwise disjoint if, whenever X and Y are distinct sets in S, X and Y are disjoint.
- Set U = Universal set or a universe = Must be explicitly given or inferred from the context
- U − X is called the complement of X = $\overline{X}$, while a subset X of U

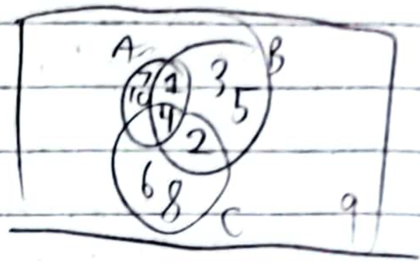
BOSS

1. a. $U = \{1,2,3,4,5,6,7,8,9,10\}$

$A = \{1,4,7,10\}$

$B = \{1,2,3,4,5\}$

$C = \{2,4,6,8\}$



a. $A \cup B = \{1,2,3,4,5,7,10\}$

b. $B - A = \{2,3,5\}$

c. $A \cup \emptyset = A = \{1,4,7,10\}$

d. $\overline{A \cap B} \cup C$

$\Rightarrow$ $\cancel{\overline{A \cap B}}$ $A \cap B = \{1,4\}$

$\overline{A \cap B} = \{2,3,5,6,7,8,9,10\}$

$\overline{A \cap B} \cup C = \{2,3,4,5,6,7,8,9,10\}$

e. $\Rightarrow (A \cup B) - (C - B) = \{1,2,3,4,5,7,10\}$

$\Rightarrow A \cup B = \{1,2,3,4,5,7,10\}$

$\Rightarrow C - B = \{6,8\}$

2. a. $C = \{1,2,3\}$ $\quad B = C \cap D$

$D = \{2,3,4\}$ $\quad = \{2,3\}$

$A = \{2,3\}$ (Terbukti)

b. $A = \{x \mid x^2 - 4x + 4 = 1\}$ $\quad B = \{1,3\}$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$ (Terbukti)

$x = 3 \lor x = 1$

3. a. $B = \{1,2,3,4\}$ $\quad A = B \cap C$

$C = \{2,4,6,8\}$ $\quad = \{2,4\}$ (Terbukti)

b. $A = \{1,2\}$ $\quad B = \{x \mid x^3 - 2x^2 - x + 2 = 0\}$

$$\begin{array}{c|cccc} & 1 & -2 & -1 & 2 \\ 2 & \downarrow & 2 & 0 & -2 \\ \hline & 1 & 0 & -1 & \boxed{0} \end{array}$$

(Terbukti) $\quad (x-2)(x^2-1) = 0$

$x = 2 \lor x = 1 \lor x = -1$

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Practice 1

Let the universe be the set $U = \{1, 2, 3, \ldots, 10\}$. Let $A = \{1, 4, 7, 10\}$, $B = \{1, 2, 3, 4, 5\}$, and $C = \{2, 4, 6, 8\}$. List the elements of each set.

a. $A \cup B$

→ $A \cup B = \{1, 2, 3, 4, 5, 7, 10\}$

b. $B - A$

→ $B - A = \{2, 3, 5\}$

c. $A \cup \emptyset$

→ $A \cup \emptyset = A = \{1, 4, 7, 10\}$

d. $\overline{A \cap B} \cup C$

→ $A \cap B = \{1, 4\}$

$\overline{A \cap B} = \{2, 3, 5, 6, 7, 8, 9, 10\}$

$\therefore \overline{A \cap B} \cup C = \{2, 3, 5, 6, 7, 8, 9, 10\}$

e. $(A \cup B) - (C - B)$

→ $A \cup B = \{1, 2, 3, 4, 5, 7, 10\}$

$C - B = \{6, 8\}$

$\therefore (A \cup B) - (C - B) = \{1, 2, 3, 4, 5, 7, 10\}$

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Practice 2

1. Show that $A = B$

a. $C = \{1,2,3\}$, $D = \{2,3,4\}$, $A = \{2,3\}$, $B = C \cap D$

→ ◦) $B = C \cap D$
$= \{2,3\}$

◦)& $A = B = \{2,3\}$ (Terbukti).

b. $A = \{x \mid x^2 - 4x + 4 = 1\}$, $B = \{1,3\}$

→ ◦) $x^2 - 4x + 4 = 1$
$x^2 - 4x + 3 = 0$
$(x-3)(x-1) = 0$
$x = 3 \vee x = 1$

◦) $A = \{x \mid x = 3 \vee x = 1\} = \{1,3\}$

◦)& $A = B = \{1,3\}$ (Terbukti)

2. Show that $A \neq B$

a. $B = \{1,2,3,4\}$, $C = \{2,4,6,8\}$, $A = B \cap C$

→ ◦) $A = B \cap C$
$= \{2,4\}$

◦) $A = \{2,4\}$
◦) $B = \{1,2,3,4\}$

◦)& $A \neq B$ (Terbukti)

b. $A = \{1,2\}$, $B = \{x \mid x^3 - 2x^2 - x + 2 = 0\}$

→ ◦) $x^3 - 2x^2 - x + 2 = 0$
$(x-2)(x^2-1) = 0$
$(x-2)(x-1)(x+1) = 0$
$x = 2 \vee x = 1 \vee x = -1$

◦)
$$\begin{array}{c|cccc} & 1 & -2 & -1 & 2 \\ 2 & \downarrow & 2 & 0 & -2 \\ \hline & 1 & 0 & -1 & \boxed{0} \end{array}$$

◦) $A = \{1,2\}$

◦) $B = \{x \mid x = -1 \vee x = 1 \vee x = 2\}$
$= \{-1,1,2\}$

◦)& $A \neq B$ (Terbukti)

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Practice 3

Show that $A \subseteq B$ or $A \not\subseteq B$

a. $A = \{1,2\}$, $B = \{x \mid x^3 - 6x^2 + 11x = 6\}$

→ ∘) $x^3 - 6x^2 + 11x = 6$

$x^3 - 6x^2 + 11x - 6 = 0$

$(x-2)(x^2 - 4x + 3) = 0$

$(x-2)(x-3)(x-1) = 0$

$x = 2 \vee x = 3 \vee x = 1$

∘) $\begin{array}{c|cccc} & 1 & -6 & 11 & -6 \\ 2 & \downarrow & 2 & -8 & 6 \\ \hline & 1 & -4 & 3 & \boxed{0} \end{array}$

∘) $A = \{1,2\}$

∘) $B = \{x \mid x = 1 \vee x = 2 \vee x = 3\}$

$= \{1,2,3\}$

∘)∘∘ $A \subseteq B$ (Terbukti)

b. $A = \{1\} \times \{1,2\}$, $B = \{1\} \times \{1,2,3\}$

→ ∘) $A = \{1\} \times \{1,2\}$

$= \{1,1\}, \{1,2\}$

∘) $B = \{1\} \times \{1,2,3\}$

$= \{1,1\}, \{1,2\}, \{1,3\}$

∘)∘∘ $A \subseteq B$ (Terbukti)

c. $A = \{x \mid x^3 - 2x^2 - x + 2 = 0\}$, $B = \{1,2\}$

→ ∘) $x^3 - 2x^2 - x + 2 = 0$

$(x-2)(x^2 - 1) = 0$

$x = 2 \vee x = 1 \vee x = -1$

∘) $A = \{x \mid x = -1 \vee x = 1 \vee x = 2\}$

$= \{-1, 1, 2\}$

∘) $\begin{array}{c|cccc} & 1 & -2 & -1 & 2 \\ 2 & \downarrow & 2 & 0 & -2 \\ \hline & 1 & 0 & -1 & \boxed{0} \end{array}$

∘) $B = \{1,2\}$

∘)∘∘ $A \not\subseteq B$ (Terbukti)

d. $A = \{1,2,3,4\}$, $C = \{5,6,7,8\}$, $B = \{n \mid n \in A \text{ and } n + m = 8 \text{ for some } m \in C\}$

→ ∘) $n + m = 8$

n	m
1	7
2	6
3	5

∘) $B = \{1,2,3\}$

∘) $A = \{1,2,3,4\}$

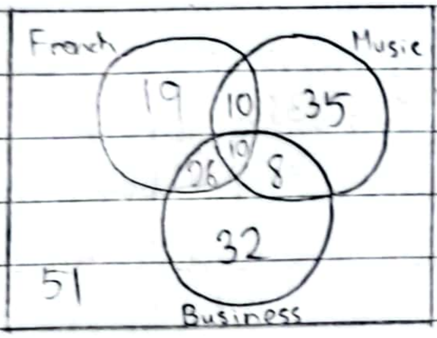
∘)∘∘ $A \not\subseteq B$

(Terbukti)

Practice 4

There is a group of 191 students, of which 10 are taking French, business, and music; 36 are taking French and business; 20 are taking French and music; 18 are taking business and music; 65 are taking French; 76 are taking business; and 63 are taking music.

a. How many are taking French and music but not business?
→ c) U c)/. 10 students



b. How many are taking business and neither French nor music?
→ c) 32 students

c. How many are taking French or business (or both)?
→ c) 19 + 32 + 26 + 10 + 10 + 8 = 105 students

d. How many are taking music or French (or both) but not business?
→ c) 19 + 35 + 10 = 64 students

e. How many are taking none of the three subjects?
→ c) 51 students

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Practice 1

Given that proposition p is false, proposition q is true, and proposition r is false, determine whether each proposition below is true or false.

a) $\neg p \vee q$

→ ∘) p = false ∘) q = true

∘) $\neg p$ = true ∘) ∴ $\neg p \vee q$ = true or true = true

b) $\neg p \vee \neg(q \wedge r)$

→ ∘) q = true ∘) p = false

∘) r = false ∘) $\neg p$ = true

∘) $q \wedge r$ = true and false = false

∘) $\neg(q \wedge r)$ = true

∘) ∴ $\neg p \vee \neg(q \wedge r)$ = true or true = true

c) $\neg(p \vee q) \wedge (\neg p \vee r)$

→ ∘) p = false ∘) $\neg p$ = true

∘) q = true ∘) r = false

∘) $p \vee q$ = false or true = true

∘) $\neg p \vee r$ = true or false = true

∘) $\neg(p \vee q)$ = false

∘) ∴ $\neg(p \vee q) \wedge (\neg p \vee r)$ = false and true = false

d) $(p \vee \neg r) \wedge \neg((q \vee r) \vee \neg(r \vee p))$

→ ∘) p = false ∘) q = true ∘) $r \vee p$ = false or false = false

∘) r = false ∘) $q \vee r$ = true or false = true

∘) $\neg r$ = true ∘) $\neg(r \vee p)$ = true

∘) $p \vee \neg r$ = false or true = true

c) $(q \vee r) \vee \neg(r \vee p)$ = true or true = true

c) $\neg((q \vee r) \vee \neg(r \vee p))$ = false

c) $(p \vee \neg r) \wedge \neg((q \vee r) \vee \neg(r \vee p))$ = true and false = false

Practice 2

Write the truth table of each proposition below:

a) $(p \wedge q) \vee (\neg p \vee q)$

p	q	$p \wedge q$	$\neg p \vee q$	$(p \wedge q) \vee (\neg p \vee q)$
T	T	T	T	T
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T

c) T = True

c) F = False

b) $\neg(p \wedge q) \vee (r \wedge \neg p)$

p	q	r	$\neg(p \wedge q)$	$r \wedge \neg p$	$\neg(p \wedge q) \vee (r \wedge \neg p)$
T	T	T	F	F	F
T	T	F	F	F	F
T	F	T	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	F	T

c) T = True

c) F = False

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c) $(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)$

→ c)

p	q	$p \vee q$	$\neg p \vee q$	$p \vee \neg q$	$\neg p \vee \neg q$	$(p\vee q)\wedge(\neg p\vee q)\wedge(p\vee\neg q)\wedge(\neg p\vee\neg q)$
T	T	T	T	T	F	F
T	F	T	F	T	T	F
F	T	T	T	F	T	F
F	F	F	T	T	T	F

c) T = True c) F = False

d) $\neg(p \wedge q) \vee (\neg q \vee r)$

→ c)

p	q	r	$\neg(p \wedge q)$	$\neg q \vee r$	$\neg(p\wedge q) \vee (\neg q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

c) T = True
c) F = False

Practice 3

For each pair of propositions P and Q below, state whether or not $P \equiv Q$.

a) $P = p$, $Q = p \vee q$

→ c) $P = p$ c) $p \vee q = Q$ c) T = True c) F = False

$P = p$	$p \vee q$	$= Q$
T	T V T	T
T	T V F	T
F	F V T	T
F	F V F	F

c) $P \not\equiv Q$

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b) $P = p \land q$, $Q = \neg p \lor \neg q$

→ c) $p \land q = P$

$p \land q$	P
$T \land T$	T
$T \land F$	F
$F \land T$	F
$F \land F$	F

p	q	$\neg p \lor \neg q$	$= Q$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

c) T = True

c) F = False

c) $P \not\equiv Q$

c) $P = p \to q$, $Q = \neg p \lor q$

→ c) $p \to q = P$

$p \to q$	P
$T \to T$	T
$T \to F$	F
$F \to T$	T
$F \to F$	T

p	q	$\neg p \lor q$	$= Q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

c) T = True

c) F = False

c) $P \equiv Q$

d) $P = p \land (\neg q \lor r)$, $Q = p \lor (q \land \neg r)$

→ c)

p	q	r	$\neg q \lor r$	$p \land (\neg q \lor r)$	$= P$	$q \land \neg r$	$p \lor (q \land \neg r)$	$= Q$
T	T	T	T	T	T	F	T	T
T	T	F	F	F	F	T	T	T
T	F	T	T	T	T	F	T	T
T	F	F	T	T	T	F	T	T
F	T	T	T	F	F	F	F	F
F	T	F	F	F	F	T	T	T
F	F	T	T	F	F	F	F	F
F	F	F	T	F	F	F	F	F

c) T = True

c) F = False

c) $P \not\equiv Q$

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Practice 4

Determine whether each argument below is valid

a) $p \to q$

$\underline{\neg p}$

$\therefore \neg q$

→ ∴) $p \to q \equiv \neg p \vee q$

$\underline{\neg p}$

$\therefore q$ (Disjunctive syllogism)

∴) ∴ $p \to q$ is invalid

$\underline{\neg p}$

$\therefore \neg q$

b) $p \to q$

$\underline{\neg q}$

$\therefore \neg p$

→ ∴) $p \to q$

$\underline{\neg q}$

$\therefore \neg p$ (Modus tollens)

∴) ∴ $p \to q$ is valid.

$\underline{\neg p}$

$\therefore \neg p$

c) $\underline{p \wedge \neg p}$

$\therefore q$

→ ∴)

p	q	$p \wedge \neg p$	$(p \wedge \neg p) \to q$
T	T	F	T
T	F	F	T
F	T	F	T
F	F	F	T (Tautology)

OR ∴) $\underline{p \wedge \neg p}$

$\therefore p$ (Simplification)

∴) ∴ $\underline{p \wedge \neg p}$

$\therefore$

d) $p \to (q \to r)$

$\underline{q \to (p \to r)}$

$\therefore (p \vee q) \to r$

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Practice 4

Determine whether each argument below is valid.

a) $p \rightarrow q$

$\neg p$

$\therefore \neg q$

∴ Truth table:

∴ T = True

∴ F = False

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge \neg p$	$((p \rightarrow q) \wedge \neg p) \rightarrow \neg q$
T	T	T	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T (Not Tautology)

∴ $p \rightarrow q$ is invalid.

$\neg p$

$\therefore \neg q$

b) $p \rightarrow q$

$\neg q$

$\therefore \neg p$

∴ Truth table:

∴ T = True

∴ F = False

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge \neg q$	$((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$
T	T	T	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	T	T (Tautology)

∴ $p \rightarrow q$

$\neg q$

$\therefore \neg p$ (Modus tollens)

∴ $p \rightarrow q$ is valid.

$\neg q$

$\therefore \neg p$

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c) $p \wedge \neg p$

$\therefore q$

→ ∴) Truth table:

∴) T = True

∴) F = False

p	q	$p \wedge \neg p$	$(p \wedge \neg p) \to q$
T	T	F	T
T	F	F	T
F	T	F	T
F	F	F	T (Tautology)

∴) $p \wedge \neg p$

$\therefore q$ (Simplification)

∴) ∴ $p \wedge \neg p$ / $\therefore q$ is valid.

d) $p \to (q \to r)$

$q \to (p \to r)$

$\therefore (p \vee q) \to r$

→ ∴) Truth table:

∴) T = True

∴) F = False

p	q	r	$q \to r$	$p \to (q \to r)$	$p \to r$	$q \to (p \to r)$	$p \vee q$	$(p \vee q) \to r$
T	T	T	T	T	T	T	T	T
T	T	F	F	F	F	F	T	F
T	F	T	T	T	T	T	T	T
T	F	F	T	T	F	T	T	F
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	F
F	F	T	T	T	T	T	F	T
F	F	F	T	T	T	T	F	T

$(p \to (q \to r)) \wedge (q \to (p \to r))$	$((p \to (q \to r)) \wedge (q \to (p \to r))) \to ((p \vee q) \to r)$
T	T
F	T
T	T
T	F
T	T
T	F
T	T
T	T (Not Tautology)

c) $p \to (q \to r)$ is invalid.

$q \to (p \to r)$

$\varepsilon_c\ (p \vee q) \to r$

Practice 5

Determine the truth value of each statement below. The domain of discourse is R. Justify your answers.

a) $\forall x (x^2 > x)$

→ c) $\forall x (x^2 > x)$ is false since, if $x = \frac{1}{2}$, then $x^2 > x$

$= (\frac{1}{2})^2 > \frac{1}{2}$

$= \frac{1}{4} > \frac{1}{2}$ (False)

b) $\exists x (x^2 > x)$

→ c) $\exists x (x^2 > x)$ is true because, if $x = 2$, then $x^2 > x$

$= 2^2 > 2$

$= 4 > 2$ (True)

c) Even though if we substitute $x = 1$ the result we get is false, the symbol $\exists$ means there is an x that has the value. Unlike the poin a), the symbol $\forall$ means for all x can determine true of $x^2 > x$ statement.

c) $\forall x (x > 1 \to x^2 > x)$

→ c) $\forall x (x > 1 \to x^2 > x)$ is true because there's no $x \in R > 1$ that makes $x^2 > x$ false. Moreover, $x^2 > x \equiv x^2 - x > 0 \equiv x(x-1) > 0$

$x > 1 \vee x < 0$

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d) $\exists x (x>1 \rightarrow x^2>x)$

→ ∴ $\exists x (x>1 \rightarrow x^2>x)$ is true because if $x=2$, then $2^2>2$ is true or if $x=3$, then $3^2>3$ is also true.

Practice 6

Determine the truth value of each statement below. The domain of discourse is R×R. Justify your answers.

a) $\forall x \forall y (x^2<y+1)$

→ ∴ $\forall x \forall y (x^2<y+1)$

↳ For every real number x and y, $x^2<y+1$

↳ The statement above is not true.

↳ Prove : $\left.\begin{matrix} x=1 \\ y=-2 \end{matrix}\right\}$ $1<-2+1$, $1<-1$

∴ $\forall x \forall y (x^2<y+1)$ is false.

b) $\forall x \exists y (x^2<y+1)$

→ ∴ $\forall x \exists y (x^2<y+1)$

↳ For every real number x, there exists a real number y such that $x^2<y+1$

↳ The statement above is true

↳ Prove : $x=3$, $y=10$

∴ $\forall x \exists y (x^2<y+1)$ is true.

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c) $\exists x \forall y (x^2 < y+1)$

→ ∴ $\exists x \forall y (x^2 < y+1)$

↳ There exists some real number x such that for every real number y, $x^2 < y+1$.

↳ The statement above is not true.

↳ Prove : $x = 3$, $y = 10$ but $x = 3$, $y = 2$

∴ $\exists x \forall y (x^2 < y+1)$ is false.

d) $\exists x \exists y (x^2 < y+1)$

→ ∴ $\exists x \exists y (x^2 < y+1)$

↳ There exists some real number x and y such that $x^2 < y+1$.

↳ The statement above is true.

↳ Prove : $x = 1$, $y = 2$ or $x = 2$, $y = 5$

∴ $\exists x \exists y (x^2 < y+1)$ is true.

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Mathematical system consists of axioms, definitions, and undefined terms. Axioms are assumed to be true. Definitions are used to create new concepts in terms of existing ones.

A theorem is a proposition that has been proved to be true. Special kinds of theorems are referred to as lemmas and corollaries. A lemma is a theorem that is usually not too interesting in its own right but is useful in proving another theorem. A corollary is a theorem that follows easily from another theorem. An argument that establis the ttuth of a theorem is called proof.

Euclidian geometry furnishes an example of a mathematical system. Among the axioms are:

- Given 2 distinct points, there is exactly one line that contains them.
- Given a line and a point not on the line, there is exactly 1 line parallel to the line through the point.

The terms point and line are undefined terms that are implicitly defined by the axioms that describe their properties. Among the definitions are:

- 2 triangles are congruent if their vertices can be paired so that the corresponding sides and corresponding angles are equal.
- 2 angles are supplementary if the sum of their measure is 180°.

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Practice 1

1. Prove that for all integers m and n, if m and n are even, then $m+n$ is even.

→ Let m and n be arbitrary integers, and suppose that m and n are even. Since m is even, there exists an integer k_1 such that $m = 2k_1$. Since n is also even, there also exists an integer k_2 such that $n = 2k_2$.
Then, the sum is $m+n = 2k_1 + 2k_2 = 2(k_1 + k_2)$.
Thus, there exists an integer k (Namely $k = k_1 + k_2$) such that $m+n = 2(k_1+k_2) = 2k$. Therefore, $m+n$ is even
(Direct Proofs)

2. Prove that for all rational numbers x and y, $x+y$ is rational.

→ If x is rational number, then there exists 2 integers a and b such that $x = \frac{a}{b}$ with $b \neq 0$. If y is rational number, then there exists another 2 integers c and d such that $y = \frac{c}{d}$ with $d \neq 0$.

Thus, $x+y = \frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ with $bd \neq 0$

Therefore, $x+y$ is rational.
(Direct Proofs)

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3. Prove that $X \cap Y \subseteq X$ for all sets X and Y

→ ◦ Assume that for all a, if $a \in X \cap Y \subseteq X$, then $a \in X$, and if $a \in X$, then $a \in X \cap Y \subseteq X$.
◦ Let $a \in X \cap Y$. Then $a \in X$ and $a \in Y$. If $a \in X$, then $a \in X \cap Y$. If $a \in Y$, then $a \in X \cap Y$, so again $a \in X \cap Y$. Since we know that $a \in X$ and $a \in X \cap Y$, there will be $n(X) \geq n(X \cap Y)$. Hence, $X \cap Y \subseteq X$. The proof is complete.
(Direct Proofs)

Practice 2

1. Prove that for every $n \in \mathbb{Z}$, if n^2 is odd, then n is odd.

→ ◦ Assume the hypothesis n^2 is even and that the conclusion is false that n is even. Since n is even, there exists an integer k such that $n = 2k$.
◦ Then, $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$
◦ Thus, n^2 is even, which contradicts the hypothesis n^2 is odd.
◦ Therefore, the proof by contradiction is complete.
(Proof by Contradiction)

2. Prove that for all $x, y \in \mathbb{R}$, if x is rational and y is irrational, then $x+y$ is irrational.

→ ◦ If x is rational, then $x+y$ is rational which means y is also rational.
◦ If x is rational, and y is irrational, then $x+y$ is also irrational
◦ This is equivalent to x is rational and y is irrational such that $x+y$ is irrational. (Contrapositive).

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∘) Assume that x is rational, then there exists 2 integers a and b such that $x = \frac{a}{b}$. If $x+y$ is also rational, then there exists 2 integers p and q such that $x+y = \frac{p}{q}$ with $b \neq 0$ and $q \neq 0$.

∘) Then, $x+y = \frac{p}{q}$

$$\frac{a}{b} + y = \frac{p}{q}$$

$$y = \frac{p}{q} - \frac{a}{b} = \frac{pb - aq}{bq}$$

And so y can be written as a fraction which means y is rational.

∘) However, we initially asserted that y was irrational and hence we have a contradiction, so $x+y$ can't be rational and must be irrational.
(Proof by Contradiction)

3. Prove that if a and b are real numbers with $a < b$, there exists a rational number x satisfying $a < x < b$.

∘) If $a < b$, then

$$a < b \qquad \text{and} \qquad a < b$$

$$\frac{a}{2} < \frac{b}{2} \qquad\qquad \frac{a}{2} < \frac{b}{2}$$

$$\frac{a}{2} + \frac{a}{2} < \frac{b}{2} + \frac{a}{2} \qquad\qquad \frac{a}{2} + \frac{b}{2} < \frac{b}{2} + \frac{b}{2}$$

$$a < \frac{a+b}{2} \qquad\qquad \frac{a+b}{2} < b$$

∘) Hence, we get $a < \frac{a+b}{2}$ and $\frac{a+b}{2} < b$ which we combine it become $a < \frac{a+b}{2} < b$. From this statement, we can conclude that $\frac{a+b}{2} = x$ which x is a rational number.

∘) Therefore, the proof is complete
(Direct Proofs)

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Practice 3

1. Use resolution to derive each conclusion

a. $\neg p \vee q \vee r$

$\neg q$

$\neg r$

$\therefore \neg p$

→ c) $\neg p \vee q \vee r$ → $\neg p \vee q \vee r$

$\neg q$ → $\neg q$

$\neg r$ → $\neg p \vee r$

$\neg r$

$\therefore \neg p$

b. $\neg p \vee t$

$\neg q \vee s$

$\neg r \vee (s \wedge t)$

$p \vee q \vee r \vee u$

$\therefore s \vee t \vee u$

→ c) $\neg p \vee t$ → $\neg p \vee t$ → $\neg p \vee t$

$\neg q \vee s$ → $\neg q \vee s$ → $\neg q \vee s$

$\neg r \vee (s \wedge t) \rightarrow (\neg r \vee s) \wedge (\neg r \vee t) \rightarrow \neg r \vee s$

$p \vee q \vee r \vee u$ → $p \vee q \vee r \vee u$ → $\neg r \vee t$

$\therefore s \vee t \vee u$ → $\therefore s \vee t \vee u$ → $p \vee q \vee r \vee u$

$\therefore s \vee t \vee u$

Practice 4

1. Using induction, verify that each equation is true for every positive integer n.

a. $1+3+5+\ldots+(2n-1)=n^2$

→ i) $1+3+5+\ldots+(2n-1)=n^2$

↳ $n=1 \rightarrow 1=1^2$

Basis step $\quad 1=1$ (True)

↳ $n=n+1 \rightarrow 1+3+5+\ldots+(2n+2-1)=(n+1)^2$

Inductive step $\quad 1+3+5+\ldots+(2n-1)+(2n+1)=n^2+2n+1$

$n^2+2n+1=n^2+2n+1$

(True)

ii). Since Basic step and Inductive step are true, hence the Principle of Mathematical Induction tells us that $1+3+5+\ldots+(2n-1)=n^2$

b. $1(1!)+2(2!)+\ldots+n(n!)=(n+1)!-1$

→ i) $1(1!)+2(2!)+\ldots+n(n!)=(n+1)!-1$

↳ $n=1 \rightarrow 1(1!)=(1+1)!-1$

Basis step $\quad 1=2-1$

$1=1$ (True)

↳ $n=n+1 \rightarrow 1(1!)+2(2!)+\ldots+(n+1)(n+1)!=(n+1+1)!-1$

Inductive step $\quad 1(1!)+2(2!)+\ldots+n(n!)+(n+1)(n+1)!=(n+2)!-1$

$(n+1)!-1+(n+1)(n+1)!=(n+2)!-1$

$(n+1)!(n+2)=(n+2)!$

(True) $(n+2)!=(n+2)!$

ii). Since Basic step and Inductive step are true, hence the Principle of Mathematical Induction tells us that $1(1!)+2(2!)+\ldots+n(n!)=(n+1)!-1$

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Hash function takes a data item to be stored or retrieved and computes the first choice for a location for the item.

$h(n) = n \bmod 11$

132			102	15	5	257		558		32
0	1	2	3	4	5	6	7	8	9	10

sisa sharusnya 4 tetapi sdh ditempati di 15

Collision resolution policy

Ceiling = $\lceil x \rceil$ = Pembulatan ke atas

Floor = $\lfloor x \rfloor$ = Pembulatan ke bawah

A function from $X \times X$ to X is called a binary operator on X. A function from X to X is called a unary operator on X.

A sequence s is increasing if $S_n < S_{n+1}$ for all n for which n and $n+1$ are in the domain of the sequence. A sequence s is decreasing if $S_n > S_{n+1}$ for all n for which n and $n+1$ are in the domain of the sequence. A sequence s is nondecreasing if $S_n \leq S_{n+1}$ for all n for which n and $n+1$ are in the domain of the sequence (A nondecreasing sequence is like an increasing sequence except that $<$ is replaced by $\leq$. A sequence s is nonincreasing if $S_n \geq S_{n+1}$ for all n for which n and $n+1$ are in the domain of the sequence (A nonincreasing sequence is like a decreasing sequence except that $>$ is replaced by $\geq$.

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A string is a finite sequence of characters. Within a computer, bit strings (strings of 0's and 1's) represent data and instructions to execute.

Since string is a sequence, order is taken into account. Repetitions in string can be specified by superscripts.

String with no elements = Null string = λ. The length of a string α is the number of elements in α. The length of $\alpha = |\alpha|$. X^* = The set of all strings over X, including the null string. X^+ = The set of all nonnull strings over X.

If $a = aabab$ and $\beta = a^3b^4a^{32}$, then $|a| = 5$ and $|b| = 39$. If α and β are 2 strings, the string consisting of α followed by β, written $\alpha\beta$ = Concatenation of α and β. If $\gamma = aab$ and $\theta = cabd$, then $\gamma\theta = aabcabd$, $\gamma\lambda = \gamma = aab = \lambda\gamma$

A substring of a string α is obtained by selecting some or all consecutive elements of α. The string $\beta = add$ is a substring $\alpha = aaaddad$. If we take $\gamma = aa$ and $\delta = ad$, we have $\alpha = \gamma\beta\delta$.

A (binary) relation R from a set X to a set Y is a subset of the Cartesian product $X \times Y$. If $(x, y) \in R$, we write $x R y$ and x is related to y. If $X = Y$, we call R a (binary) relation on X.

A relation R on a set X is reflexive (sifat 1) if $(x, x) \in R$ for every $x \in X$. A relation R on a set X is symmetric (sifat 2) if for all $x, y \in X$, if $(x, y) \in R$, then $(y, x) \in R$.

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In symbols, a relation R is symmetric if $\forall x \forall y\ [(x,y) \in R] \rightarrow [(y,x) \in R]$

A relation R on a set X is antisymmetric if for all $x, y \in X$, if $(x,y) \in R$ and $(y,x) \in R$, then $x = y$. In symbols, a relation R is antisymmetric if $\forall x \forall y\ [(x,y) \in R \wedge (y,x) \in R] \rightarrow [x = y]$.

A relation R on a set X is transitive if $\forall x \forall y \forall z \in X$, if (x,y) and $(y,z) \in R$, then $(x,z) \in R$. In symbols, a relation R is transitive if $\forall x \forall y \forall z\ [(x,y) \in R \wedge (y,z) \in R] \rightarrow [(x,z) \in R]$.

A relation R on a set X is a partial order if R is reflexive, antisymmetric, and transitive. Suppose that R is a partial order on a set X. If $x, y \in X$ and either $x \preceq y$ or $y \preceq x$, we say that x and y are comparable. If $x, y \in X$ and $x \npreceq y$ and $y \npreceq x$, we say that x and y are incomparable. If every pair of elements in X is comparable, we call R a total order.

The inverse of R, denoted R^{-1}, is the relation from Y to X defined by $R^{-1} = \{(y,x) \mid (x,y) \in R\}$.

The composition of R_1 and $R_2 = R_1 \circ R_2$, is the relation from X to Z defined by $R_2 \circ R_1 \{(x,z) \mid (x,y) \in R_1$ and $(y,z) \in R_2$ for some $y \in Y\}$.

A relation that is reflexive, symmetric, and transitive on a set X is called an equivalence relation on X.

$x \preceq y \rightarrow x$ blh lbh duluan atau sama-sama dgn y

$x \prec y \rightarrow x$ hrs lbh duluan dr y

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A matrix is a convenient way to represent a relation R from X to Y.

- $R = \{(1,b), (1,d), (2,c), (3,c), (3,b), (4,a)\}$

$$\begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

- $x R y$ if x divides y $\{2,3,4\}$ to $\{5,6,7,8\}$

$$\begin{array}{c|c} & \begin{matrix} 5 & 6 & 7 & 8 \end{matrix} \\ \hline \begin{matrix} 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

R is transitive if and only if whenever entry i,j in A^2 is nonzero, entry i,j in A is also nonzero.

(a,a) (b,b) (c,c) (d,d) (b,c) (c,b)

$$\begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \hline \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} = A \qquad A^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Nonzero

- d divides n = $d \mid n$
- d doesn't divide n = $d \nmid n$

$3 \mid 21 = 7$

divisor / factor of 21

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- $n = qd + r, \quad 0 \le r < d$

Quotient Remainder

- If $d \mid m$ and $d \mid n$, then $d \mid (m+n)$
- If $d \mid m$ and $d \mid n$, then $d \mid (m-n)$
- If $d \mid m$, then $d \mid mn$

- GCD = Greatest Common Divisor = FPB = gcd(m,n)
  gcd (30,105) = 15

- LCM = Least Common Multiple = KPK = lcm (30,105)
  lcm (30,105) = 210

- In a digital computer, data and instructions are encoded as bits 0 and 1

- Euclidean algorithm is an old, famous, and efficient algorithm for finding the GCD of 2 integers

- If a is a nonnegative integer, b is a positive integer, and $r = a \bmod b$, then $\gcd(a,b) = \gcd(b,r)$

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Date: Week 10

Practice 1

1. Four microprocessors are randomly selected from a lot of 100 microprocessors among which 10 are defective. Find the probability of obtaining no defective microprocessors.

→ $P = \frac{C(90,4)}{C(100,4)} = \frac{90!}{86!\cdot 4!} \cdot \frac{96!\cdot 4!}{100!} = \frac{90\cdot 89\cdot 88\cdot 87\cdot \cancel{86!}\cdot \cancel{96!}}{\cancel{86!}\cdot 100\cdot 99\cdot 98\cdot 97\cdot \cancel{96!}}$

≈ 0.652

2. In the California Daily 3 game, a consestant must select three numbers among 0 to 9, repetitions allowed. A "straight play" win requires that the numbers be matched in the exact order in which they are randomly drawn by a lottery representative. What is the probability of choosing the winning numbers?

→ Straight play: 012 234 456 678 & vice versa
123 345 567 789

$P = \frac{8\cdot 2}{C(10,3)} = 16\cdot\frac{7!\cdot 3!}{10!} = \frac{16\cdot 7!\cdot 3\cdot 2^1}{10\cdot 9\cdot 8\cdot 7!} = \frac{2}{15} \approx 0.1\dot{3}$

3. In the multi-state Big Game, to win the grand prize the contestant must watch five distinct numbers, in any order, among the numbers 1 through 50, and one Big Money Ball number between 1 and 36, all randomly drawn by a lottery representotive. What is the probability of choosing the winning numbers?

→ Win → 1 until 36
All → 1 until 50

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$\therefore P = \dfrac{1}{C(50,5)\cdot 36} = \dfrac{45!\cdot 5!}{50!\cdot 36} = \dfrac{\cancel{45!}\cdot 5\cdot 4\cdot 3\cdot 2}{50\cdot 49\cdot 48\cdot 47\cdot 46\cdot \cancel{45!}} \approx 0.00000001311$

Practice 2

1. Suppose that a professional wrestler is selected at random among 90 wrestlers, where 35 are over 350 pounds, 20 are bad guys, and 15 are over 350 pounds and bad guys. What is the probability that the wrestler selected is over 350 pounds or a bad guy?

→ ∴ Let A denotes the event of a wrestler that over 350 pounds.
∴ Let B denotes the event of a wrestler that is bad guy.
∴ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \dfrac{C(35,1)}{C(90,1)} + \dfrac{C(20,1)}{C(90,1)} - \dfrac{C(15,1)}{C(90,1)}$

$= \dfrac{35}{90} + \dfrac{20}{90} - \dfrac{15}{90} = \dfrac{40}{90} = \dfrac{4}{9} \approx 0.\dot{4}$

2. A company that buys computers from three vendors and tracks the number of defective machines. The following table shows the results. Let A denote the event "the computer was purchased from Acme," let D denote the event "the computer was purchased from Dot-Com," let N denote the event "the computer was purchased from Nuclear," and let B denote the event "the computer was defective." Find P(A), P(D), and P(N). Find P(B|A), P(B|D), and P(B|N). Find P(B).

	Vendor		
	Acme	Dot Com	Nuclear
Percent purchased	55	10	35
Percent defective	1	3	3

P(B|A) = Peluang Defective dr TOTAL Acme

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- $P(A) = 0.55$
- $P(D) = 0.1$
- $P(N) = 0.35$
- $P(B|A) = 0.1$
- $P(B|D) = 0.3$
- $P(B|N) = 0.3$
- $P(A|B) = \dfrac{P(B|A) \cdot P(A)}{P}$

$=$

An event is an outcome or combination of outcomes from an experiment. The sample space is the event consisting of all possible outcomes. Thus, if all outcomes in a finite sample space are equally likely, then $\boxed{P(E) = \frac{|X|}{|S|}}$, while $|X|$ is event and $|S|$ is finite sample space.

- $\boxed{0 \leq |X| \leq 1}$, $\forall x \in S$ atau $\sum_{x \in S} P(x) = 1$.
- $\boxed{P(E) = \sum_{x \in E} P(x)}$, with E is a certain event
- $\boxed{P(x) + P(\overline{x}) = 1}$
- $\boxed{P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)}$, with E_1 & E_2 are events

- Events E_1 and E_2 are mutually exclusive if E_1 and $E_2 = \emptyset$, then $\boxed{P(E_1 \cup E_2) = P(E_1) + P(E_2)}$
- Let E and F be events, and assume that $P(F) > 0$. The conditional probability of E given F is $\boxed{P(E|F) = \frac{P(E \cap F)}{P(F)}}$
- Events E and F are independent if $\boxed{P(E \cap F) = P(E)\,P(F)}$

Week 11

A recursive function is a function that invokes itself. A recursive algorithm is an algorithm that contains a recursive function. A recurrence relation for the sequence $a_0, a_1, \ldots$ is an equation that relates a_n to certain of its predecessors $a_0, a_1, \ldots, a_{n-1}$. Initial conditions for the sequence $a_0, a_1, \ldots$ are explicitly given values for a finite number of the terms of the sequence.

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The Fibonacci sequence is defined by the recurrence relation $f_n = f_{n-1} + f_{n-2}$, $n \geq 3$ and initial conditions $f_1 = 1$, $f_2 = 1$.

• There are 2 ways of solving a recurrence relation involving the sequence $a_0, a_1, \dots$ is to find an explicit formula for the general term a_n, which are:

1. Iteration

We use the recurrence relation to write the nth term a_n in terms of its predecessors $a_{n-1}, \dots, a_0$. We then successively use the recurrence relation to replace each of $a_{n-1}, \dots$ by certain of their predecessors. We continue until an explicit formula is obtained. For instance:

We can solve the recurrence relation $a_n = a_{n-1} + 3$ subject to the initial condition $a_1 = 2$.

∘) $a_{n-1} = a_{n-2} + 3$
∘) $a_{n-2} = a_{n-3} + 3$
∘) $a_n = a_{n-2} + 2 \cdot 3$
$= a_{n-3} + 3 \cdot 3$

∘) $a_n = a_{n-1} + 3$
$= a_{n-2} + 3 + 3$
$= a_{n-2} + 2 \cdot 3$

In general, we have $a_n = a_{n-k} + k \cdot 3$. If we set $k = n-1$, then we have $a_n = a_1 + 3(n-1) \equiv a_n = 2 + 3(n-1)$

We can also solve the recurrence relation $S_n = 2S_{n-1}$ subject to initial condition $S_0 = 1$.

∘) $S_n = 2S_{n-1} = 2(2S_{n-2}) = \dots = 2^n S_0 = 2^n$.

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2. Linear Homogeneous Recurrence Relations with Constant Coefficients

A linear homogeneous recurrence relation of order k with constant cofficients is a recurrence relation of the form $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}$, $c_k \neq 0$. Notice that a linear homogeneous recurrence relation of order k with constant coefficients, together with the k initial conditions $a_0 = C_0, a_1 = C_1, \dots, a_{k-1} = C_{k-1}$, uniquely defines a sequence $a_0, a_1, \dots$

The recurrence relations $S_n = 2S_{n-1}$ and $f_n = f_{n-1} + f_{n-2}$ which defines the Fibonocci sequence, are both linear homogeneous recurrence relations with constant coefficients. The first recurrence relation is of order 1 and the second is of order 2.

Examples:

- $a_n = 3a_{n-1}a_{n-2}$ is not a linear homogeneous recurrence relation with constant coefficients because terms $a_{n-1}a_{n-2}$ are not permitted (Must be ca_k). Rearrence relations such as the example above are said to be nonlinear.

- $a_n - a_{n-1} = 2n$ is not a linear homogeneous recurrence relation with constant coefficients because the RHS is not 0. Such equation is said to be inhomogeneous.

- $a_n = 3na_{n-1}$ is not a linear homogeneous recurrence relation with constant coefficients because $3n$ is not constant. It's a linear homogeneous recurrence relation w/ nonconstant coefs.

Let $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ be a second-order, linear homogereous recurrene relation with constant coefficients. If S and T are solutions of that equation, then $a = bS + dT$ is also the solution. If r is a root of $t^2 - c_1 t - c_2 = 0$, then the sequence r^n, $n = 0, 1, \ldots$, is a solution of $a_n = c_1 a_{n-1} + c_2 a_{n-2}$. If a is the sequence defined by that equation, then $a_0 = C_0$, $a_1 = C_1$, and r_1 and r_2 are roots of $t^2 - c_1 t - c_2 = 0$, then there exist constants b and d such that $a_n = b r_1^n + d r_2^n$, $n = 0, 1, \ldots$

Let $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ be a second-order linear homogeneous recurrence relation with constant coefficients. Let a be the sequence satisfying that equation and $a_0 = C_0$, $a_1 = C_1$. If both roots of $t^2 - c_1 t - c_2 = 0$, are equal to r, then there exist constants b and d such that $a_n = b r^n + d n r^n$, $n = 0, 1, \ldots$

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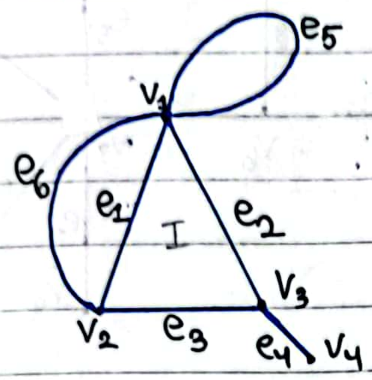
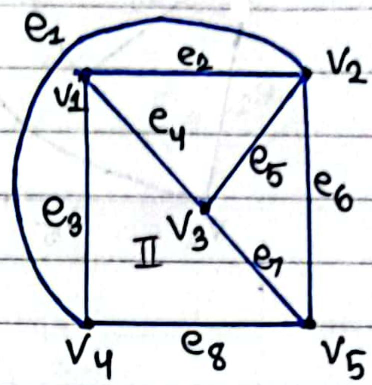
Date: Week 12

Practice 1

1. For each graph $G = (V, E)$ below, find V, E, all parallel edges, all loops, all isolated vertices, and tell whether G is a simple graph. Also, tell on which vertices edge e_1 is incident.





→ ◦ Graph I :

◦ $V = \{v_1, v_2, v_3, v_4\}$

◦ $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$

or

$E = \{(v_1, v_2), (v_2, v_3), (v_1, v_3), (v_3, v_4), (v_1, v_1)\}$

◦ Isolated vertices $= \{\}$

◦ Parallel edges $= e_6$ and e_1 are both associated with the vertex pair $\{v_1, v_2\}$.

◦ Loop $= \{e_5\}$ on vertex $\{v_1\}$

◦ G is not a simple graph because it has parallel edge and loop.

◦ v_1 and v_2 vertices are each incident with edge e_1.

◦ Graph II :

◦ $V = \{v_1, v_2, v_3, v_4, v_5\}$

◦ $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$

or

$E = \{(v_2, v_4), (v_1, v_2), (v_1, v_4), (v_1, v_3), (v_2, v_3), (v_2, v_5), (v_3, v_5), (v_4, v_5)\}$

◦ Parallel edges $= \{\}$

◦ Loop $= \{\}$

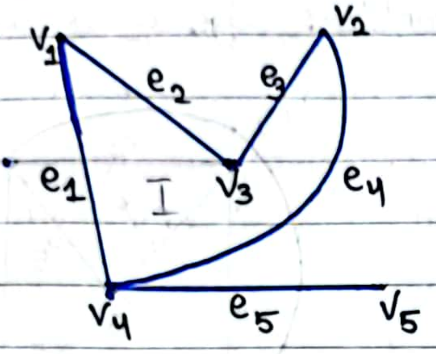
◦ Isolated vertices $= \{\}$

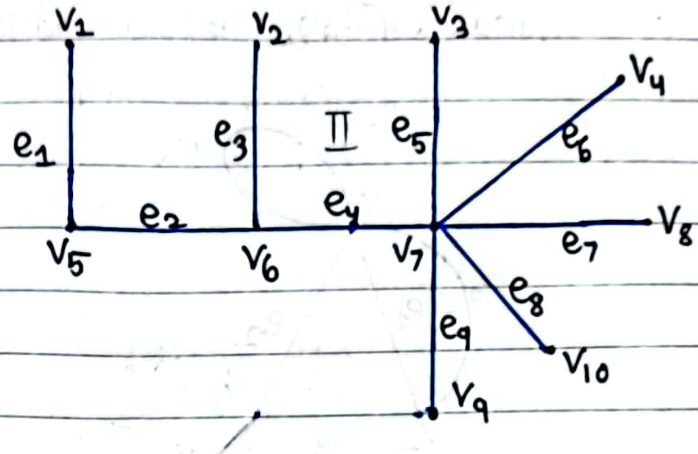
◦ G is a simple graph because it doesn't have parallel edges and loop.

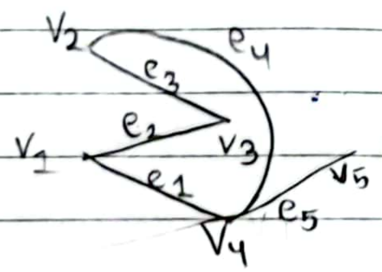
◦ v_2 and v_4 vertices are each incident with edge e_1.

Practice 2

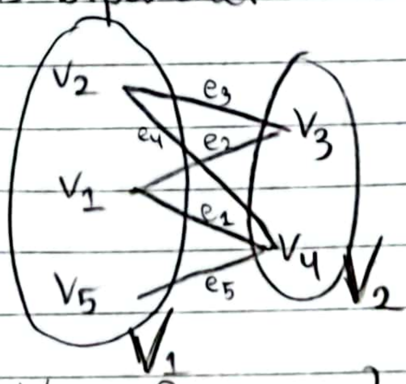
1. State which graphs below are bipartite graphs. If the graph is bipartite, specify the disjoint vertex sets.





- Graph I :
  - Suppose graph I is bipartite. Then, the vertex set can be partitioned into 2 subsets V_1 and V_2 such that each edge is incident on 1 vertex in V_1 and 1 vertex in V_2.
  - Consider the vertices v_1, v_2, v_3, v_4. v_1 are adjacent to v_3 and v_4. v_2 are adjacent to v_3 and v_4. Thus, we can assume that v_1 and v_2 are in V_1, v_3 and v_4 are in V_2. While v_5 is in V_1 too because v_5 adjacent with v_4 and it is not adjacent to v_1 or v_2, which suitable $V_1 \cap V_2 = \phi$
  - Therefore, graph I is bipartite.
  - 

    or

- Disjoint vertex sets : $V_1 = \{v_1, v_2, v_5\}$

  $V_2 = \{v_3, v_4\}$

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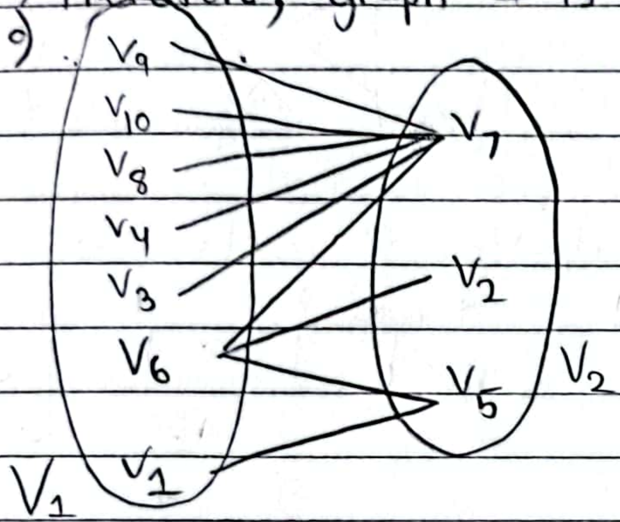
∘) Graph II :

∘) Suppose graph II is bipartite. Then, the vertex set can be partitioned into 2 subsets V_1 and V_2 such that each edge is incident on 1 vertex in V_1 and 1 vertex in V_2.

∘) Consider the vertices $v_1, v_2, v_3, v_5, v_6, v_7$. v_1 are adjacent to v_5. Then, we can assume that v_1 is in V_1 and v_5 is in V_2. From this statement, we can conclude that $v_6, v_3, v_4, v_8, v_{10}, v_9$ are in V_1 and v_2, v_7 are in V_2 which suitable $V_1 \cap V_2 = \emptyset$.

∘) Therefore, graph II is bipartite

∘)



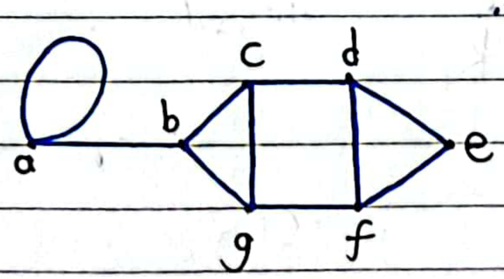
∘) Disjoint vertex sets :

$V_1 = \{v_1, v_3, v_4, v_6, v_8, v_9, v_{10}\}$

$V_2 = \{v_2, v_5, v_7\}$

Practice 3.

1. Find all the simple cycles and simple paths (from a to e) in the following graph.



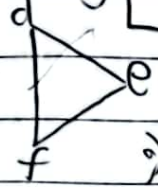
→ ∘) Simple paths (a to e) : (a, b, c, d, e)   (a, b, g, f, e)
(a, b, c, d, f, e)   (a, b, g, f, d, e)
(a, b, c, g, f, e)   (a, b, g, c, d, e)
(a, b, c, g, f, d, e)   (a, b, g, c, d, f, e)

c) Simple cycle : (a)
(a, b, a)
(b, c, g, b)
Same cycle with (b, c, g, b) { (c, b, g, c)
(g, c, b, g)

(c, d, f, g, c)
(d, f, g, c, d)
(f, g, c, d, f)
(g, c, d, f, g)

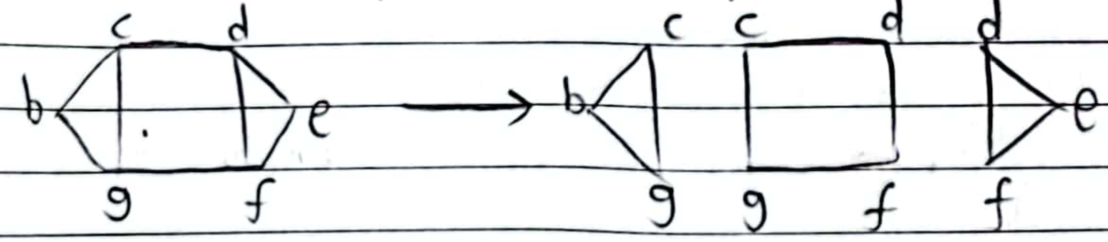
Same cycle with (c, d, f, g, c)

(d, e, f, d)
Same cycle with (d, e, f, d) { (e, d, f, e)
(f, e, d, f)

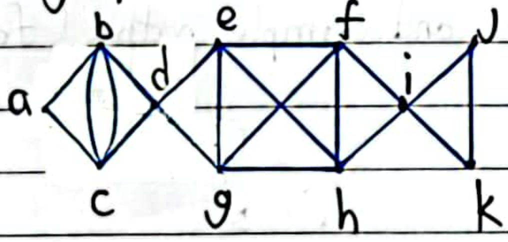


∘) Same cycle ≠ New cycle

∘) Note that (b, c, d, e, f, g, b) is not a simple cycle because it can seperate to 3 parts such as :



2. Decide whether the graph below has an Euler cycle. If the graph has an Euler cycle, exhibit one.



→ ∘) $\delta(a) = \delta(k) = \delta(j) = 2$
∘) $\delta(b) = \delta(c) = \delta(d) = \delta(e) = \delta(g) = \delta(f) = \delta(h) = \delta(i) = 4$
∘) Since the degree of every vertex is even, then we can conclude that the graph has an Euler cycle.
∘) Exhibition of Euler cycle = ijkifedbcbacdgfheghi

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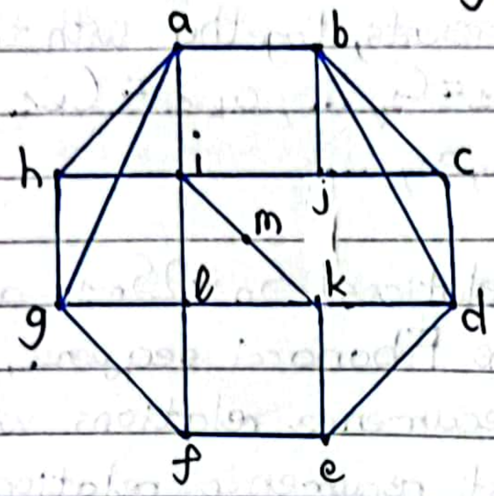
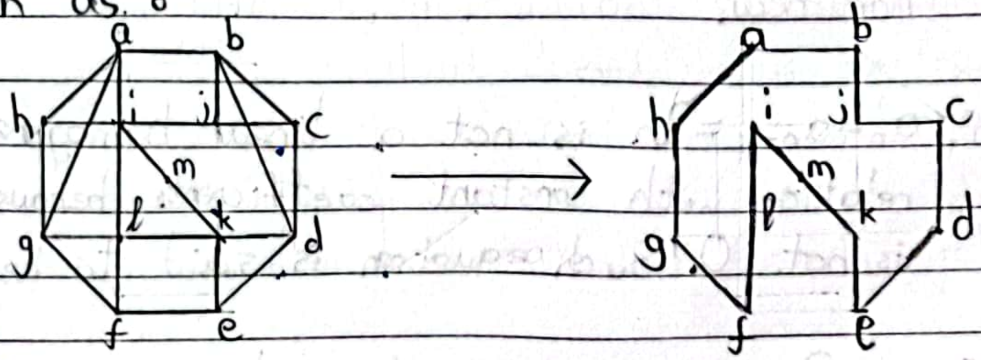
Practice 4

1. Determine whether or not the graph below contains a Hamiltonian cycle. If there is a Hamiltonian cycle, exhibit it; otherwise, give an argument that shows there is no Hamiltonian cycle.



→ ∘) The graph above contains Hamiltonian cycle. In hamiltonian cycle, the vertex incident with another different vertex. If the vertex incident with the same vertex that has been passed through before, it isn't Hamiltonian cycle. Therefore, we can conclude that the Hamiltonian cycle of the graph above is (a, b, j, c, d, e, k, m, i, l, f, g, h, a). Then, from this cycle we can draw the Hamiltonian cycle such as :



Nama : Jackson Lawrence
NIM : 00000070612

No.
Date: Week 12

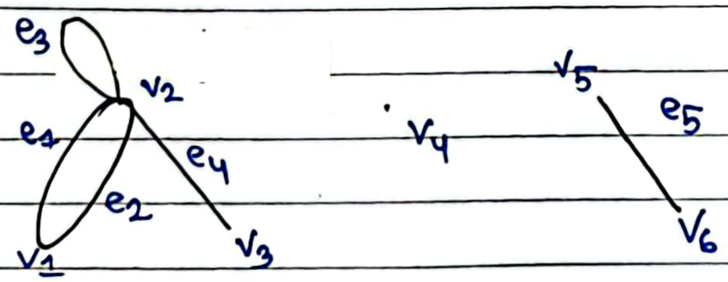
In graph, the dots are called vertices (nodes) and the lines that connect the vertices are called edges (arcs).

A graph / undirected graph G consists of a set V of vertices and a set E of edges such that each edge e associated with an unordered pair of vertices. If there is a unique edge e associated with the vertices v and w, then $e = (v,w) \vee e = (w,v)$.

A digraph / directed graph G consists of a set V of vertices and a set E of edges such that each edge e is associated with an ordered pair of vertices. If there is a unique edge e associated with the ordered pair (v,w) of vertices, then $e = (v,w)$ denotes an edge from v to w.

An edge e in a graph / digraph that is associated with the pair of vertices v and w is said to be incident on v and w, and v and w are said to be incident on e and to be adjacent vertices.
saling koneksi
edge dgn vertex

If G a graph / digraph with vertices V and E, then $G = (V, E)$. Unless specified otherwise, the sets E and V are assumed to be finite and V is assumed to be nonempty.



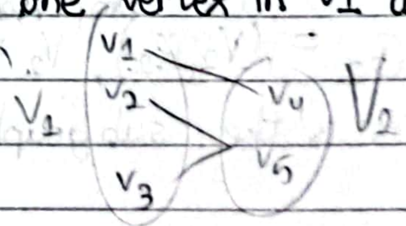
- e_1 and e_2 are associated with v_1 and v_2. Such edges are called parallel edges.

- An edge incident on a single vertex is called loop, like e_3.
- A vertex, such as vertex V_4 is called isolated vertex because doesn't incident on any edge.
- Graph with neither loops ncr parallel edges is called simple graph.

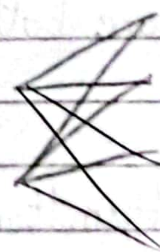
A graph with numbers on the edges is called weighted graph. If edge e is labeled k, then the weight of edge e is k. In weighted graph, the length of a path is the sum of the weights of the edges in the path.

The complete graph on n vertices, denoted K_n, is the simple graph with n vertices in which there is an edge between every pair of distinct vertices.

A graph $G = (V, E)$ is bipartite if there exist subsets V_1 and V_2 of V such that $V_1 \cap V_2 = \phi$, $V_1 \cup V_2 = V$, and each edge in E is incident on one vertex in V_1 and one in V_2. Dapat dibagi menjadi 2 bagian.

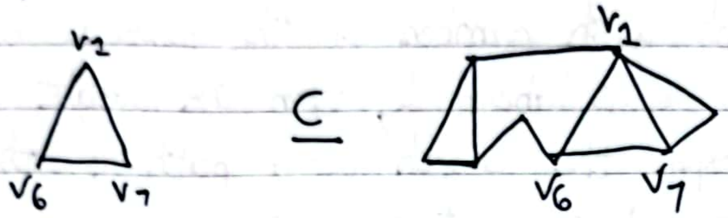


The complete bipartite graph on m and n vertices, denoted $K_{m,n}$, is the simple graph whose vertex set is partitioned into sets V_1 with m vertices and V_2 with n vertices in which the edge consists of all edges of the form (v_1, v_2) with $v_1 \in V_1$ and $v_2 \in V_2$. e.g.:

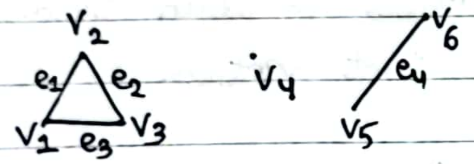


Let $G = (V, E)$ be a graph. We call (V', E') a subgraph of G if :

1) $V' \subseteq V$ and $E' \subseteq E$
2) For every edge $e' \in E'$, if e' is incident on v' and w', then $v', w' \in V'$.



Let G be the graph. The component of G containing v_3 is the subgraph.



$G_1 = (V_1, E_1)$
$V_1 = \{v_1, v_2, v_3\}$
$E_1 = \{e_1, e_2, e_3\}$

The component of G containing v_4 is the subgraph $G_2 = (V_2, E_2)$, $V_2 = \{v_4\}$, $E_2 = \emptyset$. The component of G containing v_5 is the subgraph $G_3 = (V_3, E_3)$, $V_3 = \{v_5, v_6\}$, $E_3 = \{e_4\}$

Let v and w be vertices in a graph G. A simple path from v to w is a path with no repeated vertices. A cycle/circuit is a path of nonzero length from v to v with no repeated edges. A simple cycle is a cycle from v to v in which, except for the beginning and ending vertices that are both equal to v, there are no repeated vertices.

hilang dr gansnya
Inti dari Euler cycle adlh degreenya genap. Inti dari Hamiltonian cycle adlh melewati semua vertex tanpa hrs ulang.

$\delta(v) = 4$.

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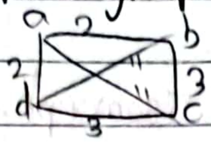
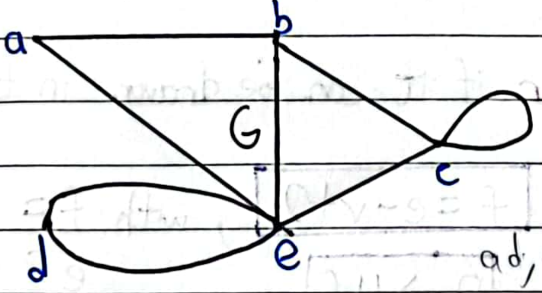
- The traveling salesperson problem is related to the problem of finding a Hamiltonian cycle in a graph. The problem is that given a weighted graph G, find a minimum-length Hamiltonian cycle in G.



$(a,b,c,d,e) \rightarrow$ minimum dr awal ke akhir dgn tdk sama

Week 13

There are some other methods to represent a graph besides by drawing it, such as by using adjacency matrix and incidence matrix. To obtain the adjacency matrix of this graph, first select an ordering of the vertices, say a,b,c,d,e. Next, label the rows and columns of a matrix with the ordered vertices. The entry in this matrix in row i, column j, $i \neq j$, is the number of edges incident on i and j. If $i = j$, the entry is twice the number of loops incident on i.

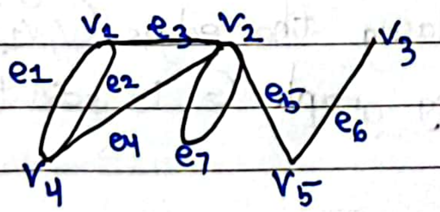


$\rightarrow$

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 \\ 1 & 1 & 1 & 2 & 0 \end{pmatrix} \end{array}$$

adjacency

We can obtain the degree of vertex in a graph G by summing row v or column v in G's adjacency matrix.



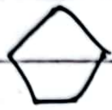
$\rightarrow$

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

$\wedge$ is the first utk diproses kecuali tanda kurung aja

x and $\bar{x}$ are literal

A graph without loops each column has 2 1's and that the sum of a row gives the degree of the vertex identified with that row.

 &  are isomorphic.

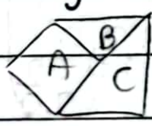
G_1 G_2

↳ di matriks nya sama $G_1 = G_2$
↳ perempatan garis kolom sama

G_1 and G_2 are isomorphic if there is a one-to-one, onto function f from the vertices of G_1 to the vertices of G_2 and a one-to-one, onto function g from the edges of G_1 to the edges of G_2, so that an edge e is incident on v and w in G_1 if and only if the edge $g(e)$ is incident on $f(v)$ and $f(w)$ in G_2. The pair of functions f and g is called an isomorphism of G_1 onto G_2.

Not isomorphic = Invariant.
~~Jlh vertex degreenya beda~~

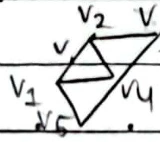
A graph is planar if it can be drawn in the plane without its edges crossing. Form :

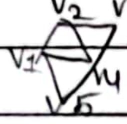


$\boxed{f = e - v + 2}$, with f = Faces
$\boxed{2e \geq 4f}$ e = Edges
v = Vertices

If a graph G has a vertex v of degree 2 and edges (v, v_1) and (v, v_2) with $v_1 \neq v_2$, then edges (v, v_1) and (v, v_2) are in series. A series reduction consists of deleting the vertex v from the graph G and replacing the edges (v, v_1) and (v, v_2) by the edge (v_1, v_2). The resulting graph G' is said to be obtained from G by a series reduction.

↗series reduction



& 

~~are series~~

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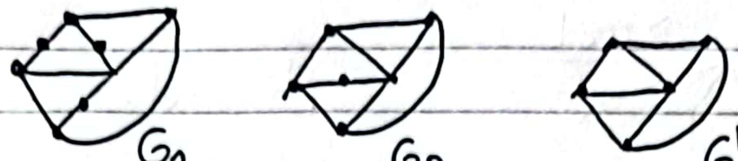
Graph G_1 and G_2 are homeomorphic if G_1 and G_2 can be reduced to isomorphic graphs by performing a sequence of series reduction.



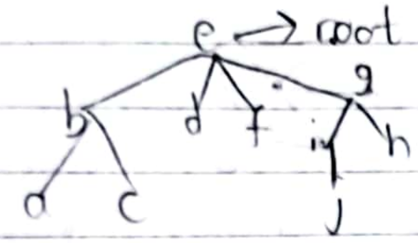
G_1 G_2 G'

G_1 & G_2 are homeomorphic since they can be reduced to the graph G by series reduc.

Pelimnasi sampai ga ada parallel edges & minimalisir edges/vertex

Kuratowski's Theorem : A graph G is planar if and only if G doesn't contain a subgraph homeomorphic to K_5 or $K_{3,3}$

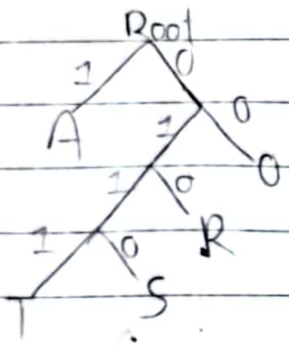
A tree T is a simple graph satisfying the following, if v and w were vertices in T, there is a unique simple path from v to w. A rooted tree is a tree in which a particular vertex is designated the root.



a c j h level 2   b d f g level 1

j level 3   e level 0

Huffman Codes



010 = R

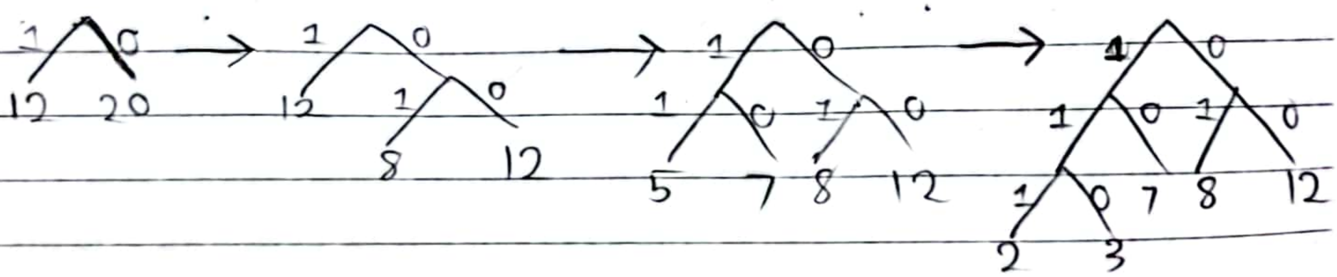
010100 = RAO

2, 3, 7, 8, 12 → 2 + 3, 7, 8, 12   Misal ada 2

5, 7, 8, 12 → 5 + 7, 8, 12   3

8, 12, 12 → 8 + 12, 12   ~~7~~ 8

12, 20   12



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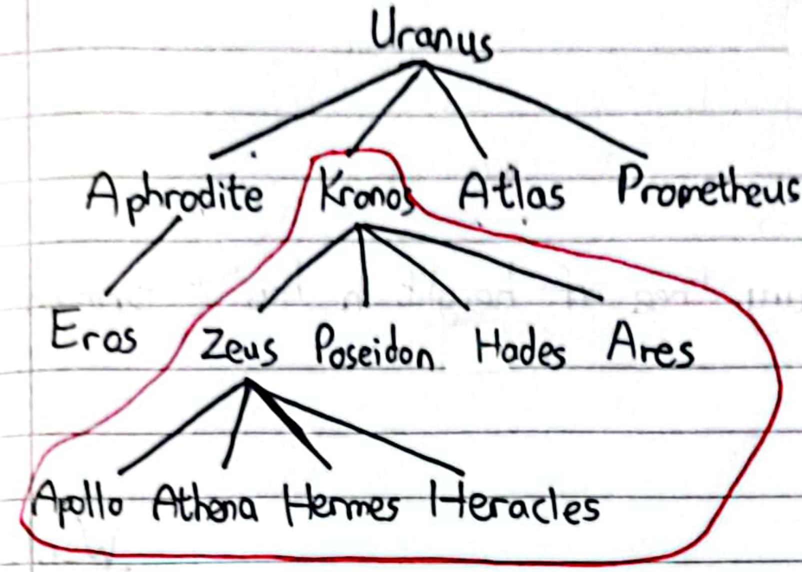
Trees' Terminologies



- The parent of Eros is Aphrodite.
- The ancestors of Hermes are Zeus, Kronos, and Uranus
- The children of Zeus are Apollo, Athena, Hermes, and Heracles
- The descendents of Kronos are Zeus, Poseidon, Hades, Ares, Apollo, Athena, Hermes, and Heracles.
- Aphrodite and Prometheus are siblings.
- The terminal vertices (no children, leaf) are Eros, Apollo, Athena, Hermes, Heracles, Poseidon, Hades, Ares, Atlas, and Prometheus
- The internal vertices (branch) are Uranus, Aphrodite, Kronos, and Zeus.
- The subtree rooted at Kronos is dilingkar

A graph with no cycles is called an acyclic graph. A tree is a connected, acyclic graph. The converse is also true, every connected, acyclic graph is a tree. Let T be a graph with n vertices. The following are equivalent.

- T is a tree
- T is connected and acyclic
- T is connected and has n-1 edges
- T is acyclic and has n-1 edges.

BOSS

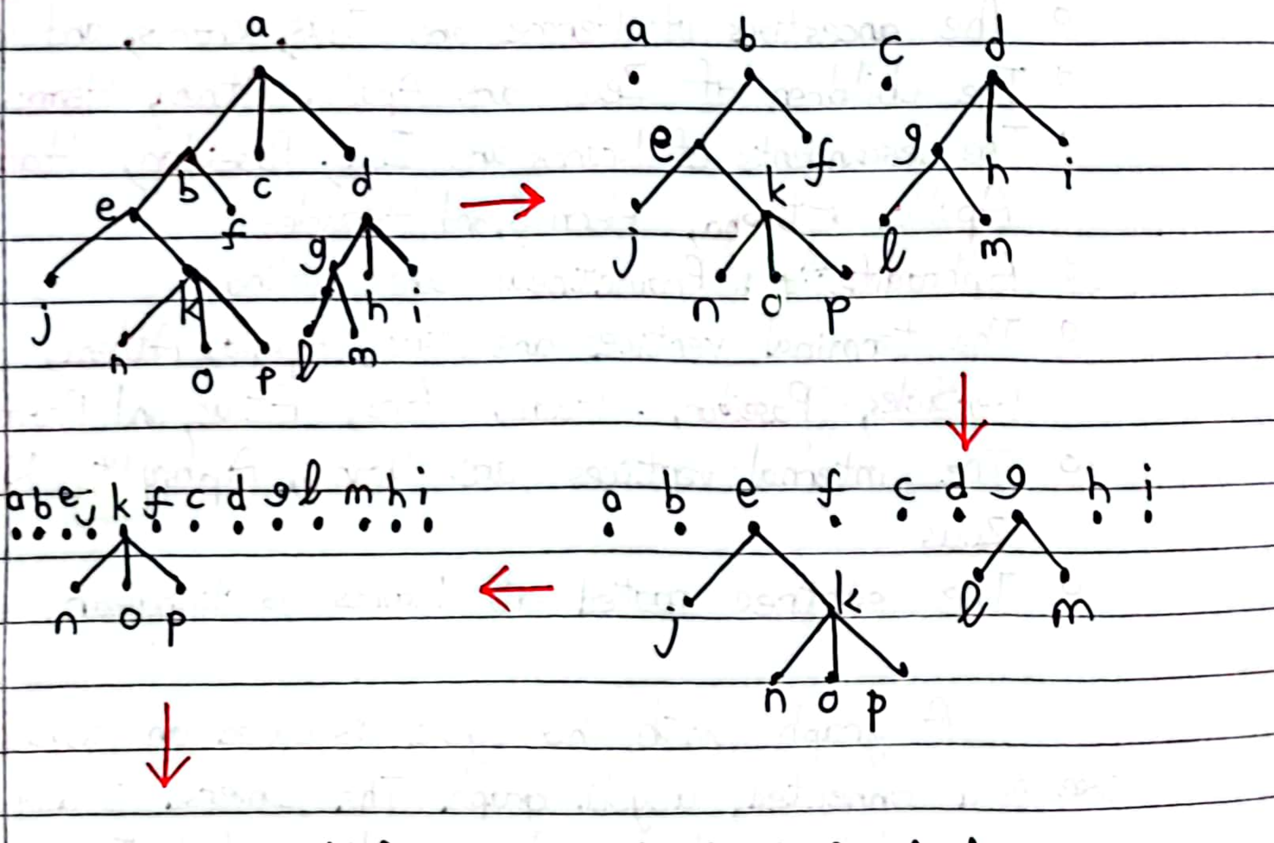
A tree T is a spanning tree of a graph G if T is a subgraph of G that contains all of the vertices of G.

Breath-first search → cari utama terus cari anaknya dan incidentnya

If a binary tree of height h has t terminal vertices, then $\boxed{{}^{2}\log t \leq h}$

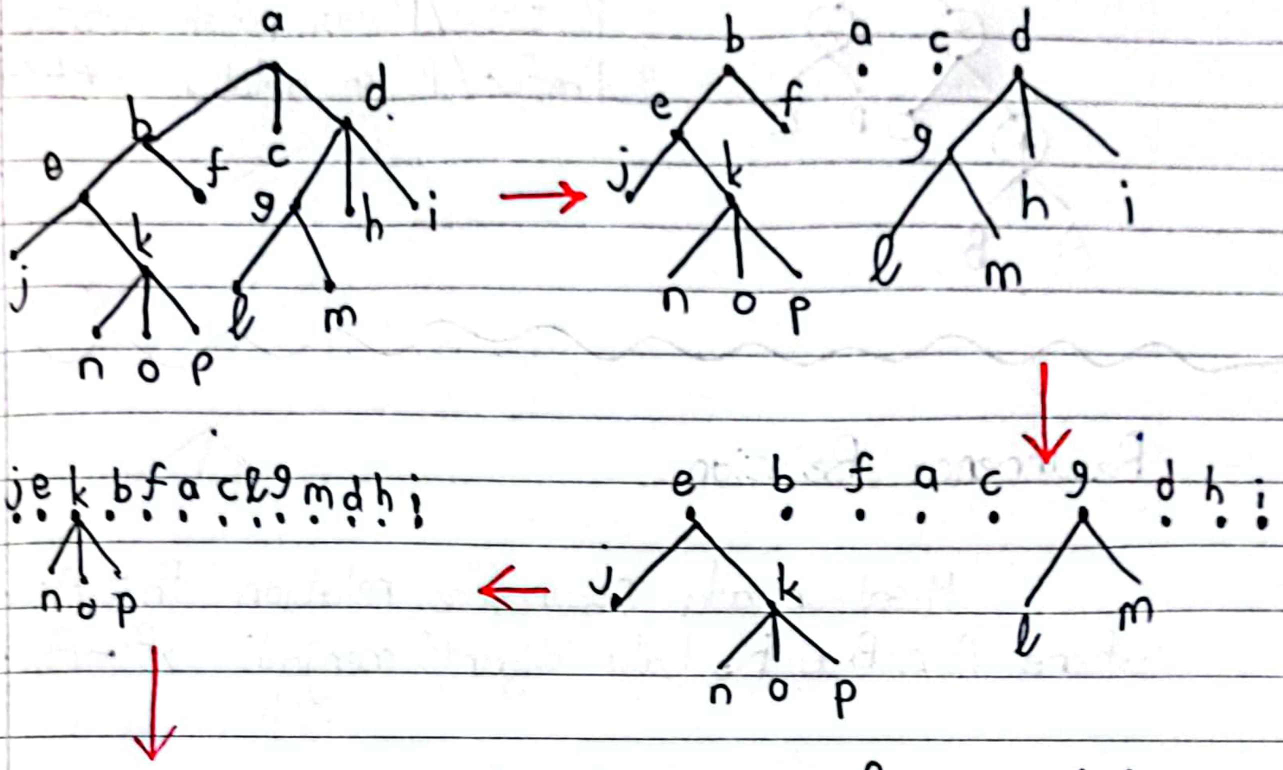
Week 15

A. Preorder Traversal



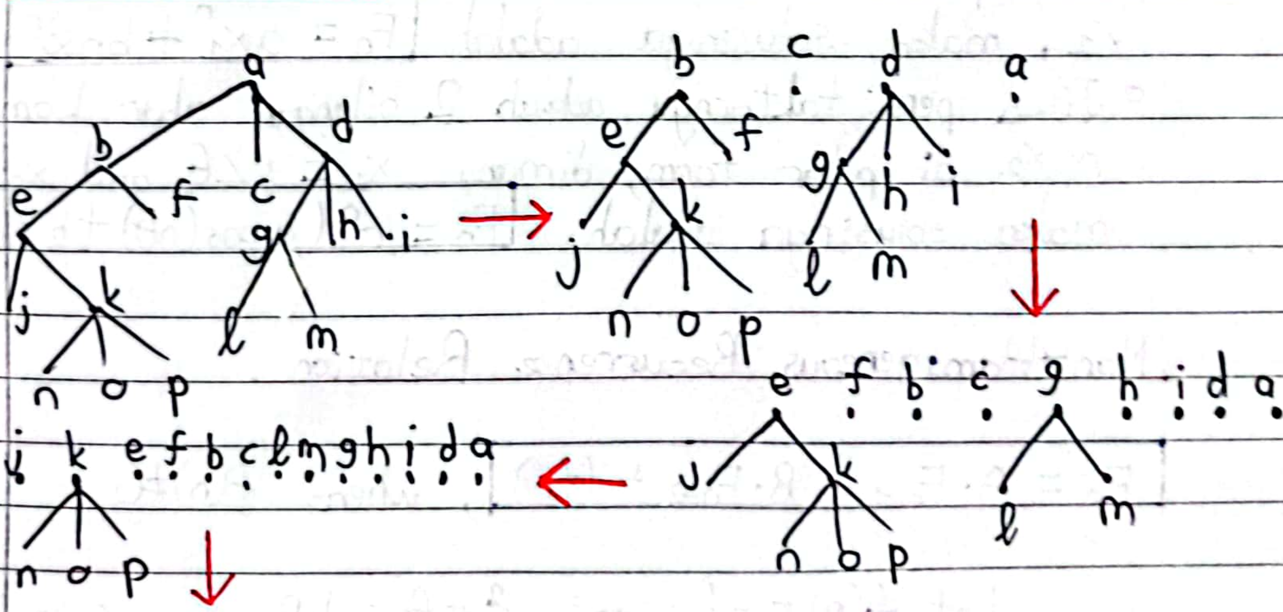
a b e j k n o p f c d g l m h i

B. Inorder Traversal



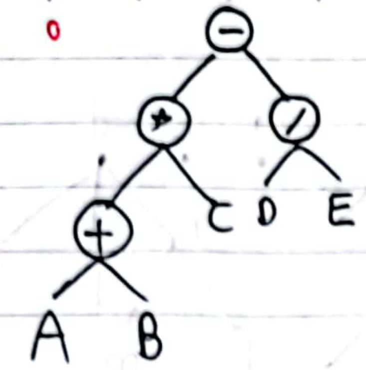
j e n k o p b f a c l g m d h i

C. Postorder Traversal



j n o p k e f b c l m g h i d a

No.
Date:



- Fully parenthesized : $(((A+B)*C)-(D/E))$
- Postfix/Reverse polish notation : $AB+C*DE/-$
- Prefix/Polish notation : $-*+ABC/DE$

Recurrence Relation

Misalkan ada recurrence relation $F_n = A\cdot F_{n-1} + B\cdot F_{n-2}$, where $A \wedge B \in \mathbb{R}$. Lalu diganti menjadi $x^2 - Ax - B = 0$

Ibarat $F_n - A\cdot F_{n-1} - BF_{n-2} = 0$

- Jika pers. faktornya $(x-x_1)(x-x_2)=0$, maka akar-akarnya $x_1 \wedge x_2$, maka solusinya adalah $\boxed{F_n = ax_1^n + bx_2^n}$
- Jika pers. faktornya $(x-x_1)^2=0$, maka akarnya adalah x_1, maka solusinya adalah $\boxed{F_n = ax_1^n + bnx_1^n}$
- Jika pers. faktornya adalah 2 bilangan akar kompleks, $x_1 \wedge x_2$ di polar form, dimana $x_1 = r\angle\theta$ and $x_2 = r\angle(-\theta)$, maka solusinya adalah $\boxed{F_n = r^n(a\cos(n\theta) + b\sin(n\theta))}$

Non-Homogeneous Recurrence Relation

$\boxed{F_n = A\cdot F_{n-1} + B\cdot F_{n-2} + f(n)}$, where $f(n) \neq 0$

Let $f(n) = cx^n$ and $x^2 = Ax + B$, $x_1 \wedge x_2$ roots, then:

- If $x \neq x_1$ and $x \neq x_2$, then $a_t = Ax^n$
- If $x = x_2$, $x \neq x_2$, then $a_t = Anx^n$
- If $x = x_1 = x_2$, then $a_t = An^2x^n$

Contd :

1. Solve the recurrence relation $F_n = 3F_{n-1} + 10F_{n-2} + 7.5^n$, where $F_0 = 4$ and $F_1 = 3$.

∘) $f(n) = 7.5^n$

∘) $x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = 5 \lor x = -2$

∘) $a_h = a \cdot 5^n + b(-2)^n$

∘) $a_t = A_n x^n = 7.5^n = A_n 5^n$

∘) $An5^n = 3A(n-1) \cdot 5^{n-1} + 10A(n-2)\, 5^{n-2} + 7.5^n$

$An5^2 = 3 \cdot A(n-1) \cdot 5 + 10A(n-2)\, 5^0 + 7.5^2$

$25An = 15An - 15A + 10An - 20A + 175$

$A = 5$

∘) $F_n = An5^n = 5n5^n = n \cdot 5^{n+1}$

∘) $F_n = a_h + a_t$

$= a \cdot 5^n + b(-2)^n + n5^{n+1}$

∘) $F_0 = 4$

$4 = a + b$

∘) $F_1 = 3$

$3 = 5a - 2b + 25$

$-22 = 5a - 2b$

∘) $2a + 2b = 8$

$5a - 2b = -22$ +

$7a = -14$

$a = -2, b = 6$

∘) ∴ $F_n = -2 \cdot 5^n + 6(-2)^n - 2.5^n$

NB :

$f(n)$	Trial Solutions
4	A
$5 \cdot 2^n$	$An2^n$
$8 \cdot 5^n$	$An \cdot 5^n$
4^n	$A \cdot 4^n$
$2n^2 + 3n + 1$	$An^2 + Bn + C$

No.
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Teorema Bayes

Rumus : $\boxed{P(A|B) = \frac{P(A) \cdot P(B|A)}{P(B)}}$, dengan

- $P(A|B)$ = Seberapa sering A terjadi setelah B terjadi
- $P(B|A)$ = Seberapa sering B terjadi setelah A terjadi
- $P(A)$ = Seberapa besar kemungkinan A
- $P(B)$ = Seberapa besar kemungkinan B

Contoh Soal :

1. Kebakaran berbahaya krn ada api jarang terjadi = 1%
Munculnya Asap cukup umum terjadi = 10%
Kebakaran berbahaya menghasilkan Asap = 90%
Kemungkinan Api berbahaya ketika ada Asap = ?

→ $P(Api|Asap) = \frac{P(Api) \cdot P(Asap|Api)}{P(Asap)} = \frac{1\% \cdot 90\%}{10\%} = 0{,}09$

2. 50% hari hujan dimulai dengan hari mendung / berawan, namun pagi hari berawan sering terjadi (40%), dan bulan ini bukan bulan penghujan (Hanya 3 dari 30 hari cenderung turun hujan -10%-). Berapa peluang hujan siang hari ini?

→ $P(Hujan|Awan) = \frac{P(Hujan) \cdot P(Awan|Hujan)}{P(Awan)} = \frac{10\% \cdot 50\%}{40\%}$
$= 0{,}125$

No.
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3. Terdapat 100 orang dalam sebuah pesta. Anda akan menghitung berapa banyak orang yang memakai baju pink dan bukan pink, dan apakah pria atau bukan berdasarkan angka berikut.

	Pink	NotPink
Man	5	35
NotMan	20	40

→ ◦ Total Man = 40 ◦ Total Pink = 25
◦ Total NotMan = 60 ◦ Total NotPink = 75
◦ $P(Man) = \frac{40}{100} = 0,4$ ◦ $P(Pink) = \frac{25}{100} = 0,25$
◦ $P(Pink \mid Man) = \frac{5}{40} = 0,125$ ◦ Peluang orang yang memakai pink adalah pria
$= P(Man \mid Pink) = \frac{P(Man) \cdot P(Pink \mid Man)}{P(Pink)}$

$= \frac{0,4 \cdot 0,125}{0,25} = 0,2$

4. Kompetisi seni diikuti oleh 3 pelukis, Adi, Rea, Ray.
Adi memasukkan 15 lukisan, 4% karyanya telah win first prize
Dea memasukkan 5 lukisan, 6% karyanya telah win first prize
Ray memasukkan 10 lukisan, 3% karyanya telah win first prize.
Berapa peluang win first prize?

→ ◦ $P(Adi \mid Pertama) = \frac{P(Adi) \cdot P(Pertama \mid Adi)}{P(Adi) \cdot P(Pertama \mid Adi) + P(Dea) \cdot P(Pertama \mid Dea) + P(Ray) \cdot P(Pertama \mid Ray)}$

$= \frac{\frac{15}{30} \cdot 4\%}{\frac{15}{30} \cdot 4\% + \frac{5}{30} \cdot 6\% + \frac{10}{30} \cdot 3\%}$

$= \frac{0,6}{0,6 + 0,3 + 0,3}$

$= 0,5$

Bass

Date:

5. 1% wanita menderita kanker payudara (Brti 99% tidak)
80% mammogram mampu mendeteksi kanker payudara atas semua pasien penderita kanker payudara (Brti 20% terlewatkan oleh mammogram)
9,6% mammogram mendeteksi kanker payudara ketika pasien sebenarnya tidak menderita kanker payudara (Brti 90,4% mammogram mendeteksi pasien yang tidak menderita kanker payudara dengan benar dengan hasil negatif)
Berapa peluang fix kena kanker?

c)

	Kanker (1%)	No kanker (99%)
Tes positif	80%	9,6%
Tes negatif	20%	90,4%

c) $P = \dfrac{80\% \cdot 1\%}{80\% \cdot 1\% + 9{,}6\% \cdot 99\%} = \dfrac{0{,}008}{0{,}008 + 0{,}09504} = \dfrac{0{,}008}{0{,}10304} \approx 0{,}0776$