

1.0) Gauss Elimination

$$\begin{aligned}3x_1 - x_2 + 4x_3 &= 2 \\17x_1 + 2x_2 + x_3 &= 14 \\x_1 + 12x_2 - 7x_3 &= 54\end{aligned}$$

Step 1: Turn equations to matrix form $Ax = y$

$$\begin{aligned}3x_1 - x_2 + 4x_3 &= 2 \\17x_1 + 2x_2 + x_3 &= 14 \\x_1 + 12x_2 - 7x_3 &= 54\end{aligned} \rightarrow \begin{bmatrix}3 & -1 & 4 \\ 17 & 2 & 1 \\ 1 & 12 & -7\end{bmatrix} \begin{bmatrix}x_1 \\ x_2 \\ x_3\end{bmatrix} = \begin{bmatrix}2 \\ 14 \\ 54\end{bmatrix}$$

Step 2: Get the augmented matrix

$$\begin{bmatrix}3 & -1 & 4 \\ 17 & 2 & 1 \\ 1 & 12 & -7\end{bmatrix} \begin{bmatrix}x_1 \\ x_2 \\ x_3\end{bmatrix} = \begin{bmatrix}2 \\ 14 \\ 54\end{bmatrix} \rightarrow [A, y] = \begin{bmatrix}3 & -1 & 4 & 2 \\ 17 & 2 & 1 & 14 \\ 1 & 12 & -7 & 54\end{bmatrix}$$

Step 3: 2nd row first element to 0

$$\begin{bmatrix}3 & -1 & 4 & 2 \\ 17 & 2 & 1 & 14 \\ 1 & 12 & -7 & 54\end{bmatrix} \xrightarrow{m_{2,1} = \frac{17}{3}} \begin{bmatrix}3 & -1 & 4 & 2 \\ 0 & \frac{23}{3} & -\frac{65}{3} & \frac{8}{3} \\ 1 & 12 & -7 & 54\end{bmatrix}$$

Step 4: 3rd row first element to 0

$$\begin{bmatrix}3 & -1 & 4 & 2 \\ 0 & \frac{23}{3} & -\frac{65}{3} & \frac{8}{3} \\ 1 & 12 & -7 & 54\end{bmatrix} \xrightarrow{m_{3,1} = \frac{1}{3}} \begin{bmatrix}3 & -1 & 4 & 2 \\ 0 & \frac{23}{3} & -\frac{65}{3} & \frac{8}{3} \\ 0 & 3 & -2 & \frac{50}{3}\end{bmatrix}$$

Step 5: 3rd row second element to 0

$$\begin{bmatrix}3 & -1 & 4 & 2 \\ 0 & \frac{23}{3} & -\frac{65}{3} & \frac{8}{3} \\ 0 & 3 & -2 & \frac{50}{3}\end{bmatrix} \xrightarrow{m_{3,2} = \frac{37}{23}} \begin{bmatrix}3 & -1 & 4 & 2 \\ 0 & \frac{23}{3} & -\frac{65}{3} & \frac{8}{3} \\ 0 & 0 & \frac{183}{69} & \frac{3384}{69}\end{bmatrix}$$

Step 6 : Obtained x_3

$$x_3 = \frac{3384}{69} = \frac{3384}{1830} = \frac{564}{305}$$

Step 7 : Insert x_3 to 2nd equation

$$\left[\begin{array}{cccc|c} 3 & -1 & 4 & 2 \\ 0 & \frac{23}{3} & -\frac{65}{3} & \frac{8}{3} \\ 0 & 0 & \frac{1830}{69} & \frac{3384}{69} \end{array} \right] \rightarrow \begin{cases} \left(\frac{23}{3} x_2 - \frac{65}{3} x_3 = \frac{8}{3} \right) \cdot 3 \\ 23x_2 - 65 \cdot \frac{564}{305} = 8 \end{cases} \cdot 305$$
$$7015x_2 - 36660 = 2440$$
$$7015x_2 = 39100$$
$$x_2 = \frac{39100}{7015} = \frac{340}{61}$$

Step 8 : Insert x_2 & x_3 to 1st equation

$$\left[\begin{array}{cccc|c} 3 & -1 & 4 & 2 \\ 0 & \frac{23}{3} & -\frac{65}{3} & \frac{8}{3} \\ 0 & 0 & \frac{1830}{69} & \frac{3384}{69} \end{array} \right] \rightarrow 3x_1 - x_2 + 4x_3 = 2$$
$$\left(3x_1 - \frac{340}{61} + 4 \cdot \frac{564}{305} = 2 \right) \cdot 61 \cdot 305$$
$$55815x_1 - 103700 + 137616 = 37210$$
$$55815x_1 = 3294$$
$$x_1 = \frac{3294}{55815} = \frac{18}{305}$$

$$\therefore x_1 = \frac{18}{305} \approx 0.059$$

$$x_2 = \frac{340}{61} \approx 5.574$$

$$x_3 = \frac{564}{305} \approx 1.849$$

2.0) Gauss - Jordan Elimination

$$\begin{aligned} 3x_1 - x_2 + 4x_3 &= 2 \\ 17x_1 + 2x_2 + x_3 &= 14 \\ x_1 + 12x_2 - 7x_3 &= 54 \end{aligned}$$

Step 1: Turn equations to matrix form $Ax = y$

$$\begin{aligned} 3x_1 - x_2 + 4x_3 &= 2 \\ 17x_1 + 2x_2 + x_3 &= 14 \\ x_1 + 12x_2 - 7x_3 &= 54 \end{aligned} \rightarrow \begin{bmatrix} 3 & -1 & 4 & | & 2 \\ 17 & 2 & 1 & | & 14 \\ 1 & 12 & -7 & | & 54 \end{bmatrix}$$

Step 2: Get the augmented matrix

$$\begin{bmatrix} 3 & -1 & 4 & | & 2 \\ 17 & 2 & 1 & | & 14 \\ 1 & 12 & -7 & | & 54 \end{bmatrix} \rightarrow [A, y] = \begin{bmatrix} 3 & -1 & 4 & 2 \\ 17 & 2 & 1 & 14 \\ 1 & 12 & -7 & 54 \end{bmatrix}$$

Step 3: First element in 1st row to 1

Pivot \rightarrow

$$\begin{bmatrix} 3 & -1 & 4 & 2 \\ 17 & 2 & 1 & 14 \\ 1 & 12 & -7 & 54 \end{bmatrix} \xrightarrow{m_1 = \frac{1}{3}} \begin{bmatrix} 1 & -\frac{1}{3} & \frac{4}{3} & \frac{2}{3} \\ 17 & 2 & 1 & 14 \\ 1 & 12 & -7 & 54 \end{bmatrix}$$

Step 4: 2nd row and 3rd row first element to 0

$$\begin{bmatrix} 1 & -\frac{1}{3} & \frac{4}{3} & \frac{2}{3} \\ 17 & 2 & 1 & 14 \\ 1 & 12 & -7 & 54 \end{bmatrix} \xrightarrow{\begin{array}{l} m_{2,1} = 17 \\ m_{3,1} = 1 \end{array}} \begin{bmatrix} 1 & -\frac{1}{3} & \frac{4}{3} & \frac{2}{3} \\ 0 & \frac{32}{3} & \frac{19}{3} & \frac{40}{3} \\ 0 & \frac{37}{3} & -\frac{25}{3} & \frac{150}{3} \end{bmatrix}$$

Step 5: 2nd element in 2nd row to 1

$$\begin{bmatrix} 1 & -\frac{1}{3} & \frac{4}{3} & \frac{2}{3} \\ 0 & \frac{32}{3} & \frac{19}{3} & \frac{40}{3} \\ 0 & \frac{37}{3} & -\frac{25}{3} & \frac{150}{3} \end{bmatrix} \xrightarrow{m_2 = \frac{3}{32}} \begin{bmatrix} 1 & -\frac{1}{3} & \frac{4}{3} & \frac{2}{3} \\ 0 & 1 & \frac{19}{32} & \frac{40}{32} \\ 0 & \frac{37}{3} & -\frac{25}{3} & \frac{150}{3} \end{bmatrix}$$

Step 6: 3rd row second element to 0

$$\begin{bmatrix} 1 & -\frac{1}{3} & \frac{4}{3} & \frac{2}{3} \\ 0 & 1 & -\frac{65}{23} & \frac{23}{23} \\ 0 & \frac{37}{3} & -\frac{25}{3} & \frac{160}{3} \end{bmatrix} \xrightarrow{m_{3,2} = \frac{37}{3}} \begin{bmatrix} 1 & -\frac{1}{3} & \frac{4}{3} & \frac{2}{3} \\ 0 & 1 & -\frac{65}{23} & \frac{23}{23} \\ 0 & 0 & \frac{830}{69} & \frac{3384}{69} \end{bmatrix}$$

Step 7: 3rd row 3rd element to 1

$$\begin{bmatrix} 1 & -\frac{1}{3} & \frac{4}{3} & \frac{2}{3} \\ 0 & 1 & -\frac{65}{23} & \frac{23}{23} \\ 0 & 0 & \frac{1830}{69} & \frac{3384}{69} \end{bmatrix} \xrightarrow{m_3 = \frac{69}{1830}} \begin{bmatrix} 1 & -\frac{1}{3} & \frac{4}{3} & \frac{2}{3} \\ 0 & 1 & -\frac{65}{23} & \frac{23}{23} \\ 0 & 0 & 1 & \frac{3384}{1830} \end{bmatrix}$$

Step 8: 3rd element in 2nd row to 0

$$\begin{bmatrix} 1 & -\frac{1}{3} & \frac{4}{3} & \frac{2}{3} \\ 0 & 1 & -\frac{65}{23} & \frac{23}{23} \\ 0 & 0 & 1 & \frac{564}{305} \end{bmatrix} \xrightarrow{m_{2,3} = -\frac{65}{23}} \begin{bmatrix} 1 & -\frac{1}{3} & \frac{4}{3} & \frac{2}{3} \\ 0 & 1 & 0 & \frac{39100}{7015} \\ 0 & 0 & 1 & \frac{564}{305} \end{bmatrix}$$

Step 9: 3rd element in 1st row to 0

$$\begin{bmatrix} 1 & -\frac{1}{3} & \frac{4}{3} & \frac{2}{3} \\ 0 & 1 & 0 & \frac{39100}{7015} \\ 0 & 0 & 1 & \frac{564}{305} \end{bmatrix} \xrightarrow{m_{1,3} = \frac{4}{3}} \begin{bmatrix} 1 & -\frac{1}{3} & 0 & -\frac{1646}{15} \\ 0 & 1 & 0 & \frac{340}{564} \\ 0 & 0 & 1 & \frac{564}{305} \end{bmatrix}$$

Step 10: 2nd element in 1st row to 0

$$\begin{bmatrix} 1 & -\frac{1}{3} & 0 & -\frac{1646}{15} \\ 0 & 1 & 0 & \frac{340}{564} \\ 0 & 0 & 1 & \frac{564}{305} \end{bmatrix} \xrightarrow{m_{1,2} = -\frac{1}{3}} \begin{bmatrix} 1 & 0 & 0 & \frac{9882}{167445} \\ 0 & 1 & 0 & \frac{340}{564} \\ 0 & 0 & 1 & \frac{564}{305} \end{bmatrix}$$

$$x_1 = \frac{9882}{167445} = \frac{18}{305} \approx 0.059$$

$$x_2 = \frac{340}{61} \approx 5.574$$

$$x_3 = \frac{564}{305} \approx 1.849$$

3c Lower triangular matrix L & Upper triangular matrix U

$$\begin{array}{l} \text{Q) } \begin{aligned} 3x_1 - x_2 + 4x_3 &= 2 \\ 17x_1 + 2x_2 + x_3 &= 14 \\ x_1 + 12x_2 - 7x_3 &= 54 \end{aligned} \rightarrow \begin{bmatrix} 3 & -1 & 4 \\ 17 & 2 & 1 \\ 1 & 12 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 14 \\ 54 \end{bmatrix} \end{array}$$

$$[A, y] = \begin{bmatrix} 3 & -1 & 4 & 2 \\ 17 & 2 & 1 & 14 \\ 1 & 12 & -7 & 54 \end{bmatrix}$$

$$\text{Q) } \begin{bmatrix} 3 & -1 & 4 & 2 \\ 17 & 2 & 1 & 14 \\ 1 & 12 & -7 & 54 \end{bmatrix} \xrightarrow{R_2 - \left(\frac{17}{3}\right)R_1} \begin{bmatrix} 3 & -1 & 4 & 2 \\ 0 & \frac{19}{3} & \frac{13}{3} & \frac{8}{3} \\ 1 & 12 & -7 & 54 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 & 4 & 2 \\ 0 & \frac{19}{3} & \frac{13}{3} & \frac{8}{3} \\ 1 & 12 & -7 & 54 \end{bmatrix} \xrightarrow{R_3 - \left(\frac{1}{3}\right)R_1} \begin{bmatrix} 3 & -1 & 4 & 2 \\ 0 & \frac{19}{3} & \frac{13}{3} & \frac{8}{3} \\ 0 & \frac{37}{3} & \frac{4}{3} & \frac{50}{3} \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 & 4 & 2 \\ 0 & \frac{19}{3} & \frac{13}{3} & \frac{8}{3} \\ 0 & \frac{37}{3} & \frac{4}{3} & \frac{50}{3} \end{bmatrix} \xrightarrow{R_3 - \left(\frac{37}{23}\right)R_2} \begin{bmatrix} 3 & -1 & 4 & 2 \\ 0 & \frac{19}{3} & \frac{13}{3} & \frac{8}{3} \\ 0 & 0 & \frac{12}{3} & \frac{183}{69} \end{bmatrix}$$

$$\text{so Upper (U)} = \begin{bmatrix} 3 & -1 & 4 & 2 \\ 0 & \frac{19}{3} & \frac{13}{3} & \frac{8}{3} \\ 0 & 0 & \frac{12}{3} & \frac{183}{69} \end{bmatrix}$$

$$\text{Lower (L)} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{17}{3} & 1 & 0 \\ \frac{1}{3} & \frac{37}{23} & 1 \end{bmatrix}$$