Practice – Week 05

Practice – 01

1. Write down the characteristic equation for matrix $A = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$.

Practice – 01 – Answer

Characteristic equation :

$$(A - \lambda I)x = 0$$

$$\downarrow$$

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} 3-\lambda & 2 \\ 5 & 3-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)(3-\lambda) - 10 = 0 \;\;\Rightarrow\;\; \lambda^2 - 6\lambda - 1 = 0$$

Practice – 02

2. Using the above characteristic equation to solve for eigenvalues and eigenvectors for matrix A.

Practice – 02 – Answer

Eigenvalue :

$$\lambda^2 - 6\lambda - 1 = 0$$

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{36 + 4}}{2} = 3 \pm \sqrt{10}$$

$$\lambda_1 = 3 + \sqrt{10}\,, \quad \lambda_2 = 3 - \sqrt{10}$$

Practice – 02 – Answer

Eigenvector :

$\lambda_1 = 3 + \sqrt{10} \quad \rightarrow$

$$\begin{bmatrix} 3-(3+\sqrt{10}) & 2 \\ 5 & 3-(3+\sqrt{10}) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\sqrt{10} & 2 \\ 5 & -\sqrt{10} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-\sqrt{10}x_1 + 2x_2 = 0, \quad 5x_1 - \sqrt{10}x_2 = 0$$

$$x_2 = \frac{\sqrt{10}}{2} x_1$$

$$\text{Eigen}vector\ 1: \ x = k_1 \begin{bmatrix} 1 \\ \frac{\sqrt{10}}{2} \end{bmatrix}$$

Practice – 02 – Answer

Eigenvector :

$\lambda_1 = 3 - \sqrt{10} \quad \rightarrow \quad \begin{bmatrix} 3-(3-\sqrt{10}) & 2 \\ 5 & 3-(3-\sqrt{10}) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} \sqrt{10} & 2 \\ 5 & \sqrt{10} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\sqrt{10}x_1 + 2x_2 = 0, \quad 5x_1 + \sqrt{10}x_2 = 0$$

$$x_2 = -\frac{\sqrt{10}}{2}x_1$$

$$\mathrm{Eigen}vector\ 2: \ x = k_1 \begin{bmatrix} 1 \\ -\frac{\sqrt{10}}{2} \end{bmatrix}$$

Practice – 03

3. Use the first eigenvector that derived from problem 2 to verify that $Ax = \lambda x$.

Practice – 03 – Answer

$$\text{Eigen}vector\ 1:\ x = k_1 \begin{bmatrix} 1 \\ \frac{\sqrt{10}}{2} \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ \frac{\sqrt{10}}{2} \end{bmatrix} = \begin{bmatrix} 3 + \sqrt{10} \\ 5 + 3.\frac{\sqrt{10}}{2} \end{bmatrix} = \left(3 + \sqrt{10}\right) \begin{bmatrix} 1 \\ \frac{\sqrt{10}}{2} \end{bmatrix}$$

$\Downarrow$

Ref.

Practice – 03 – Answer

Ref.

$$5+3.\frac{\sqrt{10}}{2}=\left(3+\sqrt{10}\right).\frac{5+3.\frac{\sqrt{10}}{2}}{3+\sqrt{10}}=\left(3+\sqrt{10}\right).\ \frac{5+3.\frac{\sqrt{10}}{2}}{3+\sqrt{10}}\ .\ \frac{3-\sqrt{10}}{3-\sqrt{10}}=$$

$$\left(3+\sqrt{10}\right).\ \frac{15-5\sqrt{10}+9.\frac{\sqrt{10}}{2}-3.\frac{10}{2}}{9-10}=\left(3+\sqrt{10}\right).\ \frac{-\frac{\sqrt{10}}{2}}{-1}=\left(3+\sqrt{10}\right).\frac{\sqrt{10}}{2}$$

Python Code – 1 : Eigenvalues & Eigenvectors

```
In [1]: import numpy as np
        from numpy.linalg import eig

        a = np.array([[3,2],
                      [5,3]])
        w,v = eig(a)

        print('Eigenvalue : ',w)
        print('Eigenvector : ',v)
```

```
Eigenvalue :  [ 6.16227766 -0.16227766]
Eigenvector :  [[ 0.53452248 -0.53452248]
 [ 0.84515425  0.84515425]]
```

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 [ 0.84515425  0.84515425]]
```

$$\lambda_1 = 3 + \sqrt{10} = 6.16227$$

$$\lambda_2 = 3 - \sqrt{10} = -0.16227$$

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 [ 0.84515425  0.84515425]]
```

$$x_1 = k_1 \begin{bmatrix} 1 \\ \frac{\sqrt{10}}{2} \end{bmatrix} = k_1 \begin{bmatrix} 1 \\ 1.58114 \end{bmatrix}$$

$$x_2 = k_2 \begin{bmatrix} 1 \\ -\frac{\sqrt{10}}{2} \end{bmatrix} = k_2 \begin{bmatrix} 1 \\ -1.58114 \end{bmatrix}$$

$$\frac{0.53452}{0.84515} = 1.58114 \qquad \frac{-0.53452}{0.84515} = -1.58114$$