Digital Signal Processing

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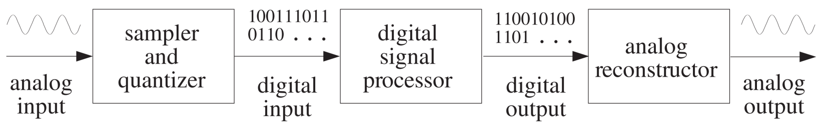
1.1 Introduction

A typical signal processing system includes 3 stages:



The analog signal is ***sampled*** and each sample is ***quantized*** to a finite number of bits (A/D converter). The digitalized samples are processed by a digital signal processor.

- The digital processor can be programmed to perform signal processing operations such as filtering, spectrum estimation.
- Digital signal processor can be a general purpose computer, DSP chip or other digital hardware.

The resulting output samples are converted back into analog by an ***analog reconstructor*** (D/A converter).

1.2 Analog to digital conversion

Analog to digital (A/D) conversion is a three-step process.

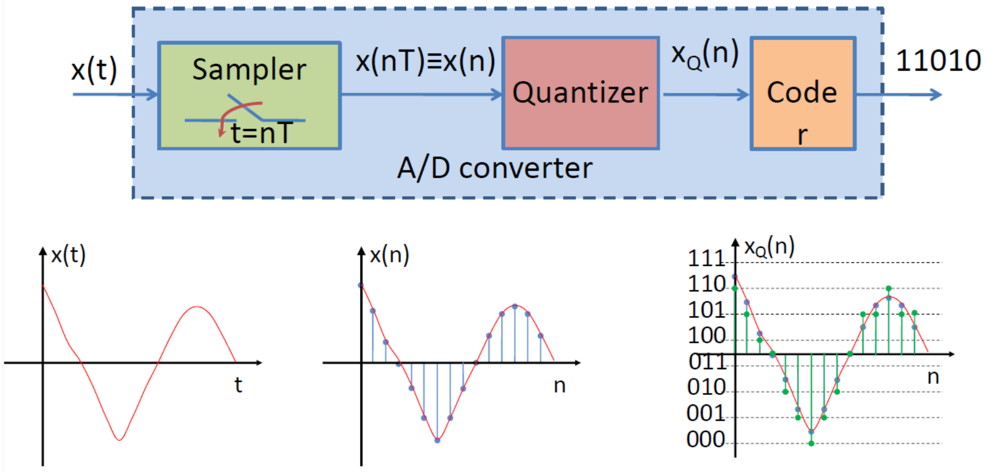
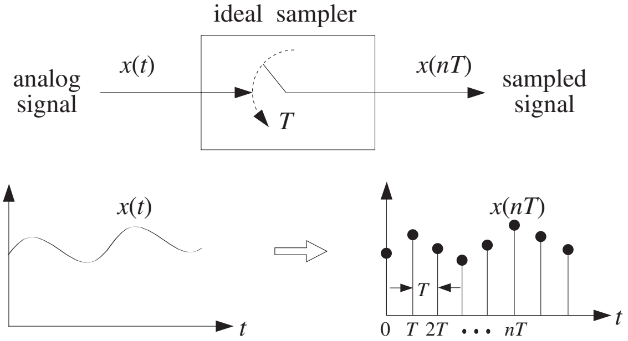


Figure 1: x(n) is discrete time signal but continus in amplitude

* Transform from blue to green is quantizer.

1.3 Sampling

Sampling is to convert a continuous time signal into a discrete time signal. The analog signal is periodically measured at every T seconds.



$$x(n) \equiv x(nT) = x(t = nT),\ n = \dots -2, -1, 0, 1, 2, 3 \dots$$

- T: sampling interval or sampling period (second);
- Fs=1/T: sampling rate or frequency (samples/second or Hz)

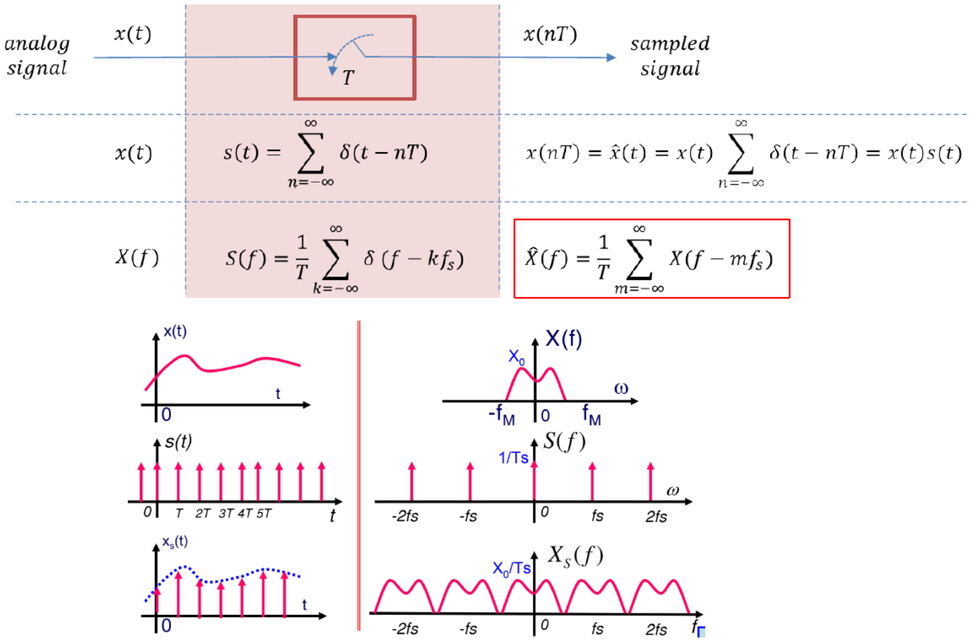
1.4 Aliasing of Sinusoids

In general, the sampling of a continuous-time sinusoidal signal $x(t) = A\cos(2\pi F_0 t + \theta)$ at a sampling rate $F_s = 1/T$ results in a discrete-time signal $x(n)$.

The sinusoids $x_k(t) = A\cos(2\pi F_k t + \theta)$ is sampled at F_s, resulting in a discrete time signal $x_k(n)$.

If $F_k = F_0 + kF_s, k = 0, \pm 1, \pm 2, \dots$, then $x(n) = x_k(n)$.

1.5 Spectrum Replication



Observation: The spectrum of discrete-time signal is a sum of the original spectrum of analog signal and its periodic replication at the interval F_s.

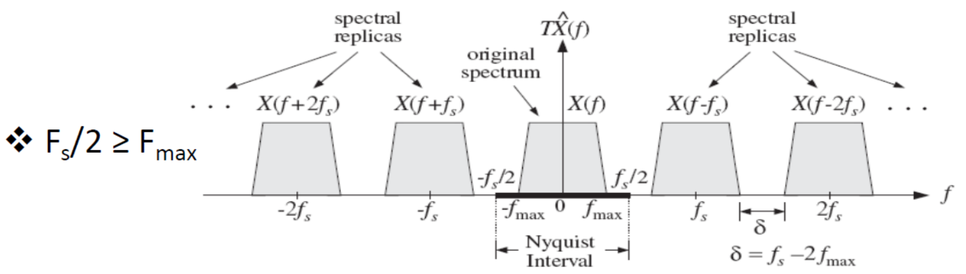


Fig: Spectrum replication caused by sampling

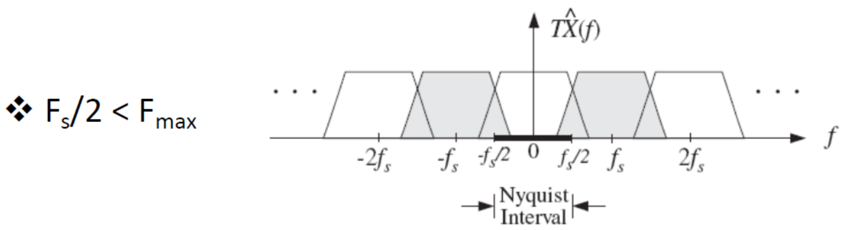


Fig: Aliasing caused by overlapping spectral replicas

1.6 Sampling Theorem

For accurate representation of a signal $x(t)$ by its time samples $x(nT)$, two conditions must be met:

1) The signal $x(t)$ must be band-limited, i.e., its frequency spectrum must be limited to $F_{\max}$.
2) The sampling rate F_s must be chosen at least twice the maximum frequency $F_{\max}$. $F_s \geq 2F_{\max}$
   - $F_s = 2F_{\max}$ is called Nyquist rate.
   - $F_s/2$ is called Nyquist frequency.
   - $[-F_s/2, F_s/2]$ is Nyquist interval.

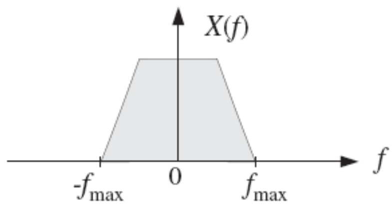


Figure 2: Typical band-limited spectrum

1.7 Ideal analog reconstruction

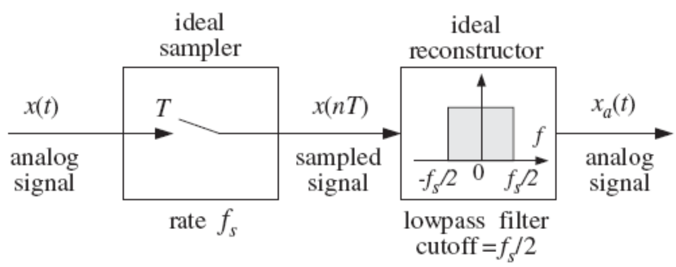


Figure 3: Ideal reconstructor as a lowpass filter

An ideal reconstructor acts as a lowpass filter with cutoff frequency equal to the Nyquist frequency $F_s/2$. An ideal reconstructor (lowpass filter) $H(F) = \begin{cases} T \in [-F_s/2, F_s/2] \\ 0 \quad\quad otherwise \end{cases}$. Then $\widehat{X}_a(F) = \widehat{X}(F)H(F) = X(F)$.

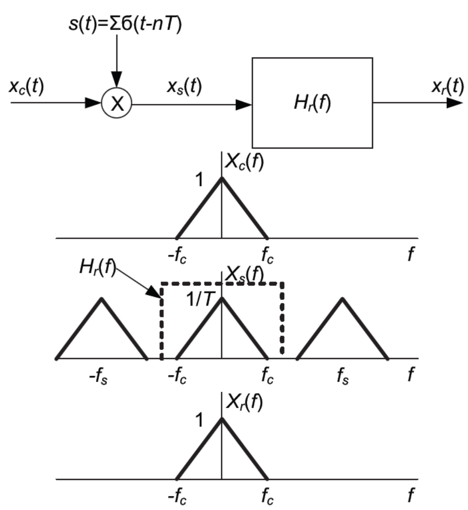


Figure 4: Example Demonstration

1.8 Ideal antialiasing prefilter

The signals in practice may not band-limited, thus they must be filtered by a lowpass filter

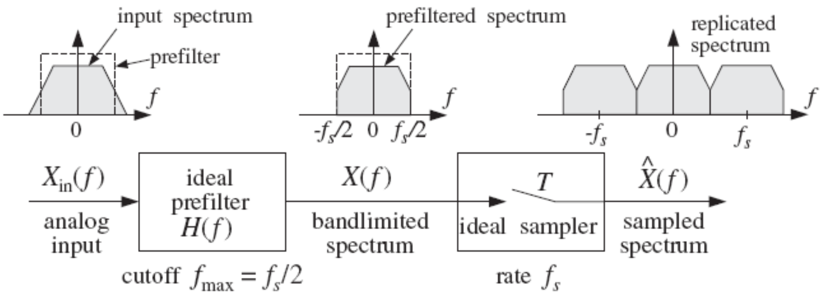


Figure 5: Ideal antialiasing prefilter

1.9 Practical antialiasing prefilter

- A lowpass filter: Passband $[-F_{pass}, F_{pass}]$ is the frequency range of interest for the application ($F_{max} = F_{pass}$).
- The stopband frequency F_{stop} and the minimum stopband attenuation Astop dB must be chosen appropriately to minimize the aliasing effects.
- The Nyquist frequency $F_s/2$ is in the middle of transition region.

$$F_s = F_{pass} + F_{stop}$$

- The attenuation of the filter in decibels is defined as (where f_0 is a convenient reference frequency, typically taken to be at DC for a lowpass filter):

$$A(F) = -2\log_{10}\left|\frac{H(F)}{H(F_0)}\right| \quad (dB)$$

- $\alpha_{10} = A(10F) - A(F)$ (dB/decade): the increase in attenuation when F is changed by a factor of ten.
- $\alpha_2 = A(2F) - A(F)$ (dB/octave): the increase in attenuation when F is changed by a factor of two.