Recitation Class for Mid II
Chapter 6 - 9

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2021.07.06

Outline

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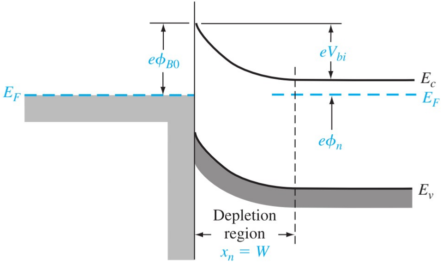
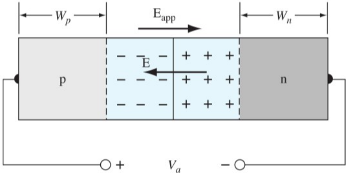
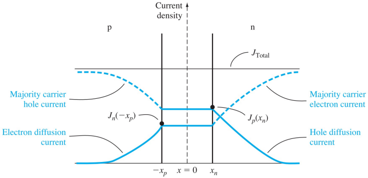
Overview

- **Chapter 9** Metal–Semiconductor and Semiconductor Heterojunctions



- **Chapter 7 & 8** The pn Junction and pn Junction Diode





Overview

- **Chapter 6** Nonequilibrium Excess Carriers in Semiconductors

$$D_n \frac{\mathrm{d}^2(\delta n)}{\mathrm{d}x^2} + \mu_n \left(E \frac{\mathrm{d}(\delta n)}{\mathrm{d}x} + n \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g_n - \frac{n}{\tau_{nt}} = \frac{\mathrm{d}(\delta n)}{\mathrm{d}t}$$

$$D_p \frac{\mathrm{d}^2(\delta p)}{\mathrm{d}x^2} - \mu_p \left(E \frac{\mathrm{d}(\delta p)}{\mathrm{d}x} + p \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g_p - \frac{p}{\tau_{pt}} = \frac{\mathrm{d}(\delta p)}{\mathrm{d}t}$$

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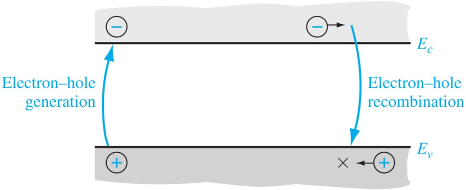
Electron-hole Generation & Recombination



$$G_{n0} = G_{p0}, \quad R_{n0} = R_{p0}$$

Thermal-equilibrium

Thermal-equilibrium: the net carrier concentrations are independent of time, which means that the generation and recombination of electrons and holes are equal.

$$G_{n0} = G_{p0} = R_{n0} = R_{p0}$$

Nonequilibrium:

$$n = n_0 + \delta n$$
$$p = p_0 + \delta p$$

Note that $np \neq n_0 p_0 = n_i^2$.

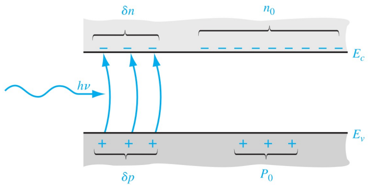


Figure: Creation of excess electron and hole densities by photons

Net Recombination Rate

n-type:

$$R'_n = R'_p = \frac{\delta p(t)}{\tau_{p0}}$$

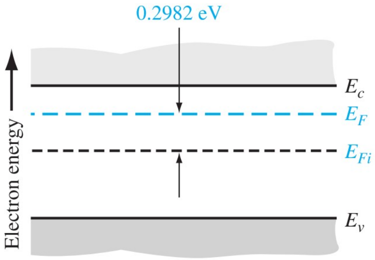
p-type:

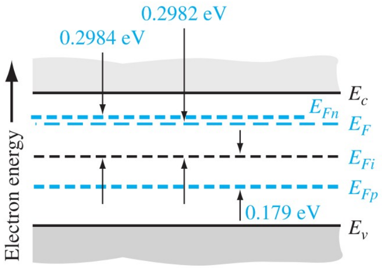
$$R'_n = R'_p = \frac{\delta n(t)}{\tau_{n0}}$$

Quasi-Fermi Energy Level

$$n_0 + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

$$p_0 + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$





Question

Why we only consider minority excess carrier?

Consider the case where we have a n-type silicon semiconductor, with $n_0 = 10^{17} cm^{-3}$, and $p_0 = n_i^2 / n_0 = 2250 cm^{-3}$. And the excess carrier $\delta n = 10^{14} cm^{-3}$, which is only 0.1% of n_0. However, $\delta p = \delta n = 10^{14} cm^{-3}$, which is greatly larger than p_0.

You can also have such feeling from the Quasi-Fermi Energy Level diagram that E_{Fn} is close to E_F while E_{Fp} changes a lot.

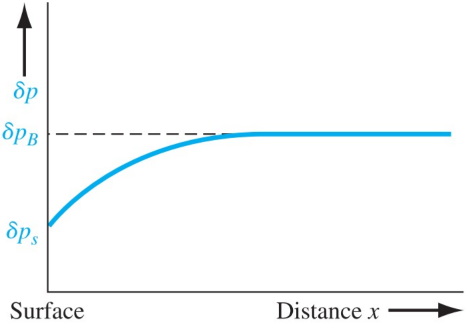
Excess Carrier Lifetime

$$\boxed{\begin{aligned} R_n = R_p &= \frac{C_n C_p N_t (np - n_i^2)}{C_n(n + n') + C_p(p + p')} \\ &= \frac{(np - n_i^2)}{\tau_{p0}(n + n') + \tau_{n0}(p + p')} \end{aligned}}$$

where

$$n' = N_c \exp\left[-\frac{E_c - E_t}{kT}\right], p' = N_v \exp\left[-\frac{E_t - E_v}{kT}\right], \text{ and } \tau_{n0} = \frac{1}{C_n N_t}$$

Surface Effects



$$\boxed{-D_p \left[\hat{n} \cdot \frac{\mathrm{d}(\delta p)}{\mathrm{d}x}\right]\Bigg|_{\text{surf}} = s\, \delta p|_{\text{surf}}}$$

$$\delta p(x) = g' \tau_{p0} \left(1 - \frac{s L_p \mathrm{e}^{-x/L_p}}{D_p + s L_p}\right)$$

Time-dependent Continuity Equation

$$\boxed{\begin{aligned} D_n \frac{\mathrm{d}^2 n}{\mathrm{d}x^2} + \mu_n \left(E \frac{\mathrm{d}n}{\mathrm{d}x} + n \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g_n - \frac{n}{\tau_{nt}} &= \frac{\mathrm{d}n}{\mathrm{d}t} \\ D_p \frac{\mathrm{d}^2 p}{\mathrm{d}x^2} - \mu_p \left(E \frac{\mathrm{d}p}{\mathrm{d}x} + p \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g_p - \frac{p}{\tau_{pt}} &= \frac{\mathrm{d}p}{\mathrm{d}t} \end{aligned}}$$

For homogeneous semiconductor, $n(x) = n_0 + \delta n(x)$, the equation can be simplified to

$$\boxed{\begin{aligned} D_n \frac{\mathrm{d}^2 (\delta n)}{\mathrm{d}x^2} + \mu_n \left(E \frac{\mathrm{d}(\delta n)}{\mathrm{d}x} + n \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g_n - \frac{n}{\tau_{nt}} &= \frac{\mathrm{d}(\delta n)}{\mathrm{d}t} \\ D_p \frac{\mathrm{d}^2 (\delta p)}{\mathrm{d}x^2} - \mu_p \left(E \frac{\mathrm{d}(\delta p)}{\mathrm{d}x} + p \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g_p - \frac{p}{\tau_{pt}} &= \frac{\mathrm{d}(\delta p)}{\mathrm{d}t} \end{aligned}}$$

$$\boxed{g_n - \frac{n}{\tau_{pt}} = g' - \frac{\delta n}{\tau_{pt}}}$$

Equation Simplification

Table 6.2 | Common ambipolar transport equation simplifications

Specification	Effect
Steady state	$\frac{\partial(\delta n)}{\partial t} = 0, \quad \frac{\partial(\delta p)}{\partial t} = 0$
Uniform distribution of excess carriers (uniform generation rate)	$D_n \frac{\partial^2(\delta n)}{\partial x^2} = 0, \quad D_p \frac{\partial^2(\delta n)}{\partial x^2} = 0$
Zero electric field	$\mathrm{E}\frac{\partial(\delta n)}{\partial x} = 0, \quad \mathrm{E}\frac{\partial(\delta p)}{\partial x} = 0$
No excess carrier generation	$g' = 0$
No excess carrier recombination (infinite lifetime)	$\frac{\delta n}{\tau_{n0}} = 0, \quad \frac{\delta p}{\tau_{p0}} = 0$

Equation General Solution

See cheating paper.

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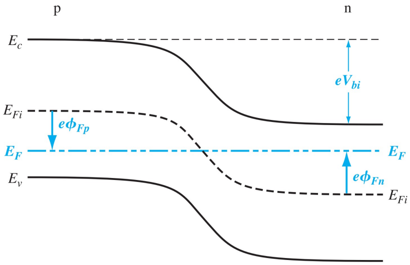
Built-in Potential Barrier



$$\boxed{\begin{aligned} V_{bi} &= |\phi_{Fn}| + |\phi_{Fp}| \\ &= \frac{kT}{e} \ln\left(\frac{N_a N_d}{n_i^2}\right) = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \end{aligned}}$$

V_{bi}: built-in potential barrier.
$V_t = kT/e$ defined as the thermal voltage.

Zero Applied Bias

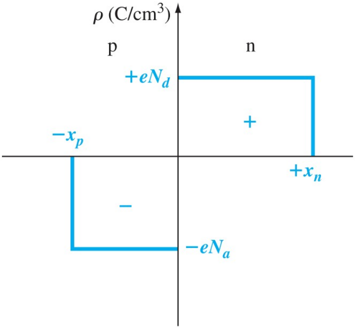


Figure: space charge density

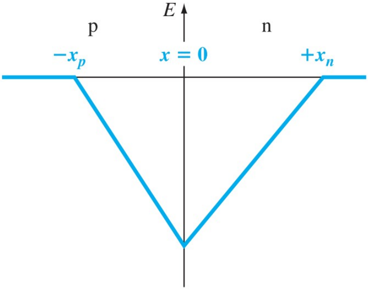


Figure: electric field

$$N_a x_p = N_d x_n$$

$$|k| = \frac{eN_{a/d}}{\varepsilon_s}$$

Zero Applied Bias

$$N_a x_p = N_d x_n$$

$$x_n = \sqrt{\frac{2\varepsilon_s (V_{bi} + V_R)}{e} \left[\frac{N_a}{N_d}\right] \left[\frac{1}{N_a + N_d}\right]}$$

$$x_p = \sqrt{\frac{2\varepsilon_s (V_{bi} + V_R)}{e} \left[\frac{N_d}{N_a}\right] \left[\frac{1}{N_a + N_d}\right]}$$

$\varepsilon_s = \varepsilon_r \varepsilon_0$, where $\varepsilon_0 = 8.85 \times 10^{-14} F \cdot cm^{-1}$.
$\varepsilon_r = 11.7$ for Si.

$$W = x_n + x_p = \sqrt{\frac{2\varepsilon_s (V_{bi} + V_R)}{e} \left[\frac{N_a + N_d}{N_a N_d}\right]}$$

Zero Applied Bias

$$E = \begin{cases} -\dfrac{eN_a}{\varepsilon_s}(x + x_p), & -x_p \leq x \leq 0 \\ \dfrac{eN_d}{\varepsilon_s}(x_n - x), & 0 \leq x \leq x_n \end{cases}$$

$$\begin{aligned} |E_{max}| &= -\frac{eN_d x_n}{\varepsilon_s} = -\frac{eN_a x_p}{\varepsilon_s} \\ &= -\frac{2(V_{bi} + V_R)}{W} \end{aligned}$$

$$\phi(x) = -\int E(x)\,\mathrm{d}x$$

Reverse Applied Bias

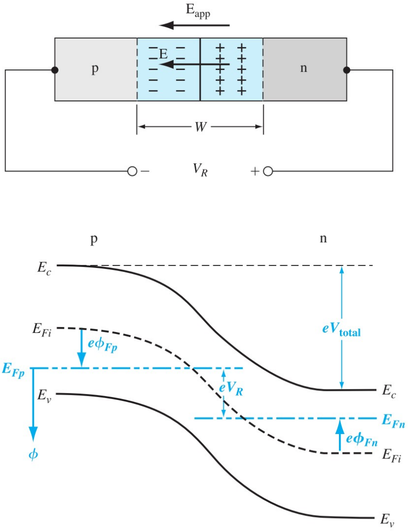


Figure: Energy-band diagram

Junction Capacitance

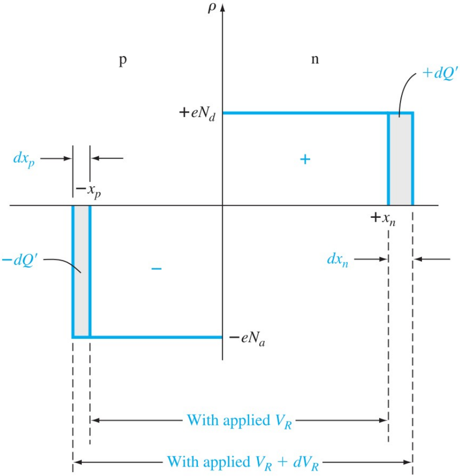


Figure: Differential change in the space charge width with a differential change in reverse-biased voltage for a uniformly doped pn junction

Junction Capacitance

$$C' = \frac{\mathrm{d}Q'}{dV_R}$$
$$dQ' = eN_d \,\mathrm{d}x_n = eN_a \,\mathrm{d}x_p$$

$\mathrm{d}Q'$ has units of C/cm^2, and C' has units of F/cm^2.

$$\boxed{C' = \sqrt{\frac{e\varepsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)}} = \frac{\varepsilon_s}{W}}$$

One-sided Junciton

$$C' \approx \sqrt{\frac{e\varepsilon_s N_d}{2(V_{bi} + V_R)}}$$

$$\Rightarrow \boxed{\left(\frac{1}{C'}\right)^2 = \frac{2(V_{bi} + V_R)}{e\varepsilon_s N_d}}$$

Experimentally determine V_{bi} and N_d.

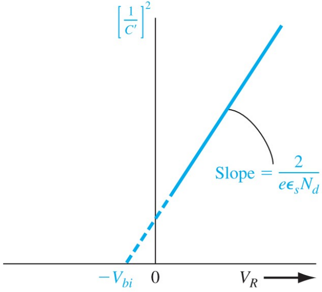


Figure: $(1/C')^2$ versus V_R of a uniformly doped pn junction

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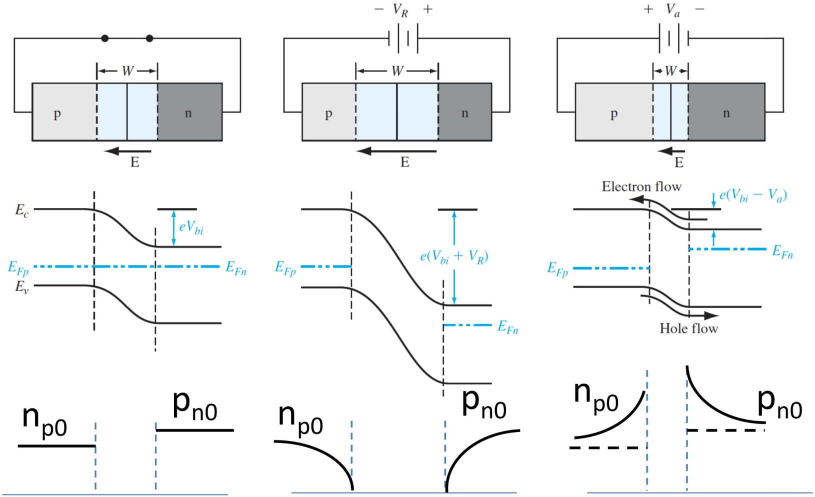
Charge Flow



$-\ V_R\ +$
$+\ V_a\ -$
W
p
n
E
E_c
eV_{bi}
E_{Fp}
E_{Fn}
E_v
$e(V_{bi}+V_R)$
Electron flow
$e(V_{bi}-V_a)$
Hole flow
n_{p0}
p_{n0}

Relation of Concentrations

$$n_{p0} = n_{n0} \exp\left(-\frac{eV_{bi}}{kT}\right)$$

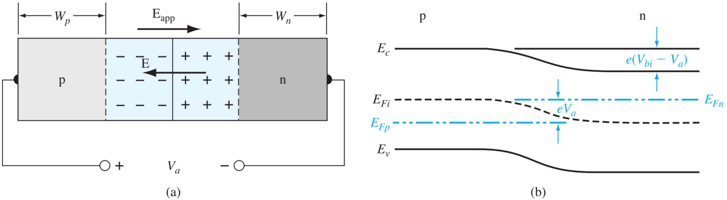
Relates the minority carrier electron concentration on the p side of the junction to the majority carrier electron concentration on the n side of the junction in thermal equilibrium.

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$n_{n0} \approx N_d$$

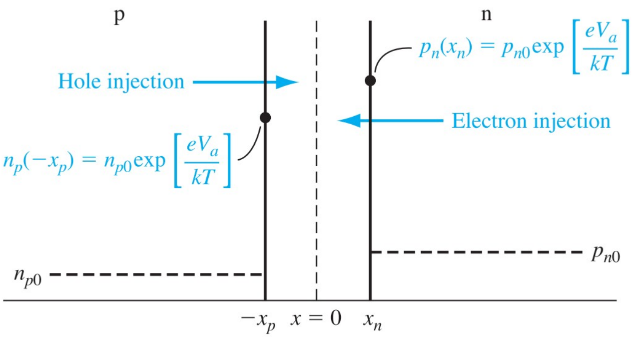
$$n_{p0} \approx \frac{n_i^2}{N_a}$$

Forward Biased



$$n_p = n_{n0} \exp\left(-\frac{e(V_{bi} - V_a)}{kT}\right)$$
$$= n_{p0} \exp\left(\frac{eV_a}{kT}\right)$$

Forward Biased



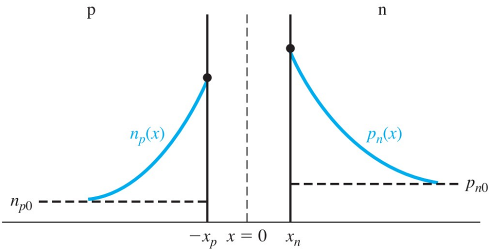
Minority Carrier Distribution

The solution is

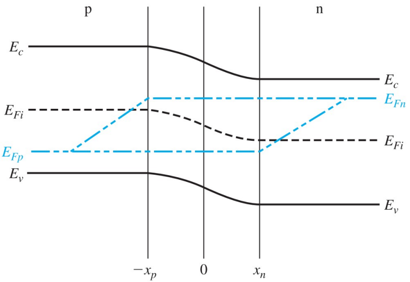
$$\boxed{\delta p_n(x) = p_n(x) - p_{n0} = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_n - x}{L_p}\right), \quad x \geq x_n}$$

Similarly,

$$\boxed{\delta n_p(x) = n_p(x) - n_{p0} = n_{p0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_p + x}{L_n}\right), \quad x \leq -x_p}$$



Quasi-Fermi Level

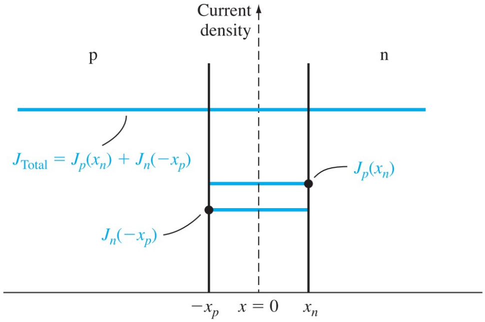


$$p = p_0 + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

Therefore the quasi-Fermi levels are linear functions of distance in the neutral p and n regions as shown in the Figure.

$$np = n_i^2 \exp\left(\frac{E_{Fn} - E_{Fp}}{kT}\right)$$

Ideal - pn Junction Current



$$J_p(x_n) = -eD_p \left. \frac{\mathrm{d}\left(\delta p_n(x)\right)}{\mathrm{d}x} \right|_{x=x_n}$$

Ideal - pn Junction Current - Continue

$$J_p(x_n) = \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

$$J_n(-x_p) = \frac{eD_n n_{p0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

Then the total current density

$$\boxed{\begin{aligned} &J = J_s\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] \\ &\qquad\qquad \text{where } J_s = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right] \end{aligned}}$$

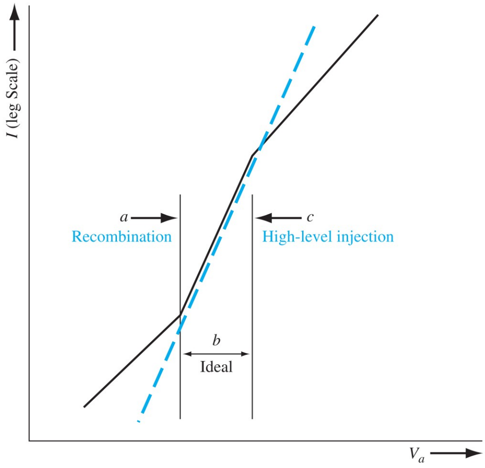
Non-Ideal I - Generation-recombination currents

$$
\begin{aligned}
R_n &= \frac{(np - n_i^2)}{\tau_p \left[n + n_i \exp\left(\frac{E_t - E_i}{kT}\right)\right] + \tau_n \left[p + n_i \exp\left(\frac{E_i - E_t}{kT}\right)\right]} \\
&= -\frac{n_i}{2\tau} = -G_0 \qquad \text{assume } E_t = E_i, \tau_n = \tau_p = \tau
\end{aligned}
$$

$$
\begin{aligned}
J_r &= \int_0^W qG_0 \, \mathrm{d}x \\
&= \frac{qWn_i}{2\tau}
\end{aligned}
$$

$$
J = J_s \left[\exp\left(\frac{qV_a}{kT}\right) - 1\right] + \frac{qWn_i}{2\tau} \left[\exp\left(\frac{qV_a}{2kT}\right) - 1\right]
$$

Non-Ideal II - High Level Injection



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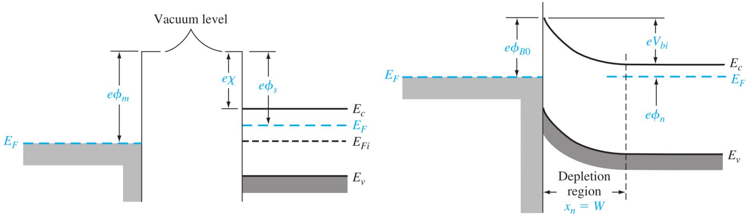
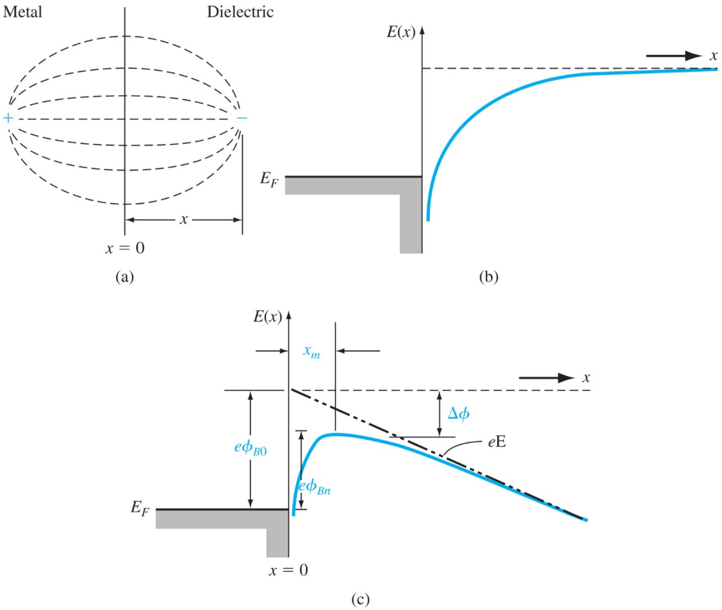
The Schottky Barrier Diode



Work function: ϕ
Electron affinity: χ
Schottky barrier: $\phi_{B0} = \phi_m - \chi$
Built-in potential barrier: $V_{bi} = \phi_{B0} - \phi_n$

Non-ideal Effects - Schottky Barrier Lowering



Non-ideal Effects - Schottky Barrier Lowering

$$F = \frac{-e^2}{4\pi\varepsilon_s(2x)^2} = -eE$$

$$-\phi(x) = +\int_x^{\infty} E \, \mathrm{d}x' = \frac{-e}{16\pi\varepsilon_s x}$$

$$\text{With electric field:} \quad -\phi(x) = \frac{-e}{16\pi\varepsilon_s x} - Ex$$

$$\frac{\mathrm{d}(e\phi(x))}{\mathrm{d}x} = 0 \quad \Rightarrow \quad \Delta\phi = \sqrt{\frac{eE}{4\pi\varepsilon_s}}$$

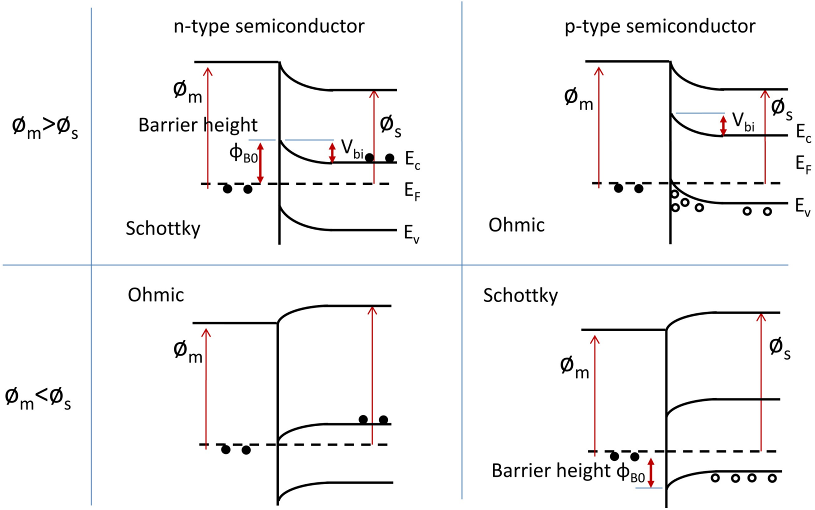
Current-Voltage Relationship

$$J = J_{sT}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

$$J_{sT} = A^* T^2 \exp\left(\frac{-e\phi_{Bn}}{kT}\right)$$

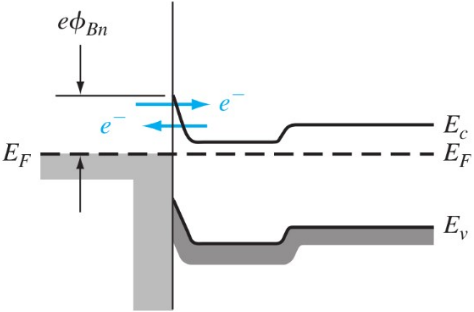
$$A^* = \frac{4\pi e m_n^* k^2}{h^3}$$

A^*: effective Richardson constant for thermionic emission.
(After plugging in all values, times 10^{-4} to get the unit $A \cdot K^{-2} \cdot cm^{-2}$)



n-type semiconductor
p-type semiconductor
Øm>Øs
Øm
Barrier height
ϕB0
Vbi
Øs
Ec
EF
Ev
Schottky
Øm
Vbi
Øs
Ec
EF
Ev
Ohmic
Øm<Øs
Ohmic
Øm
Schottky
Øm
Øs
Barrier height ϕB0

Tunneling Barrier



$$J_t \propto \exp\left(\frac{-e\phi_{Bn}}{E_{oo}}\right)$$

$$E_{oo} = \frac{e\hbar}{2}\sqrt{\frac{N_d}{\varepsilon_s M_n^*}}$$

Specific Contact Resistance

R_c defined as the reciprocal of the derivative of current density with respect to voltage evaluated at zero bias.

$$R_c = \left(\frac{\partial J}{\partial V}\right)^{-1}\Bigg|_{V=0} \qquad \Omega - cm^2$$

$$J_n = A^* T^2 \exp\left(\frac{-e\phi_{Bn}}{kT}\right)\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

$$R_c = \frac{\left(\frac{kT}{e}\right)\exp\left(\frac{+e\phi_{Bn}}{kT}\right)}{A^* T^2}$$

Good luck to your midterm exam!