Recitation Class 5

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2021.06.22

Outline

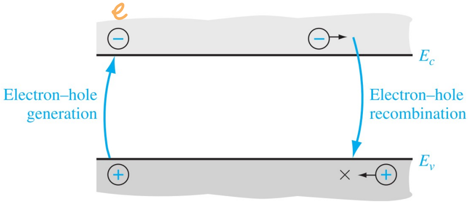
Table of Contents

Notations in Chapter 6

Table 6.1 | Relevant notation used in Chapter 6

Symbol	Definition
n_0, p_0	Thermal-equilibrium electron and hole concentrations (independent of time and also usually position)
n, p	Total electron and hole concentrations (may be functions of time and/or position)
$\delta n = n - n_0$ $\delta p = p - p_0$	Excess electron and hole concentrations (may be functions of time and/or position)
g'_n, g'_p	Excess electron and hole generation rates
R'_n, R'_p	Excess electron and hole recombination rates
τ_{n0}, τ_{p0}	Excess minority carrier electron and hole lifetimes

Electron-hole Generation & Recombination



$$G_{n0} = G_{p0}, \quad R_{n0} = R_{p0}$$

Thermal-equilibrium

Thermal-equilibrium: the net carrier concentrations are independent of time, which means that the generation and recombination of electrons and holes are equal.

$$G_{n0} = G_{p0} = R_{n0} = R_{p0}$$

Nonequilibrium:

$$n = n_0 + \delta n$$
$$p = p_0 + \delta p$$

Note that $np \neq n_0 p_0 = n_i^2$.

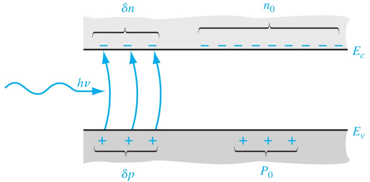


Figure: Creation of excess electron and hole densities by photons

Net Recombination Rate

n-type:

$$R'_n = R'_p = \frac{\delta p(t)}{\tau_{p0}}$$

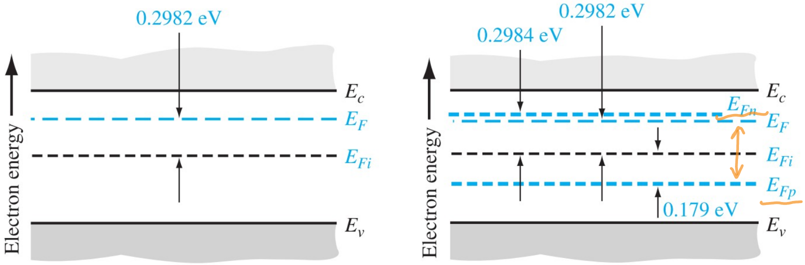
p-type:

$$R'_n = R'_p = \frac{\delta n(t)}{\tau_{n0}}$$

Quasi-Fermi Energy Level

$$n_0 + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

$$p_0 + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$



Question

Why we only consider minority excess carrier?

Consider the case where we have a n-type silicon semiconductor, with $n_0 = 10^{17} cm^{-3}$, and $p_0 = n_i^2/n_0 = 2250 cm^{-3}$. And the excess carrier $\delta n = 10^{14} cm^{-3}$, which is only 0.1% of n_0. However, $\delta p = \delta n = 10^{14} cm^{-3}$, which is greatly larger than p_0.

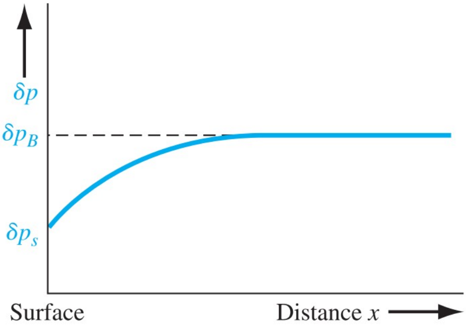
You can also have such feeling from the Quasi-Fermi Energy Level diagram that E_{Fn} is close to E_F while E_{Fp} changes a lot.

Excess Carrier Lifetime

$$\boxed{R_n = R_p = \frac{C_n C_p N_t (np - n_i^2)}{C_n(n+n') + C_p(p+p')} \equiv R}$$

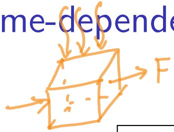
$$\text{where } n' = N_c \exp\left[-\frac{E_c - E_t}{kT}\right], \qquad p' = N_v \exp\left[-\frac{E_t - E_v}{kT}\right]$$

Surface Effects



$$\boxed{-D_p \left[\hat{n} \cdot \frac{\mathrm{d}(\delta p)}{\mathrm{d}x}\right]\Bigg|_{\text{surf}} = s\ \delta p|_{\text{surf}}}$$

Time-dependent Continuity Equation



$g' - \frac{n}{\tau_{nt}}$

$\frac{dF}{dx} = g' - \frac{n}{\tau_{nt}}$

$F = \frac{J}{-e}$ — $e\mu_p p E + D_p \frac{dn}{dx}$

$$\boxed{\begin{aligned} D_n \frac{\mathrm{d}^2 n}{\mathrm{d}x^2} + \mu_n \left(E \frac{\mathrm{d}n}{\mathrm{d}x} + n \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g_n - \frac{n}{\tau_{nt}} &= \frac{\mathrm{d}n}{\mathrm{d}t} \\ D_p \frac{\mathrm{d}^2 p}{\mathrm{d}x^2} - \mu_p \left(E \frac{\mathrm{d}p}{\mathrm{d}x} + p \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g_p - \frac{p}{\tau_{pt}} &= \frac{\mathrm{d}p}{\mathrm{d}t} \end{aligned}}$$

For homogeneous semiconductor, $\underbrace{n(x) = n_0 + \delta n(x)}_{\text{Const}}$, the equation can be simplified to

$$\boxed{\begin{aligned} D_n \frac{\mathrm{d}^2 (\delta n)}{\mathrm{d}x^2} + \mu_n \left(E \frac{\mathrm{d}(\delta n)}{\mathrm{d}x} + n \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g_n - \frac{n}{\tau_{nt}} &= \frac{\mathrm{d}(\delta n)}{\mathrm{d}t} \\ D_p \frac{\mathrm{d}^2 (\delta p)}{\mathrm{d}x^2} - \mu_p \left(E \frac{\mathrm{d}(\delta p)}{\mathrm{d}x} + p \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g_p - \frac{p}{\tau_{pt}} &= \frac{\mathrm{d}(\delta p)}{\mathrm{d}t} \end{aligned}}$$

Equation Simplification

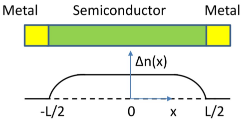
Table 6.2 | Common ambipolar transport equation simplifications

Specification	Effect
Steady state	$\dfrac{\partial(\delta n)}{\partial t} = 0, \quad \dfrac{\partial(\delta p)}{\partial t} = 0$
Uniform distribution of excess carriers (uniform generation rate)	$D_n \dfrac{\partial^2(\delta n)}{\partial x^2} = 0, \quad D_p \dfrac{\partial^2(\delta n)}{\partial x^2} = 0$
Zero electric field	$\mathrm{E}\dfrac{\partial(\delta n)}{\partial x} = 0, \quad \mathrm{E}\dfrac{\partial(\delta p)}{\partial x} = 0$
No excess carrier generation	$g' = 0$
No excess carrier recombination (infinite lifetime)	$\dfrac{\delta n}{\tau_{n0}} = 0, \quad \dfrac{\delta p}{\tau_{p0}} = 0$

Example I

Given a piece of p-type uniformly doped semiconductor in contact with two metal electrodes separated by a length of L, forming a photoconductor device. The light illumination will create electron-hole pairs at a generation rate of g. The minority carrier recombination lifetime is τ_0. Find the analytical distribution of the excess minority electrons at zero external bias. Note that light illumination will not create excess carriers in metals.

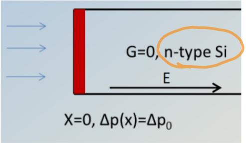
$n = \underbrace{n_0}_{c} + \delta n$



$$D_n \frac{\mathrm{d}^2(\delta n)}{\mathrm{d}x^2} + \cancel{\mu_n \left(E \frac{\mathrm{d}(\delta n)}{\mathrm{d}x} + n \frac{\mathrm{d}E}{\mathrm{d}x} \right)} + g_n - \frac{n}{\tau_{nt}} = \underbrace{\frac{\mathrm{d}(\delta n)}{\mathrm{d}t}}_{} = 0$$

Example II

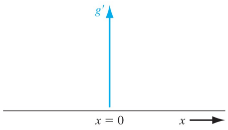
A light beam is illuminated on the surface of a silicon wafer, generating excess carriers Δp_0 at the surface ($x = 0$). The wafer is placed in a constant electric field with a known intensity E. We assume there is no external generation inside the wafer. The thickness of the wafer is infinite. Find the excess minority carriers at equilibrium as a function of the distance away from the surface ($x = 0$). Small injection condition is always maintained and the wafer is uniformly doped as N_d.



$$D_p \frac{\mathrm{d}^2(\delta p)}{\mathrm{d}x^2} - \mu_p \left(E \frac{\mathrm{d}(\delta p)}{\mathrm{d}x} + \overset{=0}{p \frac{\mathrm{d}E}{\mathrm{d}x}} \right) + \overset{=0}{g_p} - \frac{p}{\tau_{pt}} = \overset{=0}{\underset{=0}{\frac{\mathrm{d}(\delta p)}{\mathrm{d}t}}}$$

Example III

Consider a p-type semiconductor that is homogeneous and infinite in extent. Assume a zero applied electric field. For a one-dimensional crystal, assume that excess carriers are being generated at $x = 0$ only, as indicated in Figure below. The excess carriers being generated at $x = 0$ will begin diffusing in both the $+x$ and $-x$ directions. Calculate the steady-state excess carrier concentration as a function of x.



$$D_n \frac{\mathrm{d}^2(\delta n)}{\mathrm{d}x^2} + \cancel{\mu_n \left(E \frac{\mathrm{d}(\delta n)}{\mathrm{d}x} + n \frac{\mathrm{d}E}{\mathrm{d}x} \right)}^{=0} + \overset{=0}{g_n} - \frac{n}{\tau_{nt}} = \overset{=0}{\frac{\mathrm{d}(\delta n)}{\mathrm{d}t}}$$

$$=0$$

General Solutions – t

- $$\frac{\mathrm{d}(\delta p)}{\mathrm{d}t} = -\frac{\delta p}{\tau_{p0}}$$

  solution:

  $$\delta p(t) = \delta p(0)\mathrm{e}^{-t/\tau_{p0}}$$

- $$g' - \frac{\delta p}{\tau_{p0}} = \frac{\mathrm{d}(\delta p)}{\mathrm{d}t}$$

  solution:

  $$\delta p(t) = g'\tau_{p0}\left(1 - \mathrm{e}^{-t/\tau_{p0}}\right)$$

General Solutoins – x

- $$D_n \frac{\mathrm{d}^2(\delta n)}{\mathrm{d}x^2} - \frac{\delta n}{\tau_{n0}} = 0$$

  solution:

  $$\delta n(x) = A\mathrm{e}^{-x/L_n} + B\mathrm{e}^{x/L_n}, \quad L_n = \sqrt{D_n \tau_{n0}}$$

  special:

  $$\delta n(x) = \begin{cases} \delta n(0)\mathrm{e}^{-x/L_n}, x \geq 0 \\ \delta n(0)\mathrm{e}^{+x/L_n}, x \leq 0 \end{cases}$$

- $$D_p \frac{\mathrm{d}^2 \delta p}{\mathrm{d}x^2} - \frac{\delta p}{\tau} + G_{ex} = 0$$

  solution:

  $$\delta p(x) = A \exp(\lambda x) + g\tau, \quad \lambda = \pm \frac{1}{\sqrt{D_p \tau}}$$

General Solutions – E

- $$D_p \frac{\mathrm{d}^2(\delta p)}{\mathrm{d}x^2} - \mu_p E_0 \frac{\mathrm{d}(\delta p)}{\mathrm{d}x} - \frac{\delta p}{\tau_{p0}} = \frac{\mathrm{d}(\delta p)}{\mathrm{d}t}$$

  solution:

  $$\delta p(x,t) = \frac{\mathrm{e}^{-t/\tau_{p0}}}{(4\pi D_p t)^{1/2}} \exp\left[\frac{-(x-\mu_p E_0 t)^2}{4 D_p t}\right]$$

General Soluitons – E

- $$D_p \frac{\mathrm{d}^2 \delta p}{\mathrm{d}x^2} - \mu_p E \frac{\mathrm{d}\delta p}{\mathrm{d}x} - \frac{\delta p}{\tau} = 0$$

  solution:

  $$\delta p(x) = A \exp\left(\lambda x\right) + C$$

  $$\lambda = \frac{L_p(E) \pm \sqrt{L_p^2(E) + 4L_p^2}}{2L_p^2}, \quad L_p = \sqrt{\tau D_p}, \quad L_p(E) = \tau \mu_p E$$

  special:

  $$\delta p(x) = \delta p(0) \exp\left[\frac{L_p(E) \pm \sqrt{L_p^2(E) + 4L_p^2}}{2L_p^2} x\right] = \begin{cases} \delta p(0) \exp\left(-\frac{x}{L_p}\right), & \text{if } L_p(E) \ll L_p \\ \delta p(0) \exp\left(-\frac{x}{L_p(E)}\right), & \text{if } L_p(E) \gg L_p \end{cases}$$

Table of Contents

Structure

VE311

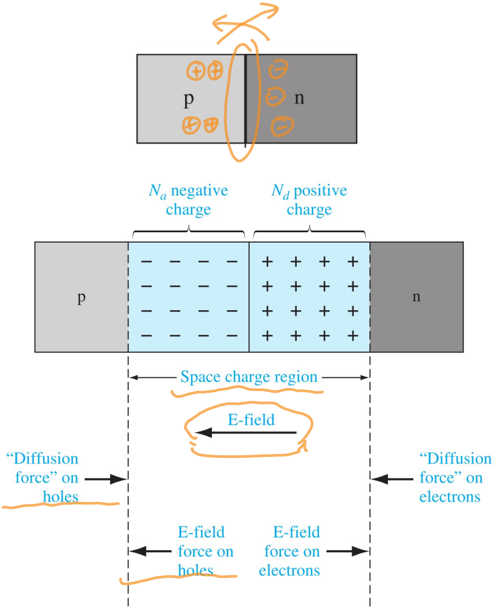


Figure: *space charge region (depletion region)*

Energy-band Diagram

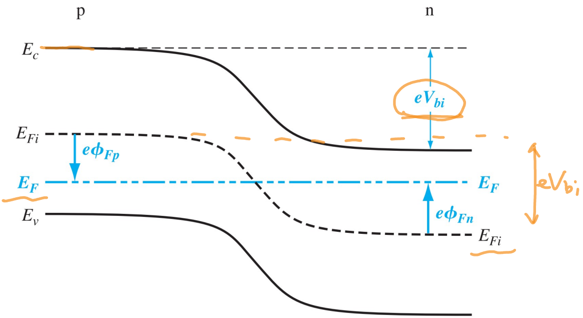
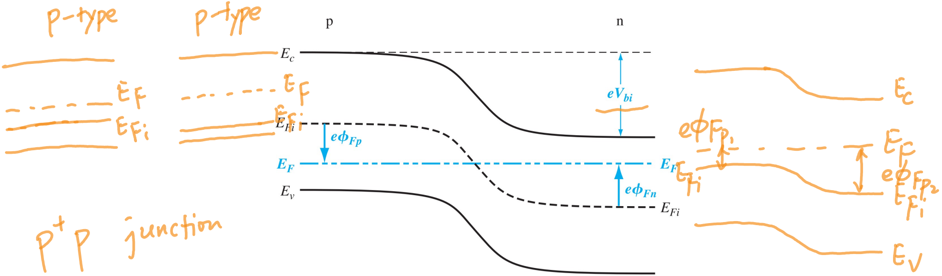


Figure: Energy-band diagram of a pn junction in thermal equilibrium

V_{bi}: built-in potential barrier.

Built-in Potential Barrier



$$V_{bi} = |\phi_{Fn}| + |\phi_{Fp}|$$

$$V_{bi} = |e\phi_{Fp_2}| - |e\phi_{Fp_1}|$$

$$= \frac{kT}{e} \ln\left(\frac{N_a N_d}{n_i^2}\right) = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$ (not recommended)

$V_t = kT/e$ defined as the thermal voltage.

Zero Applied Bias

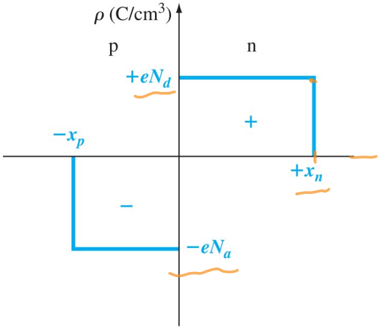


Figure: space charge density

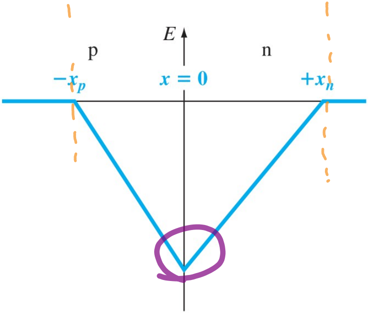


Figure: electric field

$$N_a x_p = N_d x_n$$

$$|k| = \frac{eN_{a/d}}{\varepsilon_s}$$

Zero Applied Bias

$V_R = 0$

$$N_a x_p = N_d x_n$$

$$x_n = \sqrt{\frac{2\varepsilon_s\left(V_{bi} + V_R\right)}{e}\left[\frac{N_a}{N_d}\right]\left[\frac{1}{N_a + N_d}\right]}$$

$$x_p = \sqrt{\frac{2\varepsilon_s\left(V_{bi} + V_R\right)}{e}\left[\frac{N_d}{N_a}\right]\left[\frac{1}{N_a + N_d}\right]}$$

$\frac{N_d}{N_a} x_n$

$\varepsilon_s = \varepsilon_r \varepsilon_0$, where $\varepsilon_0 = 8.85 \times 10^{-14} F \cdot cm^{-1}$.
$\varepsilon_r = 11.7$ for Si.

$$W = x_n + x_p = \sqrt{\frac{2\varepsilon_s\left(V_{bi} + V_R\right)}{e}\left[\frac{N_a + N_d}{N_a N_d}\right]}$$

Zero Applied Bias

$$E = \begin{cases} -\boxed{\dfrac{eN_a}{\varepsilon_s}}(x + x_p), & -x_p \le x \le 0 \\ \dfrac{eN_d}{\varepsilon_s}(x_n - x), & 0 \le x \le x_n \end{cases}$$

$$|E_{max}| = -\frac{eN_d x_n}{\varepsilon_s} = -\frac{eN_a x_p}{\varepsilon_s}$$

x=0

$$= -\frac{2(V_{bi} + V_R)}{W}$$

Q4

$$\phi(x) = -\int E(x)\,\mathrm{d}x$$

End