Recitation Class 2

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Outline

Chapter 3-II Introduction to the Quantum Theory of Solids

Chapter 4-I The Semiconductor in Equilibrium

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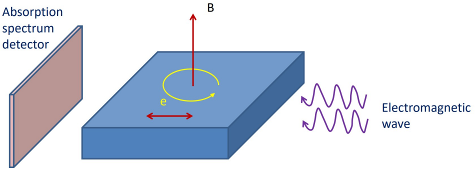
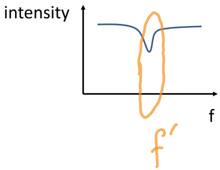
Chapter 4-I The Semiconductor in Equilibrium

Effective Mass: experimentally

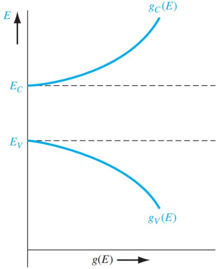
Cyclotron resonance





$$\boxed{m^* = \frac{eB\lambda}{2\pi c}}$$

Density of States Function



$$\boxed{\begin{aligned} g(E) &= \frac{4\pi(2m)^{\frac{3}{2}}}{h^3}\sqrt{E} \\ g_c(E) &= \frac{4\pi(2m_n^*)^{\frac{3}{2}}}{h^3}\sqrt{E - E_c} \\ g_v(E) &= \frac{4\pi(2m_p^*)^{\frac{3}{2}}}{h^3}\sqrt{E_v - E} \end{aligned}}$$

Related Materials

Proof (if interested):

- `https://eng.libretexts.org/Bookshelves/Materials_Science/Supplemental_Modules_(Materials_Science)/Electronic_Properties/Density_of_States`
- Textbook 3.4 Density of States Function

Example

Determine the number ($\#/\mathrm{cm}^3$) of quantum states in silicon between E_c and $E_c + kT$ at $T = 300K$.

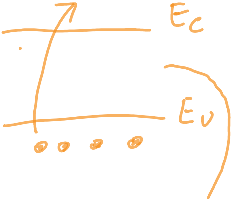
$$
\begin{aligned}
N &= \int_{E_c}^{E_c+kT} \frac{4\pi(2m^*)^{3/2}}{h^3}\sqrt{E - E_c}\,\mathrm{d}E \\
&= \frac{4\pi(2m^*)^{3/2}}{h^3}\,\frac{2}{3}\,(E - E_c)^{3/2}\Big|_{E_c}^{E_c+kT} \\
&= 2.22 \times 10^{25} m^{-3} \text{ or } 2.12 \times 10^{19} cm^{-3}
\end{aligned}
$$

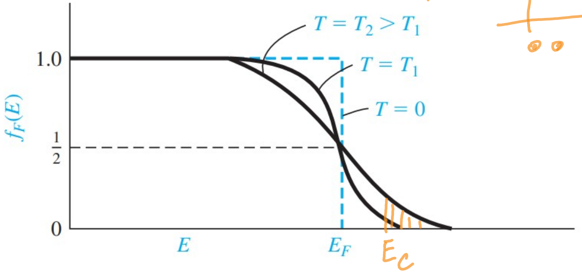
Distribution Function

- Fermi-Dirac probability function:

$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E-E_F}{kT}\right)}$$

T=0





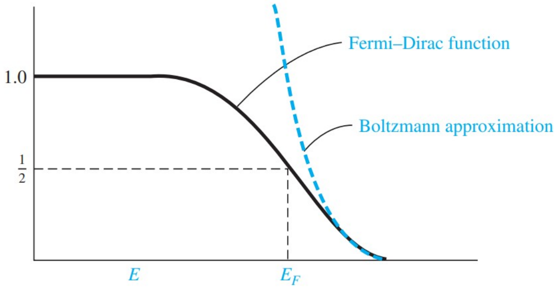
$f_F \neq 0$

$g(E) = 0$

Distribution Function

- Boltzmann distribution
  When $\exp\left(\frac{E-E_F}{kT}\right) >> 1 \Rightarrow E - E_F > 2kT$

$$f_F(E) \approx exp\left(-\frac{E - E_F}{kT}\right)$$



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n_0 and p_0 Equations

$$n_0 = \int_{E_c}^{\infty} g_c(E) f_F(E)\, \mathrm{d}E$$

$$\Rightarrow \quad \boxed{n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right], \quad N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}}$$

$$p_0 = \int_{-\infty}^{E_v} g_v(E) \left(1 - f_F(E)\right)\, \mathrm{d}E$$

$$\Rightarrow \quad \boxed{p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right], \quad N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}}$$

Example

Calculate the thermal-equilibrium hole concentration in silicon at $T = 400K$. Assume that the Fermi energy is $0.27eV$ above the valence-band energy. The value of N_v for silicon at $T = 300K$ is $N_v = 1.04 \times 10^{19} cm^{-3}$.

$$\underline{kT = 0.0259 \text{ only for } T = 300K}$$

$$kT = (0.0259)\left(\frac{400}{300}\right) = 0.03453eV$$

$$N_v = (1.04 \times 10^{19})\left(\frac{400}{300}\right)^{3/2} = 1.60 \times 10^{19} cm^{-3}$$

The hole concentration is then

$$p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right] = (2.60 \times 10^{19}) \exp\left(\frac{-0.27}{0.03453}\right)$$
$$= 6.43 \times 10^{15} cm^{-3}$$

Intrinsic Semiconductor

For intrinsic semiconductor, the Fermi energy level is called the intrinsic Fermi energy, or $E_F = E_{Fi}$. We have

$$n_0 = n_i = N_c \exp\left[\frac{-(E_c - E_{Fi})}{kT}\right]$$

$$p_0 = n_i = N_v \exp\left[\frac{-(E_{Fi} - E_v)}{kT}\right]$$

Take the product:

$$\boxed{n_0 p_0 = n_i^2 = N_c N_v \exp\left[\frac{-(E_c - E_v)}{kT}\right] = N_c N_v \exp\left[\frac{-E_g}{kT}\right]}$$

Self-consistency

$$n_i^2 = N_c N_v \exp\left[\frac{-(E_c - E_v)}{kT}\right] = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$$

For Si at $300K$:

$$\begin{aligned} n_i &= 1.5 \times 10^{10} cm^{-3}, \\ E_g &= 1.12 eV, \\ N_c &= 2.8 \times 10^{19} cm^{-3}, \\ N_v &= 1.04 \times 10^{19} cm^{-3}, \\ kT &= 0.0259 eV \end{aligned}$$

$$LHS = 2.25 \times 10^{20} \neq 4.82936 \times 10^{19} = RHS$$

Self-consistency Example

For n-doped Silicon semiconductor at $300K$, the Fermi level is $E_F = E_c - 0.3eV$. Calculate p_0.

- Approach I:

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right] = 2.61 \times 10^{14} cm^{-3}$$

$$p_0 = \frac{n_i^2}{n_0} = 8.62 \times 10^5 cm^{-3}$$

- Approach II:

$$E_F - E_v = E_g - (E_c - E_F) = 0.82eV$$

$$p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right] = 1.85 \times 10^5 cm^{-3}$$

$E_g = E_c - E_v$

$\approx 1.12eV$

E_c

$0.3eV$

E_F

E_v

$E_F = E_v + 0.82eV$

End