

Recitation Class for Mid II

Chapter 6 - 9

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Outline

Overview

Chapter 6 Nonequilibrium Excess Carriers in Semiconductors

Chapter 7 The pn junction

Chapter 8 The pn Junction Diode

Chapter 9 Metal–Semiconductor and Semiconductor
Heterojunctions

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Overview

Chapter 6 Nonequilibrium Excess Carriers in Semiconductors

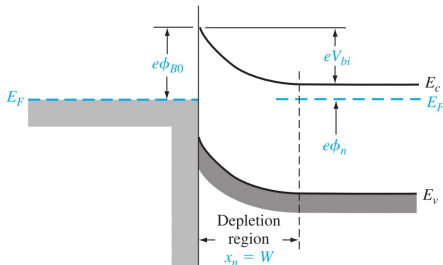
Chapter 7 The pn junction

Chapter 8 The pn Junction Diode

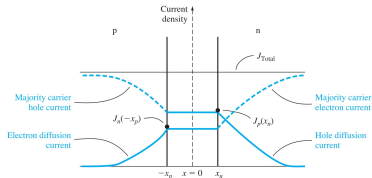
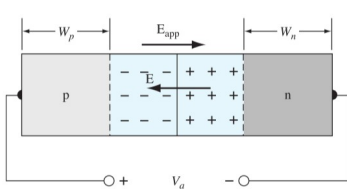
Chapter 9 Metal–Semiconductor and Semiconductor
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Overview

► Chapter 9 Metal–Semiconductor and Semiconductor Heterojunctions



► Chapter 7 & 8 The pn Junction and pn Junction Diode



► Chapter 6 Nonequilibrium Excess Carriers in Semiconductors

$$D_n \frac{d^2(\delta n)}{dx^2} + \mu_n \left(E \frac{d(\delta n)}{dx} + n \frac{dE}{dx} \right) + g_n - \frac{n}{\tau_{nt}} = \frac{d(\delta n)}{dt}$$
$$D_p \frac{d^2(\delta p)}{dx^2} - \mu_p \left(E \frac{d(\delta p)}{dx} + p \frac{dE}{dx} \right) + g_p - \frac{p}{\tau_{pt}} = \frac{d(\delta p)}{dt}$$

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Electron-hole Generation & Recombination



$$G_{n0} = G_{p0}, \quad R_{n0} = R_{p0}$$

Thermal-equilibrium

Thermal-equilibrium: the net carrier concentrations are independent of time, which means that the generation and recombination of electrons and holes are equal.

$$G_{n0} = G_{p0} = R_{n0} = R_{p0}$$

Nonequilibrium:

$$n = n_0 + \delta n$$

$$p = p_0 + \delta p$$

Note that $np \neq n_0 p_0 = n_i^2$.

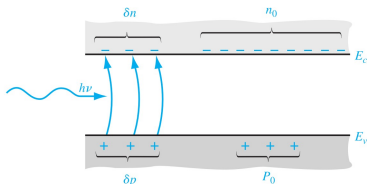


Figure: Creation of excess electron and hole densities by photons

Net Recombination Rate

$$\underline{R} = \frac{p}{\tau_{p0}} + \frac{\delta p}{\tau_{p0}} \\ = \underline{R_0 + R'_n}$$

n-type:

$$\underline{R'_n} = R'_p = \frac{\delta p(t)}{\tau_{p0}}$$

p-type:

$$R'_n = R'_p = \frac{\delta n(t)}{\tau_{n0}}$$

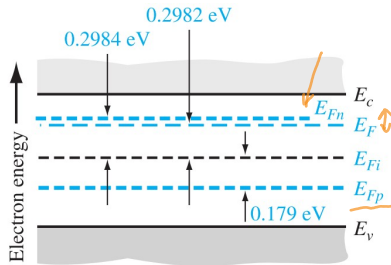
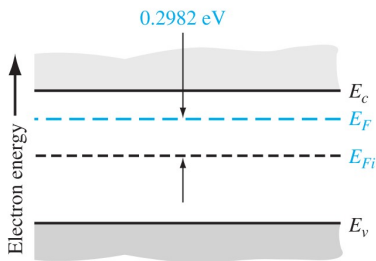
$$\underline{g - \frac{p}{\tau_{p0}}} = \underline{g' - \frac{\delta p}{\tau_{p0}}}$$

$$\underline{(g_0 + g')} - \frac{p_0 + \delta p}{\tau_{p0}}$$

Quasi-Fermi Energy Level

$$n_0 + \delta n = n_i \exp \left(\frac{E_{Fn} - E_{Fi}}{kT} \right)$$

$$p_0 + \delta p = n_i \exp \left(\frac{E_{Fi} - E_{Fp}}{kT} \right)$$



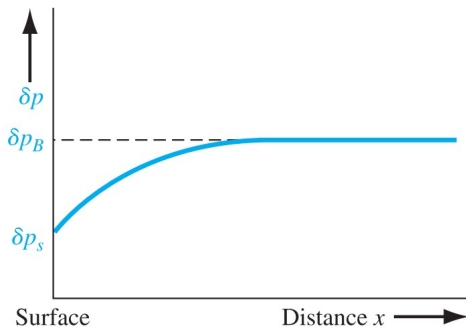
Excess Carrier Lifetime

$$\begin{aligned} R_n = R_p &= \frac{C_n C_p N_t (np - n_i^2)}{C_n (n + n') + C_p (p + p')} \\ &= \frac{(np - n_i^2)}{\tau_{p0} (n + n') + \tau_{n0} (p + p')} \end{aligned}$$

where

$$n' = N_c \exp \left[-\frac{E_c - E_t}{kT} \right], p' = N_v \exp \left[-\frac{E_t - E_v}{kT} \right], \text{ and } \tau_{n0} = \frac{1}{C_n N_t}$$

Surface Effects



$$-D_p \left[\hat{n} \cdot \frac{d(\delta p)}{dx} \right] \Big|_{\text{surf}} = s \delta p|_{\text{surf}}$$

$$\delta p(x) = g' \tau_{p0} \left(1 - \frac{s L_p e^{-x/L_p}}{D_p + s L_p} \right)$$

Time-dependent Continuity Equation

$$\begin{aligned} D_n \frac{d^2 n}{dx^2} + \mu_n \left(E \frac{dn}{dx} + n \frac{dE}{dx} \right) + g_n - \frac{n}{\tau_{nt}} &= \frac{dn}{dt} \\ D_p \frac{d^2 p}{dx^2} - \mu_p \left(E \frac{dp}{dx} + p \frac{dE}{dx} \right) + g_p - \frac{p}{\tau_{pt}} &= \frac{dp}{dt} \end{aligned}$$

For homogeneous semiconductor, $n(x) = n_0 + \delta n(x)$, the equation can be simplified to

$$\begin{aligned} D_n \frac{d^2(\delta n)}{dx^2} + \mu_n \left(E \frac{d(\delta n)}{dx} + n \frac{dE}{dx} \right) + g_n - \frac{n}{\tau_{nt}} &= \frac{d(\delta n)}{dt} \\ D_p \frac{d^2(\delta p)}{dx^2} - \mu_p \left(E \frac{d(\delta p)}{dx} + p \frac{dE}{dx} \right) + g_p - \frac{p}{\tau_{pt}} &= \frac{d(\delta p)}{dt} \end{aligned}$$

$$\underline{g_n - \frac{n}{\tau_{pt}} = g' - \frac{\delta n}{\tau_{pt}}}$$

Equation Simplification

Table 6.2 | Common ambipolar transport equation simplifications

Specification	Effect
Steady state	$\frac{\partial(\delta n)}{\partial t} = 0, \quad \frac{\partial(\delta p)}{\partial t} = 0$
Uniform distribution of excess carriers (uniform generation rate)	$D_n \frac{\partial^2(\delta n)}{\partial x^2} = 0, \quad D_p \frac{\partial^2(\delta p)}{\partial x^2} = 0$
Zero electric field	$E \frac{\partial(\delta n)}{\partial x} = 0, \quad E \frac{\partial(\delta p)}{\partial x} = 0$
No excess carrier generation	$g' = 0$
No excess carrier recombination (infinite lifetime)	$\frac{\delta n}{\tau_{n0}} = 0, \quad \frac{\delta p}{\tau_{p0}} = 0$

Equation General Solution

See cheating paper.

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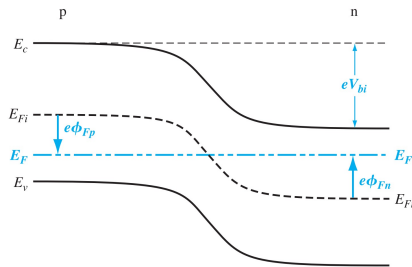
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Built-in Potential Barrier



p⁺p
n⁺n

$$V_{bi} = |\phi_{Fn}| + |\phi_{Fp}|$$

$$= \frac{kT}{e} \ln \left(\frac{N_a N_d}{n_i^2} \right) = V_t \ln \left(\frac{N_a N_d}{n_i^2} \right)$$

V_{bi} : built-in potential barrier.

$V_t = kT/e$ defined as the thermal voltage.

Zero Applied Bias

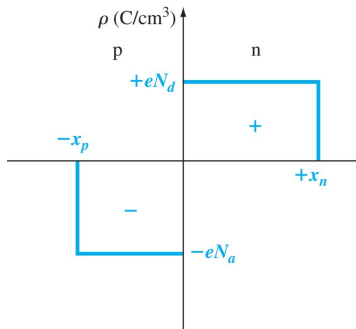


Figure: space charge density

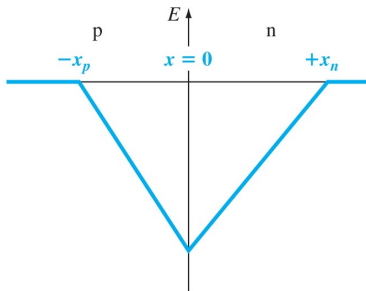


Figure: electric field

$$\underline{N_a x_p = N_d x_n}$$

$$|k| = \frac{eN_a/d}{\epsilon_s}$$

Zero Applied Bias

$$N_a x_p = N_d x_n$$

$$x_n = \sqrt{\frac{2\varepsilon_s (V_{bi} + V_R)}{11.7 \varepsilon_0 e} \left[\frac{N_a}{N_d} \right] \left[\frac{1}{N_a + N_d} \right]}$$

$$x_p = \sqrt{\frac{2\varepsilon_s (V_{bi} + V_R)}{e} \left[\frac{N_d}{N_a} \right] \left[\frac{1}{N_a + N_d} \right]}$$

$\varepsilon_s = \varepsilon_r \varepsilon_0$, where $\varepsilon_0 = 8.85 \times 10^{-14} \text{ F} \cdot \text{cm}^{-1}$.

$\varepsilon_r = 11.7$ for Si.

$$W = x_n + x_p = \sqrt{\frac{2\varepsilon_s (V_{bi} + V_R)}{e} \left[\frac{N_a + N_d}{N_a N_d} \right]}$$

Zero Applied Bias

$$E = \begin{cases} -\frac{eN_a}{\epsilon_s}(x + x_p), & -x_p \leq x \leq 0 \\ -\frac{eN_d}{\epsilon_s}(x_n - x), & 0 \leq x \leq x_n \end{cases}$$

$$|E_{max}| = -\frac{eN_d x_n}{\epsilon_s} = -\frac{eN_a x_p}{\epsilon_s}$$

$$= -\frac{2(V_{bi} + V_R)}{W}$$

V_a

$V_R = -V_a$

$(V_{bi} - V_a)$

$$\phi(x) = -\int E(x) dx$$

Reverse Applied Bias

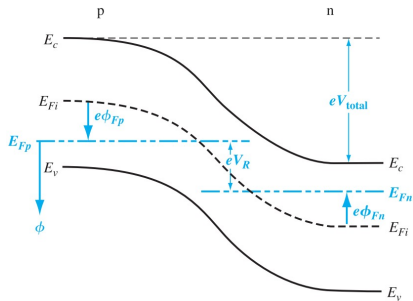
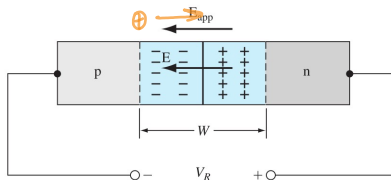


Figure: Energy-band diagram

Junction Capacitance

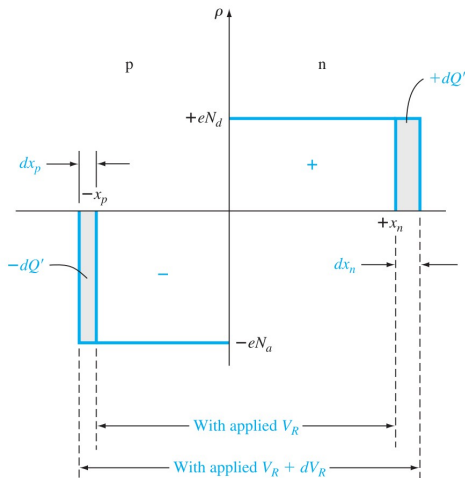


Figure: Differential change in the space charge width with a differential change in reverse-biased voltage for a uniformly doped pn junction

Junction Capacitance

$$C' = \frac{dQ'}{dV_R}$$

$$dQ' = eN_d dx_n = eN_a dx_p$$

dQ' has units of C/cm^2 , and C' has units of F/cm^2 .

$$C' = \sqrt{\frac{e\epsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)}} = \frac{\epsilon_s}{W}$$

One-sided Junction



$$C' \approx \sqrt{\frac{e\epsilon_s N_d}{2(V_{bi} + V_R)}}$$
$$\Rightarrow \left(\frac{1}{C'}\right)^2 = \frac{2(V_{bi} + V_R)}{e\epsilon_s N_d}$$

Experimentally determine V_{bi} and N_d .

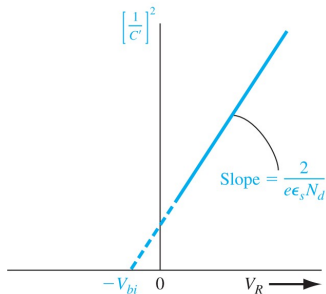


Figure: $(1/C')^2$ versus V_R of a **uniformly** doped pn junction

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$$g = g_0 + g'$$

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$$g_0 = R_0 = \frac{n_0}{\tau_{n0}}$$

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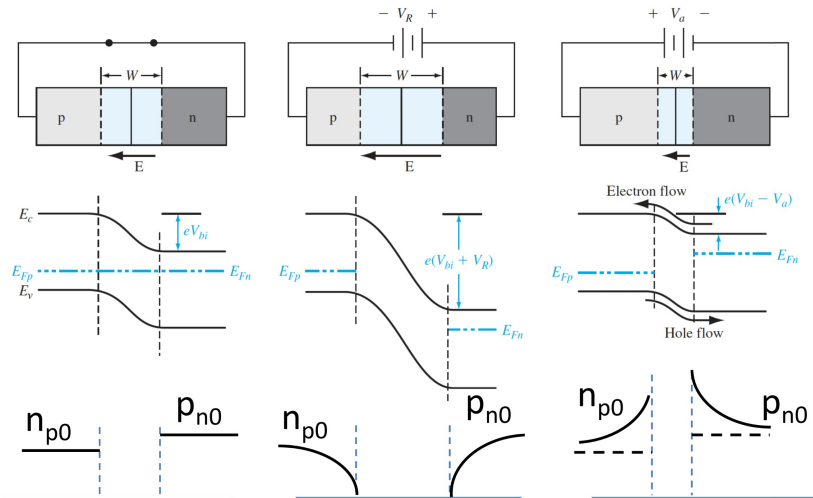
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$$I(v)$$

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Charge Flow



Relation of Concentrations

$$\underline{n_{p0} = n_{n0} \exp\left(-\frac{eV_{bi}}{kT}\right)}$$

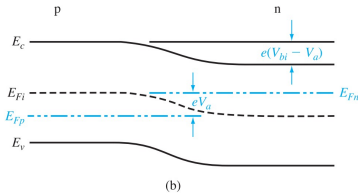
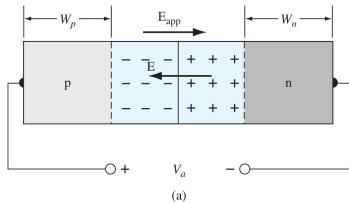
Relates the minority carrier electron concentration on the p side of the junction to the majority carrier electron concentration on the n side of the junction in thermal equilibrium.

$$\underline{V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)}$$

$$n_{n0} \approx N_d$$

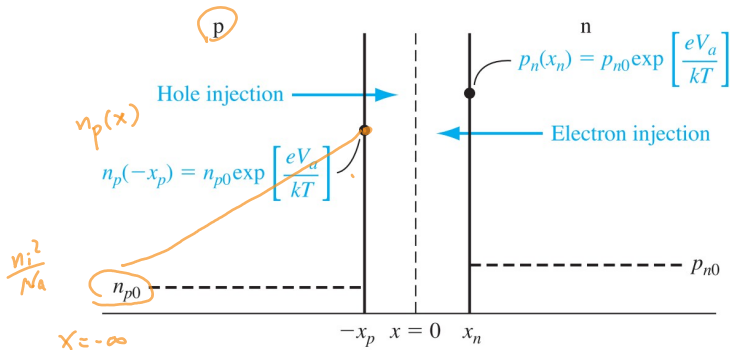
$$n_{p0} \approx \frac{n_i^2}{N_a}$$

Forward Biased



$$\begin{aligned}
 n_p &= n_{n0} \exp \left(-\frac{e(V_{bi} - V_a)}{kT} \right) \\
 &= n_{p0} \exp \left(\frac{eV_a}{kT} \right)
 \end{aligned}$$

Forward Biased



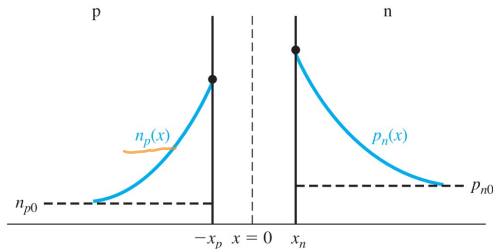
Minority Carrier Distribution

The solution is

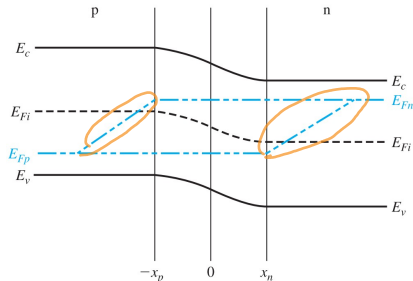
$$\delta p_n(x) = p_n(x) - p_{n0} = p_{n0} \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right] \exp\left(\frac{x_n - x}{L_p}\right), \quad x \geq x_n$$

Similarly,

$$\delta n_p(x) = n_p(x) - n_{p0} = n_{p0} \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right] \exp\left(\frac{x_p + x}{L_n}\right), \quad x \leq -x_p$$



Quasi-Fermi Level

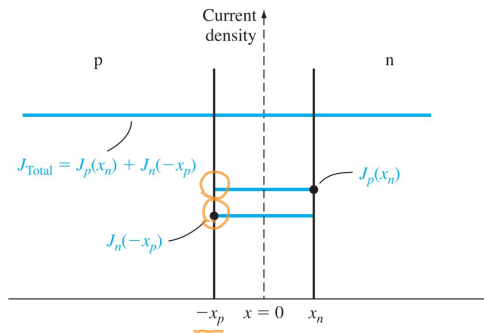


$$p = p_0 + \delta p = n_i \exp \left(\frac{E_{Fi} - E_{Fp}}{kT} \right)$$

Therefore the quasi-Fermi levels are linear functions of distance in the neutral p and n regions as shown in the Figure.

$$np = n_i^2 \exp \left(\frac{E_{Fn} - E_{Fp}}{kT} \right)$$

Ideal - pn Junction Current



$$J_p(x_n) = -eD_p \left. \frac{d(\delta p_n(x))}{dx} \right|_{x=x_n}$$

Ideal - pn Junction Current - Continue

$$J_p(x_n) = \frac{eD_p p_{n0}}{L_p} \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right]$$

$$J_n(-x_p) = \frac{eD_n n_{p0}}{L_n} \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right]$$

Then the total current density

$$J = J_s \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right]$$

where $J_s = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n} \right]$

$$J_s = \dots \sqrt{D_p} + \dots \sqrt{D_n}$$

$$\frac{D_p}{\mu_p} = \frac{kT}{e}$$

Non-Ideal I - Generation-recombination currents

$$R_n = \frac{(np - n_i^2)}{\tau_p \left[n + n_i \exp\left(\frac{E_t - E_i}{kT}\right) \right] + \tau_n \left[p + n_i \exp\left(\frac{E_i - E_t}{kT}\right) \right]}$$

$$= -\frac{n_i}{2\tau} = -G_0 \quad \text{assume } E_t = E_i, \tau_n = \tau_p = \tau$$

$$J_r = \int_0^W q G_0 dx = q G_0 W$$

$$= \frac{q W n_i}{2\tau}$$

$$J = J_s \left[\exp\left(\frac{qV_a}{kT}\right) - 1 \right] + \frac{q W n_i}{2\tau} \left[\exp\left(\frac{qV_a}{2kT}\right) - 1 \right]$$

0.027

0.03...

Non-Ideal II - High Level Injection

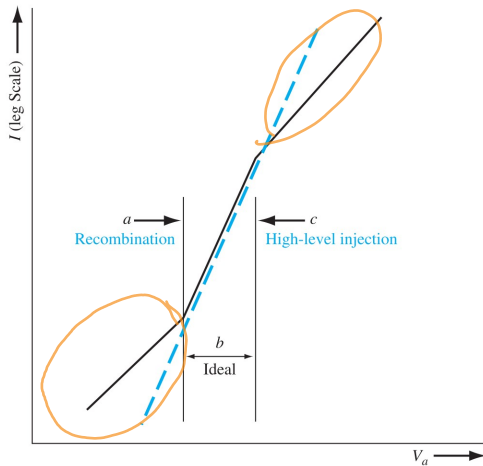


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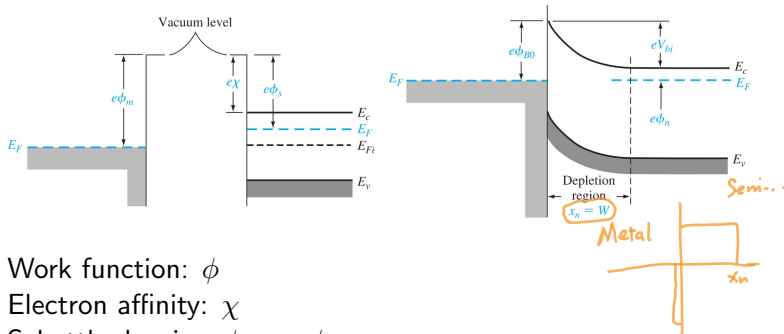
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The Schottky Barrier Diode



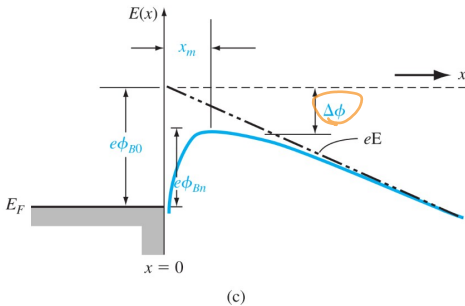
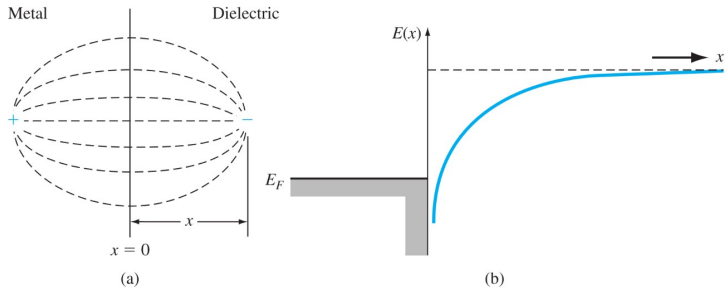
Work function: ϕ

Electron affinity: χ

Schottky barrier: $\phi_{B0} = \phi_m - \chi$

Built-in potential barrier: $V_{bi} = \phi_{B0} - \phi_n$

Non-ideal Effects - Schottky Barrier Lowering



Non-ideal Effects - Schottky Barrier Lowering

$$F = \frac{-e^2}{4\pi\epsilon_s(2x)^2} = -eE$$
$$-\phi(x) = + \int_x^\infty E \, dx' = \frac{-e}{16\pi\epsilon_s x}$$

With electric field: $-\phi(x) = \frac{-e}{16\pi\epsilon_s x} - Ex$

$$\frac{d(e\phi(x))}{dx} = 0 \quad \Rightarrow \quad \Delta\phi = \sqrt{\frac{eE}{4\pi\epsilon_s}}$$

Current-Voltage Relationship

$$J = J_{sT} \left[\exp \left(\frac{eV_a}{kT} \right) - 1 \right]$$

$$J_{sT} = A^* T^2 \exp \left(\frac{-e\phi_{Bn}}{kT} \right)$$

$$A^* = \frac{4\pi e m_n^* k^2}{h^3}$$

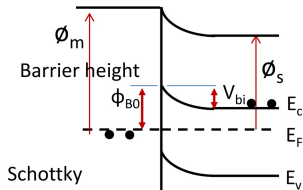
A^* : effective Richardson constant for thermionic emission.

(After plugging in all values, times 10^{-4} to get the unit

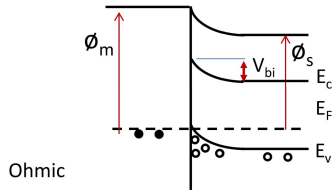
$A \cdot K^{-2} \cdot cm^{-2}$)

$$\phi_m > \phi_s$$

n-type semiconductor

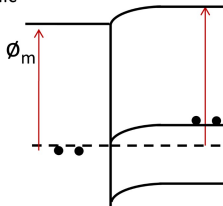


p-type semiconductor

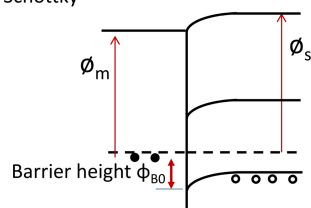


$$\phi_m < \phi_s$$

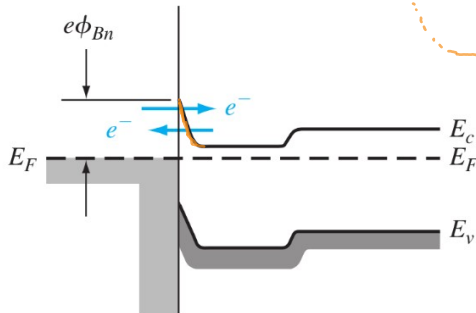
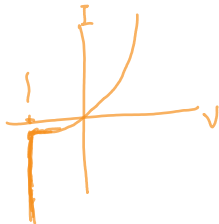
Ohmic



Schottky



Tunneling Barrier



$$\underline{J_t \propto \exp\left(\frac{-e\phi_{Bn}}{E_{oo}}\right)}$$

$$\underline{E_{oo} = \frac{e\hbar}{2} \sqrt{\frac{N_d}{\epsilon_s M_n^*}}}$$

Specific Contact Resistance

R_c defined as the reciprocal of the derivative of current density with respect to voltage evaluated at zero bias.

$$R_c = \left(\frac{\partial J}{\partial V} \right)^{-1} \bigg|_{V=0} \quad \Omega - cm^2$$

$$J_n = A^* T^2 \exp \left(\frac{-e\phi_{Bn}}{kT} \right) \left[\exp \left(\frac{eV}{kT} \right) - 1 \right]$$

$$R_c = \frac{\left(\frac{kT}{e} \right) \exp \left(\frac{+e\phi_{Bn}}{kT} \right)}{A^* T^2}$$

Good luck to your midterm exam!