

# Recitation Class 5

Zexi Li

lzx12138@sjtu.edu.cn

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# Outline

Chapter 6 Nonequilibrium Excess Carriers in Semiconductors

Chapter 7-I The pn junction

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Chapter 7-I The pn junction

# Notations in Chapter 6

**Table 6.1** | Relevant notation used in Chapter 6

Symbol	Definition
$n_0, p_0$	Thermal-equilibrium electron and hole concentrations (independent of time and also usually position)
$n, p$	Total electron and hole concentrations (may be functions of time and/or position)
$\delta n = n - n_0$ $\delta p = p - p_0$	Excess electron and hole concentrations (may be functions of time and/or position)
$g'_n, g'_p$	Excess electron and hole generation rates
$R'_n, R'_p$	Excess electron and hole recombination rates
$\tau_{n0}, \tau_{p0}$	Excess minority carrier electron and hole lifetimes

# Electron-hole Generation & Recombination



$$G_{n0} = G_{p0}, \quad R_{n0} = R_{p0}$$

# Thermal-equilibrium

**Thermal-equilibrium:** the net carrier concentrations are independent of time, which means that the generation and recombination of electrons and holes are equal.

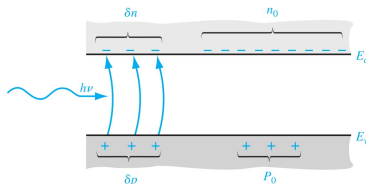
$$G_{n0} = G_{p0} = R_{n0} = R_{p0}$$

**Nonequilibrium:**

$$n = n_0 + \delta n$$

$$p = p_0 + \delta p$$

Note that  $np \neq n_0p_0 = n_i^2$ .



**Figure:** Creation of excess electron and hole densities by photons

# Net Recombination Rate

n-type:

$$R'_n = R'_p = \frac{\delta p(t)}{\tau_{p0}}$$

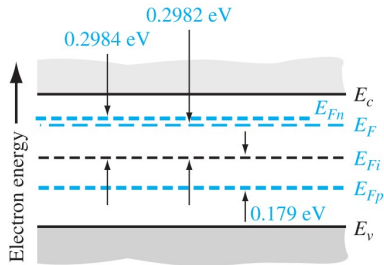
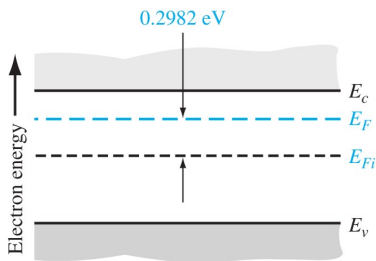
p-type:

$$R'_n = R'_p = \frac{\delta n(t)}{\tau_{n0}}$$

# Quasi-Fermi Energy Level

$$n_0 + \delta n = n_i \exp \left( \frac{E_{Fn} - E_{Fi}}{kT} \right)$$

$$p_0 + \delta p = n_i \exp \left( \frac{E_{Fi} - E_{Fp}}{kT} \right)$$





# Question

## Why we only consider minority excess carrier?

Consider the case where we have a n-type silicon semiconductor, with  $n_0 = 10^{17} \text{ cm}^{-3}$ , and  $p_0 = n_i^2 / n_0 = 2250 \text{ cm}^{-3}$ . And the excess carrier  $\delta n = 10^{14} \text{ cm}^{-3}$ , which is only 0.1% of  $n_0$ . However,  $\delta p = \delta n = 10^{14} \text{ cm}^{-3}$ , which is greatly larger than  $p_0$ .

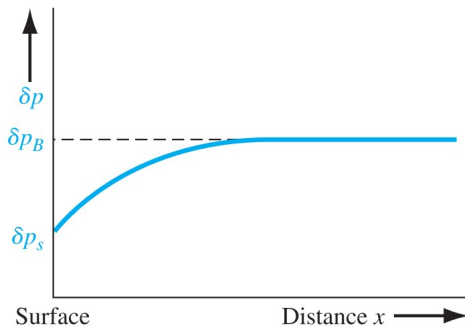
You can also have such feeling from the Quasi-Fermi Energy Level diagram that  $E_{Fn}$  is close to  $E_F$  while  $E_{Fp}$  changes a lot.

## Excess Carrier Lifetime

$$R_n = R_p = \frac{C_n C_p N_t (np - n_i^2)}{C_n (n + n') + C_p (p + p')} \equiv R$$

$$\text{where } n' = N_c \exp \left[ -\frac{E_c - E_t}{kT} \right], \quad p' = N_v \exp \left[ -\frac{E_t - E_v}{kT} \right]$$

# Surface Effects



$$\left. -D_p \left[ \hat{n} \cdot \frac{d(\delta p)}{dx} \right] \right|_{\text{surf}} = s \delta p|_{\text{surf}}$$

# Time-dependent Continuity Equation

$$\begin{aligned} D_n \frac{d^2 n}{dx^2} + \mu_n \left( E \frac{dn}{dx} + n \frac{dE}{dx} \right) + g_n - \frac{n}{\tau_{nt}} &= \frac{dn}{dt} \\ D_p \frac{d^2 p}{dx^2} - \mu_p \left( E \frac{dp}{dx} + p \frac{dE}{dx} \right) + g_p - \frac{p}{\tau_{pt}} &= \frac{dp}{dt} \end{aligned}$$

For homogeneous semiconductor,  $n(x) = n_0 + \delta n(x)$ , the equation can be simplified to

$$\begin{aligned} D_n \frac{d^2(\delta n)}{dx^2} + \mu_n \left( E \frac{d(\delta n)}{dx} + n \frac{dE}{dx} \right) + g_n - \frac{n}{\tau_{nt}} &= \frac{d(\delta n)}{dt} \\ D_p \frac{d^2(\delta p)}{dx^2} - \mu_p \left( E \frac{d(\delta p)}{dx} + p \frac{dE}{dx} \right) + g_p - \frac{p}{\tau_{pt}} &= \frac{d(\delta p)}{dt} \end{aligned}$$

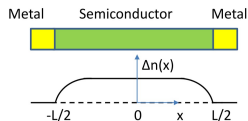
# Equation Simplification

**Table 6.2** | Common ambipolar transport equation simplifications

Specification	Effect
Steady state	$\frac{\partial(\delta n)}{\partial t} = 0, \quad \frac{\partial(\delta p)}{\partial t} = 0$
Uniform distribution of excess carriers (uniform generation rate)	$D_n \frac{\partial^2(\delta n)}{\partial x^2} = 0, \quad D_p \frac{\partial^2(\delta p)}{\partial x^2} = 0$
Zero electric field	$E \frac{\partial(\delta n)}{\partial x} = 0, \quad E \frac{\partial(\delta p)}{\partial x} = 0$
No excess carrier generation	$g' = 0$
No excess carrier recombination (infinite lifetime)	$\frac{\delta n}{\tau_{n0}} = 0, \quad \frac{\delta p}{\tau_{p0}} = 0$

## Example I

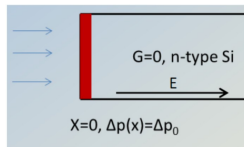
Given a piece of p-type uniformly doped semiconductor in contact with two metal electrodes separated by a length of  $L$ , forming a photoconductor device. The light illumination will create electron-hole pairs at a generation rate of  $g$ . The minority carrier recombination lifetime is  $\tau_0$ . Find the analytical distribution of the excess minority electrons at zero external bias. Note that light illumination will not create excess carriers in metals.



$$D_n \frac{d^2(\delta n)}{dx^2} + \mu_n \left( E \frac{d(\delta n)}{dx} + n \frac{dE}{dx} \right) + g_n - \frac{n}{\tau_{nt}} = \frac{d(\delta n)}{dt}$$

## Example II

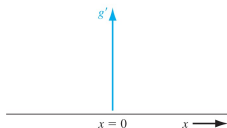
A light beam is illuminated on the surface of a silicon wafer, generating excess carriers  $\Delta p_0$  at the surface ( $x = 0$ ). The wafer is placed in a constant electric field with a known intensity  $E$ . We assume there is no external generation inside the wafer. The thickness of the wafer is infinite. Find the excess minority carriers at equilibrium as a function of the distance away from the surface ( $x = 0$ ). Small injection condition is always maintained and the wafer is uniformly doped as  $N_d$ .



$$D_p \frac{d^2(\delta p)}{dx^2} - \mu_p \left( E \frac{d(\delta p)}{dx} + p \frac{dE}{dx} \right) + g_p - \frac{p}{\tau_{pt}} = \frac{d(\delta p)}{dt}$$

## Example III

Consider a p-type semiconductor that is homogeneous and infinite in extent. Assume a zero applied electric field. For a one-dimensional crystal, assume that excess carriers are being generated at  $x = 0$  only, as indicated in Figure below. The excess carriers being generated at  $x = 0$  will begin diffusing in both the  $+x$  and  $-x$  directions. Calculate the steady-state excess carrier concentration as a function of  $x$ .



$$D_n \frac{d^2(\delta n)}{dx^2} + \mu_n \left( E \frac{d(\delta n)}{dx} + n \frac{dE}{dx} \right) + g_n - \frac{n}{\tau_{nt}} = \frac{d(\delta n)}{dt}$$



# General Solutions – t



$$\frac{d(\delta p)}{dt} = -\frac{\delta p}{\tau_{p0}}$$

solution:

$$\delta p(t) = \delta p(0)e^{-t/\tau_{p0}}$$



$$g' - \frac{\delta p}{\tau_{p0}} = \frac{d(\delta p)}{dt}$$

solution:

$$\delta p(t) = g'\tau_{p0} \left(1 - e^{-t/\tau_{p0}}\right)$$

## General Solutoins – x



$$D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{n0}} = 0$$

solution:

$$\delta n(x) = Ae^{-x/L_n} + Be^{x/L_n}, \quad L_n = \sqrt{D_n \tau_{n0}}$$

special:

$$\delta n(x) = \begin{cases} \delta n(0)e^{-x/L_n}, & x \geq 0 \\ \delta n(0)e^{+x/L_n}, & x \leq 0 \end{cases}$$



$$D_p \frac{d^2 \delta p}{dx^2} - \frac{\delta p}{\tau} + G_{ex} = 0$$

solution:

$$\delta p(x) = A \exp(\lambda x) + g\tau, \quad \lambda = \pm \frac{1}{\sqrt{D_p \tau}}$$

# General Solutions – E



$$D_p \frac{d^2(\delta p)}{dx^2} - \mu_p E_0 \frac{d(\delta p)}{dx} - \frac{\delta p}{\tau_{p0}} = \frac{d(\delta p)}{dt}$$

solution:

$$\delta p(x, t) = \frac{e^{-t/\tau_{p0}}}{(4\pi D_p t)^{1/2}} \exp \left[ \frac{-(x - \mu_p E_0 t)^2}{4D_p t} \right]$$

# General Solitons – E



$$D_p \frac{d^2 \delta p}{dx^2} - \mu_p E \frac{d\delta p}{dx} - \frac{\delta p}{\tau} = 0$$

solution:

$$\delta p(x) = A \exp(\lambda x) + C$$

$$\lambda = \frac{L_p(E) \pm \sqrt{L_p^2(E) + 4L_p^2}}{2L_p^2}, \quad L_p = \sqrt{\tau D_p}, \quad L_p(E) = \tau \mu_p E$$

special:

$$\delta p(x) = \delta p(0) \exp \left[ \frac{L_p(E) \pm \sqrt{L_p^2(E) + 4L_p^2}}{2L_p^2} x \right] = \begin{cases} \delta p(0) \exp \left( -\frac{x}{L_p} \right), & \text{if } L_p(E) \ll L_p \\ \delta p(0) \exp \left( -\frac{x}{L_p(E)} \right), & \text{if } L_p(E) \gg L_p \end{cases}$$

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# Structure

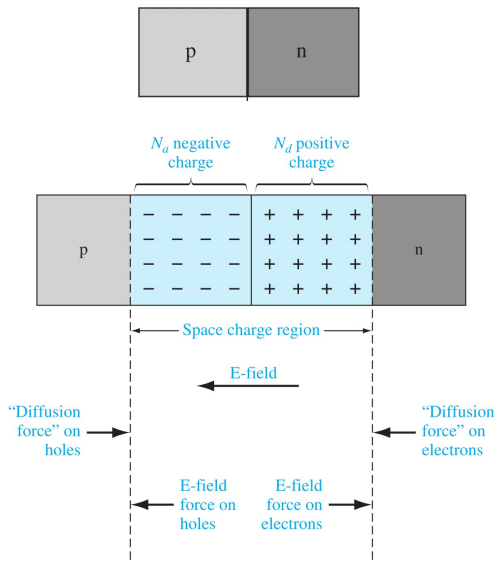


Figure: *space charge region (depletion region)*

# Energy-band Diagram

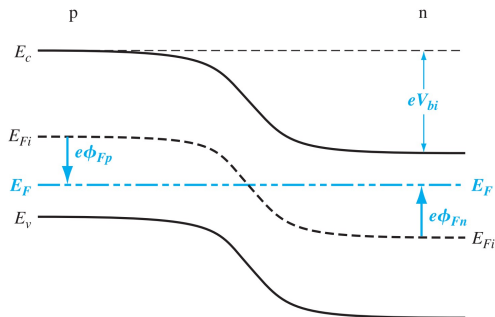
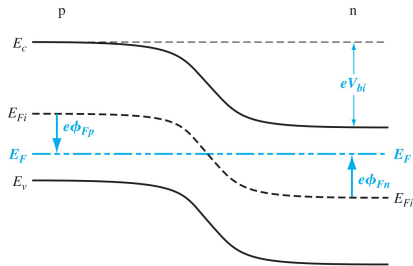


Figure: Energy-band diagram of a pn junction in thermal equilibrium

$V_{bi}$ : built-in potential barrier.

# Built-in Potential Barrier



$$\begin{aligned} V_{bi} &= |\phi_{Fn}| + |\phi_{Fp}| \\ &= \frac{kT}{e} \ln \left( \frac{N_a N_d}{n_i^2} \right) = V_t \ln \left( \frac{N_a N_d}{n_i^2} \right) \quad (\text{not recommended}) \end{aligned}$$

$V_t = kT/e$  defined as the thermal voltage.



# Zero Applied Bias

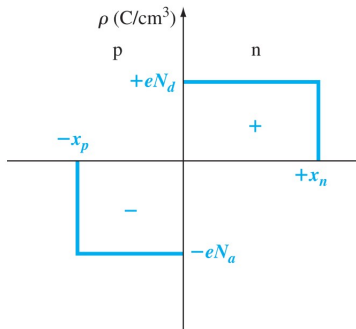


Figure: space charge density

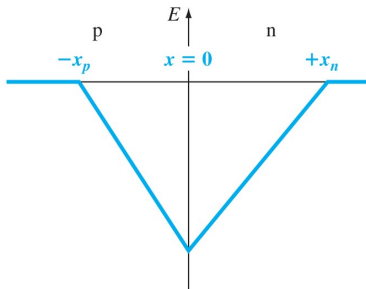


Figure: electric field

$$N_a x_p = N_d x_n$$

$$|k| = \frac{eN_d/d}{\epsilon_s}$$

# Zero Applied Bias

$$N_a x_p = N_d x_n$$

$$x_n = \sqrt{\frac{2\epsilon_s (V_{bi} + V_R)}{e} \left[ \frac{N_a}{N_d} \right] \left[ \frac{1}{N_a + N_d} \right]}$$

$$x_p = \sqrt{\frac{2\epsilon_s (V_{bi} + V_R)}{e} \left[ \frac{N_d}{N_a} \right] \left[ \frac{1}{N_a + N_d} \right]}$$

$\epsilon_s = \epsilon_r \epsilon_0$ , where  $\epsilon_0 = 8.85 \times 10^{-14} \text{ F} \cdot \text{cm}^{-1}$ .

$\epsilon_r = 11.7$  for Si.

$$W = x_n + x_p = \sqrt{\frac{2\epsilon_s (V_{bi} + V_R)}{e} \left[ \frac{N_a + N_d}{N_a N_d} \right]}$$

# Zero Applied Bias

$$E = \begin{cases} -\frac{eN_a}{\epsilon_s}(x + x_p), & -x_p \leq x \leq 0 \\ \frac{eN_d}{\epsilon_s}(x_n - x), & 0 \leq x \leq x_n \end{cases}$$

$$\begin{aligned} |E_{max}| &= -\frac{eN_dx_n}{\epsilon_s} = -\frac{eN_ax_p}{\epsilon_s} \\ &= -\frac{2(V_{bi} + V_R)}{W} \end{aligned}$$

$$\phi(x) = -\int E(x) dx$$

End