Recitation Class 6

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Outline

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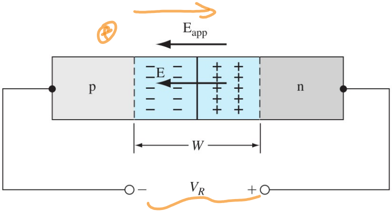
Reverse Applied Bias



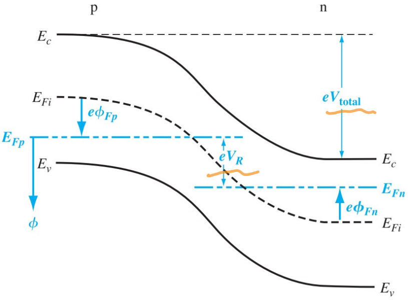


Figure: Energy-band diagram

From RC 5

$$N_a x_p = N_d x_n$$

$$x_n = \sqrt{\frac{2\varepsilon_s \left(V_{bi} + V_R\right)}{e} \left[\frac{N_a}{N_d}\right] \left[\frac{1}{N_a + N_d}\right]}$$

$$x_p = \sqrt{\frac{2\varepsilon_s \left(V_{bi} + V_R\right)}{e} \left[\frac{N_d}{N_a}\right] \left[\frac{1}{N_a + N_d}\right]}$$

$\varepsilon_s = \varepsilon_r \varepsilon_0$, where $\varepsilon_0 = 8.85 \times 10^{-14} F \cdot cm^{-1}$.
$\varepsilon_r = 11.7$ for Si.

$$W = x_n + x_p = \sqrt{\frac{2\varepsilon_s \left(V_{bi} + V_R\right)}{e} \left[\frac{N_a + N_d}{N_a N_d}\right]}$$

From RC 5

$$\boxed{\begin{aligned} E &= \begin{cases} -\dfrac{eN_a}{\varepsilon_s}(x + x_p), & -x_p \leq x \leq 0 \\ \dfrac{eN_d}{\varepsilon_s}(x_n - x), & 0 \leq x \leq x_n \end{cases} \\ |E_{max}| &= -\frac{eN_d x_n}{\varepsilon_s} = -\frac{eN_a x_p}{\varepsilon_s} \\ &= -\frac{2(V_{bi} + V_R)}{W} \\ \phi(x) &= -\int E(x)\,\mathrm{d}x \end{aligned}}$$

From RC 5

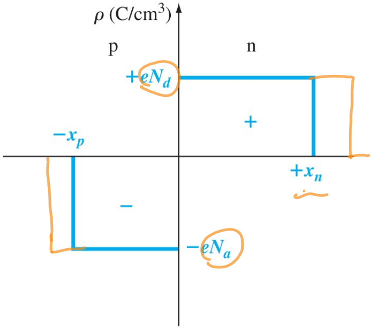


Figure: space charge density

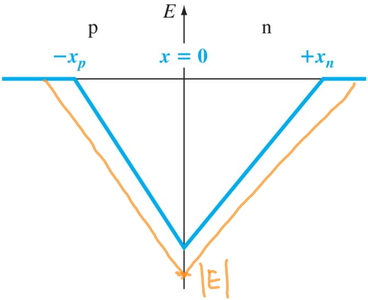


Figure: electric field

$$N_a x_p = N_d x_n$$

$$|k| = \frac{eN_{a/d}}{\varepsilon_s}$$

Junction Capacitance

$$C = \frac{dQ}{dV}$$

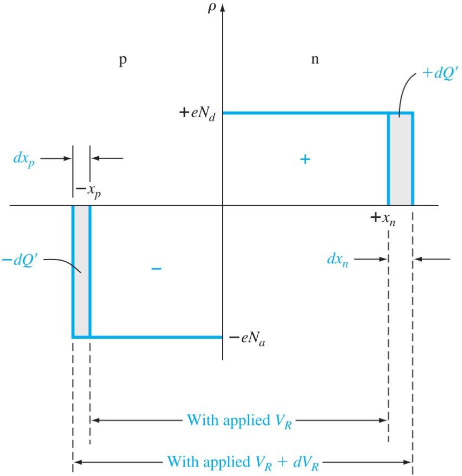


Figure: Differential change in the space charge width with a differential change in reverse-biased voltage for a uniformly doped pn junction

Junction Capacitance

$$C' = \frac{\mathrm{d}Q'}{dV_R}$$

$C = A\,C'$

$$dQ' = eN_d\,\mathrm{d}x_n = eN_a\,\mathrm{d}x_p$$

$\mathrm{d}Q'$ has units of C/cm^2, and C' has units of F/cm^2.

$$\boxed{C' = \sqrt{\frac{e\varepsilon_s N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}} = \frac{\varepsilon_s}{W}}$$

One-sided Junction

p$^+$n junction, $N_a \gg N_d$.

$$x_p \ll x_n$$
$$W \approx x_n$$
$$C' \approx \sqrt{\frac{e\varepsilon_s N_d}{2(V_{bi} + V_R)}}$$

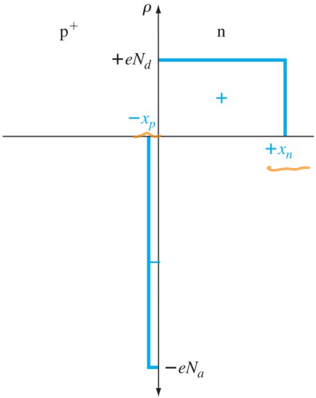


Figure: Space charge density of a one-sided p$^+$n junction.

One-sided Junciton

$$C' \approx \sqrt{\frac{e\varepsilon_s N_d}{2(V_{bi} + V_R)}}$$

$$\Rightarrow \boxed{\left(\frac{1}{C'}\right)^2 = \frac{2(V_{bi} + V_R)}{e\varepsilon_s N_d}}$$

Experimentally determine V_{bi} and N_d.

$\left(\frac{1}{C'}\right)^2 \propto V_{bi} + V_R$

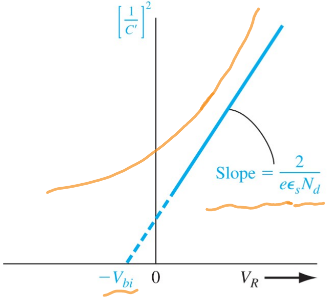


Figure: $(1/C')^2$ versus V_R of a uniformly doped pn junction

$\frac{2}{e\,\varepsilon_s N_d(x)}$

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Notations

n_{p0} (p side
electron concentration

Term	Meaning
N_a	Acceptor concentration in the p region of the pn junction
N_d	Donor concentration in the n region of the pn junction
$n_{n0} = N_d$	Thermal-equilibrium majority carrier electron concentration in the n region
$p_{p0} = N_a$	Thermal-equilibrium majority carrier hole concentration in the p region
$n_{p0} = n_i^2/N_a$	Thermal-equilibrium minority carrier electron concentration in the p region
$p_{n0} = n_i^2/N_d$	Thermal-equilibrium minority carrier hole concentration in the n region
n_p	Total minority carrier electron concentration in the p region
p_n	Total minority carrier hole concentration in the n region
$n_p(-x_p)$	Minority carrier electron concentration in the p region at the space charge edge
$p_n(x_n)$	Minority carrier hole concentration in the n region at the space charge edge
$\delta n_p = n_p - n_{p0}$	Excess minority carrier electron concentration in the p region
$\delta p_n = p_n - p_{n0}$	Excess minority carrier hole concentration in the n region

Ideal Current–Voltage Relationship

1. The abrupt depletion layer approximation applies. The space charge regions have abrupt boundaries, and the semiconductor is neutral outside of the depletion region.
2. The Maxwell–Boltzmann approximation applies to carrier statistics.
3. The concepts of low injection and complete ionization apply.
4. a) The total current is a constant throughout the entire pn structure.
   b) The individual electron and hole currents are continuous functions through the pn structure.
   c) The individual electron and hole currents are constant throughout the depletion region.

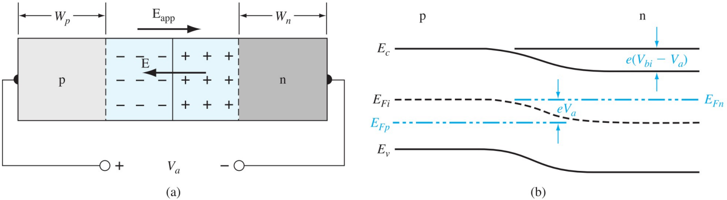
Concentration Relation on Two Sides

$$\left\{\begin{aligned} V_{bi} &= V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \\ n_{n0} &\approx N_d \\ n_{p0} &\approx \frac{n_i^2}{N_a} \end{aligned}\right.$$

$$\Longrightarrow \quad n_{p0} = n_{n0} \exp\left(-\frac{eV_{bi}}{kT}\right)$$

Relates the minority carrier electron concentration on the p side of the junction to the majority carrier electron concentration on the n side of the junction in thermal equilibrium.

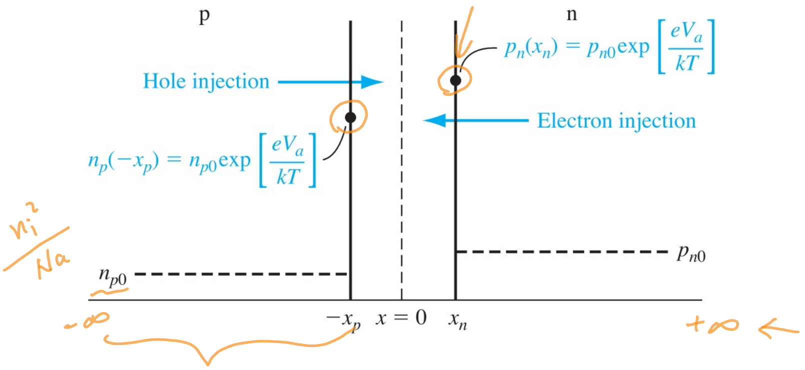
Forward Biased



$$n_p = n_{n0} \exp\left(-\frac{e(V_{bi} - V_a)}{kT}\right)$$

$$= n_{p0} \exp\left(\frac{eV_a}{kT}\right)$$

Forward Biased



Minority Carrier Distribution

$$D_p \frac{\mathrm{d}^2(\delta p_n)}{\mathrm{d}x^2} - \mu_p \left(E \frac{\mathrm{d}(\delta p_n)}{\mathrm{d}x} + p \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g' - \frac{\delta p_n}{\tau_{pt}} = 0$$

Assumption: the electric field is zero in both the neutral p and n regions.
In n region for $x > x_n$, we have $g' = 0$.
The equation becomes

$$\frac{\mathrm{d}^2(\delta p_n)}{\mathrm{d}x^2} - \frac{\delta p_n}{L_p^2} = 0, \quad (x > x_n), \quad L_n^2 = D_n \tau_{n0}$$

Solve it with boundary conditions

$$p_n(x_n) = p_{n0} \exp\left(\frac{eV_a}{kT}\right)$$

$$p_n(x \to +\infty) = p_{n0}$$

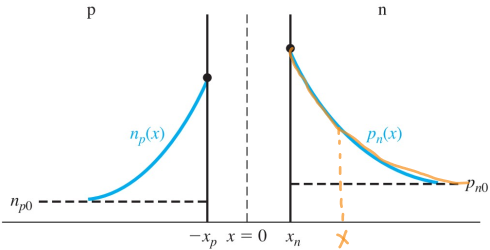
Minority Carrier Distribution - Continue

The solution is

$$\boxed{\delta p_n(x) = p_n(x) - p_{n0} = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_n - x}{L_p}\right), \quad x \geq x_n}$$

Similarly,

$$\boxed{\delta n_p(x) = n_p(x) - n_{p0} = n_{p0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_p + x}{L_n}\right), \quad x \leq -x_p}$$



Quasi-Fermi Level

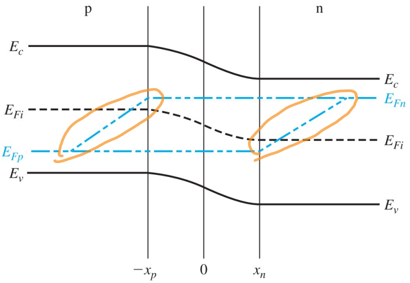


Figure: Quasi-Fermi levels through a forward-biased pn junction.

In Chapter 6 there are equations

$$p = p_0 + \delta p = \underline{n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)}$$

Therefore the quasi-Fermi levels are linear functions of distance in the neutral p and n regions as shown in the Figure.

Quasi-Fermi Level - Continue

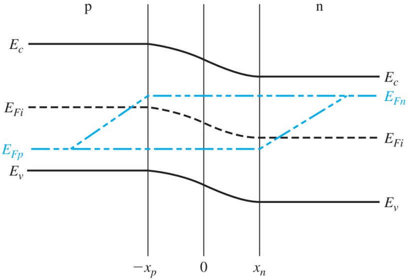


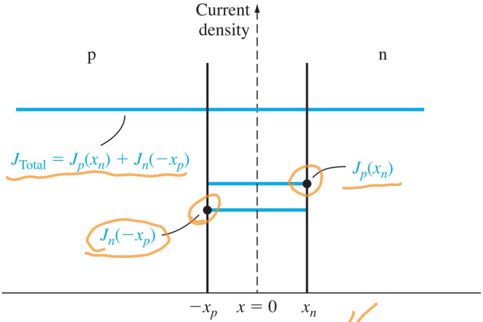
Figure: Quasi-Fermi levels through a forward-biased pn junction.

Also,

$$np = n_i^2 \exp\left(\frac{E_{Fn} - E_{Fp}}{kT}\right)$$

Ideal pn Junction Current

Assumption 4(a): The total current is a constant throughout the entire pn structure.



$$J_p(x_n) = -eD_p \left. \frac{\mathrm{d}\left(\delta p_n(x)\right)}{\mathrm{d}x} \right|_{x=x_n}$$

Ideal pn Junction Current - Continue

$$J_p(x_n) = \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

$$J_n(-x_p) = \frac{eD_n n_{p0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

Then the total current density

$$\boxed{\begin{gathered} J = J_s\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] \\ \text{where } J_s = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right] \end{gathered}}$$

Ideal pn Junction Current - Continue

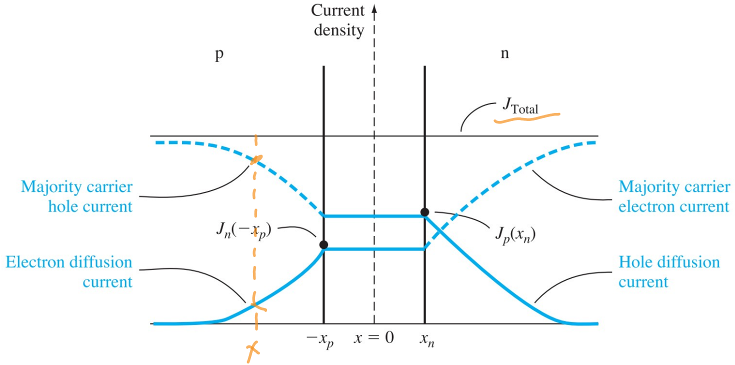


Figure: Idea electron and hole current components through a pn junction under forward bias.

Example

Consider the following parameters in a silicon pn junction at $T = 300K$:

$$N_a = N_d = 10^{16} cm^{-3} \qquad n_i = 1.5 \times 10^{10} cm^{-3}$$
$$D_n = 25 cm^2/s \qquad \tau_{p0} = \tau_{n0} = 5 \times 10^{-7} s$$
$$D_p = 10 cm^2/s \qquad \varepsilon_r = 11.7$$

a) Determine the ideal reverse-saturation current density.
b) Calculate the electric field in a neutral region of a silicon diode to produce a given majority carrier drift current density.

(Textbook Example 8.2 & 8.4)

Solution

Solution:

a) $\boxed{}$

b) $\boxed{}$

Comment:

a) The ideal reverse-biased saturation current density is very small. 10^{-11}

b) We assumed, in the derivation of the current–voltage equation, that the electric field in the neutral p and n regions was zero. Although the electric field is not zero, this example shows that the magnitude is very small—thus the approximation of zero electric field is very good.

End