Resistivity:

Conductors	Semiconductors	Insulators
$< 10^{-3}$ Ω•cm	$10^{-3} - 10^9$ Ω•cm	$> 10^9$ Ω•cm
Metals (Au, Al, Cu, Hg…)	Si, Ge, GaAs, InP…	SiO$_2$, HfO$_2$ …
Solids, liquids (Hg)	Solids	Solids, liquids gases

Unit cell: any small volume of crystal to reproduce the entire crystal.
Primitive cell: smallest unit cell
Crytalline Plane and Miller Index

Reciprocals of the intercepts: (1/3, ½, 1)

Multiply by the lowest common denominator: (2,3,6)
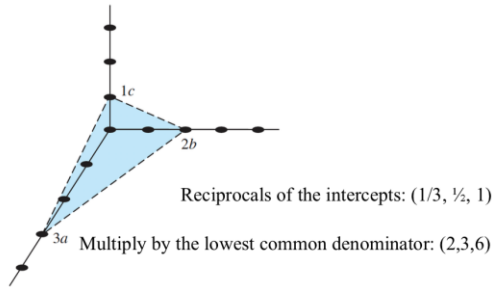
$$\frac{\partial^2 y}{\partial x^2} = k^2 y$$

General solution: $y = Ae^{bx}$

Plug into the equation: $b^2 Ae^{bx} = k^2 Ae^{bx}$

$$\Rightarrow b = \pm k$$
$$\Rightarrow y = A_1 e^{kx} + A_2 e^{-kx}$$

$$\frac{\partial^2 y}{\partial x^2} = -k^2 y$$

General solution: $y = Ae^{bx}$

Plug into the equation: $b^2 Ae^{bx} = -k^2 Ae^{bx}$

$$\Rightarrow b = \pm ki$$
$$\Rightarrow y = A_1 e^{ikx} + A_2 e^{-ikx}$$

$$K = \frac{2\pi}{\lambda}, E = mc^2 = h\nu = \frac{hc}{\lambda}, p = \frac{h}{\lambda} = mv$$

$$\frac{\partial^2 \psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V(x))\psi(x) = 0$$

$$E = \frac{k^2 \hbar^2}{2m} = \frac{n^2 \pi^2 \hbar^2}{2ma^2}$$

$$k = \frac{n\pi}{a} \quad n = 0, \pm 1, \pm 2, \dots$$

$p = \hbar k = mv$

$$\frac{dE}{dk} = \frac{\hbar^2 k}{m} \xrightarrow{mv = \hbar k} \frac{\hbar mv}{m} = \hbar v$$

$E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$

$$v = \frac{1}{\hbar}\frac{dE}{dk}$$

$$J = qNv_d = q\sum_i^N v_i$$

Conduction Band:

$$E = E(k) = E_c + \frac{\hbar^2}{2m_n^*}(k - k_1)^2$$

Valence Band:
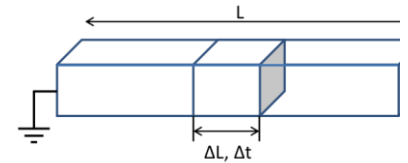
$$E = E(k) = E_c - \frac{\hbar^2}{2m_p^*}(k - k_2)^2$$

$E - E_c = C_1(k)^2$

$\frac{1}{\hbar^2}\frac{d^2 E}{dk^2} = \frac{2C_1}{\hbar^2}$

$\frac{1}{\hbar^2}\frac{d^2 E}{dk^2} = \frac{2C_1}{\hbar^2} = \frac{1}{m^*}$

n type semiconductor

$I = \frac{\Delta Q}{\Delta t} = \frac{nqA_c \Delta L}{\Delta t} = nqA_c v$

$v = \mu E = \mu V/L$

$I = \frac{\Delta Q}{\Delta t} = \frac{nqA_c \Delta L}{\Delta t} = nqA_c \mu V/L \quad \Rightarrow \quad \sigma = \frac{I}{V} = \frac{N_D qA_c \mu}{L}$

$f_F(E) = \dfrac{1}{1 + \exp\left(\dfrac{E - E_F}{kT}\right)}$

$\boxed{f_F(E) \approx \exp\left(-\dfrac{E - E_F}{kT}\right)}$

$$E = E(k) = E_c + \frac{\hbar^2}{2m_n^*}k^2$$
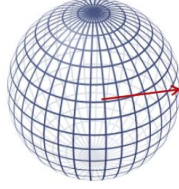
$$k = \mp\frac{\sqrt{2m_n^*(E - E_c)}}{\hbar}$$

Within ΔE, we have the number of k is $\dfrac{d(k/\pi)}{dE}\Delta E$

$$g(E) = \frac{1}{2}\frac{d(k/\pi)}{dE}$$

$$E = E(k) = E_c + \frac{\hbar^2}{2m_n^*}k^2$$

$$k = \mp \frac{\sqrt{2m_n^*(E - E_c)}}{\hbar}$$

Within ΔE, we have the number of k is $\frac{d\left(4\pi\left(\frac{k}{\pi}\right)^2/3\right)}{dE}\Delta E$

$$g(E) = \frac{1}{8}\frac{d\left(4\pi\left(\frac{k}{\pi}\right)^2/3\right)}{dE}$$

$$g_v(E) = \frac{4\pi(2m_p^*)^{3/2}}{h^3}\sqrt{E_v - E}$$

$$n_0 = \int_{E_c}^{\infty} g_c(E)f_F(E)dE$$

$$p_0 = \int_{-\infty}^{E_v} g_v(E)[1 - f_F(E)]dE$$

$$if\ \exp(x - \varepsilon) > 10 \Leftrightarrow \frac{E - E_F}{kT} > 3 \Leftrightarrow E - E_F > 3kT$$

$$n_0 = \frac{2(2\pi m_n^* kT)^{\frac{3}{2}}}{h^3}\exp\left(\frac{E_F - E_c}{kT}\right) = N_c\exp(\frac{E_F - E_c}{kT})$$

$$p_0 = \frac{2(2\pi m_p^* kT)^{\frac{3}{2}}}{h^3}\exp\left(\frac{E_v - E_F}{kT}\right) = N_v\exp(\frac{E_v - E_F}{kT})$$

$$n \times p = n_i^2 = N_cN_v\exp\left(\frac{E_v - E_c}{kT}\right) \Rightarrow n_i = \sqrt{N_cN_v}\exp(-\frac{E_g}{2kT})$$

$$n = N_c\exp(\frac{E_F - E_c}{kT}) \qquad p = N_v\exp(\frac{E_v - E_F}{kT})$$

$N_c \approx 10^{19}cm^{-3}$

$N_v \approx 10^{19}cm^{-3}$

$$n_0 = n_i\exp\left[\frac{E_F - E_{Fi}}{kT}\right] \qquad p_0 = n_i\exp\left[\frac{-(E_F - E_{Fi})}{kT}\right]$$

$n_i \approx 10^{10}cm^{-3}$

$$n = N_c\exp\left(\frac{E_{Fi} - E_c}{kT}\right) = p = N_v\exp(\frac{E_v - E_{Fi}}{kT})$$

$$E_{Fi} = \frac{1}{2}(E_c + E_v) + \frac{1}{2}kTln(\frac{N_v}{N_c})$$

$$E_{midgap} = \frac{1}{2}(E_c + E_v)$$

$$E_{Fi} = E_{midgap} + \frac{3}{4}kTln(\frac{m_p^*}{m_n^*})$$

$$n_d = N_d - N_d^+$$

$$= \frac{N_d}{1 + \frac{1}{2}\exp(\frac{E_d - E_F}{kT})}$$

$$p_a = \frac{N_a}{1 + \frac{1}{g}\exp\left(\frac{E_F - E_a}{kT}\right)} = N_a - N_a^-$$

$$n_0 + (N_a - p_a) = p_0 + (N_d - n_d)$$

$$n_0 = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

$$n_0 = \frac{N_d^+ + \sqrt{(N_d^+)^2 + 4n_i^2}}{2}\ (but\ N_d^+\ unknown)$$

$$n_0 = N_c \times \frac{-1 + \sqrt{1 + \frac{8N_D}{N_c}\exp(\frac{E_A}{kT})}}{4\exp(\frac{E_A}{kT})} = \begin{cases} \sqrt{\frac{N_DN_c}{2}}\exp(-\frac{E_A}{2kT}) & partial\ ionization, \\ N_D & complete\ ionization \end{cases}$$

$$n_0 = \frac{N_D + \sqrt{N_D^2 + 4n_i^2}}{2}\quad \text{Complete ionization at high T}$$

Intrinsic

$$E_F = E_c + kTln(\frac{\sqrt{1 + \frac{8N_D}{N_c}\exp(\frac{E_A}{kT})} - 1}{4\exp(\frac{E_A}{kT})}) = \begin{cases} \frac{E_c + E_D}{2} + \frac{kT}{2}ln\frac{N_D}{2N_c} & T\ small \\ E_c - kTln\frac{N_c}{N_D} & T\ big \end{cases}$$

$$v_d \approx \left(\frac{q\tau_{cp}}{m_{cp}^*}\right)E \Rightarrow \frac{v_d}{E} = \frac{q\tau_{cp}}{m_{cp}^*} = \mu_p\ (for\ holes)$$

$$\frac{v_d}{E} = \frac{q\tau_{cn}}{m_{cn}^*} = \mu_n\ (for\ electrons)$$

$$I_{drf} = \frac{\Delta Q}{\Delta t} = \frac{qp_0A_c\Delta L}{\Delta t} = qp_0A_cv = qp_0A_c\mu_pE = qp_0A_c\mu_p\frac{V}{L} = \sigma \cdot V$$

$$J_{drf} = q(p_0\mu_p + n_0\mu_n)E$$

$$\frac{1}{u} = \frac{1}{\mu_L} + \frac{1}{\mu_I}$$

$$v_n = \frac{v_s}{\left[1 + \left(\frac{E_{on}}{E}\right)^2\right]^{1/2}} \quad v_p = \frac{v_s}{\left[1 + \left(\frac{E_{op}}{E}\right)^2\right]^{1/2}}$$

$$J_{nx|dif} = -qF_n = qD_n\frac{dn}{dx}$$

$$J_{px|dif} = qF_p = -qD_p\frac{dp}{dx}$$

$$J = qn\mu_nE_x + qp\mu_pE_x + qD_n\nabla n - qD_p\nabla p$$

$$E_x = -\frac{d\phi}{dx} = \frac{1}{e}\frac{dE_i}{dx}$$

$$= \frac{1}{q}\frac{kT}{n(x)}\frac{dn(x)}{dx}$$

$$\boxed{D_n = \frac{\mu_n kT}{q}}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{q\mu_n n} = \frac{1}{q\mu_n N_d} \qquad \rho = \frac{1}{\sigma} = \frac{1}{q\mu_n p} = \frac{1}{q\mu_n N_a}$$
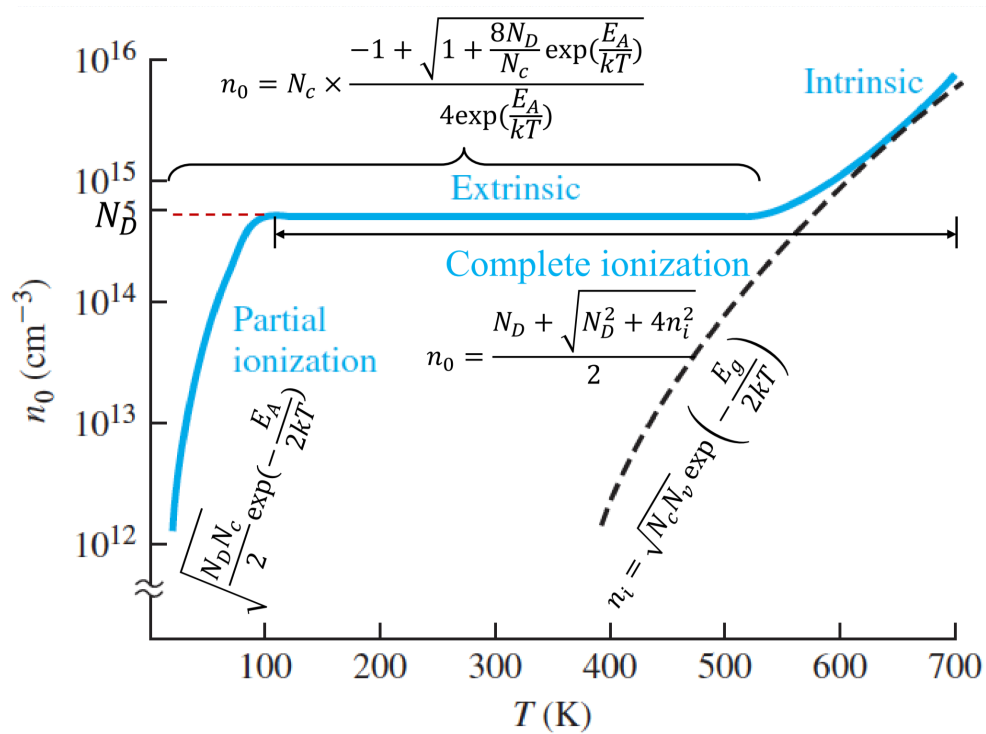
$$n_0 = N_c \times \frac{-1 + \sqrt{1 + \frac{8N_D}{N_c}\exp(\frac{E_A}{kT})}}{4\exp(\frac{E_A}{kT})}$$

Intrinsic

Extrinsic

Complete ionization

Partial ionization

$$n_0 = \frac{N_D + \sqrt{N_D^2 + 4n_i^2}}{2}$$

$$\sqrt{\frac{N_D N_c}{2}}\exp\left(-\frac{E_A}{2kT}\right)$$

$$n_i = \sqrt{N_c N_v}\exp\left(-\frac{E_g}{2kT}\right)$$

n_0 (cm^{-3})

10^{16}
10^{15}
N_D
10^{14}
10^{13}
10^{12}

T (K)

100 200 300 400 500 600 700

Table B.2 | Conversion factors

	Prefixes		
1 Å (angstrom) = 10^{-8} cm = 10^{-10} m	10^{-15}	femto-	= f
1 μm (micrometer) = 10^{-4} cm	10^{-12}	pico-	= p
1 mil = 10^{-3} in. = 25.4 μm	10^{-9}	nano-	= n
2.54 cm = 1 in.	10^{-6}	micro-	= μ
1 eV = 1.6×10^{-19} J	10^{-3}	milli-	= m
1 J = 10^{7} erg	10^{+3}	kilo-	= k
	10^{+6}	mega-	= M
	10^{+9}	giga-	= G
	10^{+12}	tera	= T

Table B.3 | Physical constants

Avogadro's number	$N_A = 6.02 \times 10^{+23}$ atoms per gram molecular weight
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J/K $= 8.62 \times 10^{-5}$ eV/K
Electronic charge (magnitude)	$e = 1.60 \times 10^{-19}$ C
Free electron rest mass	$m_0 = 9.11 \times 10^{-31}$ kg
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H/m
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-14}$ F/cm $= 8.85 \times 10^{-12}$ F/m
Planck's constant	$h = 6.625 \times 10^{-34}$ J-s $= 4.135 \times 10^{-15}$ eV-s
	$\dfrac{h}{2\pi} = \hbar = 1.054 \times 10^{-34}$ J-s
Proton rest mass	$M = 1.67 \times 10^{-27}$ kg
Speed of light in vacuum	$c = 2.998 \times 10^{10}$ cm/s
Thermal voltage (T = 300 K)	$V_t = \dfrac{kT}{e} = 0.0259$ V
	$kT = 0.0259$ eV

Table B.4 | Silicon, gallium arsenide, and germanium properties (T = 300 K)

Property	Si	GaAs	Ge
Atoms (cm^{-3})	5.0×10^{22}	4.42×10^{22}	4.42×10^{22}
Atomic weight	28.09	144.63	72.60
Crystal structure	Diamond	Zincblende	Diamond
Density (g/cm^3)	2.33	5.32	5.33
Lattice constant (Å)	5.43	5.65	5.65
Melting point (°C)	1415	1238	937
Dielectric constant	11.7	13.1	16.0
Bandgap energy (eV)	1.12	1.42	0.66
Electron affinity, χ (V)	4.01	4.07	4.13
Effective density of states in conduction band, N_c (cm^{-3})	2.8×10^{19}	4.7×10^{17}	1.04×10^{19}
Effective density of states in valence band, N_v (cm^{-3})	1.04×10^{19}	7.0×10^{18}	6.0×10^{18}
Intrinsic carrier concentration (cm^{-3})	1.5×10^{10}	1.8×10^{6}	2.4×10^{13}
Mobility (cm^2/V-s)			
Electron, μ_n	1350	8500	3900
Hole, μ_p	480	400	1900
Effective mass $\left(\dfrac{m^*}{m_0}\right)$			
Electrons	$m_l^* = 0.98$ $m_t^* = 0.19$	0.067	1.64 0.082
Holes	$m_{lh}^* = 0.16$ $m_{hh}^* = 0.49$	0.082 0.45	0.044 0.28
Density of states effective mass			
Electrons $\left(\dfrac{m_{dn}^*}{m_o}\right)$	1.08	0.067	0.55
Holes $\left(\dfrac{m_{dp}^*}{m_o}\right)$	0.56	0.48	0.37
Conductivity effective mass			
Electrons $\left(\dfrac{m_{cn}^*}{m_o}\right)$	0.26	0.067	0.12
Holes $\left(\dfrac{m_{cp}^*}{m_o}\right)$	0.37	0.34	0.21

Table B.5 | Other semiconductor parameters

Material	E_g (eV)	a (Å)	ϵ_r	χ	$\bar{n}$
Aluminum arsenide	2.16	5.66	12.0	3.5	2.97
Gallium phosphide	2.26	5.45	10	4.3	3.37
Aluminum phosphide	2.43	5.46	9.8		3.0
Indium phosphide	1.35	5.87	12.1	4.35	3.37

Table B.6 | Properties of SiO_2 and Si_3N_4 (T = 300 K)

Property	SiO_2	Si_3N_4
Crystal structure	[Amorphous for most integrated circuit applications]	
Atomic or molecular density (cm^{-3})	2.2×10^{22}	1.48×10^{22}
Density (g/cm^3)	2.2	3.4
Energy gap	≈ 9 eV	4.7 eV
Dielectric constant	3.9	7.5
Melting point (°C)	≈ 1700	≈ 1900