CS70 HW8

March 14, 2021

1 Counting, Counting, and More Counting

(a) 2^n
$\binom{n+k}{k}$. Didn't see the existence of K.

(b) $\binom{52}{13}$
No aces: $\binom{48}{13}$
Four aces: $\binom{48}{9}$
6 spades: $\binom{13}{6} \cdot \binom{39}{7}$

(c) $\frac{104!}{2^{52}}$

(d) $\binom{99}{50} \cdot 2^{49}$
We haven't carefully consider the situation where my answer has repetition. The answer shall be

$$\sum_{i=50}^{99} \binom{99}{i}.$$

And we can simplify this equation, using $\sum_{i=50}^{99} \binom{99}{i} = \sum_{i=0}^{49} \binom{99}{i}$. Thus $2 * A = 2^{99} \implies A = 2^{98}$.

Another smart way to view the problem, using symmetry, since the total number is odd, so the situation is rather there are more 1s than 0s or the reverse. The total number is $2^{99} \implies$ N(more 1s than 0s) $= 2^{98}$

(e) FLORIDA: 7!
ALASKA: $\frac{6!}{3!}$
ALABAMA: $\frac{7!}{4!}$
MONTANA: $\frac{7!}{2!2!}$

(f) (1) 5! (2) 6!/2

(g) 27^9

(h) 7^2
Didn't see the balls are identical here. After distributing each bin a ball, we are left with 2. We have 6 bars, 6 + 2 = 8 stars. Therefore $\binom{8}{2}$

(i) $\binom{35}{26}$

(j) $\frac{20!}{2^{10}10!}$

(k) $\binom{n+k}{k}$

(l) $\binom{n-3}{1}$
$\binom{n-1}{1}$. # of bars: 1. # of stars: 1 + n - 2 = 1 + n.

(m) $\binom{n}{k}$
$\binom{n-1}{k}$. # of bars: k. # of stars: k + n - k - 1 = n - 1.

2 Binomial Beads

(a) $\binom{n}{k}$.

(b) Value $= x^k \cdot y^{n-k}$.

(c) $\sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$.

(d) Since in part (c), the form summation is exactly same as RHS of the equation. And the LHS is another way of counting total value. From this induction relation,

$$V(n+1) = (x+y)V(n)$$
$$V(1) = x + y$$

Thus the answer is also $(x+y)^n$. So we can conclude the binomial theorem.

3 Minesweeper

(a) (i) 10 mines, 54 not mines. Thus $10/(54+10) = \frac{5}{32}$.

(ii) $\binom{55}{10}/\binom{64}{10}$.

(iii) $\binom{8}{k}\binom{55}{10-k}/\binom{64}{10}$. (Why 8 ? The one you pick is not a mine)

(b) Pick near: $k/8$; Pick different one: $(10-k)/55$.
When this two equals, $63k = 80$. Thus $k = 1.2xxx$. So when $k \leq 1$, picks near. When $n \geq 2$, pick different one.

(c) The possibility space for the first pick being number 1 : $\Omega = \binom{8}{1}\binom{55}{9}$. The possibility space of the event $|A| = 4 * \binom{52}{6}$. Thus $P(A) = 4 * \binom{52}{6}/\binom{8}{1}\binom{55}{9}$.

4 Playing Strategically

(a) $P(E_1) = \frac{1}{3} + \frac{2}{3} * \frac{1}{3} * \frac{1}{3} + \dots = \frac{1}{3} * [1 + \frac{2}{9} + (\frac{2}{9})^2 + \dots] = \frac{1}{3} * \frac{9}{7} = \frac{3}{7}$;

(b) $P(E_2) = 1/3 * P(E_1) = 1/7$;

(c) E_3: Bob against Carol with first shooting.
E_4: Bob against Carol with second shooting. $P(E_3) = 1/3; P(E_4) = 0$;

(d) Now we wanna show that for Eve: he will shoot Carol before Bob: Since in dual, he has bigger possibility to win, besides, if he shoots Bob and succeed, he will definitely lose. And for Carol: shoot Eve before Bob, which is obvious Eve has stronger shooting skill than Bob.

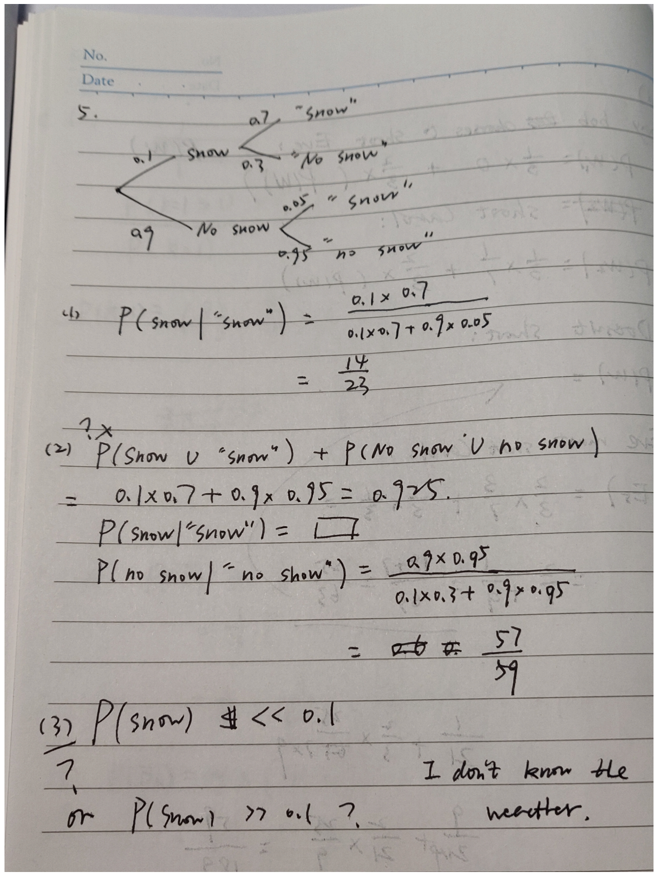
Once we have these two facts, now we compute Bob winning rate with following events:

1. Bob shoots Eve: $P(W_1) = 1/3 * 0 + 2/3 * P(W)$, where $P(W)$ is the winning rate as same as shooting air.
2. Bob shoots Carol: $P(W_2) = 1/3 * 1/7 + 2/3 * P(W)$, where $P(W)$ is the winning rate as same as shooting air.
3. Bob shoots air: now we compute $P(W)$. Since Eve will shoot Carol, with 2/3 possibility that it will become E_1; 1/3 possibility that it will become E_3, since Eve will be shot by Carol next. $P(W) = 2/7 + 1/9 = 25/63$. Now we compare winning rate, $P(W_1) < P(W)$ is obvious, $P(W_2) = 1/21 + 2/3 * P(W) = 59/189 < 26/63$.

Therefore, with above analysis, Bob better shoots air.

5 Weathermen

Since this question I draw a tree and I don't know how to rep it in LaTex. Therefore I just post here.



5.

$P(\text{snow}) = 0.1$: 0.7 "snow", 0.3 "No snow"

$P(\text{No snow}) = 0.9$: 0.05 "snow", 0.95 "no snow"

(1) $P(\text{snow} \mid \text{"snow"}) = \dfrac{0.1 \times 0.7}{0.1\times 0.7 + 0.9 \times 0.05}$

$= \dfrac{14}{23}$

(2) ?× $P(\text{Snow} \cup \text{"snow"}) + P(\text{No snow} \cup \text{no snow})$

$= 0.1 \times 0.7 + 0.9 \times 0.95 = 0.925.$

$P(\text{snow} \mid \text{"snow"}) = \boxed{}$

$P(\text{no snow} \mid \text{"no show"}) = \dfrac{0.9 \times 0.95}{0.1 \times 0.3 + 0.9 \times 0.95}$

$= \cancel{0.6} \; \dfrac{57}{59}$

(3) $P(\text{snow}) \ll 0.1$?

or $P(\text{snow}) \gg 0.1$?

I don't know the weather.

Figure 1: Handwriting of Question 5

The solution doesn't care you know the weather condition or not. But there must be some intuition behind it, since it's called PARADOX.

(c) Even though Jerry's overall accuracy is lower, it is still possible that she is a better weatherman if the weather is different.

For example, let's assume that it snows 50% of days in Alaska.

- When it snows, Jerry correctly predicts snow 80% of the time.
- When it doesn't snow, Jerry correctly predicts no snow 100% of the time.

Jerry's overall accuracy turns out to be less than Bob's even though she is better at predicting both categories!

For more info on this kind of phenomena, check out Simpson's Paradox!

Figure 2: