CS70 Dis 6B - 7A
Good Question

Mar 6, 2021

1 Hello World!

You want to determine whether a program P prints "Hello World!" before running the kth line in the program. Is there a computer program that can perform this task? Justify your answer.
Proof by contradiction.

Proof. Say such program exists, called HWTest(P, x). Then we can use this program as subroutine to def Halt program, namely Halt(P, x).

```
Halt(P, x)
#def P' as: run P without printing operations :
#Print("Hello World!")
If HWTest(P', x): return True.
If not HWTest(P', x): return False.
```

Therefore, if such HWTest exists, we can generate Halt problem, which we know doesn't exists. So HWTest doesn't exists. □

2 Computability

There is no computer program Line which takes a program P, an input x, and a line number L, and determines whether the Lth line of code is executed when the program P is run on the input x.
Proof by contradiction.

Proof. Say such program exists, called Line(P, x). Then we can use this program as subroutine to def Halt program, namely Halt(P, x).

```
Halt(P, x)
#def P' as: modify P with each exit and return statement jumping t
If Line(P', x): return True.
If not Line(P', x): return False.
```

Therefore, if such Line exists, we can generate Halt problem, which we know doesn't exists. So Line doesn't exists. □

3 Counting on Graphs

How many ways are there to color the faces of a cube using exactly 6 colors, such that each face has a different color? Note: two colorings are considered the same if one of them can be obtained by rotating the other.

If no restriction : 6! permutation.

Now we wanna fix one face, say Up. $\Longrightarrow$ There are 6 choices.

Once we fix Up, we can rotate the cube right/left four times. $\Longrightarrow$ There are another 4 choices.

Therefore, there are 24 mapping to 1.

Total number: 6!/24

4 Captain Combinatorial

Prove

$$\sum_{i=1}^{n} i\binom{n}{i}^2 = n\binom{2n-1}{n-1}$$

RHS: first we can express $\binom{n}{i}^2$ as $\binom{n}{i} * \binom{n}{n-i}$, then imagine there are n men and n women. First we choose i men, which is $\binom{n}{i}$, then we choose $(n-i)$ women, which is $\binom{n}{n-i}$. Since there is also i in the sum, we pick one man to be the captain. Therefore, the RHS is satisfied.

LHS: we know in total we choose n people, among which there is a captain. So we can also first choose the captain in men, which is n. Then in the remaining, we choose $n-1$ people, which is $\binom{2n-1}{n-1}$. So RHS is also satisfied.