HW1 1.

(a)

x	y	¬~~A~~x∨y	¬y	Res
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	F

contradiction.

(b)

x	y	¬x ∨ (x∨y)		Res
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

Tautology.

(c)

x	y	x∨y	∨	x∨¬y	Res
T	T	T		T	T
T	F	T		T	T
F	T	T		F	T
F	F	F		T	T

Tautology.

(d)

x	y	¬x∨y	∨	¬x∨¬y	Res
T	T	T		F	T
T	F	~~T~~F		T	T
F	T	T		T	T
F	F	T		T	T

Tautology.

(e)

x	y	x∨y	∧	¬(x∧y)	Res
T	T	T		F	F
T	F	~~F~~T		~~F~~T	~~F~~T
F	T	~~F~~T		~~F~~T	~~F~~T
F	F	F		T	F

~~contradiction~~

Neither

(f)

x	y	$\neg x \vee y$	$\wedge$ $x \vee y$	$\wedge$ $\neg y$	Res.
T	T	T	T	F	F
T	F	F	T	T	F
F	T	T	T	F	F
F	F	T	F	T	F

contradiction.

2.

(1) Possibly true.

Maybe $y=4$ isn't that specific value to make $G(x,y)$ True.

(2) Same as (1)

(3) Certainly True.

(4) Certainly False.

$\exists y\ G(3,y)$ is True. so $\neg G(3,y)$ is False.

(5) Possibly true.

R : same as (1)

(b)

x	y	z	R
F	F	T	T
F	T	F	T
T	F	F	T
			F
			⋮

$z \backslash xy$	00	01	11	10
0				
1	1			

$$R = (\neg x \wedge \neg y \wedge z) \vee (\neg x \wedge y \wedge \neg z) \vee (x \wedge \neg y \wedge \neg z)$$

3. (a) $(\exists n \in R)(n \notin Q)$

True like $n=\sqrt{2}$

(b) $(\forall n \in Z)((n \in N) \vee (n<0)) \wedge (\neg((x \in N) \wedge (x<0)))$

~~(c)~~ True

when $n \in Z$ and $n \geq 0$, it's natural Number

if $n<0$, Then $n<0$ is True.

(c) $(\forall n \in \mathbb{N})(6 \mid n \Rightarrow (2 \mid n) \overset{\vee}{\cancel{\wedge}} (3 \mid n))$

True.

$6 = 2 \times 3$.

(d) Real Numbers are complex number. T.

(e) if an integer is divisible by 2 ~~and~~ or 3, it's divisible by 6.

~~True~~. F. Countereample: $n=2$ and $6 \nmid 2$.

wrong

$n = 3 \times \underline{k}$ divisible by 3.

$n = 3 \times \underline{2 \cdot c}$ divisible by 2.

$= 6c$ $\Rightarrow$ so it's divisible by 6.

(f) ~~If~~ For any natural Number greater than 7, there exists a and b natural numbers which sum is equal to that number.

4.

(a) Proof by contraposition.

Proof: $((10 \mid x) \vee (10 \mid y)) \Rightarrow (10 \mid xy)$

which is quite obvious.

if one of x, y is divisible by 10, then multiplied by another is also divisible by 10.

(b) Converse: $((10 \nmid x) \wedge (10 \nmid y)) \Rightarrow (10 \nmid xy)$ which is wrong. $x=25$ $y=2$ $xy=50$ $10 \mid xy$

~~$(\forall x, y \in \mathbb{Z})$~~ $(\forall x, y \in \mathbb{Z})$

Origin: $(\forall x, y \in \mathbb{Z})\ (10 \nmid xy) \Rightarrow ((10 \nmid x) \wedge (10 \nmid y))$

$$((10 \mid xy) \vee ((10 \nmid x) \wedge (10 \nmid y)))$$

converse: $(\exists x, y \in \mathbb{Z})\ (\underline{(10 \nmid xy)} \wedge \underline{((10 \mid x) \vee (10 \mid y))})$

if $(10 \mid x) \wedge (10 \mid y) \Rightarrow 10 \mid xy$

so which is obviously wrong.

P. $\neg P$ is wrong so P is right.

5.

(a) Proof by direct prove:

n is odd $\Rightarrow$ $n = 2k+1$ $\quad k \in N$

$$n^2 + 2n = (2k+1)^2 + 2(2k+1) = \underbrace{4k^2}_{E} + \underbrace{4k}_{E} + \underbrace{2(2k+1)}_{E} + 1 = \text{odd}.$$

(b) Proof by case:

1/ $x \geq y$ $\quad (x+y-|x-y|)/2 = (x+y-(x-y))/2 = y = \min(x,y)$

2/ $x < y$ $\quad (x+y-|x-y|)/2 = x = \min(x,y)$

(c) Proof by contraposition.
$(\forall a, b \in R)$
if $a \geq 7$ and $b \geq 3$, then $a+b \geq 10$. obviously T.

(d) Proof by contraposition

if $r+1$ is rational, then $r+1 = \frac{p}{q}$ $\quad (p, q \in Z)$

$r = \frac{p}{q} - 1 = \frac{p-q}{q} = \frac{p'}{q'}$ is rational.

(e) Disprove.

counter example: $n = 10.$ $\quad 10n^2 = 1000$

$10! > 1000.$

6. No idea. Don't know How to prove set!
Peek at Sol. Just use Definition (Gain Info.)
/Use

(a) $X = f^{-1}(A \cup B)$

$Y = f^{-1}(A) \cup f^{-1}(B).$

1/ Proof: $X \subseteq Y.$

$\forall e, e \in X$ by definition, $f(e) \in (A \cup B)$

Proof by case: $f(e) \in A$ $\quad \exists e = f^{-1}(A) \in f^{-1}(A) \cup f^{-1}(B)$

$f(e) \in B$ $\quad e = f^{-1}(B) \in f^{-1}(A) \cup f^{-1}(B).$

2/ Proof: $Y \subseteq X.$

$\forall e \in f^{-1}(A) \cup f^{-1}(B)$ more concretely $f(e) \in A \in A \cup B$

proof by cases: $e \in f^{-1}(A) \in f^{-1}(A \cup B)$ $e \in f^{-1}(A \cup B)$. so, $x = y$.

$e \in f^{-1}(B) \in f^{-1}(A \cup B)$.

(b) $x = f^{-1}(A \cap B)$

$y = f^{-1}(A) \cap f^{-1}(B)$

1/ $x \subseteq y$.

$\forall e,\ e \in f^{-1}(A \cap B)$ by definition $f(e) = A \cap B$.

$f(e) \subseteq A$ and $f(e) \subseteq B$.

$e = f^{-1}(A)$ and $e \subseteq f^{-1}(B)$ $e \subseteq f^{-1}(A) \cap f^{-1}(B)$.

2/ $\forall e.\ e \in f^{-1}(A) \cap f^{-1}(B)$

By definition, $e \in f^{-1}(A)$ and $e \in f^{-1}(B)$

$f(e) \in A$ and $f(e) \in B$

$f(e) \in (A \cap B)$

$e \in f^{-1}(A \cap B)$.

(c). $x = f^{-1}(A \setminus B)$

$y = f^{-1}(A) \setminus f^{-1}(B)$.

1/ $x \subseteq y$.

$\forall e.\ e \in f^{-1}(A \setminus B) \xrightarrow{\text{by def}} f(e) \in (A \setminus B)$

$f(e) \in A$ and $f(e) \notin B$.

$e \in f^{-1}(A)$ and $e \notin f^{-1}(B)$

$e \in f^{-1}(A) \setminus f^{-1}(B)$.

2/ Reverse Order.

~~(d)~~ $x = f(A \overset{\cup}{\cancel{\cap}} B)$

$y = f(A) \cup f(B)$

1/ $x \subseteq y$

$\forall e \in \cancel{f(A\cup B)}\ A\cup B$. $e\in A$ $f(A\overset{\cup}{\cap}B) \in f(A) \in (f(A)\cup f(B))$

Proof by cases: $\cancel{e\in f(A) \in f(A)\cup f(B)}$

$\cancel{e\in f(B) \in f(A)\cup f(B)}$.

$e\in B$ $f(A\cup B) \in f(B) \in (\cdots)$

2) $Y \subseteq X$.

$\forall e \in f(A)\cup f(B)$ Proof by cases: $e\in f(A)$

by def $f^{-1}(e) \in A \in A\cup B \Rightarrow e \in f(A\cup B)$

same as $e\in f(B)$.

so $X = Y$.

~~e)~~. $X = f(A\cap B)$

$Y = f(A)\cap f(B)$

$\forall e \in X = f(A\cap B)$. $e\in f(A)$ and $e\in f(B)$

$f^{-1}(e) = A\cap B$. $\therefore e \subseteq f(A)\cap f(B)$.

$f^{-1}(e) \in A$ and $f^{-1}(e)\in B$. $X \in Y$.

✓ $f: x^2 \to x^2$ $R \to R$

$A: [1]$ $B: [-1]$

(d) (e) (f) proof is on HW page

~~f)~~. $X = f(A\setminus B)$

$Y = f(A)\setminus f(B)$

$Y \subseteq X$

$\forall e \in Y = f(A)\setminus f(B) \Rightarrow e\in f(A)$ and $e \notin f(B)$

$f^{-1}(e) \in A$ and $f^{-1}(e) \notin B$.

2.2

$f^{-1}(e) \in A\setminus B$

$e \in f(A\setminus B)$

✓ $f: x \to x^2$ $R\to R$

$A: [1]$ $B: [-1]$ where $f(A)\setminus f(B)$ is ϕ.