3 Propositional Practice

(a) $(\exists x \in \mathbb{R})\ (x \notin \mathbb{Q})$

True.

Consider $x = \pi$. $\pi \in \mathbb{R}$, and $\pi \notin \mathbb{Q}$, so the problem is true.

(b) $(\forall x \in \mathbb{Z})\ (((x \in \mathbb{N}) \vee (x < 0)) \wedge (\neg((x \in \mathbb{N}) \wedge (x < 0))))$

True.

Let $x \in \mathbb{N}$, so $x >= 0$ or $x < 0$, but not both.

If $x >= 0$, then x is a natural number; if $x < 0$, then is negative; x can't be both.

Thus, the proposition is true.

(c) $(\forall x \in \mathbb{N})\ ((6 \mid x) \Rightarrow ((2 \mid x) \vee (3 \mid x)))$

True.

Let $x \in \mathbb{N}$, $x = 6 * k$, so $x \in \mathbb{N}$

So $x = 2 * (3k)$ where $3k \in \mathbb{N}$, which means that $2 \mid x$

So $((2 \mid x) \vee (3 \mid x))$ is true, which means that the proposition is true.

(d) All real numbers are complex numbers.

True.

Let $x \in \mathbb{R}$, so $x = x + 0 * i$, and $\sin\, x, 0 \in \mathbb{R}$, So by the definition of complex numbers, x is a complex number.

(e) Integers that are divisible by 2 or 3 are divisible by 6.

False.

Give a counter example, take integer $n = 2$, $2 \mid n$ is true but $6 \mid n$ is false.

So by the definition of implication, this proposition is false.