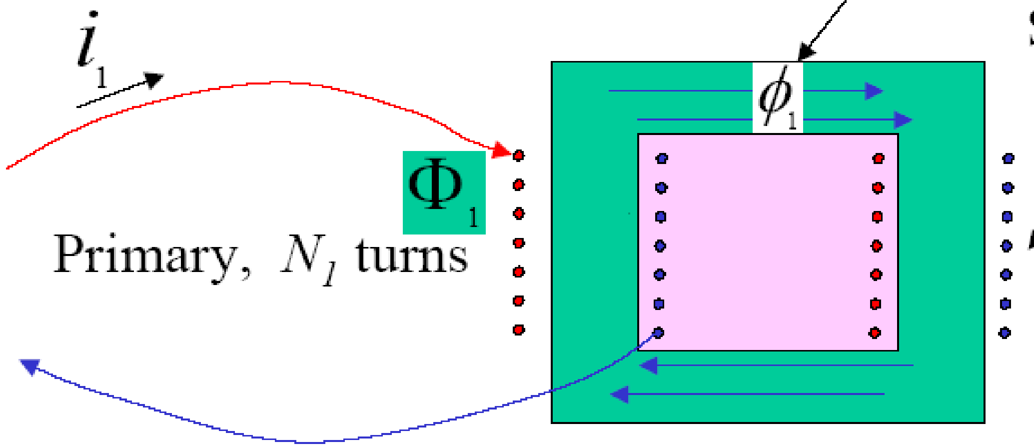
TRANSFORMER MODELING

Suppose a current i_1 flows in the primary.
The current i_2 is zero (secondary is open)

Iron core
cross-sectional area A

Secondary, N_2 turns



Total flux in primary

$$\Phi_1 = N_1 \phi_1$$

Self inductance of primary

$$L_1 = -\frac{\Phi_1}{i_1} = \frac{N_1^2}{R_{core}}$$

From Ampere's law

$$\phi_1 = -\frac{N_1 i_1}{\left(\frac{\ell_{core}}{\mu A}\right)} = -\frac{N_1 i_1}{R_{core}}$$

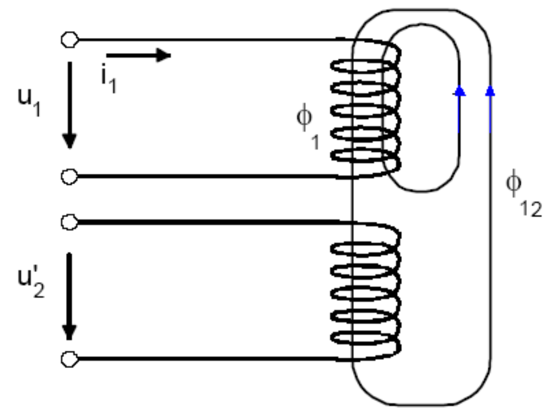
Reluctance

TRANSFORMER MODELING

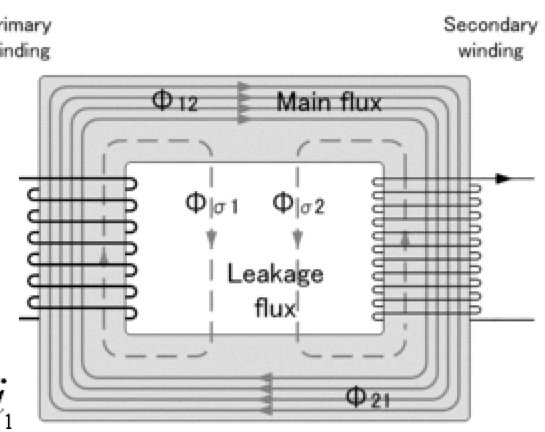
Let φ_{12} be the flux in each of the loops in the secondary resulting from the current i_1 in the primary.

Write: $\varphi_{12} = k\,\varphi_1$

Where *k is* a *flux coupling coefficient*. $k < 1$ but in good transformer $k \approx 1$.



$\varphi_1 = \Phi_1/N_1$



Total flux in secondary

$$\Phi_{12} = N_2 \phi_{12}$$

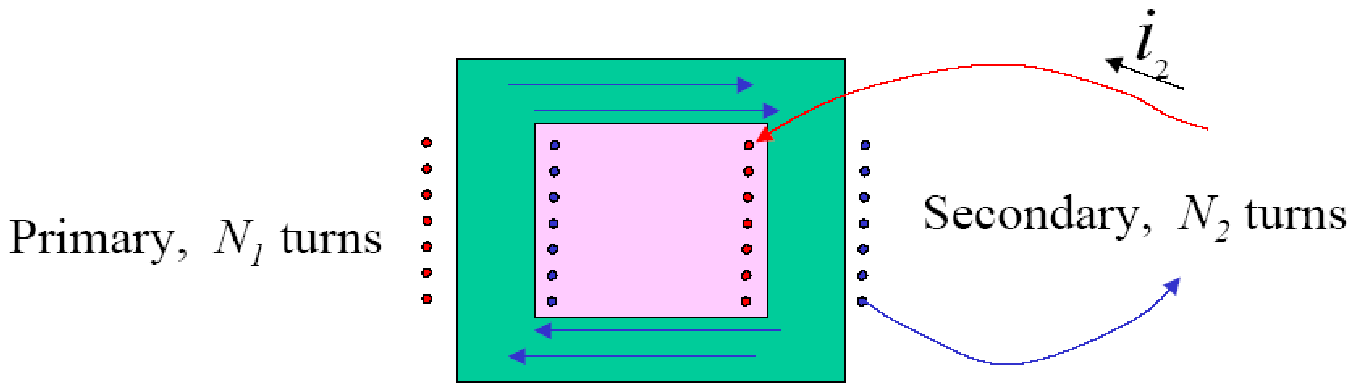
$$\Phi_{12} = N_2 k \frac{\Phi_1}{N_1}$$

$$\Phi_{12} = -\frac{N_2}{N_1} k L_1 i_1$$

$$\Phi_{12} = -M i_1$$

$$M = \frac{N_2}{N_1} k L_1$$ Mutual inductance

TRANSFORMER MODELING



$$L_2 = \frac{N_2^2}{R_{core}}$$

Reluctance

$$\boxed{\frac{L_1}{L_2} = \frac{N_1^2}{N_2^2}}$$

$$\boxed{M = k\sqrt{L_1 L_2}}$$

$$M = \frac{N_2}{N_1} k L_1 = \sqrt{\frac{L_2}{L_1}} k L_1$$

TRANSFORMER MODELING

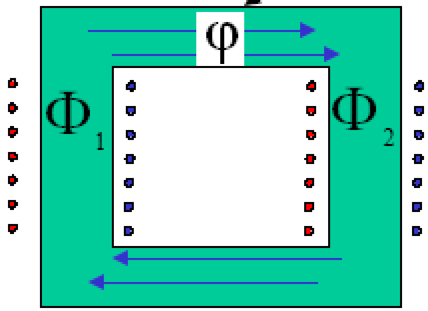
voltage transformation

Assume $k = 1$ for this transformer

Same A in all loops

Iron core
cross-sectional area A

Primary, N_1 turns



Secondary, N_2 turns

With $\dfrac{\Phi_1}{\Phi_2} = \dfrac{N_1}{N_2}$

And

$$v_1 = -\frac{d\Phi_1}{dt} \qquad v_2 = -\frac{d\Phi_2}{dt}$$

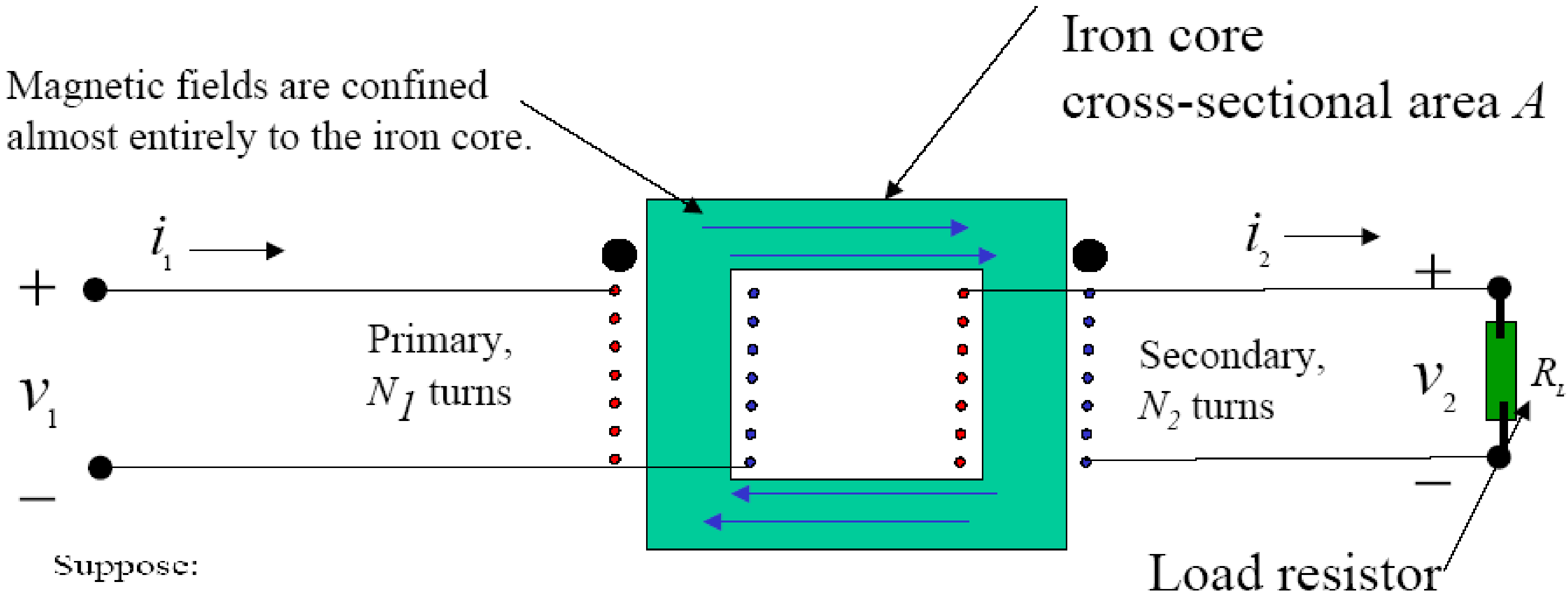
Then

$$\boxed{\frac{v_1}{v_2} = \frac{N_1}{N_2}}$$

A mutual flux links the two coils: $\varphi = \dfrac{N_1 i_1 - N_2 i_2}{\Re_{core}}$

TRANSFORMER MODELING

current transformation



Suppose:

$$v = v_o e^{j\omega t}$$

Then expect for current $i = i_o e^{j\omega t}$

$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \qquad v_1 = j\omega L_1 i_1 - j\omega M i_2$$

$$v_2 = M \frac{di_1}{dt} - L_2 \frac{di_2}{dt} \qquad v_2 = j\omega M i_1 - j\omega L_2 i_2$$

TRANSFORMER MODELING

current transformation

$$v_2 = j\omega M i_1 - j\omega L_2 i_2$$

$$v_1 = j\omega L_1 i_1 - j\omega M i_2$$

But $v_2 = i_2 R_L$

$$i_2 R_L + j\omega L_2 i_2 = j\omega M i_1$$

$$\boxed{i_2 = \frac{j\omega M i_1}{R_L + j\omega L_2}}$$

If: $j\omega L_2 >> R_L$

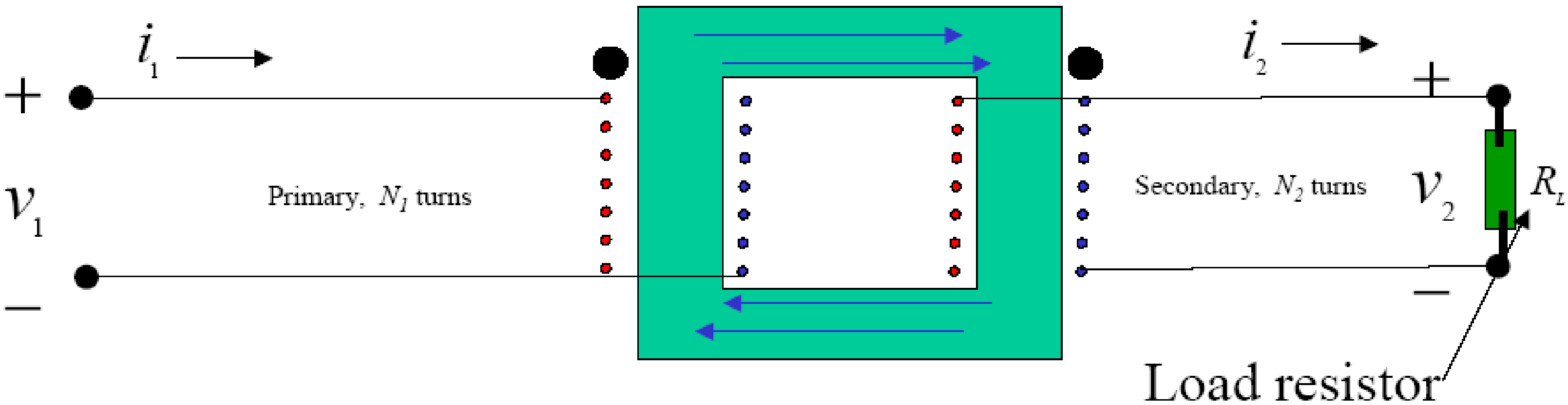
$$M = \sqrt{L_1 L_2}$$

Then $\dfrac{i_2}{i_1} = \dfrac{j\omega M}{j\omega L_2} = \dfrac{M}{L_2}$

$$\boxed{\frac{i_2}{i_1} = \frac{N_1}{N_2}}$$

TRANSFORMER MODELING

impedance transformation



$$v_1 = j\omega L_1 i_1 - j\omega M i_2$$

$$\boxed{i_2 = \frac{j\omega M i_1}{R_L + j\omega L_2}}$$

$$v_1 = j\omega L_1 i_1 - j\omega M \left(\frac{j\omega M}{R_L + j\omega L_2} \right) i_1$$

$$v_1 = \left(j\omega L_1 + \frac{\omega^2 M^2}{R_L + j\omega L_2} \right) i_1$$

TRANSFORMER MODELING

impedance transformation

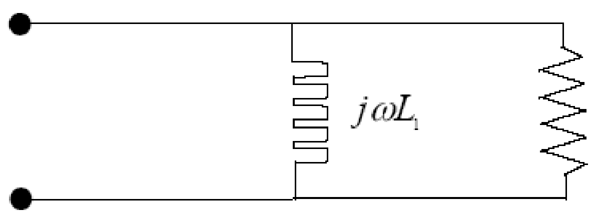
Input admittance

$$Y_{in} = \frac{1}{j\omega L_1 + \dfrac{\omega^2 M^2}{R_L + j\omega L_2}}$$

$$Y_{in} = \frac{R_L + j\omega L_2}{j\omega R_L L_1 + \omega^2 \underbrace{\left(M^2 - L_1 L_2\right)}_{0 \text{ for } k=1}}$$

Equivalent circuit for the primary

$$\boxed{Y_{in} = \frac{1}{j\omega L_1} + \frac{L_2}{R_L L_1}}$$



$$\frac{L_1}{L_2} R_L = \frac{N_1^2}{N_2^2} R_L = R_L'$$

If $j\omega L_1 >> \dfrac{L_1}{L_2} R_L$ the impedance looking into the primary is just the Transformed load resistance.

This result breaks down a suitably low frequencies.

TRANSFORMER MODELING

Equivalent circuit

$$v_1 = j\omega L_1 i_1 - j\omega M i_2$$

$$v_1 = j\omega(1-k)L_1 i_1 + j\omega k L_1 i_1 - j\omega M i_2$$

Let $N_1/N_2=a$, then $M=(N_2/N_1)kL_1=(1/a)kL_1$

$$v_1 = j\omega(1-k)L_1 i_1 + j\omega k L_1 i_1 - j\omega \frac{1}{a} k L_1 i_2$$

$$v_1 = j\omega(1-k)L_1 i_1 + j\omega k L_1 \left(i_1 - \frac{i_2}{a} \right)$$

TRANSFORMER MODELING

Equivalent circuit

For the secondary:

$$v_2 = j\omega M i_1 - j\omega L_2 i_2$$

$$v_2 = j\omega M i_1 - j\omega k L_2 i_2 - j\omega(1-k)L_2 i_2$$

$$v_2 = j\omega\left(\frac{1}{a}kL_1\right)i_1 - j\omega k\left(\frac{L_1}{a^2}\right)i_2 - j\omega(1-k)L_2 i_2$$

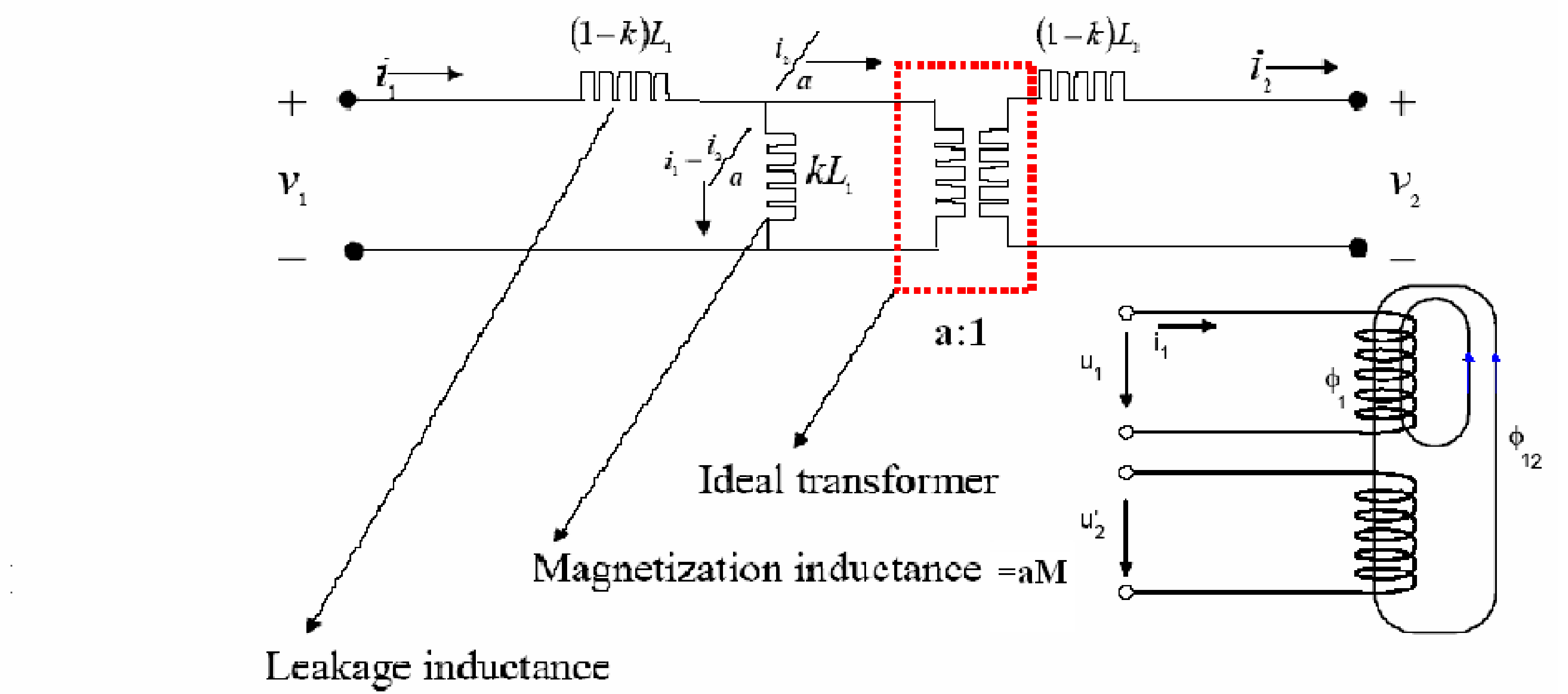
$$v_2 = \left(j\omega k L_1\left(i_1 - \frac{i_2}{a}\right)\right)\frac{1}{a} + j\omega(1-k)L_2(-i_2)$$

TRANSFORMER MODELING

Equivalent circuit

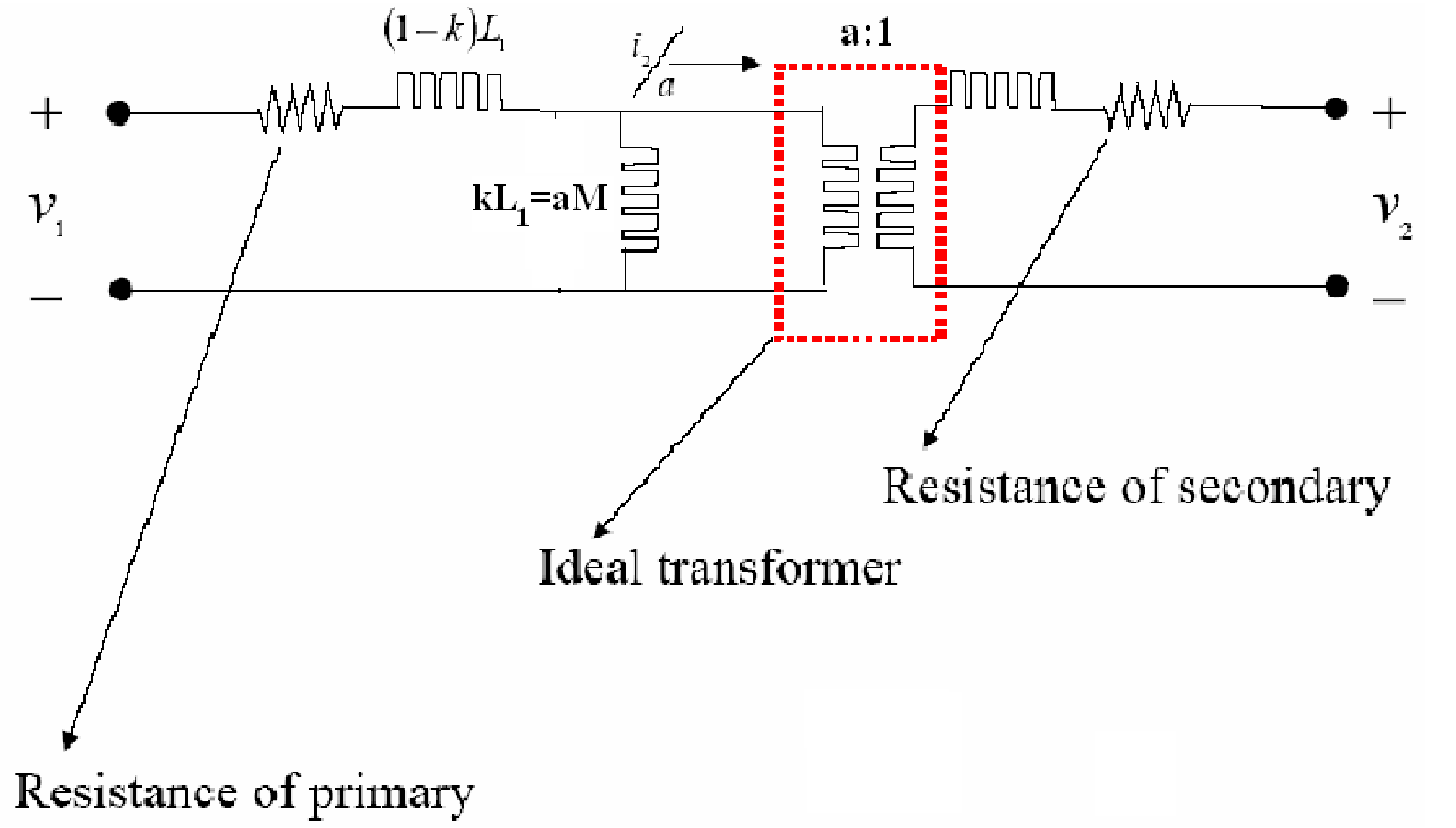
$$v_1 = j\omega(1-k)L_1 i_1 + j\omega k L_1\left(i_1 - \frac{i_2}{a}\right)$$

$$v_2 = \left(j\omega k L_1\left(i_1 - \frac{i_2}{a}\right)\right)\frac{1}{a} + j\omega(1-k)L_2(-i_2)$$



TRANSFORMER MODELING

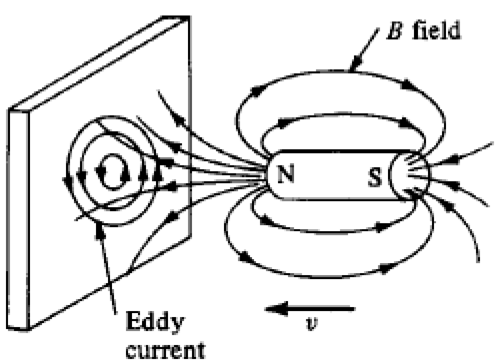
Equivalent circuit



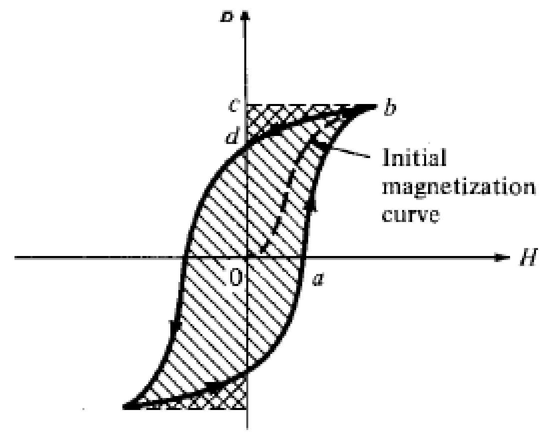
Transformer Losses

There are two dominant loss mechanisms in transformers:

•Eddy currents



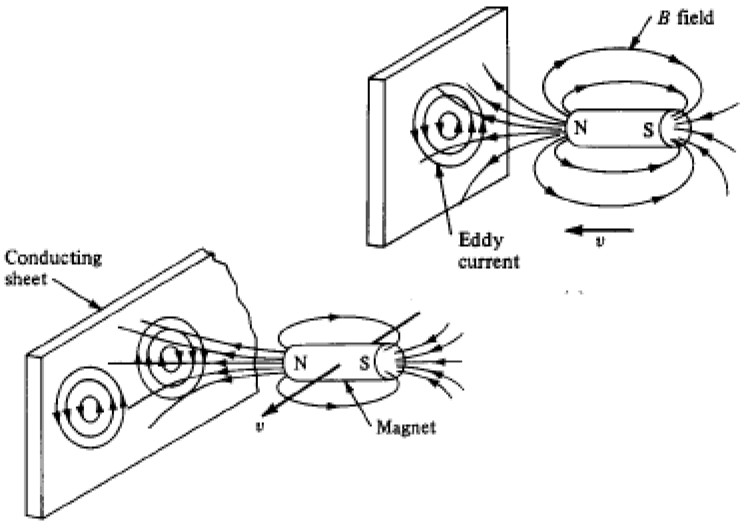
Hysteresis



Both depend upon magnetic properties of the materials used to construct the core of transformer and its design.

Transformer Losses

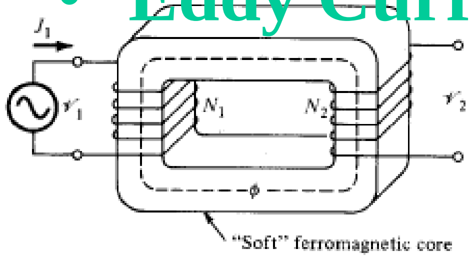
- **Eddy Currents**
- The changing magnetic field on the conducting sheet will induce current in the sheet. These currents are known as Eddy currents. Energy loss occurs through Joule's heating.

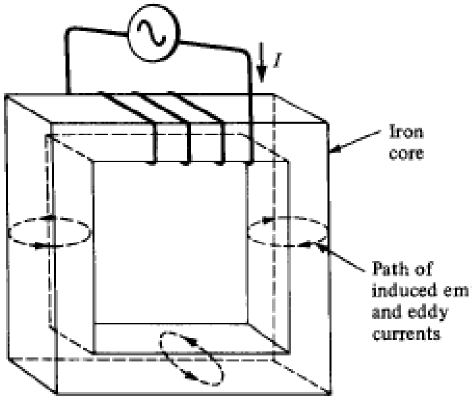


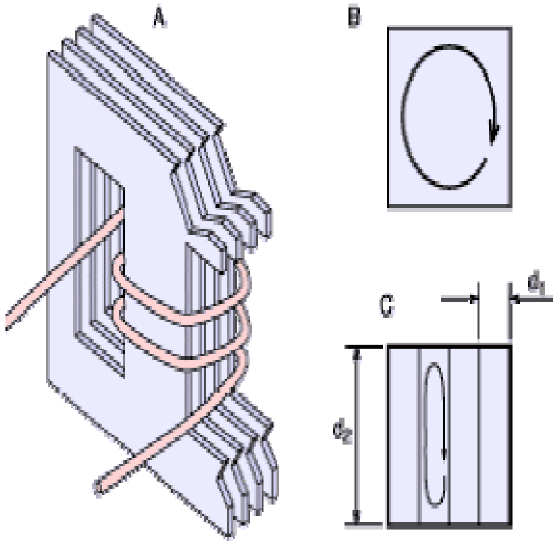
$$W_e = K_e f^2 K_f^2 B_m^2 \ watts$$

Transformer Losses

- **Eddy Currents**

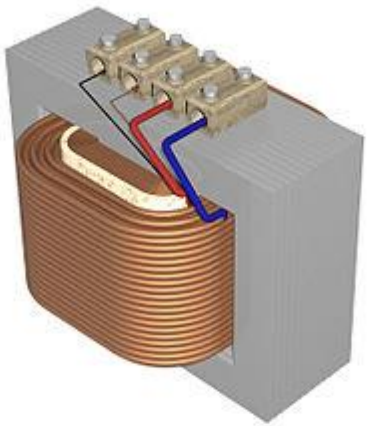


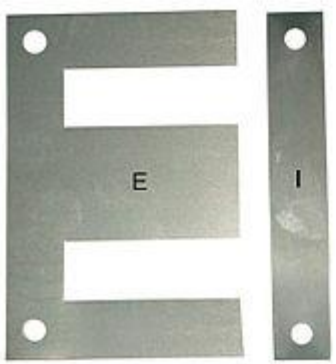




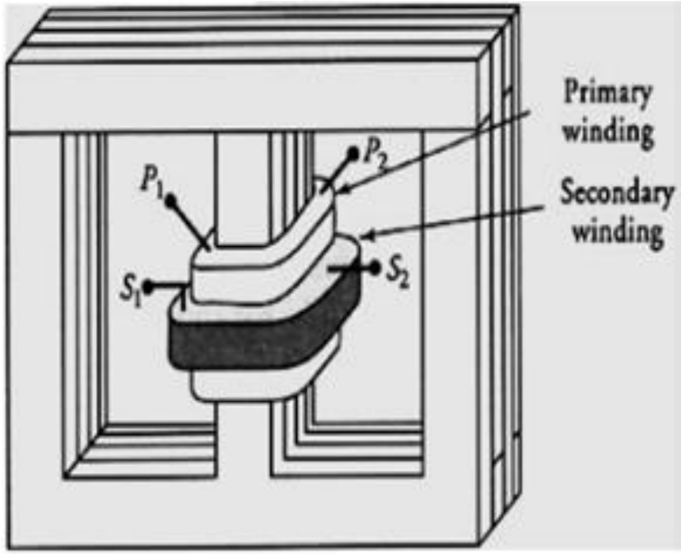
Eddy currents are ***reduced*** by lamination of the core. Lamination breaks the Eddy Current paths

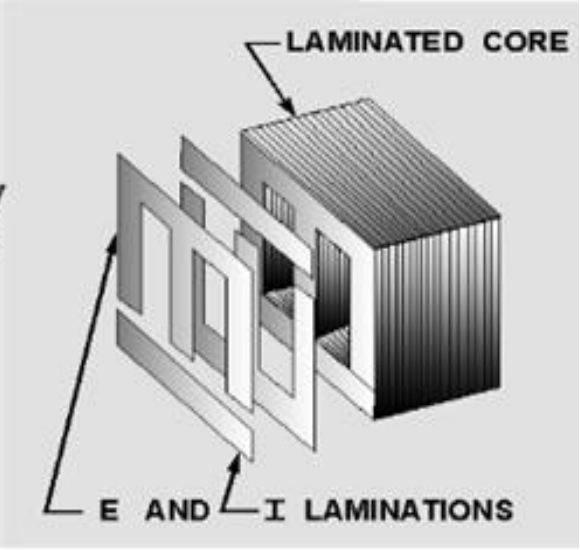
Transformer Losses





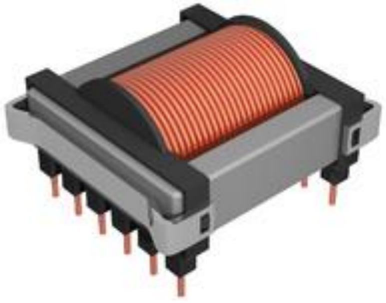
Typical EI Lamination Pair

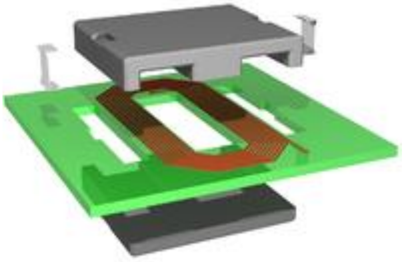




Transformer Losses

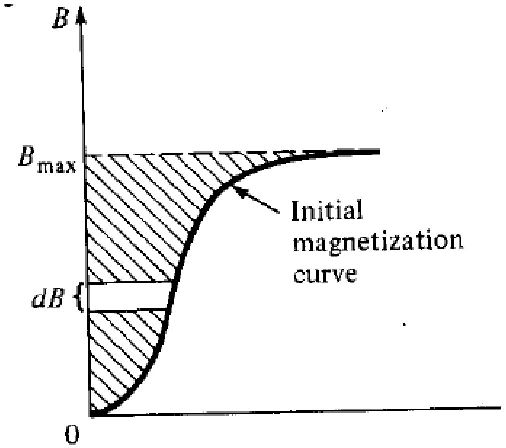
- **High frequency transformer cores**





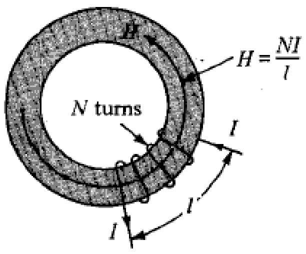
Transformer Losses

- **Hysteresis**



First consider the work which must be done by the power source supplying the primary to increase the magnetic field in the core by dB.

We shall use the model device a toroidal coil with an iron core.



(a)

$$I \Rightarrow I + dI$$
$$H \Rightarrow H + dH$$
$$B \Rightarrow B + dB$$
$$t \Rightarrow t + dt$$

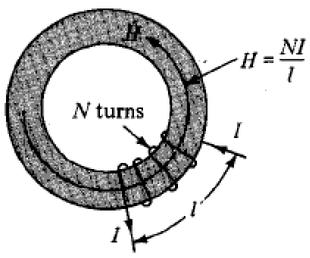
Increase the current in time interval *dt*

Transformer Losses

- **Hysteresis**

Then by Lenz's law an electromotive force will be induced in the winding tending to oppose the increase in current.

$$V = -N\frac{d\Phi}{dt}$$



(a)

$$I \Rightarrow I + dI$$
$$H \Rightarrow H + dH$$
$$B \Rightarrow B + dB$$
$$t \Rightarrow t + dt$$

To increase the current the generator must furnish energy in the amount of.

$$\Delta W = -VIdt = NId\Phi$$

Taking B as constant over the cross-section of the toroid core, an expression for the flux can be written.

$$\Phi = BA$$

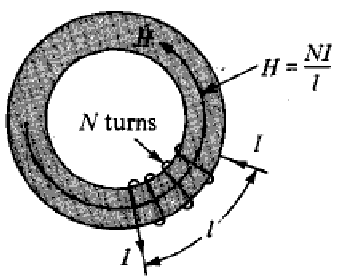
$$d\Phi = AdB$$

Transformer Losses

- **Hysteresis**

From Ampere's law we can obtain an expression for the magnetic field inside the toroid core.

$$H = \frac{NI}{2\pi r}$$



(a)

$$I \Rightarrow I + dI$$
$$H \Rightarrow H + dH$$
$$B \Rightarrow B + dB$$
$$t \Rightarrow t + dt$$

To increase the current the generator must furnish energy in the amount of.

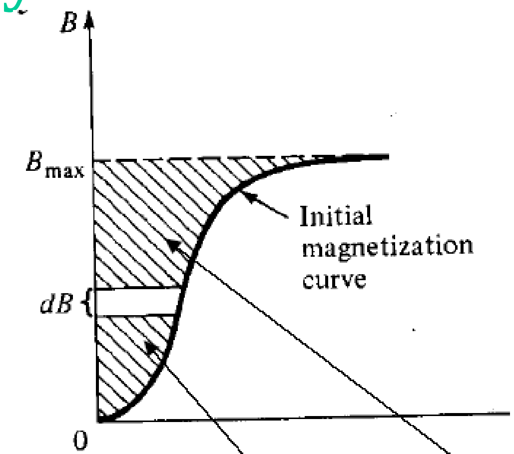
$$\Delta W = 2\pi r A H dB$$

where the volume of the toroid is:

$$vol = 2\pi r A$$

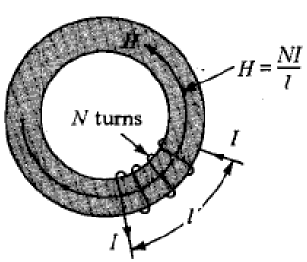
Transformer Losses

- **Hysteresis**



Now we can sum "integrate" over the initial magnetization part of the hysteresis curve in order to obtain the work done by the generator in establishing the maximum magnetic flux density in the core B_{max}

$$W = vol \int_{0}^{B_{\max}} H dB$$

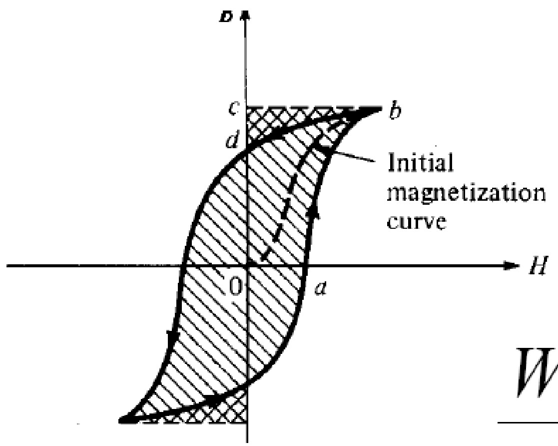


(a)

This integral is the shaded area of the above curve multiplied by the volume "*vol*" of the toroid's core.

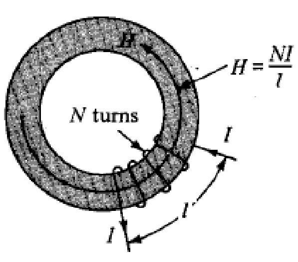
Transformer Losses

- **Hysteresis**



The work done in one cycle, W_h, is the striped area of the hysteresis cycle. The cross hatched area is the work returned to the source.

$$\frac{W}{vol} = w = \int_0^{B_{max}} H dB$$



(a)

$$W_h = \text{ Area of Hysteresis loop}$$

Power loss

$$P_h = W_h f$$

$$W_h = K_h f (B_m)^{1\cdot 6} \; watts$$

Transformer Modeling

Equivalent circuit