③

a.

$V = jL_{11}\omega I + j\omega M I + j\omega L_{22} I + j\omega M I \rightarrow$ معادل $L = L_{11} + L_{22} + 2M$

$L_{11} = 2H$
$L_{22} = 3H$
$M = 1H$

b.

$\rightarrow V = j\omega L_{11} I + j\omega M I + j\omega L_{22} I + j\omega M I$

معادل $j\omega L = L_{11} + L_{22} + 2M$

c)

$M \leftarrow L_{11} + L_{22} - 2M \leftarrow a \quad , \quad L_{22} \leftarrow jب = 11' \quad , \quad L_{11} \leftarrow jب = 22'$

④ a.

(a)

$\rightarrow L = \begin{bmatrix} L_a & M \\ M & L_b \end{bmatrix}$

(b)

$\rightarrow L = \begin{bmatrix} L_a & L_c \\ L_c & L_b \end{bmatrix}$
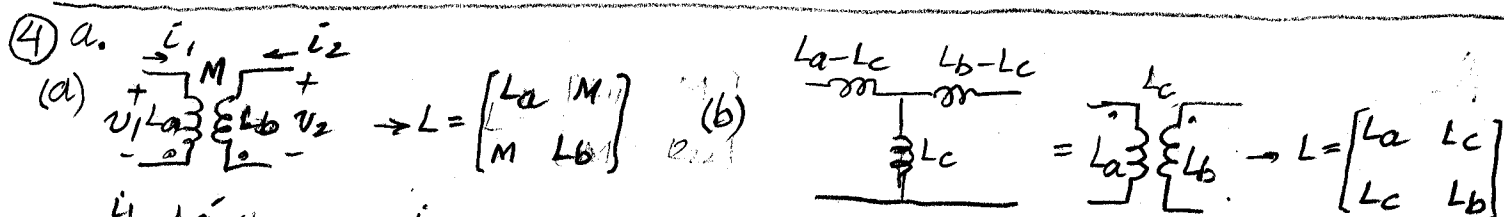
(c)

$V_1 = j\omega L_a' I_1 + \frac{n_1}{n_2} V_2$

$V_2 = \left(\frac{n_1}{n_2} I_1 + I_2\right) j\omega L_b$

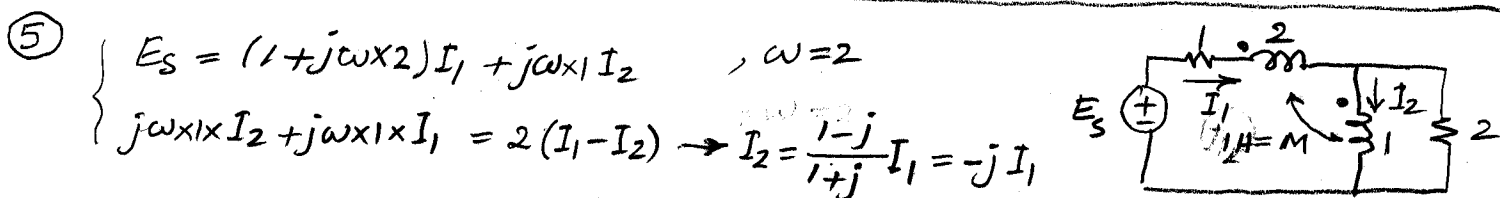$\rightarrow V_1 = j\omega L_a' I_1 + \left(\frac{n_1}{n_2}\right)^2 I_1 j\omega L_b + \frac{n_1}{n_2} I_2 j\omega L_b$

$V_2 = \frac{n_1}{n_2} j\omega L_b I_1 + j\omega L_b I_2$

$\rightarrow L = \begin{bmatrix} L_a' + \left(\frac{n_1}{n_2}\right)^2 L_b & \frac{n_1}{n_2} L_b \\ \frac{n_1}{n_2} L_b & L_b \end{bmatrix}$

b. اگر $L_c = M \rightarrow \begin{bmatrix} L_a & M \\ M & L_b \end{bmatrix}$ ✓

c. اگر $M = \frac{n_1}{n_2} L_b$, $L_a' + \left(\frac{n_1}{n_2}\right) L_b = L_a \rightarrow \frac{n_1}{n_2} = \frac{M}{L_b}$, $L_a' = L_a - M$
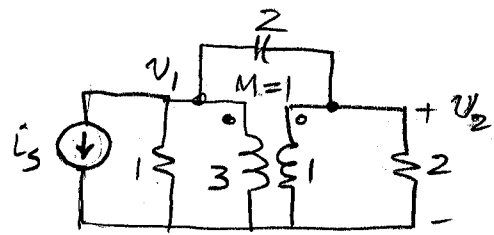
⑤

$\begin{cases} E_S = (1 + j\omega \times 2) I_1 + j\omega \times 1 I_2 \\ j\omega \times 1 \times I_2 + j\omega \times 1 \times I_1 = 2(I_1 - I_2) \end{cases}$, $\omega = 2$ $\rightarrow I_2 = \frac{1-j}{1+j} I_1 = -j I_1$

$\rightarrow E_S = (1 + j4) I_1 + j2(-j) I_1 = (3 + j4) I_1 \rightarrow I_1 = \frac{E_S}{3 + j4} = \frac{1}{5} e^{-j53.1°} E_S \rightarrow i_1 = 0.2 \cos(2t - 23.1°)$

$I_2 = -j I_1 = e^{-j90°} I_1 \rightarrow i_2 = 0.2 \cos(2t - 113.1°)$

⑥

$$\begin{cases} \dfrac{V_1}{1} + I_1 + (V_1 - V_2) \times j\omega \times 2 = I_s \quad , \omega = 1 \\[2mm] \dfrac{V_2}{2} + I_2 + (V_2 - V_1) \times j\omega \times 2 = 0 \end{cases}$$

$$V_1 = j\omega \times 3\, I_1 + j\omega \times 1\, I_2 \quad , \quad V_2 = j\omega \times 1\, I_2 + j\omega \times 1\, I_1 \quad \rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = j\omega \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \dfrac{1}{j\omega}\begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \rightarrow I_1 = \dfrac{1}{j\omega}(V_1 - V_2) \;,\; I_2 = \dfrac{1}{j\omega}(-V_1 + 3V_2)$$

$$\rightarrow V_1 + \dfrac{1}{j\omega}(V_1 - V_2) + j2\omega(V_1 - V_2) = I_s \;,\; \dfrac{V_2}{2} + \dfrac{1}{j\omega}(-V_1 + 3V_2) + (V_2 - V_1)\times j2\omega = 0$$
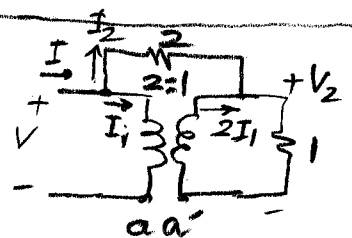
$$\begin{bmatrix} 1+j & -j \\ -j & \tfrac{1}{2}-j \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_s \\ 0 \end{bmatrix} \rightarrow V_2 = \dfrac{\begin{vmatrix} 1+j & I_s \\ -j & 0 \end{vmatrix}}{\triangle} = \dfrac{j\,I_s}{\triangle}$$

$$\triangle = (1+j)(\tfrac{1}{2}-j) + 1 = \tfrac{5}{2} - j\tfrac{1}{2} \rightarrow V_2 = \dfrac{j(\tfrac{5}{2}+j\tfrac{1}{2})}{\tfrac{25}{4}+\tfrac{1}{4}}I_s = (-\tfrac{1}{13} + j\tfrac{5}{13})I_s$$

$$V_2 = 0.392\, e^{j101.3^\circ} \rightarrow V_2 = \cos(t + 101.3^\circ)$$

⑧ $\quad W$ ذخیره شده $= \dfrac{1}{2}\times 5 \times 2^2 + \dfrac{1}{2}\times 4 \times 1^2 + \dfrac{1}{2}\times 2\times(-3)^2 = 21\,J$

⑪ a. $\quad I_2 = 0 \;,\; V_2 = 2I_1 \;,\; V = 2V_2 = 4I_1$

$$\rightarrow R_{in} = \dfrac{V}{I} = 4 \;\Omega$$

b. $\quad I_2 = \dfrac{V - V_2}{2} \;,\; V_2 = (2I_1 + I_2)\times 1 \;,\; V = 2(2I_1 + I_2)$

$$I_2 = \dfrac{V - 2I_1 - I_2}{2} \rightarrow V = 2I_1 + 3I_2 \rightarrow I_1 = \dfrac{V}{8}, I_2 = \dfrac{V}{4} \rightarrow R_{in} = \dfrac{V}{I_1 + I_2} = \dfrac{8}{3}\;\Omega$$

⑫

(a) $\quad Z_{11} = \dfrac{V_1}{I_1} = \dfrac{1}{1 + \dfrac{1}{j2\omega + \dfrac{1}{j2\omega + \dfrac{1}{2n^2}}}}$

$Z_{21} = \dfrac{V_2}{I_1} = \dfrac{\dfrac{1}{j2\omega + 1/2n^2}}{1 + j2\omega + \dfrac{1}{j2\omega + 1/2n^2}} \times \dfrac{1}{n}$

(b) $\quad Z_{22} = \dfrac{V_2}{I_2} = \dfrac{1}{\tfrac{1}{2} + j\omega \times 2 \times n^2 + \dfrac{1}{\tfrac{1}{n^2}(j\omega \times 2 + 1)}}$

$Z_{12} = \dfrac{V_1}{I_2} = \dfrac{1}{n}\times \dfrac{\dfrac{1}{j2\omega + 1/2n^2}}{\dfrac{1}{j2\omega + 1/2n^2} + j2\omega + 1} \times 1 = Z_{21}$