- **Stationary charge:**
  - $v_q = 0$
  - $E \neq 0$ $\qquad$ $B = 0$

A stationary charge produces an electric field only.

- **Moving charge:**
  - $v_q \neq 0$ and $v_q = constant$
  - $E \neq 0$ $\qquad$ $B \neq 0$

A uniformly moving charge produces an electric and magnetic field.

- **Accelerating charge:**
  - $v_q \neq 0$ and $a_q \neq 0$
  - $E \neq 0$ $\qquad$ $B \neq 0$ $\qquad$ *Radiating field*

A accelerating charge produces an electric and magnetic field and a radiating electromagnetic field.

Units and definitions:

$$1T = 10^4 G$$

tesla

Gauss

SI unit

$$1T = 1\frac{Wb}{m^2}$$

Weber

$$\vec{B}$$

Magnetic field vector

Magnetic induction

Magnetic flux density

$$\vec{H}$$

Magnetic field strength

$$\boxed{\vec{B} = \mu\vec{H}}$$

Permeability

$$\boxed{\mu = \mu_r \mu_o}$$

Permeability of free space

Relative permeability for a medium

Permeability of the medium

$$\mu_o = 4\pi 10^{-7} \left\{ \frac{H}{m} \right\}$$

Exact constant

$$\left\{ \frac{H}{m} \right\} \Leftrightarrow \left\{ \frac{Wb}{m} \right\}$$

Relative permeability $\mu_r \chi$

Bismuth	0.99983	Diamagnetic
Mercury	0.999968	
Gold	0.999964	
Silver	0.99998	
Lead	0.999983	
Copper	0.999991	
Water	0.999991	
Vacuum	1.000	
Air	1.00000036	Paramagnetic
Aluminium	1.000021	
Palladium	1.00082	
Cobalt	250	Ferromagnetic
Nickel	600	
Iron	6000	

In most atoms, electrons occur in pairs. Electrons in a pair spin in opposite directions. So, when electrons are paired together, their opposite spins cause their magnetic fields to cancel each other. Therefore, no net magnetic field exists. Alternately, materials with some unpaired electrons will have a net magnetic field and will react more to an external field. Most materials can be classified as diamagnetic, paramagnetic or ferromagnetic.

Diamagnetic materials have a weak, negative susceptibility to magnetic fields. Diamagnetic materials are slightly repelled by a magnetic field and the material does not retain the magnetic properties when the external field is removed. In diamagnetic materials all the electron are paired so there is no permanent net magnetic moment per atom. Diamagnetic properties arise from the realignment of the electron paths under the influence of an external magnetic field. Most elements in the periodic table, including copper, silver, and gold, are diamagnetic.

Paramagnetic materials have a small, positive susceptibility to magnetic fields. These materials are slightly attracted by a magnetic field and the material does not retain the magnetic properties when the external field is removed. Paramagnetic properties are due to the presence of some unpaired electrons, and from the realignment of the electron paths caused by the external magnetic field.

Ferromagnetic materials have a large, positive susceptibility to an external magnetic field. They exhibit a strong attraction to magnetic fields and are able to retain their magnetic properties after the external field has been removed. Ferromagnetic materials have some unpaired electrons so their atoms have a net magnetic moment. They get their strong magnetic properties due to the presence of magnetic domains. In these domains, large numbers of atom's moments (10^{12} to 10^{15}) are aligned parallel so that the magnetic force within the domain is strong. When a ferromagnetic material is in the unmagnitized state, the domains are nearly randomly organized and the net magnetic field for the part as a whole is zero. When a magnetizing force is applied, the domains become aligned to produce a strong magnetic field within the part. Iron, nickel, and cobalt are examples of ferromagnetic materials. Components with these materials are commonly inspected using the magnetic particle method.

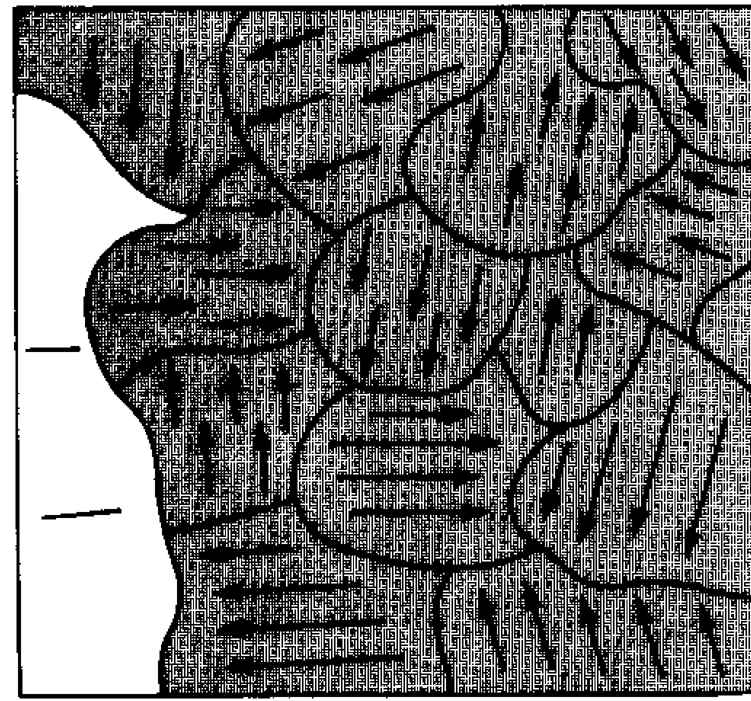
What’s going on

Diamagnetism

Ferromagnetic materials, such as iron, are strongly attracted to both poles of a magnet.
Paramagnetic materials, such as aluminum, are weakly attracted to both poles of a magnet.
Diamagnetic materials, however, are **repelled** by both poles of a magnet. The diamagnetic force of repulsion is very weak (a hundred thousand times weaker than the ferromagnetic force of attraction). Water, the main component of grapes, is diamagnetic.
When an electric charge moves, a magnetic field is created. Every electron is therefore a very tiny magnet, because electrons carry charge and they spin. Additionally, the motion of an orbital electron is an electric current, with an accompanying magnetic field.
In atoms of iron, cobalt, and nickel, electrons in one atom will align with electrons in neighboring atoms, making regions called domains, with very strong magnetization. These materials are ferromagnetic, and are strongly attracted to magnetic poles.
Atoms and molecules that have single, unpaired electrons are paramagnetic. Electrons in these materials orient in a magnetic field so that they will be weakly attracted to magnetic poles. Hydrogen, lithium, and liquid oxygen are examples of paramagnetic substances.
Atoms and molecules in which all of the electrons are paired with electrons of opposite spin, and in which the orbital currents are zero, are diamagnetic. Helium, bismuth, and water are examples of diamagnetic substances.
If a magnet is brought toward a diamagnetic material, it will generate orbital electric currents in the atoms and molecules of the material. The magnetic fields associated with these orbital currents will be oriented such that they repelled by the approaching magnet.
This behavior is predicted by a law of physics known as Lenz's Law. This law states that when a current is induced by a change in magnetic field (the orbital currents in the grape created by the magnet approaching the grape), the magnetic field produced by the induced current will oppose the change.

Ferromagnetic WHY

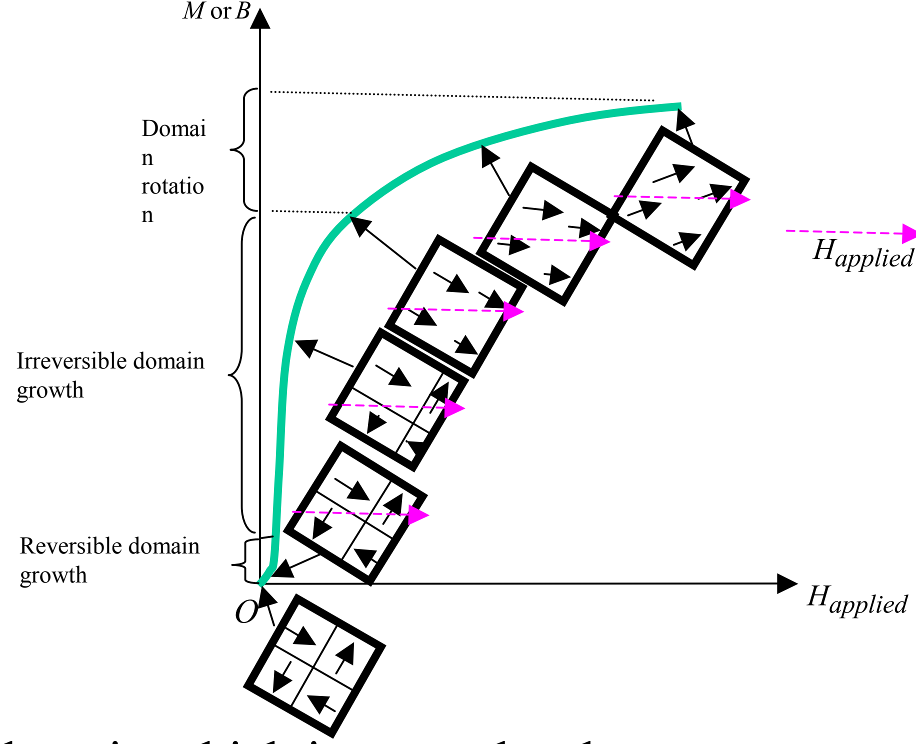
(( ))



The domain structure in a polycrystalline ferromagnetic material. The spin in each domain line up spontaneously. However, the domains are randomly oriented with respect to each other which makes the specimen as a whole appear to be un-magnetized. The external magnetic field will therefore be zero.

Ferromagnetic WHY

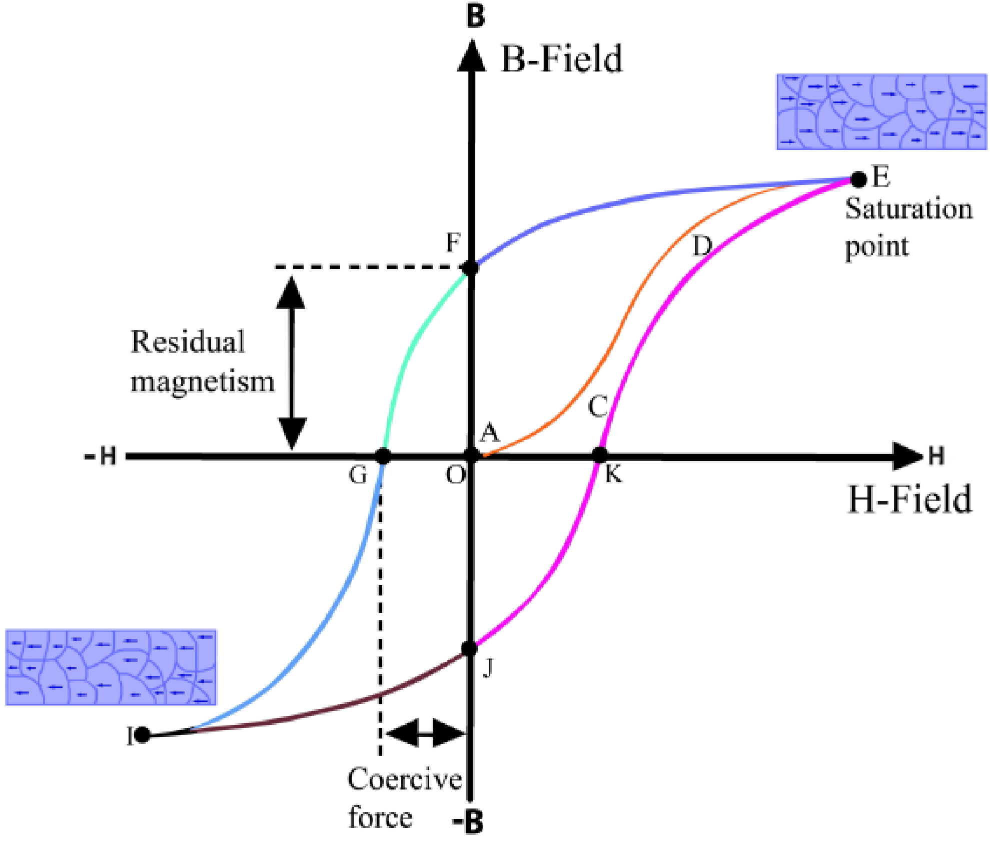
Ferromagnetic materials show hysteresis in the B versus H curve.



• As the magnetic filed is increased, the domain which is most closely aligned with the applied magnetic field will grow. This growth is at the expense of those domains not in alignment with the applied magnetic field.

• Domain growth continues until the entire material consist of one domain.

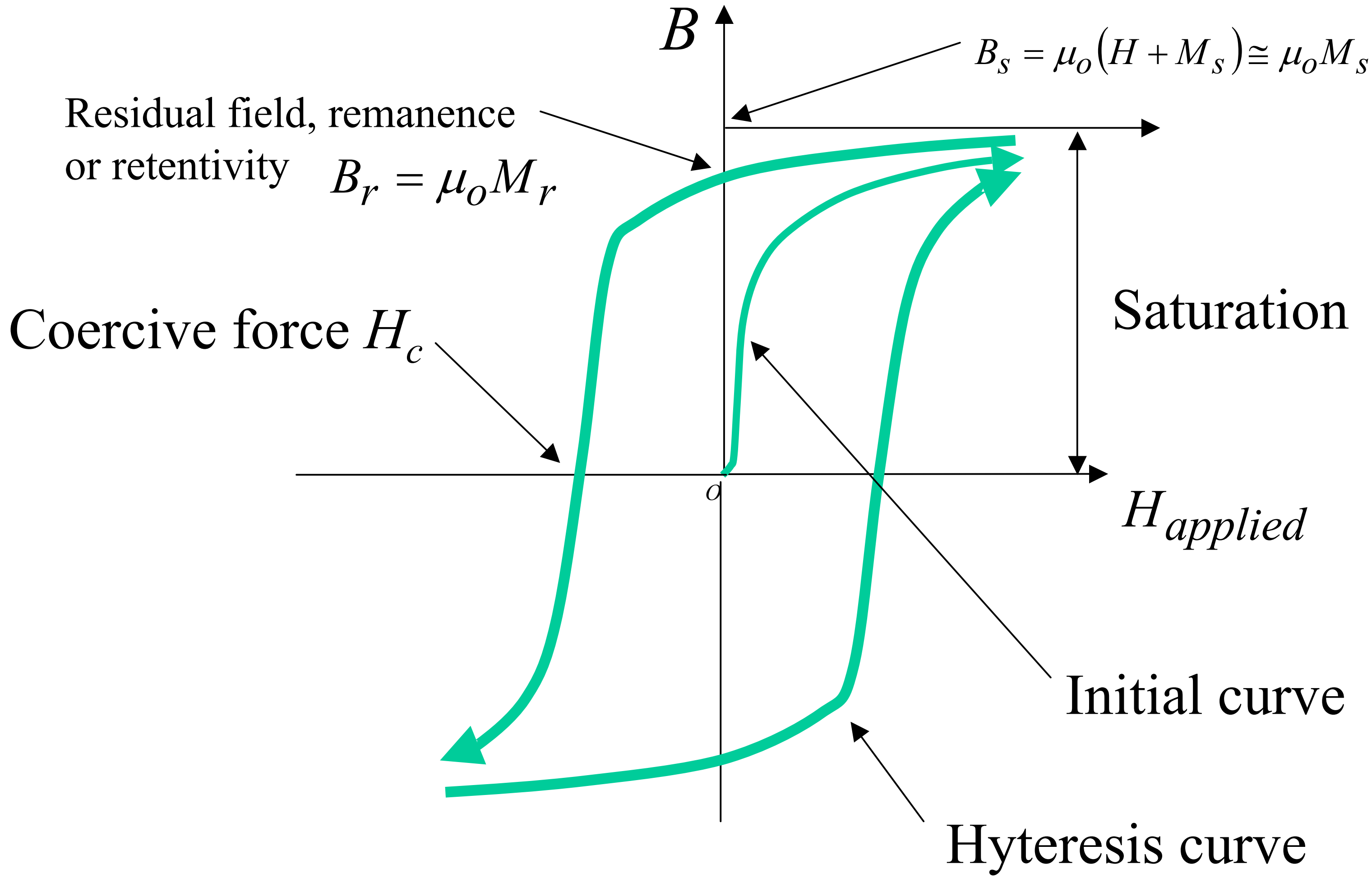
• Domain rotation will then occur in order to complete the alignment of the magnetic moment with the applied filed, saturating the effect.



B
B-Field
E
Saturation
point
F
D
Residual
magnetism
C
A
-H
H
G
O
K
H-Field
J
I
Coercive
force
-B

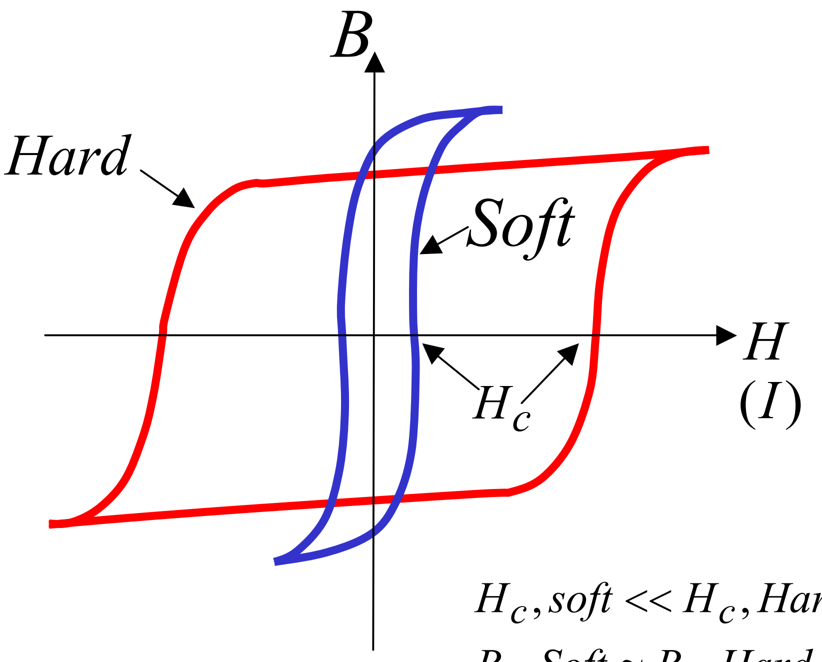
Ferromagnetic

What happens when we cycle the applied magnetic field.



Ferromagnetic

SOFT and HARD Ferromagnetic materials



$H_c, soft << H_c, Hard$

$B_s, Soft \approx B_s, Hard$

Hysteresis loops of a soft magnetic material, which is easy to magnetize and demagnetize, and those of hard magnetic materials

Soft ::: transformer cores, solenoids, ….
Hard :: permanent magnets

Area of hysteresis loop is equivalent to energy lost in one cycle.

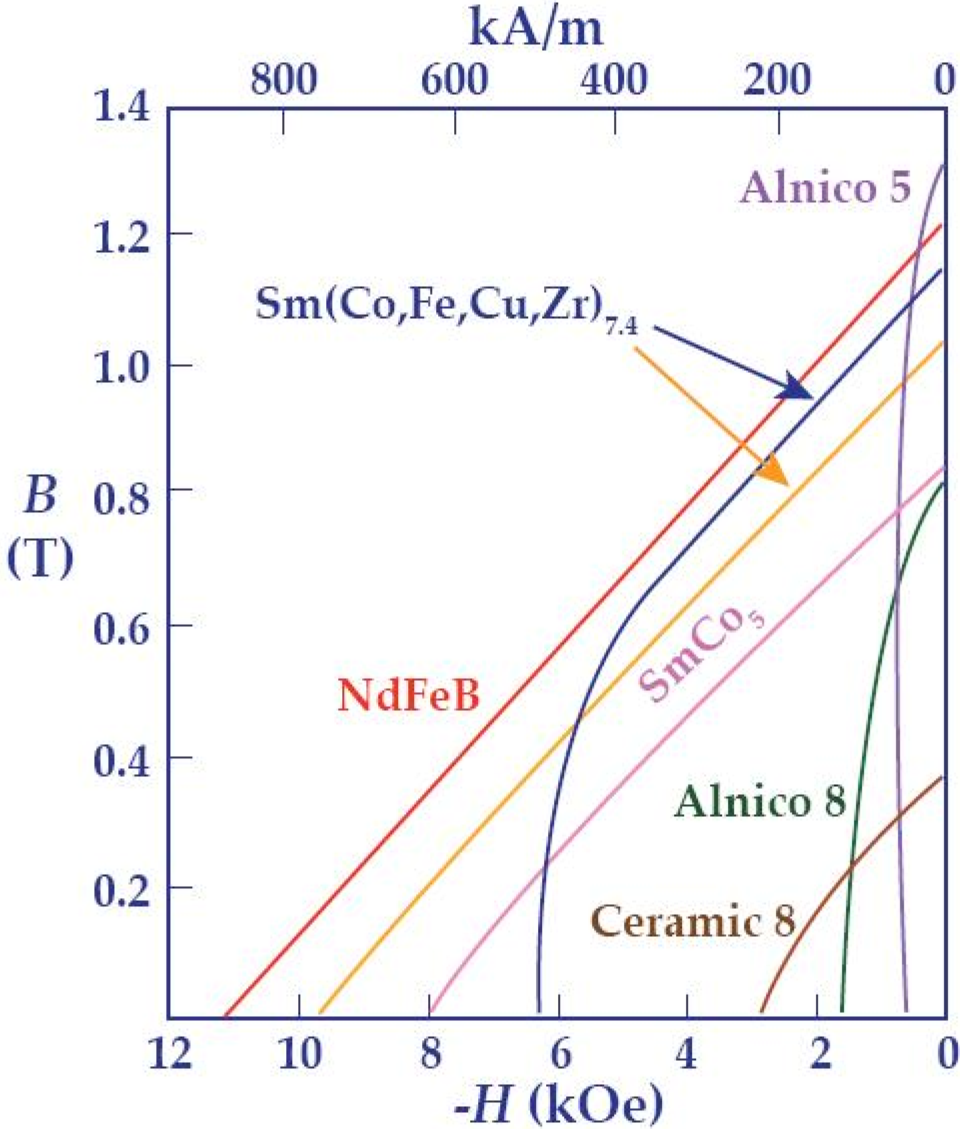
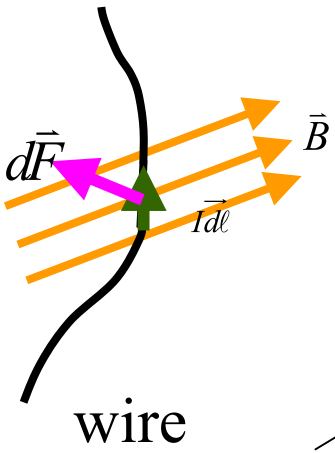


Figure 6 — Comparison of rare-earth magnets with some older magnet types. *B*,*H* demagnetization curves of average commercial magnets.

POSTULATE 1 FOR THE MAGNETIC FIELD

A current element $\vec{I}d\ell$ immersed in a magnetic field $\vec{B}$ will experience a force $d\vec{F}$ given by:



Meters (m)

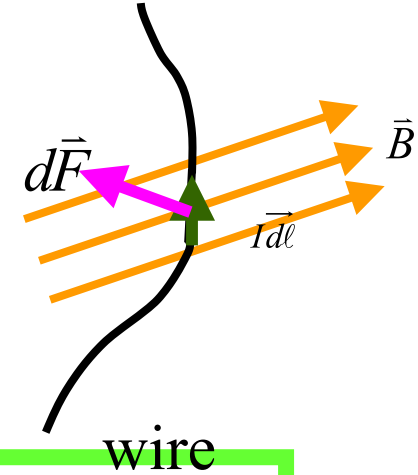
$$d\vec{F} = \vec{I} \times \vec{B}d\ell$$

Newtons {N}

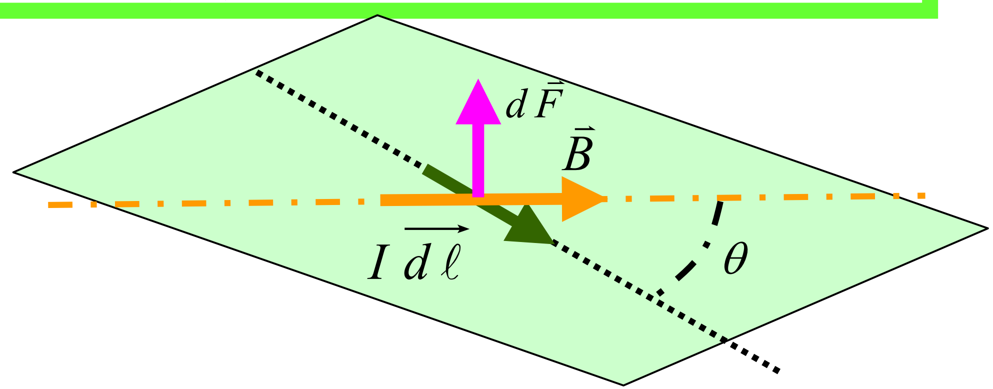
Amps (A)

Tesla (T)

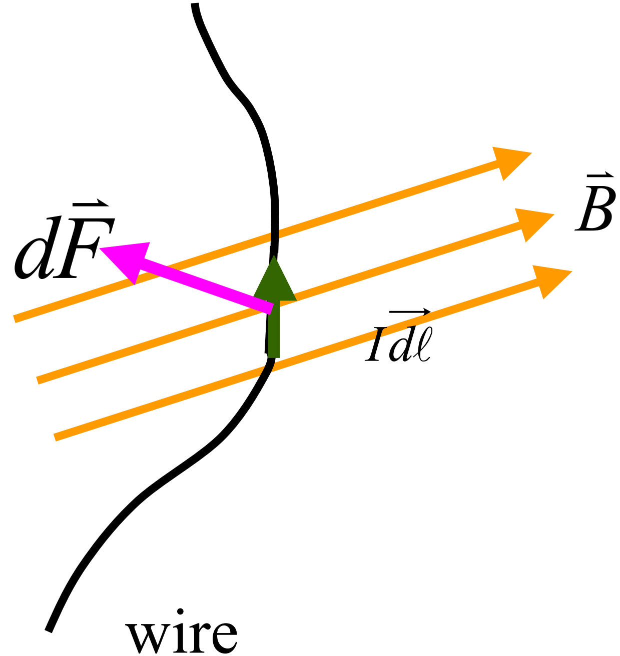
POSTULATE 1 FOR THE MAGNETIC FIELD



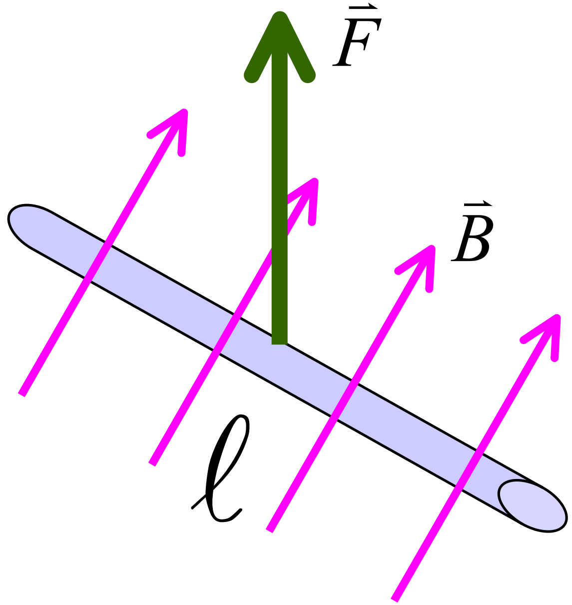
A current element experiences a force which is at right angles to the plane formed by the current element and magnetic field direction magnitude: $dF = IBd\ell\sin(\theta)$



Postulate 1 for the magnetic field



Consider a straight segment ℓ



Net force on the segment

$$F = \int_{\ell} dF = IB\ell$$

Example (Solution)

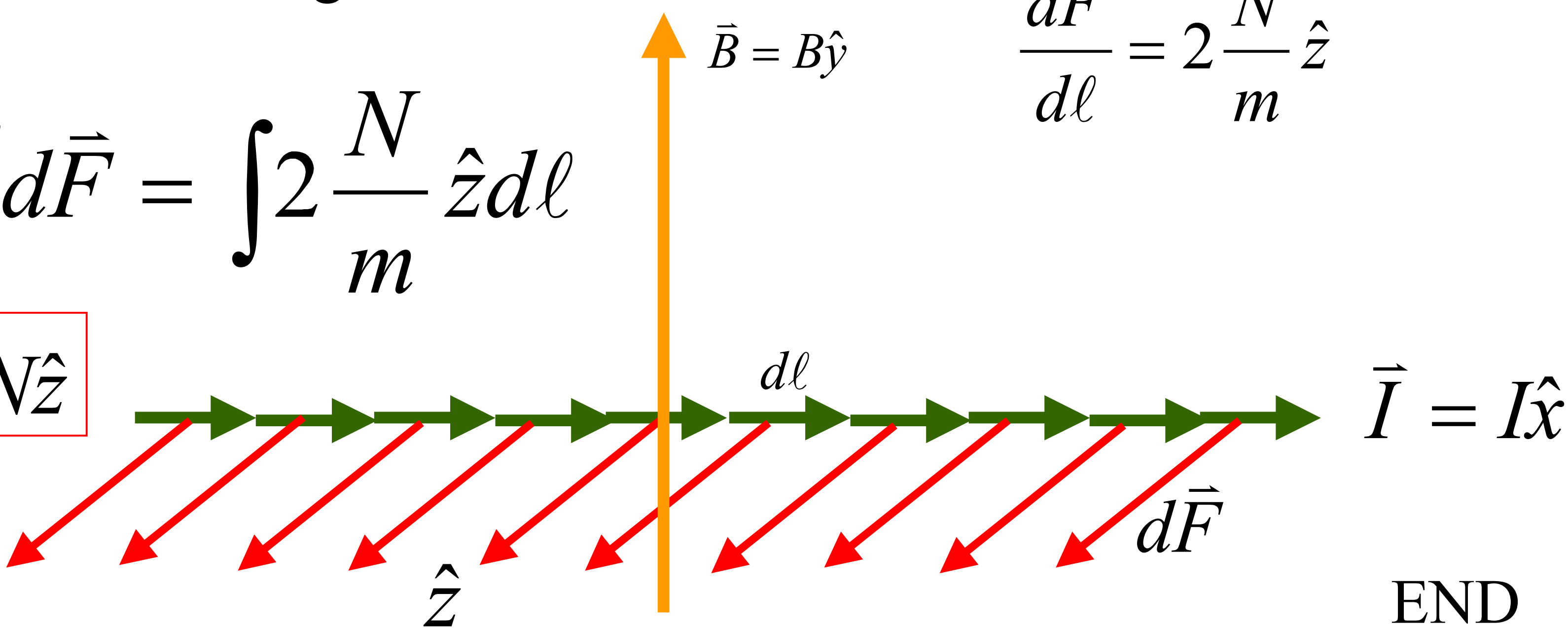
A straight segment of wire carries a current of I = 1 A in the positive x direction. It is placed in a magnetic B = 2 T which points in the positive y direction. A) Find the magnetic force vector per unit length of the wire. B) If the wire is 1 m in length what is the total magnetic force on the wire.

Taking the wire as straight as shown in the figure, the total magnetic force on the wire can be obtained by summing all force elements along the wire.

$$\frac{d\vec{F}}{d\ell} = 2\frac{N}{m}\hat{z}$$

$$\vec{F} = \int d\vec{F} = \int 2\frac{N}{m}\hat{z}\,d\ell$$

$$\boxed{\vec{F} = 2N\hat{z}}$$

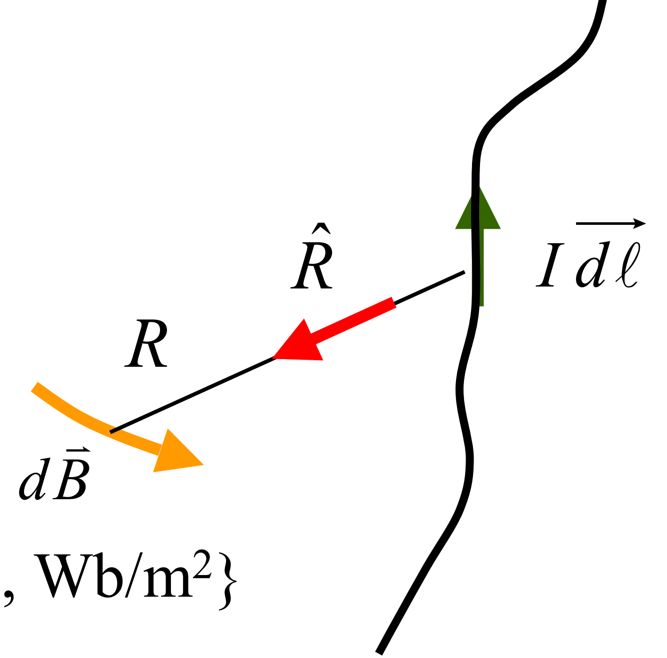


END

Postulate 2 for the magnetic field

A current element $\vec{I}d\ell$ produces a magnetic field $\vec{B}$ which at a distance R is given by:

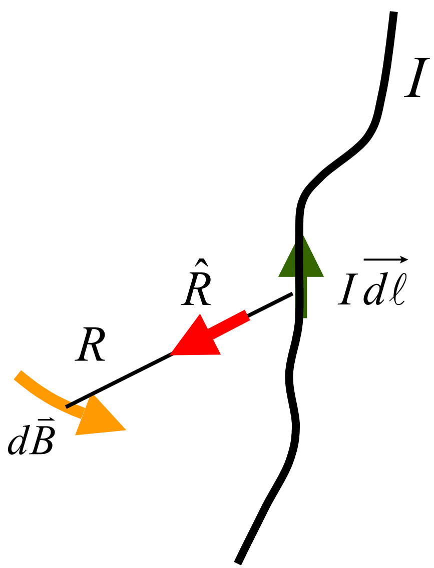
$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{\vec{I}\times\hat{R}}{R^2}d\ell$$



Units of {T, G, Wb/m^2}

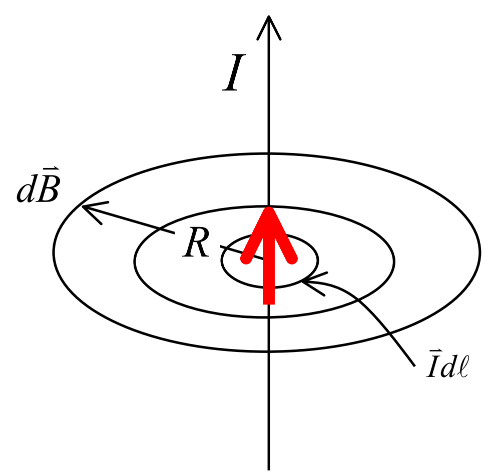
Postulate 2 for the magnetic field

Postulate 2 implies that the magnetic field is everywhere normal to the element of length $d\ell$



$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{\vec{I}\times\hat{R}}{R^2}d\ell$$

$$dB = \frac{\mu_o}{4\pi}\frac{Id\ell}{R^2}\sin(\theta)$$



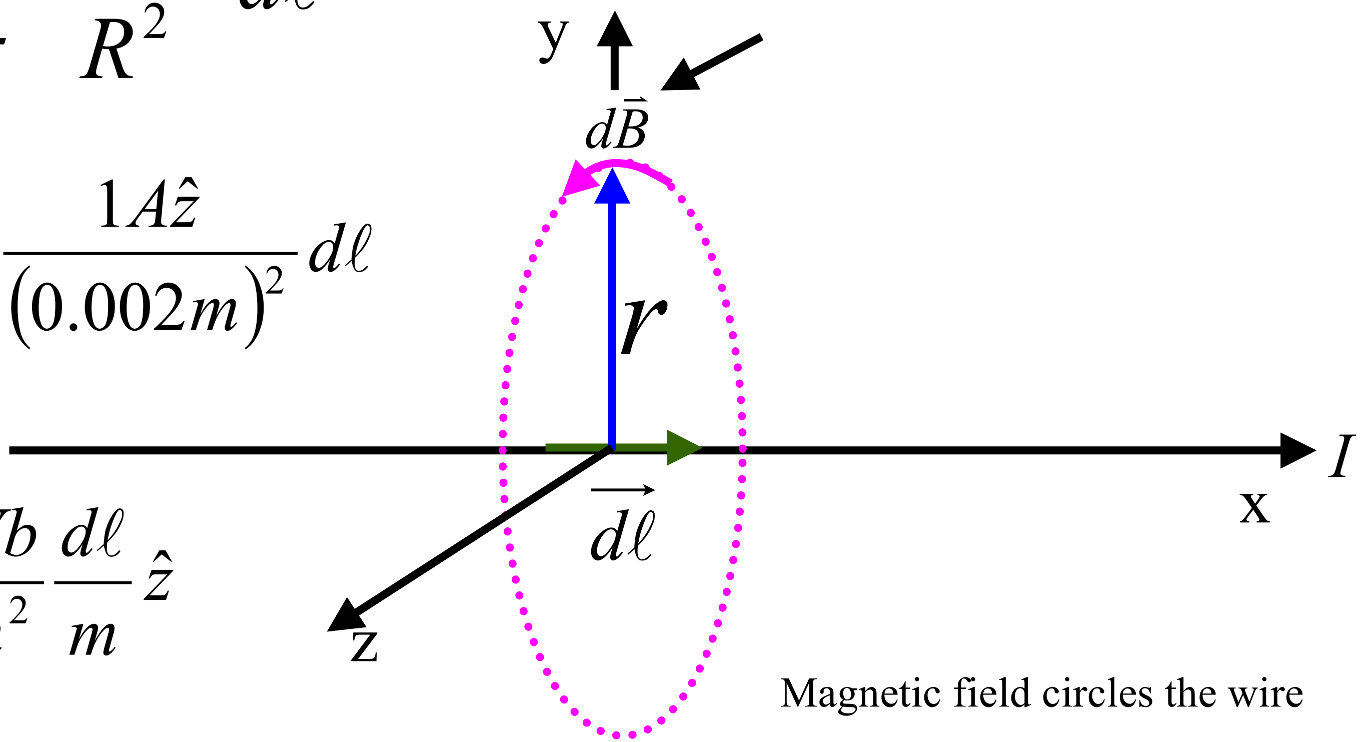
Example (Solution)

A small segment of wire carries a current of I = 1 A and is oriented such that the current is directed along the positive x direction. Calculate the magnetic field produced by a short segment of this wire at radial distance r = 2 mm. Assume free space exists about the wire.

$$d\vec{B} = \frac{\mu_o}{4\pi} \frac{\vec{I} \times \hat{R}}{R^2} d\ell$$

$$d\vec{B} = 10^{-7} \frac{H}{m} \frac{1A\hat{z}}{(0.002m)^2} d\ell$$

$$d\vec{B} = .025 \frac{Wb}{m^2} \frac{d\ell}{m} \hat{z}$$



Magnetic field circles the wire

END

Example (Solution)

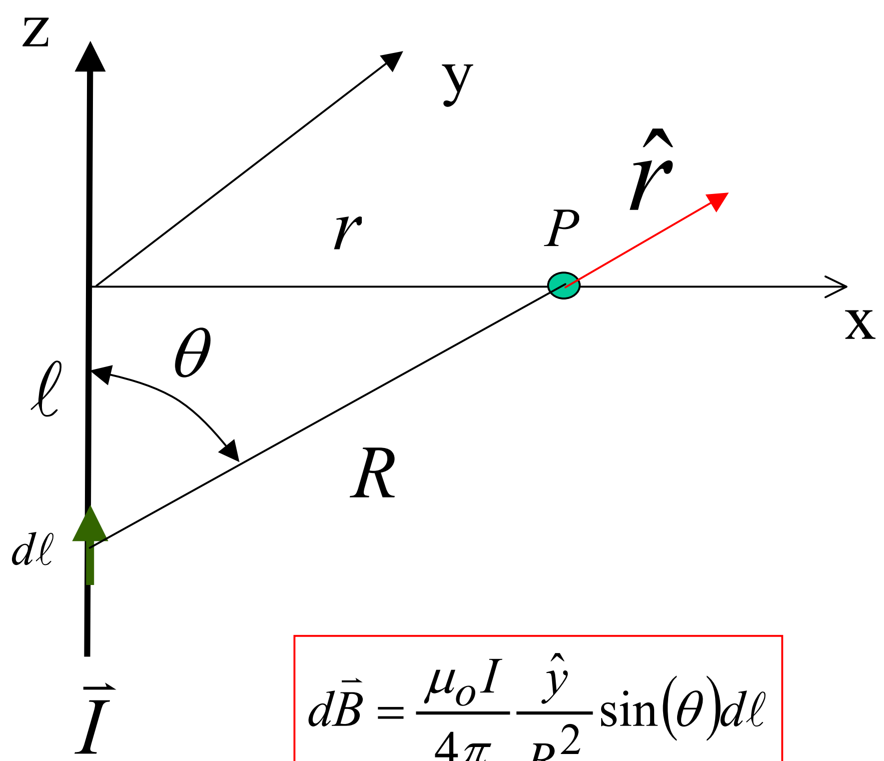
Calculate the magnetic field produced by a long straight wire carrying a current I. Take the wire as infinite in length and oriented along the Z axis as shown in the figure. The magnetic field is to be calculated at a radial distance r from the wire.

The field $d\vec{B}$ at P can be expressed as:

$$\boxed{d\vec{B} = \frac{\mu_o}{4\pi}\frac{\vec{I}\times\hat{r}}{R^2}d\ell}$$

$$\vec{I} = I\hat{z}$$

$$\hat{r} = \sin(\theta)\hat{x} + \cos(\theta)\hat{z}$$



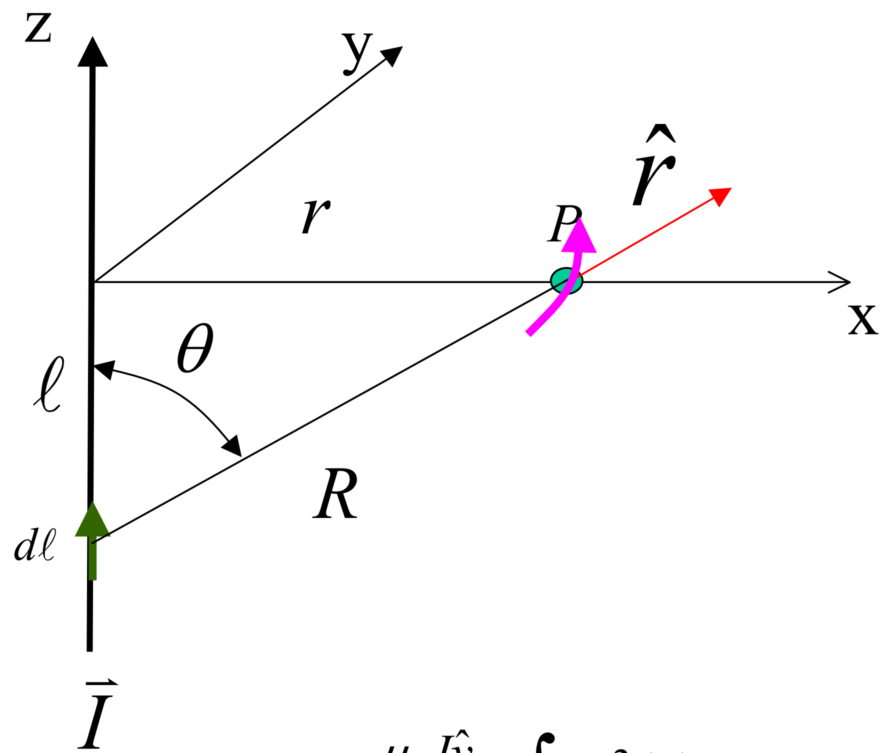
$$\boxed{d\vec{B} = \frac{\mu_o I}{4\pi}\frac{\hat{y}}{R^2}\sin(\theta)d\ell}$$

Example (Solution)

Calculate the magnetic field produced by a long straight wire carrying a current I. Take the wire as infinite in length and oriented along the Z axis as shown in the figure. The magnetic field is to be calculated at a radial distance r from the wire.

Now we sum over the length of the wire

$$\vec{B} = \int_{wire} d\vec{B} = \int_{wire} \frac{\mu_o I}{4\pi}\frac{\hat{y}}{R^2}\sin(\theta)d\ell$$



Note $$R = \frac{r}{\sin(\theta)}$$

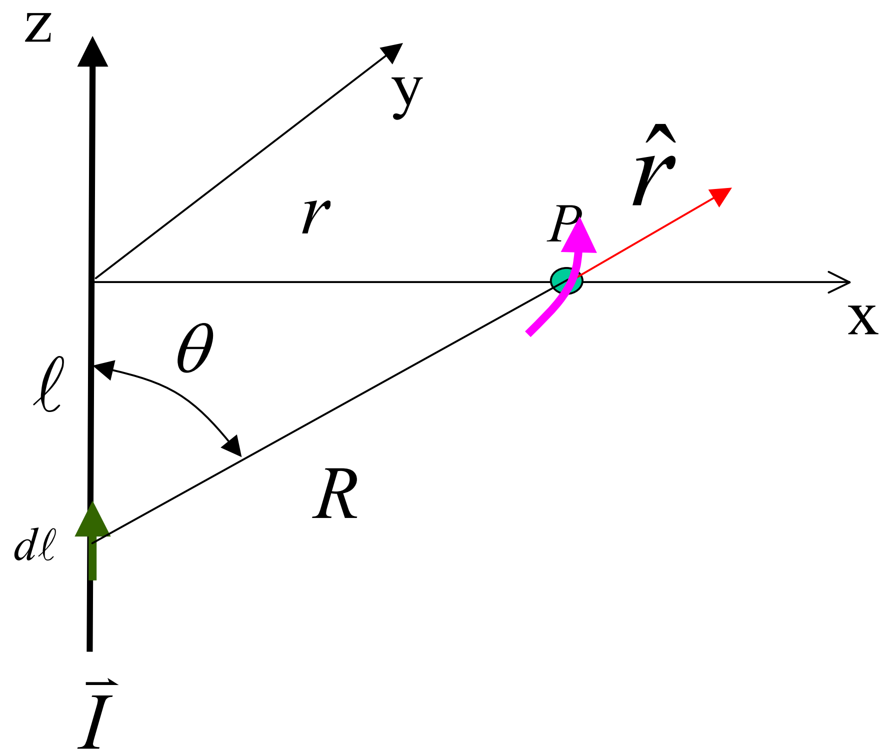
$$\vec{B} = \frac{\mu_o I\hat{y}}{4\pi r^2}\int_{wire}\sin^3(\theta)d\ell$$

Example (Solution)

Calculate the magnetic field produced by a long straight wire carrying a current I. Take the wire as infinite in length and oriented along the Z axis as shown in the figure. The magnetic field is to be calculated at a radial distance r from the wire.

Result from integration

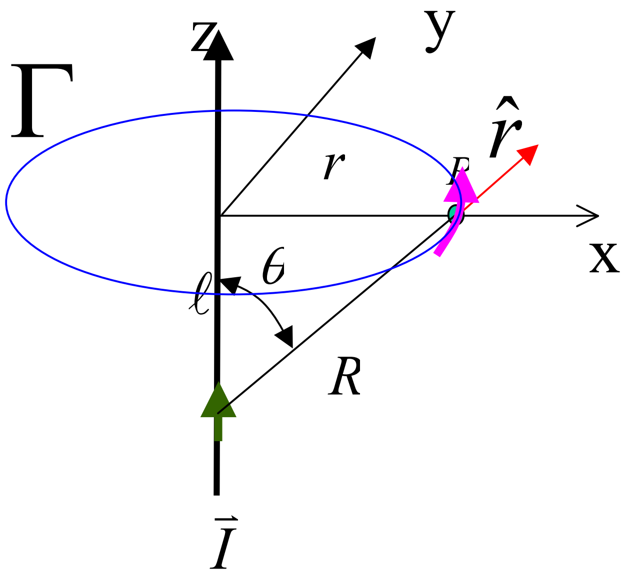
$$\boxed{\vec{B} = \frac{\mu_o I \hat{y}}{2\pi r}}$$



AMPERE'S LAW

We need a simple means of computing the magnetic field for a known current distribution.

The starting point is a modification of postulate 2.



Result for wire

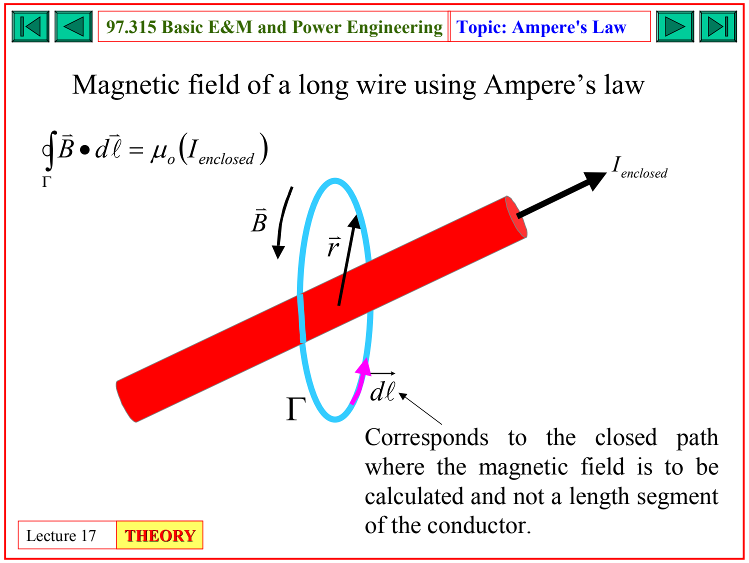
$$\boxed{\vec{B} = \frac{\mu_o I \hat{y}}{2\pi r}}$$

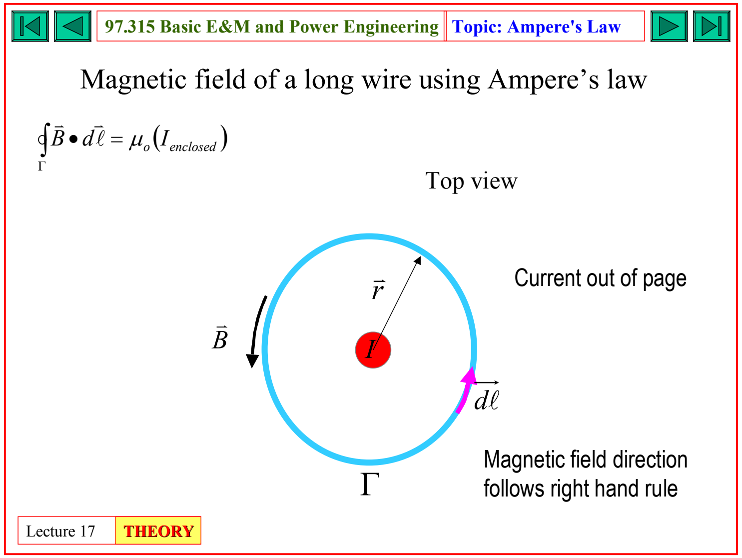
AMPERE'S LAW

$$\oint_{\Gamma} \vec{B} \bullet d\vec{\ell} = \mu_o \left(I_{enclosed} \right)$$

Current enclosed by path Γ

Line integral around closed path Γ





AMPERE'S LAW

$$\oint_\Gamma \vec{B} \bullet d\vec{\ell} = \mu_o \left(I_{enclosed} \right)$$

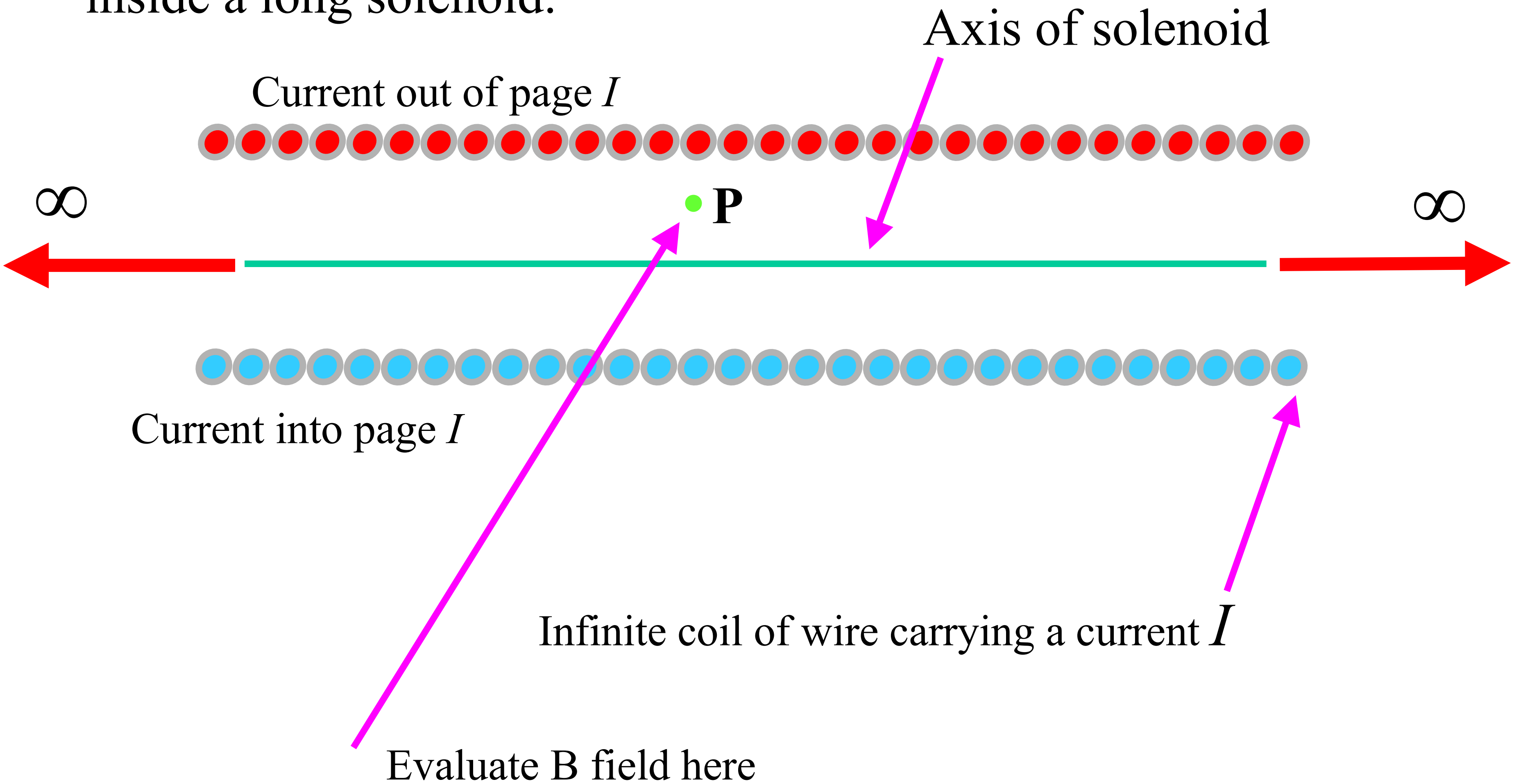
$$\oint_\Gamma \vec{B} \bullet d\vec{\ell} = B 2 \pi r$$

$$B 2 \pi r = \mu_o I$$

$$\boxed{B = \frac{\mu_o I}{2 \pi r}}$$

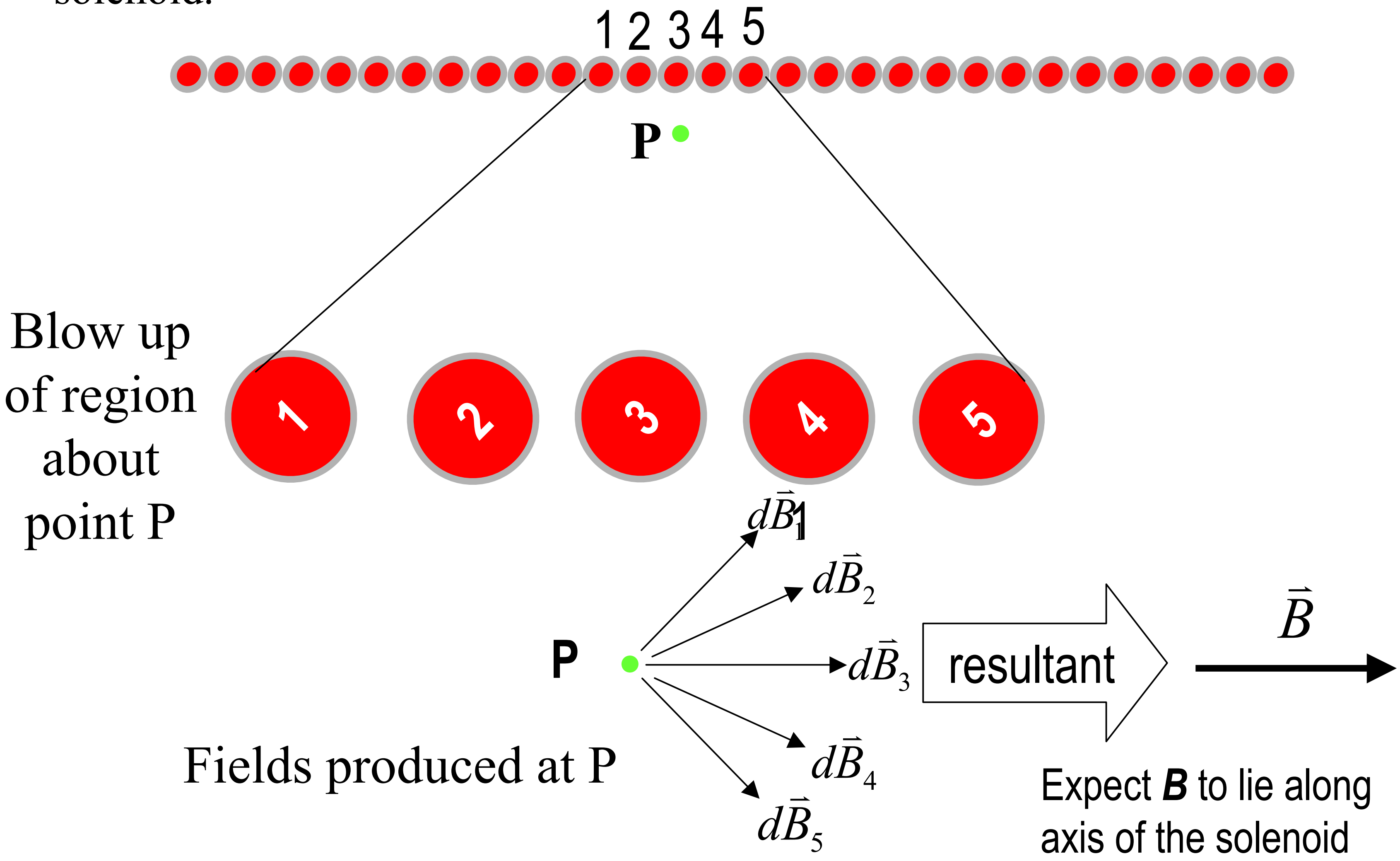
Example (Question)

Obtain an expression for the electric field at a point inside a long solenoid.



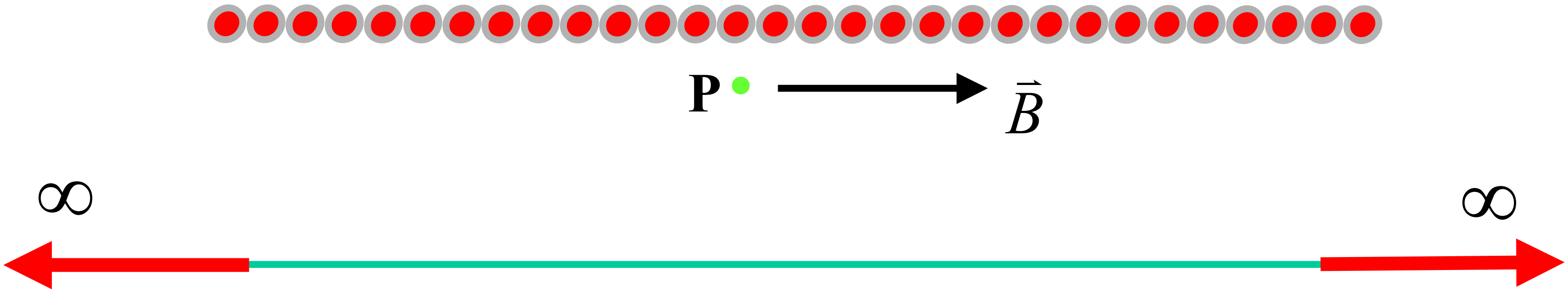
Example (Solution)

Obtain an expression for the electric field at a point inside a long solenoid.

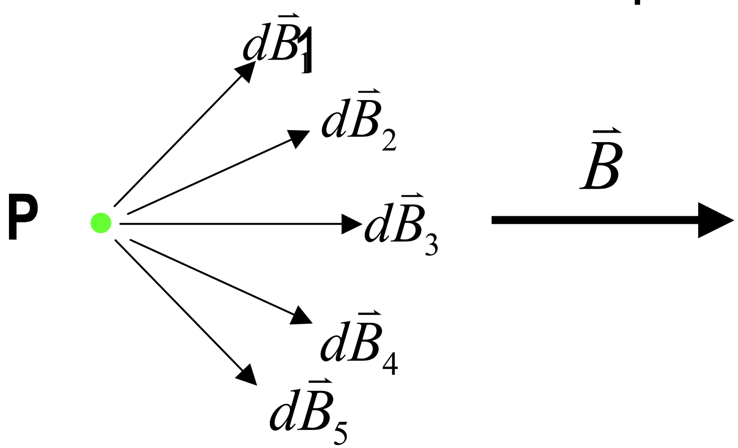


Example (Solution)

Obtain an expression for the electric field at a point inside a long solenoid.



Expect ***B*** to lie along axis of the solenoid

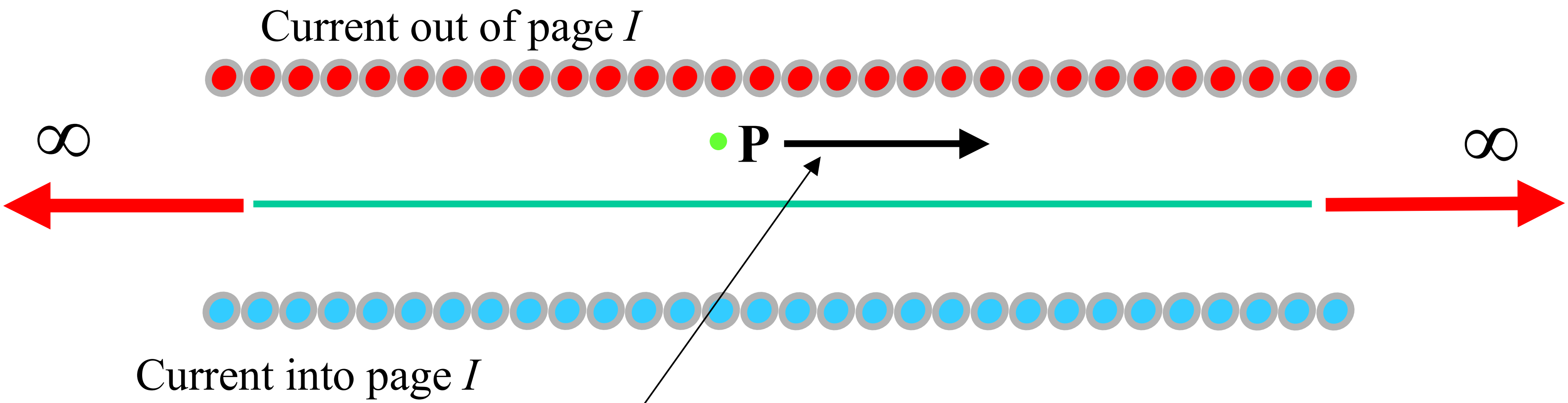


$$\vec{\nabla} \bullet \vec{B} = 0$$

Implies that ***B*** field has no radial component. I.e. no component pointing towards or away from the solenoid axis.

Example (Solution)

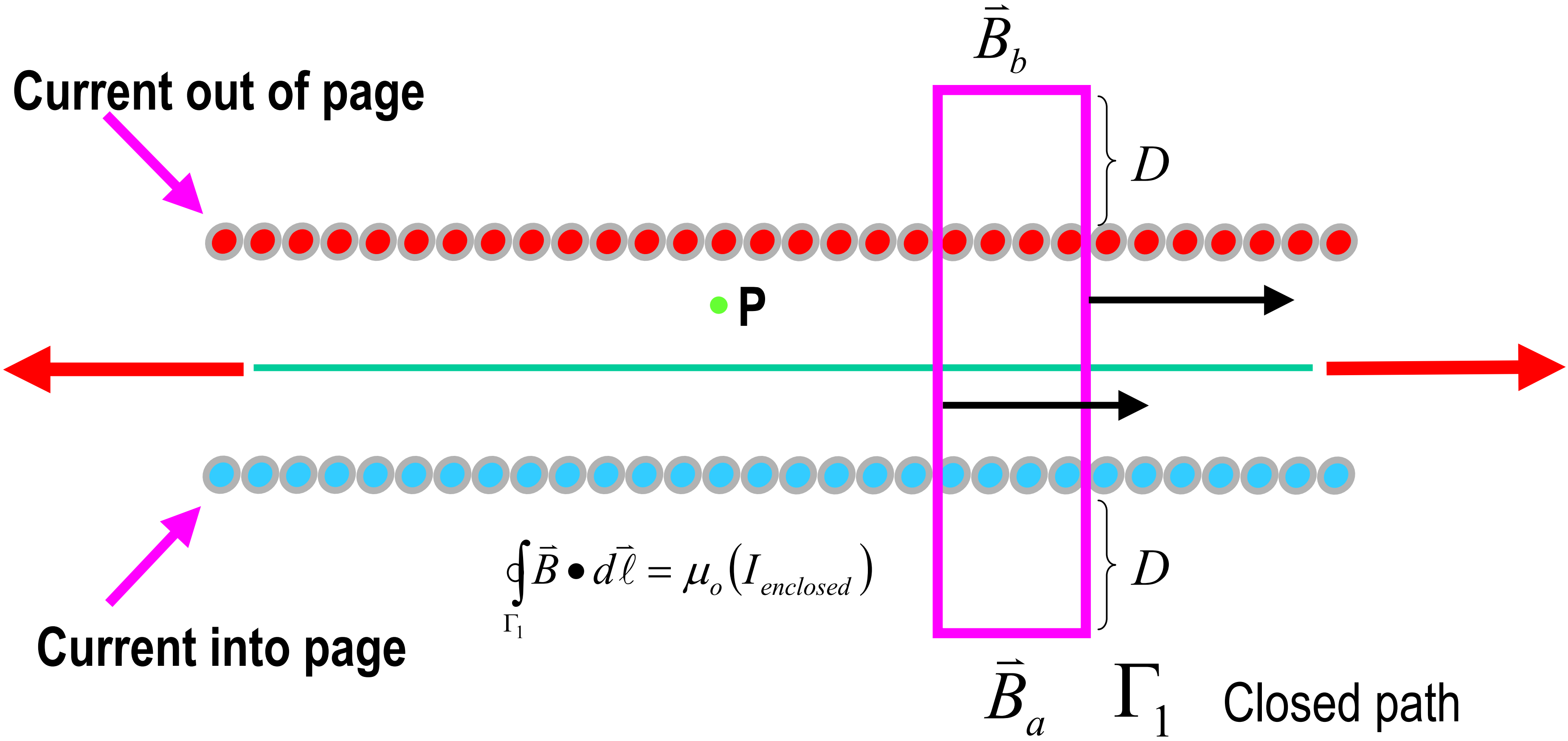
Obtain an expression for the electric field at a point inside a long solenoid.



We have established that the ***B*** field is along the axis of the solenoid. We will use this result to first show that the ***B*** field external to the solenoid is zero, then we can determine the expression for the field inside the solenoid.

Example (Solution)

Obtain an expression for the electric field at a point inside a long solenoid.

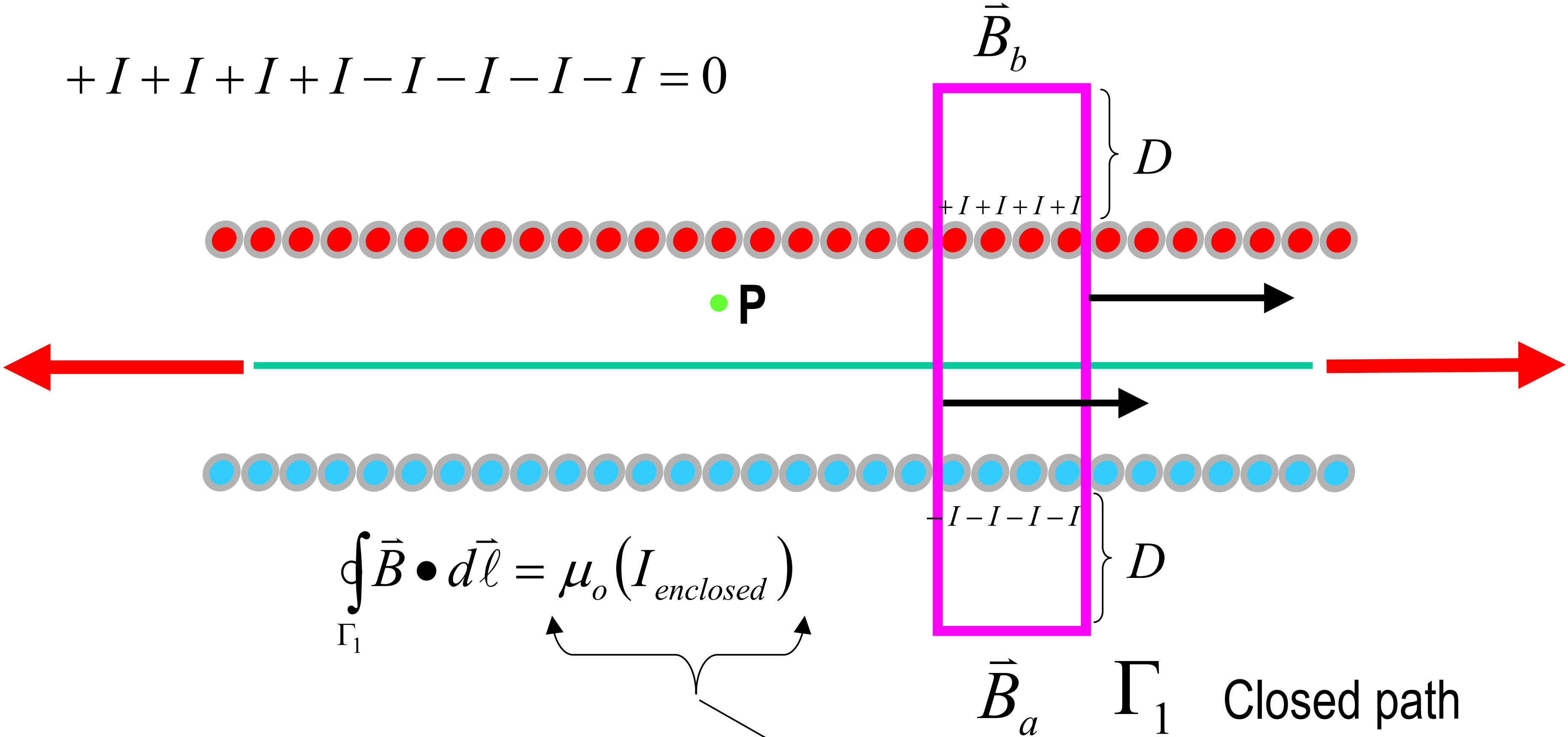


CLAIM: The magnetic field outside of the solenoid is zero.

Example (Solution)

Obtain an expression for the electric field at a point inside a long solenoid.

$$+I+I+I+I-I-I-I-I=0$$

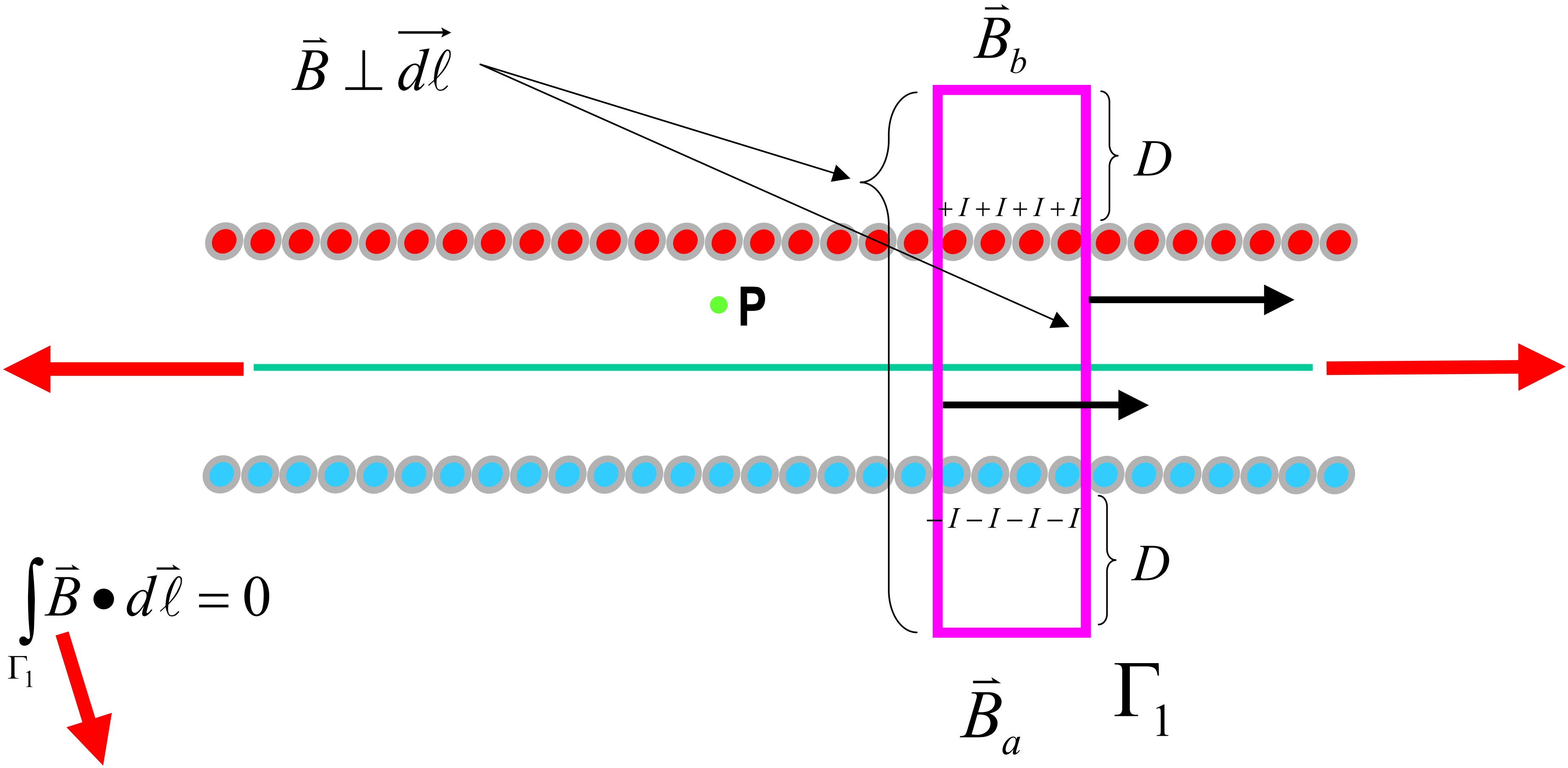


$$\oint_{\Gamma_1} \vec{B} \bullet d\vec{\ell} = \mu_o \left(I_{enclosed} \right)$$

Must have $\int_{\Gamma_1} \vec{B} \bullet d\vec{\ell} = 0$ since there is no net enclosed current by the closed path.

Example (Solution)

Obtain an expression for the electric field at a point inside a long solenoid.

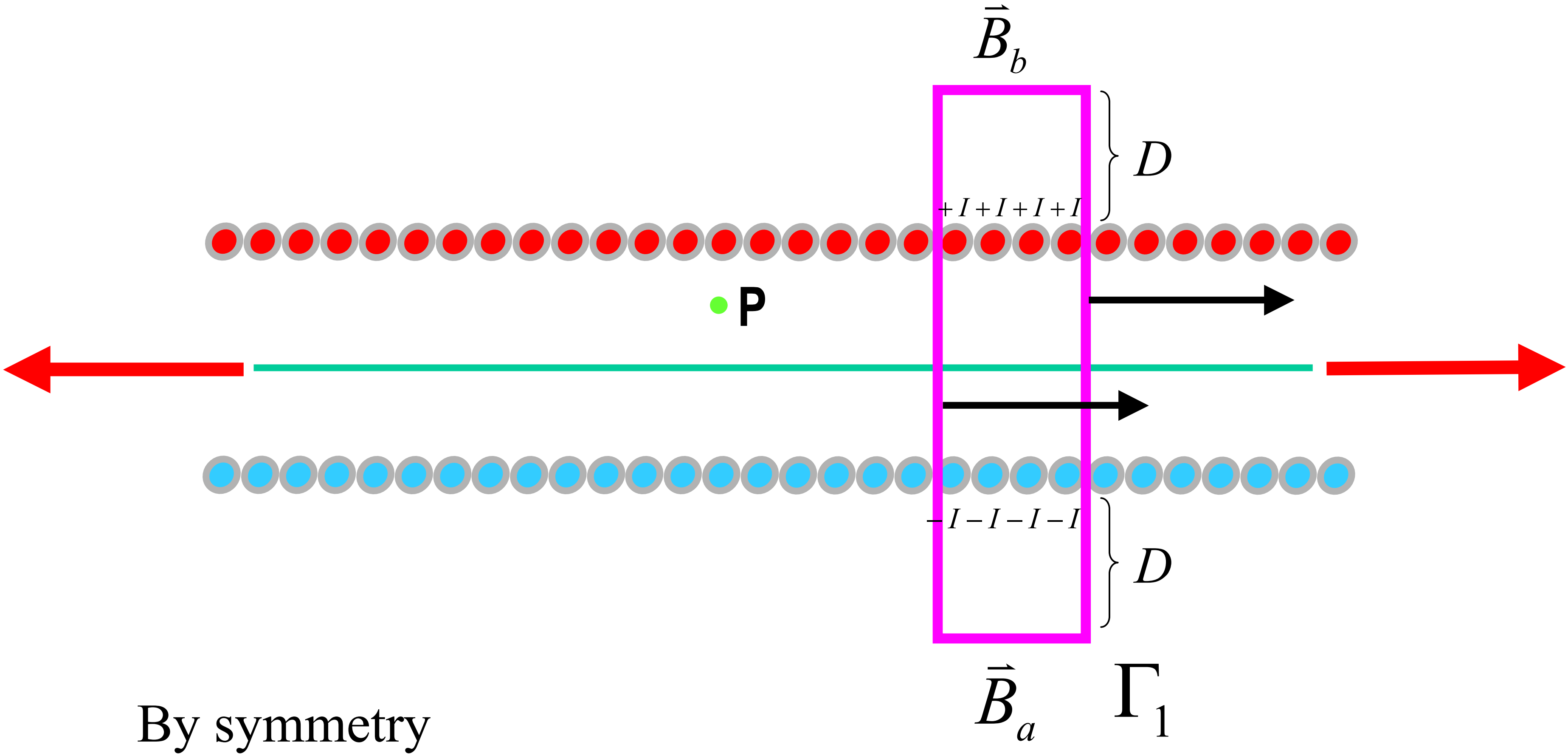


$$\oint_{\Gamma_1} \vec{B} \bullet d\vec{\ell} = 0$$

Conceivably there might be some non-zero field components outside of the solenoid, $\vec{B}_a$ and $\vec{B}_b$ as shown.

Example (Solution)

Obtain an expression for the electric field at a point inside a long solenoid.

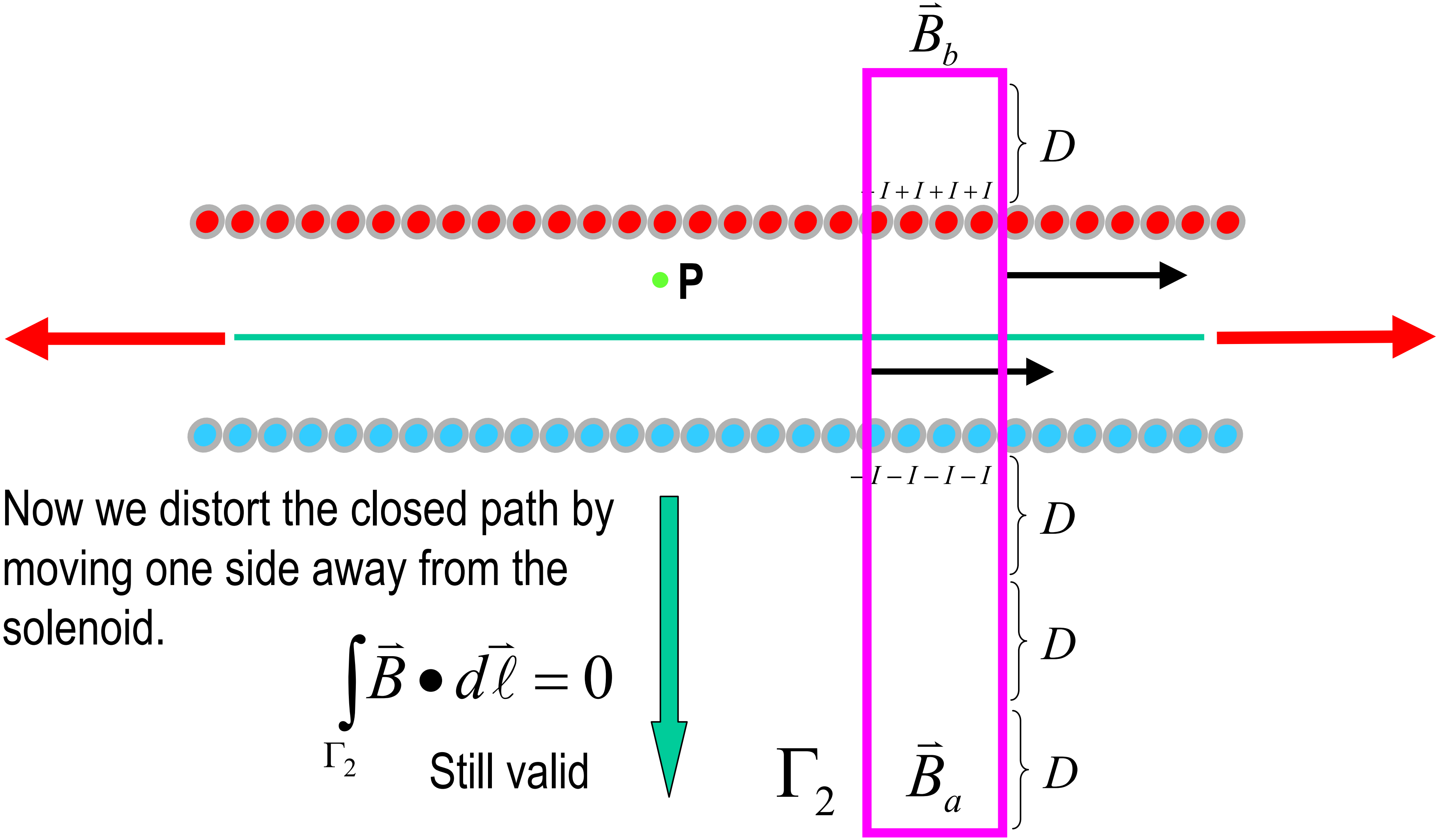


By symmetry

We would need to have $\vec{B}_a = \vec{B}_b$ to give . $\int_{\Gamma_1} \vec{B} \bullet d\vec{\ell} = 0$

Example (Solution)

Obtain an expression for the electric field at a point inside a long solenoid.



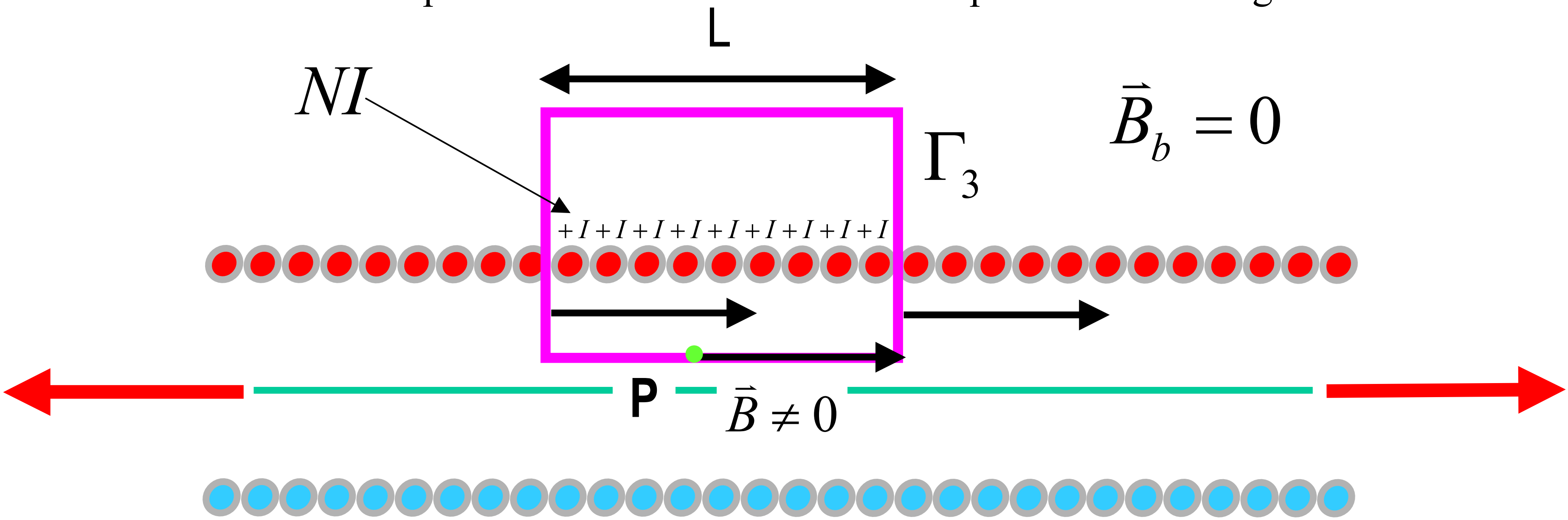
Now we distort the closed path by moving one side away from the solenoid.

$$\int_{\Gamma_2} \vec{B} \bullet d\vec{\ell} = 0$$

Still valid

Example (Solution)

Obtain an expression for the electric field at a point inside a long solenoid.



$$\oint_{\Gamma_3} \vec{B} \bullet d\vec{\ell} = \mu_o \left(I_{enclosed} \right)$$

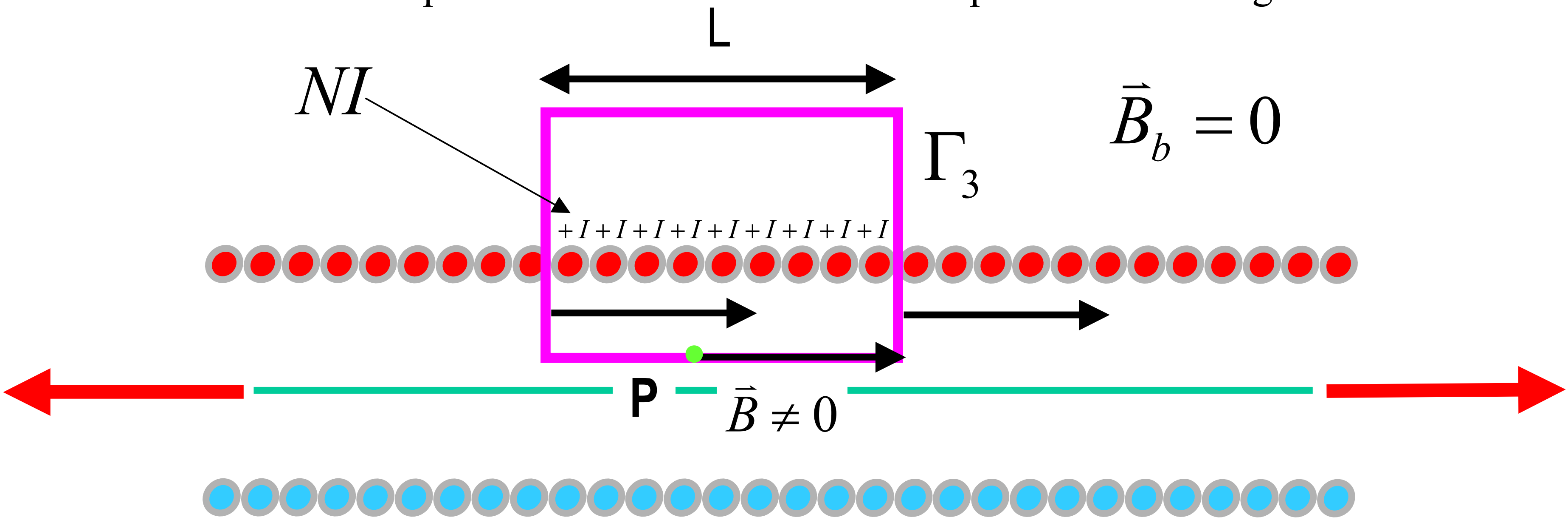
$$\int_{\Gamma_3} \vec{B} \bullet d\vec{\ell} = BL = \mu_o NI$$

Ampere's Law

$$\boxed{B = \frac{\mu_o NI}{L}}$$

Example (Solution)

Obtain an expression for the electric field at a point inside a long solenoid.



$$\oint_{\Gamma_3} \vec{B} \bullet d\vec{\ell} = \mu_o \left(I_{enclosed} \right)$$

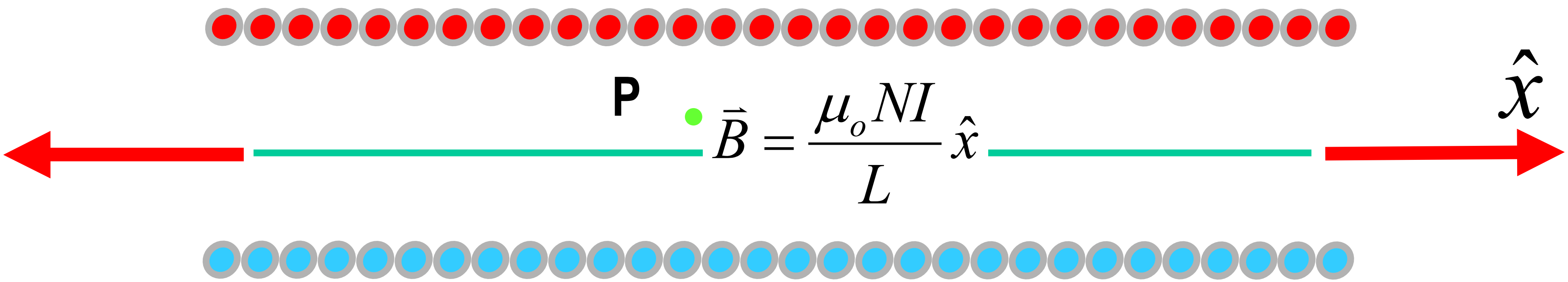
$$\int_{\Gamma_3} \vec{B} \bullet d\vec{\ell} = BL = \mu_o NI$$

Ampere's Law

$$\boxed{B = \frac{\mu_o NI}{L}}$$

$\vec{B}_b = 0$

N : number of turns enclosed by length L



$$B = \frac{\mu_o NI}{L}$$

- ***B*** is independent of distance from the axis of the long solenoid as we are inside the solenoid!
- ***B*** is uniform inside the long solenoid.
- Direction of ***B*** from right hand rule