کنترل خطی — پاسخ تمرینات سری دوم — گروه کنترل فرهاری

سؤال ۱:

با توجه به بلوک دیاگرام داریم:

$$\dot{x}_3 = -(fK+d)x_1 + ax_1 + fu$$

$$\dot{x}_1 = -Kx_2 + u$$

$$y = bx_2$$

$$\Rightarrow \quad A = \begin{bmatrix} 0 & -K \\ a & -(fK+d) \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ f \end{bmatrix}, \quad C = \begin{bmatrix} 0 & b \end{bmatrix}, \quad D = [0]$$

سؤال ۲:

$$m_1\ddot{x} + K_1(x-q) + b_1(\dot{x}-\dot{q}) = u(t)$$

$$m_2\ddot{q} + K_2 q + b_2\dot{q} + b_1(\dot{q}-\dot{x}) + K_1(q-x) = 0$$

$$x_1 = x \qquad x_2 = \dot{x} \qquad x_3 = q \qquad x_4 = \dot{q}$$

$$\Rightarrow \quad \dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_1}{m_1} & -\frac{b_1}{m_1} & \frac{K_1}{m_1} & \frac{b_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{K_1}{m_2} & \frac{b_1}{m_2} & -\frac{(K_1+K_2)}{m_2} & -\frac{(b_1+b_2)}{m_2} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix} u(t)$$

از آنجا: $y(t) = q(t) \Rightarrow C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$ x

سؤال ۳ :

$$C_1\dot{v}_1 = \frac{v_a - v_1}{R_1} + \frac{v_2 - v_1}{R_2}$$

$$C_2\dot{v}_2 = \frac{v_b - v_2}{R_3} + \frac{v_1 - v_2}{R_2}$$

$$x_1 = v_1 \quad , \quad x_2 = v_2$$

$$\Rightarrow \quad \dot{x} = \begin{bmatrix} -\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1}\right) & \frac{1}{R_2C_1} \\ -\frac{1}{R_2C_2} & -\left(\frac{1}{R_3C_2} + \frac{1}{R_2C_2}\right) \end{bmatrix} x + \begin{bmatrix} \frac{1}{R_1C_1} & 0 \\ 0 & \frac{1}{R_3C_2} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix}$$

سؤال ۴ :

$$\frac{y(s)}{U(s)} = \frac{b_0}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \Rightarrow$$

$$\frac{d^4(y/b_0)}{dt^4} + a_3 \frac{d^3(y/b_0)}{dt^3} + a_2 \frac{d^2(y/b_0)}{dt^2} + a_1 \frac{d(y/b_0)}{dt} + a_0(y/b_0) = u$$

$\Rightarrow$ تعریف متغیر حالت

$$x_1 = y/b_0$$

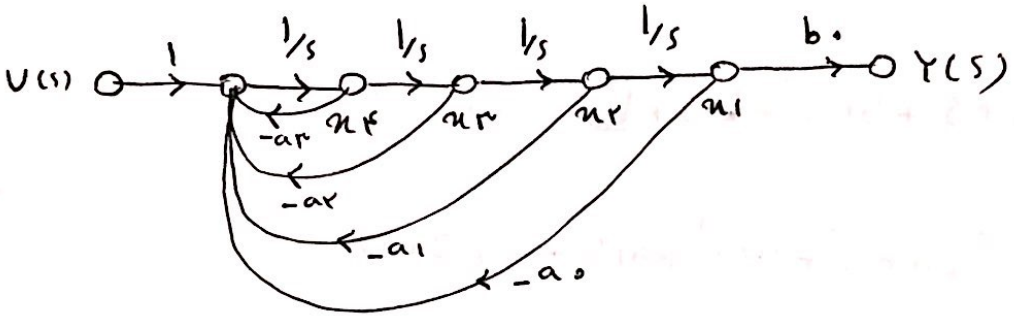
$$x_2 = \dot{x}_1 = \dot{y}/b_0$$

$$x_3 = \dot{x}_2 = \ddot{y}/b_0$$

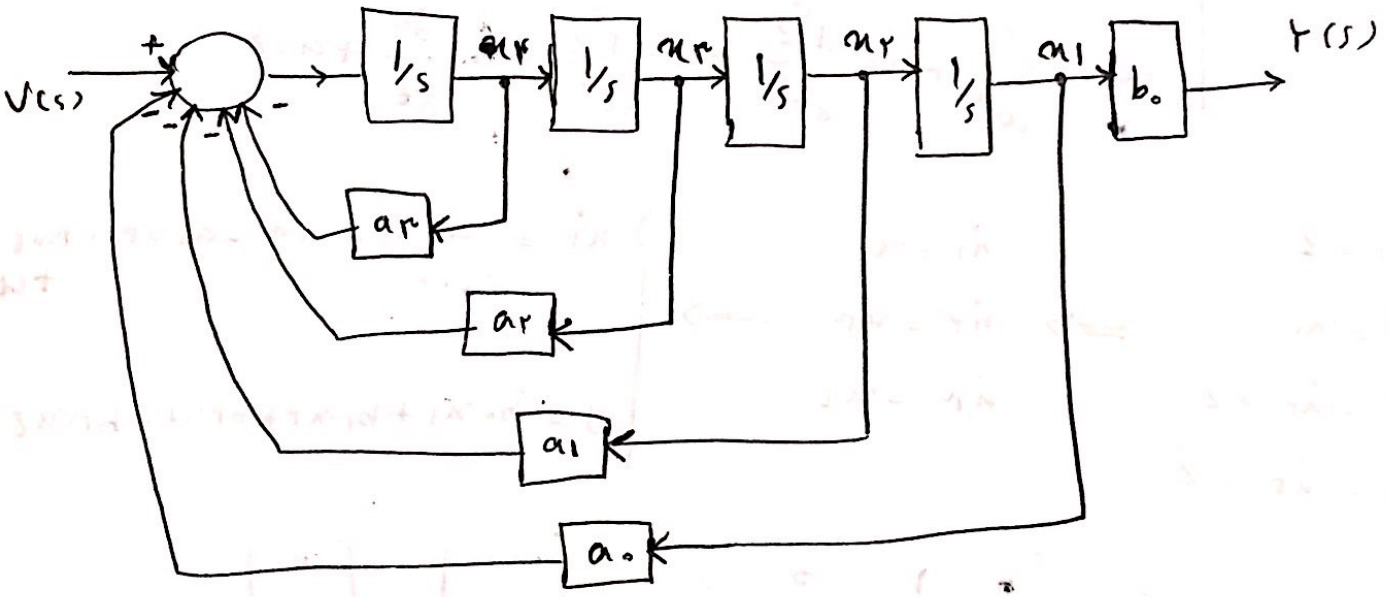
$$x_4 = \dot{x}_3 = \dddot{y}/b_0$$

ادامه سوال ۴ (الف)

سیگنال فلوگراف:



بلوک دیاگرام:



سؤال ٤ :

(ب)

$$G(s)=\frac{Y(s)}{U(s)}=\frac{b_3s^3+b_2s^2+b_1s+b_0}{s^4+a_3s^3+a_2s^2+a_1s+a_0}\;\frac{Z(s)}{Z(s)}$$

$$\Rightarrow\begin{cases}Y(s)=(b_3s^3+b_2s^2+b_1s+b_0)Z(s)\\ U(s)=(s^4+a_3s^3+a_2s^2+a_1s+a_0)Z(s)\end{cases}$$

$$\Longrightarrow\begin{cases}y=b_3\frac{d^3z}{dt^3}+b_2\frac{d^2z}{dt^2}+b_1\frac{dz}{dt}+b_0z\\ u=\frac{d^4z}{dt^4}+a_3\frac{d^3z}{dt^3}+a_2\frac{d^2z}{dt^2}+a_1\frac{dz}{dt}+a_0z\end{cases}$$

$$x_1=z \qquad \dot{x}_1=x_2$$
$$x_2=\dot{x}_1=\dot{z} \Longrightarrow \dot{x}_2=x_3$$
$$x_3=\dot{x}_2=\ddot{z} \qquad \dot{x}_3=x_4$$
$$x_4=\dot{x}_3=\dddot{z}$$

$$\Longrightarrow\begin{cases}\dot{x}_4=-a_0x_1-a_1x_2-a_2x_3-a_3x_4+u\\ \vdots\\ y=b_0x_1+b_1x_2+b_2x_3+b_3x_4\end{cases}$$

$$\Longrightarrow\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}0&1&0&0\\ 0&0&1&0\\ 0&0&0&1\\ -a_0&-a_1&-a_2&-a_3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 0\\ 1\end{bmatrix}u(t)$$

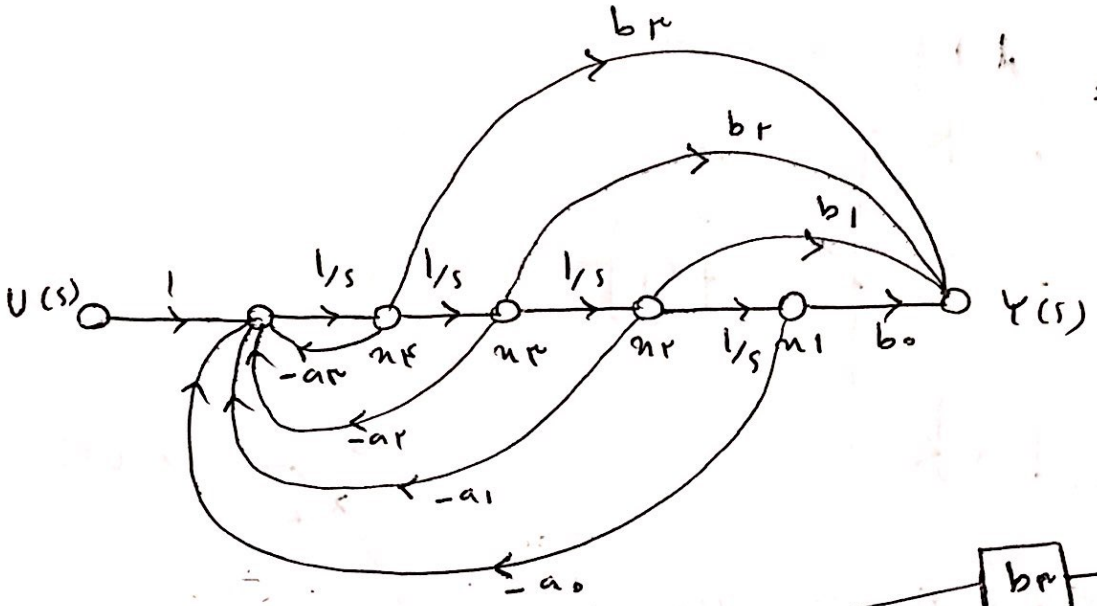
$$y(t)=Cx=\begin{bmatrix}b_0&b_1&b_2&b_3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}$$

کنترل خطی ۱

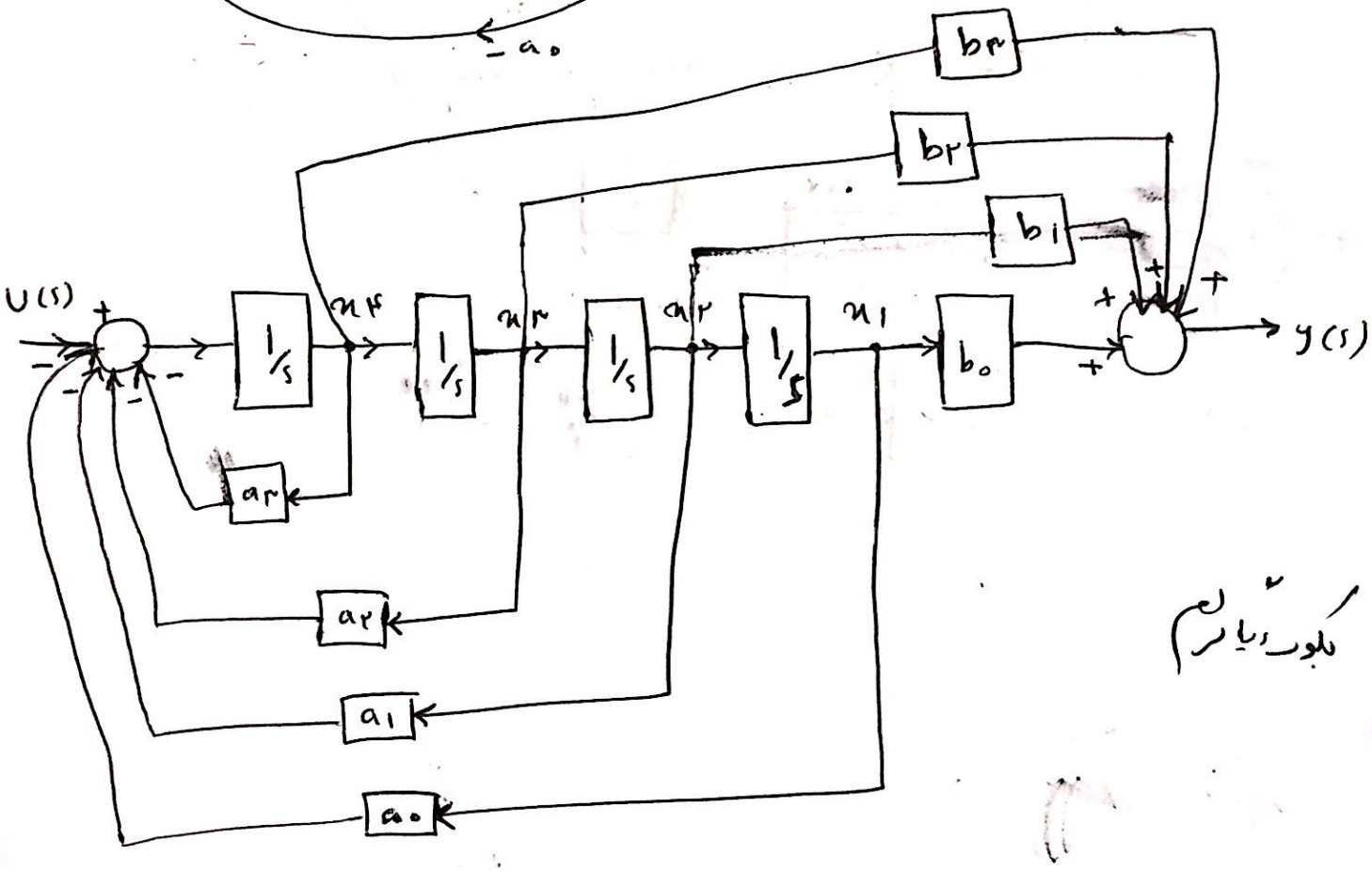
پاسخ تمرین سری دوم

مرده دلتر فرهادی

- ادامه سوال ۴ قسمت (ب):



شبیه سازی نگورلف



کلوب دیاگرام

سؤال ۵ :