Energy conversion I

Lecture 6:

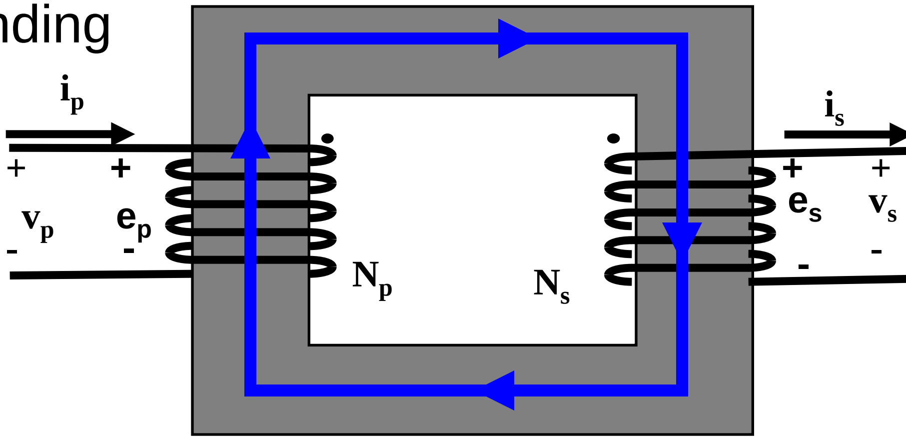
Topic 2: Transformers & its performance (S. Chapman, ch. 2)

- Introduction
- Types and Construction of Transformers.
- Ideal Transformer.
- **Theory of operation of real single-phase transformers.**
- **The Equivalent Circuit of a Transformer.**
- The Per-Unit System of Measurement.
- Transformer voltage regulation and efficiency.
- Autotransformers.
- Three phase transformers.

Conditions for an Ideal Transformer

e_p: **Induced** voltage in the primary winding

e_s: **Induced** voltage in the secondary winding



$$e_p = N_p \frac{d\varphi_p}{dt} \quad e_s = N_s \frac{d\varphi_s}{dt}$$

No flux leakage:

$$\varphi_p = \varphi_s = \varphi_M \Rightarrow \frac{e_p}{e_s} = \frac{N_p}{N_s}$$

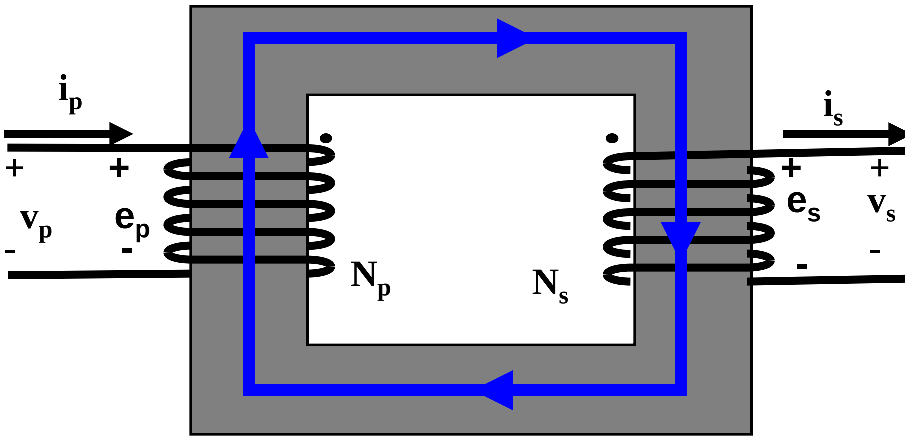
Zero-resistance windings :

$$v_p = e_p, v_s = e_s \Rightarrow \frac{v_p}{v_s} = \frac{N_p}{N_s}$$

Conditions for an Ideal Transformer

i_p: primary winding **current**

i_s: Secondary winding **current**



$$H_c l_c = N_p i_p - N_s i_s$$

$$H_c = \frac{B_c}{\mu_r \mu_0} = \frac{\varphi_M}{A_c \mu_r \mu_0}$$

A very high relative permeability:

$$\mu_r = \infty \Rightarrow H_c = 0$$

$$N_p i_p = N_s i_s$$

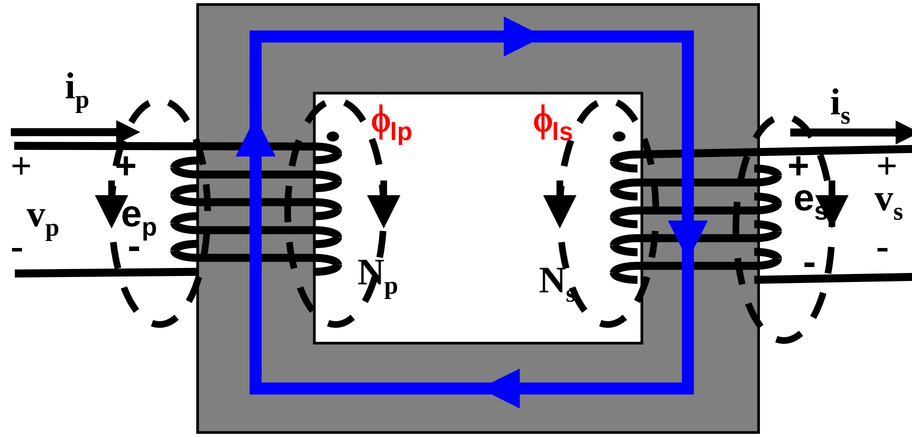
An **Ideal transformer** is a **multi winding** magnetic system with **zero winding-resistance**, **without flux leakage** and **with an infinity permeability** magnetic core

Effect of leakage flux

Considering Flux leakage in windings:

$$\varphi_p = \varphi_{lp} + \varphi_M \Rightarrow e_p = N_p \frac{d\varphi_{lp}}{dt} + N_p \frac{d\varphi_M}{dt}$$

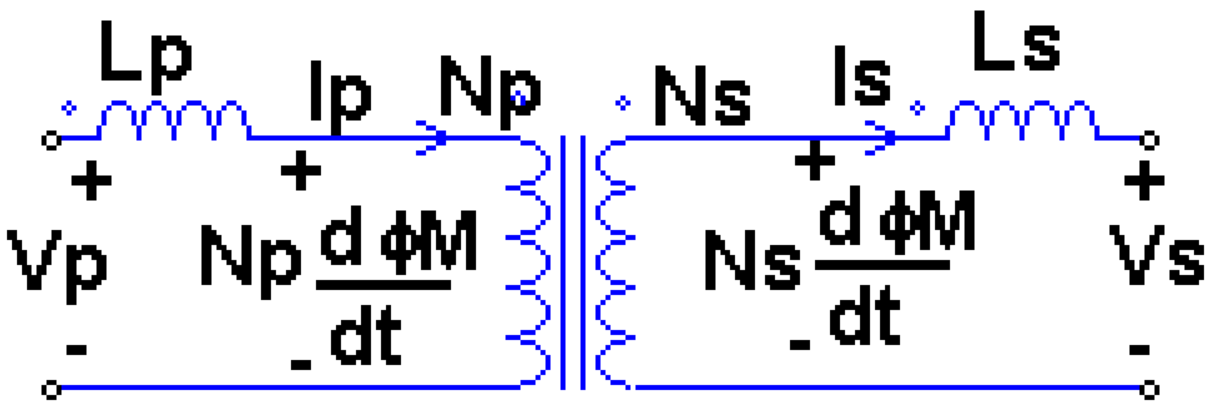
$$\varphi_s = \varphi_M - \varphi_{ls} \Rightarrow e_s = N_s \frac{d\varphi_M}{dt} - N_s \frac{d\varphi_{ls}}{dt}$$



Defining leakage inductances:

$$e_{Lp} = N_p \frac{d\varphi_{lp}}{dt} = L_p \frac{di_p}{dt}$$

$$e_{Ls} = N_s \frac{d\varphi_{ls}}{dt} = L_s \frac{di_s}{dt}$$



Note: $\varphi_{lp} << \varphi_m$, $\varphi_{ls} << \varphi_m$

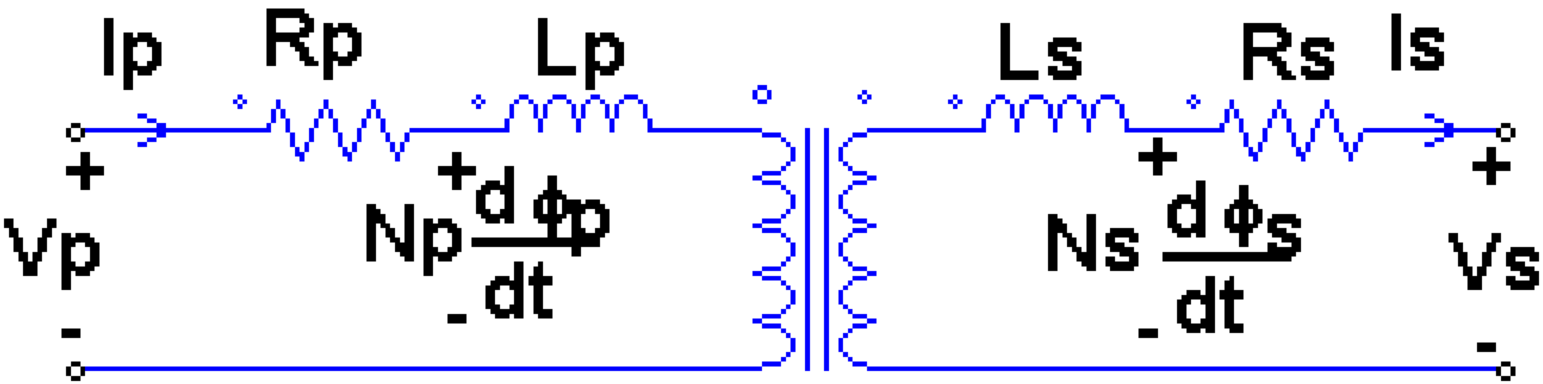
Effect of winding resistance

Considering winding wire resistance:

$$v_p = R_p i_p + e_p = R_p i_p + L_p \frac{di_p}{dt} + \boxed{N_p \frac{d\varphi_M}{dt}}$$

$$v_s = e_s - R_s i_s = \boxed{N_s \frac{d\varphi_M}{dt}} - L_s \frac{di_s}{dt} - R_s i_s$$

R_p and R_s are lumped equivalent resistances



Effect of limited permeability

No-load Transformer: $i_s = 0$

$$H_c l_c = N_p i_p - \cancel{N_s i_s}$$

$$H_c = \frac{B_c}{\mu_r \mu_0} = \frac{\varphi_M}{A_c \mu_r \mu_0}$$

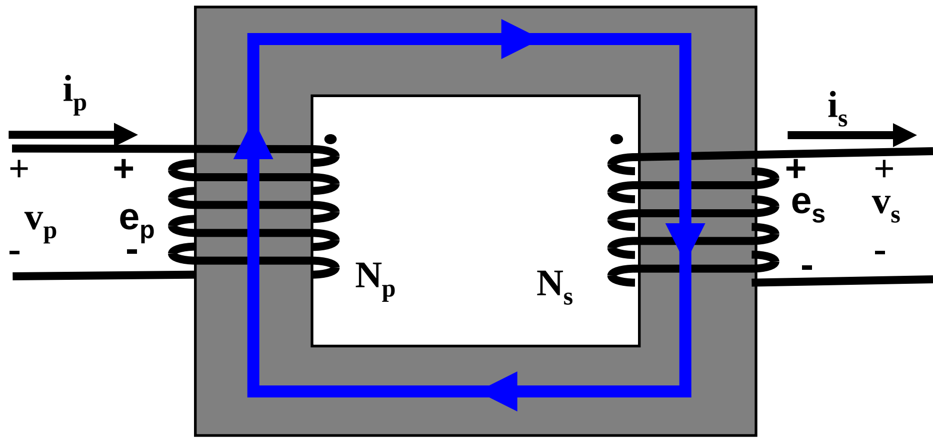
$$N_p i_p = \frac{\varphi_M}{A_c \mu_r \mu_0} l_c = \varphi_M R$$

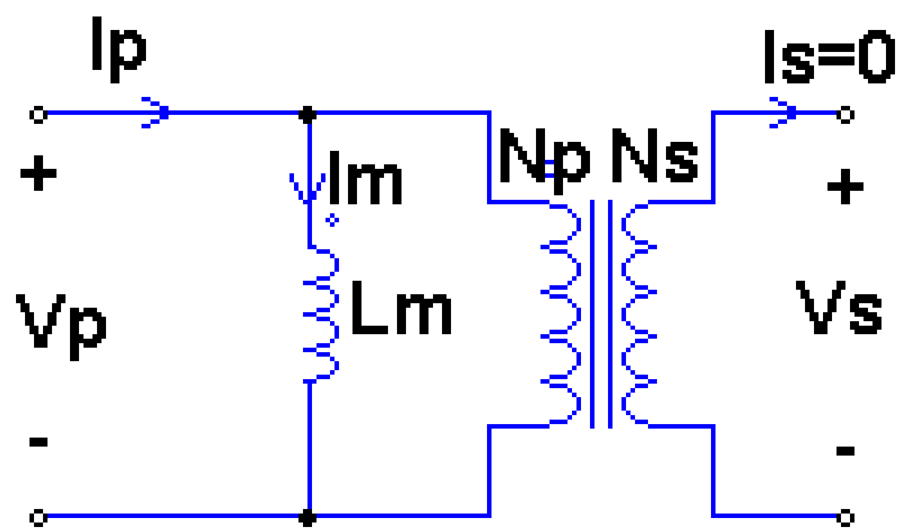
$$\lambda_p = N_p \varphi_M = \frac{N_p^2}{R} i_p = L_m i_p = L_m i_m$$

L_m: Magnetizing Inductance

I_m: Magnetizing current

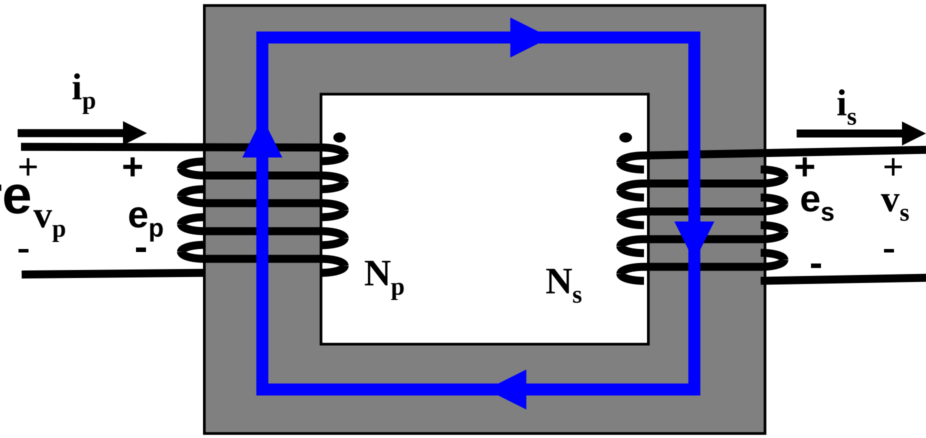
For the voltage again: $\dfrac{v_p}{v_s} = \dfrac{e_p}{e_s} = \dfrac{N_p}{N_s}$





Effect of limited permeability

Loaded Transformer: $i_s \neq 0$

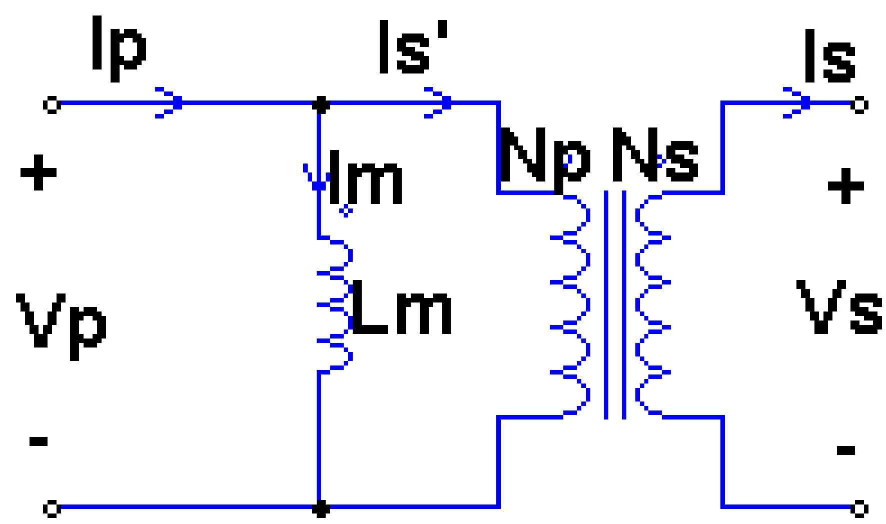


$v_p = e_p$ $\longrightarrow$ φ_m as before $\longrightarrow$ I_m as before

$$H_c l_c = N_p i_p - N_s i_s = N_p i_m$$

$$i_p - i_m = \frac{N_s}{N_p} i_s = i_s^{'}$$

$$\lambda_p = N_p \varphi_M = L_m i_m$$



For the voltage again:

$$\frac{v_p}{v_s} = \frac{e_p}{e_s} = \frac{N_p}{N_s}$$

Magnetizing current in real transformer

Loss-free core:

Sinusoidal Voltage

⬇

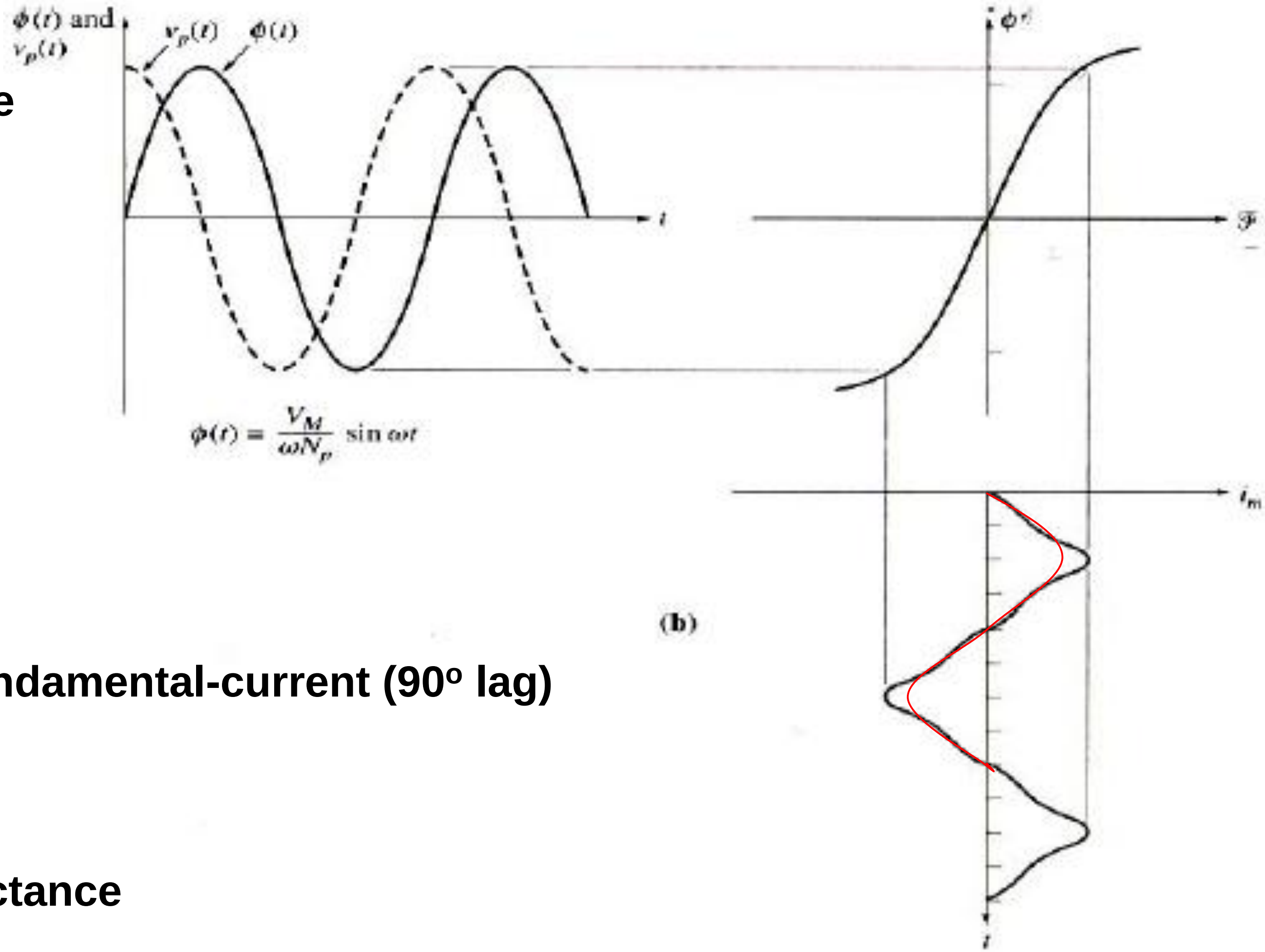
Sinusoidal flux
(90º lag)
+ nonlinear core

⬇

Harmonic current
+ flux in-phase fundamental-current (90º lag)

⬇

Magnetizing Inductance



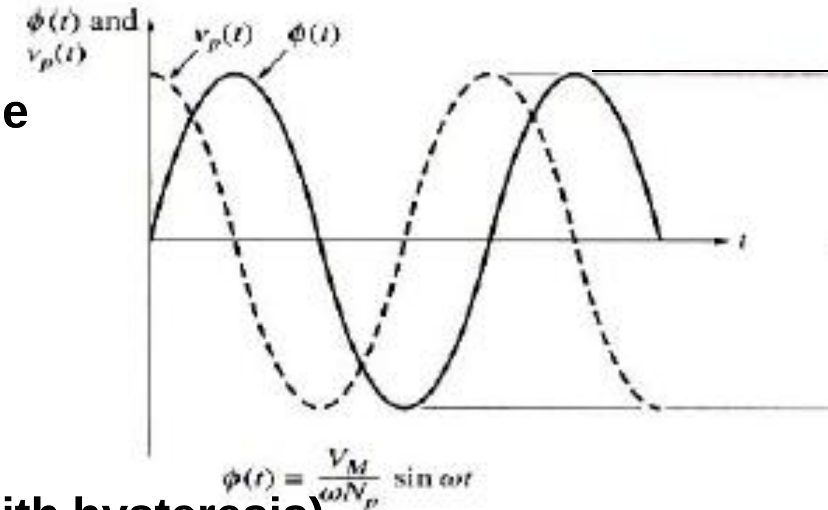
Magnetizing current in real transformer

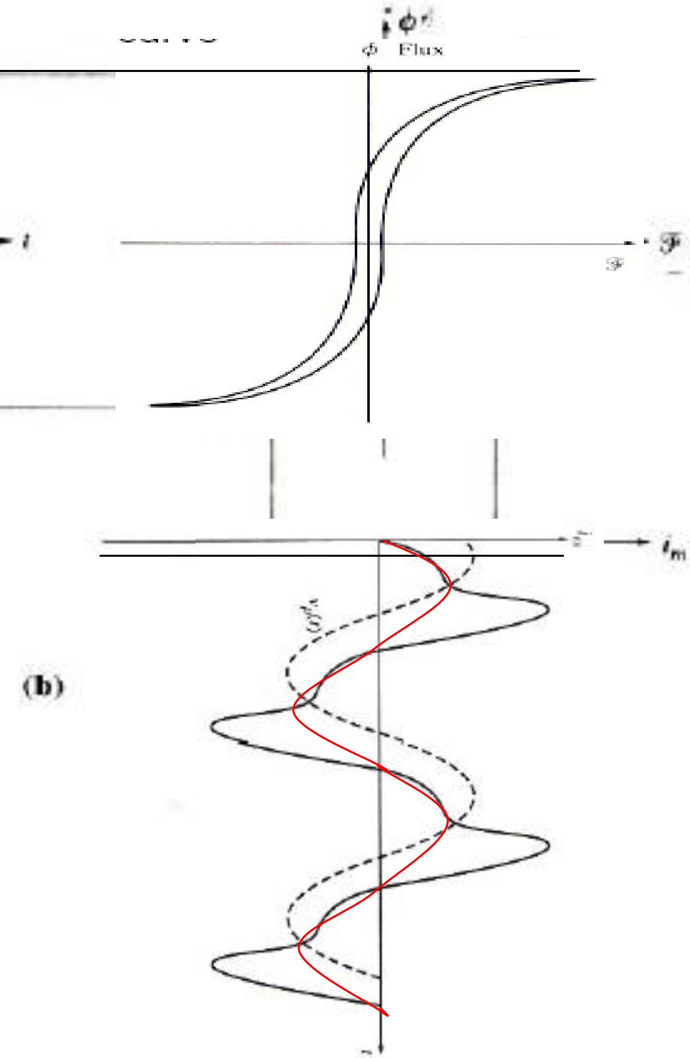
Real core:

Sinusoidal Voltage

Sinusoidal flux
(90º lag)
+
nonlinear core (with hysteresis)

Harmonic current
+ fundamental current with less than 90º lag



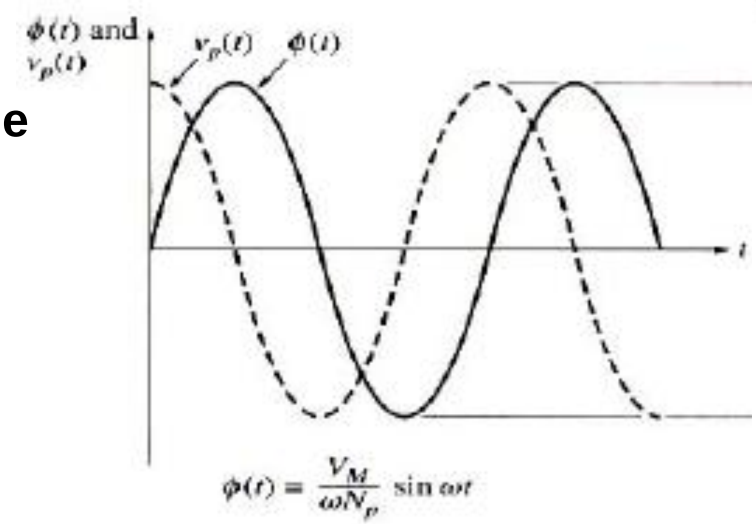


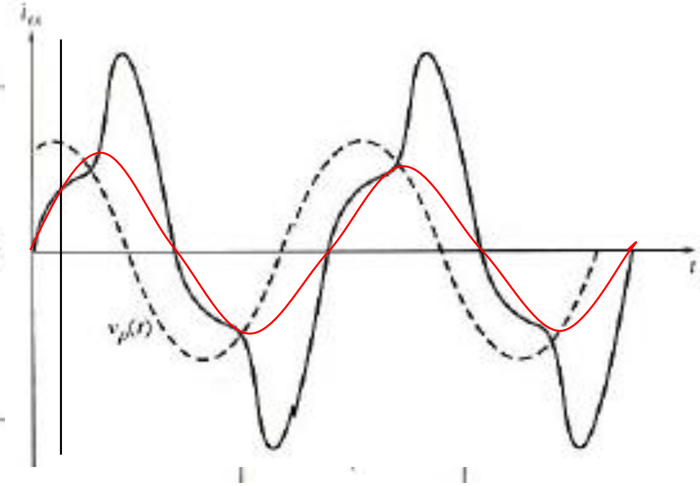
Magnetizing current in real transformer

Real core:

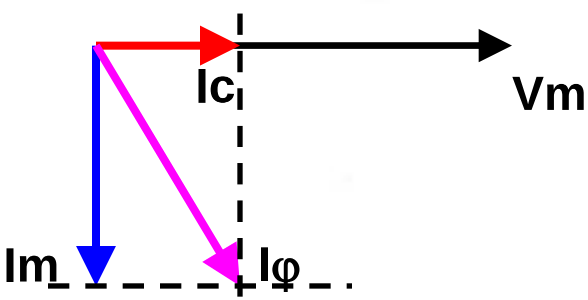
Sinusoidal Voltage

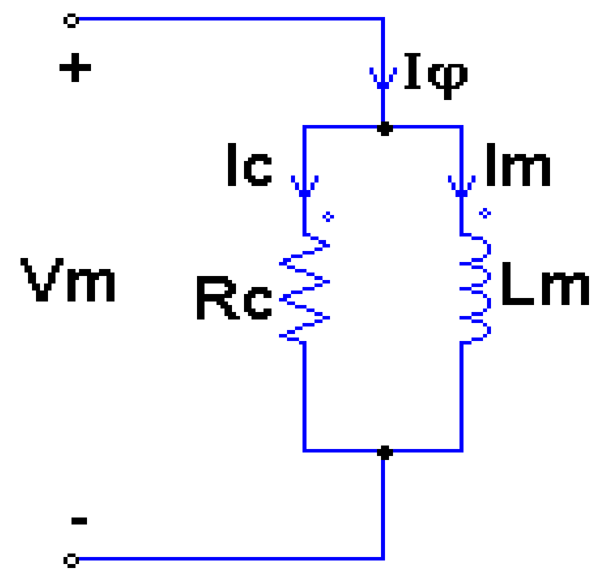
Sinusoidal flux (90º lag)





fundamental current with less than 90º lag





Equivalent circuit of a real Transformer

- Copper (winding) loss: R_p, R_s
- Leakage flux: Xp, Xs
- Core loss: R_c
- Magnetizing current (X_M)
- Transformer Turns ratio: Ideal transformer
- Electrical Isolation: Ideal transformer

$$\frac{V_p}{V_s} \approx \frac{N_p}{N_s}$$

$$\frac{I_p}{I_s} \approx \frac{N_s}{N_p}$$