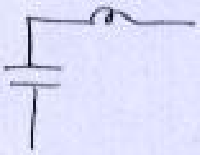
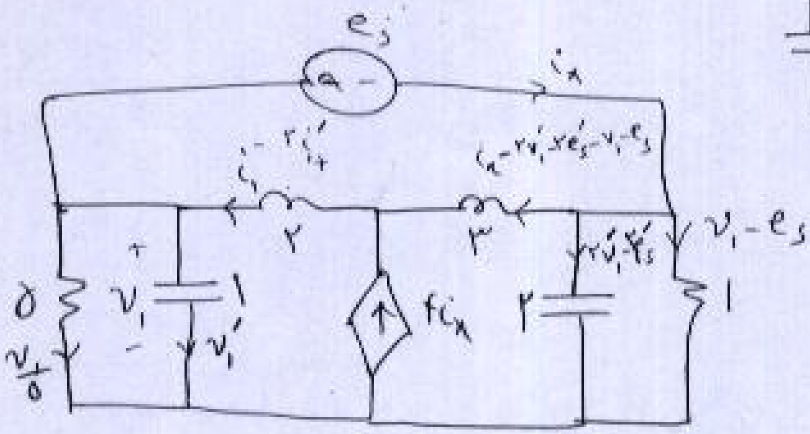
حل تمرین‌های سری ۲


[illegible]

(۴-۱)



$$X = \begin{bmatrix} v_1 \\ i_1 \end{bmatrix}$$

$$i_x - 2v_1' - 2e_s' - v_1 - e_s + 4i_x = i_1$$

$$v_1' + 2v_1' - 2e_s' + \frac{v_1}{5} + v_1 - e_s = 4i_x \Rightarrow 5i_x = \frac{15}{4}v_1' + \frac{3}{2}v_1 - \frac{5}{2}e_s' - \frac{5}{4}e_s$$

$$\Rightarrow \frac{7}{4}v_1' - \frac{9}{2}e_s' + \frac{1}{2}v_1 - \frac{9}{4}e_s = i_1 \Rightarrow v_1' = \frac{4}{7}\left(-\frac{1}{2}v_1 + i_1 + \frac{9}{2}e_s' + \frac{9}{4}e_s\right)$$

$$v_1' = \frac{-2}{7}v_1 + \frac{4}{7}i_1 + \frac{1}{7}e_s + \frac{9}{7}e_s'$$

$$\Rightarrow v_1'' = \frac{-2}{7}v_1' + \frac{4}{7}i_1' + \frac{1}{7}e_s' + \frac{9}{7}e_s'' \Rightarrow$$

$$e_s + 3(i_x - 2v_1' - 2e_s' - v_1 - e_s)' + 2i_1' = 0$$

$$\begin{bmatrix} v_1' \\ i_1' \end{bmatrix} = \begin{bmatrix} -\frac{2}{7} & \frac{4}{7} \\ \frac{72}{427} & -\frac{144}{427} \end{bmatrix} \begin{bmatrix} v_1 \\ i_1 \end{bmatrix} + \begin{bmatrix} \frac{1}{7} \\ \frac{48}{427} \end{bmatrix} e_s + \begin{bmatrix} \frac{9}{7} \\ \frac{348}{427} \end{bmatrix} e_s' + \begin{bmatrix} 0 \\ \frac{120}{427} \end{bmatrix} e_s''$$

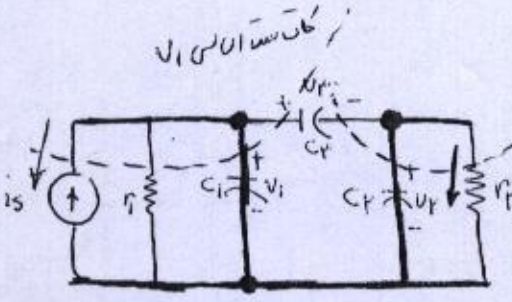
(ب) در این حالت واضح است که معادله v_1' فرق نمی‌کند اما در رابطه i_1' داریم:

$$\text{KVL} \quad -\frac{i_1'}{2} - \frac{i_1' - 4i_x'}{2} \Rightarrow -i_1' + 2i_x' = -i_1' + \frac{3}{4}v_1'' - e_s'' + \frac{3}{5}v_1' - \frac{e_s'}{2}$$

$$= -i_1' - \frac{3}{7}v_1' + \frac{3}{7}i_1' + \frac{3}{14}e_s' + \frac{21}{14}e_s' - e_s'' + \frac{3}{5}v_1' - \frac{e_s'}{2}$$

$$= -\frac{1}{7}i_1' + \frac{2}{35}\left(-\frac{2}{7}v_1 + \frac{4}{7}i_1 + \frac{1}{7}e_s + \frac{9}{7}e_s'\right) + \frac{3}{14}e_s' + \frac{21}{14}e_s' - e_s'' + \frac{e_s'}{2}$$

۶-(۴)



متغیرهای حالت $\{v_1, v_2\}$ $\quad X = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$$-i_s + \frac{v_1}{R_1} + C_1\dot{v}_1 + C_2(\dot{v}_1 - \dot{v}_2) = 0$$

$$C_2(\dot{v}_2 - \dot{v}_1) + C_2\dot{v}_2 + \frac{v_2}{R_2} = 0$$

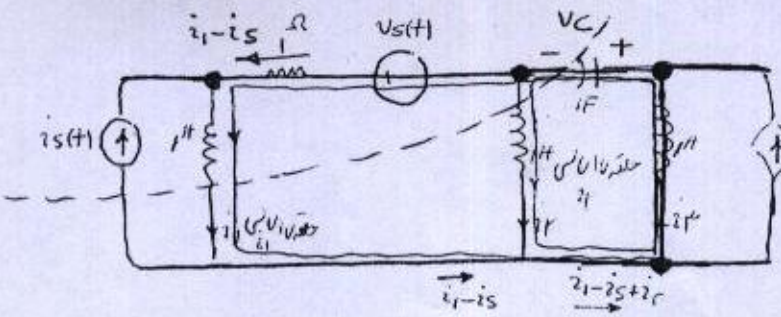
$$\dot{X} = AX + BU$$

$$\begin{cases} (C_1 + C_2)\dot{v}_1 - C_2\dot{v}_2 = -\frac{v_1}{R_1} + i_s \\ -C_2\dot{v}_1 + (C_2 + C_2)\dot{v}_2 = -\frac{v_2}{R_2} \end{cases}$$

$$\begin{bmatrix} C_1+C_2 & -C_2 \\ -C_2 & C_2+C_2 \end{bmatrix} \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1} & 0 \\ 0 & -\frac{1}{R_2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} i_s \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \frac{1}{C_1C_2 + C_1C_2 + C_2C_2} \begin{bmatrix} C_2+C_2 & C_2 \\ C_2 & C_1+C_2 \end{bmatrix} \left(\begin{bmatrix} -\frac{1}{R_1} & 0 \\ 0 & -\frac{1}{R_2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} i_s \\ 0 \end{bmatrix} \right)$$

۷- (۳)



گره v_C: $-\dot{v}_C + \dot{i}_1 + \dot{i}_2 - \dot{i}_S = 0$

حلقه i_1: $\dot{i}_1 + \dot{i}_2 + v_C - v_S + i_1 - i_S = 0$

حلقه i_2: $\dot{i}_2 + \dot{i}_2 + v_C = 0$

$$\dot{v}_C = \dot{i}_1 + \dot{i}_2 - \dot{i}_S$$

$$\dot{i}_1 = -i_1 - v_C + v_S + i_S + \dot{i}_S$$

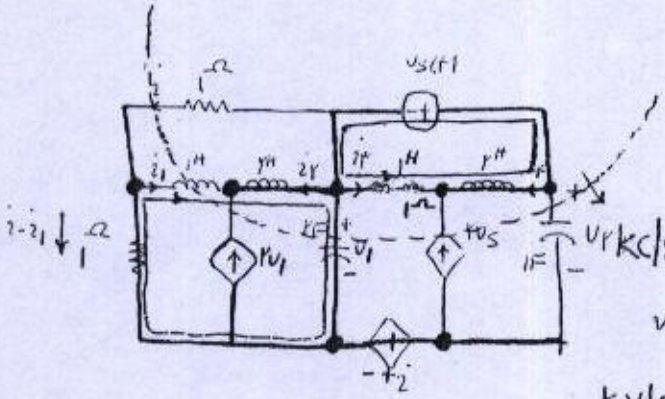
$$\dot{i}_2 = -v_C + \dot{i}_S$$

$$\dot{X} = \begin{bmatrix} \dot{v}_C \\ \dot{i}_1 \\ \dot{i}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ -1 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_S \\ v_S \end{bmatrix}$$

3×2 $\quad 2\times 1$

$$\dot{X} = AX + BU + E\dot{U} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{i}_S \\ \dot{v}_S \end{bmatrix}$$

۷- (۴)



$\{v_1, i_1, i_2\}$

KCL: $2\dot{v}_1 - 2v_S + \dot{v}_2 - 2v_1 - i_1 + \dot{i} = 0$

$v_2 = v_1 + v_S$ ، $i + i - i_1 - v_1 = 0$ $\quad i = \frac{i_1 + v_1}{2}$

KVL: $\dot{i}_1 - 2\dot{i}_2 + v_1 + \frac{\dot{i}_1}{2} - \frac{v_1}{2} = 0$

$$i_2 = -2v_1 - i_1$$

KVL: $\dot{i}_2 + \dot{i}_2 - 2\dot{i}_2 + v_S = 0$

$$i_2 = -2v_S - i_2$$

$$4\dot{v}_1 = \frac{4}{2}v_1 + \frac{i_1}{2} + 2v_S - \dot{v}_S$$

$$4\dot{v}_1 + 4\dot{i}_1 = -\frac{v_1}{2} - \frac{i_1}{2}$$

$$4\dot{i}_2 = -i_2 - v_S - 4\dot{v}_S$$

$$4\dot{i}_1 = -\frac{4}{4}\left(\frac{4}{2}v_1 + \frac{i_1}{2} + 2v_S - \dot{v}_S\right) - \frac{v_1}{2} - \frac{i_1}{2} = -\frac{5}{2}v_1 - \frac{7}{4}i_1 - \frac{8}{4}v_S + \frac{4}{4}\dot{v}_S$$

$$\dot{X} = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & 0 \\ -\frac{5}{4} & -\frac{7}{16} & 0 \\ 0 & 0 & -\frac{1}{4} \end{bmatrix} X + \begin{bmatrix} \frac{2}{4} \\ -\frac{8}{4} \\ -\frac{1}{4} \end{bmatrix} v_S + \begin{bmatrix} -\frac{1}{4} \\ \frac{4}{4} \\ -\frac{4}{4} \end{bmatrix} \dot{v}_S$$