

# Energy conversion I

## Lecture 4:

### Topic 1: Magnetic materials and Circuits (S. Chapman, ch. 1)

- Magnetic Field production and Mag. Circuits modeling
- Ferromagnetic Materials behavior
- Faraday's law
- **Electrical Equivalent Cct. For magnetic Ccts**
- **Permanent Magnet Materials**
- **Force applied on a wire by external magnetic field**
- **Voltage induced in on moving conductor in magnetic field**

# Electrical Equivalent Circuit For magnetic Circuits

Neglecting winding resistance:

$$V = e = \frac{d\lambda}{dt}$$

$$\lambda = Li \quad \text{L: Inductance (Henry)}$$

Neglecting Flux leakage:

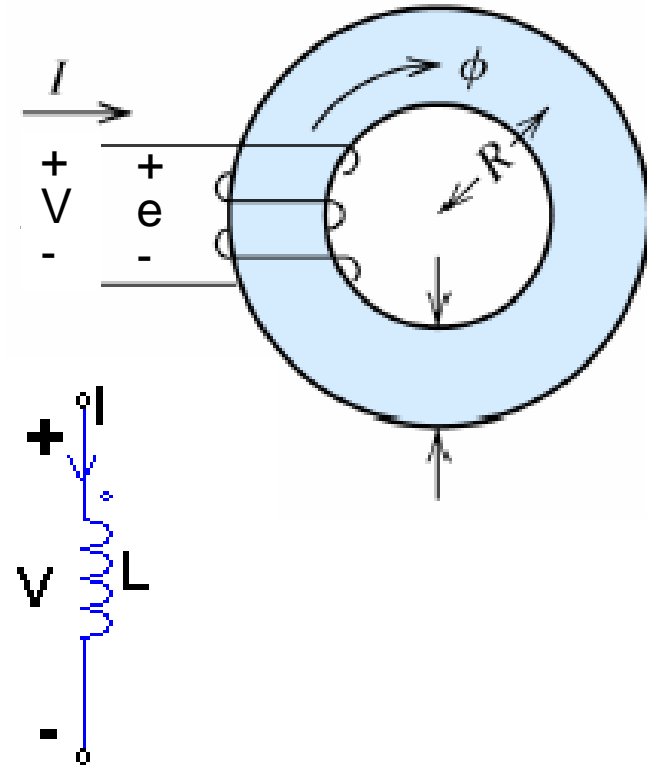
$$V = e = \frac{d\lambda}{dt} = \frac{d(Li)}{dt} = L \frac{di}{dt} + i \frac{dL}{dt}$$

$$V = L \frac{di}{dt} \quad !$$

$$\begin{aligned} \lambda &= N\phi = NBA = N\mu HA \\ &= N\mu \frac{NI}{L_c} A = LI \end{aligned}$$



$$L = \mu \frac{N^2}{L_c} A = \frac{N^2}{R_M}$$



# Electrical Equivalent Circuit For magnetic Circuits

**Modelling leakage flux:**

$$\phi = \phi_l + \phi_M$$

$$\lambda = N\phi = N\phi_l + N\phi_M$$

$$= N\mu_0 \frac{NI}{l_l} A_l + N\mu \frac{NI}{l_M} A_M$$

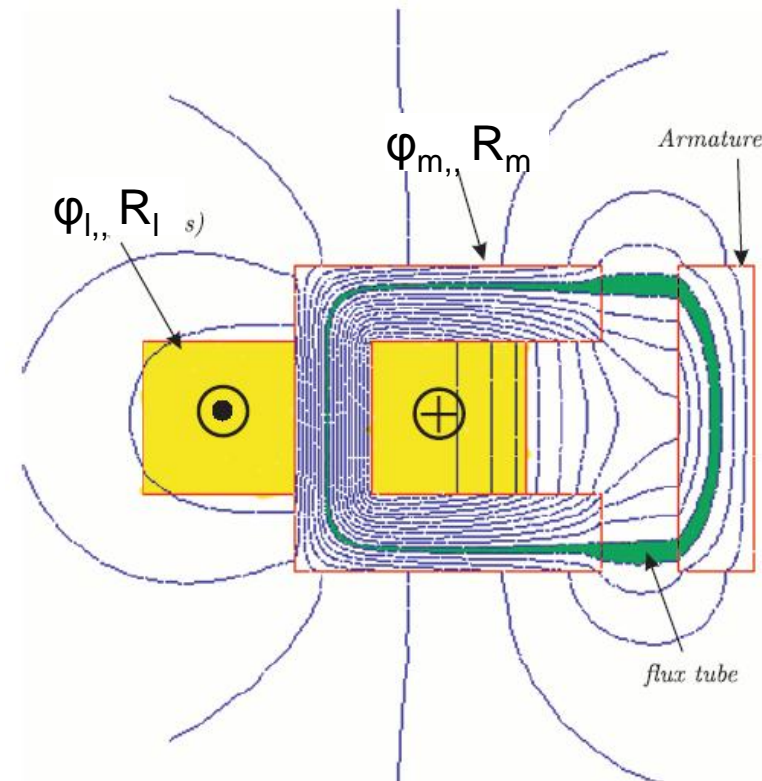
$$= \frac{N^2}{\frac{l_l}{\mu_0 A_l}} I + \frac{N^2}{\frac{l_M}{\mu A_M}} I = (L_l + L_M) I$$

$$L_{eq} = L_l + L_M \quad \text{Series Inductances}$$

**Note:**  $\phi = \phi_l + \phi_M$

$$= \frac{NI}{R_l} + \frac{NI}{R_M} = NI \left( \frac{1}{R_l} + \frac{1}{R_M} \right)$$

$$= \frac{F}{R_{eq}} \Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_l} + \frac{1}{R_M}$$



**Parallel reluctances**

**What about winding resistance**

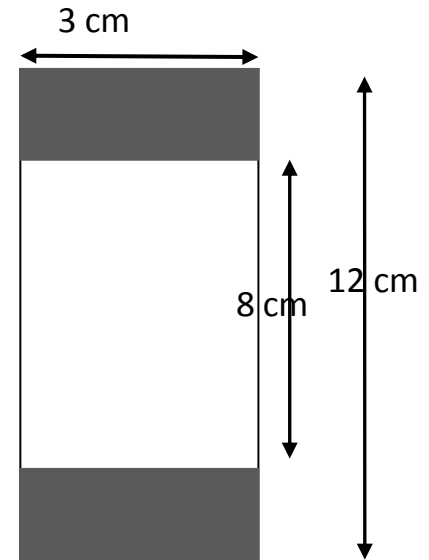
## Example:

Following figure shows the cross section of a 100 turns iron core coil. For  $B < 1 \text{ T}$ ,  $\mu = 0.01$

The wire resistance is  $10 \Omega$ .

A- what is the maximum current if the core is not saturated.

B- What is the maximum magnitude (rms) of a sinusoidal 50 Hz Voltage if the core is not saturated.



**Solution:**

**A:**

$$H \cdot l_c = N i \quad \Rightarrow \quad i = \frac{H \cdot l_c}{N} = \frac{\frac{1}{0.01} \times 0.1 \pi}{100} = 0.314 \text{ A}$$

**B: The equivalent circuit is an R-L:**

$$L = \frac{N^2}{R_m} = \frac{N^2}{\frac{l_c}{\mu A_c}} = \frac{100^2}{\frac{0.1 \pi}{0.01 (0.03 \times 0.02)}} = 0.191 \text{ H}$$

$$V_{\max} = (R + j \omega L) I_{\max}$$

$$I_{\max} = \frac{0.314}{\sqrt{2}} = 0.222 \text{ A}$$

**Repeat B for DC voltage!**

$$|V_{\max}| = |(10 + j \times 100 \pi \times 0.191)| \times 0.222 = 13.5 \text{ V}$$

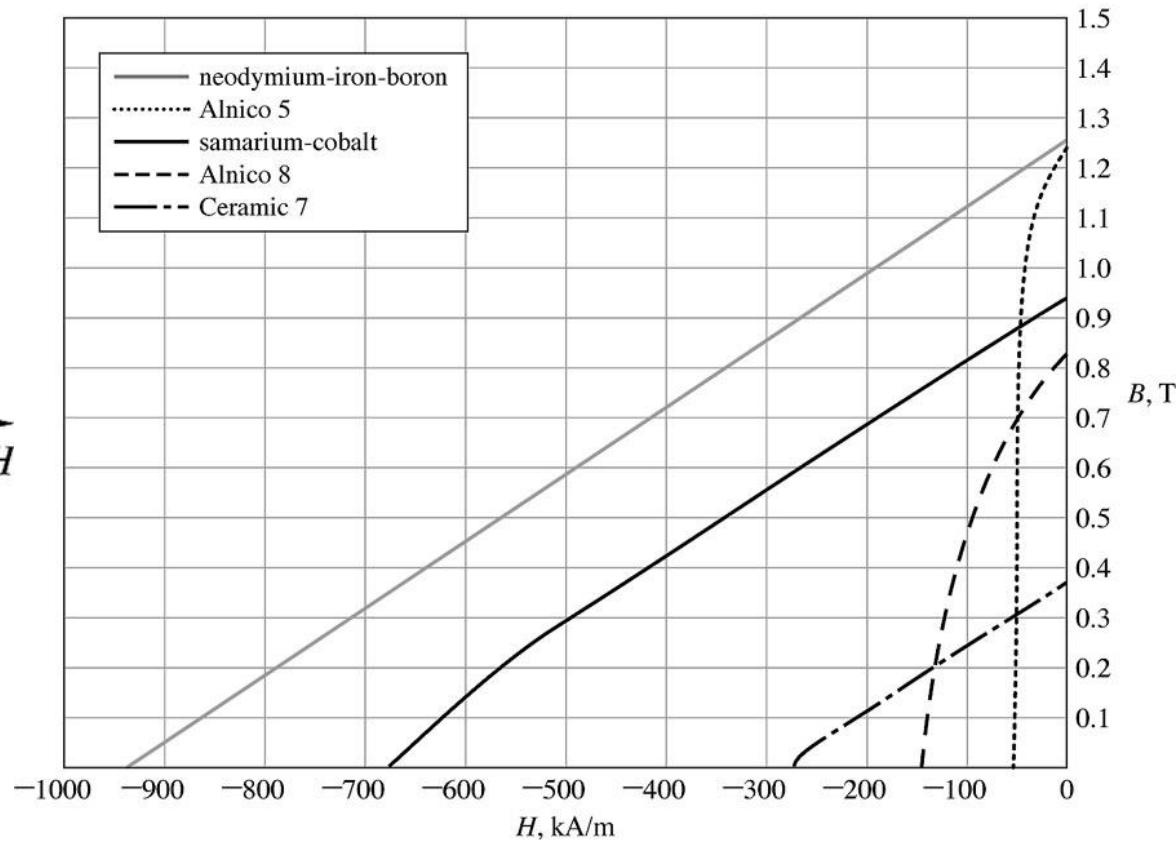
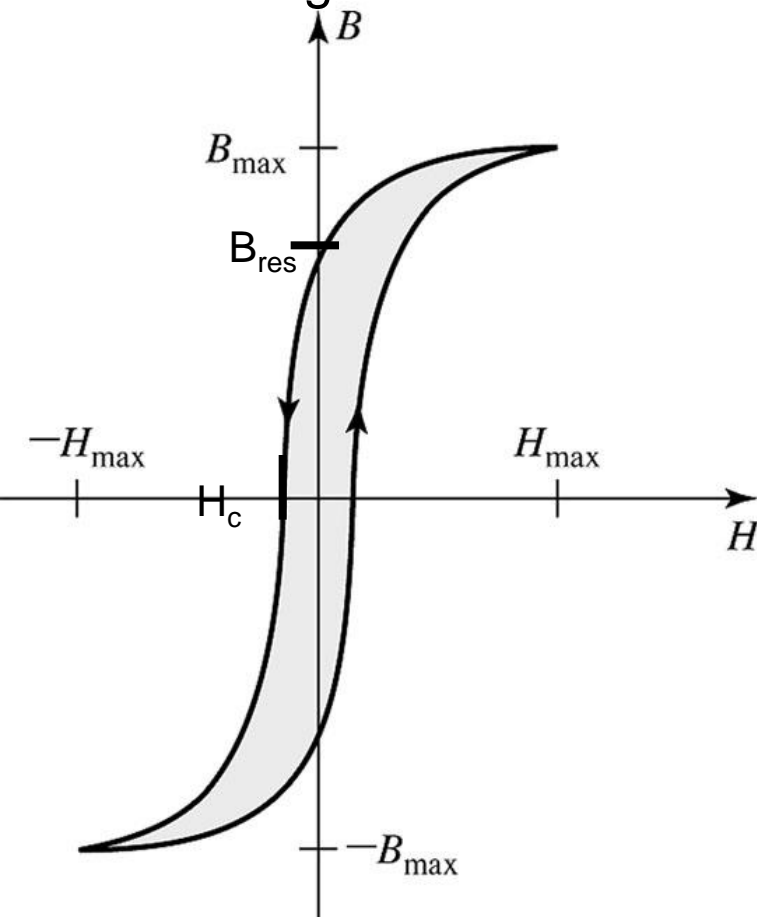
# Permanent Magnet Materials

$B_{\text{res}}$  : Residual flux generating magnetic flux in the magnetic systems

$H_c$ : Coercive magnetizing intensity (required to demagnetize the core)

Having as large a  $B_{\text{res}}$  and  $H_c$  as possible is better

External magnetic fields and excessive heating can demagnetize PMs



# Permanent Magnet Systems

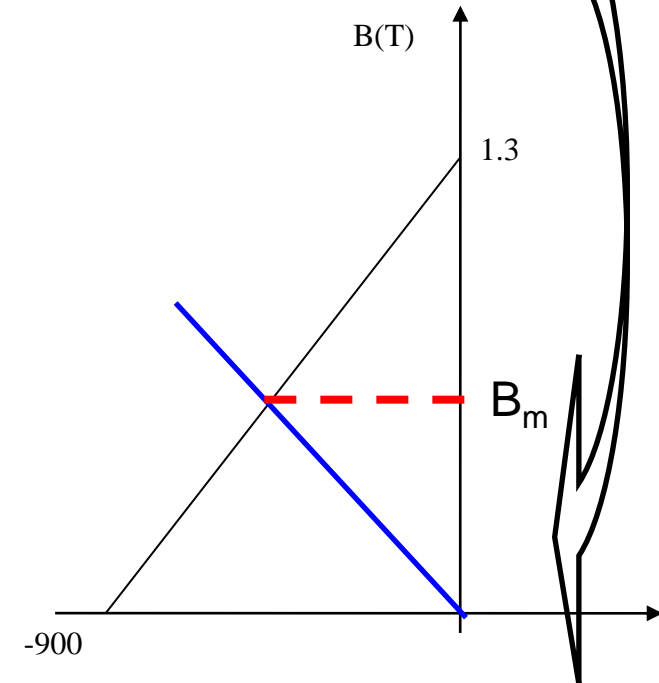
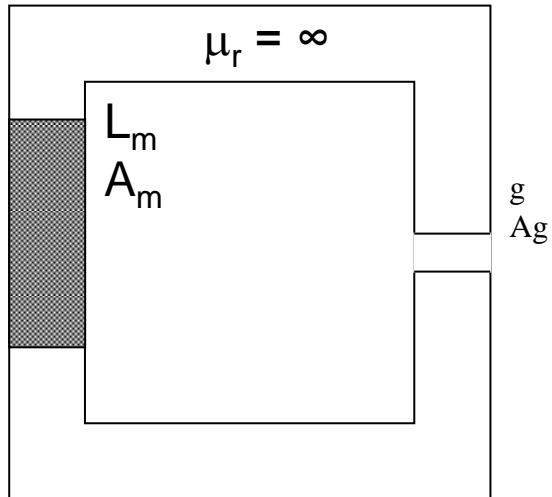
$$H_m L_m + H_g g = 0 \text{ (Amp law)}$$

$$B_m A_m = B_g A_g = \mu_0 H_g A_g$$



$$H_g = -H_m L_m / g$$

$$B_m = -\mu_0 A_g H_m L_m / (g \times A_m) \text{ \& PM}$$



Minimizing PM volume to have a required B  
(usually  $B_g$ )

$$V_m = L_m \times A_m = -\frac{H_g g}{H_m} \times \frac{B_g A_g}{B_m} = -\frac{B_g^2 V_g}{\mu_0 B_m \times H_m}$$

Minimizing PM volume if  $B_m \times H_m$   
Is maximum

# Force applied on a wire by ext. magnetic field

**Force** will be exerted to a **current carrying conductor** present in a **magnetic field of flux density B**.

$$\mathbf{F} = i (\mathbf{l} \times \mathbf{B})$$

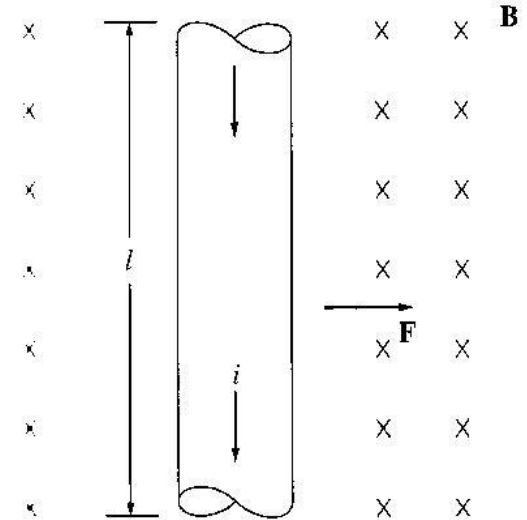
i: current,

l: length of wire (vector), in direction of current flow

B: Flux density (Vector)

Very **simple** case when **l** is perpendicular to **B**:

$$\mathbf{F} = i\mathbf{l}\mathbf{B}$$



# Voltage induced in on moving conductor in magnetic field

**Voltage** will be **induced** on the terminals of a **moving** conductor **cutting** a **magnetic field**.

$$e_{\text{ind}} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l}$$

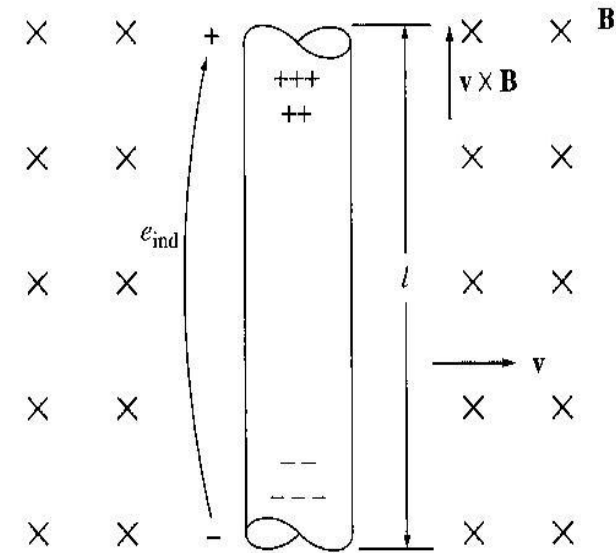
$\mathbf{v}$  – velocity of the wire (Vector)

$\mathbf{B}$  – magnetic field density (vector)

$\mathbf{l}$  – length of the wire in the magnetic field (Vector)

Very simple case when  $\mathbf{v}$  is perpendicular to  $\mathbf{B}$  and  $\mathbf{l}$ :

$$e_{\text{ind}} = vBl$$





# Example: Linear DC motor

Turning on the switch → current → force → movement → induced voltage

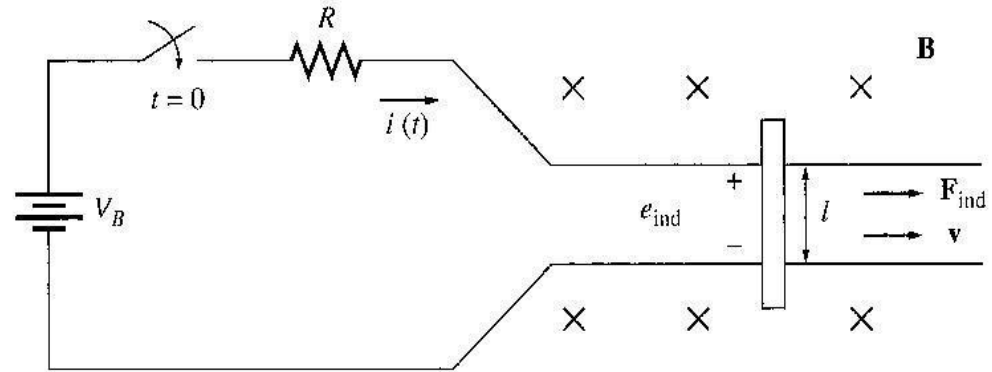
If no-load:

Induced voltage = Applied voltage

Speed = No-load speed

If loaded:

Steady state current to generate required Force (load) → Induced voltage < Applied voltage → Speed < No-load speed



**How does no-load speed changes with input voltage variations?**  
**What is the difference between no-load and loaded speed?**

**Think about Linear DC generator**