Energy conversion I

Lecture 2:

Topic 1: Magnetic materials and Circuits (S. Chapman, ch. 1)

- **Magnetic Field production and Mag. Circuits modeling**
- Ferromagnetic Materials behavior
- Electrical Equivalent Cct. For magnetic Ccts
- Faraday's law
- Permanent Magnet Materials
- Force applied on a wire by ext. magnetic field
- Voltage induced in on moving conductor in magnetic field

Magnetic Field

Magnetic Fields is the fundamental Mechanism by which **energy converts** from one form to another in **Motors**, **Generators** & **Transformers.**

Basic Principles of energy conversion using magnetic fields:

1- current carrying wire produces magnetic field.

2- time-varying magnetic field induces voltage. in coil

3- current carrying wire in an external magnetic field experience a force.

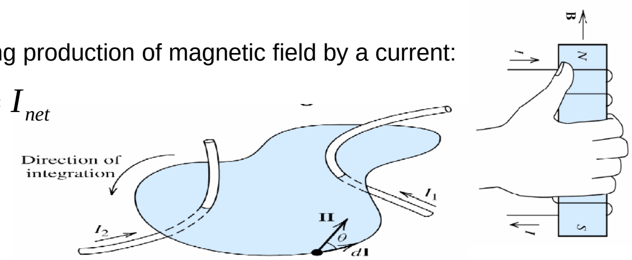
4- voltage is induced in a wire moving in an external magnetic field.

Magnetic Field

Ampere's law

Amper's Law is governing production of magnetic field by a current:

$$\oint_c H.dl = \int_s j.ds = I_{net}$$



the line integral of the tangential component of the **magnetic field intensity** **H** around a closed contour C is equal to the total current passing through any surface S linking that contour.

H : M**agnetic field intensity** produced by $\mathbf{I_{net}}$

dl : Differential element of the length of the closed path of integration

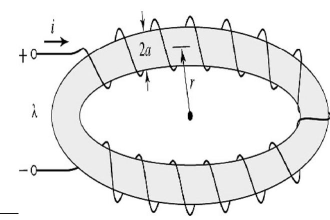
J : Current density

ds: Differential element of the area of integration

In SI **I** measured in **amperes (A)** and **H** in **ampere-turns per meter At/m**

Magnetic Field

Example:



EXAMPLE:

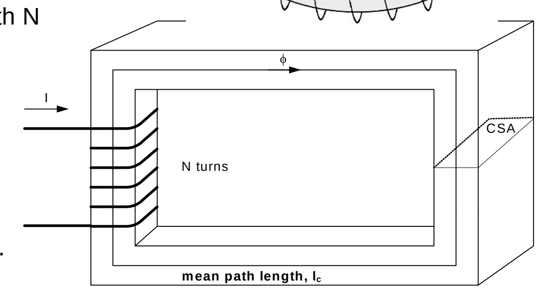
a rectangular ferromagnetic core with N turns of winding on one leg.

assumption:

- path of integration would be mean path of core lc.

- H and lc are in the same direction.

- all magnetic field produced by current in winding confined to core boundary (flux Calculation)



$$Hl_c = Ni$$

$$\therefore H = \frac{Ni}{l_c}$$

Magnetic Field

Magnetic Flux and Magnetic Flux Density

Magnetic field **flux** also depends on core **material.**

B = μ . H

μ: magnetic permeability of material, Henrys per meter H/m <u>(show how?)</u>

B: magnetic flux density produced, Tesla (Webers per sq.meter) Wb/m^2

μ_0= free space permeability (μ_0= $4\pi\times10^{-7}$)

μ = μ_r μ_0 (μr relative permeability) Used to compare magnetizing ability

for steels used in modern machines 2000-6000

Magnetic materials are used in transformer or motor core to increase & concentrate magnetic flux (similar to the role of conductors in circuits)

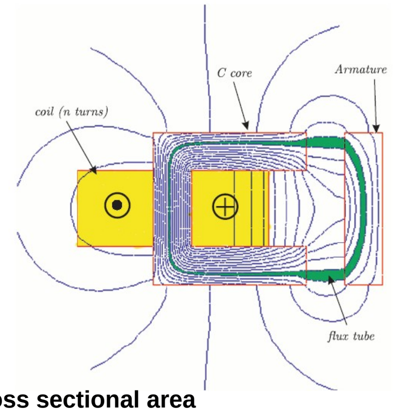
Magnetic Field

Magnetic Flux and Magnetic Flux Density

Much higher permeability of core w.r.t. air cause great majority of flux in core

Total flux passing an area A :

$$\varphi = \int_A B \cdot dA$$



dA: differential unit of area

If flux density vector perpendicular to plane of A & const.

➔ **φ = B A**

Total flux in core of Example:

φ = B A = μ H A= μ N i A / lc **A: core cross sectional area**

Magnetic Circuits Modeling

In example of magnetic field production, current in coil producing flux, is an analogous to voltage in electric circuit producing current.

Magnetic Equivalent Circuit defined as an analogous to Equivalent Electric Circuit.

In Electric circuit voltage source V drives I through R (V=R.I),

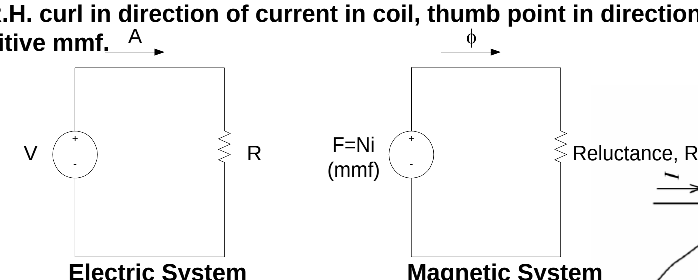
while :

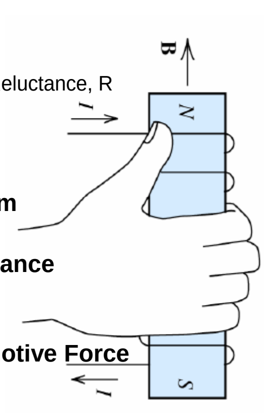
In Magnetic circuit Magneto-motive force (mmf) F = NI drives flux through reluctance of the core path.

Magnetic Circuits Modeling

mmf of magnetic circuit has also a polarity:

Positive end is associated to end from which flux exits if fingers of R.H. curl in direction of current in coil, thumb point in direction of positive mmf.





Electric System		Magnetic System
V = IR	⇔	F = φ R
R : Resistance		R : Reluctance
I : current		φ : flux
V : voltage(EMF)		F : MMF
EMF: Electromotive Force		MMF: Magnetomotive Force

Magnetic Circuits Modeling

Magnetic Reluctance:

Magnetic Reluctance: the measure of material resistance to the flow of magnetic flux.

$$\phi = BA = \mu HA$$

$$= \mu \frac{Ni}{l_c} A = F \frac{\mu A}{l_c} = \frac{F}{R}$$

$$\therefore R = \frac{l_c}{\mu A}$$

l_c: length of the magnetic path
A: cross section of magnetic flux path
μ: magnetic permeability

Reluctance very similar to resistance!

Equiv. reluctance of reluct.s in series (conducting the same flux):

Req = R1+R2 + R3 + …

For parallel reluctances (parallel path of fluxes):

1/Req = 1/R1+1/R2 + 1/R3 +.. …

Magnetic Circuits Modeling

Employing equivalent cct. **Flux** in core is approximated (within 5 percent of real value at best)

Reasons of inaccuracy:

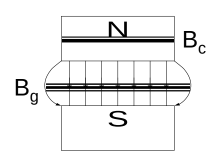
1- against the assumption of confining flux to core boundary, **small fraction** of flux circled through **air (leakage flux).**

2- assuming a **certain mean path** length and **cross section** in reluctance calculation (not exact specially at corners).

3- in ferromagnetic materials, **μ varies** with existing flux in material (**a nonlinear effect**).

4- Air gaps in flux path of the core, causes **fringing effect** on magnetic field in air gap . Therefore effective cross sectional area of air gap is larger than cross sectional area of iron core: $\mathbf{B_g < B_c}$

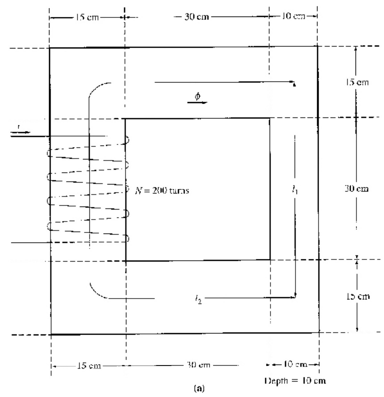
Exact calculation using Maxwell's equations too difficult & not needed in many cases.



Magnetic Circuits Modeling

Example 1:

A ferromagnetic core shown below.
Its three sides has uniform width, while fourth side is somewhat thinner.
depth of core (into page): 10 cm
Others shown , 200 turn coil on left leg, μ_r= 2500, Passing 1 A, **flux**?



(a)

Solution:

Core can be divided into 2 regions:

- (1) thinner side with a mean path length of 45 cm, & a cross-sectional area of10x10 cm^2
- (2) other three sides with a mean path of 130 cm (5+30+7.5+7.5+30+7.5+7.5+30+5) & a cross sectional area of 15x10 cm

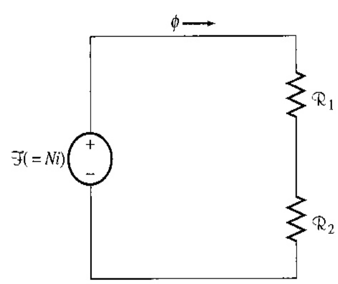
Magnetic Circuits Modeling

Reluctance of region 1: $\mathbf{R_1}$
R_1 = 0.45/[2500x4πx10^{-7}x0.01] =14300 Atr/ Wb

Reluctamce of region 2: $\mathbf{R_2}$
R_2=1.3/[2500x4πx10^{-7}x0.015]=27600 Atr/ Wb

Neglecting flux leakage : Flux is the same in both regions: **$\mathbf{R_1}$ in series with $\mathbf{R_2}$**

flux in core: φ=F/ (R_1+R_2) = 200x1 / (14300+27600)
= 0.0048 Wb



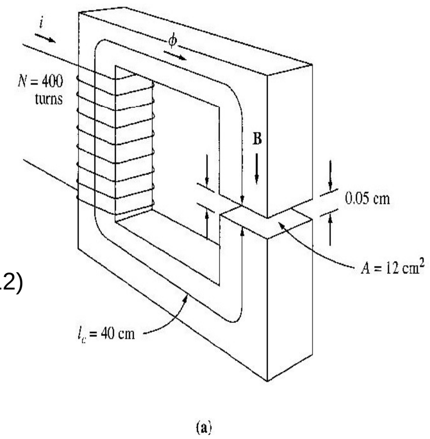
(b)

Magnetic Circuits Modeling

Example 2:

Ferromagnetic core of figure below:
mean path length 40cm, Gap 0.05 cm
A:12 cm^2, N:400 t.s, μ_r= 4000
fringing increase A in air gap by 5%

a) total R ?
b) I required for B=0.5T



(a)

Solution:

a) Rc=lc / (μ Ac) = 0.4 /(4000x4πx10^{-7}x0.0012)
= 66300 At/ Wb

Effective air gap area: 12 x 1.05 =12.6 cm^2
Ra=la/[μ_0Aa]=0.0005/(4πx10^{-7}x0.00126)
=316000 A.t/Wb
R_{eq}=66300+316000=382300 At./Wb

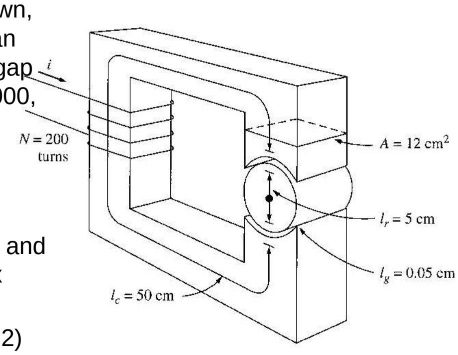
b) F =φ R , φ=B.A , F= N i ➔ NI =B.A .R
I =BAR/N=0.5x0.00126x383200/400=0.602 A

Magnetic Circuits Modeling

Example 3:

A simplified rotor stator of dc motor as shown, mean path of stator 50 cm, A=12 cm^2, mean length of rotor 5 cm, air gaps 0.05 cm, Air gap area (including fringing) 14 cm^2, Iron μr=2000, N=200

If I=1A, Ba=?



Solution:

Three different parts: stator (Rs), rotor (Rr) and air gap (Ra) in two parts with the same flux (series resiatnces)

Rs=ls/(μ_r μ_0 As)=0.5/ (2000x4πx10^{-7}x0.0012)
=166000 At/ Wb

Rr =lr/(μ_r μ_0 Ar)=0.05/(2000x4πx10^{-7}x0.0012)
=16600 At/ Wb

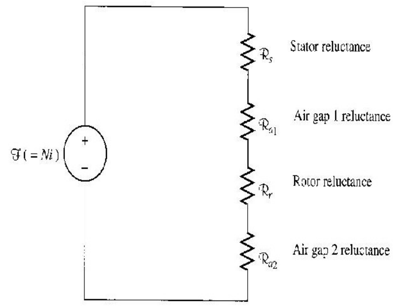
Ra = la/(μrμ0Aa)=0.0005/(4πx10^{-7}x0.0014)
=284000 At/ Wb

Magnetic Circuits Modeling

Equivalent magnetic circuit as shown :

R_{eq}=Rs+R_{a1}+R_r+R_{a2} =
166000+284000+16600+
284000=751000 A.tr / Wb

Net mmf applied:
F = NI = 200 x 1.0 =200 A.tr
Φ = F / R = 200 / 751000
=0.00266 Wb
B = φ / A = 0.000266 / 0.0014 =0.19 T



(b)