Subject:
Year. Month. Date. ( )

HW7 - ۹۲۱۰۲۸۴۶ محمد یاسر

تمرین سری ۷ تئوری کنترل استاد رحیمی

1- تبدیل لاپلاس هر تابع را بدست آورید:

$$f(t) = e^{-3t} + e^{-2t}\cos 4t + \frac{\sin t}{t}$$

$$\mathcal{L}(f(t)) = \frac{1}{s+3} + \frac{s+2}{(s+2)^2+14} - \tan^{-1}(s)$$

$$h(s) = \int_0^\infty \frac{\sin t}{t} e^{-st} ds \Rightarrow h'(s) = -\int_0^\infty \sin t\, e^{-st} ds = -\frac{a}{s^2+a^2} = -\frac{d}{ds}\tan^{-1}(s) \Rightarrow h(s) = -\tan^{-1}(s)$$

$$g(t) = t\cos t + u(t-2) + \sin(2t-3)$$

$$t f(t) = -\frac{dF(s)}{ds}$$

$$\mathcal{L}(t\cos t) = -\frac{d}{ds}\left(\frac{s}{s^2+1}\right) = -\frac{s^2+1-2s^2}{(s^2+1)^2} = \frac{s^2-1}{(s^2+1)^2}$$

$$u(t-2) \rightarrow F(s) = \int_{0^-}^{\infty} u(t-2)e^{-st} ds = e^{-2s}$$

$$\sin(2t-3) = \sin 2t \cdot \cos 3 - \cos 2t \sin 3$$
$$= \frac{2}{s^2+4}\cos 3 - \frac{s}{s^2+4}\sin 3$$

$$\mathcal{L}(g(t)) = \frac{s^2-1}{(s^2+1)^2} + e^{-2s} + \frac{2}{s^2+4}\cos 3 - \frac{s}{s^2+4}\sin 3$$

PAPCO

Year. Month. Date. ( )

2- عکس تبدیل لاپلاس هر تابع را بدست آورید:

$$F_1(s) = \frac{2s^2-4}{(s-2)(s+1)(s-3)} = \frac{4}{3(s-2)} - \frac{1}{6(s+1)} + \frac{7}{2(s-3)}$$

$$\mathcal{L}^{-1}(F_1) = \frac{4}{3}e^{2t} - \frac{1}{6}e^{-t} + \frac{7}{2}e^{3t}$$

$$F_2(s) = \frac{s^2+3s+5}{(s+1)^2(s+2)} = \frac{3}{(s+1)^2} + \frac{-2}{s+1} + \frac{3}{s+2}$$

$$3e^{-t}\frac{1}{t} - 2e^{-t} + 3e^{-2t} = \mathcal{L}^{-1}(F_2(s))$$

$$F_3(s) = \frac{s^2+3s+7}{((s+2)^2+4)(s+1)} = \frac{-1}{((s+2)^2+4)} + \frac{1}{s+1}$$

$$= \frac{-1}{2}u^{-2t}\sin 2t + u^{-t} = \mathcal{L}^{-1}(F_3(s))$$

PAPCO

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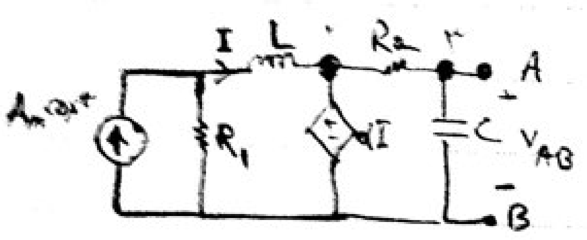
3 - امپدانس معادل تونن هر مدار را از دو سر A و B در حوزه ی لاپلاس بدست آورید.



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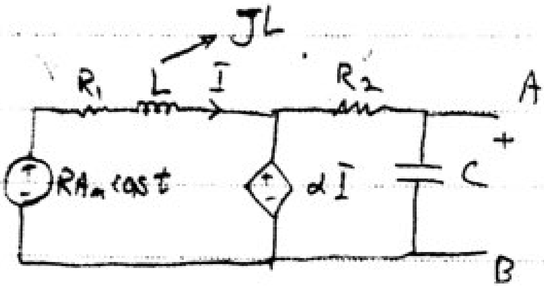
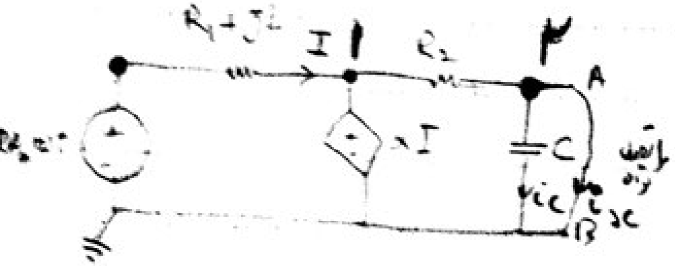




$$I = \frac{V_1 - RA_m \cos t}{R_1 + JL}$$

$$V_1 = \alpha I \Rightarrow I = \frac{\alpha I - RA_m \cos t}{R_1 + JL} \Rightarrow (R_1 - \alpha + JL) I = -RA_m \cos t$$

$$\Rightarrow I = \frac{-RA_m \cos t}{R_1 - \alpha + JL}$$

$$V_{AB} = \frac{1}{C} \int i_C \cdot dt$$

$$i_C = \frac{V_{AB} - \alpha I}{R_2} \Rightarrow C V'_{AB} = \frac{V_{AB} - \alpha I}{R_2}$$

$$\Rightarrow R_2 C V'_{AB} - V_{AB} - \alpha \frac{RA_m \cos t}{R_1 - \alpha + JL} = 0$$

اتصال کوتاه: $i_{sc} = \frac{\alpha I}{R_2} = -\alpha \frac{RA_m \cos t}{R_2 (R_1 - \alpha + JL)}$

$$Z_{th} = \frac{V_{AB}}{i_{sc}}$$

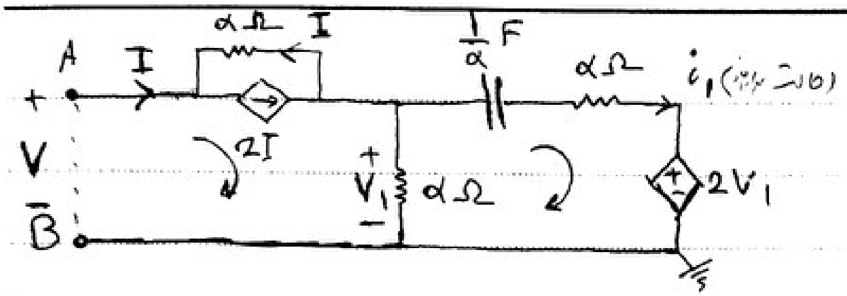
حالت پایدار

$$V(t) = -\frac{\alpha A_m R_1 \sin t}{(C^2 R_2^2 + 1)(\alpha - JL - R_1)} + \frac{\alpha A_m R_1 \cos t}{R_2 (C^2 R_2^2 + 1)(\alpha - JL - R_1)}$$

$$i_{sc} = -\frac{\alpha R_1 A_m \cos t}{R_2 (R_1 - \alpha + JL)}$$

$$\mathcal{L}(Z_{th}) = \frac{\mathcal{L}(\tan t)}{(C^2 R_2^2 + 1)(\alpha - JL - R_1) R_2 (R_1 - \alpha + JL)} - \frac{1}{s} \times \frac{1}{R_2 (C^2 R_2^2 + 1)(\alpha - JL - R_1)}$$

$$\Rightarrow Z_{th} = \frac{\tan t}{(C^2 R_2^2 + 1)(\alpha - JL - R_1) R_2 (R_1 - \alpha + JL)} - \frac{1}{R_2 (C^2 R_2^2 + 1)(\alpha - JL - R_1)}$$



$$V_A - \alpha I + \alpha(I - i_1) = 0 \Rightarrow V_A - \alpha i_1 = 0$$

$$\alpha \int i_1 \cdot dt + \alpha i_1 + 2\alpha(I - i_1) + \alpha(i_1 - I) = 0$$

$$\Rightarrow \alpha \int i_1 \cdot dt + \alpha I = 0$$

sc $$-\alpha I_{sc} + \alpha(i_{sc} - i_2) = 0 \Rightarrow i_2 = 0$$

$$\alpha \int i_2 \cdot dt + \alpha i_2 + 2\alpha(i_{sc} - i_2) + \alpha(i_2 - i_{sc}) = 0$$

$$\alpha \int i_2 \cdot dt + \alpha i_{sc} = 0$$