

# Energy conversion I

## Lecture 14:

### Topic 4: Synchronous Machines (S. Chapman ch. 5&6)

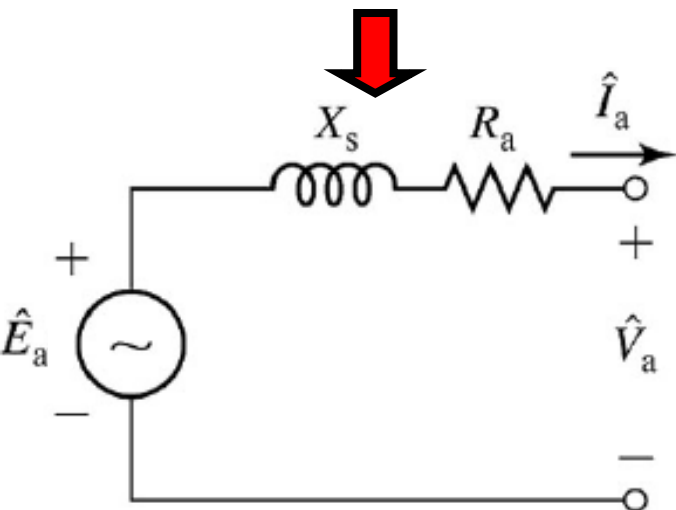
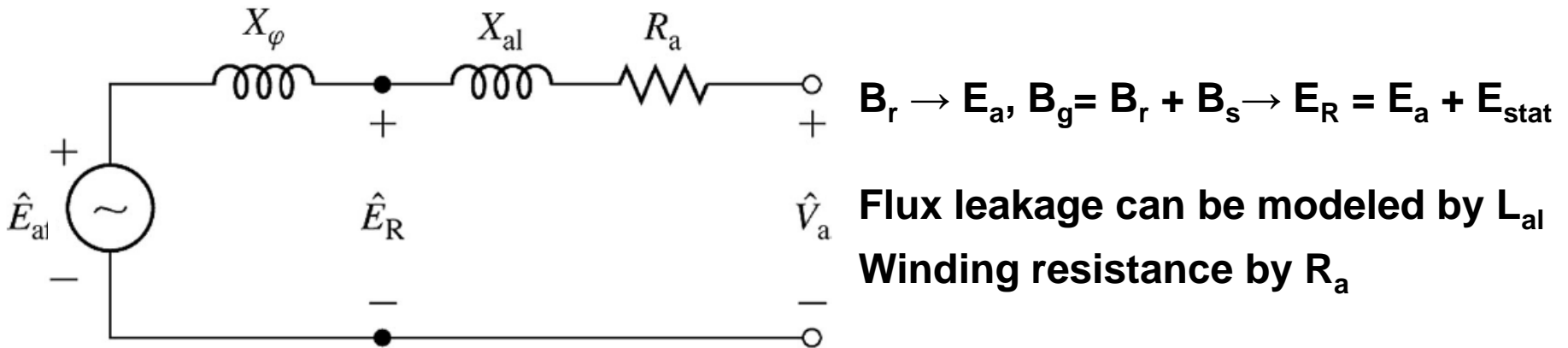
- Introduction
- Synchronous Generators Construction
- **Steady state equivalent circuit**
- **Power and Torque**
- Grid connected Synchronous machines
- Synchronous motors
- Power factor correction
- Start-up of synchronous motors

# Steady state equivalent circuit

Rotating stator flux induces three phase voltages in stator three phase winding too (armature reaction).

This induced voltage is similar to the induced voltage in inductances.

Using Single phase equivalent circuit:



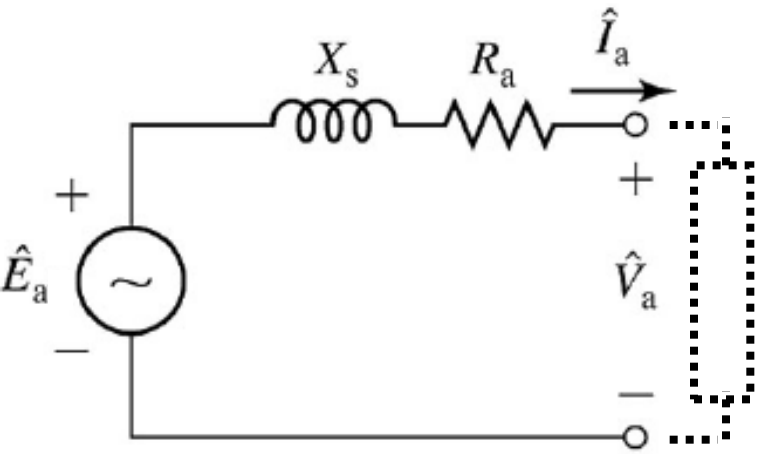
Using generator form for equivalent circuit:

$$V_a = E_a - jX_s I_a - R_a I_a$$

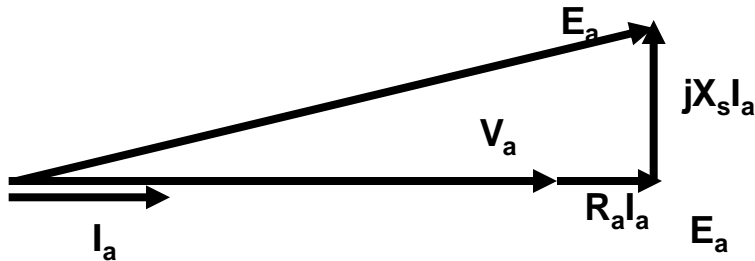
Think about equivalent circuit of Rotor in steady state!!

What about salient pole machines

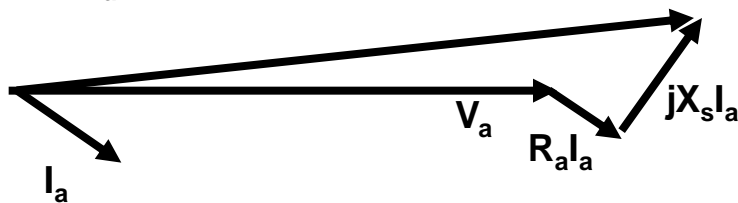
# Phasor diagram of a synchronous generator



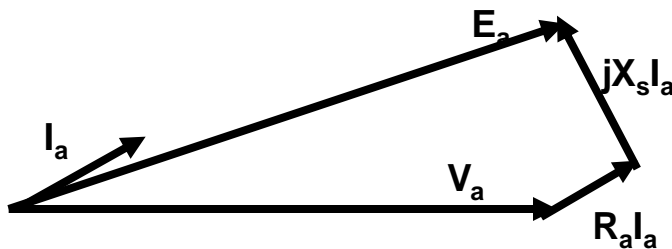
$$V_a = E_a - jX_s I_a - R_a I_a$$



**Unity Power factor (Resistive load)**



**Lagging current (Inductive load)  
Generator supplies reactive power**

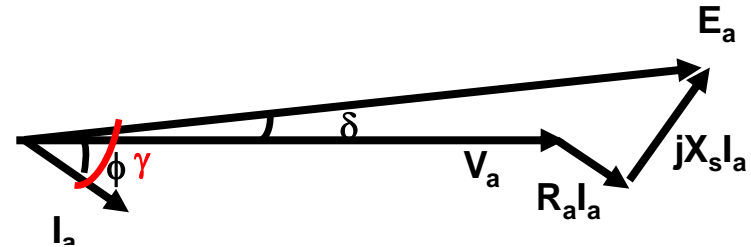


**Leading current (Capacitive load)  
Generator absorbs reactive power**

**Compare  $E_a$  ( $/I_f$ ) in these three cases!**

# Power and Torque of a synchronous generator

$$V_a = E_a - jX_s I_a - R_a I_a$$



**Output power:** Electrical delivered to the load (/ Grid) connected to the stator terminals:

$$P_{\text{out}} = V_a I_a \cos\phi \text{ (per phase)}$$

$$Q_{\text{out}} = V_a I_a \sin\phi \text{ (per phase) (positive if current is lagging)}$$

**Power loss:**

$$\text{Copper loss} = R_a I_a^2 \text{ (per phase)}$$

**Iron Loss :** (not included in the equivalent circuit)

**Where and why iron loss?**

**Converted Power:** Mechanical power converted to electrical power (Generator)

$$P_{\text{conv}} = E_a I_a \cos\gamma \text{ (per phase) (Electrical)}$$

$$P_{\text{conv}} = T_{\text{ind}} \Omega_s \text{ (three phase) (Mechanical)} \quad \Omega_s = \omega_s \cdot 2/p$$

**Mechanical power loss:**

**Friction and windage losses**

**Input Power: Mechanical**

**Don't forget field power!**

# Load / Torque angle

Usually:  $R_a \ll X_s$  and therefore:  $V_a = E_a - jX_s I_a$

$$X_s I_a \cos \phi = E_a \sin \delta$$

$$p = \frac{V_a E_a}{X_s} \sin \delta \quad (\text{per phase})$$

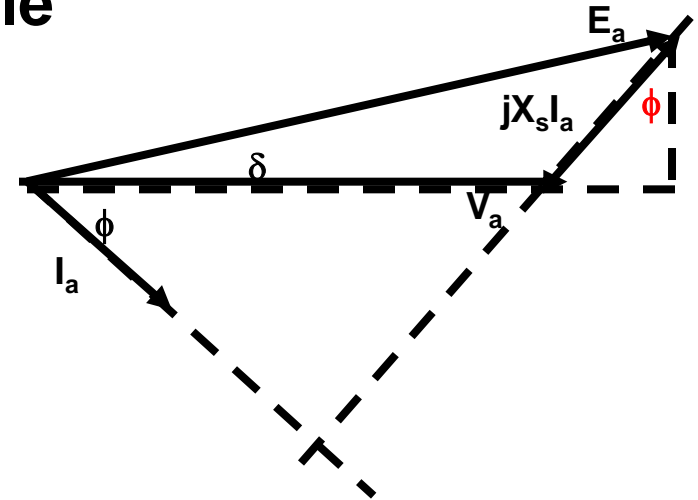
$$p_{3ph} = 3 \frac{V_a E_a}{X_s} \sin \delta = T \Omega_s$$

$$T_{ind} = \frac{3}{2 \omega_s} \frac{V_a E_a}{X_s} \sin \delta$$

$\delta$  : Torque( / Load) angle

$\delta > 0$   $T_{ind} > 0$  : Generator

$\delta < 0$   $T_{ind} < 0$  : Motor



$E_a$  is induced by  $B_r$

$V_a$  is induced by  $B_g$  (total flux density)



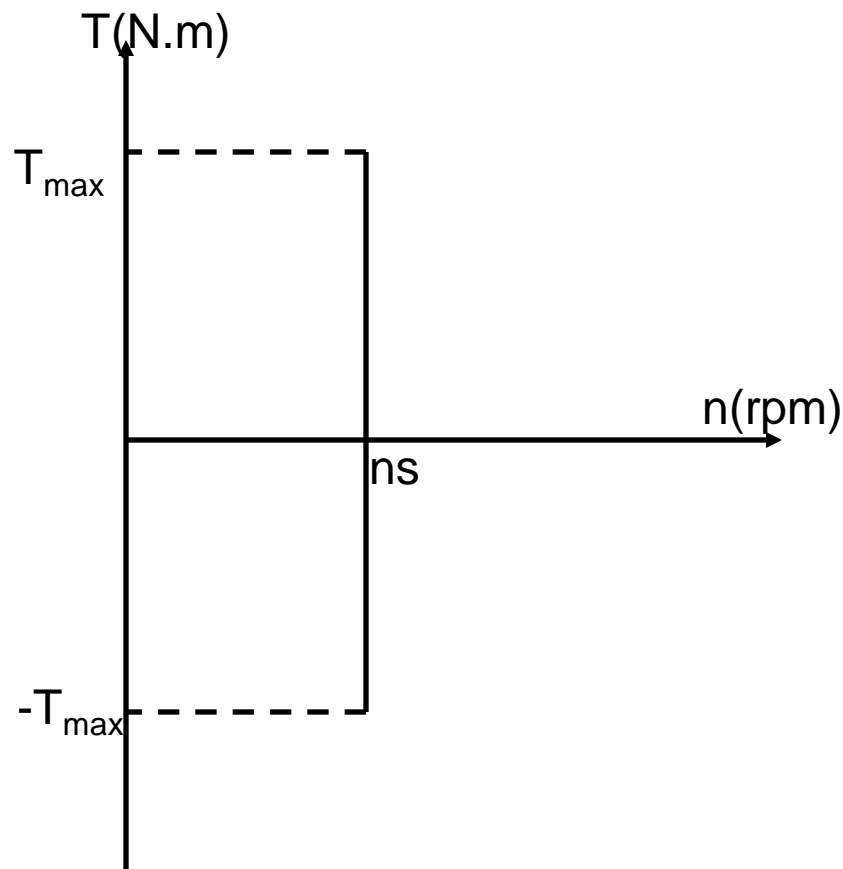
$\delta$  is the angle between  $B_r$  and  $B_g$



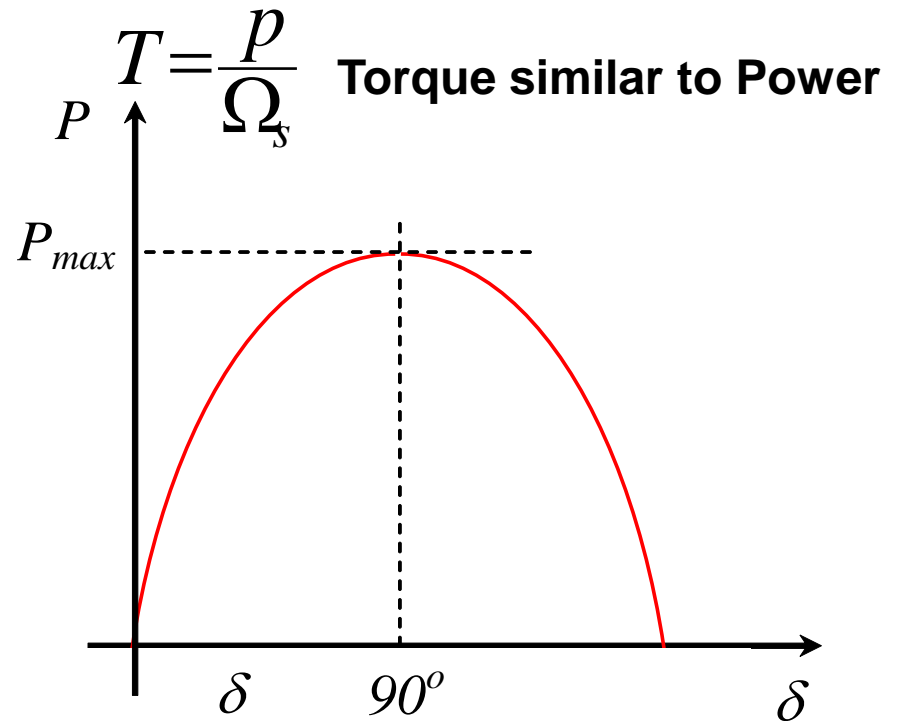
$B_g$  follows  $B_r$  in generating mode  
 $B_r$  follows  $B_g$  in motoring mode

# Power and Torque Characteristics

**Synchronous Machine:  
Torque at Synchronous  
Speed (Torque-speed  
characteristics)**



$$P = 3 \frac{E_a V_a}{X_s} \sin \delta$$



**Can machine work with  $\delta > 90^\circ$   
Or  $\delta < -90^\circ$ ?**

## Example:

A three phase, 5kW, 208 V, four-pole, 60 Hz, synchronous machine has negligible stator winding resistance and a synchronous reactance of  $8\Omega$ . The machine is connected to a 3phase 208V, 60Hz power supply.

a- Determine the excitation voltage and torque angle when the machine is delivering rated kVA at 0.8 PF lagging.

$$V_a = \frac{208}{\sqrt{3}} = 120V$$

$$I_a = \frac{5000}{\sqrt{3} \times 208} \angle -\cos^{-1}(0.8) = 139 \angle -36.9^\circ$$

$$E_a = V_a + jX_s I_a = 120 + j8 \times 139 \angle -36.9^\circ = 206.9 \angle 25.5^\circ$$

**Excitation voltage: 206.9 V/phase**

**Power Angle: 25.5°**

b- What is the maximum power can be delivered?

$$P_{\max} = 3 \frac{V_a \times E_a}{X_s} = 3 \frac{120 \times 206.9}{8} = 9.3 \text{ kW}$$

c: If the field excitation is now increased by 20%(without changing the prime mover power), find the stator current, power factor and reactive kVA supplied by the machine.

Power is the same as before, therefore:

$$\frac{V_a E_{a1}}{X_s} \sin \delta_1 = \frac{V_a E_{a2}}{X_s} \sin \delta_2$$

$$E_{a1} \sin \delta_1 = E_{a2} \sin \delta_2$$

$$\sin \delta_2 = \frac{E_{a1}}{E_{a2}} \sin \delta_1$$

$$\delta_2 = 21^\circ$$

$$I_a = \frac{E_a - V_a}{jX_s} = \frac{2482 \angle 21^\circ - 1200 \angle 0^\circ}{j8}$$

$$= 17.86 \angle -51.5^\circ$$

Power factor =  $\cos 51.5^\circ = 0.621$  lag

Reactive VA =  $3V_a I_a \sin 51.5^\circ$

$$= 5.03 \text{ kVAR}$$

Having  $E_{a2}$  and  $\delta_2$  current can be calculated:

Increasing field current has increased Reactive power delivered