In the Name of God

Assignment #2 EE, Signals & Systems 1393-94

Due: Saturday 9-12-93

Q1. In the previous homework you proved that for a linear system, causality is equivalent to the following condition: For every input that is zero up to a time t_0 the output is zero up to the time t_0 (initial rest condition).

a) For a linear system, prove that causality implies that the impulse response $h(t,\tau)$ is zero for all $t<\tau$
b) For a linear system, prove that if the impulse response $h(t,\tau)$ is zero for all $t<\tau$ then the system is causal
c) What can you say about the impulse response of an LTI system that is causal?

Q2. Prove each of the following cases using the definition of convolution and state what property of an LTI system it conveys if we consider h[n] to be the impulse response of an LTI system and x[n] to be the input to this system.

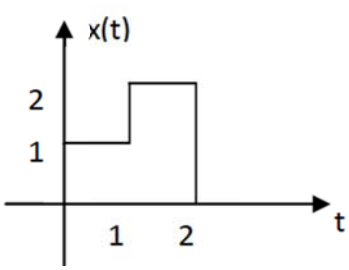
a) $h1[n] * h2[n] = h2[n] * h1[n]$

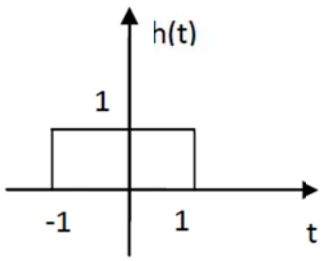
b) $h[n] * (a.x1[n] + b.x2[n]) = a.(h[n] * x1[n]) + b.(h[n] * x2[n])$

c) $h[n] * x[n - n_0] = (h * x)[n - n_0]$

Q3. In each case, find the convolution of the two functions:

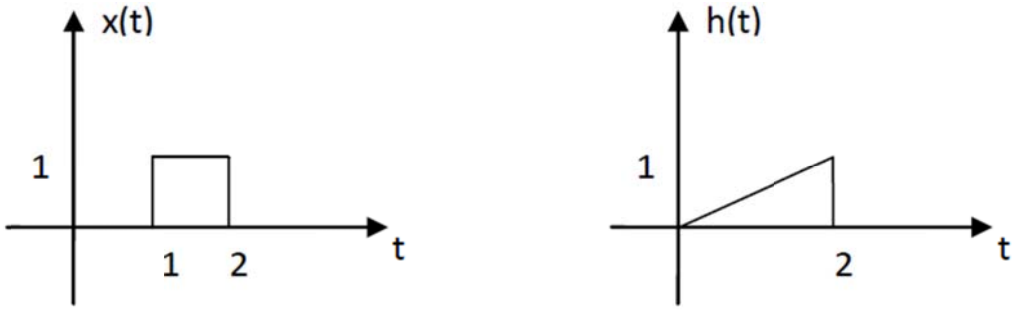
a)



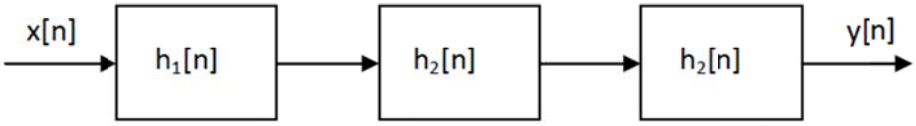


b)$x[n] = \left(-\frac{1}{2}\right)^n u[n-4]$ $h[n] = 4^n u[2-n]$

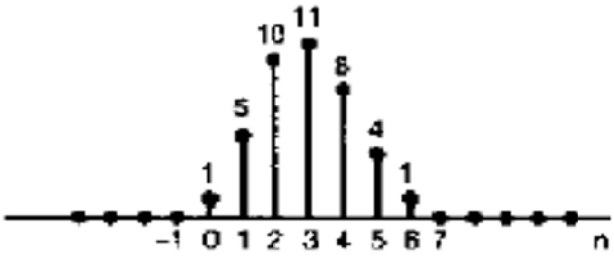
c)



Q4. Consider the LTI system depicted below:



$h2[n] = u[n] - u[n-2]$ and the impulse response of the whole system is shown in the figure below



a) Find the impulse response of the first system h1[n]
b) Find the response of the system to the input $x[n] = \delta[n] + \delta[n-2]$

Q5. The following are the impulse responses of LTI systems. Determine whether each system is causal and/or stable:

a) $h[n] = (0.3)^n u[n+5]$
b) $h[n] = 7^n u[4-n]$
c) $n * (0.1)^n u[n-2]$

d) $h(t) = t * e^{-t}u(t)$
e) $h(t) = e^{3t}u(-2-5t)$

Q6. If the input - output relationship of an LTI system is $y(t) = \int_{-inf}^{t} x(\tau+2)e^{-(t-\tau)}d\tau$, find the impulse response of the system.

Q7. Determine whether each of the following statements concerning LTI systems is true or false. Justify your answers.

a) If the input to an LTI system in periodic, so is the output

b) If h(t) is the impulse response of an LTI system and h(t) is periodic and nonzero, the system is unstable.

c) The inverse of a causal system is always causal

d) If $|h[n]| \leq K$ for all n, then the LTI system with h[n] as its impulse response is stable

e) If a discrete- time LTI system has an impulse response h[n] of finite duration, the system is stable.

Q8. Show that if the response of an LTI system to x(t) is the output y(t), then the response of the system to $\frac{dx}{dt} = x'(t)$ is $y'(t)$ in three different ways:

a) Directly from the properties of linearity and TI using the definition of derivative.

$$x'(t) = \lim_{h\to 0} \frac{x(t) - x(t-h)}{h}$$

b) By differentiating the convolution integral

c) By using the continuous counterpart of the result in you proved in 2-a