Energy conversion I

Lecture 8:

Topic 2: Transformers & its performance (S. Chapman, ch. 2)

- Introduction
- Types and Construction of Transformers.
- Ideal Transformer.
- Theory of operation of real single-phase transformers.
- The Equivalent Circuit of a Transformer.
- **The Per-Unit System of Measurement.**
- **Transformer voltage regulation and efficiency.**
- Autotransformers.
- Three phase transformers.

The Per Unit System of Measurements

Measuring each electrical quantity as a decimal fraction of some base level.

$$Quantity\ \ per\ unit = \frac{actual\ \ value}{base\ value\ of\ \ quantity}$$

Is very common in Computations relating to machines, transformers, and Power systems.

Advantages:

- **A reasonably narrow numerical range for machines and transformers parameter values in a per-unit system based upon their rating.**
- **Using per-unit values for transformer equivalent-circuit parameters, the ideal transformer can be eliminated.**

The Per Unit System of Measurements

To have similar relation between P.U. quantities, two base quantities can be selected independently: Voltage and power

Other base quantities can be calculated as:

$P_{base} = Q_{base} = S_{base}$

$I_{base} = S_{base} / V_{base}$

$Z_{base} = V_{base} / I_{base} = V^2_{base} / S_{base}$

In power system S_{base} and V_{base} are defined for a specific point. Others are calculated using them.

While S_{base} does not change due a transformer, V_{base} changes based on the voltage ratio of the transformer.

Changing base values, there is no need to take care about effect of ideal transformer!

The Per Unit System of Measurements

Using Transformer own ratings (rated kVA, rated voltage) for power transformers:

$R_{eq} \approx 0.01$ P.U., $X_{eq} = [0.02 – 0.10]$ P.U. (smaller for larger transformers)

Xm = [10 – 40] P.U., Rc = [50 – 200] P.U.

For systems including different transformers or machines, the entire system must have the same base.

Per-unit value can be converted to the new base recall that the actual values are not changed:

$$(P, Q, S)_{\text{pu on base 2}} = [\,(P, Q, S)_{\text{pu on base 1}} \times Sbase_1\,]/Sbase2$$

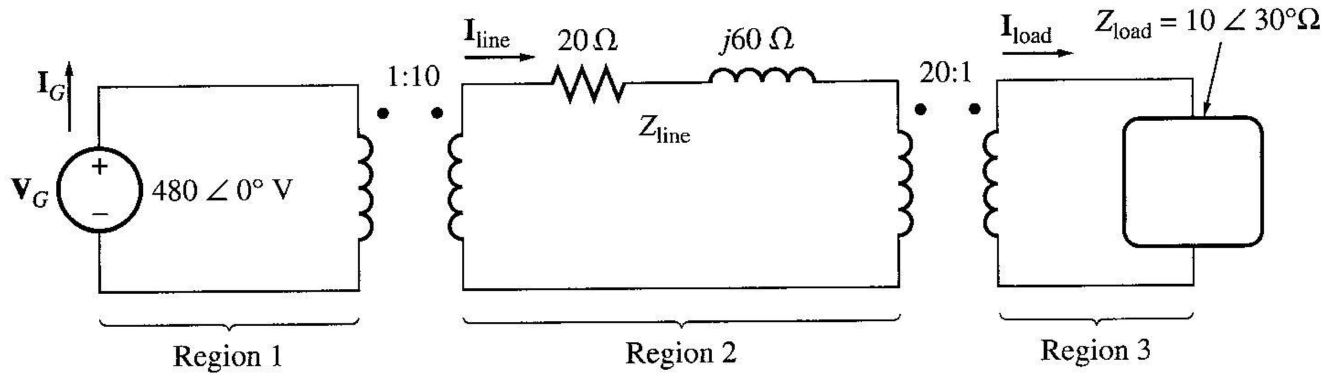
$$V_{\text{pu on base 2}} = [\,V_{\text{pu on base 1}} \times V_{\text{base1}}\,]/V_{\text{base2}}$$

$$(R, X, Z)_{\text{pu on base 2}} = (R, X, Z)_{\text{pu on base 1}} \times (V^2_{\text{base1}} S_{\text{base2}}) \,/\, (V^2_{\text{base2}} S_{\text{base1}})$$

The Per Unit System of Measurements

Example:

For the following system, the base values for the system @region 1 are 480 V, 10 kVA:



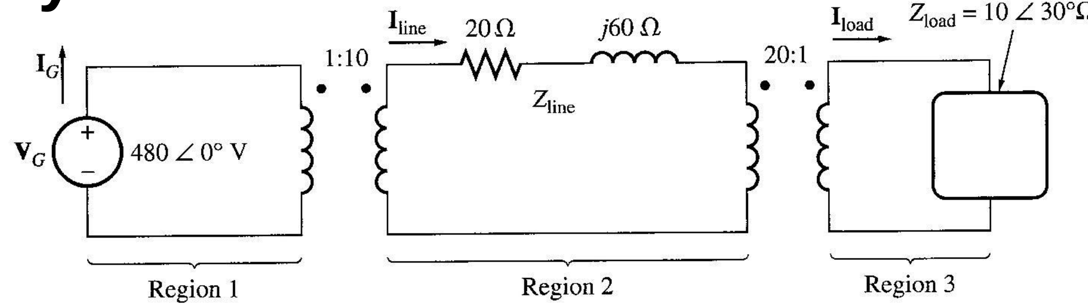
A: Find the base voltage, current, impedance, and apparent power at every point in the power system

B: Convert this system to its per-unit equivalent circuit

C: Find the power supplied to the load in this system

D: Find the power lost in the transmission line.

The Per Unit System of Measurements



Solution:

A- In region 1(Generator):

S_{base1} = 10kVA, V_{base1} = 480V → I_{base1}= S_{base1} / V_{base1} = 10000 / 480 = 20.83 A

$Z_{base1} = V_{base1} / I_{base1} = 480 / 20.83 = 23.04\ \Omega$

In region 2(transmission):

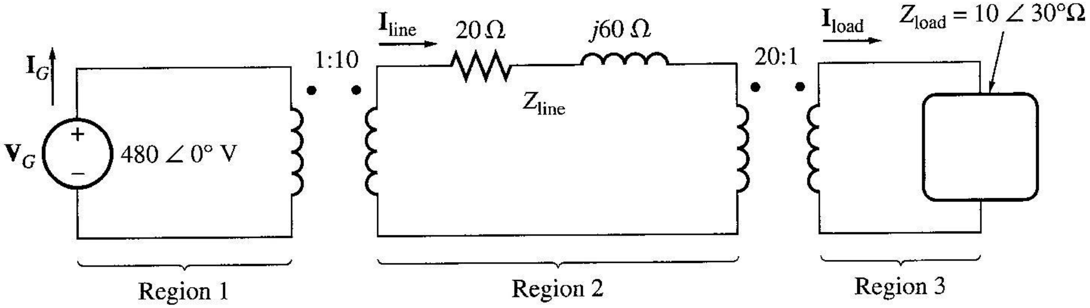
S_{base} is the same but V_{base} changes due to transformer voltage ratio, Therefore

S_{base2} = 10kVA, $V_{base2} = V_{base1} / a_{T1} = 480 / 0.1 = 4800$ V

$I_{base2} = S_{base2} / V_{base2} = 10000 / 4800 = 2.083$ A,

$Z_{base2} = V_{base2} / I_{base2} = 4800 / 2.083 = 2304\ \Omega$

The Per Unit System of Measurements



Solution:

Similarly in Region 3 (load):

S_{base3} = 10kVA, $V_{base3} = V_{base2} / a_{T2}$ = 4800 / 20 = 240 V

$I_{base3} = S_{base3} / V_{base3}$ = 10000 / 240 = 41.67 A,

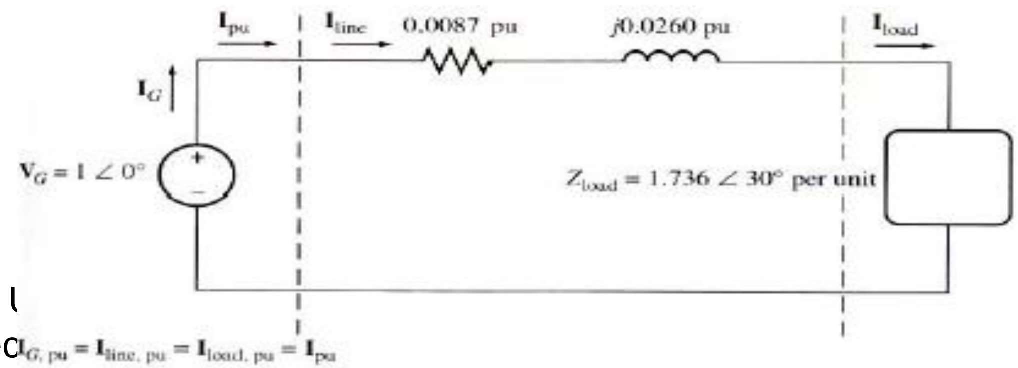
$Z_{base3} = V_{base3} / I_{base3}$ = 240 / 41.67 = 5.76 Ω

B- P.U. equivalent circuit:

Each component must be divided to its base value in its corresponding region:

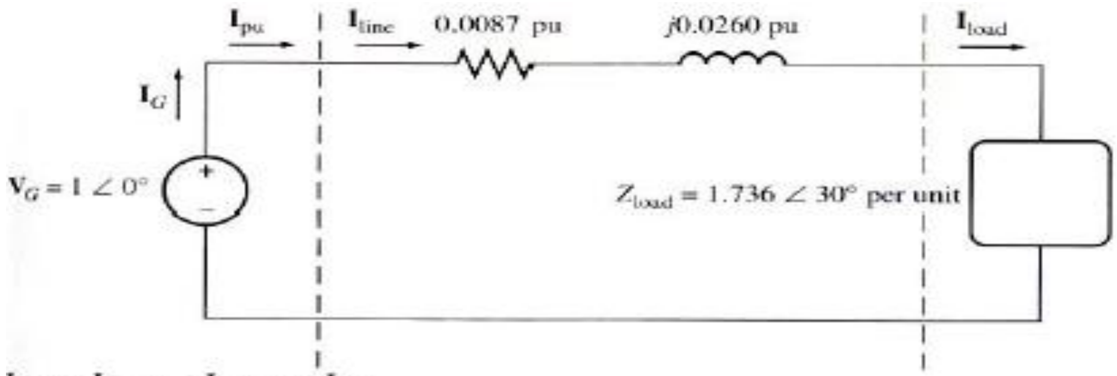
$$V_{G,pu} = \frac{480\angle 0}{480} = 1.0\angle 0\,\text{pu}, \qquad Z_{line,pu} = \frac{20 + j60}{2304} = 0.0087 + j0.0260\ \text{pu}$$

$$Z_{load,pu} = \frac{10\angle 30^o}{5.76} = 1.736\angle 30^o\ \text{pu}$$



The Per Unit System of Measurements

Solution:



C- Power delivered to the load

From the equivalent circuit:

$$I_{pu} = \frac{V_{pu}}{Z_{tot.pu}} = \frac{1\angle 0}{(0.0087 + j0.0260) + 1.736\angle 30} = 0.569\angle -30.6\ pu$$

$$P_{load,pu} = R_{pu} \cdot I_{pu}^2 = 1.503 * 0.569^2 = 0.487\ \ pu$$

$$P_{load} = P_{load,pu} \cdot P_{base} = 0.487 \cdot 10kW = 4870\ W$$

D- transmission power loss

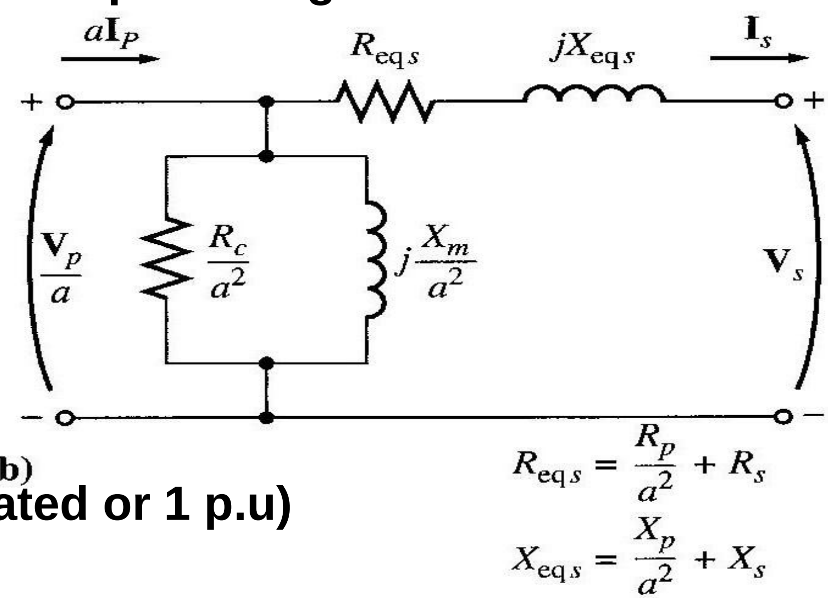
$$P_{loss,pu} = R_{line,pu} \cdot I_{pu}^2 = 0.0087 * 0.569^2 = 0.00282\ \ pu$$

$$P_{loss} = P_{loss,pu} \times P_{base} = 0.00282 \times 10000 = 28.2W$$

Transformer voltage regulation

Voltage regulation:

Series transformer impedance is the origin of **output voltage variations** even if the input voltage is constant **(load regulation)**



$$VR \equiv \frac{V_{S,nl} - V_{S,fl}}{V_{S,fl}} \times 100\,\%$$

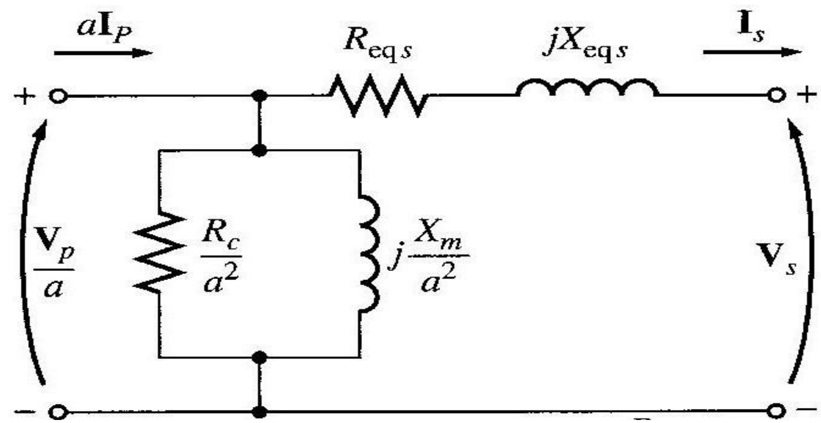
VR : Voltage regulation (percent)
$V_{s,nl}$: No-load Secondary voltage
$V_{s,fl}$: full-load Secondary voltage (usually rated or 1 p.u)

Using Simple equivalent circuit and p.u values:

$$VR = \frac{V_{p,pu} - V_{S,fl,pu}}{V_{S,fl,pu}}\, x\, 100\,\%$$

Series equivalent impedance limits short circuit current

Transformer voltage regulation

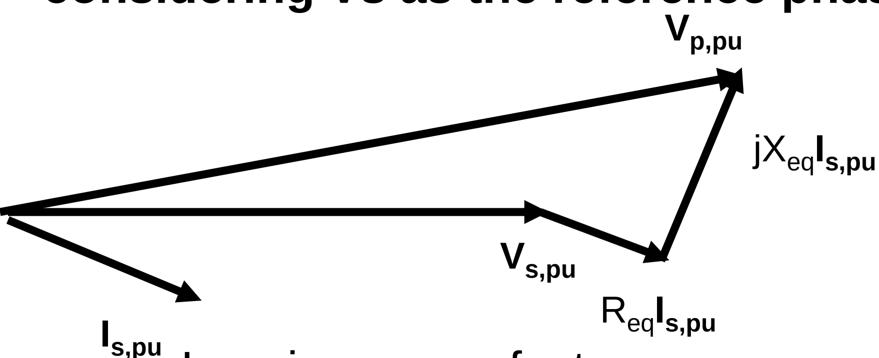


Load effect :

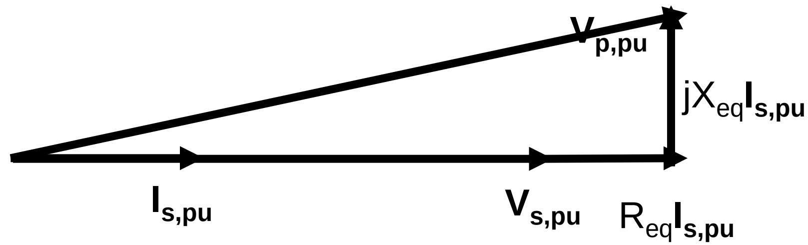
Using phasor diagram:

$$\mathbf{V}_{p,pu} = \mathbf{V}_{s,pu} + R_{eq}\mathbf{I}_{s,pu} + jX_{eq}\mathbf{I}_{s,pu}$$

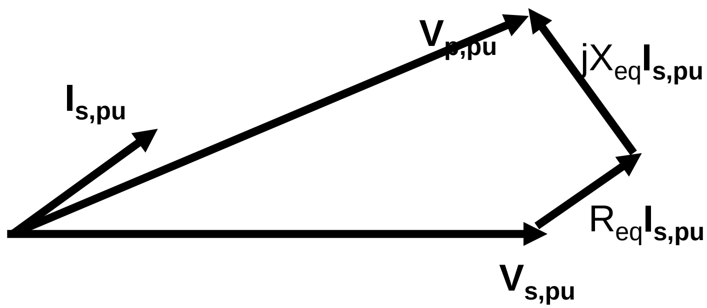
considering Vs as the reference phasor:



Lagging power factor



Unity power factor



Leading power factor

Note: Voltage regulation <u>can be</u> negative for capacitive loads.

When is VR maximum?

Transformer Tap changer

Transformer Tap changer is a device to **change the voltage ratio** for **regulation of the output voltage:**

On load tap changer: can change the voltage ratio while transformer is loaded

Off load tap Changer: changes the voltage ratio of a no-load transformer

Example: having 4 taps of $\pm$ 2.5% in HV for a 13200/480 V transformer means:

+5.0% tap :	**13860/480 V**
+2.5% tap :	**13530/480 V**
Nominal :	**13200/480 V**
-2.5% tap :	**12870/480 V**
-5.0% tap :	**12540/480 V**

Transformer Efficiency

Transformer efficiency:

$$\eta \equiv \frac{P_{out}}{P_{in}} x100\%$$

Considering power Losses:

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}} x100\% = \frac{V_S I_S \cos\theta}{P_{Cu} + P_{core} + V_S I_S \cos\theta} x100\% = \frac{V_S I_S \cos\theta}{V_p I_p \cos\theta_p} x100\%$$

P_{cu}: copper (winding RI^2) Loss (proportional to load current).

P_{core}: Core (Eddy current and Hystersis) loss (proportional to voltage).

θ_p : Phase shift between primary voltage and current

Show that for a given output voltage and power factor, maximum efficiency happens if $P_{cu} = P_{core}$

Transformer voltage regulation and efficiency

Example:

The equivalent Circuit parameters of a 15kVA, 2300/230 V transformer referred to low voltage side are:

R_{eq} = 0.0445 Ω, X_{eq} = 0.0645 Ω, R_c = 1050 Ω, X_M = 110 Ω

A- Calculate the full load voltage regulation @ 0.8 lagging power factor, 1.0 power factor and 0.8 leading power factor.

B- What is the efficiency at full load with 0.8 lagging power factor.

Solution:

The full-load current in the LV side is: $I_{s,n} = S_n / V_{s,n}$ = 15000/230 = 65.2 A

For 0.8 lagging power factor: $\mathbf{I_s} = 65.2\angle -36.9^o$

$\mathbf{V_p}' = \mathbf{V_s} + \mathbf{I_s}(R_{eq}+jX_{eq}) = 230\angle 0 + 65.2\angle -36.9(0.0445 + j0.0645) = 234.85\angle 0.4^o$

Therefore: VR= $\dfrac{234.85-230}{230}\times 100\% = 2.1\%$

For unity power factor: $\mathbf{I_s} = 65.2\angle 0^o$

$\mathbf{V_p}' = \mathbf{V_s} + \mathbf{I_s}(R_{eq}+jX_{eq}) = 230\angle 0 + 65.2(0.0445 + j0.0645) = 232.94\angle 1.04^o$

Therefore: VR= $\dfrac{232.94-230}{230}\times 100\% = 1.28\%$

For 0.8 leading power factor: $\mathbf{I_s} = 65.2\angle 36.9^o$

$\mathbf{V_p}' = \mathbf{V_s} + \mathbf{I_s}(R_{eq}+jX_{eq}) = 230\angle 0 + 65.2\angle 36.9(0.0445 + j0.0645) = 229.85\angle 1.27^o$

Therefore: VR= $\dfrac{229.85-230}{230}\times 100\% = -0.062\%$

B- efficiency at full load with 0.8 lagging power factor.

Losse can be calculated as:

$P_{cu} = R_{eq}\, I_s^2 = 0.0445(65.2)^2 = 189\ W$

$P_{core} = (V_p^{'})^2 / Rc = (234.85)^2 / 1050 = 52.5\ W$

Output power is : $P_{out} = V_s I_s \cos\theta = 230 \times 65.2 \times \cos(36.9^o) = 12000\ W$

$\eta = P_{out} / (P_{out} + P_{loss}) \times 100\% = 12000 / (12000+189+52.5) \times 100\% = 98.03\%$

Think about daily (24hours) efficiency of transformers!