

Energy conversion I

Lecture 7

Topic 2: Transformers & its performance (S. Chapman, ch. 2)

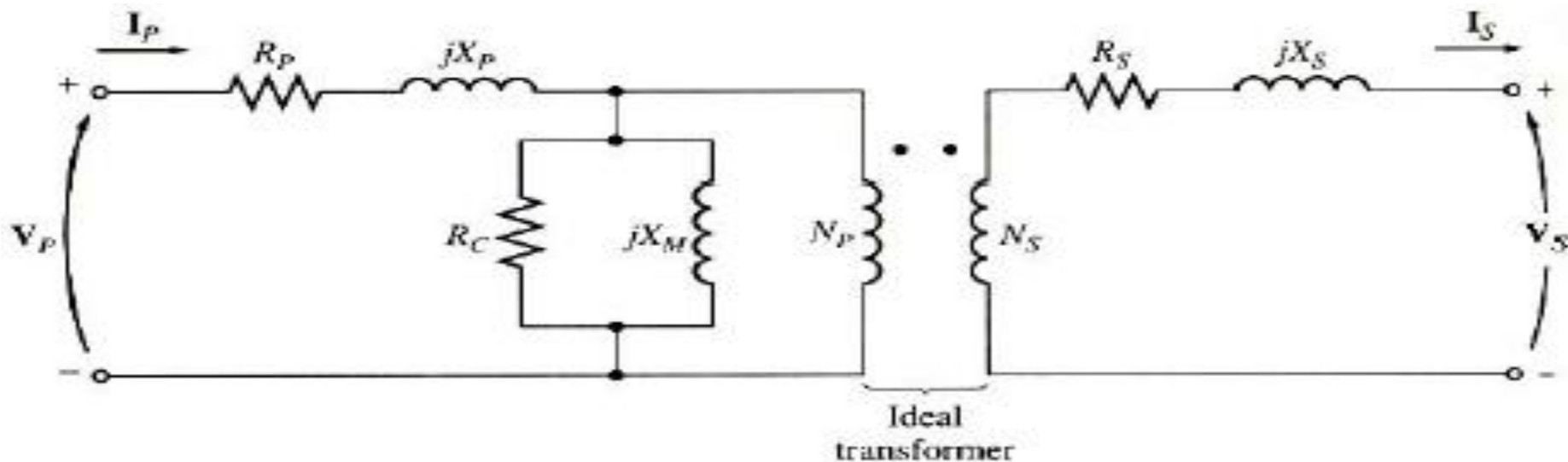
- Introduction
- Types and Construction of Transformers.
- Ideal Transformer.
- Theory of operation of real single-phase transformers.
- **The Equivalent Circuit of a Transformer.**
- The Per-Unit System of Measurement
- Transformer voltage regulation and efficiency
- Autotransformers
- Three phase transformers

Equivalent circuit of a real Transformer

- Copper (winding) loss: R_p, R_s
- Leakage flux: $X_p(L_p), X_s(L_s)$
- Core loss: R_c
- Magnetizing current (X_M)
- Transformer Turns ratio: Ideal transformer
- Electrical Isolation: Ideal transformer

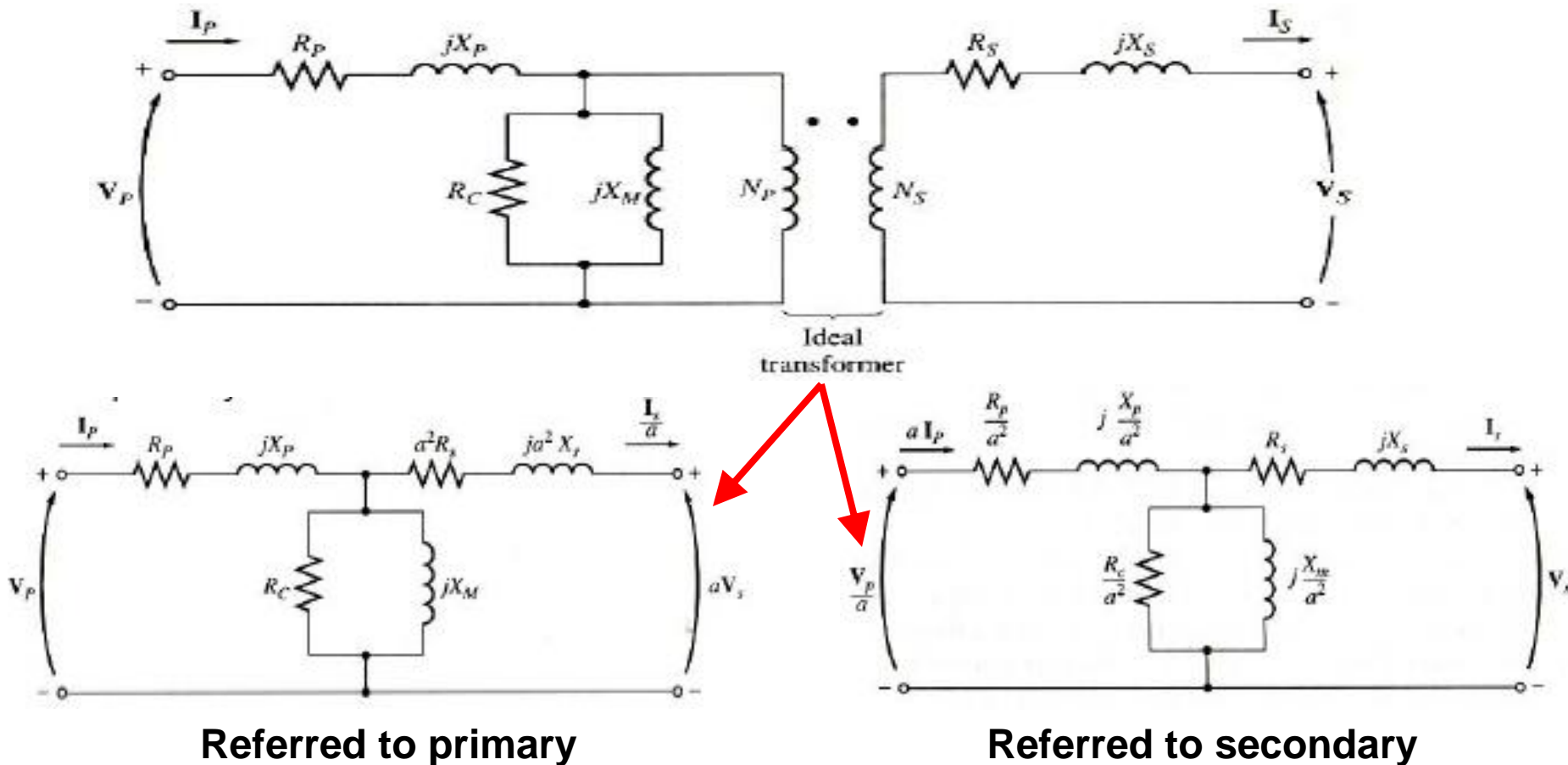
$$\frac{V_p}{V_s} \approx \frac{N_p}{N_s} = a$$

$$\frac{I_p}{I_s} \approx \frac{N_s}{N_p} = \frac{1}{a}$$



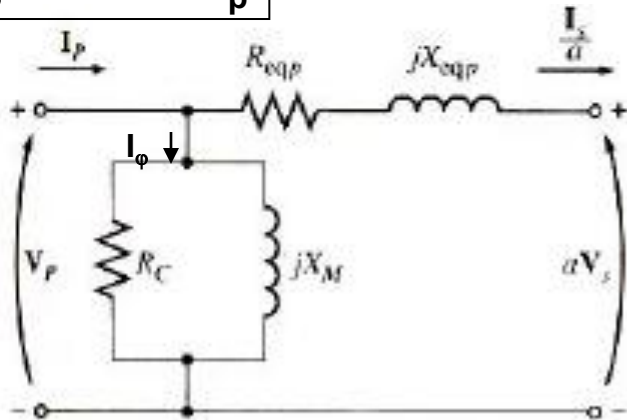
Other equivalent circuits of a real transformer

Having ideal transformer in the Model different models can be considered:



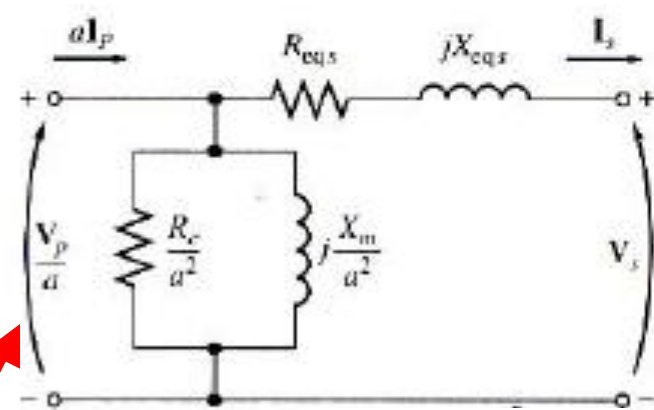
Approximate equivalent circuits of a real transformer

rated $I_\phi \ll \text{rated } I_p$



$$R_{eqp} = R_p + a^2 R_s$$

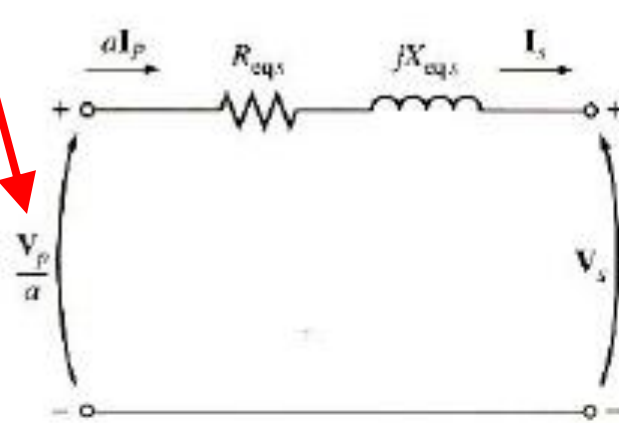
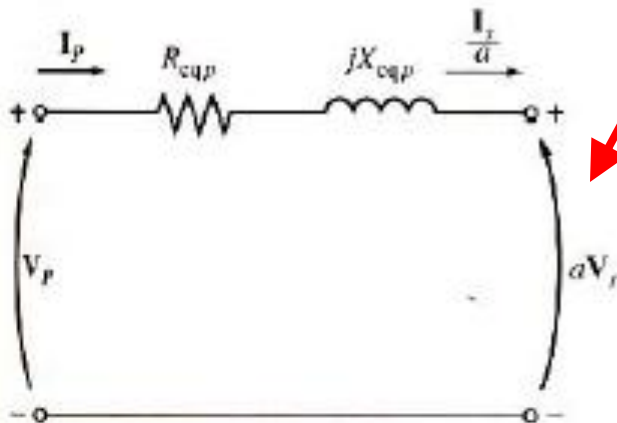
$$X_{eqp} = X_p + a^2 X_s$$



$$R_{eqs} = R_s + R_p/a^2$$

$$X_{eqs} = X_s + X_p/a^2$$

Ideal Transformer



Equivalent circuit of a real Transformer

Example:

Approximate equivalent circuit parameters of a 20kVA, 8000/240V, 60Hz transformer are listed:

$$R_{c,HV} = 160 \text{ k}\Omega, X_{m,HV} = 38.4 \text{ k}\Omega, R_{eq,HV} = 38.4 \text{ }\Omega, X_{eq,HV} = 192 \text{ }\Omega$$

What is the ratio of magnetizing current to rated current?

What is the ratio of voltage drop across the transformer series impedance to rated voltage?

Solution:

Rated high-voltage winding current is:

$$I_{HV,n} = 20000/8000 = 2.5 \text{ A}$$

$$I_{\phi} = 8000(1/160 - j1/38.4) = 50 - j208.3 = 214.2e^{-j76} \text{ mA (small lagging power factor)}$$

$$I_m / I_{HV,n} = 214.2 \cdot 10^{-3} / 2.5 = 0.085 = \mathbf{8.5\%}$$

$$\text{Voltage drop: } V = 2.5 \cdot (38.4 + j192) = 96 + j480 = 489e^{j78.7} \text{ V}$$

$$V / V_{HV,n} = 489/8000 = 0.061 = \mathbf{6.1\%}$$

Determination of Equivalent circuit components

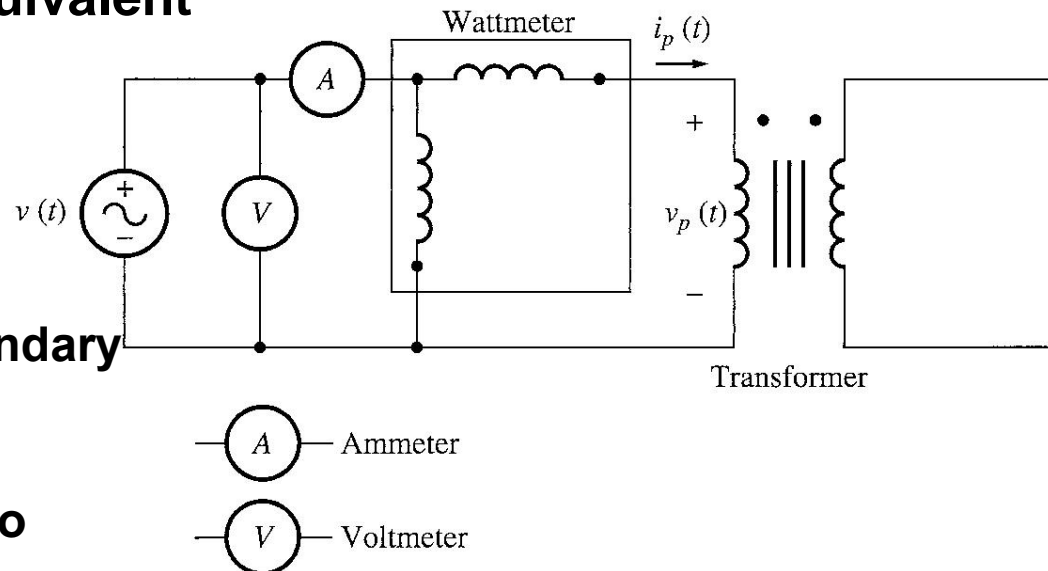
**Experimental test to get the equivalent
Circuit parameters:**

Open Circuit test:

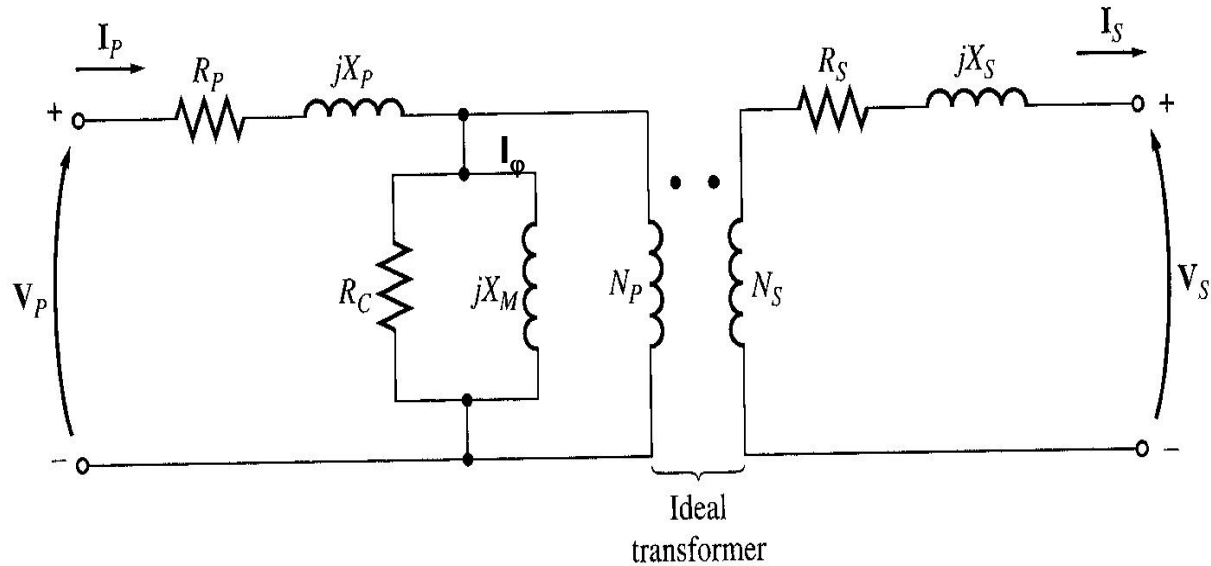
**open-circuited transformer's secondary
winding (high voltage)**

**connecting full-rated line voltage to
primary winding (low voltage)**

Measuring V, I and P



Open-circuit test



$$I_p = I_\phi \ll \text{rated } I_p$$



Negligible voltage drop
Across: $R_p + j X_p$



Rated voltages across magnetizing branch



Operating close to rated magnetic operating point



R_c : origin of power loss (core loss)
 X_m : origin of lagging power factor

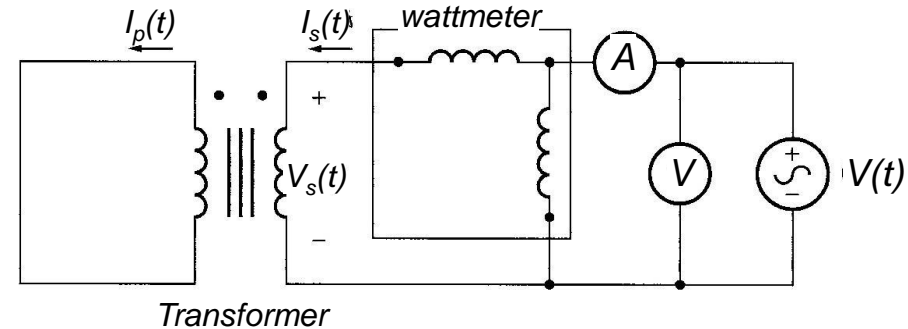
$$R_c = \frac{V_{oc}^2}{P_{oc}}$$

$$\frac{I_{oc}}{V_{oc}} = \sqrt{\frac{1}{R_c^2} + \frac{1}{X_m^2}}$$

Short-circuit test

Short-circuited transformer's primary winding (low voltage)

connecting full-rated-current voltage to secondary winding (high voltage)



Measuring V, I and P

$V_{sc} = V_s \ll \text{rated } V_s \Rightarrow$ Negligible voltage drop across: R_c & $j X_m \Rightarrow$

Negligible magnetizing current \Rightarrow rated current in primary & secondary winding

R_{eq} : origin of power loss (winding loss)

X_{eq} : origin of lagging power factor

$$R_{eq} = \frac{P_{sc}}{I_{sc}^2}$$

$$\frac{V_{sc}}{I_{sc}} = \sqrt{R_{eq}^2 + X_{eq}^2}$$

Determination of Equivalent circuit components

Notes:

Open-circuit and Short-circuit test can be done either in the low-voltage or high-voltage winding.

Based on the measuring winding, the parameters are calculated in that side (Z_{eq} in High voltage side and R_c and X_m in low voltage).

Usually Open circuit test is measured in low voltage (with rated voltage) and Short circuit test is measured in high voltage side (with rated current).

To have the detailed equivalent circuit we can suppose that:

$$R_p = a^2 R_s$$

$$X_p = a^2 X_s$$

Determination of Equivalent circuit components

Example:

The open circuit test and the short circuit test results of a 20kVA, 8000/240V, 60Hz transformer are listed.

Find the approximate equivalent circuit referred to the high voltage side.

Open circuit test (primary)	Short circuit test
$V_{OC} = 8000 \text{ V}$	$V_{SC} = 489 \text{ V}$
$I_{OC} = 0.214 \text{ A}$	$I_{SC} = 2.5 \text{ A}$
$P_{OC} = 400 \text{ W}$	$P_{SC} = 240 \text{ W}$

Solution:

O.C. test is measured in High voltage side! (How can it be found out)

S.C. test is measured in high voltage side. ($I_{hv, \text{ rated}} = 20000/8000=2.5 \text{ A}$)

Using O.C. test results:

$$R_{c,HV} = \frac{V_{oc}^2}{P_{oc}} = \frac{8000^2}{400} = 160 k\Omega$$

$$\frac{I_{oc}}{V_{oc}} = \sqrt{\frac{1}{R_c^2} + \frac{1}{X_m^2}} \Rightarrow X_m^2 = \left(\left(\frac{I_{oc}}{V_{oc}} \right)^2 - \frac{1}{R_c^2} \right)^{-1}$$

$$X_{m,HV} = 38.4 k\Omega$$

Open circuit test (primary)	Short circuit test
$V_{OC} = 8000 \text{ V}$	$V_{SC} = 489 \text{ V}$
$I_{OC} = 0.214 \text{ A}$	$I_{SC} = 2.5 \text{ A}$
$P_{OC} = 400 \text{ W}$	$P_{SC} = 240 \text{ W}$

Using S.C. test results:

$$R_{eq,HV} = \frac{P_{sc}}{I_{sc}^2} = \frac{240}{2.5^2} = 38.4 \Omega,$$

$$\frac{V_{sc}}{I_{sc}} = \sqrt{R_{eq}^2 + X_{eq}^2} \Rightarrow X_{eq}^2 = \left(\frac{V_{sc}}{I_{sc}} \right)^2 - R_{eq}^2,$$

$$X_{eq,HV} = 192 \Omega$$

What would be the Open circuit measurement results if done in LV side?

Try to find the detailed equivalent circuit having magnetizing branch in the HV side.