Logic Circuits

The Problem Collection For Mid-term Exam

This collection consists of 20 questions
and their answers which seem to cover all the
topics the students may need to know for mid-term exam…

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A) Base Conversion, Binary Arithmetic Operations, Binary Codes :

1) Determine the value of x if $(211)_x = (152)_8$.

$2x^2+x+1 = 8^2+5\times8+2 \rightarrow 2x^2 +x -105=0 \rightarrow x=7$

2) In Gray Code system if we attribute the 0 code to the 0 value then what is the code which shows the value of 6?

Gray	Decimal
000	0
001	1
011	2
010	3
110	4
111	5
101	6
100	7

→.................... (101→6)

3) In the floating point system of showing the numbers what is the ratio of the biggest number to the very close number after that?

The biggest number is : $(FFFFFFFF)_{16}= (0.111\ldots1)\times2^{64} = a$

The next number is : $(FFFFFFFE)_{16}=(0.111\ldots10)\times2^{64} = b$

$a/b= (2^{64}-2^{32})/ (2^{64}-2^{33})$

B) Error Detection and Correction Using Parity, Hamming1, Hamming2:

4) In an odd parity system we have received the Hexadecimal number $(6E)_{16}$. Is there any error in the received number? If yes, in which digit? Correct the number.

$(6E)_{16}=1101110 \rightarrow S_0=\text{odd parity}\{ P_1 , X_3 , X_5 , X_7 \} = \{ 1 , 0, 1, 0 \} = 1$

$S_1=\text{odd parity}\{ P_2 , X_3 , X_6 , X_7 \} = \{ 1 , 0, 1, 0 \} = 1$

S_2=odd parity$\{P_4,X_5,X_6,X_7\}$ = $\{1,1,1,0\}$ = 0

$(S_2S_1S_0)_2 = (011)_2 = 3$ →It is clear that there is an error in the 3rd bit so :

$X_3 = 1$ → The original correct number is : $(X_3X_5X_6X_7)_2 = (1110)_2$

5) Suppose that in a code the distance between the two closest codes is d ,the numbers of error detection is s and the numbers of error correction is t, find the relationship between these parameters for any code depending on the number of digits which differ from a code to the nearest code.(neglect the probability of receiving two errors)

It can be seen from examination that a single error detection code(s=1,t=0)requires a minimum distance of d_{min}=2,a single error correction code(s=0,t=1)requires a minimum distance of d_{min}=3 and a code with both single error correction and a double error detection(s=t=1) requires d_{min}=4.so by examination we can receive an empirical relationship like :

$$2t + s + 1 \leq d_{min}$$

C) Boolean Function, Minimization Using both Karnaugh Map and QM Tables, Glitch-Free Design, Logic Circuits Timings, Logic Design of Several Interesting Problems :

6) Simplify the following boolean expressions to the minimum number of literals :

a) $ABC + \overline{A}B + AB\overline{C}$

b) $xy + x(\overline{y}z+\overline{yz})$

a) $= AB(C+\overline{C}) + \overline{A}B = (A+\overline{A})B = B$

b) $= xy + x(\overline{y}(z+\overline{z})) = xy + x\overline{y} = x(y+\overline{y}) = x$

7) For the Boolean function F given I the truth table, find the following :

a) List the minterms of the function.

b) List the minterms of $\overline{F}$.

c) Express F in sum of minterms in algebraic form.

d) Simplify the function to an expression with a minimum number of literals.

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

a) $F = \sum m\,(2,3,6,7)$

b) $\overline{F} = \sum m\,(0,1,4,5)$

c) $F = m\,(2) + m\,(3) + m\,(6) + m\,(7) = \overline{X}Y\overline{Z} + \overline{X}YZ + XY\overline{Z} + XYZ$

d) $F = \overline{X}Y(Z+\overline{Z}) + XY(Z+\overline{Z}) = \overline{X}Y + XY = Y(\overline{X}+X) = Y$

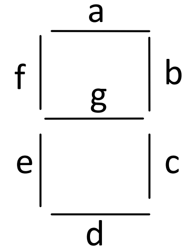
8) Write the following function in a SOP (sum of product) and simplest form using Karnaugh map :

$f(A,B,C,D) = \sum m(0,1,3,4,6,7,9,11,15)$

AB \ CD			
1	1	1	0
1	0	1	1
0	0	1	0
0	1	1	0

$\rightarrow f = CD + \overline{B}D + \overline{A}B\overline{D} + \overline{A}\overline{C}\overline{D}$

9) For the following seven segment find the Boolean Function for the d LED (f_d) as a function of w,x,y,z?



$f_d = \sum m\,(0,2,3,5,6,8,9) + d\,(10,11,12,13,14,15)$

wx \ yz	00	01	11	10
00	1	0	1	1
01	0	1	0	1
11	X	X	X	X
10	1	1	X	X

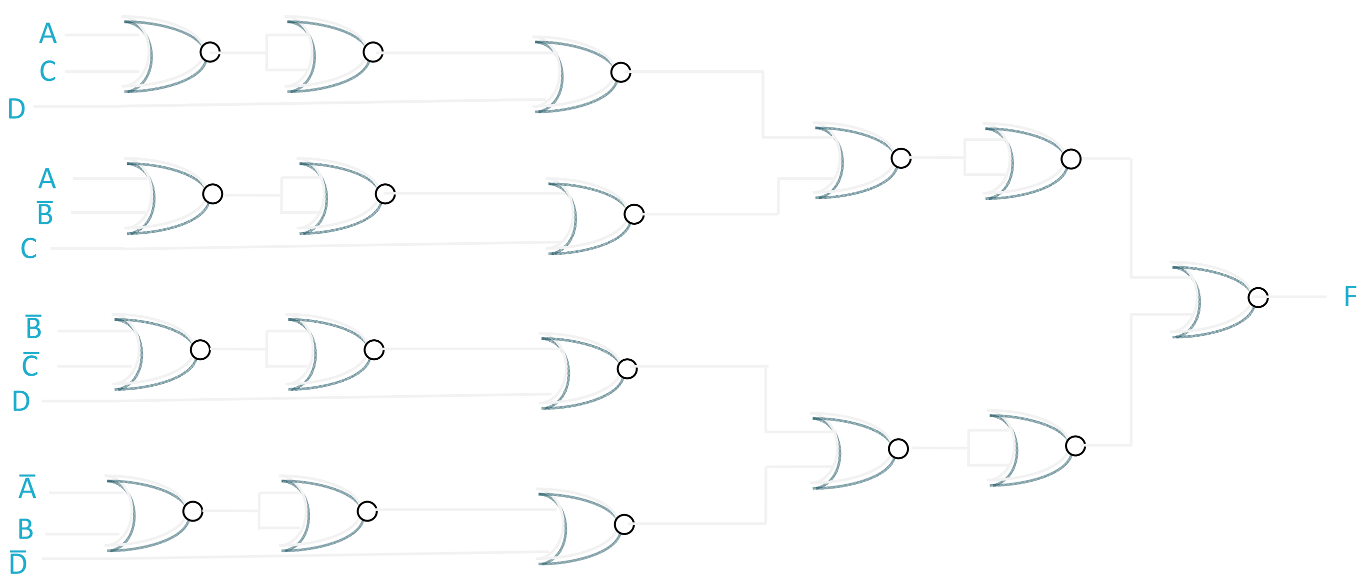
$\rightarrow \quad f_d = w + y\bar{z} + \bar{x}y + x\bar{y}z + \bar{w}\bar{x}\bar{z}$

10) Fabricate the following function with only NOR gates and plot the outcome.(Limitation: there have to be just two inputs)

f (A,B,C,D) = ∑ m (1,3,4,5,8,11,12,14,15) A=LSB

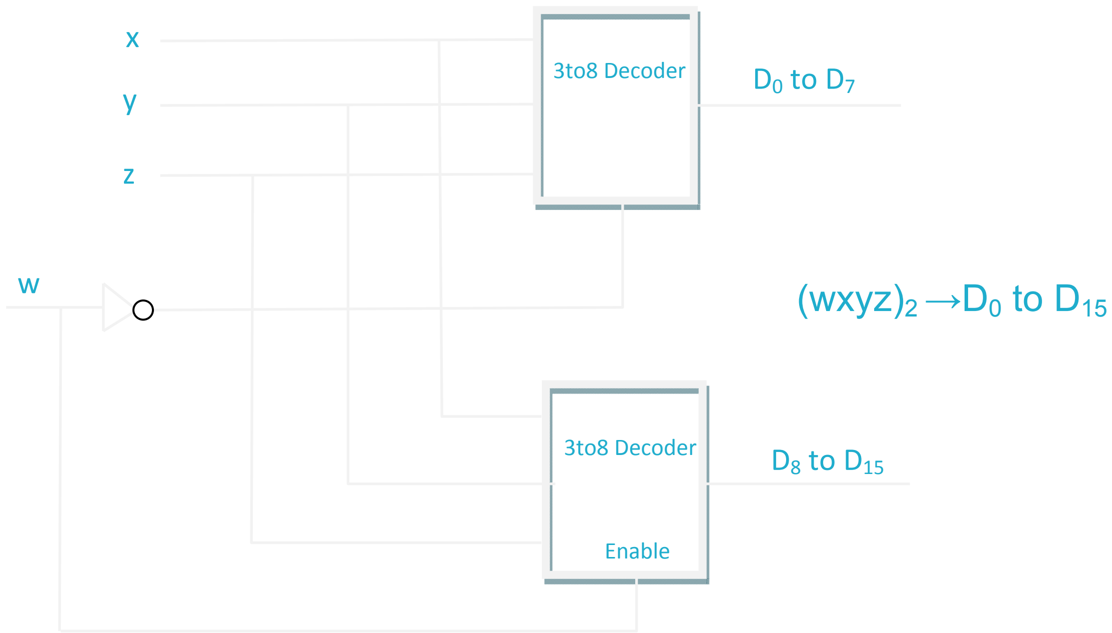
$\bar{f} = \sum m\,(0,2,6,7,9,10,13) \rightarrow \bar{f} = \bar{A}C\bar{D} + \bar{A}B\bar{C} + BC\bar{D} + A\bar{B}D$

$f = (A+C+D) \times (A+\bar{B}+C) \times (\bar{B}+\bar{C}+D) \times (\bar{A}+B+\bar{D})$

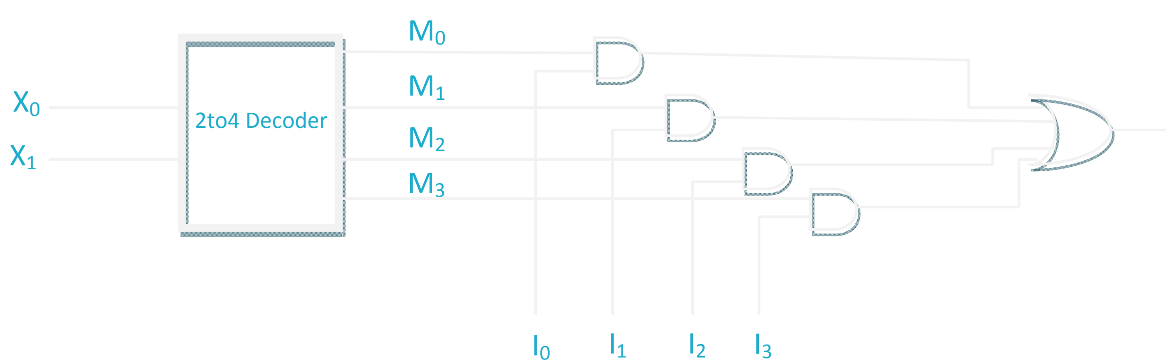


D) Modular combinational circuit design (MUX, ADDER, ENCODER,...) :

11) Design a 4 to 16 Decoder by means of two 3 to 8 Decoder.(note that you can also use the Enable pins)



12) Design and implant a MultiPlexer(with two selector) Using a 2 to 4 Decoder and minimum gates.

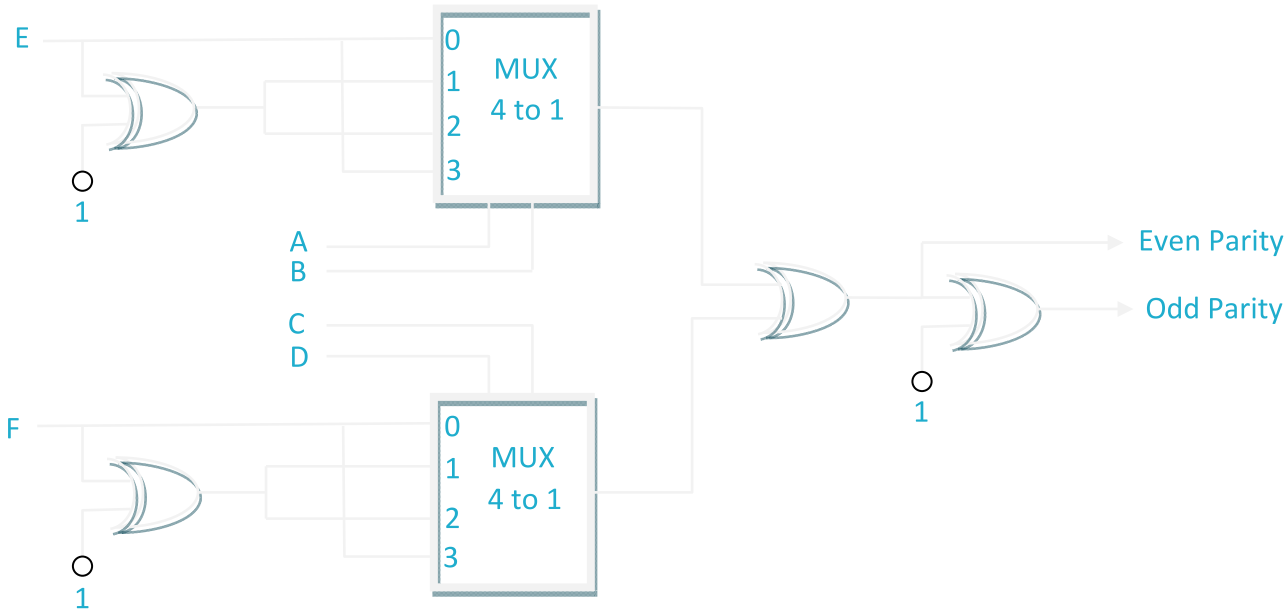


13) Desing a combinational circuit which can find the odd parity for 6 bits (A,B,C,D,E,F) Using just two 4 to 1 MUX and XOR gates.

odd parity :

For example if we have (ABCDEF) 001011 then the output should be 0.

Or if 000110 then the output should be 1.



14) Design a comparator (which compares two binary number A and B) by means of a 4 bit Adder and logic gates.

We can use the Adder as a comparator by inverting B’s digits and adding A and B which leads to A-B. From the sign of A-B we can compare their magnitudes.