

Energy conversion I

Lecture 10:

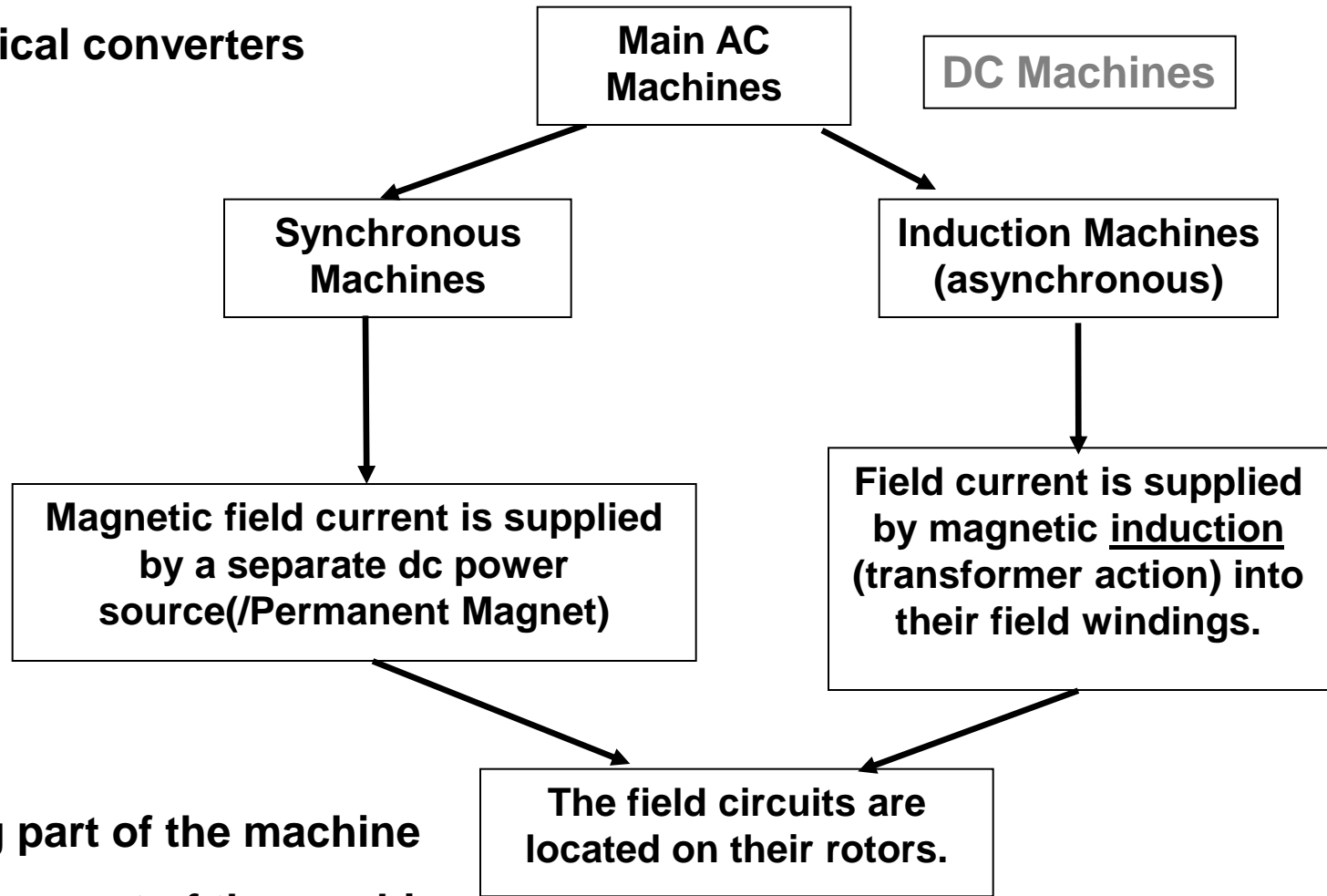
Topic 3: Fundamentals of AC machines steady state operation (S. Chapman, ch. 4)

- **Introduction**
- **Voltage of a loop in a uniform magnetic Field.**
- **Torque of a loop in a uniform magnetic Field.**
- **Rotating magnetic field.**
- Magnetomotive force and flux distribution in AC machines.
- Induced voltage in AC machines.
- Induced torque in AC machines.

Introduction

Electric machines:

Electromechanical converters



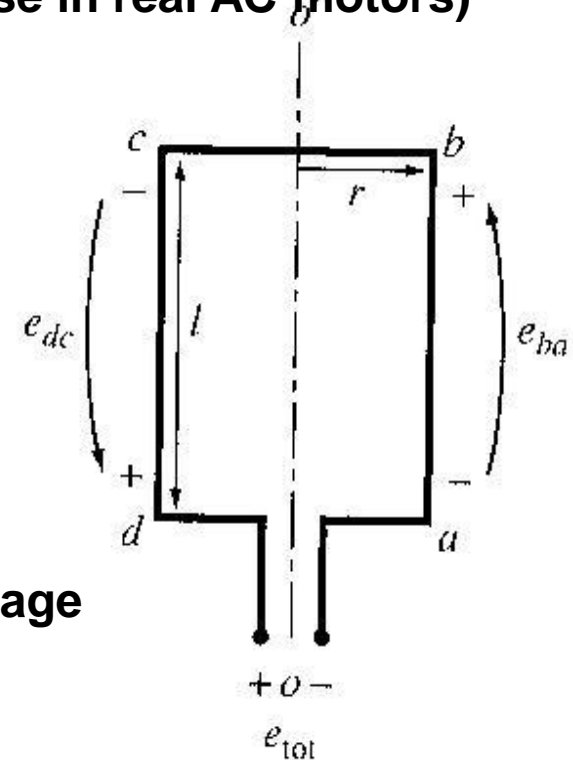
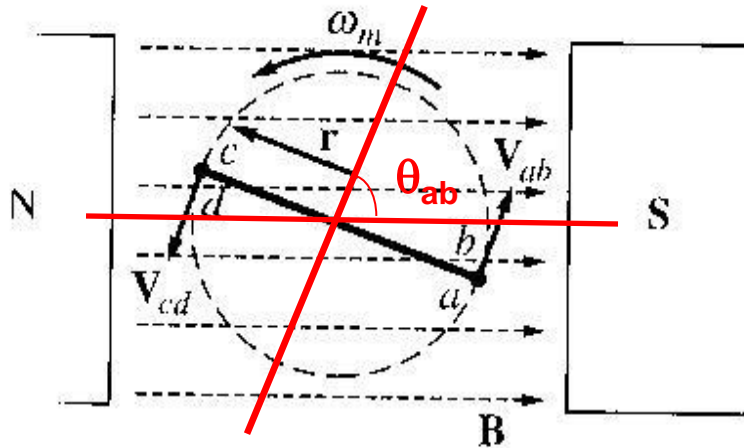
Rotor: Rotating part of the machine

Stator: Stationary part of the machine

Voltage of a loop in a uniform magnetic Field

Induced Voltage:

Moving loop in a uniform magnetic field (not the case in real AC motors)



Moving conductor in a magnetic field : induced voltage

$$e_{\text{ind}} = (\mathbf{V} \times \mathbf{B}) \cdot \mathbf{l}$$

$$e_{\text{ind,loop}} = e_{ba} + e_{cb} + e_{dc} + e_{ad}$$

$\mathbf{V} \times \mathbf{B}$: is in the direction of $ba (/dc)$ for $I_{ba} (/I_{dc})$: $e_{ba} = e_{dc} = v B l \sin \theta_{ab}$

$\mathbf{V} \times \mathbf{B}$: is perpendicular to $cb (/ad)$ for $I_{cb} (/I_{ad})$: $e_{cb} = e_{ad} = 0$

Voltage of a loop in a uniform magnetic Field

Induced Voltage:

$$e_{\text{ind,loop}} = e_{ba} + e_{cb} + e_{dc} + e_{ad} = 2v B l \sin\theta_{ab}$$

$$v = r\omega, \theta_{ab} = \omega t \text{ (rotating loop)} \rightarrow e_{\text{ind,loop}} = 2 r \omega B l \sin \omega t$$

$$\text{Loop Area: } A = 2 r l \rightarrow e_{\text{ind,loop}} = A B \omega \sin \omega t$$

$$\phi_{\text{max}} = A B \rightarrow e_{\text{ind,loop}} = \phi_{\text{max}} \omega \sin \omega t$$

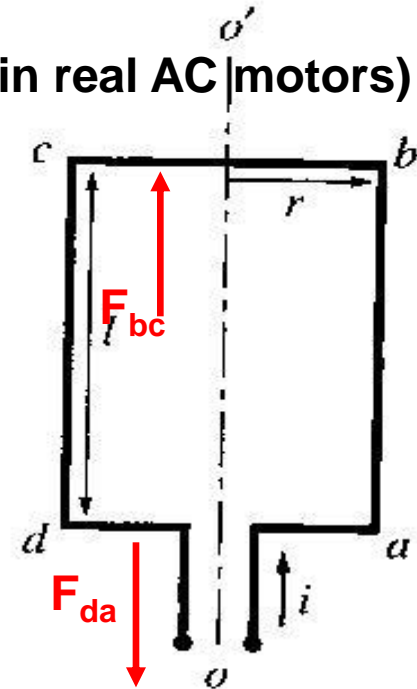
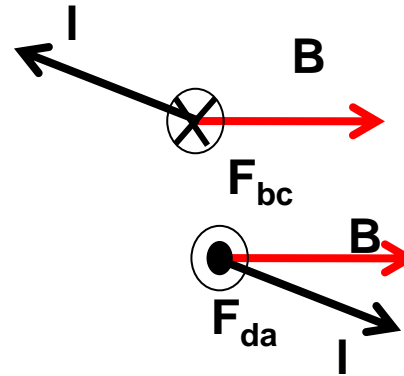
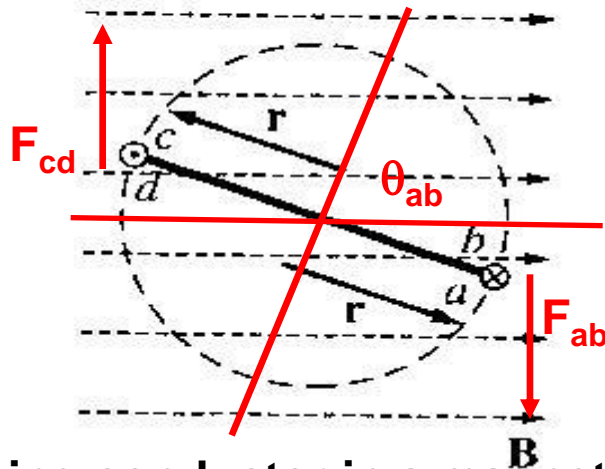
Sinusoidal Induced voltage with a magnitude that depends on machine flux, and speed of rotation (similar to real AC machines)

Think about: what happens if the loop (coil) is constant and flux rotates!!
what happens if we had several coils located in different
positions while flux rotates!!!

Torque of a loop in a uniform magnetic Field

Induced torque in a current-carrying loop:

current-carrying loop in a uniform magnetic field (not the case in real AC motors)



current-carrying conductor in a magnetic field : Force (/torque)

$$\mathbf{F} = i(\mathbf{l} \times \mathbf{B})$$

$\mathbf{l} \times \mathbf{B}$: is perpendicular to the surface including conductor ba(/dc) and \mathbf{B} :

$$\mathbf{F}_{ab} = -\mathbf{F}_{cd} = i l_{ab} \mathbf{B} \text{ (can produce torque)}$$

$\mathbf{l} \times \mathbf{B}$: is perpendicular to the surface including conduct cb(/da) and \mathbf{B} :

$$\mathbf{F}_{bc} = -\mathbf{F}_{da} = i l_{bc} \mathbf{B} \text{ (no torque)}$$

Torque of a loop in a uniform magnetic Field

Induced torque :

$$F_{ab} = -F_{cd} = i l_{ab} B \quad \Rightarrow \quad \Gamma_{ab} = \Gamma_{cd} = r i l B \sin\theta_{ab}$$

$$\Gamma_{ind,loop} = \Gamma_{ab} + \cancel{\Gamma_{bc}} + \Gamma_{cd} + \cancel{\Gamma_{da}} = 2 r i l B \sin\theta_{ab}$$

Alternative way to present torque:

$B_{loop} = \mu i / G$, G (in general a function of the geometry of the loop)

Loop Area: $A = 2 r l$, replacing for i : $\Rightarrow \Gamma_{ind,loop} = AG/\mu B_{loop} B_s \sin\theta_{ab}$

B_s is stator flux density (B)



$$\Gamma_{ind,loop} = k B_{loop} B_s \sin\theta_{ab}$$

- Induced torque depends on rotor magnetic field, stator magnetic field and the angle between them, plus a constant coming from the machine geometry.
- Induced torque intends to align the magnetic fields

What is the conditions to have a constant torque while rotor is rotating !!

Rotating magnetic field

While rotor is rotating to have a constant torque stator magnetic field should rotate!!

Solution1: Three phase winding conducting three phase current

$$I_{aa'} = I_m \cos \omega t$$

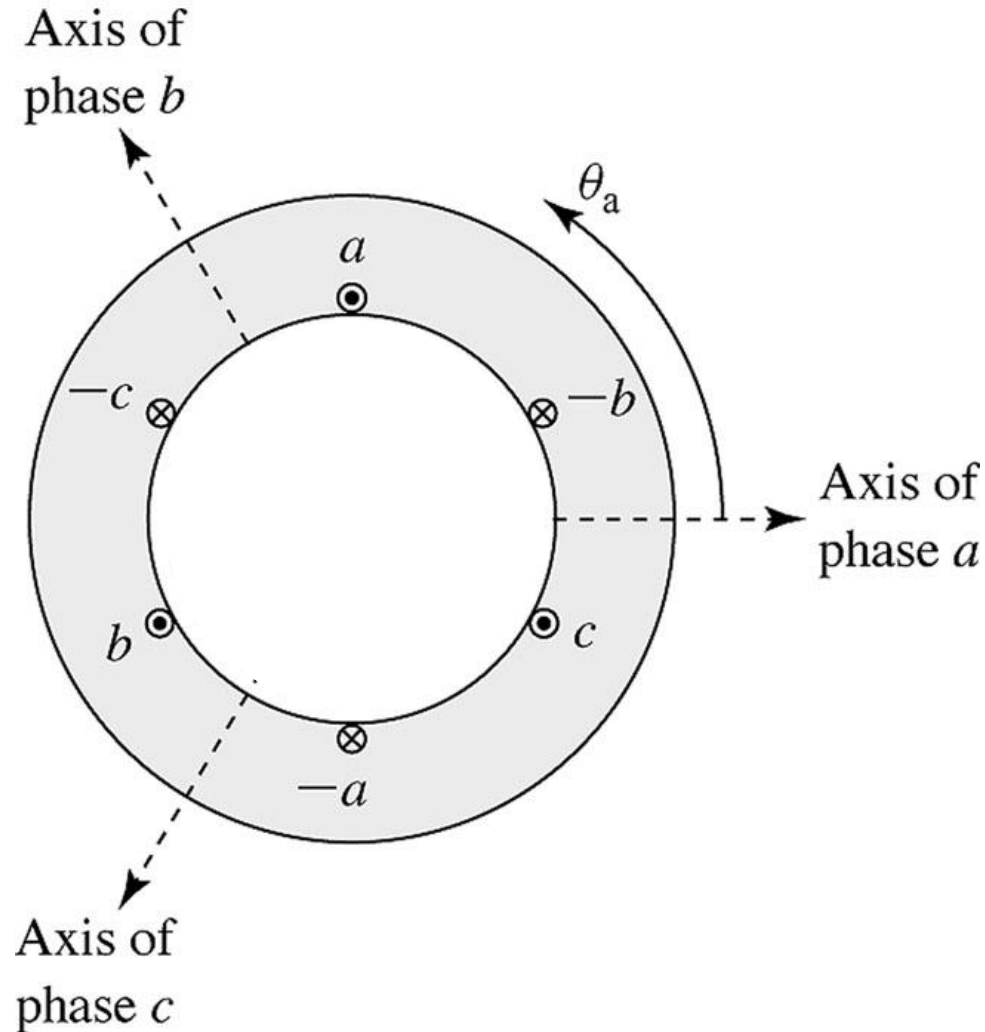
$$I_{bb'} = I_m \cos (\omega t - 120^\circ)$$

$$I_{cc'} = I_m \cos (\omega t + 120^\circ)$$

$$H_{aa'} = H_m \cos \omega t \angle 0^\circ$$

$$H_{bb'} = H_m \cos (\omega t - 120^\circ) \angle 120^\circ$$

$$H_{cc'} = H_m \cos (\omega t + 120^\circ) \angle -120^\circ$$



Rotating magnetic field

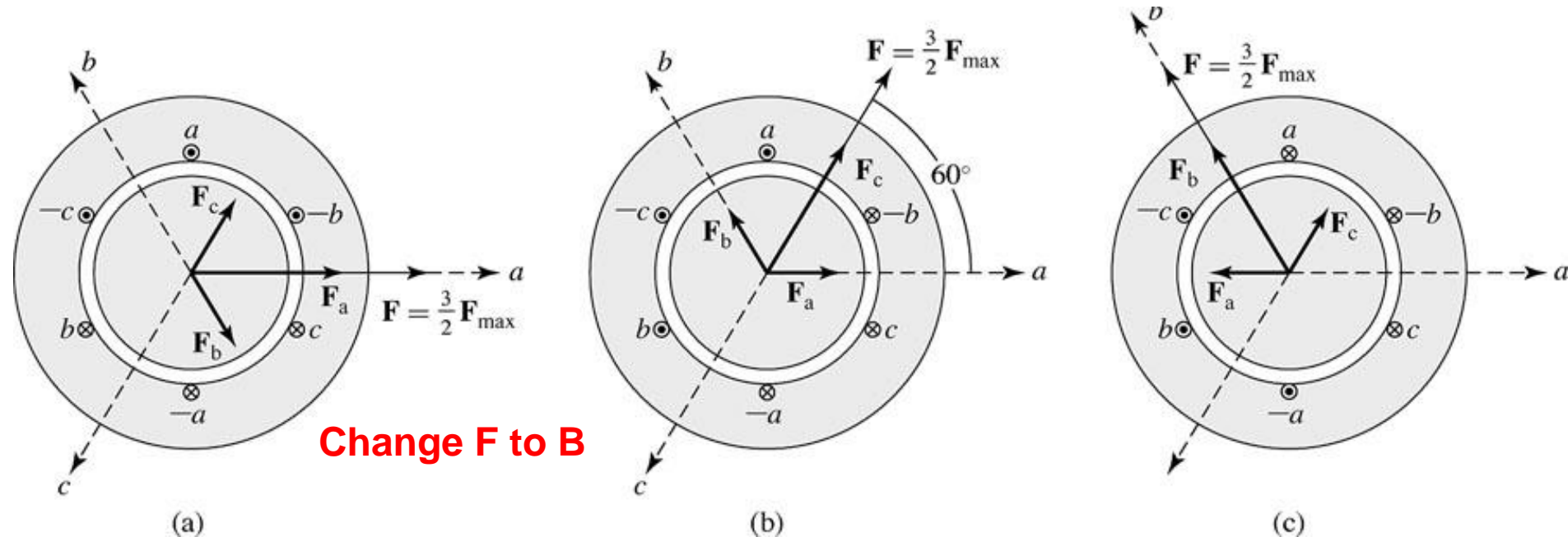
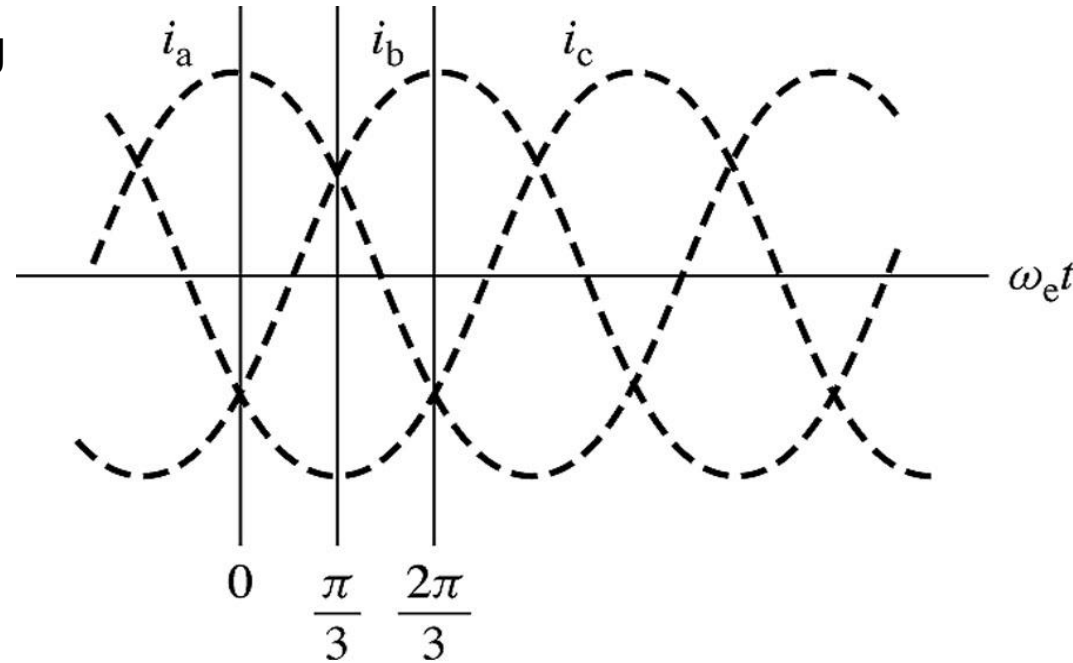
Solution1: Three phase winding

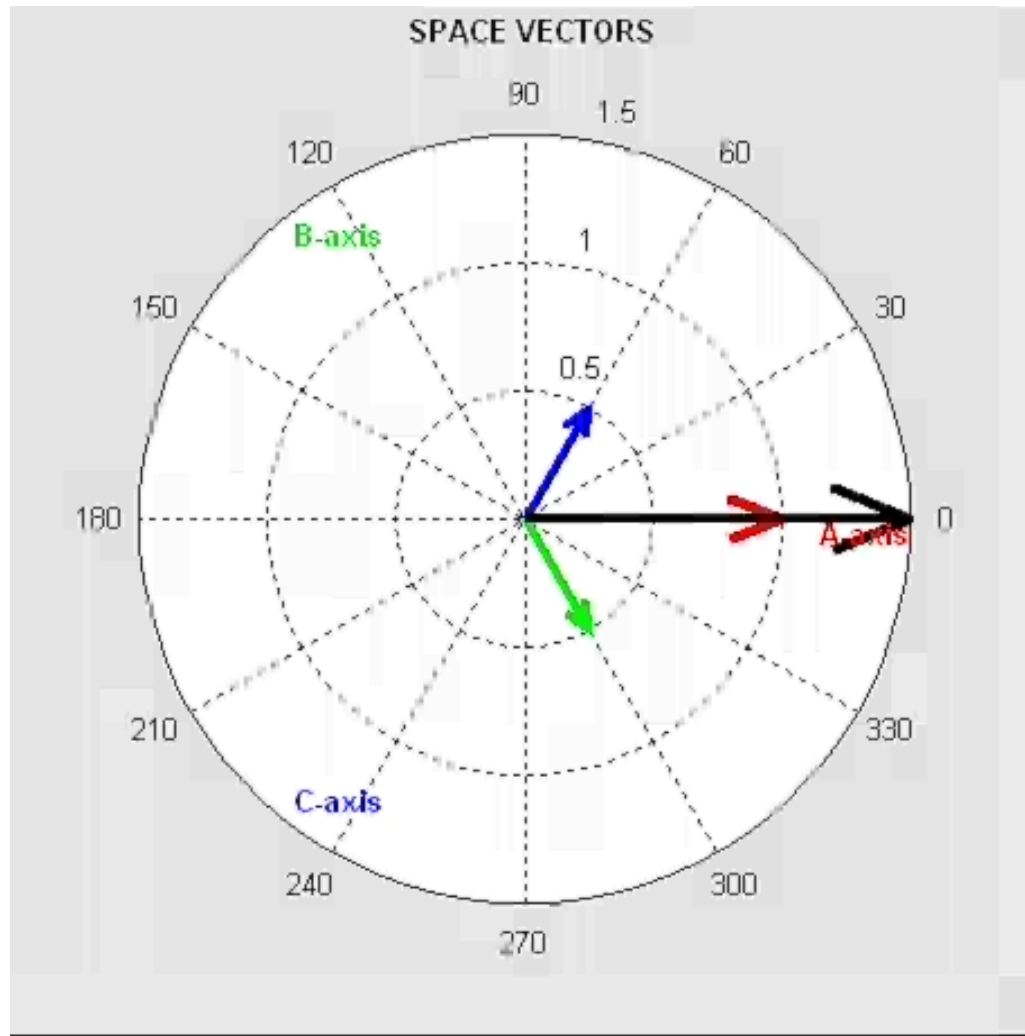
$$B_{aa'} = B_m \cos \omega t \angle 0^\circ$$

$$B_{bb'} = B_m \cos (\omega t - 120^\circ) \angle 120^\circ$$

$$B_{cc'} = B_m \cos (\omega t + 120^\circ) \angle -120^\circ$$

$$B_{s,\text{net}} = B_{aa'} + B_{bb'} + B_{cc'}$$





Thanks to Dr Mahmoud Riaz from University of Minnesota

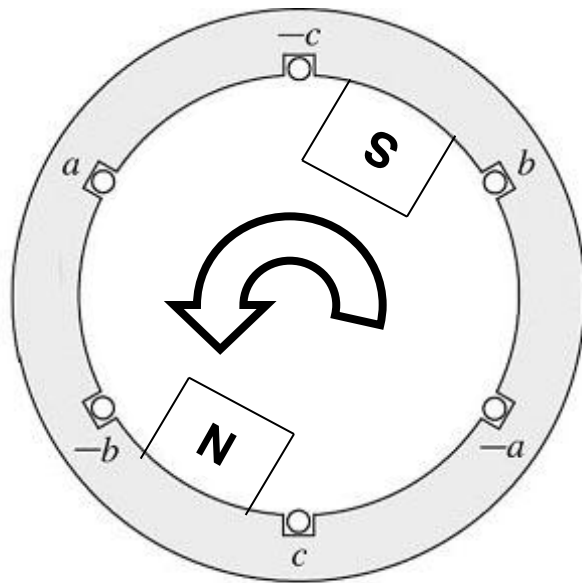
Effect of pole-pairs in magnetic field rotation speed

In 2 poles machines magnetic field rotates once every period of current:

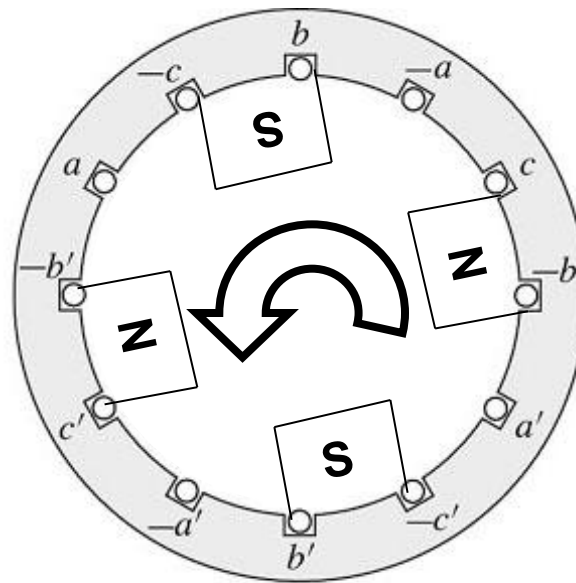
$$\omega_e = \omega_m \quad (\omega_e \text{ frequency of current, } \omega_m \text{ frequency of rotation of magnetic field})$$

In 4 poles machines magnetic field rotates once every two period of current:

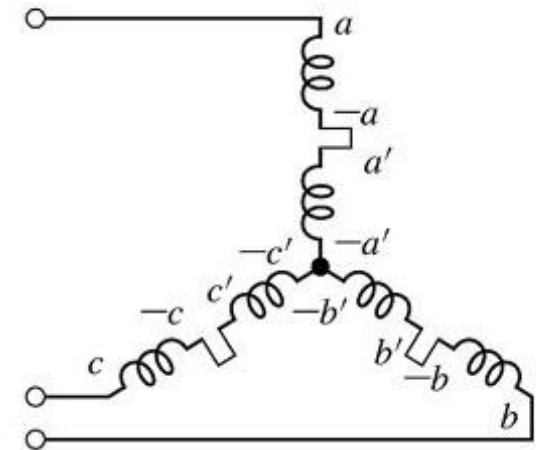
$$\omega_e = 2\omega_m$$



2 poles



4 poles



Winding connection

In general: $\omega_e = \frac{p}{2} \omega_m$

$$\theta_e = \frac{p}{2} \theta_m \quad n_m = \frac{120 \times f_e}{P}$$

P: Number of poles