

Due: Monday 18 Esfand

Q1. Find the Laplace transform and region of convergence for the following functions.

a) $x(t) = e^{-at}u(t) + 5 \cos(4t)e^{4t}u(t)$

b) $x(t) = t \frac{d}{dt}(\sin(t) u(t))$

c)

d) $x(t) = |t|e^{-5|t|}$

e) $\begin{cases} t & 0 \leq t < 1 \\ 2 - t & 1 \leq t < 2 \end{cases}$

Q2. Find all the possible $x(t)$ s assuming different ROCs

a) $X(s) = \ln\left(\frac{s^2+1}{s(s+1)}\right)$

b) $X(s) = \frac{d}{ds}\left(\frac{s^3+s}{(s(s-1))^2}\right)$

Q3. We know $h(t)$ is the impulse response of a real LTI system with exactly two poles and no finite zeros. One of the poles is located at $s=-1+2j$. Moreover the signal $e^{2t}x(t)$ is not absolutely integrable. And we know that $H(0)=4$. Find the transfer function $H(s)$ and ROC of the system.

Q4. An LTI system is described by the differential equation

$$2 \frac{d^2y}{dt^2} + \frac{dy}{dt} - 10y = \frac{dx}{dt}$$

Find $H(s)$ then find $h(t)$ that satisfies the following conditions

- a) System is stable
- b) System is causal
- c) System is neither causal nor stable

Q5. Using the properties of the Laplace transform, find the Laplace transform and ROC's of the following function:

$$x(t) = \frac{tdx}{dt} a(t) + \int_{-\infty}^t b(\tau - 3) d\tau \text{ where: } a(t) = e^t u(-t) \quad b(t) = \sin(t) u(t)$$

Q6. An LTI system has a transfer function in the form of $H(s) = k \frac{s-a}{s+b}$ where $a \geq 0$ and $b > 0$. If we know that the response of the system to $x(t) = \cos(t)$ is equal to $y(t) = \sin(t)$, what values can a and b take?