Energy conversion I

Lecture 17:

Topic 5: Induction Motors (S. Chapman ch. 7)

- Induction Motor Construction
- Basic Induction Motor Concepts
- **The Equivalent Circuit of an Induction Motor.**
- **Power and Torque in Induction Motor.**
- Induction Motor Torque-Speed Characteristics
- Starting Induction Motors
- Speed Control of Induction Motor
- Determining Circuit Model Parameters

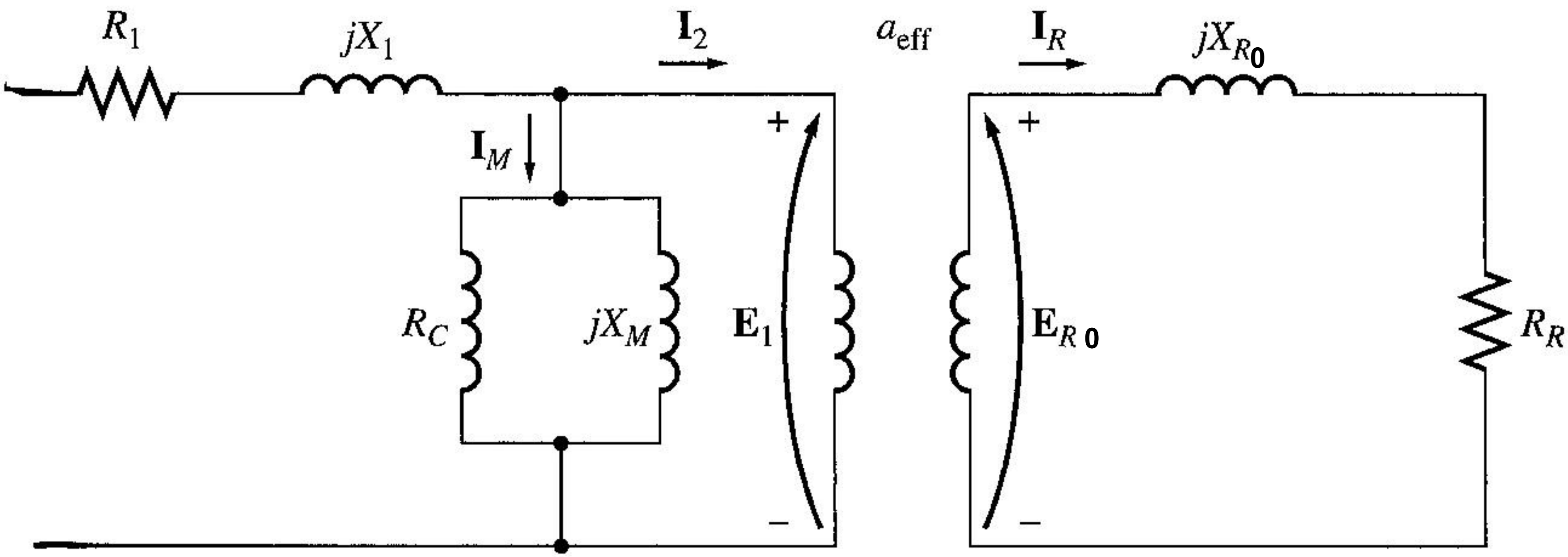
Transformer Model of blocked Rotor Induction motor:

- **Rotating magnetic flux induces voltage in both stator and rotor windings**
- **Stator and rotor winding have flux leakage modeled by leakage inductances**
- **Stator and rotor winding have resistance modeled by winding resistances**
- **Iron core loss (both in stator and rotor)**
- **Magnetizing inductance having magnetizing current**
- **Magnetizing current in magnetizing inductance is the origin of air-gap flux**
- **Iron loss depends on the magnetic operating point and can be modeled by a resistor in parallel with magnetizing inductance**

As can be seen is very similar to a transformer!!

Transformer Model of blocked Rotor Induction motor:

And therefore the per phase equivalent circuit is :



$$\frac{E_1}{E_{R0}} = \frac{N_{se}}{N_{re}} = a_{eff}$$

$$I_R = \frac{E_{R0}}{R_R + jX_{R0}}$$

X_1 and X_R leakage reactances

Effect of Rotor Rotation:

When rotor is moving with n_m rpm :

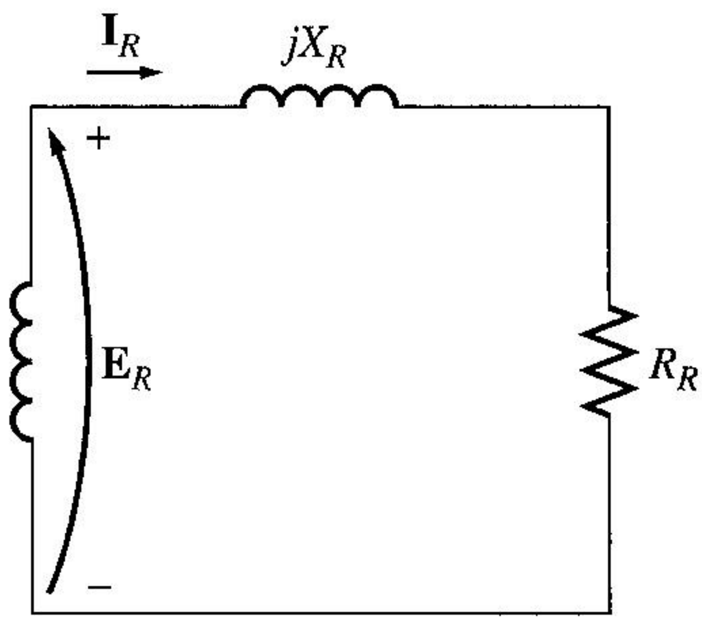
Frequency of induced voltage : $f_r = sf_s$

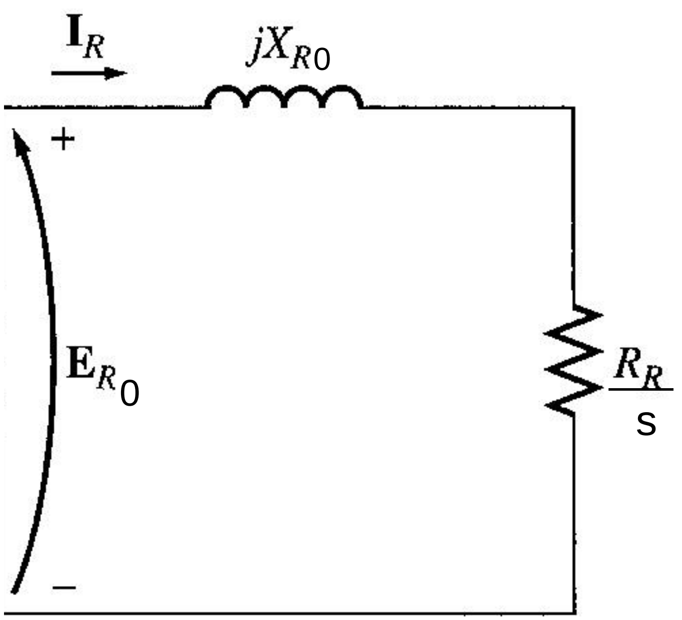
Amplitude of induced voltage : $E_R = sE_{R0}$

Leakage reactance: $X_R = s\,X_{R0}$

$$I_R = \frac{E_R}{R_R + jX_R} = \frac{sE_{R0}}{R_R + jsX_{R0}} = \frac{E_{R0}}{\frac{R_R}{s} + jX_{R0}}$$

Attention: Both torque and Rotor field depend on the amplitude of I_R

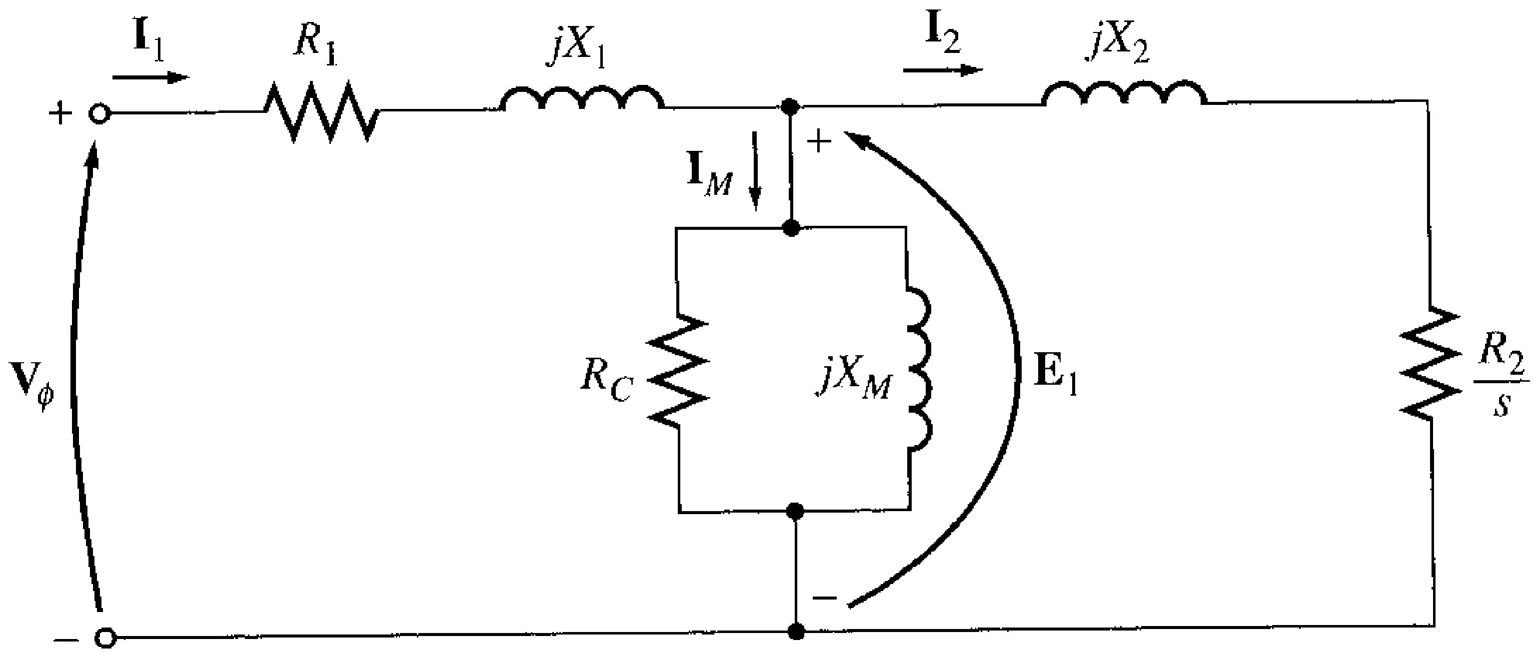




The Equivalent Circuit of an Induction Motor

Rotating rotor equivalent circuit is in the secondary of an ideal transformer with a turn ration of: $a_{eff} = N_{se} / N_{re}$

Rotor Equivalent circuit can be seen from the primary (stator):

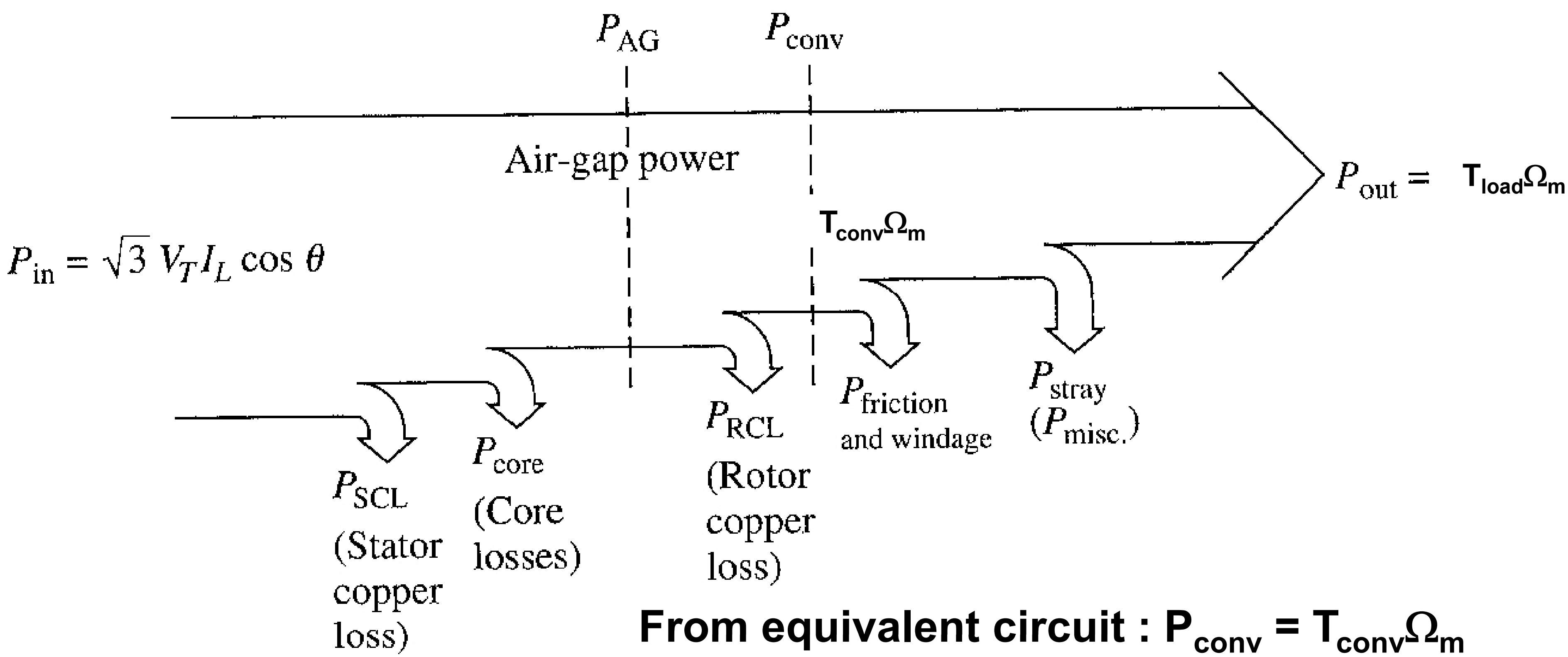


$$R_2 = a_{eff}^2 R_R \qquad X_2 = a_{eff}^2 X_R \qquad I_2 = \frac{I_R}{a_{eff}} \qquad E_1 = E_R^{'} = a_{eff} E_{R0}$$

Power Flow of an Induction Motor

Input power (Electrical) : $P_{in} = \sqrt{3} V_T I_L \cos\theta$

Output power (Mechanical) : $P_{out} = T_{load}\Omega_m$



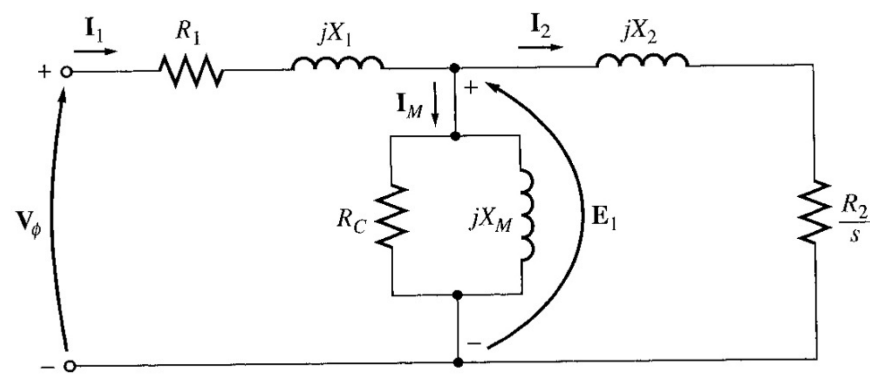
From equivalent circuit : $P_{conv} = T_{conv}\Omega_m$

Core Losses are usually lumped with mechanical losses

Power of an Induction Motor

Using per-phase equivalent circuit :

$$I_1 = \frac{V_\varphi}{Z_{eq}} \qquad Z_{eq} = R_1 + jX_1 + (R_c \text{ II } jX_M \text{ II } (R_2/s + jX_2)$$



Stator Copper Loss : $P_{SCL} = 3R_1 I_1^2$

Air-gap power : $P_{AG} = 3\frac{R_2}{s} I_2^2$

Rotor Copper Loss : $P_{RCL} = 3R_2 I_2^2$

Developed Mechanical Power : $P_{conv} = P_{AG} - P_{RCL} = 3R_2 I_2^2 (\frac{1}{s} - 1)$

Other equations : $P_{RCL} = sP_{AG}$ $\qquad P_{conv} = (1-s)P_{AG}$

What is start up converted power?

What happens to the Air-gap power?

Torque of an Induction Motor

Output power : $P_{out} = P_{conv} - P_{F\&W} - P_{misc}$

Output Torque : $T_{load} = \dfrac{P_{out}}{\Omega_m}$

Using Equivalent circuit $\mathbf{P_{conv}}$ and consequently $\mathbf{T_{ind}}$ can be calculated:

$$T_{ind} = \frac{P_{conv}}{\Omega_m}$$

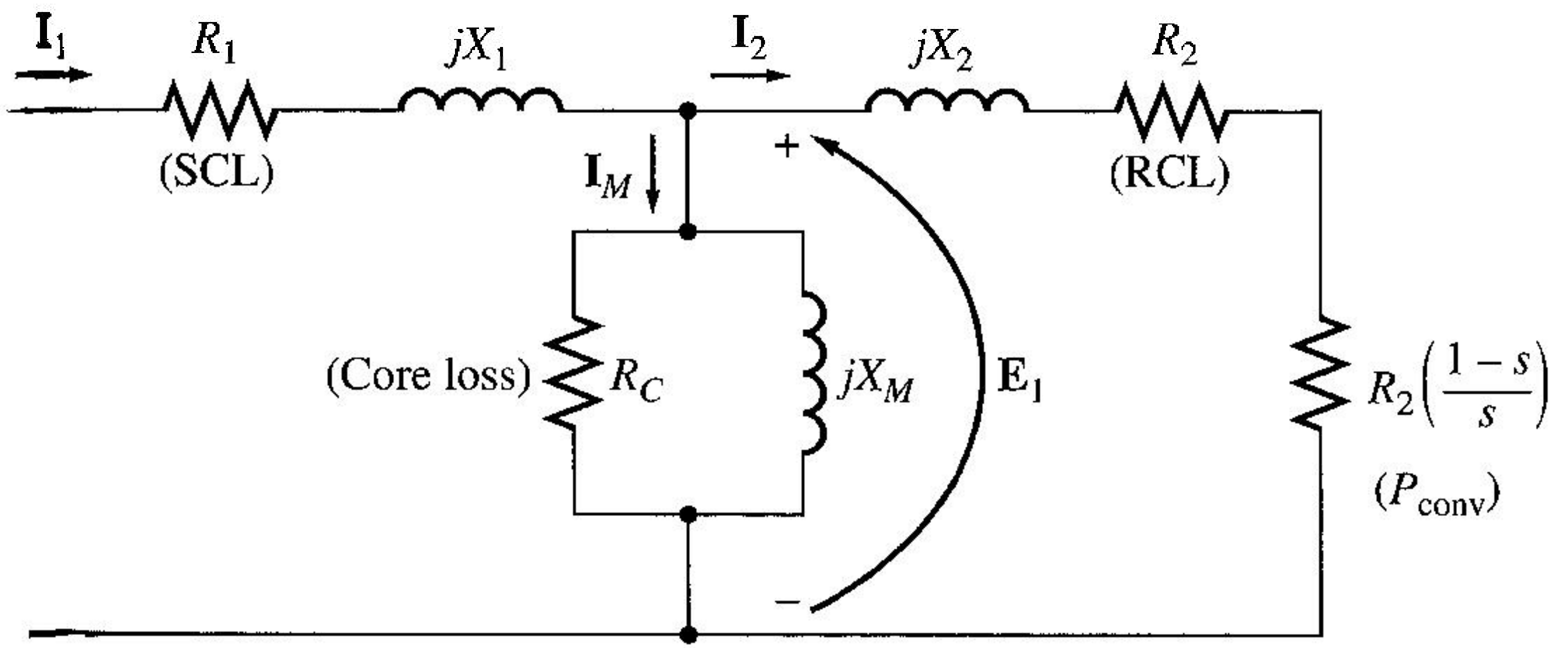
Since : $P_{conv} = (1-s)P_{AG}$, $\Omega_m = (1-s)\Omega_s$

Then : $T_{ind} = \boxed{\dfrac{P_{conv}}{\Omega_m}} = \dfrac{(1-s)P_{AG}}{(1-s)\Omega_s} = \boxed{\dfrac{P_{AG}}{\Omega_s}}$

$\Omega_s = \dfrac{\omega_s}{\frac{p}{2}}$ **Mechanical** Synchronous speed

Rotor Modified Equivalent Circuit of an Induction Motor

Separating rotor cupper loss and converted power we can have the following equivalent circuit :



Example:

A 460V, 25hp, 60Hz, 4 pole, Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

$R_1 = 0.641\ \Omega$ $R_2 = 0.332\ \Omega$

$X_1 = 1.106\ \Omega$ $X_2 = 0.464\ \Omega$ $X_m = 26.3\ \Omega$

The total rotational losses are 1100W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2% at the rated voltage and rated frequency, find the motor's

A- Speed

B- Stator current

C- Power factor

D- P_{conv} and P_{out}

E- τ_{ind} and τ_{load}

F- Efficiency

Solution:

A- Speed

The synchronous speed : $n_{sync} = \dfrac{120 f_e}{p} = \dfrac{120 \times 60}{4} = 1800 \text{ rpm}$

Therefore : $n = (1 - s) n_{sync} = (1 - 0.022) \times 1800 = 1760 \text{ rpm}$

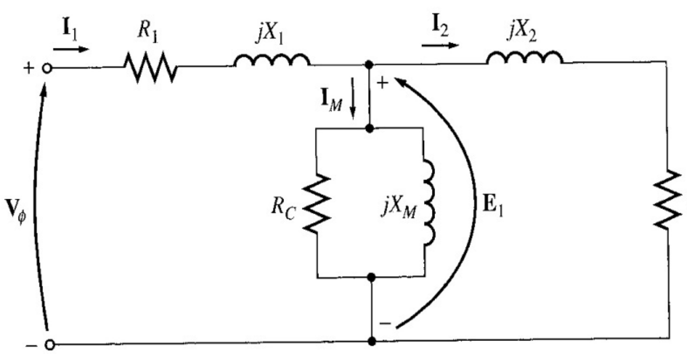
B- Stator current

To find the stator current, input impedance should be calculated :

$$Z_{in} = R_1 + jX_1 + (jX_M \text{ II } (R_2/s + jX_2))$$

$$= 0.641 + j1.106 + \frac{j26.3(\frac{0.332}{0.022} + j0.464)}{\frac{0.332}{0.022} + j(26.3 + 0.464)}$$

$$= 11.72 + j7.79 = 14.07\angle 33.6^o$$



Therefore the current is : $I_1 = \dfrac{V_\varphi}{Z_{in}} = \dfrac{460/\sqrt{3}}{14.07\angle 33.6^o} = 18.88\angle -33.6^o$

C- Power factor : $PF = \cos 33.6^{o} = 0.833$ **lagging**

D- P_{conv} and P_{out} :

$$P_{conv} = P_{AG}(1-s)$$

$$P_{AG} = P_{in} - P_{SCL} = \sqrt{3} \times 460 \times 18.88 \times \cos 33.6^{o} - 3 \times (18.88)^{2} \times 0.641 = 11845 \text{ W}$$

$$P_{conv} = 11845 \times (1-0.022) = 11585 \text{ W}$$

$$P_{out} = P_{conv} - P_{rot} = 11585 - 1100 = 10485 \text{ W} = \frac{10485}{746} = 14.1 \text{ hp}$$

E- τ_{ind} and τ_{load} :

$$T_{ind} = \frac{P_{AG}}{\Omega_s} = \frac{11845}{1800 \times \frac{2\pi}{60}} = 62.8 \text{ N.m} \qquad T_{load} = \frac{P_{out}}{\Omega_m} = \frac{10485}{1760 \times \frac{2\pi}{60}} = 56.9 \text{ N.m}$$

F- Efficiency :

$$P_{AG} = \frac{Pout}{P_{in}} \times 100\% = \frac{10485}{\sqrt{3} \times 460 \times 18.88 \times \cos 33.6} = \frac{10485}{12530} \times 100\% = 83.7\%$$