Energy conversion I

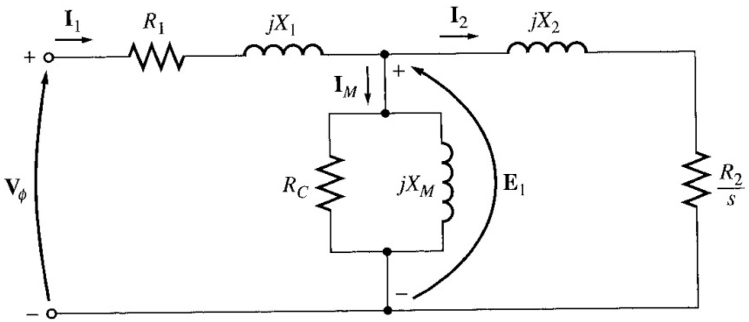
Lecture 18:

Topic 5: Induction Motors (S. Chapman ch. 7)

- Induction Motor Construction
- Basic Induction Motor Concepts
- The Equivalent Circuit of an Induction Motor.
- Power and Torque in Induction Motor.
- **Induction Motor Torque-Speed Characteristics**
- Starting Induction Motors
- Speed Control of Induction Motor
- Determining Circuit Model Parameters

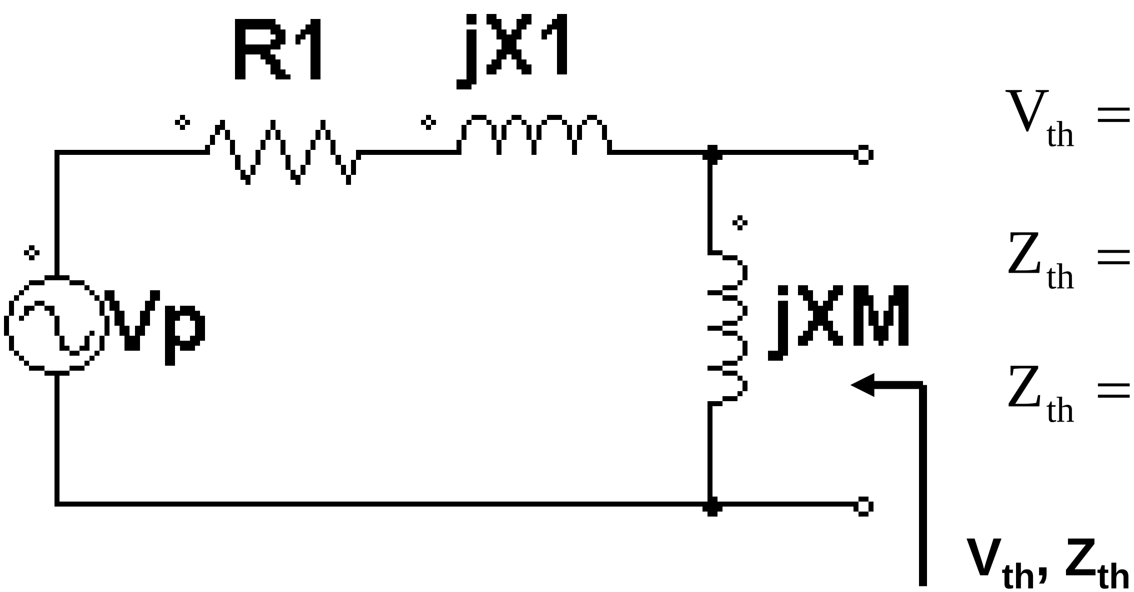
Thevenin Equivalent Circuit

Mechanical Torque: $$T_{ind} = \frac{P_{conv}}{\Omega_m} = \frac{P_{AG}}{\Omega_s}$$



Air-gap Power : $$P_{AG} = 3\frac{R_2}{s} I_2^2$$

I_2 can be calculated directly from Stator Thevenin equivalent Circuit:



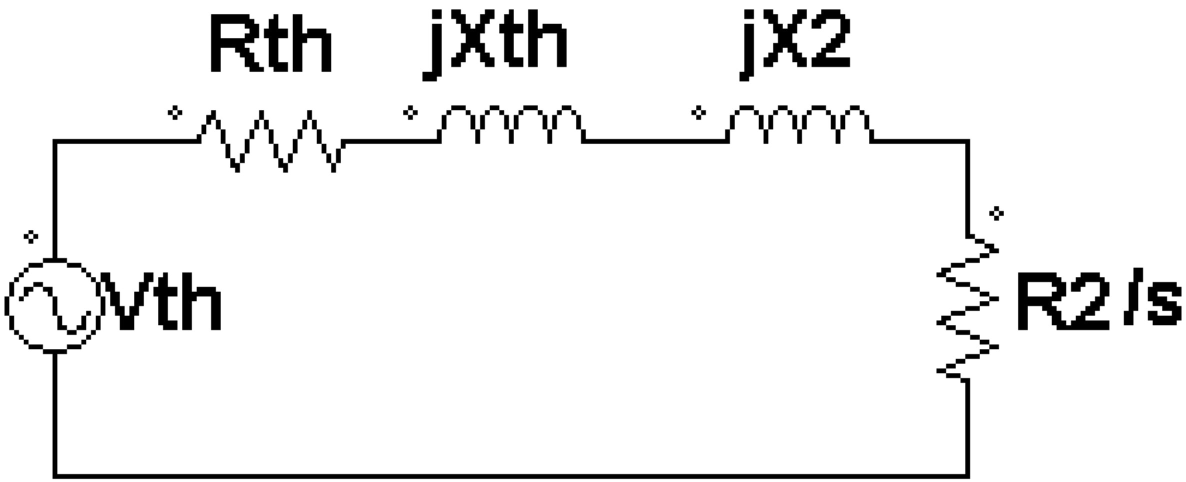
$$V_{th} = V_p \frac{jX_M}{R_1 + j(X_1 + X_M)}$$

$$Z_{th} = R_{th} + jX_{th} = jX_M \;\mathrm{II}\; (R_1 + jX_1)$$

$$Z_{th} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)}$$

Think about approximate values for V_{th}, R_{th}, X_{th}

Torque calculation using Thevenin equivalent circuit



$$\frac{V_{th}}{R_{th} + \frac{R_2}{s} + j(X_2 + X_{th})}$$

Air-gap Power : $P_{AG} = 3\frac{R_2}{s}\frac{V_{th}^2}{(R_{th} + \frac{R_2}{s})^2 + (X_2 + X_{th})^2}$

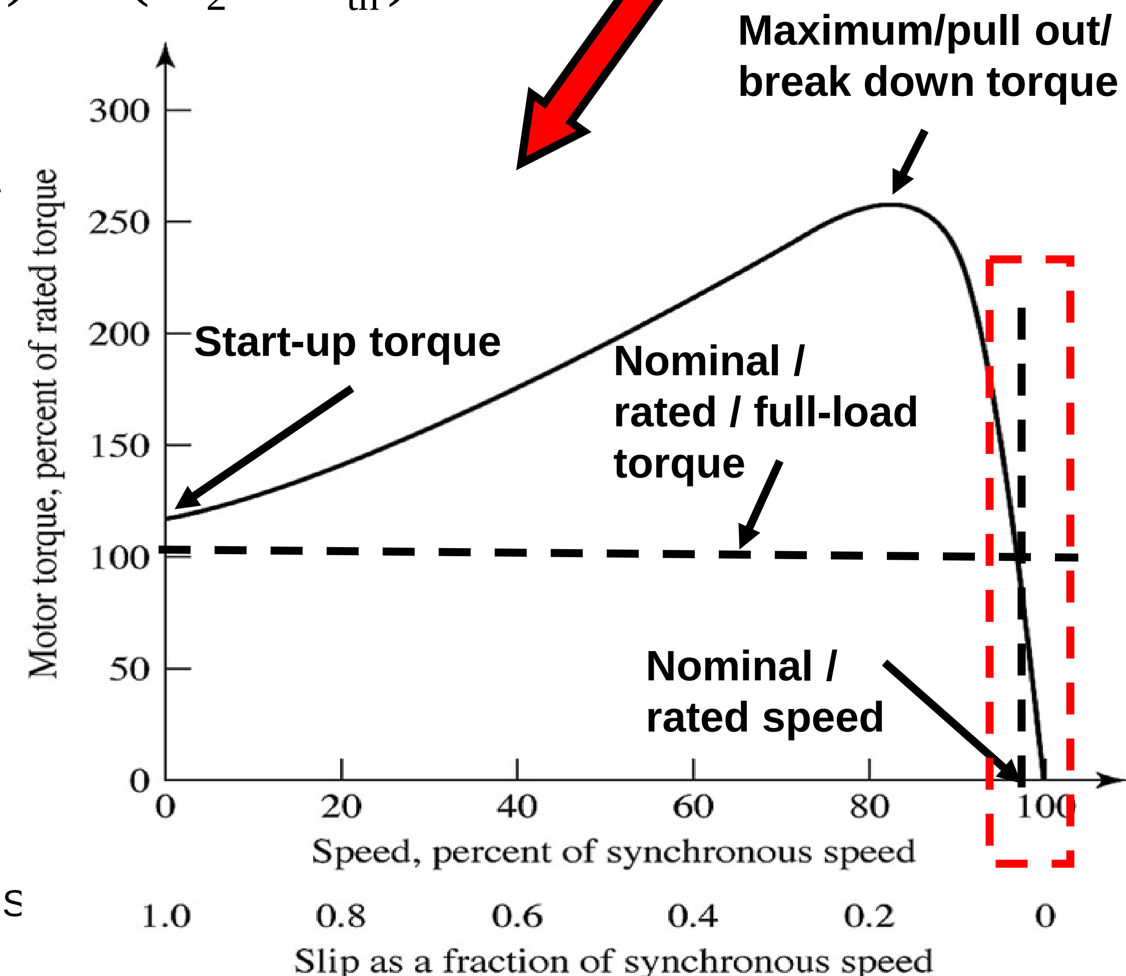
Mechanical Torque: $T_{ind} = \frac{3}{2}\frac{p}{\omega_s}\frac{R_2}{s}\frac{V_{th}^2}{(R_{th} + \frac{R_2}{s})^2 + (X_2 + X_{th})^2}$

Torque Speed Characteristic

Torque-slip for a given voltage and frequency :

$$T_{ind} = \frac{3}{2}\frac{p}{\omega_s}\frac{R_2}{\boxed{s}}\frac{V_{th}^2}{(R_{th} + \frac{R_2}{\boxed{s}})^2 + (X_2 + X_{th})^2}$$

$$s = \frac{n_s - n}{n_s}$$



For very small s(0-more than rated)

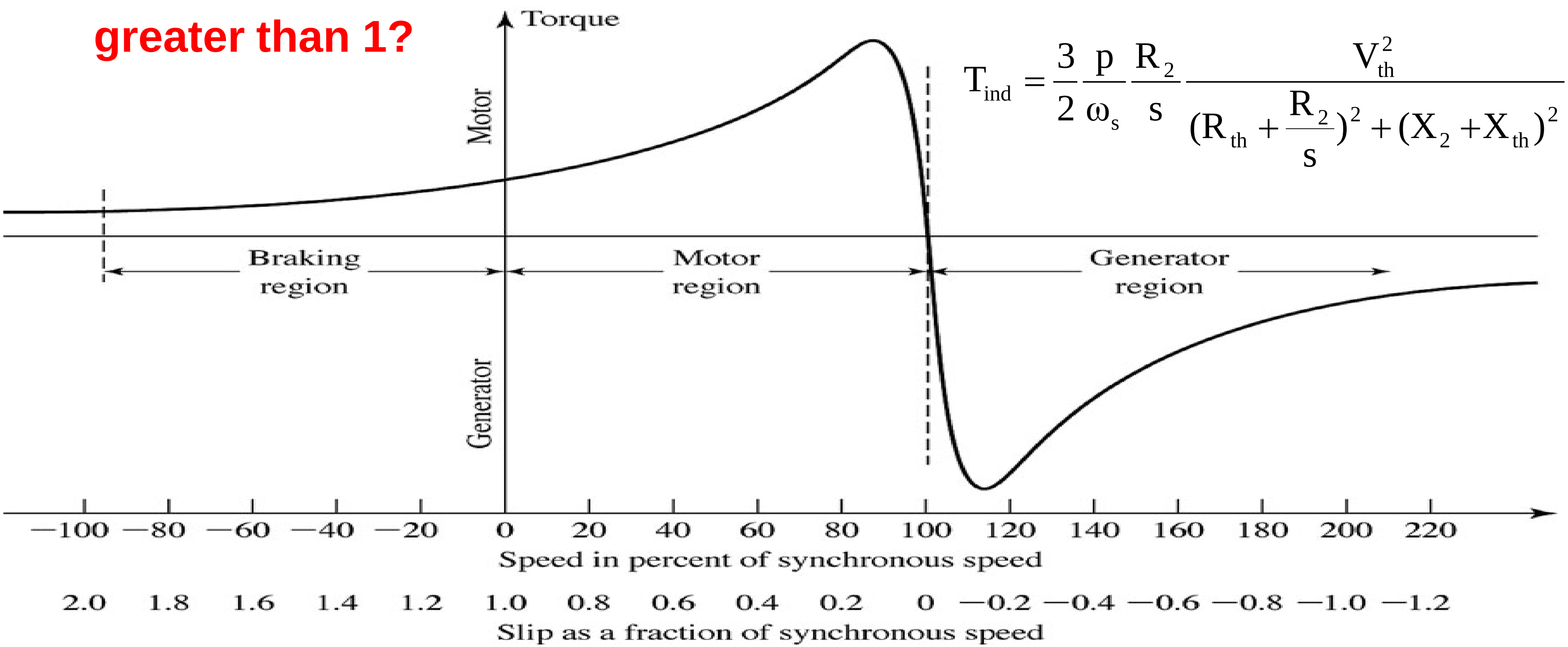
$$T_{ind} \approx \frac{3}{2}\frac{p}{\omega_s}V_{th}^2\frac{\boxed{s}}{R_2}$$

Linear torque-slip (/speed)

Some comments on torque speed Characteristics

- Start-up torque (s=1) is usually greater than rated torque
- Rated speed and no-load speed are very close to synchronous speed
- Pull-out torque is usually more than twice rated torque
- For s<0 torque is negative (generator mode). what is the speed of rotation?
- For s>1 torque is positive (Braking/ rotor plugging). How can s be greater than 1?

$$T_{ind} = \frac{3}{2}\frac{p}{\omega_s}\frac{R_2}{s}\frac{V_{th}^2}{(R_{th}+\frac{R_2}{s})^2+(X_2+X_{th})^2}$$



Example:

A 2 pole, 50 Hz induction motor supplies 15kW to a load at a speed of 2950 r/min.

A. What is the motor's slip?

B. What is the induced torque in the motor under these conditions?

C. What will the operating speed of the motor be if its torque is doubled?

D. How much power will be supplied by the motor when the torque is doubled?

Solution:

A.

$$n_{sync} = \frac{120 f_e}{p} = \frac{120 \times 50}{2} = 3000 \text{ rpm}$$

$$s = \frac{n_{sync} - n}{n_{sync}} = \frac{3000 - 2950}{3000} = 0.0167$$

B. What is the induced torque in the motor in Nm under these conditions?

Assuming $P_{conv} = P_{load}$

$$T_{ind} = \frac{p_{conv}}{\omega} = \frac{15\times10^3}{2950/60\times 2\pi} = 48.6\ \text{N.m}$$

C. What will the operating speed of the motor be if its torque is doubled?

Assuming a linear torque-slip relation close to rated torque

$s_2 = 2\times s_1 = 0.033$ $\qquad$ $n_2 = (1-s_2)\times n_{sync} = (1-0.033)\times 3000 = 2900$ rpm

D. How much power will be supplied by the motor when the torque is doubled?

$P_{conv} = T_{ind}\ \omega_m = 2\times 48.6\times 2900/60\times 2\pi = 29.5$ kW **!!**

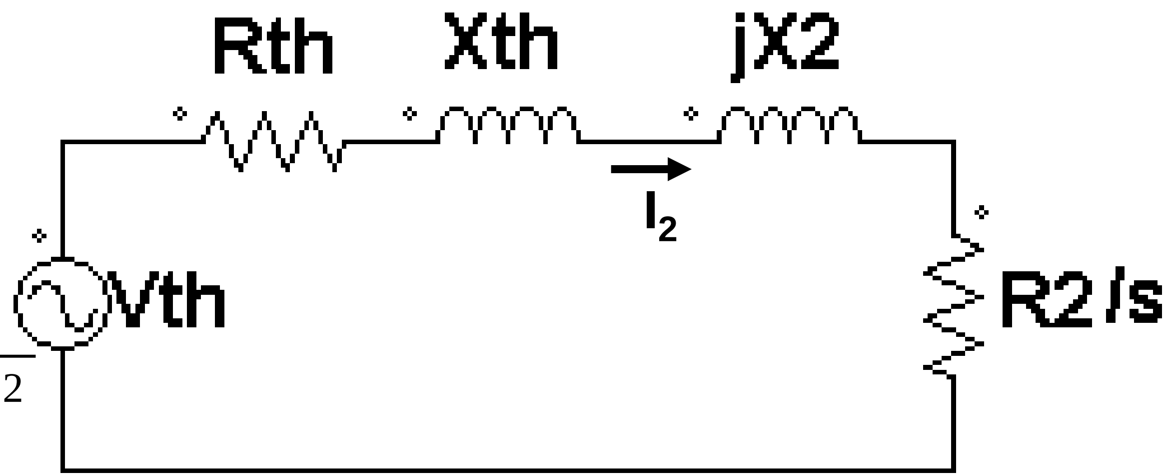
Pull-out Torque

Pull-out torque is the maximum torque:

$$\frac{\partial T_{ind}}{\partial s} = 0 \Rightarrow s_{Tmax} \Rightarrow T_{max}$$

Looking air-gap power: $T_{ind} = \frac{P_{AG}}{\Omega_s}$ **maximum torque happens if air-gap power is maximum**

Maximum power transfer theorem



$$\frac{R_2}{s_{Tmax}} = \sqrt{R_{th}^2 + (X_{th} + X_2)^2}$$

$$T_{max} = \frac{3}{4} \frac{p}{\omega_s} \frac{V_{th}^2}{Rth + \sqrt{R_{th}^2 + (X_{th} + X_2)^2}}$$

How R_2 affects S_{Tmax} and T_{max}?

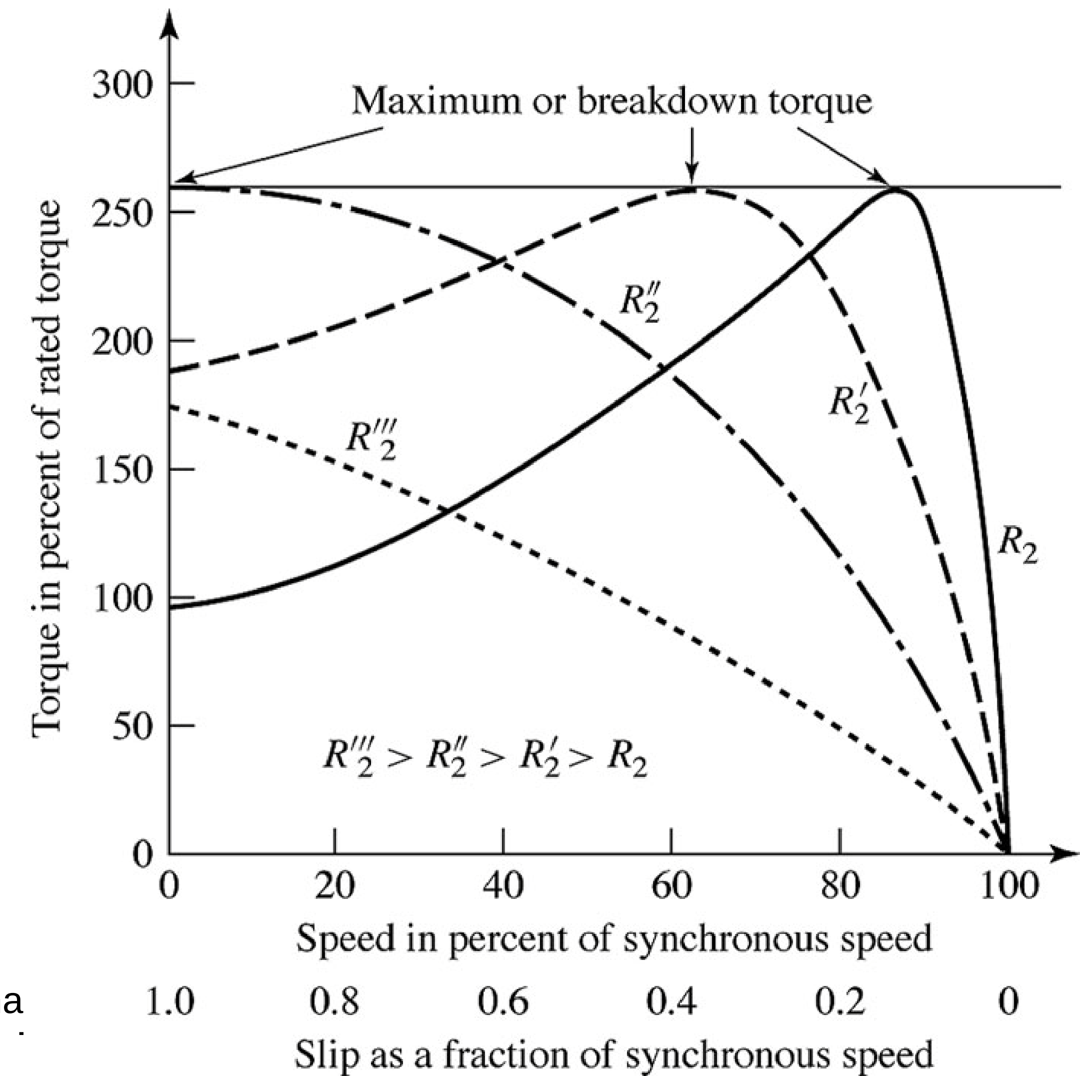
Rotor Resistance Effects

$$\frac{R_2}{s_{Tmax}} = \sqrt{R_{th}^2 + (X_{th} + X_2)^2} \quad T_{max} = \frac{3}{4}\frac{p}{\omega_s}\frac{V_{th}^2}{Rth + \sqrt{R_{th}^2 + (X_{th} + X_2)^2}}$$

Increasing R_2,

- **Increases S_{Tmax}**
- **Increases Z_{in}**
- **Decreases start-up current !**
- **No effect on T_{max}**

Can be implemented using external resistors in wound rotor motors



Example:

A 460V, 25hp, 60Hz, 4-pole, Y-connected wound rotor induction motor has the following impedances in ohms per-phase referred to the stator circuit:

$R_1 = 0.641\ \Omega$ **$R_2 = 0.332\ \Omega$**

$X_1 = 1.106\ \Omega$ **$X_2 = 0.464\ \Omega$** **$X_m = 26.3\ \Omega$**

A. **What is the maximum torque of this motor? At what speed and slip does it occur?**
B. **What is the starting torque?**
C. **When the rotor resistance is doubled, what is the speed at which the max torque now occurs? What is the new starting torque?**

$$V_{th} = V_p \frac{jX_M}{R_1 + j(X_1 + X_M)} = \frac{460}{\sqrt{3}} \frac{j26.3}{0.641 + j(26.3 + 1.106)} = 255.2\angle\alpha$$

$$Z_{th} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)} = \frac{j26.3(0.641 + j1.106)}{0.641 + j(26.3 + 1.106)} = 0.59 + j1.10$$

A. What is the max torque of this motor? At what speed and slip does it occur?

$$T_{max} = \frac{3}{4}\frac{p}{\omega_s}\frac{V_{th}^2}{Rth + \sqrt{R_{th}^2 + (X_{th} + X_2)^2}}$$

$$= \frac{3}{4}\frac{4}{120\pi}\frac{255.2^2}{0.59 + \sqrt{0.59^2 + (1.1 + 0.464)^2}} = 229 \text{ N.m}$$

$$s_{Tmax} = \frac{R_2}{\sqrt{R_{th}^2 + (X_{th} + X_2)^2}} = \frac{0.332}{\sqrt{0.59^2 + (1.1 + 0.464)^2}} = 0.198$$

$$n_{Tmax} = (1 - s_{Tmax})n_{sync} = (1 - 0.198) \times 1800 = 1444 \text{ rpm}$$

B. What is the starting torque? (s=1)

$$T_{ind} = \frac{3}{2}\frac{p}{\omega_s}\frac{R_2}{s}\frac{V_{th}^2}{(R_{th} + \frac{R_2}{s})^2 + (X_R + X_{th})^2} = 104 \text{ N.m}$$

C. When the rotor resistance is doubled, what is the speed at which the max torque now occurs? What is the new starting torque?

s_{Tmax} is proportional to R_2 $\qquad$ $s_{2,Tmax} = 0.396$

$N_{2,Tmax} = (1 - s_{2,Tmax}) \times 1800 = 1087$ rpm

For the starting torque:

$s = 1$, $R_2 = 2 \times 0.332 = 0.662\ \Omega$

$$T_{ind} = \frac{3}{2}\frac{p}{\omega_s}\frac{R_2}{s}\frac{V_{th}^2}{(R_{th} + \frac{R_2}{s})^2 + (X_R + X_{th})^2} = 170\ \text{N.m}$$

How can maximum torque happen in start-up?

Compare start up currents!