Energy conversion I

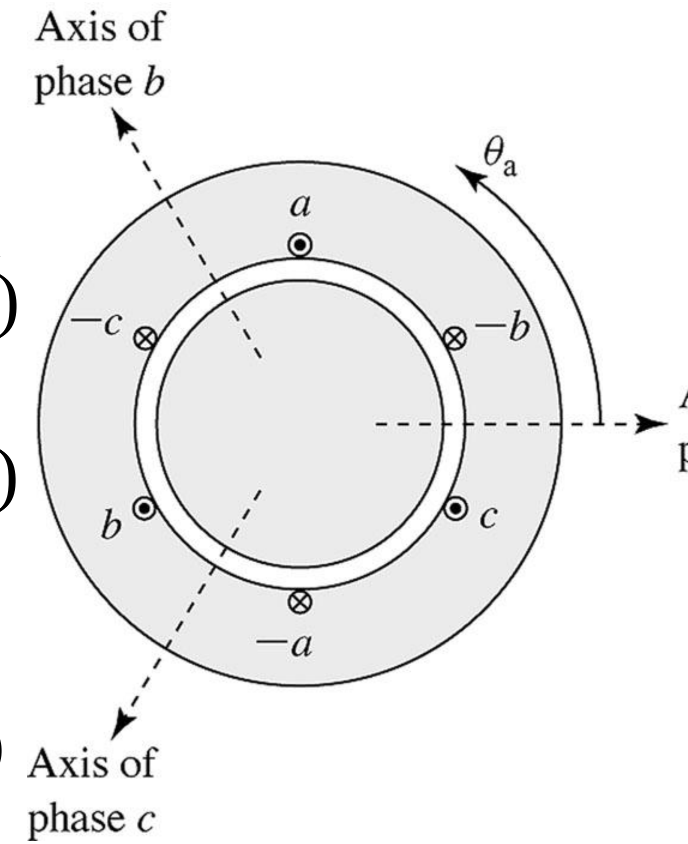
Lecture 12:

Topic 3: Fundamentals of AC machines steady state operation (S. Chapman, ch. 4)

- Introduction
- Voltage and torque of a loop in a uniform magnetic Field
- Rotating magnetic field
- Magnetomotive force and flux distribution in AC machines
- **Induced voltage in AC machines**
- **Induced torque in AC machines**

Induced voltage due to rotating flux(flux approach)

For Concentrated phase windings:



$$\varphi_a = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} B_M \cos(\omega t - \theta_a)\, l r\, d\theta_a = 2rlB_M \cos \omega t$$

$$\varphi_b = \int_{-\frac{\pi}{2}+\frac{2\pi}{3}}^{\frac{\pi}{2}+\frac{2\pi}{3}} B_M \cos(\omega t - \theta_a)\, l r\, d\theta_a = 2rlB_M \cos(\omega t - \frac{2\pi}{3})$$

$$\varphi_c = \int_{-\frac{\pi}{2}-\frac{2\pi}{3}}^{\frac{\pi}{2}-\frac{2\pi}{3}} B_M \cos(\omega t - \theta_a)\, l r\, d\theta_a = 2rlB_M \cos(\omega t + \frac{2\pi}{3})$$

$$\varphi_M = 2rlB_M$$

$e_{ind,loop}$ **= dφ/dt=2rω l $B_M$cos(ωt – θ_a) =φ_Mω cos(ωt – θ_a)**

$$\theta a = \begin{cases} \frac{\pi}{2} & \text{for winding a} \\ \frac{\pi}{2} + \frac{2\pi}{3} & \text{for winding b} \\ \frac{\pi}{2} - \frac{2\pi}{3} & \text{for winding c} \end{cases}$$

Induced voltage due to rotating flux in three-phase coils

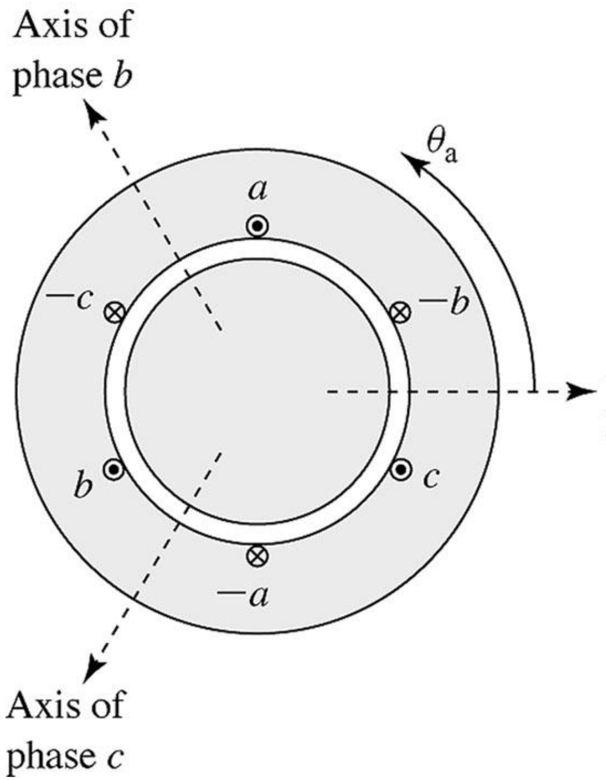
Three phase coils: three equivalent coils of N_c turns with 120º displacement

$e_{ind,loop} = 2r\omega\, l\, B_M\cos(\omega t - \theta_a) = \varphi_M\omega \cos(\omega t - \theta_a)$

$e_a = N_C\, \varphi_M\omega \sin \omega t$

$e_b = N_C\, \varphi_M\omega \sin (\omega t-120^o)$

$e_c = N_C\, \varphi_M\omega \sin (\omega t+120^o)$



A **sinusoidally** distributed **rotating magnetic field**

can generate **three phase voltage** in **three phase winding.**

Reminder: Three phase current in **three phase winding** generates **sinusoidally**

distributed **rotating magnetic field.**

What about the induced voltage in distributed windings?
What happens if a three phase winding is connected to a three phase voltage?
What happens if three phase winding rotates with ω_m?

Phase voltage of a three-phase machine

$e_a = N_{ce}\ \varphi_M \omega \sin \omega t = E_{max} \sin \omega t = \sqrt{2}\, E \sin \omega t$

$e_b = N_{ce}\ \varphi_M \omega \sin (\omega t\text{-}120^o) = E_{max} \sin (\omega t\text{-}120^o) = \sqrt{2}\, E \sin (\omega t\text{-}120^o)$

$e_c = N_{ce}\ \varphi_M \omega \sin (\omega t+120^o) = E_{max} \sin (\omega t+120^o) = \sqrt{2}\, E \sin (\omega t+120^o)$

N_e: Effective turns of phase winding

$E_{max} = N_{ce}\ \varphi_M \omega,$

$$E = \frac{N_{ce}\varphi_M \omega}{\sqrt{2}} = \frac{N_{ce}\varphi_M 2\pi}{\sqrt{2}} f = 4.44 N_{ce}\varphi_M f$$ **(similar to a transformer)**

What happens if the magnitude of the rotating flux changes?

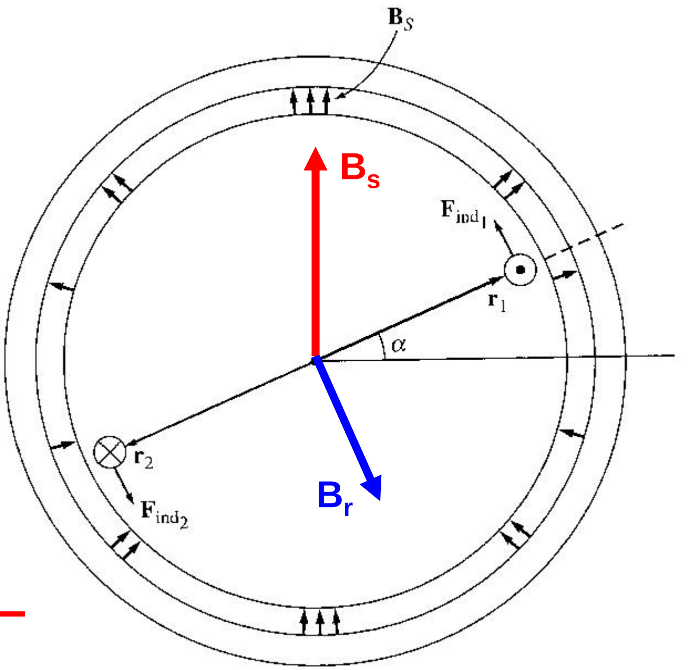
Induced Torque in an AC Machines

Sinusoidally-distributed radial rotating flux density ($B_s(\alpha)=B_s \sin\alpha$)

Loop current

Induced force applied to different parts of the loop

$F = i(l \times B)$



$|B_S(\alpha)| = B_S \sin\alpha$

i perpendicular to B for l1 and l2

$T = 2rl\ i\ B = 2rl\ i\ B_s \sin\alpha$ (B_s peak of sinusoidally -distributed stator flux density)

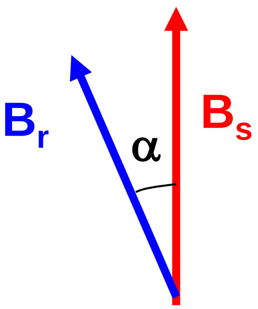
Using first harmonic approximate for rotor MMF, H and B

B_r : a sinusoidally distributed magnetic flux density, with a peak at $3\pi/2+\alpha$

$T = 2rl\ i\ B_s \sin\alpha = K\ B_r\ B_s \sin\alpha$

Constant torque in AC Machines

$T = K\, B_r\, B_s \sin\alpha$



B_s: peak of stator sinusoidally-distributed flux density
B_r: peak of rotor sinusoidally-distributed flux density
α: angle between these two peaks.

To have a constant torque:

Amplitude of B_s should be constant
Amplitude of B_r should be constant
α should be constant

$\Downarrow$

If **B_s** is a **rotating magnetic flux** then **B_r** should be a **rotating magnetic flux** rotating with the **same speed** to be **stationary** with **B_s**

Constant torque in three phase AC Machines

In AC machines with **three phase stator current** in **three phase stator winding**

B_s is a **sinusoidally-distributed rotating** flux-density.

B_r produced by a **Permanent magnet** or **DC current** (Electromagnet) is **stationary** relative to **rotor.**

To have a **constant torque, rotor** should **rotate** with the speed of **rotating stator flux.**

(Synchronous Machines)

Rotating B_s Induces **three phase voltage** in **three phase** winding of **rotor if** it is **not rotating** with the **rotation speed of B_s .**

Three phase current will be produced if **rotor windings** are **short circuited.**

Three phase induced current generates **rotating magnetic field** of **rotor (B_r).**

Explain why B_r is stationary relative to B_s!

(Induction / asynchronous Machine)

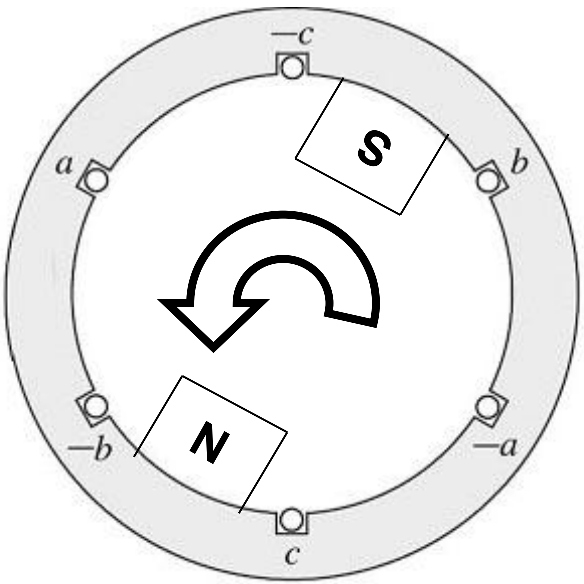
Effect of pole-pairs in magnetic field rotation speed

In 2 poles machines magnetic field rotates once every period of current:

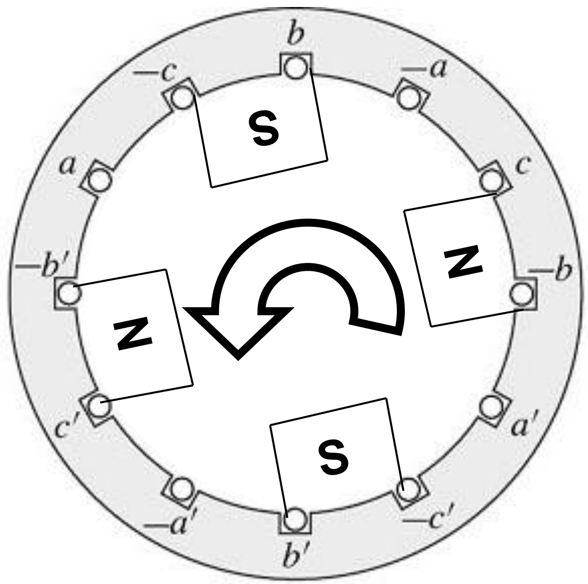
$\omega_e = \omega_m$ (ω_e frequency of current, ω_m frequency of rotation of magnetic field)

In 4 poles machines magnetic field rotates once every two period of current:

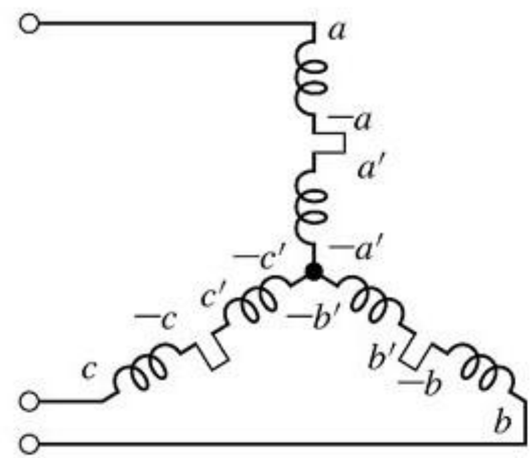
$\omega_e = 2\omega_m$ (Attention: $F_a = F_{aa'}\cos(2\theta_a)$, ... , $F = 3/2 \times N_{se}/2 \times I_m \cos(\omega t - 2\theta_a)$)



2 poles



4 poles



Winding connection

In general: $\omega_e = \frac{p}{2}\omega_m$ $\theta_e = \frac{p}{2}\theta_m$ $n_m = \frac{120 \times f_e}{P}$

P: Number of poles