بسمه ۹۲۱۰۲۸۴۶ - تمرین سری ۷ ریاضی مهندسی - [illegible]

1) $u(x,y) = x^4 + ax^2y^2 + 2x^3 + by^4 + cxy^2$

$\nabla^2 u = 0 \rightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \Rightarrow 12x^2 + 2ay^2 + 12x + 2ax^2 + 12by^2 + 2cx = 0$

$a = -6 \;,\; b = 1 \;,\; c = -6$

$f(z) = u + jv$

$x^4 - 6x^2y^2 + 2x^3 + y^4 - 6y^2x = x^4 + y^4 - 6x^2y^2 + 2x^3 - 6y^2x = u(x,y)$

$u_x = v_y = 4x^3 - 12xy^2 + 6x^2 - 6y^2$

$v = \int v_y\,dy = 4x^3y - 4xy^3 + 6x^2y - 2y^3 + k_1(x)$

$-u_y = v_x = 12x^2y - 4y^3 + 12xy \rightarrow v = \int v_x\,dx = 4x^3y - 4xy^3 + 6x^2y + k_2(y)$

$k_2(y) = -2y^3 + k_1(x) \rightarrow k_1 = c \;,\; k_2 = -2y^3 + c$

$\rightarrow v = 4x^3y - 4xy^3 + 6x^2y - 2y^3 + c$

2) $\sinh^{-1}(z) = \overbrace{\log\left[z + (z^2+1)^{\frac{1}{2}}\right]}^{A}$

$\sinh x = \frac{e^{+x} - e^{-x}}{2} \rightarrow e^{x} \sinh x = \frac{1}{2}\left(e^{2x} - \frac{1}{\;}\right)$

$\rightarrow \frac{e^{2x}}{2} - e^{x} y - \frac{1}{2} = 0$

$\rightarrow e^{x} = \frac{y \pm \sqrt{y^2+1}}{1} \Rightarrow x = \ln\left[y + \sqrt{y^2+1}\right]$

3– $w = \frac{1}{z} = \frac{1}{x+iy} = \underbrace{\frac{x}{x^2+y^2}}_{X} + i\underbrace{\frac{-y}{x^2+y^2}}_{Y}$

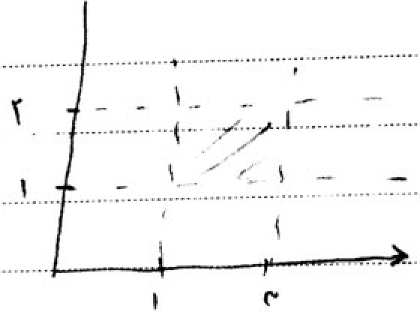
$\frac{X}{X^2+Y^2} = x \qquad \frac{-Y}{X^2+Y^2} = y$

$A\left(\left(\frac{X}{X^2+Y^2}\right)^2 + \left(\frac{-Y}{X^2+Y^2}\right)^2 \right) + B\left(\frac{X}{X^2+Y^2}\right) + C\left(\frac{-Y}{X^2+Y^2}\right) + D = 0$

$\frac{A + BX - CY + DX^2 + DY^2}{X^2+Y^2} = 0$

$\longrightarrow A + BX - CY + D(X^2+Y^2) = 0$

x y نمی‌توانند همزمان صفر شوند



$1 \leq x \leq ۳ \rightarrow x^2+y^2 \leq X \leq 3(x^2+y^2)$

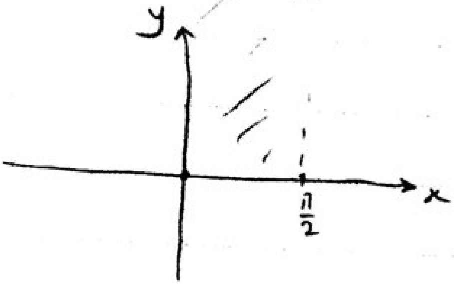
$1 \leq y \leq ۲ \quad x^2+y^2 \leq -Y \leq 2x^2+2y^2$

4– A. $z = x+iy \qquad x > 1 \ , \ y < -۲$

$0 < \frac{1}{x} < 1$ ← تمام شعاع‌ها را می‌سازد $\qquad e^{iy}$ ← تمام نقاط

B. $\frac{1}{x} e^{iy} = \frac{۲}{z+\bar{z}} e^{i\left(\frac{z-\bar{z}}{2}\right)}$

5. $0 \leqslant x \leqslant \frac{\pi}{2}$, $y \geqslant 0$, $\omega = \log(\sin(z))$



$$\sin(x+iy) = \frac{e^{x+iy} - e^{-x-iy}}{2i} = e$$

$$e^{\omega} = \sin z \rightarrow \arcsin e^{\omega} = z = x+iy$$

$$\sin(x+iy) = \sin x \cosh y + i \cos x \sinh(y) \; -$$

b) $x \geqslant 0\ ,\ y \geqslant 0,\quad \omega = (z^2 - 1)^{\frac{1}{2}}$

$\omega = (x^2 - y^2 + 2ixy - 1)^{\frac{1}{2}}$

6) $L(z) = \dfrac{z-J}{z+J}$

$$\frac{z-J}{z+J} = \frac{x+J(y-1)}{x+J(y+1)} = \frac{(x+J(y-1))(x-J(y+1))}{x^{۲}+(y+1)^{2}}$$

$$= \frac{x^{2}+y^{2}-1-2Jx}{x^{2}+(y+1)^{2}} \longrightarrow X = \frac{x^{2}+y^{2}-1}{x^{2}+(y+1)^{2}} \quad Y = \frac{-2x}{x^{2}+(y+1)^{2}}$$

$$X^{2}+Y^{2} \leqslant 1 \iff \frac{x^{۴}+y^{۴}+1+۲x^{۲}y^{۲}-۲x^{2}-۲y^{۲}+۴x^{۲}}{x^{۴}+y^{۴}+۴y^{۳}+۶y^{۲}+۴y+1+۲x^{۲}y^{۲}+۴x^{۲}y+x^{۲}} \leqslant 1$$

$$\iff x^{۴}+y^{۴}+\cancel{1}+۲x^{۲}y^{۲}-۲x^{۲}-۲y^{۲}+\cancel{۴x^{۲}} \leqslant x^{۴}+y^{۴}+۴y^{۳}+۶y^{۲}+۴y+\cancel{1}+\cancel{۲x^{۲}y^{۲}}+۴x^{۲}y+\cancel{x^{۲}}$$

$$\iff ۰ \leqslant ۴y^{۳}+۸y^{۲}+۴y+۴x^{۲}y = \underbrace{۴y}_{y \geq ۰}\big(\underbrace{(y+1)^{۲}+x^{۲}}_{\geq ۰}\big)$$

$y \geq ۰$ ← نیم‌صفحه بالایی