Transformer

Per-unit system

Why perunit= meaningful data

- Consider: “the armature resistance is 0.1 Ω”

Then ask yourself if this a high, typical or a low value?
Answer depends on size of machine:
In a large machine 0.1 Ω is excessive, small machine too low

Transformer

Per-unit system

Why Use the Per Unit System?

• Multiple voltage levels: 400kV, 275kV, 132kV, 11kV, 400V makes circuit analysis rather confusing

• Ideal transformer winding can be eliminated (assumes proper specification of base values)

• Voltages, currents and impedances expressed in perunit do not change when referred from primary to secondary

• Perunit impedances of equipment of similar type are usually similar if equipment ratings are used as base values

• Digital implementation of control systems is easier with Perunit values

Transformer

Per-unit system

- The voltage and impedance conversions make calculation in circuit with a transformer tedious
- Per-unit method eliminates such conversions

$$\text{quantity in per unit} = \frac{\text{actual quantity}}{\text{base quantity}}$$

- 2 base quantities are selected: voltage and power
- Other base values can be related through electrical laws

$$S_{base} = V_{base} I_{base}$$

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{\left(V_{base}\right)^2}{S_{base}}$$

Note: V_{base} changes at every transformer according to the turn ratio

Transformer

Per-unit system

- Per-unit value can be converted to the new base
- Recall that the actual values are not changed
- Power

$$(P,Q,S)_{p.u.}^{new} = (P,Q,S)_{p.u.}^{old} \frac{S_{base}^{old}}{S_{base}^{new}}$$

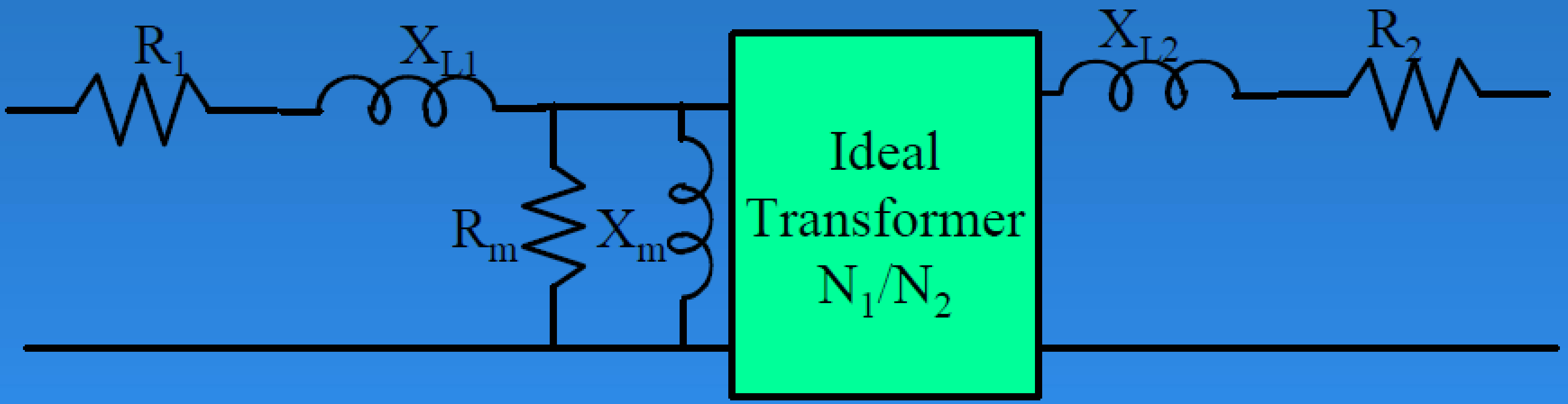
- Voltage

$$V_{p.u.}^{new} = V_{p.u.}^{old} \frac{V_{base}^{old}}{V_{base}^{new}}$$

- Impedance

$$Z_{p.u.}^{new} = Z_{p.u.}^{old} \frac{Z_B^{old}}{Z_B^{new}} = Z_{p.u.}^{old} \left[\frac{V_B^{old}}{V_B^{new}} \right]^2 \frac{S_B^{new}}{S_B^{old}}$$

Real Transformers



- R_1, R_2: ohmic losses in conductors
- X_{L1}, X_{L2}: leakage flux
- X_m: imperfect magnetisation
- R_m: core losses (eddy currents, hysteresis)

Per Unit Voltages in Transformers

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$Choose:$$

$$\frac{V_{1B}}{V_{2B}} = \frac{V_{1Nom}}{V_{2Nom}} = \frac{N_1}{N_2}$$

$$\frac{V_1^{pu}}{V_2^{pu}} = \frac{\dfrac{V_1}{V_{1B}}}{\dfrac{V_2}{V_{2B}}} = \frac{V_1}{V_2} \cdot \frac{V_{2B}}{V_{1B}} = \frac{N_1}{N_2} \cdot \frac{N_2}{N_1} = 1$$

$$\boxed{V_1^{pu} = V_2^{pu}}$$

Per Unit Currents in Transformers

$$\frac{I_1^{pu}}{I_2^{pu}} = \frac{\dfrac{I_1}{I_1^B}}{\dfrac{I_2}{I_2^B}} = \frac{I_1}{I_2} \cdot \frac{I_2^B}{I_1^B}$$

$$I_1^B = \frac{S^B}{V_1^B}$$

$$I_2^B = \frac{S^B}{V_2^B}$$

$$\frac{I_2^B}{I_1^B} = \frac{V_1^B}{V_2^B} = \frac{N_1}{N_2}$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

$$\frac{I_1^{pu}}{I_2^{pu}} = \frac{N_2}{N_1} \cdot \frac{N_1}{N_2} = 1$$

$$\boxed{I_1^{pu} = I_2^{pu}}$$

Per Unit Impedance in Transformers

$$Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

$$Z_1^{pu} \cdot Z_1^B = \left(\frac{N_1}{N_2}\right)^2 Z_2^{pu} \cdot Z_2^B$$

$$Z_1^B = \frac{V_1^B}{I_1^B} = \frac{V_1^{B\,2}}{S^B}$$

$$Z_2^B = \frac{V_2^B}{I_2^B} = \frac{V_2^{B\,2}}{S^B}$$

$$Z_1^{pu} \cdot \frac{V_{1B}^2}{S_B} = \left(\frac{N_1}{N_2}\right)^2 Z_2^{pu} \cdot \frac{V_2^{B\,2}}{S^B}$$

$$\frac{V_1^{B\,2}}{V_2^{B\,2}} = \left(\frac{N_1}{N_2}\right)^2$$

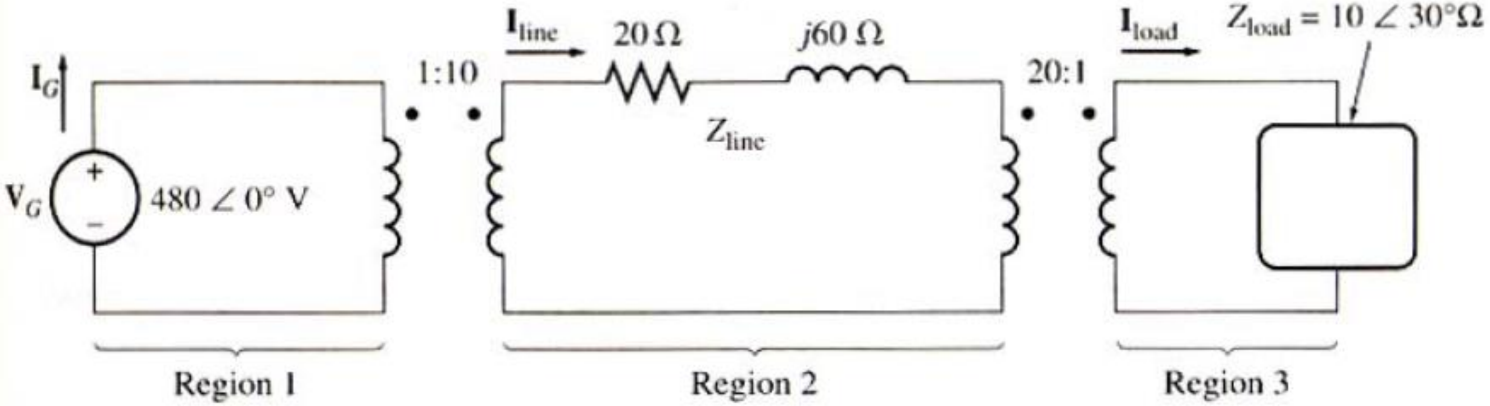
$$\boxed{Z_1^{pu} = Z_2^{pu}}$$

Transformer

Per-unit system

Example

- Select 480V and 10 kVA as base values for the following system



- Find the base voltage, current, impedance and apparent power at every point
- Convert the system to its per-unit equivalent circuit
- Find the power supplied to the load
- Find transmission line loss

Transformer

Per-unit system

Example

- Generator: V_{base1} = 480V, S_{base} = 10kVA

$$I_{base1} = \frac{S_{base}}{V_{base1}} = \frac{10kVA}{480V} = 20.83A$$

$$Z_{base1} = \frac{V_{base1}}{I_{base1}} = \frac{480V}{20.83A} = 23.04\Omega$$

- Transmission line: V_{base2} = 4800V, S_{base} = 10kVA

$$I_{base2} = \frac{S_{base}}{V_{base2}} = \frac{10kVA}{4800V} = 2.083A$$

$$Z_{base2} = \frac{V_{base2}}{I_{base2}} = \frac{4800V}{2.083A} = 2304\Omega$$

Transformer

Per-unit system

Example

- Load: V_{base3} = 240V, S_{base} = 10kVA

$$I_{base3} = \frac{S_{base}}{V_{base3}} = \frac{10kVA}{240V} = 41.67A$$

$$Z_{base3} = \frac{V_{base3}}{I_{base3}} = \frac{240V}{41.67A} = 5.76\Omega$$

- Convert to p.u. system

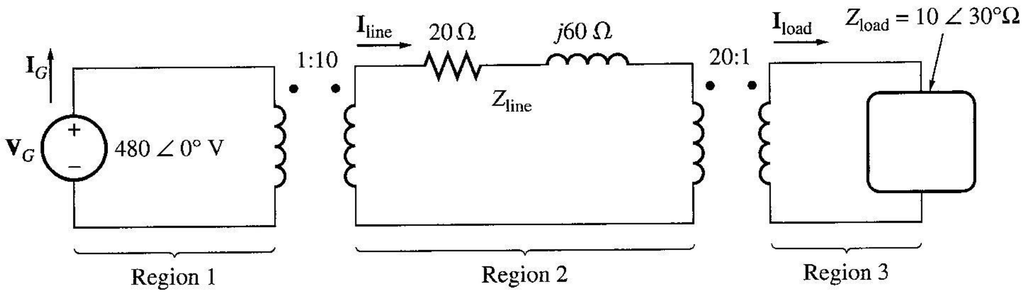
$$V_{G,p.u.} = \frac{480\angle 0}{480} = 1.0\angle 0$$

$$Z_{line,p.u.} = \frac{20 + j60}{2034} = 0.0087 + j0.0260$$

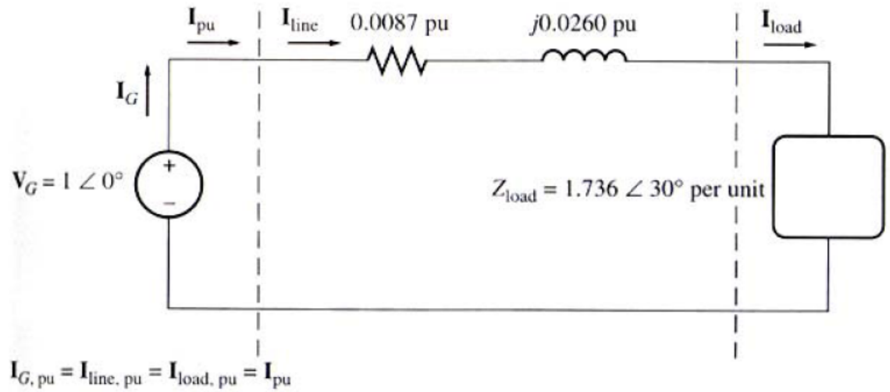
$$Z_{load,p.u.} = \frac{10\angle 30}{5.76} = 1.736\angle 30$$

Transformer

Example **Per-unit system**



- Equivalent circuit



- Current

$$I_{p.u.} = \frac{V_{p.u.}}{Z_{tot,p.u.}} = \frac{1\angle 0}{(0.0087 + j0.0260) + (1.736\angle 30)} = 0.569\angle -30.6$$

Transformer

Example **Per-unit system**

- Load power

$$P_{load,p.u.} = I_{p.u.}^2 R_{p.u.} = (0.569)^2 (1.503) = 0.487$$

$$P_{load} = P_{load,p.u.} \times S_{base} = (0.487)(10kVA) = 4870W$$

- Transmission line loss

$$P_{lineloss,p.u.} = I_{p.u.}^2 R_{p.u.} = (0.569)^2 (0.0087) = 0.00282$$

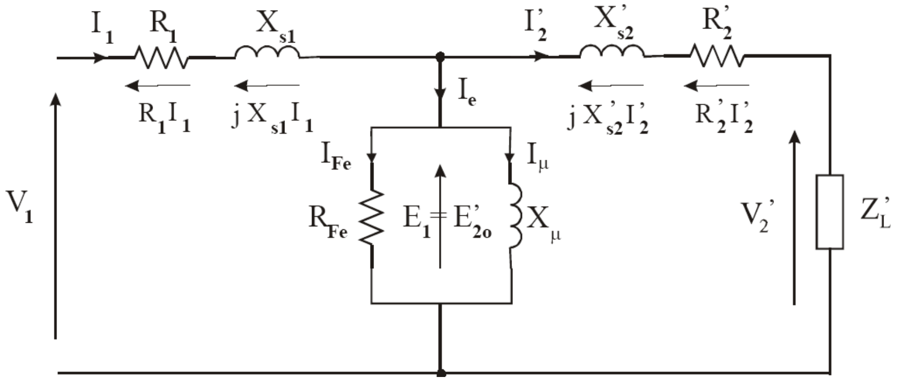
$$P_{lineloss} = P_{lineloss,p.u.} \times S_{base} = (0.00282)(10kVA) = 28.2W$$

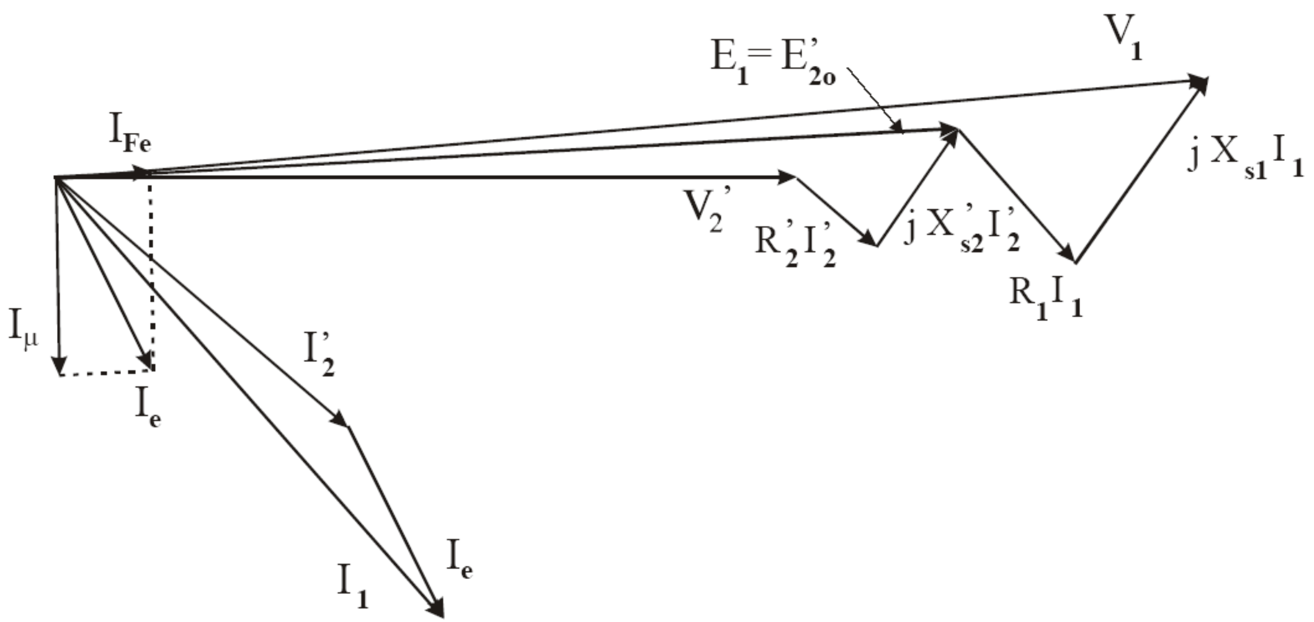
Note: If there is one device, use its own rating as base

If there are more than one device, use the largest one

Transformer

Phasor Diagram





Transformer

Voltage Regulation

- Due to series impedance, the transformer output voltage varies with the load even if the input voltage remains constant
- The voltage regulation (VR)

$$VR = \frac{V_{S,\mathrm{nl}} - V_{S,\mathrm{fl}}}{V_{S,\mathrm{fl}}} \times 100\%$$

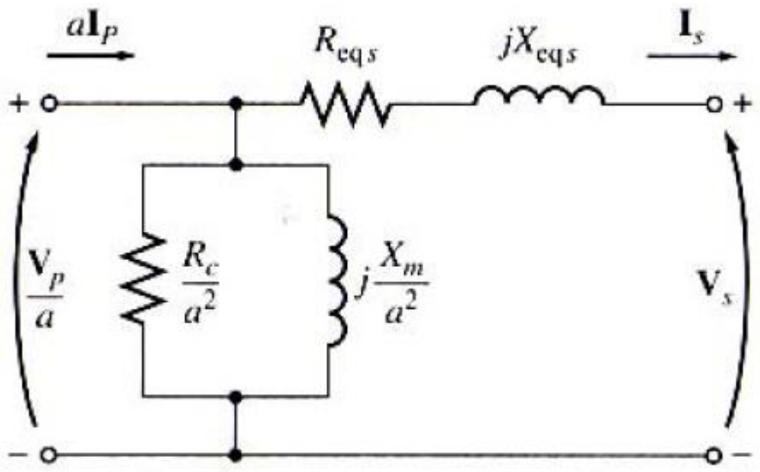
- In p.u. system

$$VR = \frac{V_{P,\mathrm{p.u.}} - V_{S,\mathrm{fl,p.u.}}}{V_{S,\mathrm{fl,p.u.}}} \times 100\%$$

- Small voltage regulation means small voltage drop and low loss

Transformer

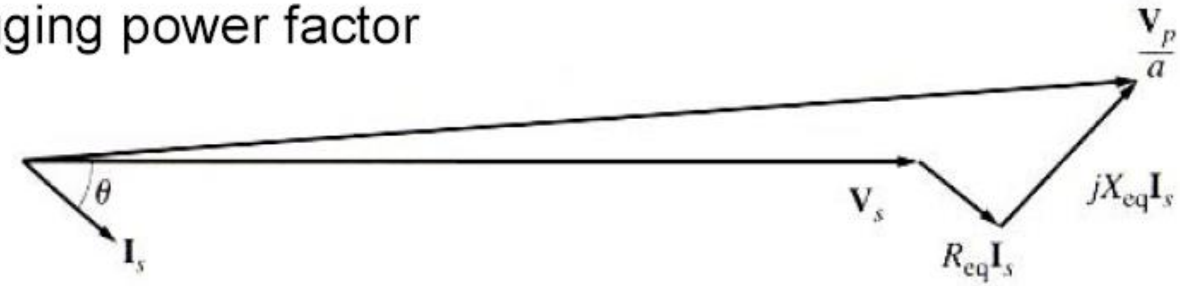
Voltage Regulation



Ignoring
excitation branch

$$\frac{\mathbf{V}_P}{a} = \mathbf{V}_S + R_{eq}\mathbf{I}_S + jX_{eq}\mathbf{I}_S$$

Lagging power factor

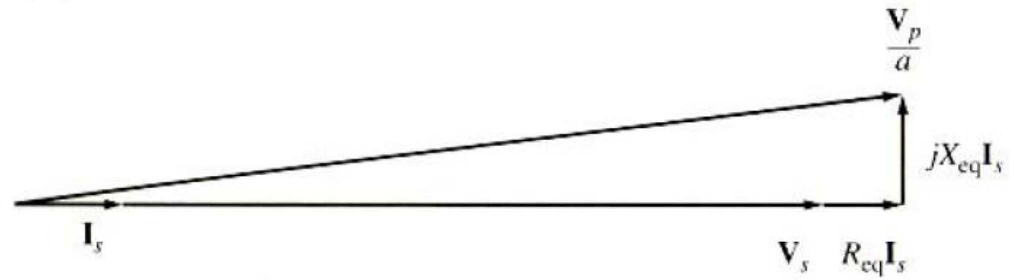


voltage regulation with lagging loads is > 0

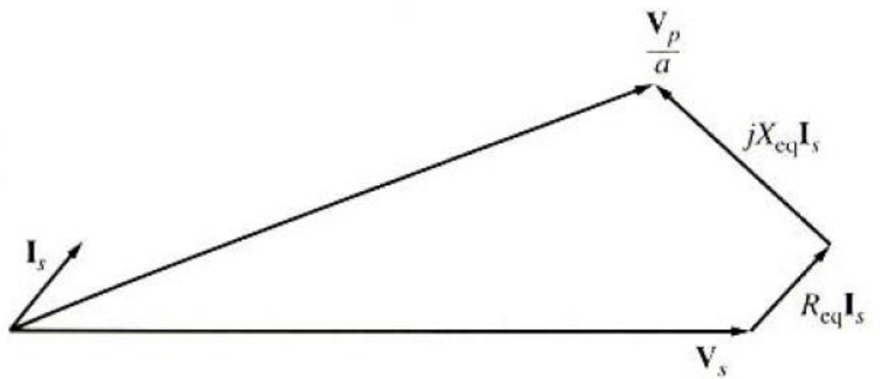
Transformer

Voltage Regulation

- Unity power factor



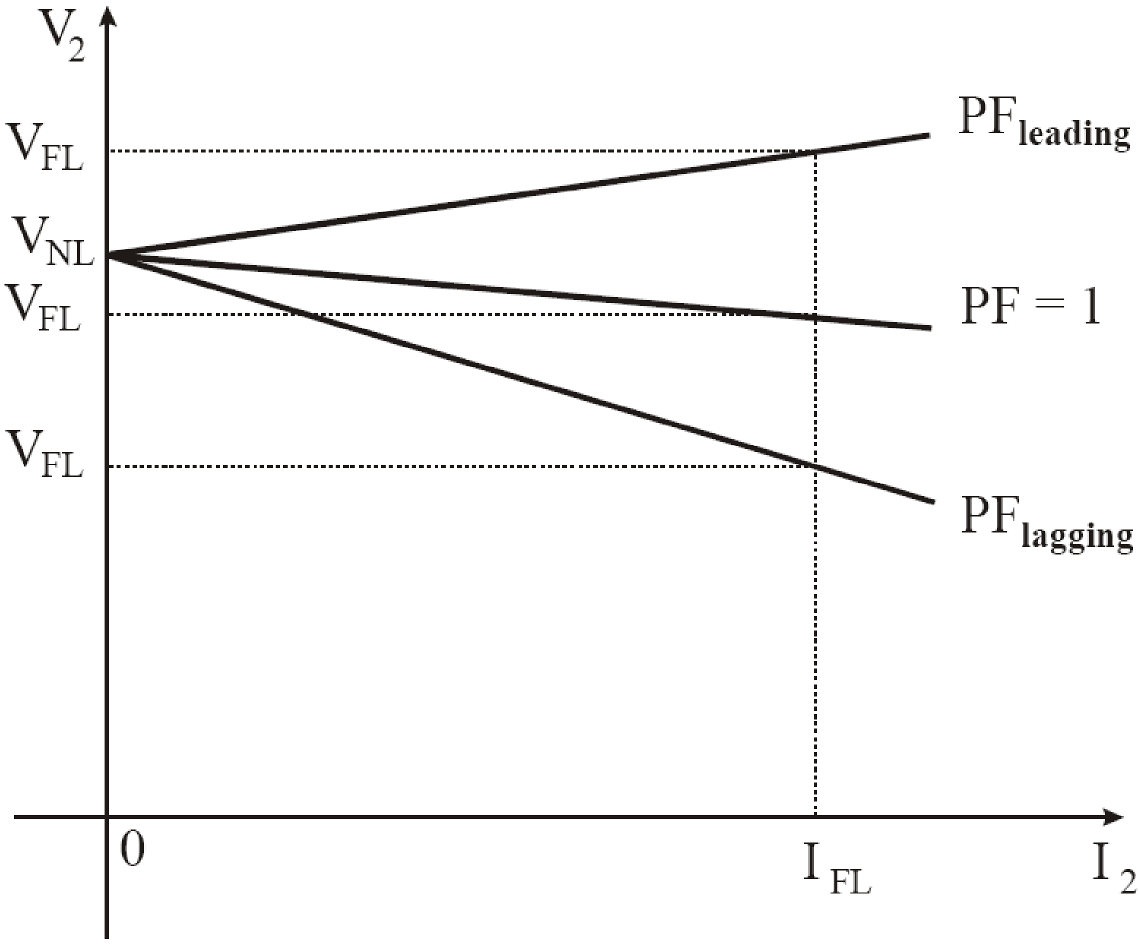
- Leading power factor



When the power factor is unity VR > 0
With a leading power factor usually VR < 0

Transformer

Voltage Regulation

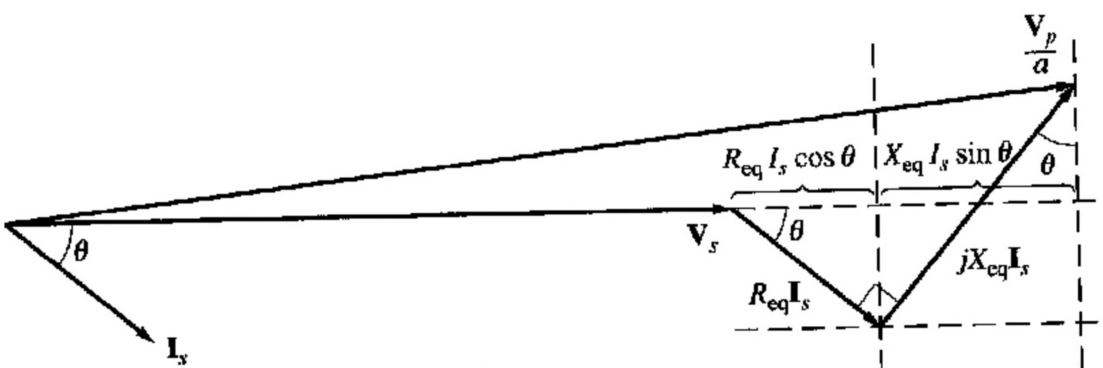


Transformer

Voltage Regulation

Simplified Voltage Regulation Calculation

For lagging loads, the vertical components of Req and Xeq will partially cancel each other. Due to that, the angle of VP/a will be very small.



$$\frac{V_p}{a} \approx \mathrm{V}_s + R_{eq} I_s \cos\theta + X_{eq} I_s \sin\theta$$

$$\text{Voltage regulation} = \frac{(V_1') - (V_2)}{(V_2)}$$

$$= \frac{I_2 R_{e2}}{V_2} \cos\theta_2 + \frac{I_2 X_{e2}}{V_2} \sin\theta_2$$

$$= R_{pu} \cos\theta_2 + X_{pu} \sin\theta_2$$

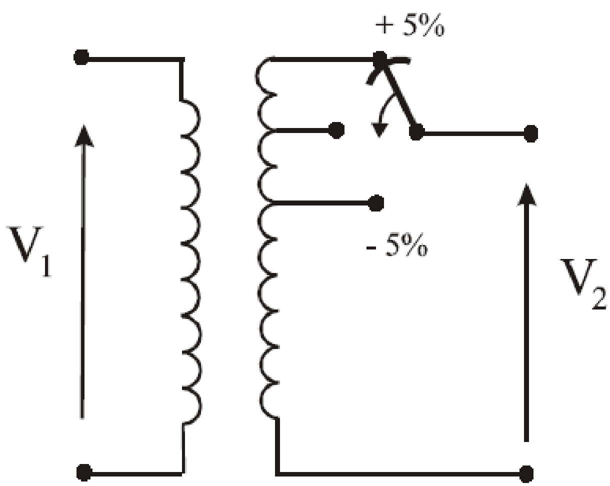
$$V.R. = R_{eq,p.u.} \cos\theta + X_{eq,p.u.} \sin\theta \qquad with\, V_{base} = V_S\,, \quad S_{base} = V_S I_S$$

Transformer

Tap changing

We assumed before that the transformer turns ratio is a fixed (constant) for the given transformer. Frequently, distribution transformers have a series of taps in the windings to permit small changes in their turns ratio. Typically, transformers may have 4 taps in addition to the nominal setting with spacing of 2.5 % of full-load voltage. Therefore, adjustments up to 5 % above or below the nominal voltage rating of the transformer are possible.

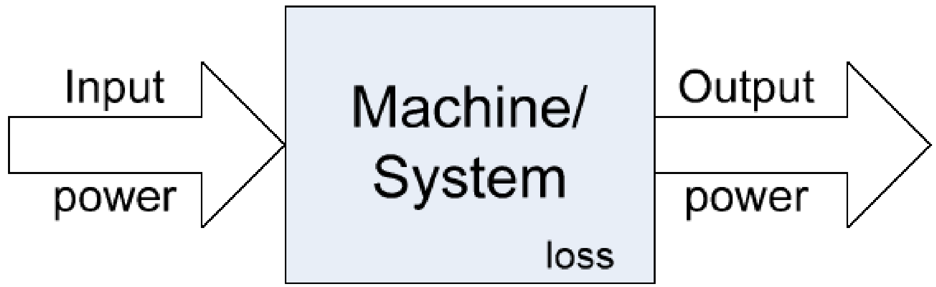
Example 4.6: A 500 kVA, 13 200/480 V transformer has four 2.5 % taps on its primary winding. What are the transformer's voltage ratios at each tap setting?



+ 5.0% tap	13 860/480 V
+ 2.5% tap	13 530/480 V
Nominal rating	13 200/480 V
- 2.5% tap	12 870/480 V
- 5.0% tap	12 540/480 V

Transformer

Efficiency



$$\boxed{\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%}$$

- Losses
  - Copper (Cu, I^2R) losses: series resistance
  - Hysteresis losses: R_h
  - Eddy current losses: R_e

$$\left.\begin{array}{l}\text{Hysteresis losses: } R_h \\ \text{Eddy current losses: } R_e\end{array}\right\} \text{Core losses} = R_{h+e} \text{ or } R_C$$

Transformer

Efficiency

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$$

$$\eta = \frac{V_S I_S \cos\theta}{P_{Cu} + P_{core} + V_S I_S \cos\theta} \times 100\%$$

Transformer

Efficiency

The transformer core loss is due to hysteresis and eddy currents. Both of these are essentially due to core of transformer and do not depend on loading, hence, *the transformer core loss is substantially independent of load current*. Core loss depends on voltage and frequency. Copper loss depend on load current.

$$P_{core} = P_{coreN}\left(\frac{V_p}{V_{pn}}\right)^2$$

$$P_{Cu} = P_{CuN}\left(\frac{I_s}{I_{sN}}\right)^2$$

Transformer

Efficiency

$$\eta = \frac{V_2 \cos\theta_2}{V_2 \cos\theta_2 + I_2 R_{e2} + P_i/I_2}$$

Assuming V_2 is constant. Therefore efficiency is maximum when the denominator is minimum, i.e. when

$$\frac{d}{dI_2}\left(V_2 \cos\theta_2 + I_2 R_{e2} + \frac{P_i}{I_2}\right) = 0$$

or
$$R_{e2} - \frac{P_i}{I_2^2} = 0$$

or
$$I_2^2 R_{e2} = P_i$$

Thus for a given value of V_2 and $\cos\theta_2$, the efficiency of a transformer is maximum when the load current is such that copper-losses are equal to the iron-losses.

Transformer

Efficiency

$$Efficiency = \frac{Input - losses}{Input} = \frac{V_1 I_1 cos\emptyset_1 - Pc - Pi}{V_1 I_1 cos\emptyset_1} = \frac{V_1 I_1 cos\emptyset_1 - I_1^2 Req - Pi}{V_1 I_1 cos\emptyset_1}$$

Simplifying;

$$Efficiency = 1 - \frac{I_1 R_{eq}}{V_1 cos\emptyset_1} - \frac{Pi}{V_1 I_1 cos\emptyset_1}$$

For efficiency to be maximum; $\frac{d(efficiency)}{dI_1} = 0$

$$\frac{d(efficiency)}{dI_1} = 0 - \frac{R_{eq}}{V_1 cos\emptyset_1} + \frac{Pi}{V_1 I_1^2 cos\emptyset_1}$$

Which then equates to

$$Pi = I_1^2 Req \ or \ Pi = Pc$$

Thus, to have maximum efficiency, the copper loss should be equal to the core loss

Transformer

Efficiency - All-day efficiency

$$\eta = \frac{P_{out}}{P_{out} + P_c + P_{cu}}$$

$$\eta = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_c + I_2^2 R_{eq2}}$$

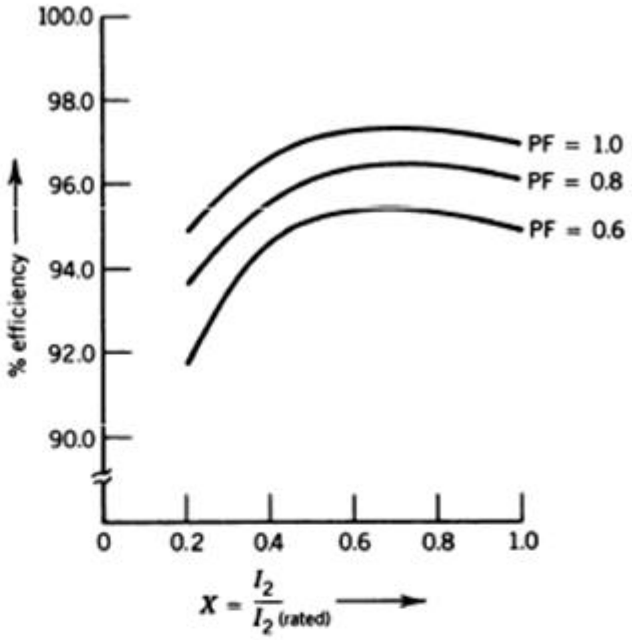
Max efficiency occurs for:

- fixed V_2 and θ_2

$$Pc = I_2^2 R_{eq2}$$

- fixed V_2 and I_2

$$\cos\theta_2 = 1$$



At any particular I_2 **maximum efficiency** happens at unity power factor.

- All-day efficiency:

$$\eta_{AD} = \frac{enrgy\ output\ over\ 24\ hours}{enrgy\ input\ over\ 24\ hours}$$

The transformer efficiency: Example

Example 4.5: A 15 kVA, 2300/230 V transformer was tested to by open-circuit and closed-circuit tests. The following data was obtained:

$V_{OC} = 2300\ V$	$V_{SC} = 47\ V$
$I_{OC} = 0.21\ A$	$I_{SC} = 6.0\ A$
$P_{OC} = 50\ W$	$P_{SC} = 160\ W$

a. Find the equivalent circuit of this transformer referred to the high-voltage side.

b. Find the equivalent circuit of this transformer referred to the low-voltage side.

c. Calculate the full-load voltage regulation at 0.8 lagging power factor, at 1.0 power factor, and at 0.8 leading power factor.

d. Plot the voltage regulation as load is increased from no load to full load at power factors of 0.8 lagging, 1.0, and 0.8 leading.

e. What is the efficiency of the transformer at full load with a power factor of 0.8 lagging?

The transformer efficiency: Example

a. The excitation branch values of the equivalent circuit can be determined as:

$$\theta_{oc} = \cos^{-1}\frac{P_{oc}}{V_{oc}I_{oc}} = \cos^{-1}\frac{50}{2300\cdot 0.21} = 84°$$

The excitation admittance is:

$$Y_E = \frac{I_{oc}}{V_{oc}}\angle -84° = \frac{0.21}{2300}\angle -84° = 0.0000\,095 - j0.0000\,908\ S$$

The elements of the excitation branch referred to the primary side are:

$$R_c = \frac{1}{0.0000095} = 105\,k\Omega$$

$$X_M = \frac{1}{0.0000908} = 11\,k\Omega$$

The transformer efficiency: Example

From the short-circuit test data, the short-circuit impedance angle is

$$\theta_{SC} = \cos^{-1}\frac{P_{SC}}{V_{SC}I_{SC}} = \cos^{-1}\frac{160}{47\cdot 6} = 55.4°$$

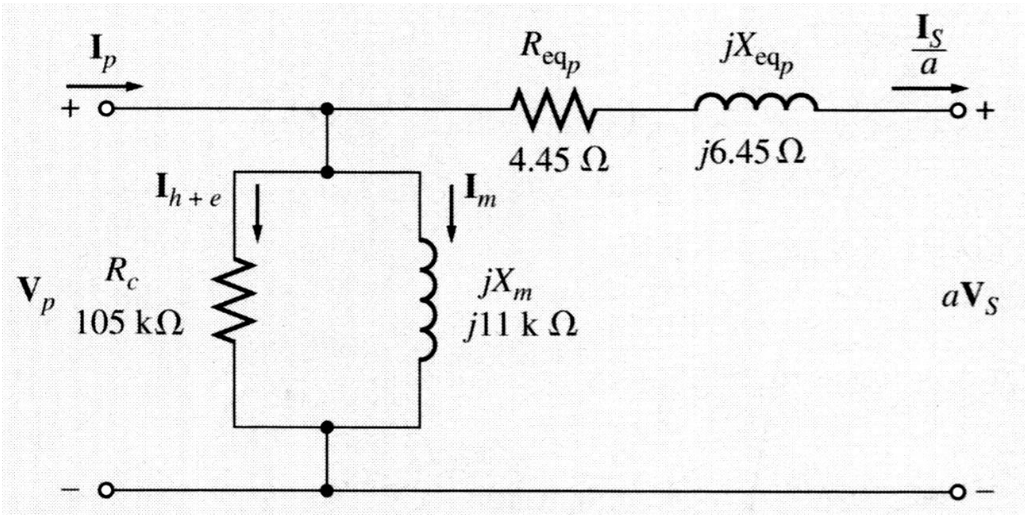
The equivalent series impedance is thus

$$Z_{SE} = \frac{V_{SC}}{I_{SC}}\angle\theta_{SC} = \frac{47}{6}\angle 55.4° = 4.45 + j6.45\ \Omega$$

The series elements referred to the primary winding are:

$$R_{eq} = 4.45\,\Omega; \qquad X_{eq} = 6.45\,\Omega$$

The equivalent circuit

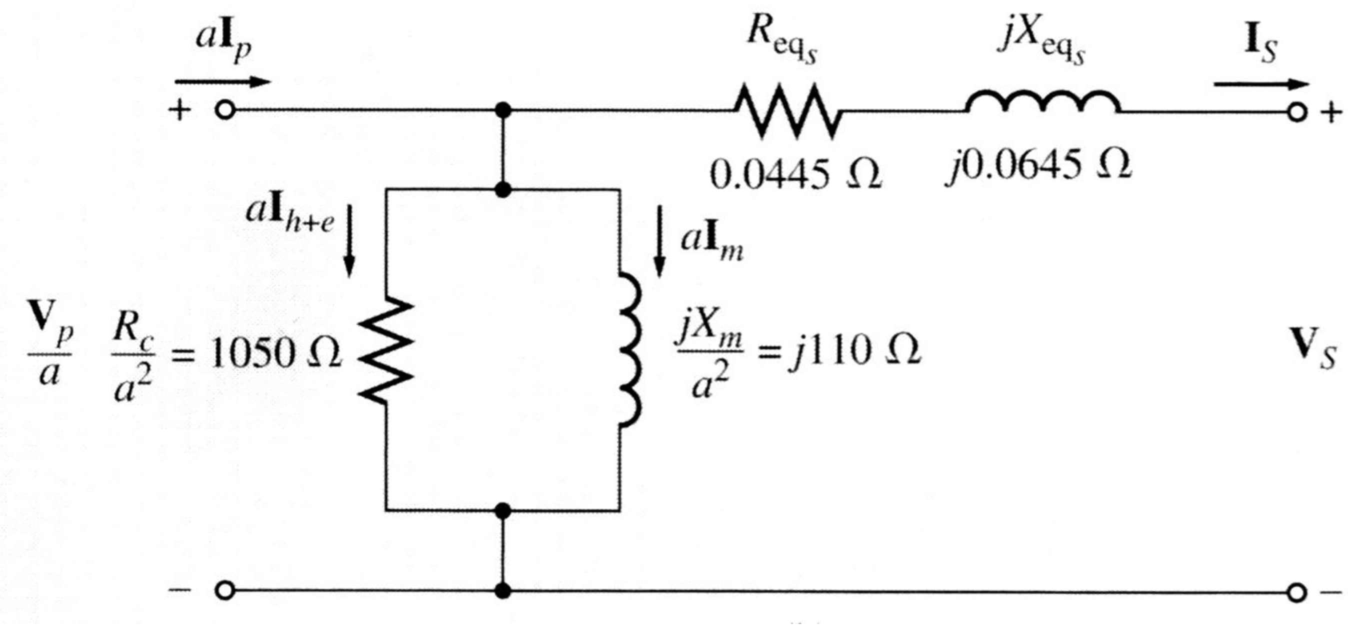


The transformer efficiency: Example

b. To find the equivalent circuit referred to the low-voltage side, we need to divide the impedance by a^2. Since a = 10, the values will be:

$$R_C = 1050\,\Omega \quad X_M = 110\,\Omega \quad R_{eq} = 0.0445\,\Omega \quad X_{eq} = 0.0645\,\Omega$$

The equivalent circuit will be



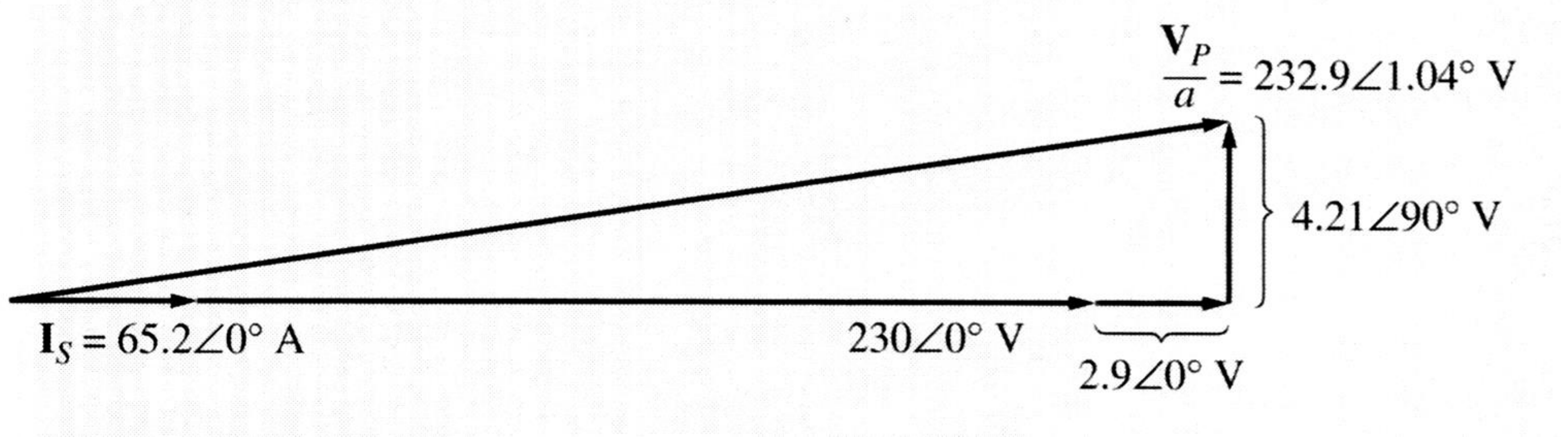
The transformer efficiency: Example

At PF = 1.0, current $I_s = 65.2\angle\cos^{-1}(1.0) = 65.2\angle 0°\ A$

and $\dfrac{V_p}{a} = 230\angle 0° + 0.0445\cdot\left(65.2\angle 0°\right) + j0.0645\cdot\left(65.2\angle 0°\right) = 232.94\angle 1.04°\,V$

The resulting voltage regulation is, therefore:

$$VR = \frac{\left|V_p/a\right| - V_{S,fl}}{V_{S,fl}}\cdot 100\% = \frac{232.94-230}{230}\cdot 100\% = 1.28\%$$



The transformer efficiency: Example

c. The full-load current on the secondary side of the transformer is

$$I_{S,\,rated} = \frac{S_{rated}}{V_{S,\,rated}} = \frac{15\,000}{230} = 65.2\ A$$

Since:

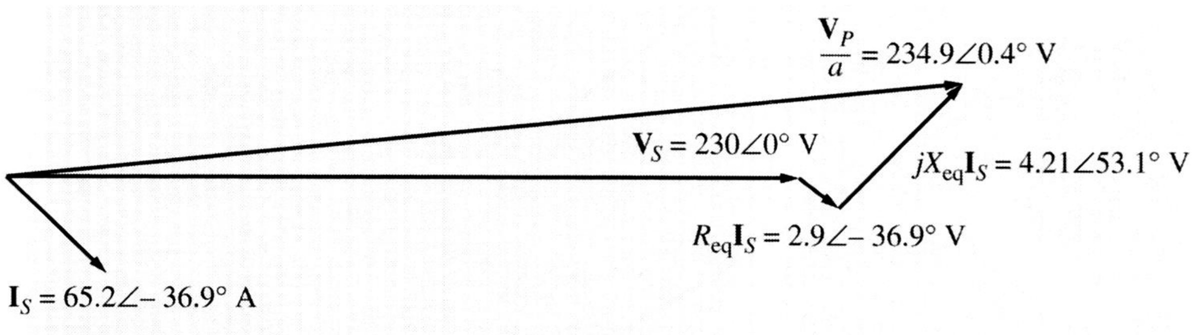
$$\frac{V_p}{a} = V_S + R_{eq} I_S + jX_{eq} I_S$$

At PF = 0.8 lagging, current $I_s = 65.2\angle -\cos^{-1}(0.8) = 65.2\angle -36.9°\ A$

and $\frac{V_p}{a} = 230\angle 0° + 0.0445\cdot\left(65.2\angle -36.9°\right) + j0.0645\cdot\left(65.2\angle -36.9°\right) = 234.85\angle 0.40°\ V$

The resulting voltage regulation is, therefore:

$$\begin{aligned} VR &= \frac{\left|V_p/a\right| - V_{S,\,fl}}{V_{S,\,fl}}\cdot 100\% \\ &= \frac{234.85 - 230}{230}\cdot 100\% \\ &= 2.1\% \end{aligned}$$



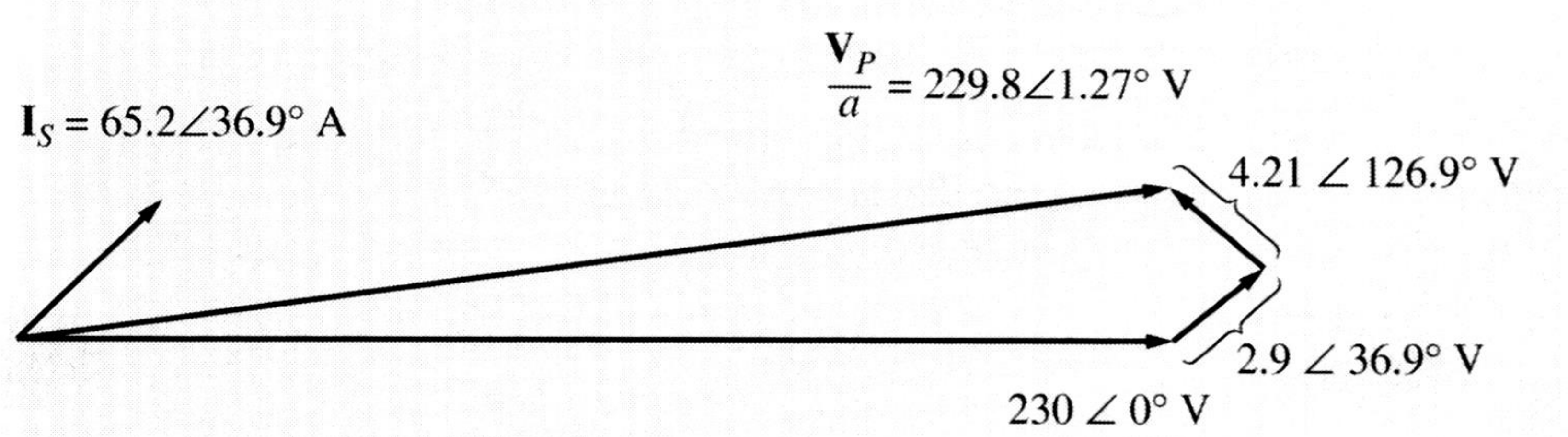
The transformer efficiency: Example

At PF = 0.8 leading, current $I_s = 65.2\angle\cos^{-1}(0.8) = 65.2\angle 36.9°\ A$

and $$\frac{V_p}{a} = 230\angle 0° + 0.0445\cdot\left(65.2\angle 36.9°\right) + j0.0645\cdot\left(65.2\angle 36.9°\right) = 229.85\angle 1.27°\ V$$

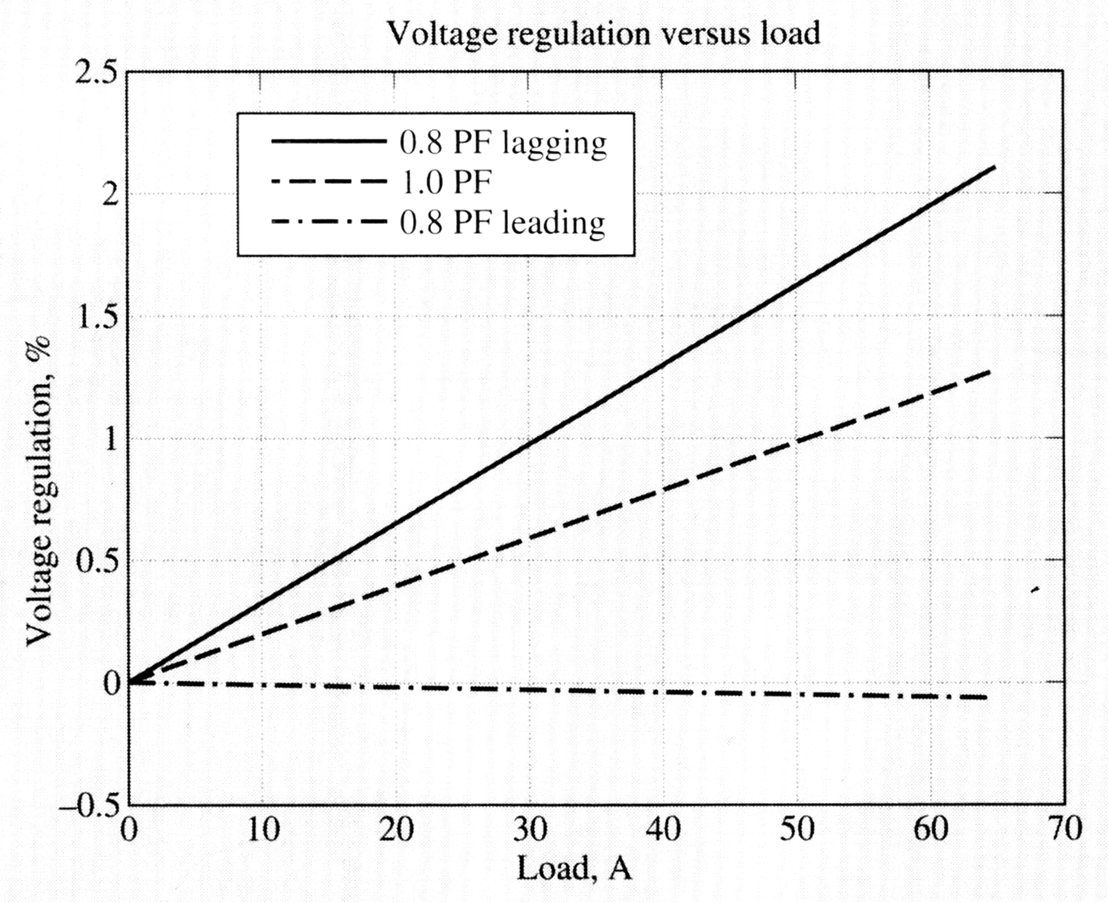
The resulting voltage regulation is, therefore:

$$VR = \frac{\left|V_p/a\right| - V_{S,fl}}{V_{S,fl}}\cdot 100\% = \frac{229.85 - 230}{230}\cdot 100\% = -0.062\%$$



The transformer efficiency: Example

Similar computations can be repeated for different values of load current. As a result, we can plot the voltage regulation as a function of load current for the three Power Factors.



The transformer efficiency: Example

e. To find the efficiency of the transformer, first calculate its losses.

The copper losses are:

$$P_{Cu} = I_S^2 R_{eq} = 65.2^2 \cdot 0.0445 = 189\,W$$

The core losses are:

$$P_{core} = \frac{\left(V_p / a\right)^2}{R_C} = \frac{234.85^2}{1050} = 52.5\,W$$

The output power of the transformer at the given Power Factor is:

$$P_{out} = V_S I_S \cos\theta = 230 \cdot 65.2 \cdot \cos 36.9^\circ = 12\,000\,W$$

Therefore, the efficiency of the transformer is

$$\eta = \frac{P_{out}}{P_{Cu} + P_{core} + P_{out}} \cdot 100\% = 98.03\%$$