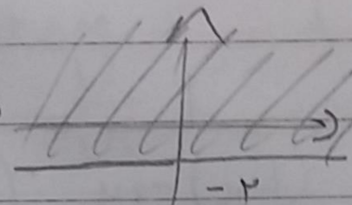
Subject: Date: .   .

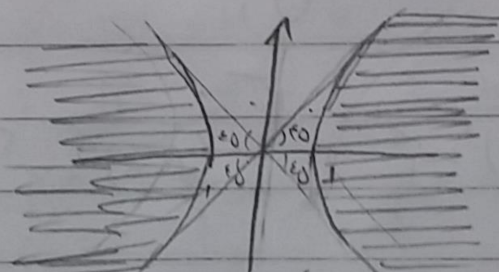
۱-۹

$z = x + iy$

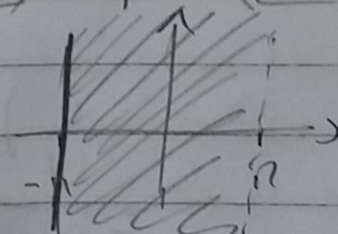
الف) $y \geq -۲$



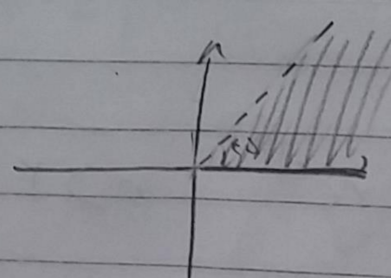
ب) $x^2 - y^2 \geq 1$



پ) $-\pi \leq x < \pi$



ت) $0 \leq \arctan \frac{y}{x} < \frac{\pi}{4}$



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۲-۶

الف) $\frac{1}{1-z} = \frac{1}{(1-x)-iy} = \frac{(1-x)+(iy)}{(1-x)^2+y^2}$

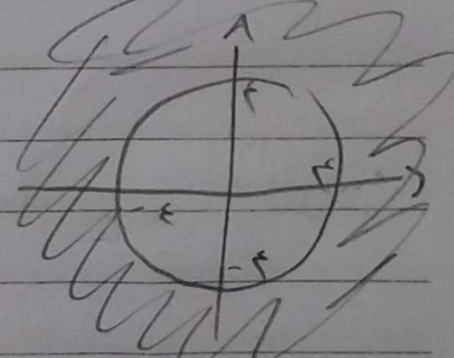
$u = \frac{1-x}{(1-x)^2+y^2} \quad v = \frac{y}{(1-x)^2+y^2}$

ب) $f(z) = \underbrace{x^2 + x - y^2}_{u} + i\underbrace{(2xy + y)}_{v}$

۳-۶

$z = re^{j\theta} \quad r > 2$

$w = r^2 e^{j2\theta} \longrightarrow$



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۶-۳ الف)

$$f(z) = \underbrace{x + \sqrt{x^2+y^2}}_{u} + i\underbrace{y}_{v}$$

$$u_x = 1 + \frac{x}{\sqrt{x^2+y^2}} \qquad u_y = \frac{y}{\sqrt{x^2+y^2}}$$

$$-v_x = 0 \qquad v_y = 1$$

$$\longrightarrow \quad x = 0 \,, \quad y = 0$$

فقط در $z=0$ مشتق پذیر است ← غیر تحلیلی

ب) ✓

پ) به جز $z = \pm i$ تحلیلی است

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$$\underbrace{x+y}_{u} + i\underbrace{y}_{v}$$

$u_x = 1 \qquad u_y = 1$

$-v_x = 0 \qquad v_y = 1$

$\times \leftarrow$

$$\frac{z+1}{\sin \pi z} = \frac{z+1}{\pi z - \frac{(\pi z)^3}{3!} + \frac{(\pi z)^5}{5!} \cdots}$$

$\rightarrow \sin \pi z \neq 0$

$e^{i2\pi z} = 1 = e^{i2n\pi} \rightarrow z = n \in \mathbb{Z}$

$z = 0, \pm 1, \pm 2, \ldots$ قطب ساده اند

ALMAS

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$z = re^{i\theta}$ $\quad f(z) = \underbrace{\theta}_{u} + i\underbrace{o}_{\bar{v}}$ (ج

$u_r = \cdot \quad \frac{1}{r}u_\theta = \frac{1}{r} \longrightarrow \times$

$-\bar{v}_r = \cdot \quad \frac{1}{r}\bar{v}_\theta = \cdot$

ق-۴

(الف $z^4 = -16 \rightarrow r^4 e^{i4\theta} = -16$

$r = 2 \quad e^{i4\theta} = e^{i(\pi + 2n\pi)} \longrightarrow \theta = \frac{\pi}{4} + \frac{n\pi}{2} \quad n \in \mathbb{Z}$

(ب $r^3 e^{i3\theta} = 1 \longrightarrow r = 1$

$e^{i3\theta} = e^{i2n\pi} \longrightarrow \theta = \frac{2}{3}n\pi \quad n \in \mathbb{Z}$

(ج $r^2 e^{i2\theta} = \sqrt{2} e^{i(\frac{\pi}{4} + 2n\pi)}$

ALMAS $\longrightarrow r = \sqrt[4]{2} \;, \theta = \frac{\pi}{8} + n\pi \quad n \in \mathbb{Z}$

Subject: Date: . .

$z^4 = -1 \rightarrow r = 1$ (ب

$e^{i4\theta} = e^{i(\pi + 2n\pi)} \rightarrow \frac{\pi}{4} + \frac{n\pi}{2} = \theta$

$e^{x-iy} = e^{x} e^{i(-y)}$ ٤-٩

$$= \underbrace{e^{x}\cos y}_{u} + \underbrace{i(-1)e^{x}\sin y}_{v}$$

$u_x = e^{x}\cos y \qquad u_y = -e^{x}\sin y$

$-v_x = e^{x}\sin y \qquad v_y = -e^{x}\cos y$

$\longrightarrow$ X

Subject:

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V-4

(الف

$u_x = \cos x \cosh y \quad u_y = \sin x \sinh y$

$-v_x = \sin x \sinh y \quad v_y = \cos x \cosh y \longrightarrow$

$u_{xx} + u_{yy} = -\sin x \cosh y + \sin x \cosh y = 0$

$v_{xx} + v_{yy} = -\cos x \sinh y + \cos x \sinh y = 0$

۱۲-۶

$$\left|\nabla|f(z)|\right|^2 = \left|\frac{\partial\sqrt{u^2+v^2}}{\partial x}\hat{x} + \frac{\partial\sqrt{u^2+v^2}}{\partial y}\hat{y}\right|^2$$

$$= \left(\frac{uu_x+vv_x}{\sqrt{u^2+v^2}}\right)^2 + \left(\frac{uu_y+vv_y}{\sqrt{u^2+v^2}}\right)^2$$

$$\frac{(uu_x+vv_x)^2+(-uv_x+vu_x)^2}{u^2+v^2}$$ کوشی ریمان

$$= u_x^2+v_x^2 = |u_x+iv_x| = |f'(z)|^2$$

NOTE.

Dollar: Euro: Gold: Oil:

۱۳-۶

$$u_r = \frac{1}{r} v_\theta$$

$$v_r = -\frac{1}{r} u_\theta$$

$$\int r u_r \, d\theta = v_\theta = \int 3 r^3 \sin\theta (\cos^2\theta - 2\cos^2\theta) \, d\theta$$

$$-3r^3 \int \left(\underbrace{\sin\theta \cos 2\theta + \sin 2\theta \cos\theta}_{\sin 3\theta} \right) d\theta$$

$$= r^3 \cos 3\theta + f(r)$$

$$v = \int -\frac{1}{r} u_\theta \, dr = \int 3r^2 \cos 3\theta \, dr$$

$$= r^3 \cos 3\theta + g(\theta)$$

NOTE:

$$\longrightarrow v = r^3 \cos 3\theta + A$$

Dollar: Euro: Gold: Oil:

۱۵-۶

$$\nabla^2 \|f(z)\|^2 = \nabla^2 (u^2+v^2)$$

$$= \frac{\partial(2uu_x + 2vv_x)}{\partial x} + \frac{\partial(2uu_y + 2vv_y)}{\partial y}$$

$$= 2\left((u_x)^2 + (v_x)^2 + (u_y)^2 + (v_y)^2 + u\cancel{(u_{xx}+u_{yy})} + v\cancel{(v_{xx}+v_{yy})}\right)$$

$$= 4u_x^2 + 4v_x^2 = 4|f'(z)|$$

۲۵ جمعه ۱۳۹۱ اسفند
15 March 2013
۳ جمادی الاول ۱۴۳۴

NOTE.

۸ $\int r u_r \, d\theta = \int \left(\frac{\cos\theta}{r} + 1 \right) d\theta$ ۱۶-۶

۹ $= -\frac{\sin\theta}{r} + \theta + f(r)$

۱۱ $\int -\frac{1}{r} u_\theta \, dr = \int \left(\frac{\sin\theta}{r^2} \right) dr = -\frac{\sin\theta}{r} + g(\theta)$

۱۲ $\longrightarrow V = -\frac{\sin\theta}{r} + \theta + A$

۱۳

۱۴ $\frac{e^{iz} - e^{-iz}}{2i} = 2 \rightarrow \left(e^{iz} \right)^2 - 4i\left(e^{iz} \right) - 1 = 0$ ۱۷-۶

۱۵ $e^{iz} = \frac{4i \pm \sqrt{-16+4}}{2} = (2 \pm \sqrt{3})\, i$

۱۶ $z = -i \ln \left(2 \pm \sqrt{3} \right) + \frac{\pi}{2} + 2n\pi$ ← الف)

۱۷

۱۸ $e^{iz} = e^{-y} (\cos x + i \sin x)$ $x = \frac{\pi}{2} + 2n\pi$ ب)

NOTE

$e^{-y} = \ln 2 \pm \sqrt{3}$

۱۸-۹

الف) $\cos z = 1 \to e^{iz} + e^{-iz} = 2$

$(e^{iz})^2 - 2(e^{iz}) + 1 = 0 \to e^{iz} = 1 = e^{i2n\pi}$

$\to z = 2n\pi$

ب) $e^{z} + e^{-z} = 0 \to e^{2z} = -1 = e^{i(2n\pi + \pi)}$

$\to z = in\pi + i\frac{\pi}{2}$

ج) $e^{z} + e^{-z} = 2 \to e^{z} = 1 = e^{i2n\pi}$

$z = i2n\pi$

د) $e^{iz} + e^{-iz} = 0 \to e^{i2z} = -1$

$= e^{i(2n\pi + \pi)}$

$z = n\pi + \frac{\pi}{2}$

NOTE:

۱۹-۶ الف)

$$(1+i)^i = e^{i\ln(1+i)} = e^{i\left(\ln\sqrt{2} + i\frac{\pi}{4}\right)}$$

$$= e^{i\ln\sqrt{2} - \frac{\pi}{4}} = e^{-\frac{\pi}{4}}\left(\cos(\ln\sqrt{2}) + i\sin(\ln\sqrt{2})\right)$$

ب)

$$\tan z = i \rightarrow \frac{\sin z}{\cos z} = i$$

$$\rightarrow \frac{e^{iz} - e^{-iz}}{e^{iz} + e^{-iz}} = -1 \rightarrow e^{iz} = -e^{iz}$$

$$\rightarrow e^{iz} = 0 \longrightarrow \times$$

پ)

$$\cosh z = 1 \rightarrow \frac{e^{z} + e^{-z}}{2} = 1$$

$$e^{2z} - 2e^{z} + 1 = 0 \rightarrow e^{z} = 1 = e^{2n\pi i}$$

$$z = 2n\pi i$$

ت)

$$\ln(-1) = \ln 1 + 2n\pi i + \pi i$$

$$2n\pi i + \pi i$$

۲۰-۶

$$\lim_{\substack{\Delta r \to 0 \\ \Delta\theta = 0}} \left(\frac{u(r+\Delta r,\theta)-u(r,\theta)}{\Delta r e^{i\theta}} + i\frac{v(r+\Delta r,\theta)-v(r,\theta)}{\Delta r e^{i\theta}} \right)$$

$$= \frac{u_r + i v_r}{e^{i\theta}} \quad *$$

$$\lim_{\substack{\Delta r = 0 \\ \Delta\theta \to 0}} \frac{u(r,\theta+\Delta\theta)-u(r,\theta)}{r(e^{i(\theta+\Delta\theta)}-e^{i\theta})} + i\frac{v(r,\theta+\Delta\theta)-v(r,\theta)}{r(e^{i(\theta+\Delta\theta)}-e^{i\theta})}$$

$$= \lim_{\substack{\Delta r = 0 \\ \Delta\theta \to 0}} \frac{\frac{u(r,\theta+\Delta\theta)-u(r,\theta)}{\Delta\theta}}{r\left(\frac{e^{i(\theta+\Delta\theta)}-e^{i\theta}}{\Delta\theta}\right)} + i\frac{\frac{v(r,\theta+\Delta\theta)-v(r,\theta)}{\Delta\theta}}{r\left(\frac{e^{i(\theta+\Delta\theta)}-e^{i\theta}}{\Delta\theta}\right)}$$

$$= \frac{u_\theta}{r\,i e^{i\theta}} + \frac{v_\theta}{r e^{i\theta}} \quad ** \longrightarrow \begin{cases} v_r = -\dfrac{u_\theta}{r} \\[2ex] u_r = \dfrac{v_\theta}{r} \end{cases}$$

NOTE.

Dollar: Euro: Gold: