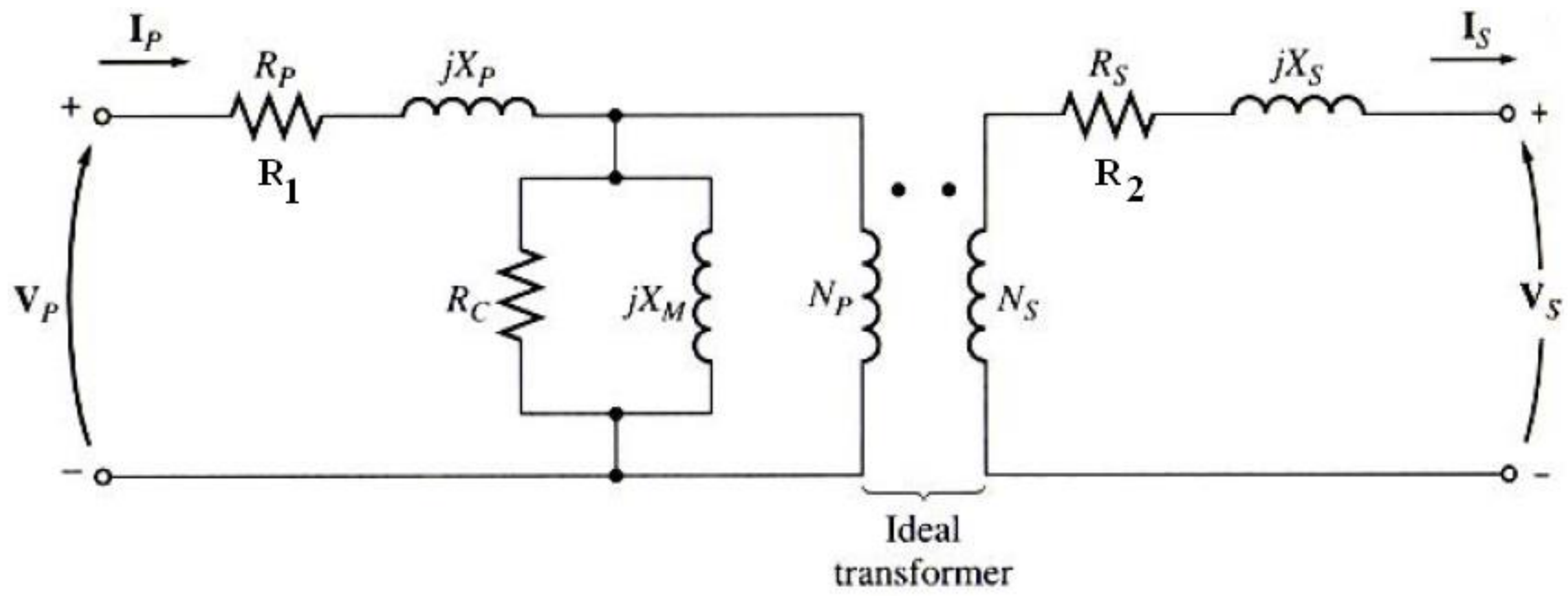
Transformer Modeling

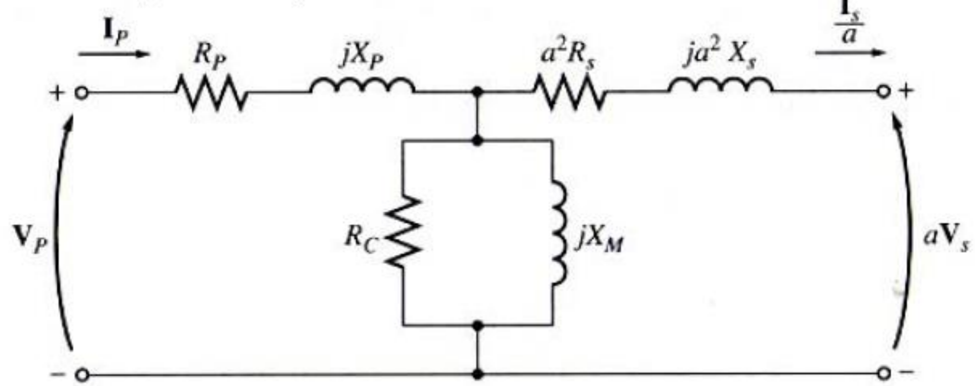
The equivalent circuit



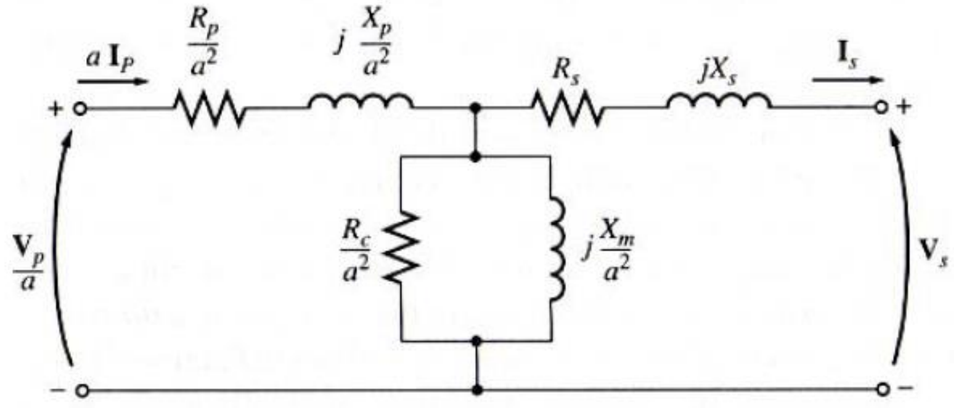
Transformer Modeling

The equivalent circuit

Referred to primary



Referred to secondary

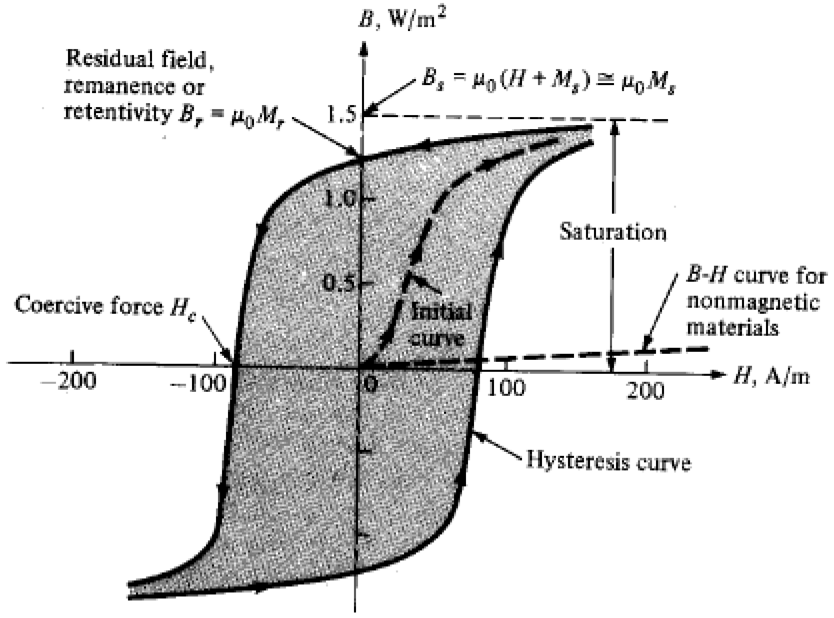


Magnetizing current

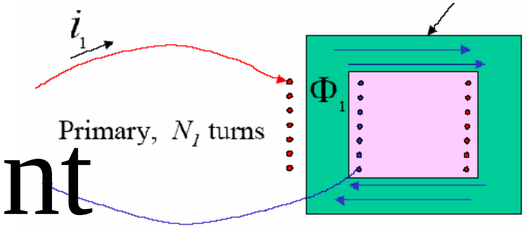
Recall the Hysteresis curve B versus H

At a sufficiently high *H* the core saturates and *B* is essentially constant.

The usual design principle is to have $B = B_{sat}$ at the voltage peaks in the primary. (This minimizes the amount of iron needed in the core)



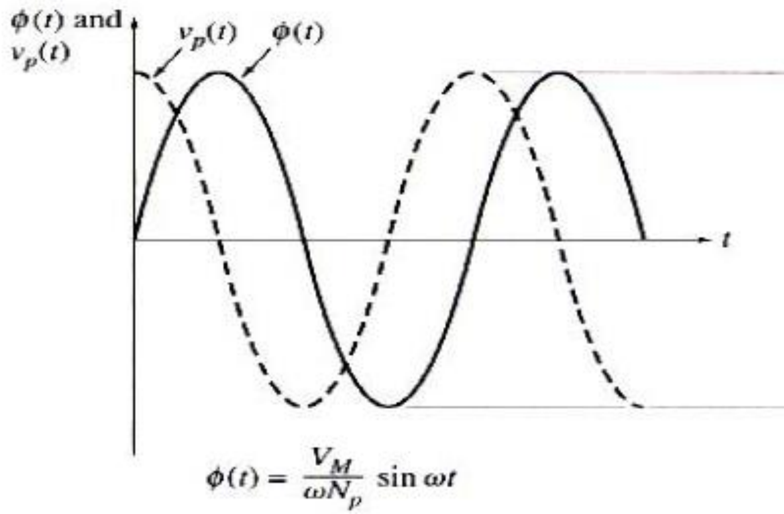
Magnetizing current

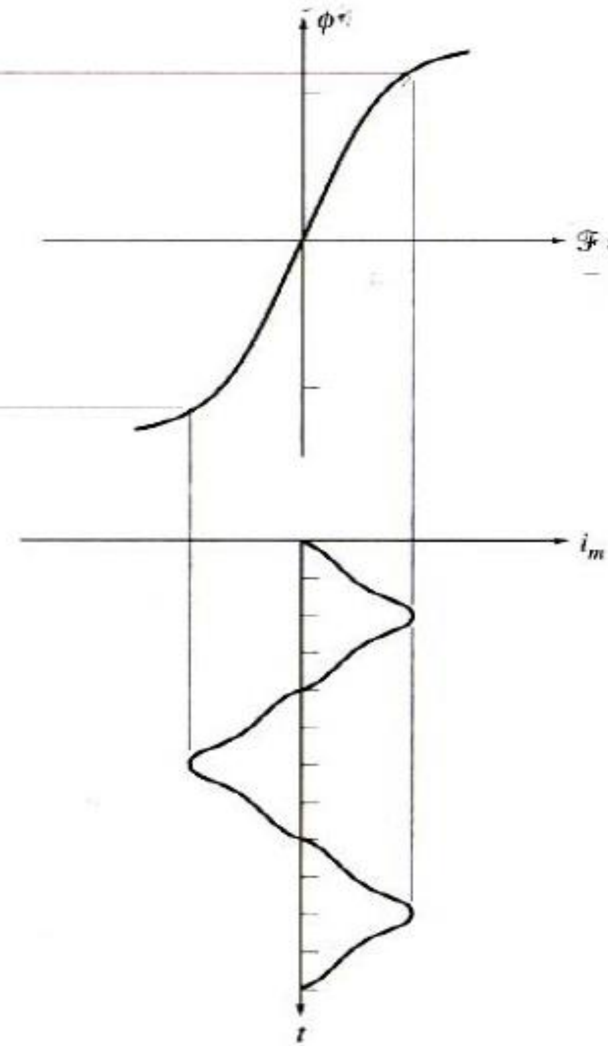


$$v_{1,\max} = -\frac{d\Phi_{1,\max}}{dt}$$

$$v_{1,peak} = \omega B_{sat} A$$

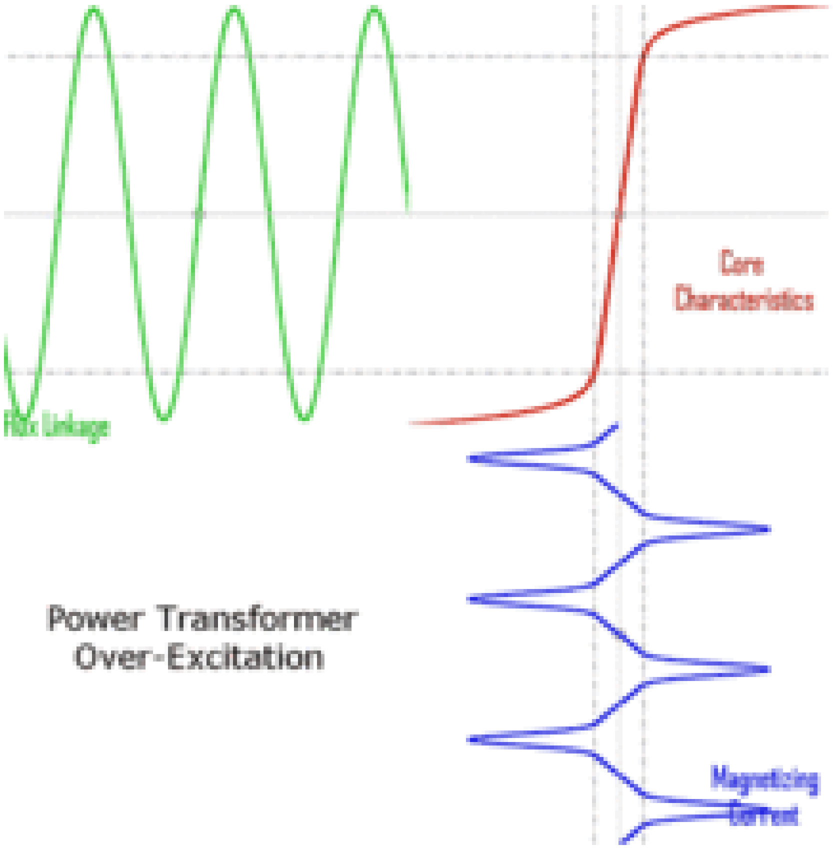
$$i_1 = \frac{H l_{core}}{N_1}$$



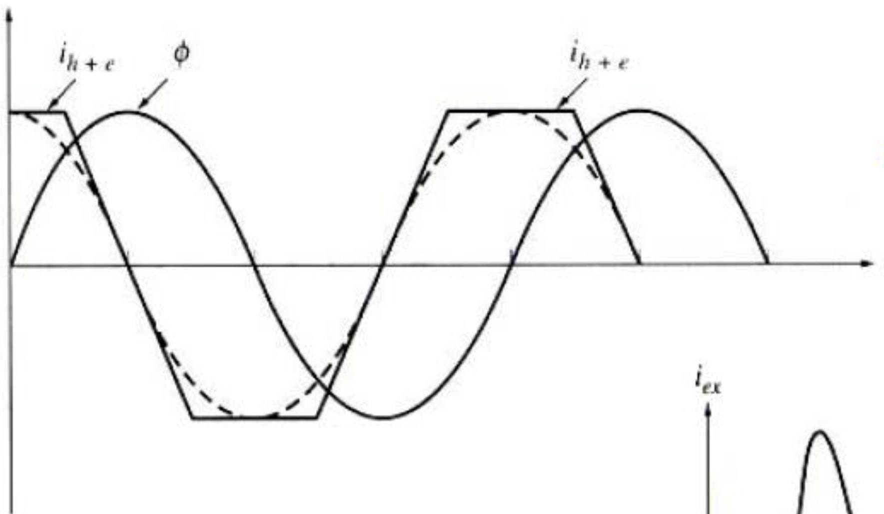


If v_1 goes beyond this range, however, H is above H_{sat}, and the effective inductance seen at the primary becomes small. The flux is also not well-confined to the core. Because of the reduced inductance, the current in the primary becomes large in these peak parts of the cycle.

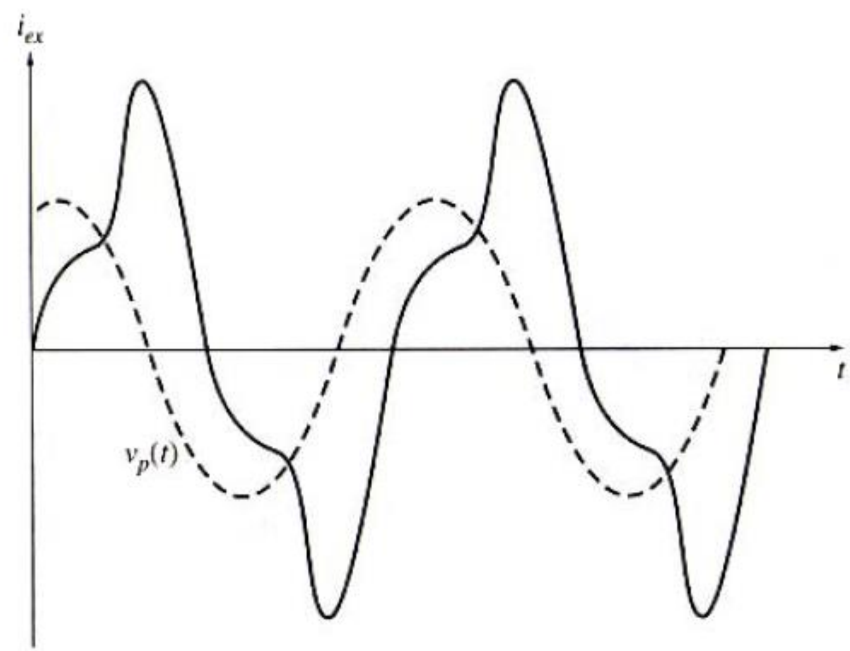
Magnetizing current



Excitation current

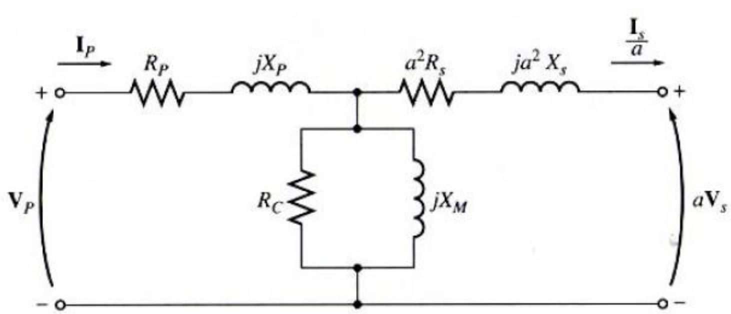


- Core-loss current



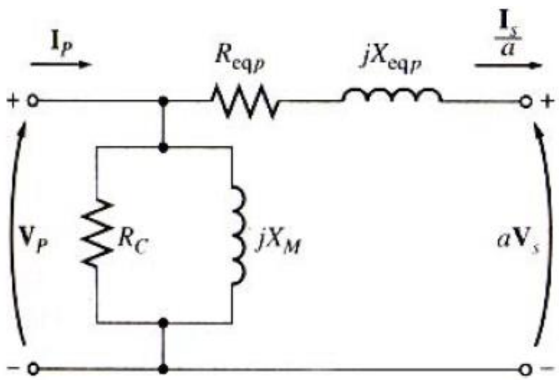
Total excitation current

$$i_{ex} = i_m + i_{h+e}$$

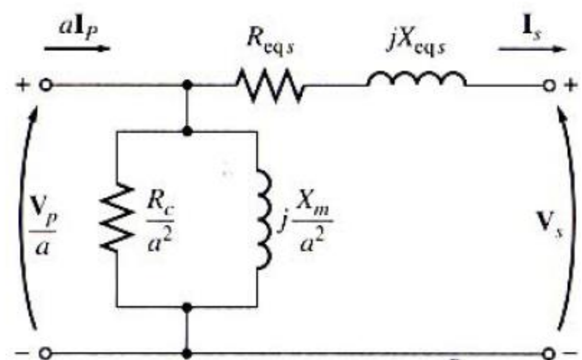


Transformer Modeling

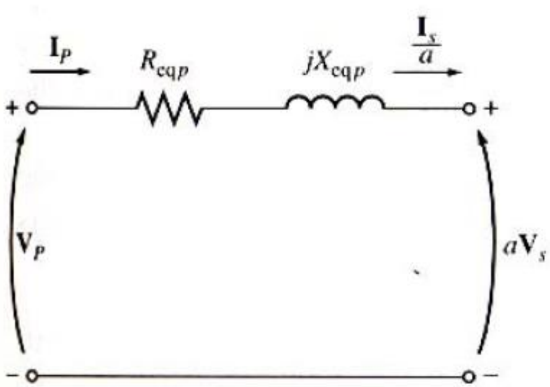
Approximate equivalent circuits

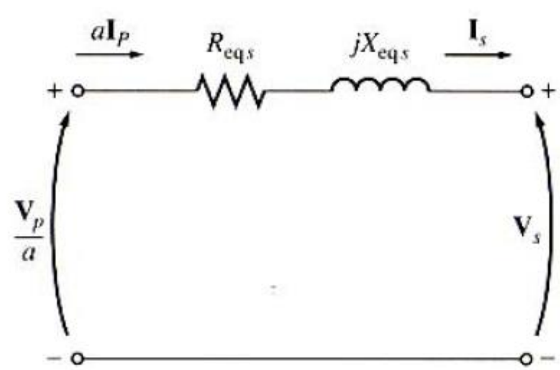


$$R_{eqp} = R_p + a^2 R_s$$
$$X_{eqp} = X_p + a^2 X_s$$



$$R_{eqs} = \frac{R_p}{a^2} + R_s$$
$$X_{eqs} = \frac{X_p}{a^2} + X_s$$

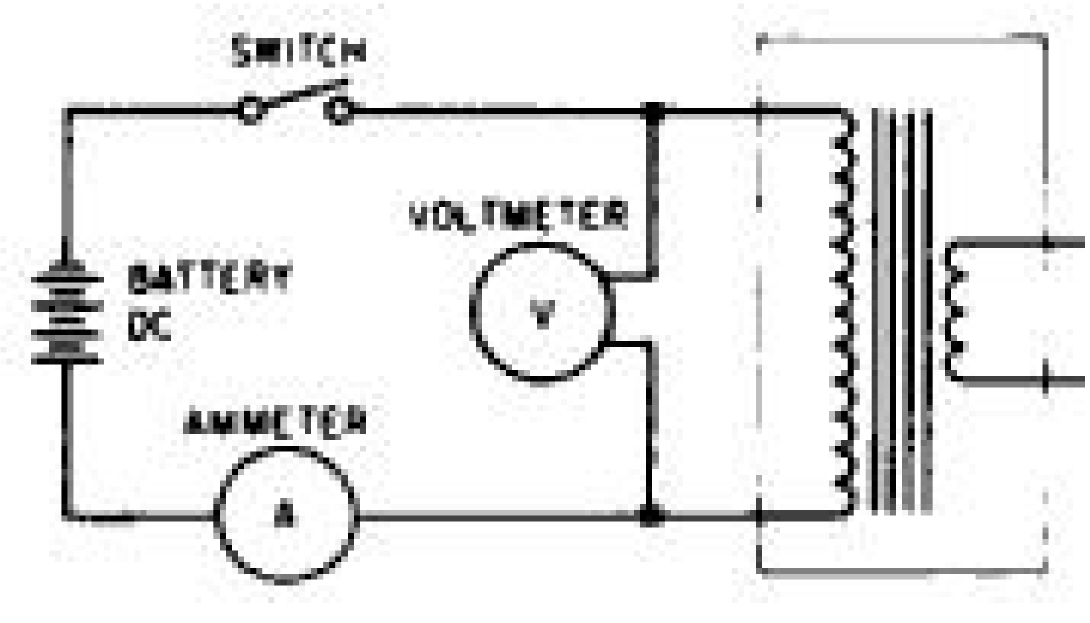




Determination of Equivalent Circuit

1-D.C. test

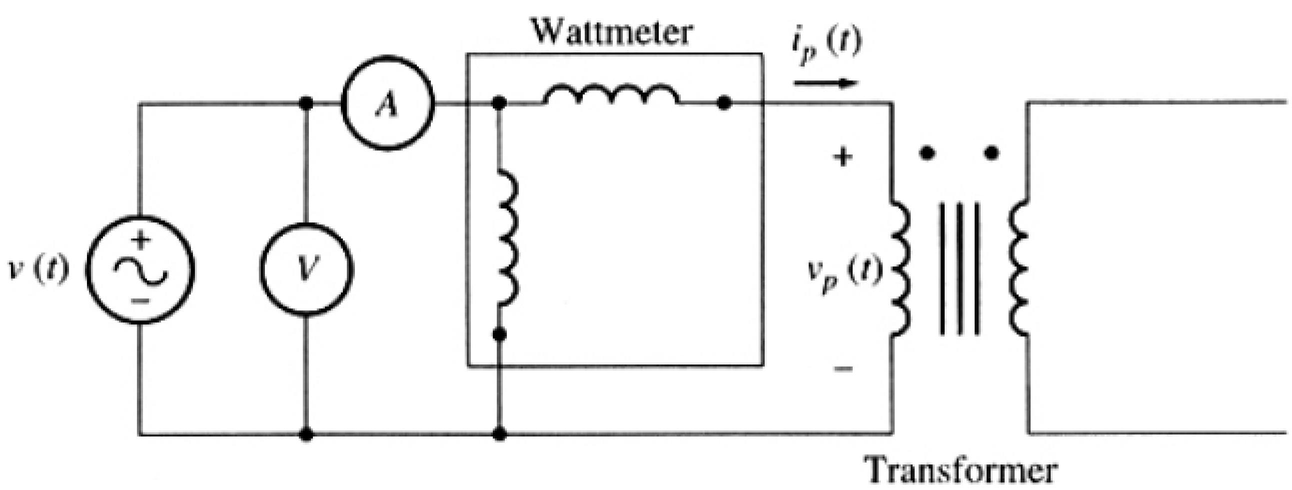
- This test is carried out to measure the resistance of each winding
- The resistance of each winding is obtained from the Ohms Law $R = E/I$



Determination of Equivalent Circuit

2- Open Circuit Test

- The secondary winding is left open
- The primary winding is connected to a full-rated voltage
- Measure V, I, P
- Want to find R_C and X_M



Determination of Equivalent Circuit

2- Open Circuit Test

- Conductance of the core-loss resistor

$$G_C = 1/R_C$$

- Susceptance of the magnetizing inductor

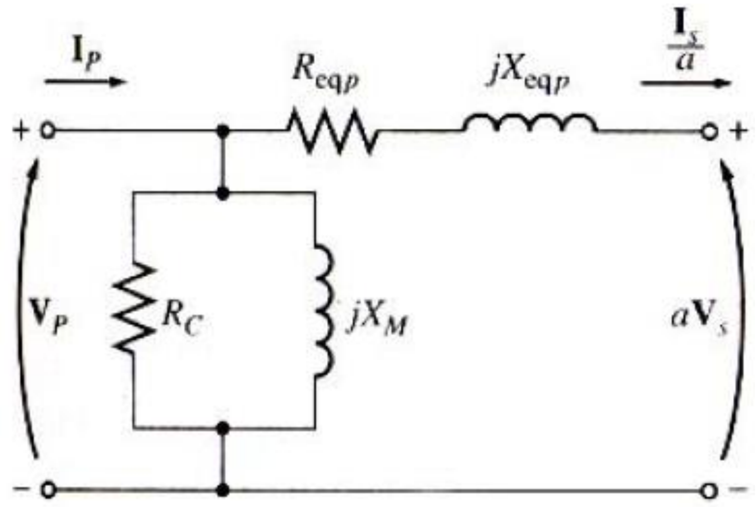
$$B_M = 1/X_M$$

- Total admittance

$$Y_E = G_C - jB_M = \frac{1}{R_C} - j\frac{1}{X_M}$$

- The admittance magnitude

$$|Y_E| = \frac{I_{OC}}{V_{OC}}$$



Determination of Equivalent Circuit

2- Open Circuit Test

- The angle
  - First we find power factor $PF = \cos\theta = \dfrac{P_{OC}}{V_{OC} I_{OC}}$
  - Then the power-factor angle is given by

$$\theta = \cos^{-1}\left(\frac{P_{OC}}{V_{OC} I_{OC}}\right)$$

- In a real transformer, the PF is always lagging

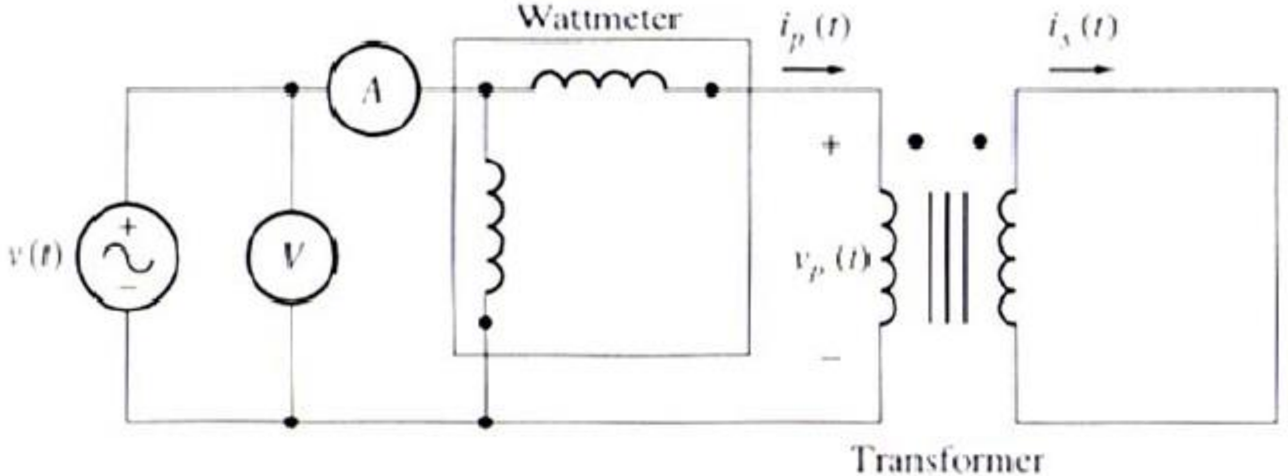
$$Y_E = \frac{I_{OC}}{V_{OC}} \angle -\theta = \frac{I_{OC}}{V_{OC}} \angle -\cos^{-1} PF$$

- The values of R_C and X_M can be determined

Determination of Equivalent Circuit

3- Short Circuit Test

- The secondary terminals are shorted
- The input voltage is adjusted until the current is equal to its rated value
- Measure V, I, P
- Want to find R_{eq} and X_{eq}



Determination of Equivalent Circuit

3- Short Circuit Test

- Series impedance referred to primary

$$|Z_{SE}| = \frac{V_{SC}}{I_{SC}}$$

- Power factor

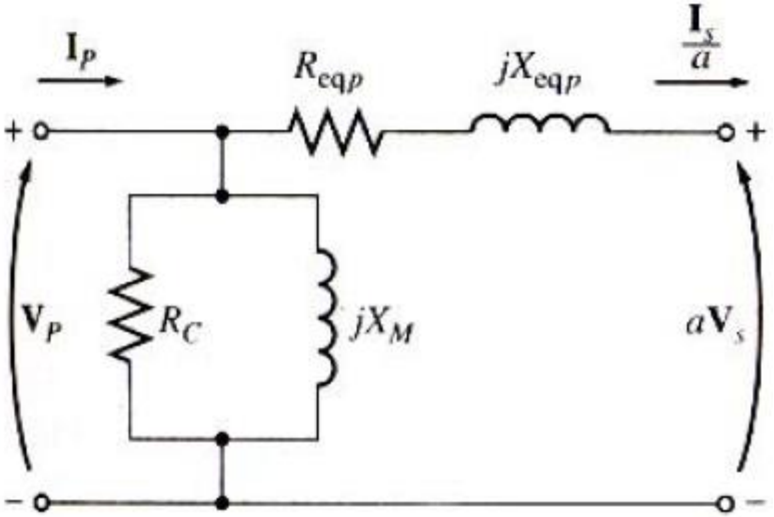
$$PF = \cos\theta = \frac{P_{SC}}{V_{SC} I_{SC}}$$

- The impedance angle

$$Z_{SE} = \frac{V_{SC}\angle 0^\circ}{I_{SC}\angle -\theta^\circ} = \frac{V_{SC}}{I_{SC}}\angle\theta^\circ$$

- and

$$Z_{SE} = R_{eq} + jX_{eq}$$

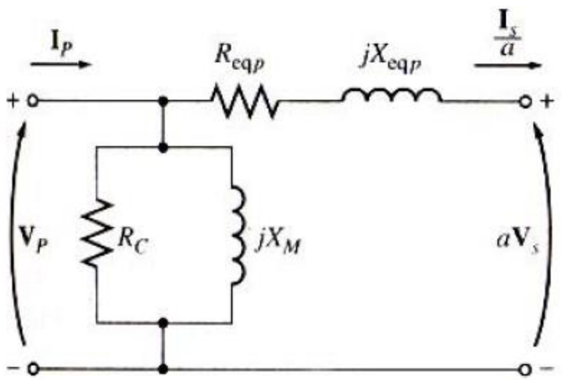


$$R_{eqp} = R_p + a^2 R_s$$

$$X_{eqp} = X_p + a^2 X_s$$

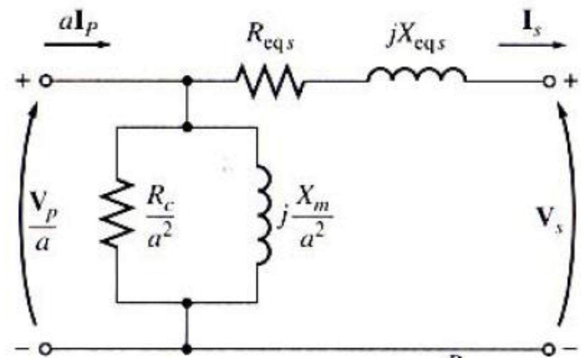
Determination of Equivalent Circuit

Approximate equivalent circuits



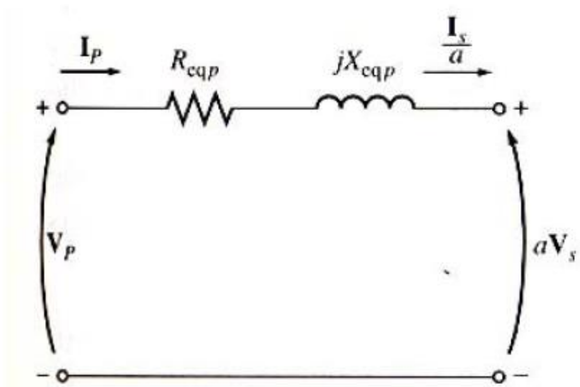
$$R_{\text{eq}p} = R_p + a^2R_s$$

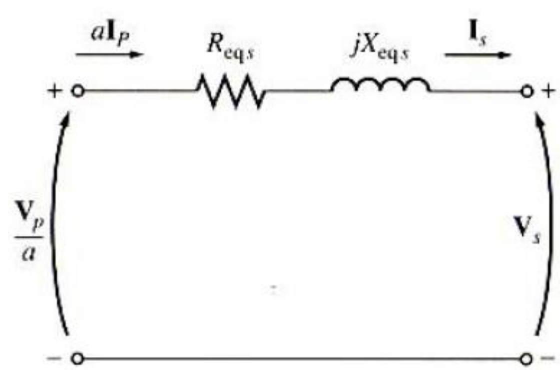
$$X_{\text{eq}p} = X_p + a^2X_s$$



$$R_{\text{eq}s} = \frac{R_p}{a^2} + R_s$$

$$X_{\text{eq}s} = \frac{X_p}{a^2} + X_s$$





Determination of Equivalent Circuit

Example

- A 20 kVA, 8000/240 V, 60 Hz transformer has been tested with the following data

Open-circuit test (on primary)	Short-circuit test (on primary)
Voc = 8000 V	Vsc = 489 V
Ioc = 0.214 A	Isc = 2.5 A
Poc = 400 W	Psc = 240 W

- Find the equivalent circuit referred to the primary side

Determination of Equivalent Circuit

Example

- Open-circuit test
  - Power factor

$$PF = \cos\theta = \frac{P_{OC}}{V_{OC} I_{OC}} = \frac{400\text{W}}{(8000\text{V})(0.214\text{A})} = 0.234 \text{ lagging}$$

  - Excitation admittance

$$Y_E = \frac{I_{OC}}{V_{OC}} \angle -\cos^{-1} PF = \frac{0.214}{8000} \angle -\cos^{-1} 0.234$$

$$= 0.0000063 - j0.0000261 = \frac{1}{R_C} - j\frac{1}{X_M}$$

  - Therefore

$$R_C = 159 \text{ k}\Omega$$

$$X_M = 38.4 \text{ k}\Omega$$

Determination of Equivalent Circuit

Example

- Short-circuit test
  - Power factor

$$PF = \cos\theta = \frac{P_{SC}}{V_{SC} I_{SC}} = \frac{240\text{W}}{(489\text{V})(2.5\text{A})} = 0.196 \text{ lagging}$$

  - Series impedance

$$Z_{SE} = \frac{V_{SC}}{I_{SC}} \angle \cos^{-1} PF = \frac{489}{2.5} \angle 78.7$$
$$= 38.4 + j192$$

  - Therefore

$$R_{eq} = 38.4\ \Omega$$
$$X_{eq} = 192\ \Omega$$

Determination of Equivalent Circuit

Example

- The equivalent circuit