1. Chart, scatter chart

   Description automatically generateda) k\_1(E) = 6.829636e+03

Average = 2.582577e+02

Median = 1.962242e+00

Maximum = 3.487956e+03

From what we can see, the upper bound k\_1(E) holds and the maximum is under the amount.

Because E is less random, and more constant numbers, and perturbation is much than A=H, which we can see below.

Graphical user interface, chart, scatter chart

Description automatically generated

k\_1(H) = 2.907028e+07

Average = 7.429396e+05

Median = 2.561129e+00

Maximum = 1.495106e+07

From what we can see, the upper bound k\_1(H) holds and the maximum is under the amount.

The amount of error is much higher that A=E probably due to rounding of the small decimal places.

Chart, scatter chart

Description automatically generated

1. a. cond(E,1) = 6.829636363636511e+03

1/rcond(E) = 6.829636363636509e+03

Average = 6.844433e+02

Median = 7.460096e+02

Maximum = 1.572475e+03

Here, we noticed that when calculating the difference between cond(E,1) and 1/rcond(E), there should have been the same number. But instead, they are slightly different, probably because some sort of computer error. They remain the upper bound of the system as expected.

Chart, scatter chart

Description automatically generated b. cond(H,1) = 2.907027900294877e+07

1/rcond(H) = 2.907027900294877e+07

Average = 1.397161e+06

Median = 8.115551e+05

Maximum = 6.259241e+06

Here, the errors cond(H,1) and 1/rcond(H) is much less, probably because the errors in H is much higher than A=E, just like what we experiences in question 1a. They theory of the upper bound holds.

2. a. (Operations performed on M1 Macbook)

For A=E

norm(A\*AINV-I,1) = 1.492139745096210e-13

norm(AC-A,1) = 1.286970530145481e-12

b.

For A=H

norm(A\*AINV-I,1) = 4.336470738053322e-09

norm(AC-A,1) = 3.164271622502213e-11

norm(AINV-HI,1) = 1.203581589834357e-03

c.

norm(A\*AINV-I,1) = 5.997717380523682e-06

norm(AC-A,1) = 5.834695104844911e-08

norm(AINV-HI8,1) = 1.010997486004489e+02

cond(H8,1) = 3.387279082022742e+10

From this, we can conclude that A=E is more accurate than A=H as predicted from Q1.

For Q2c, we noticed that the conditional number is high, which means calculations are probably not very accurate. They is probably the reason why norm(AINV-HI8,1) is such an unusually high number.

Table

Description automatically generatedOne this that was surprising was the error I was getting using different hardware. Below is the result of running E\*inv(E) on different systems.

Table

Description automatically generatedAbove: Matlab Online Below: M1 Macbook

Table

Description automatically generated with medium confidence

Above: Windows 10 (AMD Ryzen)

Due to this incredibly small difference, the final answers won’t be the same, and the difference will be incredibly small. If we round the matrix, we can get the theoretical expected matrix for A\*inv(A)=I , which Matlab is unable to do.

Table

Description automatically generated

1A: A=E

n=100;

X = zeros(n,6);

Y = zeros(n,6);

ERR = zeros(n,1);

iteration = 1;

for j = 1:10

b = B(:,j);

for k = 1:10

d = D(:,k);

x = (E\b);

y = E\(b+epsilon\*d);

ERR(iteration) = (norm(y-x,1)/norm(x,1))/(norm(epsilon\*d,1)/norm(b,1));

iteration = iteration + 1;

end

end

fprintf('k\_1(E) = %e\n',cond(E,1));

fprintf('Average = %e\n', mean(ERR));

fprintf('Median = %e\n', median(ERR));

fprintf('Maximum = %e\n', max(ERR));

hold on;

yline(cond(E,1),'-','k\_1(E)');

title('Perturbations in the right-hand side A=E');

ylabel('Amplification');

xlabel('Experiment');

plot(ERR, 'linestyle', 'none', 'marker', '.');

set(gca, 'YScale', 'log');

1A: A=H

n=100;

X = zeros(n,6);

Y = zeros(n,6);

ERR = zeros(n,1);

iteration = 1;

for j = 1:10

b = B(:,j);

for k = 1:10

d = D(:,k);

x = (H\b);

y = H\(b+epsilon\*d);

ERR(iteration) = (norm(y-x,1)/norm(x,1))/(norm(epsilon\*d,1)/norm(b,1));

iteration = iteration + 1;

end

end

fprintf('k\_1(H) = %e\n',cond(H,1));

fprintf('Average = %e\n', mean(ERR));

fprintf('Median = %e\n', median(ERR));

fprintf('Maximum = %e\n', max(ERR));

hold on;

yline(cond(H,1),'-','k\_1(H)');

title('Perturbations in the right-hand side A=H');

ylabel('Amplification');

xlabel('Experiment');

plot(ERR, 'linestyle', 'none', 'marker', '.');

set(gca, 'YScale', 'log');

1B: A=E

format long;

n=60;

X = zeros(n,6);

Y = zeros(n,6);

ERR = zeros(n,1);

iteration = 1;

for j = 1:10

b = B(:,j);

for k = 1:6

C = BIGC(:,:,k);

x = (E\b);

z = (E+epsilon\*C)\b;

ERR(iteration) = (norm(z-x,1)/norm(x,1))/(norm(epsilon\*C,1)/norm(E));

iteration = iteration + 1;

end

end

fprintf('cond(E,1) = %.15e\n',cond(E,1));

fprintf('1/rcond(E) = %.15e\n',1/rcond(E));

fprintf('Average = %e\n', mean(ERR));

fprintf('Median = %e\n', median(ERR));

fprintf('Maximum = %e\n', max(ERR));

hold on;

yline(cond(E,1),'-', 'displayname', 'k\_1(E)');

yline(1/rcond(E),'-','displayname','1/k\_1(E)');

legend;

plot(ERR, 'linestyle', 'none', 'marker', '.');

title('Perturbations of the matrix A=E');

ylabel('Amplification');

xlabel('Experiment');

set(gca, 'YScale', 'log');

1B: A=H

n=60;

ERR = zeros(n,1);

iteration = 1;

for j = 1:10

b = B(:,j);

for k = 1:6

C = BIGC(:,:,k);

x = (H\b);

z = (H+epsilon\*C)\b;

ERR(iteration) = (norm(z-x,1)/norm(x,1))/(norm(epsilon\*C,1)/norm(H));

iteration = iteration + 1;

end

end

fprintf('cond(H,1) = %.15e\n',cond(H,1));

fprintf('1/rcond(H) = %.15e\n',1/rcond(H));

fprintf('Average = %e\n', mean(ERR));

fprintf('Median = %e\n', median(ERR));

fprintf('Maximum = %e\n', max(ERR));

hold on;

yline(cond(H,1),'-', 'displayname', 'k\_1(H)');

yline(1/rcond(H),'-','displayname','1/k\_1(H)');

legend;

plot(ERR, 'linestyle', 'none', 'marker', '.');

title('Perturbations of the matrix A=H');

ylabel('Amplification');

xlabel('Experiment');

set(gca, 'YScale', 'log');

2A

format shortE;

A = E;

AINV = inv(A);

AC = inv(AINV);

I = eye(6);

disp('For A=E');

fprintf('norm(A\*AINV-I,1) = %.15e\n',norm(A\*AINV-I,1));

fprintf('norm(AC-A,1) = %.15e\n', norm(AC-A,1));

2B

format shortE;

A = H;

AINV = inv(A);

AC = inv(AINV);

I = eye(6);

disp('For A=H');

fprintf('norm(A\*AINV-I,1) = %.15e\n',norm(A\*AINV-I,1));

fprintf('norm(AC-A,1) = %.15e\n',norm(AC-A,1));

fprintf('norm(AINV-HI,1) = %.15e\n',norm(AINV-HI,1));

2C

format shortE;

A = H8;

AINV = inv(A);

AC = inv(AINV);

I = eye(8);

fprintf('norm(A\*AINV-I,1) = %.15e\n',norm(A\*AINV-I,1));

fprintf('norm(AC-A,1) = %.15e\n',norm(AC-A,1));

fprintf('norm(AINV-HI8,1) = %.15e\n',norm(AINV-HI8,1));

fprintf('cond(H8,1) = %.15e\n',cond(H8,1));