1. Find the maximum value of $3x - y + 6$ on the circle $x^2 + y^2 = 4$. (by Lagrange Multipliers.)

解：设 $f(x,y)=3x-y+6$，$g(x,y)=x^2+y^2-4=0$.

$\nabla f=\lambda\nabla g$，$\nabla f=(3,-1)$，$\nabla g=(2x,2y)$

$\therefore \begin{cases} 3=2\lambda x \\ -1=2\lambda y \\ x^2+y^2=4 \end{cases} \Rightarrow x=\frac{3}{2\lambda},\ y=-\frac{1}{2\lambda},\ \frac{9}{4\lambda^2}+\frac{1}{4\lambda^2}=4 \Rightarrow \lambda=\pm\frac{\sqrt{10}}{4}.$

$\Rightarrow$ 当 $\lambda=\frac{\sqrt{10}}{4}$ 时，$x=\frac{6}{\sqrt{10}}$，$y=-\frac{2}{\sqrt{10}}$；当 $\lambda=-\frac{\sqrt{10}}{4}$ 时，$x=-\frac{6}{\sqrt{10}}$，$y=\frac{2}{\sqrt{10}}$

$f(\frac{6}{\sqrt{10}},-\frac{2}{\sqrt{10}})=2\sqrt{10}+6$，$f(-\frac{6}{\sqrt{10}},\frac{2}{\sqrt{10}})=-2\sqrt{10}+6$

$\therefore$ 最大值为 $2\sqrt{10}+6$.

2.Use Taylor's formula for $f(x,y)=xe^y$ at the origin to find quadratic approximations of f near the origin.

解：$f_x=e^y$，$f_y=xe^y$，$f_{xx}=0$，$f_{yy}=xe^y$，$f_{xy}=e^y$

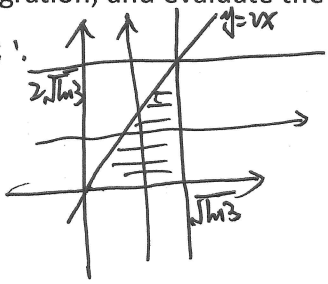
$f(0,0)=0$，$f_x(0,0)=1$，$f_y(0,0)=0$，$f_{xx}(0,0)=0$，$f_{yy}(0,0)=0$，$f_{xy}(0,0)=1$

$\therefore$ 二次拟合多项式 $Q(x,y)=f(0,0)+f_x(0,0)x+f_y(0,0)y+\frac{1}{2!}[f_{xx}(0,0)x^2+2f_{xy}(0,0)xy+f_{yy}(0,0)y^2]=0+x+0+\frac{1}{2}[0+2xy+0]$

$=x+xy$

3. Sketch the region of integration for the integral $\int_0^{2\sqrt{\ln 3}}\int_{y/2}^{\sqrt{\ln 3}} e^{x^2}\,dx\,dy$, reverse the order of integration, and evaluate the integral.

解：



$$\int_0^{2\sqrt{\ln 3}}\int_{y/2}^{\sqrt{\ln 3}} e^{x^2}\,dx\,dy$$

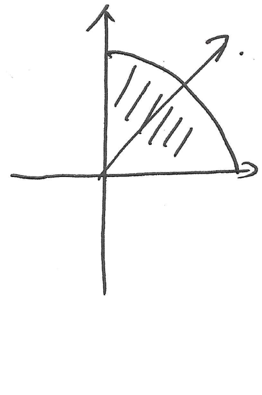
$$=\int_0^{\sqrt{\ln 3}}\int_0^{2x} e^{x^2}\,dy\,dx=\int_0^{\sqrt{\ln 3}}\left[e^{x^2}y\right]_{y=0}^{y=2x}dx$$

$$=\int_0^{\sqrt{\ln 3}}(e^{x^2}2x)\,dx=\int_0^{\sqrt{\ln 3}}e^{x^2}\,dx^2=\left[e^{x^2}\right]_0^{\sqrt{\ln 3}}=3-1$$

$$=2$$

4. Change the Cartesian integral into an equivalent polar integral for the integral $\int_0^1\int_0^{\sqrt{1-y^2}}(x^2+y^2)\,dx\,dy$. Then evaluate the polar integral.

解：



$\begin{cases} x=r\cos\theta \\ y=r\sin\theta \end{cases}$

$$\int_0^1\int_0^{\sqrt{1-y^2}}(x^2+y^2)\,dx\,dy$$

$$=\int_0^{\frac{\pi}{2}}\int_0^1 r^2\cdot r\,dr\,d\theta$$

$$=\int_0^{\frac{\pi}{2}}\left[\frac{r^4}{4}\right]_{r=0}^{r=1}d\theta$$

$$=\frac{1}{4}\cdot\frac{\pi}{2}=\frac{\pi}{8}.$$