1.(a) Find the series' radius and the interval of convergence; (b)Find what values of x does the series converge absolutely; (c)Find what values of x does the series converge conditionally. (8 points)

(1) $\sum\limits_{n=0}^{\infty} \frac{3^n x^n}{n!}$ (4 points)

Solution: $\lim\limits_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| < 1 \Rightarrow |x| \lim\limits_{n\to\infty}\left|\frac{3^{n+1}\cdot n!}{3^n\cdot(n+1)!}\right| < 1 \Rightarrow |x|\left(\lim\limits_{n\to\infty}\frac{3}{n+1}\right) < 1$ for all x.

(a) the radius $R=\infty$; the series converges for all x.

(b) the series converges absolutely for all x.

(c) there are no values for which the series converges conditionally.

(2) $\sum\limits_{n=2}^{\infty} \frac{x^n}{n(\ln n)^2}$ (4 points)

Solution: $\lim\limits_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| < 1 \Rightarrow |x| \lim\limits_{n\to\infty} \frac{n(\ln n)^2}{(n+1)[\ln(n+1)]^2} < 1 \Rightarrow |x|\left(\lim\limits_{n\to\infty}\frac{n}{n+1}\right)\left(\lim\limits_{n\to\infty}\frac{\ln n}{\ln(n+1)}\right)^2 < 1$

$\Rightarrow |x|\left(\lim\limits_{n\to\infty}\frac{1}{1+\frac{1}{n}}\right)\cdot\left(\lim\limits_{n\to\infty}\frac{\frac{1}{n}}{\frac{1}{n+1}}\right)^2 < 1 \Rightarrow |x|<1 \Rightarrow -1<x<1.$

when $x=-1$, $\sum\limits_{n=2}^{\infty}(-1)^n\cdot\frac{1}{n(\ln n)^2}$, let $f(x)=\frac{1}{x(\ln x)^2}$ is positive, continuous, and decreasing for $x\geq 2$

$\int_2^{\infty}\frac{dx}{x(\ln x)^2} = \int_2^{\infty}\frac{d(\ln x)}{(\ln x)^2} = \lim\limits_{b\to\infty} -\frac{1}{\ln x}\Big]_2^{\infty} = \frac{1}{\ln 2} \Rightarrow \sum\limits_{n=2}^{\infty}\frac{1}{n(\ln n)^2}$ converges $\Rightarrow \sum\limits_{n=2}^{\infty}(-1)^n\frac{1}{n(\ln n)^2}$ converges absolutely

when $x=1$, $\sum\limits_{n=2}^{\infty}\frac{1}{n(\ln n)^2}$ converges.

(a) the radius $R=1$; the interval of convergence is $-1\leq x\leq 1$; (b) the interval of absolute convergence is $-1\leq x\leq 1$;

(c) there are no values for which the series converges conditionally

2.Find the Taylor series at x=a of the following functions. (7 points)

(1) $f(x) = \frac{1}{x}$, $a = 3$ (4 points)

Solution: $\frac{1}{1+x} = \sum\limits_{n=0}^{\infty}(-1)^n x^n = 1-x+x^2-\cdots+(-x)^n+\cdots$, $-1<x<1$

$f(x)=\frac{1}{x}=\frac{1}{3+(x-3)} = \frac{1}{3}\cdot\frac{1}{1+\frac{x-3}{3}} = \frac{1}{3}\sum\limits_{n=0}^{\infty}(-1)^n\left(\frac{x-3}{3}\right)^n = \sum\limits_{n=0}^{\infty}(-1)^n\cdot\frac{(x-3)^n}{3^{n+1}}$,

$=\frac{1}{3}-\frac{x-3}{3^2}+\frac{(x-3)^2}{3^3}-\cdots+\frac{(-1)^n(x-3)^n}{3^{n+1}}+\cdots$

$-1<\frac{x-3}{3}<1 \Rightarrow -3<x-3<3 \Rightarrow 0<x<6$.

(2) $f(x) = \ln(1+x^2)$, $a = 0$ (3 points)

Solution: $\ln(1+x) = \sum\limits_{n=1}^{\infty}\frac{(-1)^{n-1}x^n}{n} = x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots+(-1)^{n-1}\frac{x^n}{n}+\cdots$, $-1<x\leq 1$.

$f(x)=\ln(1+x^2)=\sum\limits_{n=1}^{\infty}\frac{(-1)^{n-1}x^{2n}}{n} = x^2-\frac{x^4}{2}+\frac{x^6}{3}-\cdots+(-1)^{n-1}\frac{x^{2n}}{n}+\cdots$, $-1\leq x\leq 1$.

when $x=1$, $\sum\limits_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}$ converges conditionally; when $x=-1$, $\sum\limits_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}$ converges conditionally