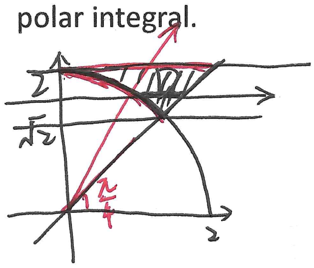
1. Change the Cartesian integral $\int_{\sqrt{2}}^{2}\int_{\sqrt{4-y^2}}^{y} dx\,dy$ into an equivalent polar integral. Then evaluate the polar integral.



解: $\int_{\sqrt{2}}^{2}\int_{\sqrt{4-y^2}}^{y} dx\,dy = \iint_D dx\,dy = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_{2}^{\frac{2}{\sin\theta}} r\,dr\,d\theta$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left[\frac{r^2}{2}\right]_{2}^{\frac{2}{\sin\theta}} d\theta$

$= \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\left(\frac{4}{\sin^2\theta} - 4\right) d\theta$

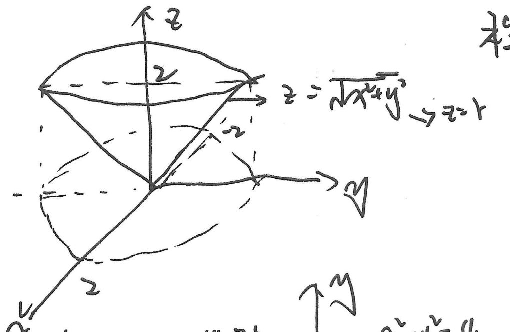
$= 2\left[\left[-\cot\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} - \frac{\pi}{4}\right]$

$= 2 - \frac{\pi}{2}$.

∵从原点出发的射线穿过D时，从 $x=\sqrt{4-y^2}$，即 $x^2+y^2=4$，即 $r=2$ 进入，从 $y=2$，即 $r\sin\theta=2$，$r=\frac{2}{\sin\theta}$ 出来.

令 $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

2. Convert the integral $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2+y^2)\,dz\,dy\,dx$ to an equivalent integral in cylindrical coordinates and evaluate the result.



柱坐标: $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z. \end{cases}$

$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2+y^2)\,dz\,dy\,dx = \int_0^{2\pi}\int_0^2\int_r^2 r^2\cdot r\,dz\,dr\,d\theta$

$= 2\pi\int_0^2 r^3[z]_{z=r}^{z=2}\,dr = 2\pi\int_0^2 r^3(2-r)\,dr$

$= 2\pi\left(\left[\frac{2r^4}{4}\right]_0^2 - \left[\frac{r^5}{5}\right]_0^2\right) = \frac{16}{5}\pi.$

在xy平面投影