# 11 Minimum Spanning Trees (February 27)

## 11.1 Introduction

Suppose we are given a connected, undirected, *weighted* graph. This is a graph $G = (V, E)$ together with a function $w\colon E \to \mathbb{R}$ that assigns a *weight* $w(e)$ to each edge $e$. For this lecture, we'll assume that the weights are real numbers. Our task is to find the *minimum spanning tree* of $G$, *i.e.*, the spanning tree $T$ minimizing the function

$$w(T) = \sum_{e \in T} w(e).$$

To keep things simple, I'll assume that all the edge weights are distinct: $w(e) \neq w(e')$ for any pair of edges $e$ and $e'$. Distinct weights guarantee that the minimum spanning tree of the graph is unique. Without this condition, there may be several different minimum spanning trees. For example, if all the edges have weight 1, then *every* spanning tree is a minimum spanning tree with weight $V - 1$.