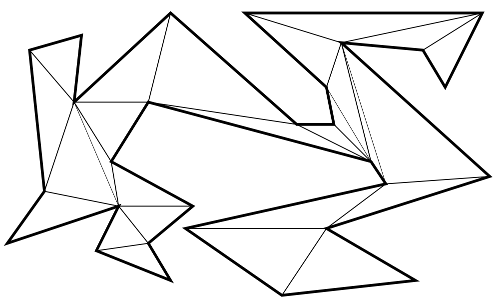
C Polygon Triangulation

C.1 Introduction

Recall from last time that a *polygon* is a region of the plane bounded by a cycle of straight edges joined end to end. Given a polygon, we want to decompose it into triangles by adding *diagonals*: new line segments between the vertices that don't cross the boundary of the polygon. Because we want to keep the number of triangles small, we don't allow the diagonals to cross. We call this decomposition a *triangulation* of the polygon. Most polygons can have more than one triangulation; we don't care which one we compute.

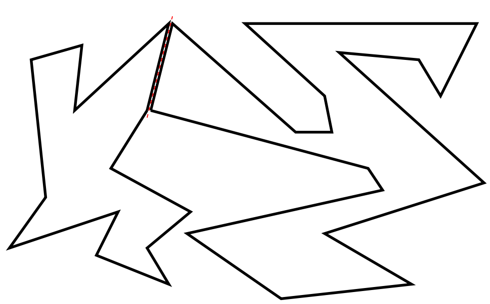


Two triangulations of the same polygon.

Before we go any further, I encourage you to play around with some examples. Draw a few polygons (making sure that the edges are straight and don't cross) and try to break them up into triangles.

C.2 Existence and Complexity

If you play around with a few examples, you quickly discover that every triangulation of an n-sided has $n-2$ triangles. You might even try to prove this observation by induction. The base case $n = 3$ is trivial: there is only one triangulation of a triangle, and it obviously has only one triangle! To prove the general case, let P be a polygon with n edges. Draw a diagonal between two vertices. This splits P into two smaller polygons. One of these polygons has k edges of P plus the diagonal, for some integer k between 2 and $n-2$, for a total of $k+1$ edges. So by the induction hypothesis, this polygon can be broken into $k-1$ triangles. The other polygon has $n-k+1$ edges, and so byt the induction hypothesis, it can be broken into $n-k-1$ tirangles. Putting the two pieces back together, we have a total of $(k-1)+(n-k-1) = n-2$ triangles.



Breaking a polygon into two smaller polygons with a diagonal.