

Obie looked at the seein' eye dog. Then at the twenty-seven 8 by 10 color glossy pictures with the circles and arrows and a paragraph on the back of each one. . . and then he looked at the seein' eye dog. And then at the twenty-seven 8 by 10 color glossy pictures with the circles and arrows and a paragraph on the back of each one and began to cry.

Because Obie came to the realization that it was a typical case of American blind justice, and there wasn't nothin' he could do about it, and the judge wasn't gonna look at the twenty-seven 8 by 10 color glossy pictures with the circles and arrows and a paragraph on the back of each one explainin' what each one was, to be used as evidence against us.

And we was fined fifty dollars and had to pick up the garbage. In the snow.

But that's not what I'm here to tell you about.

— Arlo Guthrie, “Alice’s Restaurant” (1966)

10 Basic Graph Properties (February 22)

10.1 Definitions

A *graph* G is a pair of sets (V, E) . V is a set of arbitrary objects which we call *vertices*¹ or *nodes*. E is a set of vertex pairs, which we call *edges* or occasionally *arcs*. In an *undirected* graph, the edges are unordered pairs, or just sets containing two vertices. In a *directed* graph, the edges are ordered pairs of vertices. We will only be concerned with *simple* graphs, where there is no edge from a vertex to itself and there is at most one edge from any vertex to any other.

Following standard (but admittedly confusing) practice, I’ll also use V to denote the *number* of vertices in a graph, and E to denote the *number* of edges. Thus, in an undirected graph, we have $0 \leq E \leq \binom{V}{2}$, and in a directed graph, $0 \leq E \leq V(V - 1)$.

We usually visualize graphs by looking at an *embedding*. An embedding of a graph maps each vertex to a point in the plane and each edge to a curve or straight line segment between the two vertices. A graph is *planar* if it has an embedding where no two edges cross. The same graph can have many different embeddings, so it is important not to confuse a particular embedding with the graph itself. In particular, planar graphs can have non-planar embeddings!

