

You must turn in this question sheet with your answers.**1. Déjà vu**

Prove that any positive integer can be written as the sum of distinct *nonconsecutive* Fibonacci numbers—if F_n appears in the sum, then neither F_{n+1} nor F_{n-1} will. For example: $42 = F_9 + F_6$, $25 = F_8 + F_4 + F_2$, and $17 = F_7 + F_4 + F_2$. *You must give a complete, self-contained proof, not just a reference to the posted homework solutions.*

2. L'esprit d'escalier

Recall that the *staircase* of a set of points consists of the points with no other point both above and to the right. Describe a method to maintain the staircase as new points are added to the set. Specifically, describe and analyze a data structure that stores the staircase of a set of points, and an algorithm $\text{INSERT}(x, y)$ that adds the point (x, y) to the set and returns `TRUE` or `FALSE` to indicate whether the staircase has changed. Your data structure should use $O(n)$ space, and your INSERT algorithm should run in $O(\log n)$ amortized time.

