# CS 373: Combinatorial Algorithms, Fall 2000

## Homework 1 (due November 16, 2000 at midnight)

| Name: | | |
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| Net ID: | Alias: | U $^3\!/\!_4$ 1 |

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Starting with Homework 1, homeworks may be done in teams of up to three people. Each team turns in just one solution, and every member of a team gets the same grade. Since 1-unit graduate students are required to solve problems that are worth extra credit for other students, **1-unit grad students may not be on the same team as 3/4-unit grad students or undergraduates.**

Neatly print your name(s), NetID(s), and the alias(es) you used for Homework 0 in the boxes above. Please also tell us whether you are an undergraduate, 3/4-unit grad student, or 1-unit grad student by circling U, $^3\!/\!_4$, or 1, respectively. Staple this sheet to the top of your homework.

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## Required Problems

1. Give an $O(n^2 \log n)$ algorithm to determine whether any three points of a set of $n$ points are collinear. Assume two dimensions and *exact* arithmetic.

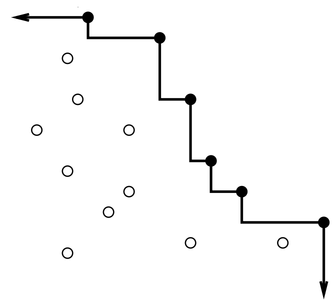
2. We are given an array of $n$ bits, and we want to determine if it contains two consecutive 1 bits. Obviously, we can check every bit, but is this always necessary?

   (a) (4 pts) Show that when $n \bmod 3 = 0$ or 2, we must examine every bit in the array. that is, give an adversary strategy that forces any algorithm to examine every bit when $n = 2, 3, 5, 6, 8, 9, \ldots$.

   (b) (4 pts) Show that when $n = 3k+1$, we only have to examine $n-1$ bits. That is, describe an algorithm that finds two consecutive 1s or correctly reports that there are none after examining at most $n-1$ bits, when $n = 1, 4, 7, 10, \ldots$.

   (c) (2 pts) How many $n$-bit strings are there with two consecutive ones? For which $n$ is this number even or odd?

3. You are given a set of points in the plane. A point is *maximal* if there is no other point both both above and to the right. The subset of maximal points of points then forms a *staircase*.



The staircase of a set of points. Maximal points are black.

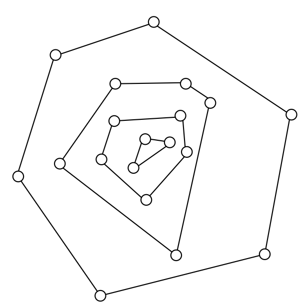
   (a) (0 pts) Prove that maximal points are *not* necessarily on the convex hull.

   (b) (6 pts) Give an $O(n \log n)$ algorithm to find the maximal points.

   (c) (4 pts) Assume that points are chosen uniformly at random within a rectangle. What is the average number of maximal points? Justify. Hint: you will be able to give an exact answer rather than just asymptotics. You have seen the same analysis before.

4. Given a set $Q$ of points in the plane, define the *convex layers* of $Q$ inductively as follows: The first convex layer of $Q$ is just the convex hull of $Q$. For all $i > 1$, the $i$th convex layer is the convex hull of $Q$ after the vertices of the first $i - 1$ layers have been removed.

   Give an $O(n^2)$-time algorithm to find all convex layers of a given set of $n$ points.



A set of points with four convex layers.

5. Prove that finding the second smallest of $n$ elements takes $n + \lceil \lg n \rceil - 2$ comparisons in the worst case. Prove for both upper and lower bounds. Hint: find the (first) smallest using an elimination tournament.