

# Countable

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# TOC

**Slides is posted on Canvas:Files. If you find any typos or have any concerns, please contact me ASAP!**

Review Cardinality

# Concepts

- ▶ **Definitions:** countable, uncountable
- ▶ Typical countably infinite sets:  $\mathbb{N}$ ,  $\mathbb{Z}$  (Example 3), odd integers (Example 1),  $\mathbb{Q}^+$  (Example 4).
- ▶ Typical uncountable sets:  $\mathbb{R}$  (Example 5).

# Function

## Theorem

*If  $A$  and  $B$  are countable sets, then  $A \cup B$  is countable.*

## Theorem (Schroder-Bernstein Theorem)

*If  $A$  and  $B$  are sets with  $|A| \leq |B|$  and  $|B| \leq |A|$ , then  $|B| = |A|$ .*

- ▶ Subset of a countable set is countable (Exercise 16).
- ▶ Superset of an uncountable set is uncountable (Exercise 15).
- ▶ Intersection of a countable set and an uncountable set is countable.
- ▶ Any interval on real numbers is uncountable (Exercise 33, Exercise 34).

# Exercise

- ▶ 1, 2, 3, 4, 11, 18, 20, 28
- ▶ For 28, also see Exercise 31 for another perspective.
- ▶ 34: Show that  $(0, 1)$  and  $\mathbb{R}$  have the same cardinality.
- ▶ 33: Open interval  $(0, 1)$  has the same cardinality to closed interval  $[0, 1]$ .