Deformation structures and norm coherence

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We generalize Ando's construction of H_{∞} complex orientations on Morava E-theories associated to Honda formal group laws over \mathbb{F}_p . We show the existence and uniqueness of such an orientation on any Morava E-theory associated to a formal group law over an algebraic extension of \mathbb{F}_p .

1 Introduction

fg vs fgl, coords, notations F_x and F_x/H (induced by Lubin isog, def, defo of Frob) Lubin-Tate E_n [Ando95, thm 2.3.1], Strickland A_r [iph, thm 4.4], univ ex of quot $F^{(p^r)}$ contravariant, omitting Spec

1.1 Acknowledgements

I thank

2 Deformation structures

2.1 Wide categories of formal groups

2.2 Deformation of isogenies and pushforward of deformation structures

bivariant: pullback and pushforward of defo strs and coords

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3 Norm coherence

3.1 Norm-coherent coordinates

Let k be an algebraic extension of \mathbb{F}_p , and G be a formal group over k of finite height n. Let F/E_n be the universal deformation of G/k, and F[p] be its subgroup of p-torsions. The latter is defined over an extension \overline{E}_n of E_n obtained by adjoining the roots of the p-series of F. Note that $F[p](\overline{E}_n)$ is stable under the action of $\operatorname{Aut}(\overline{E}_n/E_n)$. Thus, given a coordinate x on F lifting any preferred coordinate on G, the Lubin isogeny

$$f_p^x \colon F_x \to F_x/F[p]$$

can be defined over E_n (cf. [?, Theorem 1.4] and see [?, proof of Corollary 3.2] for an explicit example).

Remark 3.1 On the special fiber $\mathcal{O}_{G^{(p^n)}} = k[\![y]\!]$ over k, this Lubin isogeny is k-linear, sending g(y) to $g(x^{p^n})$. It is the relative p^n -power Frobenius and is not an endomorphism unless $k \subset \mathbb{F}_{p^n}$ (cf. [?, proof of Proposition 2.5.1]).

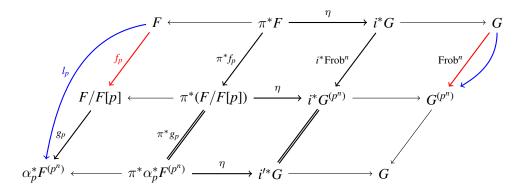
By Theorem $\ref{eq:constraints}$, in view of the above remark, we have a unique local homomorphism $\alpha_p \colon A_n \to E_n$ and a unique \star -isomorphism $g_p^x \colon F_x/F[p] \to \alpha_p^* F_x^{(p^n)}$ that classify f_p^x as a deformation of a degree- p^n isogeny. Define

$$l_p^x \colon F_x \to \alpha_p^* F_x^{(p^n)}$$

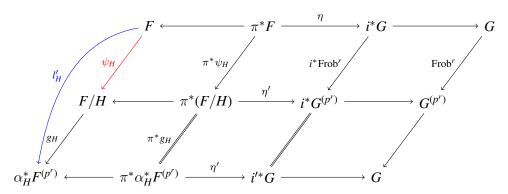
to be the composite $g_p^x \circ f_p^x$. It is uniquely characterized by the following properties (cf. [?, Proposition 2.5.4]).

- (i) The isogeny l_p^x of formal group laws has source F_x and target of the form $\alpha^* F_x^{(p^n)}$ for some ring homomorphism $\alpha \colon A_n \to E_n$.
- (ii) The kernel of l_p^x applied to $F_x(\overline{E}_n)$ is x(F[p]).
- (iii) Reducing coefficients to the residue field transforms l_p^x to the relative p^n -power Frobenius.

Explicitly, f_p^x and l_p^x fit into the following commutative diagram, where their restrictions over the residue field are highlighted in corresponding colors and thick arrows indicate homomorphisms.



More generally, let H be a subgroup of F of order p^r , and $\psi_H \colon F \to F/H$ be an isogeny with kernel H. For comparison, there are classifying maps α_H and g_H giving rise to the diagram below, where by definition $l'_H := g_H \circ \psi_H$ (the prime distinguishes ψ_H from the particular Lubin isogeny f_H).



3.2 Ando's construction of norm-coherent coordinates in greater generality

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References