R - torsion free

$\psi_p : R \to R$

$\psi_p(x) \equiv x^p \mod p$

$E = K$

λ-schemes

λ-spaces (étale) ~~fpqc fppf~~

Spec $\mathbb{F}_1$ = Spec $\mathbb{Z}$ w/ λ-structure

$\cancel{\mathrm{Spec}\,\mathbb{Z} \rightleftarrows \mathrm{Spec}\,\mathbb{F}_1}$

$X \times_{\mathbb{F}_1} \mathbb{Z}$

forget λ-structure = base change from $\mathbb{F}_1$ to $\mathbb{Z}$

applying Witt vectors functor = base-forgetting

$$\begin{tikzcd} X \arrow[r] \arrow[dr] & \mathrm{Spec}\,\mathbb{Z} \arrow[d] \\ & \mathrm{Spec}\,\mathbb{F}_1 \end{tikzcd}$$

$$\begin{tikzcd} \mathrm{Spec}\,\mathbb{F}_1 \arrow[r, shift left, "- \times_{\mathbb{F}_1} \mathbb{Z}"] & \mathrm{Spec}\,\mathbb{Z} \arrow[l, shift left, "\text{base-forget}"] \arrow[l, bend right, "W_*"'] \end{tikzcd}$$

base-forget $= W^*$

If $A \to B$ is étale

$W(A) \to W(B)$ is étale

BUT if $A \to B$ is flat, smooth, - -

$W(A) \to W(B)$ might not be.

$$\begin{tikzcd} & \mathcal{M}_p(n) \arrow[d] \\ E_* X \arrow[ur] \arrow[r] & \mathcal{M}_{fg}^{\le n} \end{tikzcd}$$

E E_∞-ring spectrum
p-complete, complex oriented

X e.g. $K(n)$-local

$\mathcal{M}_p(n)$ is a λ-stack
power ops

$$\begin{tikzcd} E_* X \arrow[r] \arrow[d] & \mathcal{M}_p(n) \arrow[r] \arrow[d] & \operatorname{Spec} \mathbb{Z} \arrow[d] \\ E_* X \arrow[r] & \mathcal{M}_p(n) \arrow[r] & \operatorname{Spec} \mathbb{F}_p \end{tikzcd}$$

<u>Thm (Lurie).</u> If $\mathcal{X} \to \mathcal{M}_p(n)$ is formally étale, $\mathcal{X}$ D-M stack, then there is a sheaf of spectra $\mathcal{O}$ on $\mathcal{M}_{fg}$ whose htopy groups

$$\pi_* \mathcal{O} = \omega_{\mathcal{X} \to \mathcal{M}_{fg}}^{\otimes(*/2)}$$

James Borger - "The Geometry of Witt vectors