Teaching statement

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In her book *Sleeping in Temples*, the pianist Susan Tomes writes about how the ideas and practices *in* music can be effectively communicated through teaching, and how vital this particular form of communication is *to* music as an art and discipline. Mathematics is also an art and a discipline, and I've found that it naturally imposes a rhythm and direction to how I can learn better and communicate more.

"Tracing the curve of the waves with the tip of his violin bow"

That was how the mighty Hungarian violinist Sándor Végh, looking out to sea, often spoke to his young students in Prussia Cove about the ebb and flow of music, the type of movement inside a piece. There seems to me a similar mechanism in communicating mathematics.

Math 230, differential multivariable calculus, is a class taught frequently. A great feature of it that I like, as I always told my students, is that we get to draw a lot of pictures. In the first class, to motivate the passage from 2-D to 3-D, I start by drawing curves—a circle, a parabola, two branches of a hyperbola—and rotating each of them to form a surface. Already in the last case, two distinct "hyperboloids" are born depending on which axis we choose to rotate a hyperbola along. Now here's a question: What kind of surfaces can we obtain from two straight lines? Over the years, it took varying time for the students to "jump out of the blackboard" and begin thinking about the possibility of two lines that are neither parallel nor intersecting. Meanwhile, new ideas and insights kept popping up: "a washer," which I didn't initially get, and, more recently, "a DNA chain!" To me, such responses were illuminating, both in the math that was being communicated, and in *how* it could be communicated.

More recently I taught Math 300, a transition course from calculus to "higher mathematics" that emphasized how to write proofs and centered on foundational notions such as sets, functions, and equivalence relations. Instead of the traditional form of lectures, most of the time it was the students who gave presentations of their proofs to flesh out various abstract concepts. There was a livelier exchange of ideas between the speakers and their audience. To visualize partial orderings using trees one has just learned from a graph theory course, to imagine an infinite set with greater cardinality as particles that are expanding faster, . . . it was invigorating to see and listen to these ideas being explained, questioned, and further explained.

A new dimension came along: how to *present* your math? By the end of the first round of presentations, I encouraged everyone to try and free themselves from referring to notes, by extracting and absorbing the key ideas and organizing their proofs in a more structural way. That was actually a challenge. Then, after the second round, I proposed the speakers be more relaxed and try to engage their audience better. Almost immediately, we all noticed how changes happened, small and big. One of the last presentations was so well-paced—slow but crystal clear—that it reminded me of the style of a mathematician in my field, a charismatic speaker. It seemed that no one wanted to interrupt the talk with a question because we were all taken with the clarity and flow. Besides their substantial work behind this individually (and with me), the progress of the class as a whole had to do with the dynamics that potentially underlie every experience of learning (and of teaching).

"It was all part of that great 'chain of musicians' . . . "

When I first began lecturing, consciously and unconsciously I tried to recall what my own teachers had done and how I'd reacted as their student. I was lucky to take a differential geometry course with an inspiring professor, who was a student of Shiing-Shen Chern. One particularly memorable thing from his class was the quick drawing of four arcs to form a curved surface—that seemed to be the arena on which everything developed. The professor would draw it over and over again, which generated a certain aesthetic aspect as well, and this picture was gradually built into my mind so that things around it felt concrete and organic. Now, in the multivariable calculus class I teach, the four-arc surface has become an imagery thread that affords and runs through "tangent planes," "chain rules," "directional derivatives," and "gradient vectors." It made its way from his blackboard to mine.

Sometimes things happened less visibly. On the first day of Math 300, to give an idea of the course, I drew a curve in the shape of $y = e^x$, and flagged the point $(0, 1)$ as "We are here"—a transitional stage between the culmination of calculus and a vastly increasing amount of more advanced mathematics. I then told the students that actually this picture wasn't quite true, and drew a second non-smooth curve in the shape of a heart rate graph that kept climbing up. I explained to them that, given my own experience, the accumulation of one's math knowledge and sophistication could be a highly nonlinear and rough-edged process. Specifically, I pointed to the first big jump on the graph, which corresponded to my first rigorous training of writing proofs in a mathematical analysis course using, in particular, a challenging textbook chosen by my professor.

Preparing for this class brought back to mind my struggle as an undergraduate: how I slowly trudged through the exercises in the chapter "Topology" of my textbook,[1] formulating one proof and then another, and how I gradually found the effort deeply rewarding and my "future" subject particularly beautiful. It also brought to mind the man who taught me this book—his physical largeness and his resonant voice, especially the accentuated way he pronounced the names of the mathematical giants, "David Hilbert," "Hermann Weyl," as he referred every once in a while during his lectures. His sound waves hit me strong enough to indicate that each of those names was the tip of an iceberg. These recollections prompted me to think about how to orient my own teaching from a long-term perspective for my students, who more or less were in the same phase as I had.

A surprise came later in Math 300. One of my students emailed me a six-page paper he wrote and typed up, titled "The exclusiveness of partial and linear orderings," in which he developed a different system of axioms

[1] A mimeographed copy of [Browder 1996].

than the one introduced by the textbook, and explained, from several perspectives, the intuition behind his axioms. What he acknowledged as "challenging me to think" was, honestly, an email I'd written to him after his in-class presentation. In it, I explained my thoughts on points where confusion had arisen, in a way a particular professor did to me when I was his student, for years. So it simply echoed when I read [Tomes 2014, p.151]:

> *Not so very long afterwards, it seemed, we were recalling his words in a more sober spirit of passing them on to our own students. We began to understand why it was important to say things over and over again. And there came a point when we realised that his speech about 'knowing you were a link in a chain going back to Brahms' was not just amusing but actually true.*

References

[Browder 1996] Andrew Browder, *Mathematical analysis*, Undergraduate Texts in Mathematics, Springer-Verlag, New York, 1996, An introduction. MR1411675(97g:00001)

[Tomes 2014] Susan Tomes, *Sleeping in Temples*, The Boydell Press, Woodbridge, 2014.