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Unstable localization and periodicity.
Algebraic topology: new trends in localization and periodicity (*Sant Feliu de Guíxols*, 1994), 33–50, *Progr. Math.*, 136, *Birkhäuser*, *Basel*, 1996.

In recent years, there have been significant advances in the study of unstable localization functors by the author, E. Dror Farjoun, and others [see A. K. Bousfield, J. Amer. Math. Soc. **7** (1994), no. 4, 831–873; MR1257059; E. Dror Farjoun, *Cellular spaces, null spaces and homotopy localization*, Lecture Notes in Math., 1622, Springer, Berlin, 1996 MR1392221 (98f:55010)]. To recall the basic construction, given a fixed space W, a space Y is called W-periodic if $W \to *$ induces an equivalence $Y \simeq \mathrm{Map}(W,Y)$, and a map $f\colon A \to B$ is called a W-periodic equivalence if $f^*\colon \mathrm{Map}(B,Y) \simeq \mathrm{Map}(A,Y)$ for each W-periodic Y. There then exists a natural W-periodization $\alpha\colon X \to P_W X$, a W-periodic equivalence from X to a W-periodic space $P_W X$.

To a great extent, the paper under review is a survey of known results. Some new results arise by developing a parallel theory for spectra, in which $\mathrm{Map}(X,Y)$, the mapping space between two spaces, is replaced by $F^c(X,Y)$, the connective cover of the function spectrum between two spectra. Then, for example, one gets Theorem 2.10: $P_W \Omega^\infty E \to \Omega P_{\Sigma^*} E$ is an equivalence for spaces W and spectra E.

Of particular interest is the case when W is a finite complex admitting a v_n self-map. Letting n vary leads to a natural unstable chromatic tower for a space X, $P_0X \leftarrow P_1X \leftarrow P_2X \leftarrow \cdots$, and a stable chromatic tower for a spectrum E, $L_0^f X \leftarrow L_1^f X \leftarrow L_2^f X \leftarrow \cdots$. If we let $M_n^f Ho^s$ denote the full subcategory of spectra whose objects are nth fibers in the stable tower, and $\widetilde{P}_n Ho$ denote the full subcategory of pointed spaces whose objects are nth fibers in the unstable tower, the author proves Theorem 6.1: For $n \geq 1$, the functor $\widetilde{P}_n \Omega^\infty\colon M_n^f Ho^s \to \widetilde{P}_n Ho$ has a left inverse. In both proof and statement, this theorem is a variation on older results of the reviewer [in *Algebraic topology (Arcata, CA, 1986)*, 243–257, Lecture Notes in Math., 1370, Springer, Berlin, 1989; MR1000381] and the author [Pacific J. Math. **129** (1987), no. 1, 1–31; MR0901254].

{For the collection containing this paper see MR1397717}

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