

Write $\tilde{T}_p = \sum_{\text{subgroups } A \text{ of order } p} \psi_A$

$$p = 2$$

$$y^2 + axy + y = x^3$$

$$\Delta = a^3 - 27$$

$$c_4 = a^4 - 24a$$

$$c_6 = -a^6 + 36a^3 - 216$$

$$\psi^2(a) = a^2 + 3d - ad^2$$

$$d^3 - ad - 2 = 0$$

$$(d' = \psi^2(d) = a - d^2, \quad a = d^2 + d')$$

$$\tilde{T}_2(\Delta) = \Delta(a^3 - 3)$$

$$\tilde{T}_2(c_4) = a^2(354 - 40a^3 + a^6)$$

$$\tilde{T}_2(c_6) = -3564 + 6534a^3 - 1128a^6 + 60a^9 - a^{12}$$

$$p = 3$$

$$y^2 + axy + ay = x^3 + x^2$$

$$\Delta = h^2 - 18h + 17$$

$$c_4 = h^2 - 18h + 33$$

$$c_6 = -h^3 + 27h^2 - 171h + 81 = -(h - 9)(h^2 - 18h + 9)$$

$$\psi^3(h) = h^3 - 27h^2 + 201h - 342 + (-6h^2 + 108h - 334)\alpha + (3h - 27)\alpha^2 + (h^2 - 18h + 57)\alpha^3$$

$$\alpha^4 - 6\alpha^2 + (h - 9)\alpha - 3 = 0$$

$$(\alpha' = \psi^3(\alpha) = -\alpha^3 + 6\alpha - h + 9, \quad -h \equiv \alpha^3 + \alpha' \pmod{3})$$

$$\tilde{T}_3(\Delta) = \Delta(2413 - 1908h + 430h^2 - 36h^3 + h^4)$$

$$\tilde{T}_3(c_4) = c_4(1245 - 1620h + 414h^2 - 36h^3 + h^4)$$

$$\tilde{T}_3(c_6) = -(h - 9)(921351 - 2452032h + 2399040h^2 - 1091232h^3 + 251928h^4 - 31104h^5 + 2088h^6 - 72h^7 + h^8)$$

$$(\tilde{T}_3(h^2 - 18h + 16) = 41017 - 75870h + 44067h^2 - 10260h^3 + 1095h^4 - 54h^5 + h^6)$$