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The L_2-localization of $W(n)$. (English summary)

Trans. Amer. Math. Soc. **350** (1998), *no. 5,* 1931–1944.

Let $W(n)$ be the fiber of the double suspension map $S^{2n-1} \to \Omega^2 S^{2n+1}$. The purpose of this article is to relate $L_2W(n)$, the $E(2)_*$-localization of $W(n)$, to the L_2 localizations of various infinite loopspaces. This is analogous to earlier very successful work by A. K. Bousfield, M. Mahowald, the author, and others on unstable L_1-localizations [see, e.g., M. E. Mahowald and R. D. Thompson, Topology **31** (1992), no. 1, 133–141; MR1153241].

To explain what the author does, localize at an odd prime p. Let G_n be the fiber of the composite $QM^{2np-1} \xrightarrow{j_p} QD_pM^{2np-1} \xrightarrow{\pi} QC$, where M^k is a mod p Moore space with top cell in dimension k, j_p is the pth James-Hopf map, and C is the p-adic construction D_pM^{2np-1} with a certain 4-cell subcomplex collapsed to a point. B. Gray [Topology **27** (1988), no. 3, 301–310; MR0963632] has shown that $W(n)$ is the loop space of a space designated $BW(n)$.

The author's first theorem is that, for $p \geq 5$, there exist (i) maps $BW(n) \to \Omega^{2p}BW(n+1)$, (ii) a homotopy equivalence of infinite loop spaces $\operatorname{hocolim}_k \Omega^{2kp}BW(n+k) \simeq QM^{2np-1}$, and (iii) a map $\lambda\colon BW(n) \to G^n$, so that all of these are compatible in the obvious way. The constructions use Gray's work as well as properties of the James-Hopf maps.

The author then proves that λ induces an isomorphism on v_m-periodic homotopy groups for $m \leq 2$, and thus (by a result in [A. K. Bousfield, J. Amer. Math. Soc. **7** (1994), no. 4, 831–873; MR1257059]) $\Omega^k\lambda\colon \Omega^k BW(n) \to \Omega^k G^n$ will be an $E(2)_*$-equivalence if $2np-2-k$ is "sufficiently large". This part of the paper uses modified Adams spectral sequence techniques as in previous work of Mahowald and the author, and Gray.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.