

Algebraic geometry of ring spectra and multiplicative invariants for families of manifolds

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Algebraic and geometric topology workshop 2017

The Witten genus and its refinements

Definition

A *genus* is a function which assigns to each closed manifold M of some type an element $g(M) \in R$ of a commutative ring R , satisfying

- $g(M_1 \amalg M_2) = g(M_1) + g(M_2)$
- $g(M_1 \times M_2) = g(M_1) \cdot g(M_2)$
- $g(\partial N) = 0$

This is the same as giving a ring homomorphism from a suitable cobordism ring, e.g., $g: MSO_* \rightarrow R$, $g: MU_* \rightarrow R$.

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The Witten genus (Witten '87): $MString_* \rightarrow MF_*$

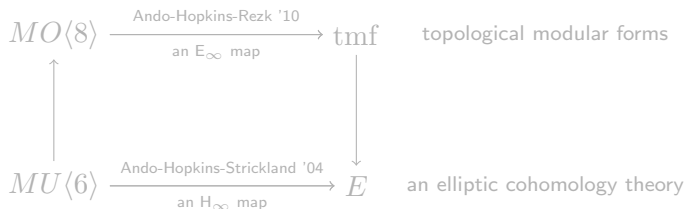
$MString_* = MO\langle 8 \rangle_* :=$ the cobordism ring of *string manifolds*

$$0 = w_1(M) = w_2(M) = p_1(M)/2$$

$$MF_* \cong \mathbb{Z}[c_4, c_6, (c_4^3 - c_6^2)/(12)^3]$$

$$MF_k := H^0(\overline{\mathcal{M}}_{\text{Ell}}, \omega^{\otimes k})$$

$MO\langle 8 \rangle^{-*}(X) \rightsquigarrow$ genus of a *family* parametrized by X (orientation)



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 MU\langle 6 \rangle & \xrightarrow[\text{an } H_\infty \text{ map}]{\text{Ando-Hopkins-Strickland '04}} & E & \text{an elliptic cohomology theory}
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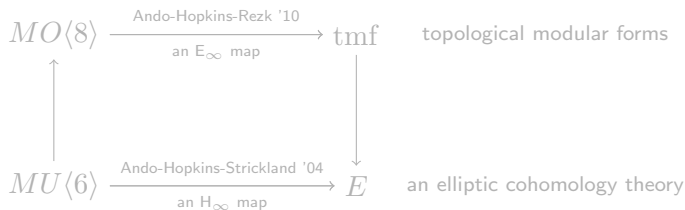
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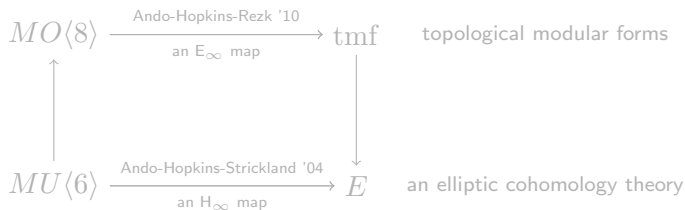
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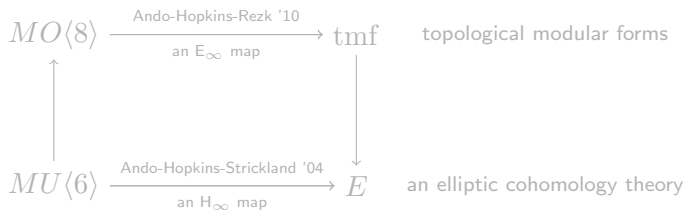
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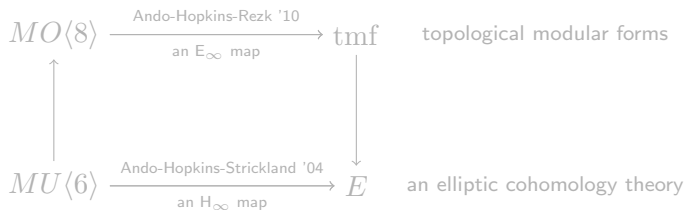
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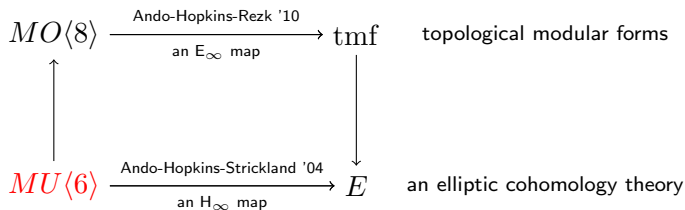
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Multiplicative structures and symmetries of the geometry

$$\begin{array}{ccc}
 D_n MU\langle 6 \rangle & \xrightarrow{D_n g} & D_n E \\
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 \quad D_n(-) := (-)^{\wedge n} / \Sigma_n, \quad n \geq 0$$

H_∞ means commutative up to homotopy

The vertical maps correspond to *power operations*; e.g., given

$$E^0 X \cong \pi_0 E^{\Sigma_+^\infty X} =: \pi_0 A = [S, A]_S \cong [E, A]_E \ni f$$

we have

$$E \xrightarrow{\alpha} D_n E \xrightarrow{D_n f} D_n A \rightarrow A$$

and thus a power operation $Q_\alpha: E^0 X \rightarrow E^0 X$. When $E = KU$, this is the n -fold tensor product of \mathbb{C} -vector bundles over X .

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 \quad D_n(-) := (-)^{\wedge n} / \Sigma_n, \quad n \geq 0$$

H_∞ means commutative up to homotopy

The vertical maps correspond to *power operations*; e.g., given

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Theorem (Morava '78, Goerss-Hopkins-Miller '90s-'04)

$\mathcal{E}: \{\text{formal groups over perfect fields, isos}\} \rightarrow \{E_\infty\text{-ring spectra}\}$

- $\mathrm{Spf} E^0(\mathbb{CP}^\infty) =$ universal deformation of a fg Γ of height n over a perfect field k of char p
- $E_* \cong \mathbb{W}(k)[[u_1, \dots, u_{n-1}]] [u^{\pm 1}]$, $|u| = 2$

A deformation (G, i, η) of Γ/k to R (Lubin-Tate '66):

$$\begin{array}{ccccccc}
 G & \xleftarrow{\quad} & \pi^* G & \xrightarrow{\eta} & i^* \Gamma & \xrightarrow{\quad} & \Gamma \\
 \downarrow & & \downarrow & & \downarrow & & \downarrow \\
 \mathrm{Spf} R & \xleftarrow{\pi} & \mathrm{Spf} R/\mathfrak{m} & \xlongequal{\quad} & \mathrm{Spf} R/\mathfrak{m} & \xrightarrow{i} & \mathrm{Spf} k
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Theorem (Morava '78, Goerss-Hopkins-Miller '90s-'04)

$\mathcal{E}: \{\text{formal groups over perfect fields, isos}\} \rightarrow \{E_\infty\text{-ring spectra}\}$

- $\mathrm{Spf} E^0(\mathbb{CP}^\infty) = \text{universal deformation of a fg } \Gamma \text{ of height } n \text{ over a perfect field } k \text{ of char } p$
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Theorem (Z.)

Let k be an algebraic extension of \mathbb{F}_p , Γ be a formal group over k of height n , and E be the Morava E-theory associated to Γ/k . Given any coordinate x_Γ on Γ , there exists a unique coordinate x on G_E lifting x_Γ such that $MU\langle 0 \rangle \xrightarrow{x} E$ is an H_∞ map.

Remark

- (Ando '95) $k = \mathbb{F}_p$, $\Gamma = \text{Honda fg}$, $E \neq \text{elliptic cohomology}$
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Remark

A *level structure* on G is a choice of finite subgroup. This theorem gives universal examples of “descent data” for level structures:

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Fix Γ/k . Then deformations of Frobenius $(G, i, \eta) \rightarrow (G', i', \eta')$ are classified by rings A_r , $r \geq 0$, with p^r the order of the subgroup scheme $\ker(G \rightarrow G') \subset G$.

Remark

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$H = \text{finite subgroup of } G \quad f_H: G \rightarrow G/H \quad x = \text{coord on } G$

$\implies x_H := \text{Norm}_{f_H^*}(x) = \det(\cdot x)$ is a coord on G/H

$$\mathcal{O}_{G/H} \xrightarrow{f_H^*} \mathcal{O}_G \xrightarrow{\text{Norm}_{f_H^*}} \mathcal{O}_{G/H}$$

Explicitly,

$$f_H^*(x_H) = \prod_{Q \in H} (x + x(Q))$$

Definition

A coordinate x on G is *norm-coherent* if

$$\psi_H(x) = f_H^*(x_H) \quad \text{for any finite } H \subset G$$

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Theorem (Ando '95, Ando-Hopkins-Strickland '04)

Let E be the Morava E-theory associated to Γ/k as before. Then the orientation $MU\langle 0 \rangle \rightarrow E$ is an H_∞ map if and only if its corresponding coordinate on G_E is norm-coherent.

Theorem (Z.)

- Any coordinate on Γ over k extends uniquely to a norm-coherent coordinate on G_E over $\pi_0 E$.
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Remark A connection to Coleman's norm operator from explicit local class field theory is still to be understood.

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