Math 240 Final Exam

Name: _____

Instructions: Calculators, course notes and textbooks are **NOT** allowed on the worksheet. All numerical answers **MUST** be exact; e.g., you should write π instead of 3.14..., $\sqrt{2}$ instead of 1.414..., and $\frac{1}{3}$ instead of 0.3333... Explain your reasoning using complete sentences and correct grammar, spelling, and punctuation.
Show ALL of your work!

	Score
1	
2	
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Total	

Question 1. (10 points) Solve the matrix equation $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}.$$

Question 2. (10 points) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation that reflects each vector through the plane $x_2 = 0$. Find the standard matrix of T.

Question 3. (10 points) Find a basis for Nul A, where

$$A = \begin{bmatrix} -3 & 9 & -2 & -7 \\ 2 & -6 & 4 & 8 \\ 3 & -9 & -2 & 2 \end{bmatrix}$$

What is the dimension of Nul A?

Question 4. (10 points) True or false? Justify your answer.

Given any $m \times n$ matrix A, the number of linearly independent columns equals the number of linearly independent rows.

Question 5. (10 points) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}.$$

Question 6. (10 points) Compute the determinant of the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 & 6 \\ 1 & -7 & -5 & 0 \\ 3 & 8 & 6 & 0 \\ 0 & 7 & 5 & 4 \end{bmatrix}$$

Is A invertible?

Question 7. (10 points) Consider the following matrix.

$$A = \begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

(i) Compute the characteristic polynomial of A.

(ii) Find all the eigenvalues and eigenvectors of A.

(iii) Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

Question 8. (10 points) Let $\mathbf{v}$ be any nonzero vector in $\mathbb{R}^n$ with $n \geq 2$. Find all the eigenvalues of the $n \times n$ matrix $\mathbf{v}\,\mathbf{v}^T$.

Question 9. (10 points) Find an orthonormal basis for Col A, where

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Question 10. (10 points) Suppose that a data set consists of points $(-6, -1)$, $(-2, 2)$, $(1, 1)$ and $(7, 6)$ on the xy-plane. Find an equation for the line that best models the relation between the x and y coordinates of these sample values. Hint: Compute a least-squares solution for $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} -6 & 1 \\ -2 & 1 \\ 1 & 1 \\ 7 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 6 \end{bmatrix}.$$