Algebraic topology and arithmetic

Yifei Zhu

Southern University of Science and Technology

2019 Young Mathematician Forum, BICMR

Motivation: cohomology theories and their operations

Generalized cohomology theory $\{h^n\}\colon \text{Spaces} \to \text{AbGroups}$

Cup product $\rightsquigarrow h^*(X)$ a graded commutative algebra over $h^*(\text{pt})$

Cohomology operation $Q^i\colon\ h^*(-) \to h^{*+i}(-)$

Example (ordinary cohomology with $\mathbb{Z}/2$-coefficients)

Steenrod squares $\mathrm{Sq}^i\colon\ H^*(-;\mathbb{Z}/2) \to H^{*+i}(-;\mathbb{Z}/2)$

Power operation $\mathrm{Sq}^i(x) = x^2$ if $i = |x|$

Steenrod algebra

Motivation: cohomology theories and their operations

Generalized cohomology theory $\{h^n\}\colon \text{Spaces} \to \text{AbGroups}$

Cup product $\rightsquigarrow h^*(X)$ a graded commutative algebra over $h^*(\text{pt})$

Cohomology operation $Q^i\colon\ h^*(-) \to h^{*+i}(-)$

Example (ordinary cohomology with $\mathbb{Z}/2$-coefficients)

Steenrod squares $\mathrm{Sq}^i\colon\ H^*(-;\mathbb{Z}/2) \to H^{*+i}(-;\mathbb{Z}/2)$

Power operation $\mathrm{Sq}^i(x) = x^2$ if $i = |x|$

Steenrod algebra

Motivation: cohomology theories and their operations

Generalized cohomology theory $\{h^n\}\colon \text{Spaces} \to \text{AbGroups}$

Cup product $\rightsquigarrow h^*(X)$ a graded commutative algebra over $h^*(\text{pt})$

Cohomology operation $Q^i\colon\ h^*(-) \to h^{*+i}(-)$

Example (ordinary cohomology with $\mathbb{Z}/2$-coefficients)

Steenrod squares $\mathrm{Sq}^i\colon\ H^*(-;\mathbb{Z}/2) \to H^{*+i}(-;\mathbb{Z}/2)$

Power operation $\mathrm{Sq}^i(x) = x^2$ if $i = |x|$

Steenrod algebra

Motivation: cohomology theories and their operations

Generalized cohomology theory $\{h^n\}\colon \text{Spaces} \to \text{AbGroups}$

Cup product $\rightsquigarrow h^*(X)$ a graded commutative algebra over $h^*(\text{pt})$

Cohomology operation $Q^i\colon\ h^*(-) \to h^{*+i}(-)$

Example (ordinary cohomology with $\mathbb{Z}/2$-coefficients)

Steenrod squares $\text{Sq}^i\colon\ H^*(-;\mathbb{Z}/2) \to H^{*+i}(-;\mathbb{Z}/2)$

Power operation $\text{Sq}^i(x) = x^2$ if $i = |x|$

Steenrod algebra

Motivation: cohomology theories and their operations

Generalized cohomology theory $\{h^n\}\colon \text{Spaces} \to \text{AbGroups}$

Cup product $\rightsquigarrow h^*(X)$ a graded commutative algebra over $h^*(\text{pt})$

Cohomology operation $Q^i\colon\ h^*(-) \to h^{*+i}(-)$

Example (ordinary cohomology with $\mathbb{Z}/2$-coefficients)

Steenrod squares $\text{Sq}^i\colon\ H^*(-;\mathbb{Z}/2) \to H^{*+i}(-;\mathbb{Z}/2)$

Power operation $\text{Sq}^i(x) = x^2$ if $i = |x|$

Steenrod algebra

Motivation: cohomology theories and their operations

Generalized cohomology theory $\{h^n\}\colon \mathrm{Spaces} \to \mathrm{AbGroups}$

Cup product $\rightsquigarrow h^*(X)$ a graded commutative algebra over $h^*(\mathrm{pt})$

Cohomology operation $Q^i\colon\ h^*(-) \to h^{*+i}(-)$

Example (ordinary cohomology with $\mathbb{Z}/2$-coefficients)

Steenrod squares $\mathrm{Sq}^i\colon\ H^*(-;\mathbb{Z}/2) \to H^{*+i}(-;\mathbb{Z}/2)$

Power operation $\mathrm{Sq}^i(x) = x^2$ if $i = |x|$

Steenrod algebra

Motivation: cohomology theories and their operations

Generalized cohomology theory $\{h^n\}\colon \mathrm{Spaces} \to \mathrm{AbGroups}$

Cup product $\rightsquigarrow h^*(X)$ a graded commutative algebra over $h^*(\mathrm{pt})$

Cohomology operation $Q^i\colon\ h^*(-) \to h^{*+i}(-)$

Example (ordinary cohomology with $\mathbb{Z}/2$-coefficients)

Steenrod squares $\mathrm{Sq}^i\colon\ H^*(-;\mathbb{Z}/2) \to H^{*+i}(-;\mathbb{Z}/2)$

Power operation $\mathrm{Sq}^i(x) = x^2$ if $i = |x|$

Adem relations $\mathrm{Sq}^i\mathrm{Sq}^j = \sum_{k=0}^{[\frac{i}{2}]} \binom{j-k-1}{i-2k} \mathrm{Sq}^{i+j-k}\mathrm{Sq}^k$, $0 < i < 2j$

Cartan formula $\mathrm{Sq}^i(xy) = \sum_{k=0}^{i} \mathrm{Sq}^{i-k}(x)\,\mathrm{Sq}^k(y)$

Motivation: cohomology theories and their operations

Generalized cohomology theory $\{h^n\}\colon\ \mathrm{Spaces} \to \mathrm{AbGroups}$

Cup product $\rightsquigarrow h^*(X)$ a graded commutative algebra over $h^*(\mathrm{pt})$

Cohomology operation $Q^i\colon\ h^*(-) \to h^{*+i}(-)$

Example (ordinary cohomology with $\mathbb{Z}/2$-coefficients)

Steenrod squares $\mathrm{Sq}^i\colon\ H^*(-;\mathbb{Z}/2) \to H^{*+i}(-;\mathbb{Z}/2)$

Power operation $\mathrm{Sq}^i(x) = x^2$ if $i = |x|$

Adem relations $\mathrm{Sq}^i\mathrm{Sq}^j = \sum_{k=0}^{[\frac{i}{2}]} \binom{j-k-1}{i-2k} \mathrm{Sq}^{i+j-k}\mathrm{Sq}^k$, $0 < i < 2j$

Cartan formula $\mathrm{Sq}^i(xy) = \sum_{k=0}^{i} \mathrm{Sq}^{i-k}(x)\,\mathrm{Sq}^k(y)$

Motivation: cohomology theories and their operations

Generalized cohomology theory $\{h^n\}$: $\mathrm{Spaces} \to \mathrm{AbGroups}$

Cup product $\rightsquigarrow h^*(X)$ a graded commutative algebra over $h^*(\mathrm{pt})$

Cohomology operation $Q^i\colon\ h^*(-) \to h^{*+i}(-)$

Example (ordinary cohomology with $\mathbb{Z}/2$-coefficients)

Steenrod squares $\mathrm{Sq}^i\colon\ H^*(-;\mathbb{Z}/2) \to H^{*+i}(-;\mathbb{Z}/2)$

Power operation $\mathrm{Sq}^i(x) = x^2$ if $i = |x|$

Adem relations $\mathrm{Sq}^i\mathrm{Sq}^j = \sum_{k=0}^{\left[\frac{i}{2}\right]} \binom{j-k-1}{i-2k} \mathrm{Sq}^{i+j-k}\mathrm{Sq}^k$, $0 < i < 2j$

Cartan formula $\mathrm{Sq}^i(xy) = \sum_{k=0}^{i} \mathrm{Sq}^{i-k}(x)\,\mathrm{Sq}^k(y)$

Motivation: cohomology theories and their operations

Generalized cohomology theory $\{h^n\}\colon \mathrm{Spaces} \to \mathrm{AbGroups}$

Cup product $\rightsquigarrow h^*(X)$ a graded commutative algebra over $h^*(\mathrm{pt})$

Cohomology operation $Q^i\colon\ h^*(-) \to h^{*+i}(-)$

Example (complex K-theory)

Adams operations $\psi^i\colon\ K(-) \to K(-)$

Power operation $\psi^p(x) \equiv x^p \mod p$

$\psi^i\psi^j = \psi^{ij}$ $\qquad$ $\psi^i(xy) = \psi^i(x)\psi^i(y)$

J. F. Adams, *Vector fields on spheres*, Ann. of Math. (2) **75** (1962)

Motivation: cohomology theories and their operations

Generalized cohomology theory $\{h^n\}$: $\mathrm{Spaces} \to \mathrm{AbGroups}$

Cup product $\rightsquigarrow$ $h^*(X)$ a graded commutative algebra over $h^*(\mathrm{pt})$

Cohomology operation $Q^i\colon\ h^*(-) \to h^{*+i}(-)$

Example (more – a sample)

Voevodsky, *Reduced power operations in motivic cohomology*, 2003.

Lipshitz and Sarkar, *A Steenrod square on Khovanov homology*, 2014.

Feng, *Étale Steenrod operations and the Artin–Tate pairing*, 2018.

Seidel, *Formal groups and quantum cohomology*, 2019.

A connection between Topology and Arithmetic (Quillen '69)

$$\begin{array}{rcl} \text{stable homotopy theory} & \leftrightsquigarrow & \text{1-dim formal group laws} \\ \text{complex-oriented } h^*(-) & & F(x,y) \text{ over } h^*(\mathrm{pt}) \\ c_1(L_1 \otimes L_2) & = & F\big(c_1(L_1), c_1(L_2)\big) \end{array}$$

Example

$$H^*(-;\mathbb{Z}) \leftrightsquigarrow \mathbb{G}_a(x,y) = x+y$$

$$K^*(-) \leftrightsquigarrow \mathbb{G}_m(x,y) = x+y-xy = 1-(1-x)(1-y)$$

A connection between Topology and Arithmetic (Quillen '69)

$$\begin{array}{rcl} \text{stable homotopy theory} & \leftrightsquigarrow & \text{1-dim formal group laws} \\ \text{complex-oriented } h^*(-) & & F(x,y) \text{ over } h^*(\mathrm{pt}) \\ c_1(L_1 \otimes L_2) & = & F\big(c_1(L_1), c_1(L_2)\big) \end{array}$$

Example

$$H^*(-;\mathbb{Z}) \leftrightsquigarrow \mathbb{G}_a(x,y) = x + y$$

$$K^*(-) \leftrightsquigarrow \mathbb{G}_m(x,y) = x + y - xy = 1 - (1-x)(1-y)$$

A connection between Topology and Arithmetic (Quillen '69)

stable homotopy theory $\leftrightsquigarrow$ 1-dim formal group laws

complex-oriented $h^*(-)$ $F(x,y)$ over $h^*(\mathrm{pt})$

$$c_1(L_1 \otimes L_2) = F\big(c_1(L_1), c_1(L_2)\big)$$

Example

$$H^*(-;\mathbb{Z}) \leftrightsquigarrow \mathbb{G}_a(x,y) = x + y$$

$$K^*(-) \leftrightsquigarrow \mathbb{G}_m(x,y) = x + y - xy = 1 - (1-x)(1-y)$$

A connection between Topology and Arithmetic (Quillen '69)

stable homotopy theory $\leftsquigarrow\!\rightsquigarrow$ 1-dim formal group laws

complex-oriented $h^*(-)$ $F(x, y)$ over $h^*(\mathrm{pt})$

$$c_1(L_1 \otimes L_2) = F\big(c_1(L_1), c_1(L_2)\big)$$

Example

$$H^*(-;\mathbb{Z}) \leftrightsquigarrow \mathbb{G}_a(x, y) = x + y$$

$$K^*(-) \leftrightsquigarrow \mathbb{G}_m(x, y) = x + y - xy = 1 - (1 - x)(1 - y)$$

A connection between Topology and Arithmetic (Quillen '69)

stable homotopy theory $\leftrightsquigarrow$ 1-dim formal group laws

complex-oriented $h^*(-)$ $F(x, y)$ over $h^*(\mathrm{pt})$

$$c_1(L_1 \otimes L_2) = F\big(c_1(L_1), c_1(L_2)\big)$$

Example

$$H^*(-;\mathbb{Z}) \leftrightsquigarrow \mathbb{G}_a(x, y) = x + y$$

$$K^*(-) \leftrightsquigarrow \mathbb{G}_m(x, y) = x + y - xy = 1 - (1 - x)(1 - y)$$

Background: Chromatic Homotopy Theory 色展同伦论

A connection between Topology and Arithmetic (Quillen '69)

stable homotopy theory $\leftrightsquigarrow$ 1-dim formal group laws

complex-oriented $h^*(-)$ $F(x,y)$ over $h^*(\mathrm{pt})$

$$c_1(L_1 \otimes L_2) = F(c_1(L_1), c_1(L_2))$$

Example

$$H^*(-;\mathbb{Z}) \leftrightsquigarrow \mathbb{G}_a(x,y) = x + y$$

$$K^*(-) \leftrightsquigarrow \mathbb{G}_m(x,y) = x + y - xy = 1 - (1-x)(1-y)$$

A connection between Topology and Arithmetic (Quillen '69)

$$\begin{array}{rcl} \text{stable homotopy theory} & \leftrightsquigarrow & \text{1-dim formal group laws} \\ \text{complex-oriented } h^*(-) & & F(x,y) \text{ over } h^*(\mathrm{pt}) \\ c_1(L_1 \otimes L_2) & = & F\big(c_1(L_1), c_1(L_2)\big) \end{array}$$

Example

$$H^*(-;\mathbb{Z}) \leftrightsquigarrow \mathbb{G}_a(x,y) = x + y$$

$$K^*(-) \leftrightsquigarrow \mathbb{G}_m(x,y) = x + y - xy = 1 - (1-x)(1-y)$$

A connection between Topology and Arithmetic (Quillen '69)

$$\begin{array}{ccc} \text{stable homotopy theory} & \leftrightsquigarrow & \text{1-dim formal group laws} \\ \text{complex-oriented } h^*(-) & & F(x,y) \text{ over } h^*(\mathrm{pt}) \\ c_1(L_1 \otimes L_2) & = & F\big(c_1(L_1), c_1(L_2)\big) \end{array}$$

Example

$$H^*(-;\mathbb{Z}) \leftrightsquigarrow \mathbb{G}_a(x,y) = x + y$$

$$K^*(-) \leftrightsquigarrow \mathbb{G}_m(x,y) = x + y - xy = 1 - (1-x)(1-y)$$

Elliptic cohomology and Morava E-theory

Definition (Ando–Hopkins–Strickland '01, Lurie '09, '18)

$$\text{elliptic cohomology theory} = \left\{ \begin{array}{ll} E, & C_{E^0(\mathrm{pt})}, \\ \alpha\colon & \operatorname{Spf} E^0(\mathbb{CP}^\infty) \xrightarrow{\sim} \widehat{C} \end{array} \right\}$$

Theorem (Morava '78, Goerss–Hopkins–Miller '90s–'04)

$$\mathcal{E}\colon \{\text{formal groups over perfect fields, isos}\} \to \{\mathbb{E}_\infty\text{-ring spectra}\}$$

- $\operatorname{Spf} E^0(\mathbb{CP}^\infty) =$ the univ deformation of a fg F of height n over a perfect field k of char p
- $\pi_* E \cong \mathbb{W}(k)[\![u_1, \ldots, u_{n-1}]\!][u^{\pm 1}], \quad |u| = -2$

Elliptic cohomology and Morava E-theory

Definition (Ando–Hopkins–Strickland '01, Lurie '09, '18)

$$\text{elliptic cohomology theory} = \left\{ \begin{array}{ll} E, & C_{/E^0(\mathrm{pt})}, \\ \alpha\colon & \operatorname{Spf} E^0(\mathbb{CP}^\infty) \xrightarrow{\sim} \widehat{C} \end{array} \right\}$$

Theorem (Morava '78, Goerss–Hopkins–Miller '90s–'04)

$\mathcal{E}\colon$ {formal groups over perfect fields, isos} $\to$ {$\mathbb{E}_\infty$-ring spectra}

- $\operatorname{Spf} E^0(\mathbb{CP}^\infty) =$ the univ deformation of a fg F of height n over a perfect field k of char p
- $\pi_* E \cong \mathbb{W}(k)[\![u_1, \ldots, u_{n-1}]\!][u^{\pm 1}], \quad |u| = -2$

Elliptic cohomology and Morava E-theory

Definition (Ando–Hopkins–Strickland '01, Lurie '09, '18)

$$\text{elliptic cohomology theory} = \left\{ \begin{array}{ll} E, & C_{/E^0(\mathrm{pt})}, \\ \alpha\colon & \operatorname{Spf} E^0(\mathbb{CP}^\infty) \xrightarrow{\simeq} \widehat{C} \end{array} \right\}$$

Theorem (Morava '78, Goerss–Hopkins–Miller '90s–'04)

$\mathcal{E}\colon$ {formal groups over perfect fields, isos} $\to$ {$\mathbb{E}_\infty$-ring spectra}

- $\operatorname{Spf} E^0(\mathbb{CP}^\infty) =$ the univ deformation of a fg F of height n over a perfect field k of char p
- $\pi_* E \cong \mathbb{W}(k)[\![u_1, \ldots, u_{n-1}]\!][u^{\pm 1}], \quad |u| = -2$

Elliptic cohomology and Morava E-theory

Definition (Ando–Hopkins–Strickland '01, Lurie '09, '18)

$$\text{elliptic cohomology theory} = \left\{ \begin{array}{ll} E, & C_{E^0(\mathrm{pt})}, \\ \alpha\colon & \operatorname{Spf} E^0(\mathbb{CP}^\infty) \xrightarrow{\sim} \widehat{C} \end{array} \right\}$$

Theorem (Morava '78, Goerss–Hopkins–Miller '90s–'04)

$\mathcal{E}\colon$ {formal groups over perfect fields, isos} $\to$ {$\mathbb{E}_\infty$-ring spectra}

- $\operatorname{Spf} E^0(\mathbb{CP}^\infty) =$ the univ deformation of a fg F of height n over a perfect field k of char p
- $\pi_* E \cong \mathbb{W}(k)[\![u_1, \ldots, u_{n-1}]\!][u^{\pm 1}], \quad |u| = -2$

Elliptic cohomology and Morava E-theory

Definition (Ando–Hopkins–Strickland '01, Lurie '09, '18)

$$\text{elliptic cohomology theory} = \left\{ \begin{array}{ll} E, & C_{E^0(\mathrm{pt})}, \\ \alpha\colon & \operatorname{Spf} E^0(\mathbb{CP}^\infty) \xrightarrow{\sim} \widehat{C} \end{array} \right\}$$

Theorem (Morava '78, Goerss–Hopkins–Miller '90s–'04)

$\mathcal{E}\colon$ {formal groups over perfect fields, isos} $\to$ {$\mathbb{E}_\infty$-ring spectra}

- $\operatorname{Spf} E^0(\mathbb{CP}^\infty)$ = the univ deformation of a fg F of height n over a perfect field k of char p
- $\pi_* E \cong \mathbb{W}(k)[\![u_1, \ldots, u_{n-1}]\!][u^{\pm 1}], \quad |u| = -2$

Elliptic cohomology and Morava E-theory

Definition (Ando–Hopkins–Strickland '01, Lurie '09, '18)

$$\text{elliptic cohomology theory} = \left\{ \begin{array}{ll} E, & C_{E^0(\mathrm{pt})}, \\ \alpha\colon & \mathrm{Spf}\, E^0(\mathbb{CP}^\infty) \xrightarrow{\sim} \widehat{C} \end{array} \right\}$$

Theorem (Morava '78, Goerss–Hopkins–Miller '90s–'04)

$$\mathcal{E}\colon \{\text{formal groups over perfect fields, isos}\} \to \{\mathbb{E}_\infty\text{-ring spectra}\}$$

- $\mathrm{Spf}\, E^0(\mathbb{CP}^\infty)$ = the univ deformation of a fg F of height n over a perfect field k of char p
- $\pi_* E \cong \mathbb{W}(k)[\![u_1, \ldots, u_{n-1}]\!][u^{\pm 1}], \quad |u| = -2$

Elliptic cohomology and Morava E-theory

Definition (Ando–Hopkins–Strickland '01, Lurie '09, '18)

$$\text{elliptic cohomology theory} = \left\{ \begin{array}{ll} E, & C_{E^0(\mathrm{pt})}, \\ \alpha\colon & \operatorname{Spf} E^0(\mathbb{CP}^\infty) \xrightarrow{\sim} \widehat{C} \end{array} \right\}$$

Theorem (Morava '78, Goerss–Hopkins–Miller '90s–'04)

$\mathcal{E}\colon$ {formal groups over perfect fields, isos} $\to$ {$\mathbb{E}_\infty$-ring spectra}

- $\operatorname{Spf} E^0(\mathbb{CP}^\infty)$ = the univ deformation of a fg F of height n over a perfect field k of char p
- $\pi_* E \cong \mathbb{W}(k)[\![u_1, \ldots, u_{n-1}]\!][u^{\pm 1}], \quad |u| = -2$

Elliptic cohomology and Morava E-theory

Definition (Ando–Hopkins–Strickland '01, Lurie '09, '18)

$$\text{elliptic cohomology theory} = \left\{ \begin{array}{ll} E, & C_{E^0(\mathrm{pt})}, \\ \alpha\colon & \operatorname{Spf} E^0(\mathbb{CP}^\infty) \xrightarrow{\sim} \widehat{C} \end{array} \right\}$$

Theorem (Morava '78, Goerss–Hopkins–Miller '90s–'04)

$$\mathcal{E}\colon \ \{\text{formal groups over perfect fields, isos}\} \to \{\mathbb{E}_\infty\text{-ring spectra}\}$$

- $\operatorname{Spf} E^0(\mathbb{CP}^\infty) =$ the univ deformation of a fg F of height n over a perfect field k of char p
- $\pi_* E \cong \mathbb{W}(k)[\![u_1, \ldots, u_{n-1}]\!][u^{\pm 1}], \quad |u| = -2$

Elliptic cohomology and Morava E-theory

Definition (Ando–Hopkins–Strickland '01, Lurie '09, '18)

$$\text{elliptic cohomology theory} = \left\{ \begin{array}{ll} E, & C_{E^0(\mathrm{pt})}, \\ \alpha\colon & \operatorname{Spf} E^0(\mathbb{CP}^\infty) \xrightarrow{\sim} \widehat{C} \end{array} \right\}$$

Theorem (Morava '78, Goerss–Hopkins–Miller '90s–'04)

$$\mathcal{E}\colon \{\text{formal groups over perfect fields, isos}\} \to \{\mathbb{E}_\infty\text{-ring spectra}\}$$

- $\operatorname{Spf} E^0(\mathbb{CP}^\infty)$ = the univ deformation of a fg F of height n over a perfect field k of char p
- $\pi_* E \cong \mathbb{W}(k)[\![u_1, \ldots, u_{n-1}]\!][u^{\pm 1}], \quad |u| = -2$

Elliptic cohomology and Morava E-theory

Definition (Ando–Hopkins–Strickland '01, Lurie '09, '18)

$$\text{elliptic cohomology theory} = \left\{ \begin{array}{ll} E, & C_{E^0(\mathrm{pt})}, \\ \alpha\colon & \operatorname{Spf} E^0(\mathbb{CP}^\infty) \xrightarrow{\sim} \widehat{C} \end{array} \right\}$$

Theorem (Morava '78, Goerss–Hopkins–Miller '90s–'04)

$$\mathcal{E}\colon \{\text{formal groups over perfect fields, isos}\} \to \{\mathbb{E}_\infty\text{-ring spectra}\}$$

- $\operatorname{Spf} E^0(\mathbb{CP}^\infty) =$ the univ deformation of a fg F of height n over a perfect field k of char p
- $\pi_* E \cong \mathbb{W}(k)[\![u_1, \ldots, u_{n-1}]\!][u^{\pm 1}], \quad |u| = -2$

Elliptic cohomology and Morava E-theory

Definition (Ando–Hopkins–Strickland '01, Lurie '09, '18)

$$\text{elliptic cohomology theory} = \left\{ \begin{array}{ll} E, & C_{E^0(\mathrm{pt})}, \\ \alpha\colon & \operatorname{Spf} E^0(\mathbb{CP}^\infty) \xrightarrow{\sim} \widehat{C} \end{array} \right\}$$

Theorem (Morava '78, Goerss–Hopkins–Miller '90s–'04)

$$\mathcal{E}\colon \{\text{formal groups over perfect fields, isos}\} \to \{\mathbb{E}_\infty\text{-ring spectra}\}$$

- $\operatorname{Spf} E^0(\mathbb{CP}^\infty) =$ the univ deformation of a fg F of height n over a perfect field k of char p
- $\pi_* E \cong \mathbb{W}(k)[\![u_1, \ldots, u_{n-1}]\!][u^{\pm 1}], \quad |u| = -2$

Elliptic cohomology and Morava E-theory

Definition (Ando–Hopkins–Strickland '01, Lurie '09, '18)

$$\text{elliptic cohomology theory} = \left\{ \begin{array}{ll} E, & C_{E^0(\mathrm{pt})}, \\ \alpha\colon & \operatorname{Spf} E^0(\mathbb{CP}^\infty) \xrightarrow{\sim} \widehat{C} \end{array} \right\}$$

Theorem (Morava '78, Goerss–Hopkins–Miller '90s–'04)

$$\mathcal{E}\colon \{\text{formal groups over perfect fields, isos}\} \to \{\mathbb{E}_\infty\text{-ring spectra}\}$$

$E =$ Morava E-theory of height n at the prime p

Goal explore the structure on $E^*(-)$. Topology $\leftrightsquigarrow$ Arithmetic

Elliptic cohomology and Morava E-theory

Definition (Ando–Hopkins–Strickland '01, Lurie '09, '18)

$$\text{elliptic cohomology theory} = \left\{ \begin{array}{ll} E, & C_{E^0(\mathrm{pt})}, \\ \alpha\colon & \mathrm{Spf}\, E^0(\mathbb{CP}^\infty) \xrightarrow{\sim} \widehat{C} \end{array} \right\}$$

Theorem (Morava '78, Goerss–Hopkins–Miller '90s–'04)

$$\mathcal{E}\colon \{\text{formal groups over perfect fields, isos}\} \to \{\mathbb{E}_\infty\text{-ring spectra}\}$$

E = Morava E-theory of height n at the prime p

Goal explore the structure on $E^*(-)$. Topology $\rightsquigarrow$ Arithmetic

Elliptic cohomology and Morava E-theory

Definition (Ando–Hopkins–Strickland '01, Lurie '09, '18)

$$\text{elliptic cohomology theory} = \left\{ \begin{array}{ll} E, & C_{E^0(\mathrm{pt})}, \\ \alpha\colon & \operatorname{Spf} E^0(\mathbb{CP}^\infty) \xrightarrow{\sim} \widehat{C} \end{array} \right\}$$

Theorem (Morava '78, Goerss–Hopkins–Miller '90s–'04)

$$\mathcal{E}\colon \{\text{formal groups over perfect fields, isos}\} \to \{\mathbb{E}_\infty\text{-ring spectra}\}$$

E = Morava E-theory of height n at the prime p

<u>Goal</u> explore the structure on $E^*(-)$. Topology $\leftrightsquigarrow$ Arithmetic

Power operations for Morava E-theory

M = E-module $\qquad \pi_0 M = [S, M]_S \cong [E, M]_E$

$$\mathbb{P}_E(M) = \bigvee_{i\geq 0} \mathbb{P}_E^i(M) = \bigvee_{i \geq 0} (\underbrace{M \wedge_E \cdots \wedge_E M}_{i\text{-fold}})_{h\Sigma_i}$$

A = commutative E-algebra

= algebra for the monad $\mathbb{P}_E$ with $\mu \colon \mathbb{P}_E(A) \to A$

$$\left.\begin{array}{l} \text{total power operation } \psi^i \colon \pi_0 A \to \pi_0\left(A^{B\Sigma_i^+}\right) \\ \forall \eta \in \pi_0 \mathbb{P}_E^i(E), \text{individual po } Q_\eta \colon \pi_0 A \to \pi_0 A \end{array}\right\} \overset{/I}{\rightsquigarrow} \text{additive}$$

$$E \xrightarrow{f_\eta} \mathbb{P}_E^i(E) \xrightarrow{\mathbb{P}_E^i(f_x)} \mathbb{P}_E^i(A) \hookrightarrow \mathbb{P}_E(A) \xrightarrow{\mu} A$$

Power operations for Morava E-theory

$M = E$-module $\qquad \pi_0 M = [S, M]_S \cong [E, M]_E$

$$\mathbb{P}_E(M) = \bigvee_{i \geq 0} \mathbb{P}_E^i(M) = \bigvee_{i \geq 0} (\underbrace{M \wedge_E \cdots \wedge_E M}_{i\text{-fold}})_{h\Sigma_i}$$

$A =$ commutative E-algebra
$\quad =$ algebra for the monad $\mathbb{P}_E$ with $\mu \colon \mathbb{P}_E(A) \to A$

$$\left.\begin{array}{l} \text{total power operation } \psi^i \colon \ \pi_0 A \to \pi_0\left(A^{B\Sigma_i^+}\right) \\ \forall \eta \in \pi_0 \mathbb{P}_E^i(E), \text{individual po } Q_\eta \colon \ \pi_0 A \to \pi_0 A \end{array}\right\} \overset{/I}{\rightsquigarrow} \text{additive}$$

$$E \xrightarrow{f_\eta} \mathbb{P}_E^i(E) \xrightarrow{\mathbb{P}_E^i(f_x)} \mathbb{P}_E^i(A) \hookrightarrow \mathbb{P}_E(A) \xrightarrow{\mu} A$$

M = E-module $\qquad \pi_0 M = [S, M]_S \cong [E, M]_E$

$$\mathbb{P}_E(M) = \bigvee_{i\geq 0} \mathbb{P}_E^i(M) = \bigvee_{i\geq 0} (\underbrace{M \wedge_E \cdots \wedge_E M}_{i\text{-fold}})_{h\Sigma_i}$$

A = commutative E-algebra
= algebra for the monad $\mathbb{P}_E$ with $\mu\colon \mathbb{P}_E(A) \to A$

total power operation $\psi^i\colon \pi_0 A \to \pi_0\left(A^{B\Sigma_i^+}\right)$
$\forall \eta \in \pi_0 \mathbb{P}_E^i(E)$, individual po $Q_\eta\colon \pi_0 A \to \pi_0 A$
$\Big\} \overset{/I}{\rightsquigarrow}$ additive

$$E \xrightarrow{f_\eta} \mathbb{P}_E^i(E) \xrightarrow{\mathbb{P}_E^i(f_x)} \mathbb{P}_E^i(A) \hookrightarrow \mathbb{P}_E(A) \xrightarrow{\mu} A$$

Power operations for Morava E-theory

$M = E$-module $\qquad \pi_0 M = [S, M]_S \cong [E, M]_E$

$$\mathbb{P}_E(M) = \bigvee_{i \geq 0} \mathbb{P}_E^i(M) = \bigvee_{i \geq 0} (\underbrace{M \wedge_E \cdots \wedge_E M}_{i\text{-fold}})_{h\Sigma_i}$$

$A =$ commutative E-algebra
$\quad =$ algebra for the monad $\mathbb{P}_E$ with $\mu \colon \mathbb{P}_E(A) \to A$

$$\left.\begin{array}{l} \text{total power operation } \psi^i \colon \ \pi_0 A \to \pi_0\big(A^{B\Sigma_i^+}\big) \\ \forall \eta \in \pi_0 \mathbb{P}_E^i(E), \text{individual po } Q_\eta \colon \ \pi_0 A \to \pi_0 A \end{array}\right\} \overset{/I}{\rightsquigarrow} \text{additive}$$

$$E \xrightarrow{f_\eta} \mathbb{P}_E^i(E) \xrightarrow{\mathbb{P}_E^i(f_x)} \mathbb{P}_E^i(A) \hookrightarrow \mathbb{P}_E(A) \xrightarrow{\mu} A$$

Power operations for Morava E-theory

$M = E$-module $\qquad \pi_0 M = [S, M]_S \cong [E, M]_E$

$$\mathbb{P}_E(M) = \bigvee_{i \geq 0} \mathbb{P}_E^i(M) = \bigvee_{i \geq 0} (\underbrace{M \wedge_E \cdots \wedge_E M}_{i\text{-fold}})_{h\Sigma_i}$$

$A =$ commutative E-algebra
$\quad =$ algebra for the monad $\mathbb{P}_E$ with $\mu\colon \mathbb{P}_E(A) \to A$

$$\left.\begin{array}{l} \text{total power operation } \psi^i\colon \pi_0 A \to \pi_0\big(A^{B\Sigma_i^+}\big) \\ \forall \eta \in \pi_0 \mathbb{P}_E^i(E), \text{individual po } Q_\eta\colon \pi_0 A \to \pi_0 A \end{array}\right\} \overset{/I}{\rightsquigarrow} \text{additive}$$

$$E \xrightarrow{f_\eta} \mathbb{P}_E^i(E) \xrightarrow{\mathbb{P}_E^i(f_x)} \mathbb{P}_E^i(A) \hookrightarrow \mathbb{P}_E(A) \xrightarrow{\mu} A$$

$M = E$-module $\qquad \pi_0 M = [S, M]_S \cong [E, M]_E$

$$\mathbb{P}_E(M) = \bigvee_{i \geq 0} \mathbb{P}_E^i(M) = \bigvee_{i \geq 0} (\underbrace{M \wedge_E \cdots \wedge_E M}_{i\text{-fold}})_{h\Sigma_i}$$

$A =$ commutative E-algebra
$\quad =$ algebra for the monad $\mathbb{P}_E$ with $\mu \colon \mathbb{P}_E(A) \to A$

$$\left.\begin{array}{l} \text{total power operation } \psi^i \colon \pi_0 A \to \pi_0\left(A^{B\Sigma_i^+}\right) \\ \forall \eta \in \pi_0 \mathbb{P}_E^i(E), \text{individual po } Q_\eta \colon \pi_0 A \to \pi_0 A \end{array}\right\} \overset{/I}{\rightsquigarrow} \text{additive}$$

$$E \xrightarrow{f_\eta} \mathbb{P}_E^i(E) \xrightarrow{\mathbb{P}_E^i(f_x)} \mathbb{P}_E^i(A) \hookrightarrow \mathbb{P}_E(A) \xrightarrow{\mu} A$$

Power operations for Morava E-theory

$M = E$-module $\qquad \pi_0 M = [S, M]_S \cong [E, M]_E$

$$\mathbb{P}_E(M) = \bigvee_{i \geq 0} \mathbb{P}_E^i(M) = \bigvee_{i \geq 0} (\underbrace{M \wedge_E \cdots \wedge_E M}_{i\text{-fold}})_{h\Sigma_i}$$

$A =$ commutative E-algebra
$\quad =$ algebra for the monad $\mathbb{P}_E$ with $\mu\colon \mathbb{P}_E(A) \to A$

$$\left.\begin{array}{l}\text{total power operation } \psi^i\colon \pi_0 A \to \pi_0\big(A^{B\Sigma_i^+}\big) \\ \forall \eta \in \pi_0 \mathbb{P}_E^i(E), \text{individual po } Q_\eta\colon \pi_0 A \to \pi_0 A\end{array}\right\} \overset{/I}{\rightsquigarrow} \text{additive}$$

$$E \xrightarrow{f_\eta} \mathbb{P}_E^i(E) \xrightarrow{\mathbb{P}_E^i(f_x)} \mathbb{P}_E^i(A) \hookrightarrow \mathbb{P}_E(A) \xrightarrow{\mu} A$$

Power operations for Morava E-theory

$M = E$-module $\qquad \pi_0 M = [S, M]_S \cong [E, M]_E$

$$\mathbb{P}_E(M) = \bigvee_{i \geq 0} \mathbb{P}_E^i(M) = \bigvee_{i \geq 0} (\underbrace{M \wedge_E \cdots \wedge_E M}_{i\text{-fold}})_{h\Sigma_i}$$

$A =$ commutative E-algebra
$\quad =$ algebra for the monad $\mathbb{P}_E$ with $\mu\colon \mathbb{P}_E(A) \to A$

$$\left.\begin{array}{l} \text{total power operation } \psi^i\colon\ \pi_0 A \to \pi_0\big(A^{B\Sigma_i^+}\big) \\ \forall \eta \in \pi_0 \mathbb{P}_E^i(E), \text{individual po } Q_\eta\colon\ \pi_0 A \to \pi_0 A \end{array}\right\} \overset{/I}{\rightsquigarrow} \text{additive}$$

$$E \xrightarrow{f_\eta} \mathbb{P}_E^i(E) \xrightarrow{\mathbb{P}_E^i(f_x)} \mathbb{P}_E^i(A) \hookrightarrow \mathbb{P}_E(A) \xrightarrow{\mu} A$$

Power operations for Morava E-theory

$M = E$-module $\qquad \pi_0 M = [S, M]_S \cong [E, M]_E$

$$\mathbb{P}_E(M) = \bigvee_{i \geq 0} \mathbb{P}_E^i(M) = \bigvee_{i \geq 0} (\underbrace{M \wedge_E \cdots \wedge_E M}_{i\text{-fold}})_{h\Sigma_i}$$

$A =$ commutative E-algebra
$\quad =$ algebra for the monad $\mathbb{P}_E$ with $\mu\colon \mathbb{P}_E(A) \to A$

$$\left.\begin{array}{l} \text{total power operation } \psi^i\colon\ \pi_0 A \to \pi_0\big(A^{B\Sigma_i^+}\big) \\ \forall \eta \in \pi_0 \mathbb{P}_E^i(E), \text{individual po } Q_\eta\colon\ \pi_0 A \to \pi_0 A \end{array}\right\} \overset{/I}{\rightsquigarrow} \text{additive}$$

$$E \xrightarrow{f_\eta} \mathbb{P}_E^i(E) \xrightarrow{\mathbb{P}_E^i(f_x)} \mathbb{P}_E^i(A) \hookrightarrow \mathbb{P}_E(A) \xrightarrow{\mu} A$$

Power operations for Morava E-theory

M = E-module $\qquad \pi_0 M = [S, M]_S \cong [E, M]_E$

$$\mathbb{P}_E(M) = \bigvee_{i \geq 0} \mathbb{P}^i_E(M) = \bigvee_{i \geq 0} (\underbrace{M \wedge_E \cdots \wedge_E M}_{i\text{-fold}})_{h\Sigma_i}$$

A = commutative E-algebra
= algebra for the monad $\mathbb{P}_E$ with $\mu \colon \mathbb{P}_E(A) \to A$

$$\left.\begin{array}{l}\text{total power operation } \psi^i \colon \pi_0 A \to \pi_0\left(A^{B\Sigma_i^+}\right) \\ \forall \eta \in \pi_0 \mathbb{P}^i_E(E), \text{individual po } Q_\eta \colon \pi_0 A \to \pi_0 A\end{array}\right\} \overset{/I}{\rightsquigarrow} \text{additive}$$

$$E \xrightarrow{f_\eta} \mathbb{P}^i_E(E) \xrightarrow{\mathbb{P}^i_E(f_x)} \mathbb{P}^i_E(A) \hookrightarrow \mathbb{P}_E(A) \xrightarrow{\mu} A$$

Power operations for Morava E-theory

$M = E$-module $\qquad \pi_0 M = [S, M]_S \cong [E, M]_E$

$$\mathbb{P}_E(M) = \bigvee_{i \geq 0} \mathbb{P}_E^i(M) = \bigvee_{i \geq 0} (\underbrace{M \wedge_E \cdots \wedge_E M}_{i\text{-fold}})_{h\Sigma_i}$$

$A =$ commutative E-algebra
$\quad =$ algebra for the monad $\mathbb{P}_E$ with $\mu\colon \mathbb{P}_E(A) \to A$

$$\left.\begin{array}{l} \text{total power operation } \psi^i\colon \ \pi_0 A \to \pi_0\big(A^{B\Sigma_i^+}\big) \\ \forall \eta \in \pi_0 \mathbb{P}_E^i(E), \text{individual po } Q_\eta\colon \ \pi_0 A \to \pi_0 A \end{array}\right\} \overset{/I}{\rightsquigarrow} \text{additive}$$

$$E \xrightarrow{f_\eta} \mathbb{P}_E^i(E) \xrightarrow{\mathbb{P}_E^i(f_x)} \mathbb{P}_E^i(A) \hookrightarrow \mathbb{P}_E(A) \xrightarrow{\mu} A$$

Power operations for Morava E-theory

M = E-module $\qquad \pi_0 M = [S, M]_S \cong [E, M]_E$

$$\mathbb{P}_E(M) = \bigvee_{i \geq 0} \mathbb{P}_E^i(M) = \bigvee_{i \geq 0} (\underbrace{M \wedge_E \cdots \wedge_E M}_{i\text{-fold}})_{h\Sigma_i}$$

A = commutative E-algebra
= algebra for the monad $\mathbb{P}_E$ with $\mu\colon \mathbb{P}_E(A) \to A$

$$\left.\begin{array}{l}\text{total power operation } \psi^i\colon\ \pi_0 A \to \pi_0\big(A^{B\Sigma_i^+}\big) \\ \forall \eta \in \pi_0 \mathbb{P}_E^i(E), \text{individual po } Q_\eta\colon\ \pi_0 A \to \pi_0 A\end{array}\right\} \overset{/I}{\rightsquigarrow} \text{additive}$$

$$E \xrightarrow{f_\eta} \mathbb{P}_E^i(E) \xrightarrow{\mathbb{P}_E^i(f_x)} \mathbb{P}_E^i(A) \hookrightarrow \mathbb{P}_E(A) \xrightarrow{\mu} A$$

$M = E$-module $\qquad \pi_0 M = [S, M]_S \cong [E, M]_E$

$$\mathbb{P}_E(M) = \bigvee_{i \geq 0} \mathbb{P}_E^i(M) = \bigvee_{i \geq 0} (\underbrace{M \wedge_E \cdots \wedge_E M}_{i\text{-fold}})_{h\Sigma_i}$$

$A =$ commutative E-algebra
$\quad =$ algebra for the monad $\mathbb{P}_E$ with $\mu\colon \mathbb{P}_E(A) \to A$

$$\left.\begin{array}{l}\text{total power operation } \psi^i\colon \ \pi_0 A \to \pi_0\big(A^{B\Sigma_i^+}\big) \\ \forall \eta \in \pi_0 \mathbb{P}_E^i(E), \text{individual po } Q_\eta\colon \ \pi_0 A \to \pi_0 A\end{array}\right\} \overset{/I}{\rightsquigarrow} \text{additive}$$

$$E \xrightarrow{f_\eta} \mathbb{P}_E^i(E) \xrightarrow{\mathbb{P}_E^i(f_x)} \mathbb{P}_E^i(A) \hookrightarrow \mathbb{P}_E(A) \xrightarrow{\mu} A$$

Power operations for Morava E-theory

$M = E$-module $\qquad \pi_0 M = [S, M]_S \cong [E, M]_E$

$$\mathbb{P}_E(M) = \bigvee_{i \geq 0} \mathbb{P}_E^i(M) = \bigvee_{i \geq 0} (\underbrace{M \wedge_E \cdots \wedge_E M}_{i\text{-fold}})_{h\Sigma_i}$$

$A =$ commutative E-algebra
$\quad =$ algebra for the monad $\mathbb{P}_E$ with $\mu\colon \mathbb{P}_E(A) \to A$

$$\left.\begin{array}{l} \text{total power operation } \psi^i\colon \pi_0 A \to \pi_0\big(A^{B\Sigma_i^+}\big) \\ \forall \eta \in \pi_0 \mathbb{P}_E^i(E), \text{individual po } Q_\eta\colon \pi_0 A \to \pi_0 A \end{array}\right\} \overset{/I}{\rightsquigarrow} \text{additive}$$

$$E \xrightarrow{f_\eta} \mathbb{P}_E^i(E) \xrightarrow{\mathbb{P}_E^i(f_x)} \mathbb{P}_E^i(A) \hookrightarrow \mathbb{P}_E(A) \xrightarrow{\mu} A$$

Theorem (Rezk '09, Barthel-Frankland '13)

If $A = K(n)$-local commutative E-algebra, then

$$\pi_* A = \text{graded amplified } L\text{-complete } \Gamma\text{-ring}$$

- $\Gamma =$ twisted bialgebra over E_0 (Dyer–Lashof algebra)
- $\exists\ Q_0 \in \Gamma$ with $Q_0(x) \equiv x^p \bmod p$ (Frobenius congruence)

Goal make this structure explicit just as for Dyer-Lashof/Steenrod operations in ordinary homology.

The case of $n = 2$ has been worked out. $\leftrightsquigarrow$ Arithmetic

Theorem (Rezk '09, Barthel-Frankland '13)

If $A = K(n)$-local commutative E-algebra, then

$$\pi_* A = \text{graded amplified } L\text{-complete } \Gamma\text{-ring}$$

- $\Gamma =$ twisted bialgebra over E_0 (Dyer–Lashof algebra)
- $\exists\ Q_0 \in \Gamma$ with $Q_0(x) \equiv x^p \bmod p$ (Frobenius congruence)

Goal make this structure explicit just as for Dyer-Lashof/Steenrod operations in ordinary homology.

The case of $n = 2$ has been worked out. $\leftrightsquigarrow$ Arithmetic

Theorem (Rezk '09, Barthel-Frankland '13)

If $A = K(n)$-local commutative E-algebra, then

$$\pi_* A = \text{graded amplified } L\text{-complete } \Gamma\text{-ring}$$

- $\Gamma =$ twisted bialgebra over E_0 (Dyer–Lashof algebra)
- $\exists\ Q_0 \in \Gamma$ with $Q_0(x) \equiv x^p \bmod p$ (Frobenius congruence)

Goal make this structure explicit just as for Dyer-Lashof/Steenrod operations in ordinary homology.
The case of $n = 2$ has been worked out. $\leftsquigarrow$ Arithmetic

Power operations for Morava E-theory (height n prime p)

Theorem (Rezk '09, Barthel-Frankland '13)

If $A = K(n)$-local commutative E-algebra, then

$$\pi_* A = \text{graded amplified } L\text{-complete } \Gamma\text{-ring}$$

- $\Gamma =$ twisted bialgebra over E_0 (Dyer–Lashof algebra)
- $\exists\ Q_0 \in \Gamma$ with $Q_0(x) \equiv x^p \bmod p$ (Frobenius congruence)

Goal make this structure explicit just as for Dyer-Lashof/Steenrod operations in ordinary homology.

The case of $n = 2$ has been worked out. $\leftsquigarrow$ Arithmetic

Power operations for Morava E-theory (height n prime p)

Theorem (Rezk '09, Barthel-Frankland '13)

If $A = K(n)$-local commutative E-algebra, then

$$\pi_* A = \text{graded amplified } L\text{-complete } \Gamma\text{-ring}$$

- $\Gamma =$ twisted bialgebra over E_0 (Dyer–Lashof algebra)
- $\exists\, Q_0 \in \Gamma$ with $Q_0(x) \equiv x^p \mod p$ (Frobenius congruence)

Goal make this structure explicit just as for Dyer-Lashof/Steenrod operations in ordinary homology.

The case of $n = 2$ has been worked out. $\leftrightsquigarrow$ Arithmetic

Power operations for Morava E-theory (height n prime p)

Theorem (Rezk '09, Barthel-Frankland '13)

If $A = K(n)$-local commutative E-algebra, then

$$\pi_* A = \text{graded amplified } L\text{-complete } \Gamma\text{-ring}$$

- $\Gamma =$ twisted bialgebra over E_0 (Dyer–Lashof algebra)
- $\exists\, Q_0 \in \Gamma$ with $Q_0(x) \equiv x^p \bmod p$ (Frobenius congruence)

<u>Goal</u> make this structure explicit just as for Dyer-Lashof/Steenrod operations in ordinary homology.

The case of $n = 2$ has been worked out. $\leftsquigarrow$ Arithmetic

Power operations for Morava E-theory (height n prime p)

Theorem (Rezk '09, Barthel-Frankland '13)

If $A = K(n)$-local commutative E-algebra, then

$$\pi_* A = \text{graded amplified } L\text{-complete } \Gamma\text{-ring}$$

- $\Gamma =$ twisted bialgebra over E_0 (Dyer–Lashof algebra)
- $\exists\ Q_0 \in \Gamma$ with $Q_0(x) \equiv x^p \mod p$ (Frobenius congruence)

Goal make this structure explicit just as for Dyer-Lashof/Steenrod operations in ordinary homology.
The case of $n = 2$ has been worked out. $\leftsquigarrow$ Arithmetic

Power operations for Morava E-theory (height $n = 2$)

Theorem (Z. '19)

Given any Morava E-theory E of height 2 at a prime p, there is an explicit presentation for its algebra of power operations, in terms of generators $Q_i \colon E^0(-) \to E^0(-)$, $0 \le i \le p$, and quadratic relations

$$Q_i Q_0 = -\sum_{k=1}^{p-i} w_0^k \, Q_{i+k} Q_k - \sum_{k=1}^{p} \sum_{m=0}^{k-1} w_0^m \, d_{i,\,k-m} \, Q_m Q_k$$

for $1 \le i \le p$, where the coefficients w_0 and $d_{i,\,k-m}$ arise from certain modular equations for elliptic curves.

<u>Remark</u> The first example, for $p = 2$, was calculated by Rezk '08. These have been applied to computations in unstable v_2-periodic homotopy theory (Z. '18 and ongoing joint work with G. Wang).

Power operations for Morava E-theory (height $n = 2$)

Theorem (Z. '19)

Given any Morava E-theory E of height 2 at a prime p, there is an explicit presentation for its algebra of power operations, in terms of generators $Q_i\colon\ E^0(-) \to E^0(-)$, $0 \le i \le p$, and quadratic relations

$$Q_i Q_0 = -\sum_{k=1}^{p-i} w_0^k\, Q_{i+k} Q_k \; - \sum_{k=1}^{p} \sum_{m=0}^{k-1} w_0^m\, d_{i,\,k-m}\, Q_m Q_k$$

for $1 \le i \le p$, where the coefficients w_0 and $d_{i,\,k-m}$ arise from certain modular equations for elliptic curves.

Remark The first example, for $p = 2$, was calculated by Rezk '08. These have been applied to computations in unstable v_2-periodic homotopy theory (Z. '18 and ongoing joint work with G. Wang).

Power operations for Morava E-theory (height $n = 2$)

Theorem (Z. '19)

Given any Morava E-theory E of height 2 at a prime p, there is an explicit presentation for its algebra of power operations, in terms of generators $Q_i \colon E^0(-) \to E^0(-)$, $0 \leq i \leq p$, and quadratic relations

$$Q_i Q_0 = -\sum_{k=1}^{p-i} w_0^k \, Q_{i+k} Q_k \; - \sum_{k=1}^{p} \sum_{m=0}^{k-1} w_0^m \, d_{i,\,k-m} \, Q_m Q_k$$

for $1 \leq i \leq p$, where the coefficients w_0 and $d_{i,\,k-m}$ arise from certain modular equations for elliptic curves.

Remark The first example, for $p = 2$, was calculated by Rezk '08. These have been applied to computations in unstable v_2-periodic homotopy theory (Z. '18 and ongoing joint work with G. Wang).

Power operations for Morava E-theory (height $n = 2$)

Theorem (Z. '19)

Given any Morava E-theory E of height 2 at a prime p, there is an explicit presentation for its algebra of power operations, in terms of generators $Q_i\colon E^0(-) \to E^0(-)$, $0 \leq i \leq p$, and quadratic relations

$$Q_i Q_0 = -\sum_{k=1}^{p-i} w_0^k \, Q_{i+k} Q_k \; - \sum_{k=1}^{p} \sum_{m=0}^{k-1} w_0^m \, d_{i,\,k-m} \, Q_m Q_k$$

for $1 \leq i \leq p$, where the coefficients w_0 and $d_{i,\,k-m}$ arise from certain modular equations for elliptic curves.

Remark The first example, for $p = 2$, was calculated by Rezk '08. These have been applied to computations in unstable v_2-periodic homotopy theory (Z. '18 and ongoing joint work with G. Wang).

Power operations for Morava E-theory (height $n = 2$)

Theorem (Z. '19)

Given any Morava E-theory E of height 2 at a prime p, there is an explicit presentation for its algebra of power operations, in terms of generators $Q_i \colon E^0(-) \to E^0(-)$, $0 \leq i \leq p$, and quadratic relations

$$Q_i Q_0 = -\sum_{k=1}^{p-i} w_0^k \, Q_{i+k} Q_k - \sum_{k=1}^{p} \sum_{m=0}^{k-1} w_0^m \, d_{i,\,k-m} \, Q_m Q_k$$

for $1 \leq i \leq p$, where the coefficients w_0 and $d_{i,\,k-m}$ arise from certain modular equations for elliptic curves.

<u>Remark</u> The first example, for $p = 2$, was calculated by Rezk '08. These have been applied to computations in unstable v_2-periodic homotopy theory (Z. '18 and ongoing joint work with G. Wang).

Power operations for Morava E-theory (height $n = 2$)

Theorem (Z. '19)

Given any Morava E-theory E of height 2 at a prime p, there is an explicit presentation for its algebra of power operations, in terms of generators $Q_i \colon E^0(-) \to E^0(-)$, $0 \leq i \leq p$, and quadratic relations

$$Q_i Q_0 = -\sum_{k=1}^{p-i} w_0^k \, Q_{i+k} Q_k \; - \sum_{k=1}^{p} \sum_{m=0}^{k-1} w_0^m \, d_{i,\,k-m} \, Q_m Q_k$$

for $1 \leq i \leq p$, where the coefficients w_0 and $d_{i,\,k-m}$ arise from certain modular equations for elliptic curves.

Remark The first example, for $p = 2$, was calculated by Rezk '08. These have been applied to computations in unstable v_2-periodic homotopy theory (Z. '18 and ongoing joint work with G. Wang).

Theorem (Z. '19)

Given any Morava E-theory E of height 2 at a prime p, there is an explicit presentation for its algebra of power operations, in terms of generators $Q_i\colon\ E^0(-) \to E^0(-)$, $0 \le i \le p$, and quadratic relations

$$Q_i Q_0 = -\sum_{k=1}^{p-i} w_0^k\, Q_{i+k} Q_k \; - \sum_{k=1}^{p} \sum_{m=0}^{k-1} w_0^m\, d_{i,\,k-m}\, Q_m Q_k$$

for $1 \le i \le p$, where the coefficients w_0 and $d_{i,\,k-m}$ arise from certain modular equations for elliptic curves.

Remark The first example, for $p = 2$, was calculated by Rezk '08. These have been applied to computations in unstable v_2-periodic homotopy theory (Z. '18 and ongoing joint work with G. Wang).

Recall E-theory at height n and prime p has an underlying model

$$\begin{array}{ccc} F_k \xleftarrow{\text{univ defo}} \Gamma_{\mathbb{W}(k)[\![u_1,\ldots,u_{n-1}]\!]} & \leftrightsquigarrow & E \\ \circlearrowleft & & \circlearrowleft \\ \text{Frobenius isogenies} & & \text{power operations} \end{array}$$

An equivalence of cats (Ando–Hopkins–Strickland '04, Rezk '09)

$$\left\{ \begin{array}{c} \text{qcoh sheaves of grd comm algs} \\ \text{over the moduli problem of} \\ \text{defos of } F/k \text{ and Frob isogs} \end{array} \right\} \cong \left\{ \begin{array}{c} \text{grd comm algs over} \\ \text{the Dyer–Lashof algebra} \\ \text{for } E \end{array} \right\}$$

Goal Compute one side explicitly to get the other side.

Recall E-theory at height n and prime p has an underlying model

$$F_k \xleftarrow{\text{univ defo}} \Gamma_{\mathbb{W}(k)[\![u_1,\ldots,u_{n-1}]\!]} \quad \rightsquigarrow \quad E$$

Frobenius isogenies   power operations

An equivalence of cats (Ando–Hopkins–Strickland '04, Rezk '09)

$$\left\{ \begin{array}{c} \text{qcoh sheaves of grd comm algs} \\ \text{over the moduli problem of} \\ \text{defos of } F/k \text{ and Frob isogs} \end{array} \right\} \cong \left\{ \begin{array}{c} \text{grd comm algs over} \\ \text{the Dyer–Lashof algebra} \\ \text{for } E \end{array} \right\}$$

Goal Compute one side explicitly to get the other side.

Recall E-theory at height n and prime p has an underlying model

$$F_k \xleftarrow{\text{univ defo}} \Gamma_{\mathbb{W}(k)[\![u_1,\ldots,u_{n-1}]\!]} \quad \rightsquigarrow \quad E$$

Frobenius isogenies ↺ power operations ↺

An equivalence of cats (Ando–Hopkins–Strickland '04, Rezk '09)

$$\left\{\begin{array}{c}\text{qcoh sheaves of grd comm algs}\\ \text{over the moduli problem of}\\ \text{defos of } F/k \text{ and Frob isogs}\end{array}\right\} \cong \left\{\begin{array}{c}\text{grd comm algs over}\\ \text{the Dyer–Lashof algebra}\\ \text{for } E\end{array}\right\}$$

Goal Compute one side explicitly to get the other side.

Moduli of formal groups and algebras of power operations

Recall E-theory at height n and prime p has an underlying model

$$F_k \xleftarrow{\text{univ defo}} \Gamma_{\mathbb{W}(k)[\![u_1,\ldots,u_{n-1}]\!]} \quad \rightsquigarrow \quad E$$

$$\circlearrowleft \qquad\qquad\qquad\qquad \circlearrowleft$$

Frobenius isogenies power operations

An equivalence of cats (Ando–Hopkins–Strickland '04, Rezk '09)

$$\left\{ \begin{array}{c} \text{qcoh sheaves of grd comm algs} \\ \text{over the moduli problem of} \\ \text{defos of } F/k \text{ and Frob isogs} \end{array} \right\} \cong \left\{ \begin{array}{c} \text{grd comm algs over} \\ \text{the Dyer–Lashof algebra} \\ \text{for } E \end{array} \right\}$$

Goal Compute one side explicitly to get the other side.

Moduli of formal groups and algebras of power operations

Recall E-theory at height n and prime p has an underlying model

$$F_k \xleftarrow{\text{univ defo}} \Gamma_{\mathbb{W}(k)[\![u_1,\ldots,u_{n-1}]\!]} \circlearrowleft \qquad \leftrightsquigarrow \qquad E \circlearrowleft$$

Frobenius isogenies power operations

An equivalence of cats (Ando–Hopkins–Strickland '04, Rezk '09)

$$\left\{ \begin{array}{c} \text{qcoh sheaves of grd comm algs} \\ \text{over the moduli problem of} \\ \text{defos of } F/k \text{ and Frob isogs} \end{array} \right\} \cong \left\{ \begin{array}{c} \text{grd comm algs over} \\ \text{the Dyer–Lashof algebra} \\ \text{for } E \end{array} \right\}$$

Goal Compute one side explicitly to get the other side.

Recall E-theory at height n and prime p has an underlying model

$$F_k \xleftarrow{\text{univ defo}} \Gamma_{\mathbb{W}(k)[\![u_1,\ldots,u_{n-1}]\!]} \circlearrowleft \quad \leftrightsquigarrow \quad E \circlearrowleft$$

Frobenius isogenies   power operations

An equivalence of cats (Ando–Hopkins–Strickland '04, Rezk '09)

$$\left\{ \begin{array}{c} \text{qcoh sheaves of grd comm algs} \\ \text{over the moduli problem of} \\ \text{defos of } F/k \text{ and Frob isogs} \end{array} \right\} \cong \left\{ \begin{array}{c} \text{grd comm algs over} \\ \text{the Dyer–Lashof algebra} \\ \text{for } E \end{array} \right\}$$

Goal Compute one side explicitly to get the other side.

Recall E-theory at height n and prime p has an underlying model

$$F_k \xleftarrow{\text{univ defo}} \Gamma_{\mathbb{W}(k)[\![u_1,\ldots,u_{n-1}]\!]} \qquad \rightsquigarrow \qquad E$$

$\circlearrowleft$ Frobenius isogenies $\qquad$ $\circlearrowleft$ power operations

An equivalence of cats (Ando–Hopkins–Strickland '04, Rezk '09)

$$\left\{\begin{array}{c}\text{qcoh sheaves of grd comm algs}\\ \text{over the moduli problem of}\\ \text{defos of } F/k \text{ and Frob isogs}\end{array}\right\} \cong \left\{\begin{array}{c}\text{grd comm algs over}\\ \text{the Dyer–Lashof algebra}\\ \text{for } E\end{array}\right\}$$

Goal Compute one side explicitly to get the other side.

Recall E-theory at height n and prime p has an underlying model

$$F_k \xleftarrow{\text{univ defo}} \Gamma_{\mathbb{W}(k)[\![u_1,\ldots,u_{n-1}]\!]} \quad \rightsquigarrow \quad E$$

Frobenius isogenies — power operations

An equivalence of cats (Ando–Hopkins–Strickland '04, Rezk '09)

$$\left\{ \begin{array}{c} \text{qcoh sheaves of grd comm algs} \\ \text{over the moduli problem of} \\ \text{defos of } F/k \text{ and Frob isogs} \end{array} \right\} \cong \left\{ \begin{array}{c} \text{grd comm algs over} \\ \text{the Dyer–Lashof algebra} \\ \text{for } E \end{array} \right\}$$

Goal Compute one side explicitly to get the other side.

Recall E-theory at height n and prime p has an underlying model

$$F_k \xleftarrow{\text{univ defo}} \Gamma_{\mathbb{W}(k)[\![u_1,\ldots,u_{n-1}]\!]} \quad\rightsquigarrow\quad E$$

Frobenius isogenies — power operations

An equivalence of cats (Ando–Hopkins–Strickland '04, Rezk '09)

$$\left\{ \begin{array}{c} \text{qcoh sheaves of grd comm algs} \\ \text{over the moduli problem of} \\ \text{defos of } F/k \text{ and Frob isogs} \end{array} \right\} \cong \left\{ \begin{array}{c} \text{grd comm algs over} \\ \text{the Dyer–Lashof algebra} \\ \text{for } E \end{array} \right\}$$

Goal Compute one side explicitly to get the other side.

Recall E-theory at height n and prime p has an underlying model

$$\begin{array}{ccc} F_k \xleftarrow{\text{univ defo}} \Gamma_{\mathbb{W}(k)[\![u_1,\ldots,u_{n-1}]\!]} & \rightsquigarrow & E \\ \circlearrowleft & & \circlearrowleft \\ \text{Frobenius isogenies} & & \text{power operations} \end{array}$$

An equivalence of cats (Ando–Hopkins–Strickland '04, Rezk '09)

$$\left\{ \begin{array}{c} \text{qcoh sheaves of grd comm algs} \\ \text{over the moduli problem of} \\ \text{defos of } F/k \text{ and Frob isogs} \end{array} \right\} \cong \left\{ \begin{array}{c} \text{grd comm algs over} \\ \text{the Dyer–Lashof algebra} \\ \text{for } E \end{array} \right\}$$

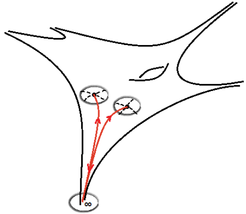
Goal Compute one side explicitly to get the other side.

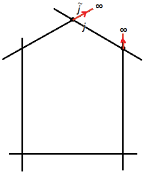
Moduli of formal groups and moduli of ell. curves (Serre–Tate '64)

p-adically, defo thy of an ec $\cong$ defo thy of its p-divisible gp

$[\Gamma_0(p)]$ as an *open arithmetic surface* (Katz–Mazur '85)

parameters for its local ring at a supersingular point, chosen from specific modular forms





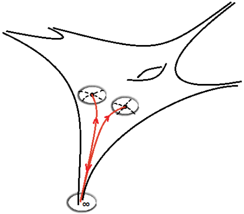
- Compactify the moduli
- Compute with explicit q-expansions
- Transport from cusps to s.sing. pt

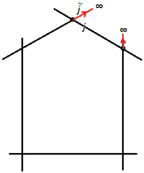
Theorem (Z. '19)

A choice of such parameters, h and α, satisfies the equation

$$(\alpha - 1)^p(\alpha - p) - \big((-1)^p + (-1)^{p-1}(-p)p + h\big)\alpha = 0$$

Moduli of formal groups and moduli of ell. curves (Serre–Tate '64)
p-adically, defo thy of an ec $\cong$ defo thy of its p-divisible gp
$[\Gamma_0(p)]$ as an *open arithmetic surface* (Katz–Mazur '85)
parameters for its local ring at a supersingular point, chosen from specific modular forms





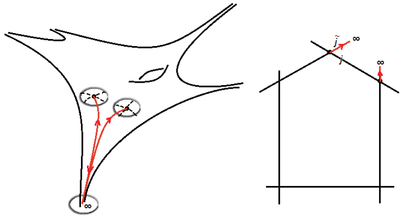
- Compactify the moduli
- Compute with explicit q-expansions
- Transport from cusps to s.sing. pt

Theorem (Z. '19)

A choice of such parameters, h and α, satisfies the equation
$$(\alpha-1)^p(\alpha-p)-\big((-1)^p+(-1)^{p-1}(-p)p+h\big)\alpha=0$$

Moduli of elliptic curves and D.–L. algebras at height 2

Moduli of formal groups and moduli of ell. curves (Serre–Tate '64)
p-adically, defo thy of a s.sing. ec $\cong$ defo thy of its p-divisible gp
$[\Gamma_0(p)]$ as an *open arithmetic surface* (Katz–Mazur '85)
parameters for its local ring at a supersingular point, chosen from specific modular forms



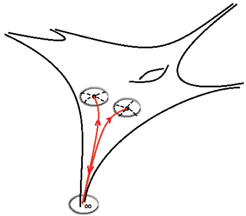
- Compactify the moduli
- Compute with explicit q-expansions
- Transport from cusps to s.sing. pt

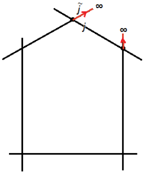
Theorem (Z. '19)

A choice of such parameters, h and α, satisfies the equation
$$(\alpha-1)^p(\alpha-p)-\big((-1)^p+(-1)^{p-1}(-p)p+h\big)\alpha=0$$

Moduli of formal groups and moduli of ell. curves (Serre–Tate '64)
p-adically, defo thy of a s.sing. ec $\cong$ defo thy of its formal gp

$[\Gamma_0(p)]$ as an *open arithmetic surface* (Katz–Mazur '85)
parameters for its local ring at a supersingular point, chosen from specific modular forms





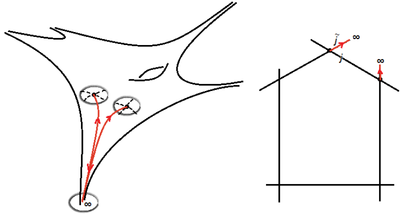
- Compactify the moduli
- Compute with explicit q-expansions
- Transport from cusps to s.sing. pt

Theorem (Z. '19)

A choice of such parameters, h and α, satisfies the equation
$$(\alpha-1)^p(\alpha-p)-\big((-1)^p+(-1)^{p-1}(-p)p+h\big)\alpha=0$$

Moduli of elliptic curves and D.–L. algebras at height 2

Moduli of formal groups and moduli of ell. curves (Serre–Tate '64)
p-adically, defo thy of a s.sing. ec $\cong$ defo thy of its formal gp
$[\Gamma_0(p)]$ as an *open arithmetic surface* (Katz–Mazur '85)
parameters for its local ring at a supersingular point, chosen from specific modular forms



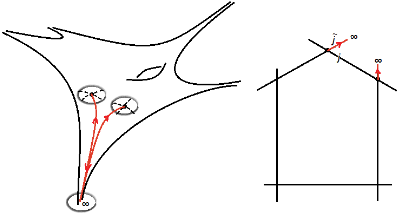
- Compactify the moduli
- Compute with explicit q-expansions
- Transport from cusps to s.sing. pt

Theorem (Z. '19)

A choice of such parameters, h and α, satisfies the equation
$$(\alpha-1)^p(\alpha-p)-\big((-1)^p+(-1)^{p-1}(-p)p+h\big)\alpha=0$$

Moduli of elliptic curves and D.–L. algebras at height 2

Moduli of formal groups and moduli of ell. curves (Serre–Tate '64)
p-adically, defo thy of a s.sing. ec $\cong$ defo thy of its formal gp
$[\Gamma_0(p)]$ as an *open arithmetic surface* (Katz–Mazur '85)
parameters for its local ring at a supersingular point, chosen from specific modular forms

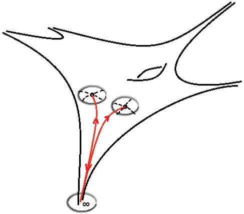


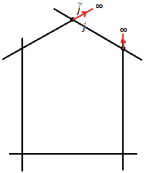
- Compactify the moduli
- Compute with explicit q-expansions
- Transport from cusps to s.sing. pt

Theorem (Z. '19)

A choice of such parameters, h and α, satisfies the equation
$$(\alpha-1)^p(\alpha-p)-\big((-1)^p+(-1)^{p-1}(-p)p+h\big)\alpha=0$$

Moduli of formal groups and moduli of ell. curves (Serre–Tate '64)
p-adically, defo thy of a s.sing. ec $\cong$ defo thy of its formal gp
$[\Gamma_0(p)]$ as an *open arithmetic surface* (Katz–Mazur '85)
parameters for its local ring at a supersingular point, chosen from specific modular forms





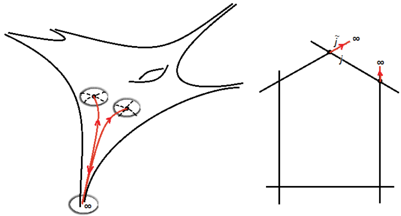
- Compactify the moduli
- Compute with explicit q-expansions
- Transport from cusps to s.sing. pt

Theorem (Z. '19)

A choice of such parameters, h and α, satisfies the equation
$(\alpha-1)^p(\alpha-p) - \big((-1)^p + (-1)^{p-1}(-p)p + h\big)\alpha = 0$

Moduli of elliptic curves and D.–L. algebras at height 2

Moduli of formal groups and moduli of ell. curves (Serre–Tate '64)
p-adically, defo thy of a s.sing. ec $\cong$ defo thy of its formal gp
$[\Gamma_0(p)]$ as an *open arithmetic surface* (Katz–Mazur '85)
parameters for its local ring at a supersingular point, chosen from specific modular forms



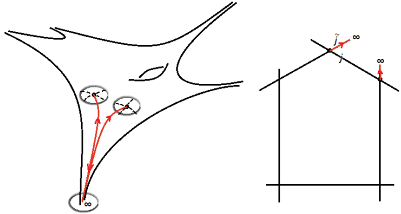
- Compactify the moduli
- Compute with explicit q-expansions
- Transport from cusps to s.sing. pt

Theorem (Z. '19)

A choice of such parameters, h and α, satisfies the equation
$$(\alpha-1)^p(\alpha-p)-\big((-1)^p+(-1)^{p-1}(-p)p+h\big)\alpha=0$$

Moduli of elliptic curves and D.–L. algebras at height 2

Moduli of formal groups and moduli of ell. curves (Serre–Tate '64)
p-adically, defo thy of a s.sing. ec $\cong$ defo thy of its formal gp
$[\Gamma_0(p)]$ as an *open arithmetic surface* (Katz–Mazur '85)
parameters for its local ring at a supersingular point, chosen from specific modular forms



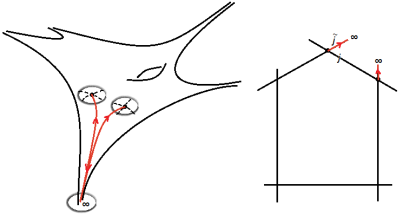
- Compactify the moduli
- Compute with explicit q-expansions
- Transport from cusps to s.sing. pt

Theorem (Z. '19)

A choice of such parameters, h and α, satisfies the equation
$$(\alpha-1)^p(\alpha-p)-\big((-1)^p+(-1)^{p-1}(-p)p+h\big)\alpha=0$$

Moduli of elliptic curves and D.–L. algebras at height 2

Moduli of formal groups and moduli of ell. curves (Serre–Tate '64)
p-adically, defo thy of a s.sing. ec $\cong$ defo thy of its formal gp
$[\Gamma_0(p)]$ as an *open arithmetic surface* (Katz–Mazur '85)
parameters for its local ring at a supersingular point, chosen from specific modular forms



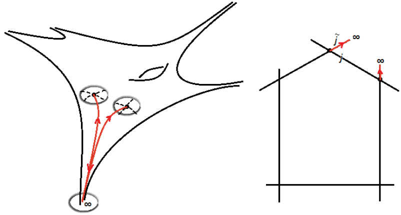
- Compactify the moduli
- Compute with explicit q-expansions
- Transport from cusps to s.sing. pt

Theorem (Z. '19)

A choice of such parameters, h and α, satisfies the equation
$$(\alpha-1)^p(\alpha-p)-\big((-1)^p+(-1)^{p-1}(-p)p+h\big)\alpha=0$$

Moduli of elliptic curves and D.–L. algebras at height 2

Moduli of formal groups and moduli of ell. curves (Serre–Tate '64)
p-adically, defo thy of a s.sing. ec $\cong$ defo thy of its formal gp
$[\Gamma_0(p)]$ as an *open arithmetic surface* (Katz–Mazur '85)
parameters for its local ring at a supersingular point, chosen from specific modular forms



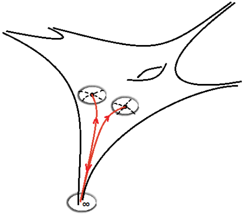
- Compactify the moduli
- Compute with explicit q-expansions
- Transport from cusps to s.sing. pt

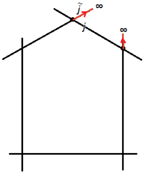
Theorem (Z. '19)

A choice of such parameters, h and α, satisfies the equation

$$(\alpha-1)^p(\alpha-p)-\big((-1)^p+(-1)^{p-1}(-p)p+h\big)\alpha=0$$

Moduli of formal groups and moduli of ell. curves (Serre–Tate '64)
p-adically, defo thy of a s.sing. ec $\cong$ defo thy of its formal gp
$[\Gamma_0(p)]$ as an *open arithmetic surface* (Katz–Mazur '85)
parameters for its local ring at a supersingular point, chosen from specific modular forms





- Compactify the moduli
- Compute with explicit q-expansions
- Transport from cusps to s.sing. pt

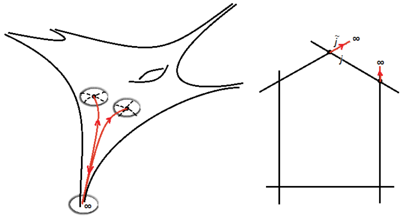
Theorem (Z. '19)

A choice of such parameters, h and α, satisfies the equation

$$(\alpha-1)^p(\alpha-p)-\big((-1)^p+(-1)^{p-1}(-p)p+h\big)\alpha=0$$

Moduli of elliptic curves and D.–L. algebras at height 2

Moduli of formal groups and moduli of ell. curves (Serre–Tate '64)
p-adically, defo thy of a s.sing. ec $\cong$ defo thy of its formal gp
$[\Gamma_0(p)]$ as an *open arithmetic surface* (Katz–Mazur '85)
parameters for its local ring at a supersingular point, chosen from specific modular forms



- Compactify the moduli
- Compute with explicit q-expansions
- Transport from cusps to s.sing. pt

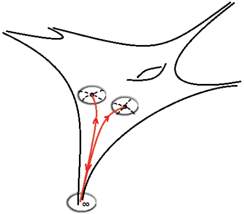
Theorem (Z. '19)

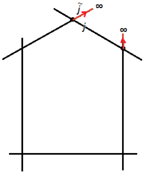
A choice of such parameters, h and α, satisfies the equation

$$(\alpha - 1)^p(\alpha - p) - \big((-1)^p + (-1)^{p-1}(-p)p + h\big)\alpha = 0$$

Moduli of elliptic curves and D.–L. algebras at height 2

Moduli of formal groups and moduli of ell. curves (Serre–Tate '64)
p-adically, defo thy of a s.sing. ec $\cong$ defo thy of its formal gp
$[\Gamma_0(p)]$ as an *open arithmetic surface* (Katz–Mazur '85)
parameters for its local ring at a supersingular point, chosen from specific modular forms





- Compactify the moduli
- Compute with explicit q-expansions
- Transport from cusps to s.sing. pt

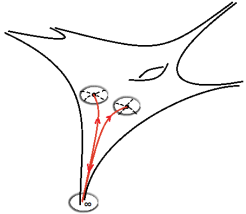
Theorem (Z. '19)

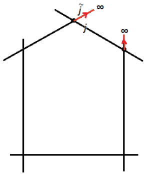
A choice of such parameters, h and α, satisfies the equation

$$(\alpha-1)^p(\alpha-p)-\big((-1)^p+(-1)^{p-1}(-p)p+h\big)\alpha \equiv \alpha(\alpha^p-h) \mod p$$

Moduli of elliptic curves and D.–L. algebras at height 2

Moduli of formal groups and moduli of ell. curves (Serre–Tate '64)
p-adically, defo thy of a s.sing. ec $\cong$ defo thy of its formal gp
$[\Gamma_0(p)]$ as an *open arithmetic surface* (Katz–Mazur '85)
parameters for its local ring at a supersingular point, chosen from specific modular forms





- Compactify the moduli
- Compute with explicit q-expansions
- Transport from cusps to s.sing. pt

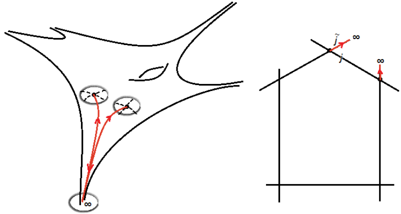
Theorem (Z. '19)

A choice of such parameters, h and α, satisfies the equation

$$(\alpha-1)^p(\alpha-p)-\big((-1)^p+(-1)^{p-1}(-p)p+h\big)\alpha \equiv \alpha(\tilde{j}^p-j) \mod p$$

Moduli of elliptic curves and D.–L. algebras at height 2

Moduli of formal groups and moduli of ell. curves (Serre–Tate '64)
p-adically, defo thy of a s.sing. ec $\cong$ defo thy of its formal gp
$[\Gamma_0(p)]$ as an *open arithmetic surface* (Katz–Mazur '85)
parameters for its local ring at a supersingular point, chosen from specific modular forms



- Compactify the moduli
- Compute with explicit q-expansions
- Transport from cusps to s.sing. pt

Theorem (Z. '19)

A choice of such parameters, h and α, satisfies the equation

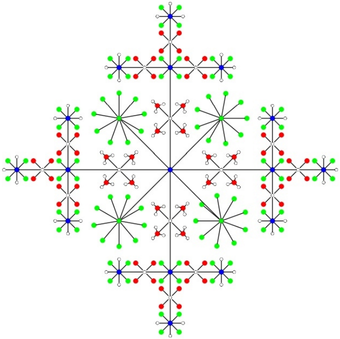
$$(\alpha-1)^p(\alpha-p)-\big((-1)^p+(-1)^{p-1}(-p)p+h\big)\alpha \equiv \alpha(\alpha^p-h) \mod p$$

Moduli of elliptic curves and D.–L. algebras at height 2

Question At height $n > 2$, can we get an explicit presentation for the Dyer–Lashof algebra of Morava E-theory?

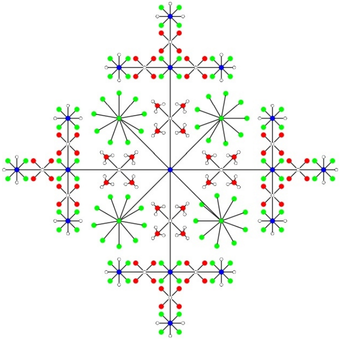
Investigating J. Weinstein's approach to integral models for modular curves via the *infinite* Lubin–Tate tower.

A picture from Jared Weinstein, Semistable models for modular curves of arbitrary level



Question At height $n > 2$, can we get an explicit presentation for the Dyer–Lashof algebra of Morava E-theory?
Investigating J. Weinstein's approach to integral models for modular curves via the *infinite* Lubin–Tate tower.

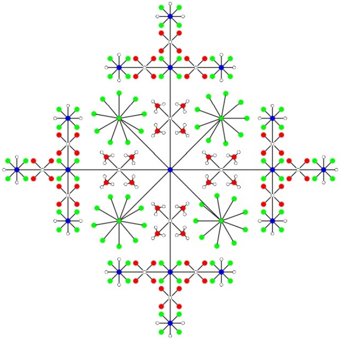
A picture from Jared Weinstein, Semistable models for modular curves of arbitrary level



Moduli of elliptic curves and D.–L. algebras at height 2

Question At height $n > 2$, can we get an explicit presentation for the Dyer–Lashof algebra of Morava E-theory?
Investigating J. Weinstein's approach to integral models for modular curves via the *infinite* Lubin–Tate tower.

A picture from Jared Weinstein, Semistable models for modular curves of arbitrary level



- Established in 2012, a public research-oriented university funded by the municipal government of Shenzhen, China's innovation center.
- Over 800 faculty members, 4205 undergrad students, 2214 postgrad/doctoral students, international students from 15 countries, student:teacher = 10:1.
- Set on five hundred acres of wooded landscape in the picturesque South Mountain area.
- Department of Mathematics founded in June 2015.
  28 research-and-teaching-line faculty members, 6 teaching-line faculty members, 101 grad students, 205 undergrad majors.
- International Center for Mathematics founded in February 2019.

Thank you.