

## 1. Maximum Sum subarray

Find the contiguous subarray within an array (containing at least one number) which has the largest sum.

For example, given the array [2,3,-2,-3, 4],  
the contiguous subarray [2,3] has the largest sum = 5.

Solution:

We need keep 2 Sum values, one is for current sum, one is for the max sum in the global loop.

For the i'th item  $a[i]$  comes, the current sum, denoted as  $curSum[i]$ , is defined as the max subarray sum value ends with  $a[i]$ :

$$curSum[i] = \max\{curSum[i-1]+a[i], a[i]\}$$

And the max sum, denoted as  $maxSum$ , is defined as

$$maxSum = \max\{curSum[i], maxSum\}$$

For examples, array= {1, -2, 3, 10, -4, 7, 2, -5}, then

curSum : 1 -1 3 13 9 16 18 13

maxSum : 1 1 3 13 13 16 18 18

## 2. Maximum Product Subarray

Find the contiguous subarray within an array (containing at least one number) which has the largest product.

For example, given the array [2, 3, 0.5, -2, 4],  
the contiguous subarray [2,3] has the largest product = 6.

Solution:

**If the elements in the array are positive**, it is same as the max sum subarray:

For the i'th item  $a[i]$  comes, the current product, denoted as  $curProd[i]$ , is defined as the max subarray product value ends with  $a[i]$ :

$$curProd[i] = \max\{curProd[i-1]*a[i], a[i]\}$$

And the max product, denoted as  $maxProd$ , is defined as

$$maxProd = \max\{curProd[i], maxProd\}$$

**If the elements in the array are positive or negative**, it is more complex than single polarity.

For the i'th item  $a[i]$  comes, max product ends with  $a[i]$  is determined by  $a[i]$  and max product/min product for item  $i-1$ . For example,  $a[i] = -4$ . Max product of item  $i-1 = 6$ , min product of item  $i-1 = -5$ . Then the max product of item  $i$  is  $-5 * (-4) = 20$ . If  $a[i] = 4$ , the max is  $4 * 6 = 24$ .

So we keep two states at item  $i-1$ : the max product and min product. If  $a[i] = 4$ , max product of item  $i-1$  is 0.25, then max product at item  $i$  is 4.

Hence, we obtain the transfer function. Denote  $\text{maxCurProd}[i]$  as the max subarray product value ends with  $a[i]$ ,  $\text{minCurProd}[i]$  as the min subarray product value ends with  $a[i]$ .

**To calculate max product:**

$$\text{temp} = \max\{\text{maxCurProd}[i-1] * a[i], \text{minCurProd}[i-1] * a[i]\}$$
$$\text{maxCurProd}[i] = \max\{a[i], \text{temp}\}$$

**To calculate min product:**

$$\text{temp} = \min\{\text{maxCurProd}[i-1] * a[i], \text{minCurProd}[i-1] * a[i]\}$$
$$\text{minCurProd}[i] = \min\{a[i], \text{temp}\}$$

For examples, array= {1, -2, 3, 10, -4}, then

maxCurProd:    1   -2   3   30   240

minCurProd:    1   -2   -6   -60   -120

Note the subarray with max product ending with  $a[i]$  can be different from the subarray with min product ending with  $a[i]$ .