

M apples N plates problem

We want M apples to be placed in N plates. It allows empty plates. How many choices to distribute apples to plates? And it does not allow repeatable combinations.

For example, $M=4$, $N=3$. The combinations are (0, 0, 4), (0, 1, 3), (0, 2, 2), (1, 1, 2). So 4 choices are acceptable. (1, 1, 2), (1, 2, 1) and (2, 1, 1) are repeatable combinations, they are regarded as one choice.

Define $f(m, n)$ as the number of choices.

1) if $n > m$, there must exist $n-m$ empty plates. $f(m, n) = f(m, m)$.

2) if $n \leq m$,

a) if there exists at least one empty plates, $f(m, n) = f(m, n-1)$

b) if all plates are placed with apples, suppose we remove one apple from N plates, $m-n$ apples now are placed into N plates. So $f(m, n) = f(m-n, n)$.

So in summary, $f(m, n) = f(m, n-1) + f(m-n, n)$.

The initial condition: $n=1$, $f(m, 1) = 1$; $m=0$ or 1 , $f(m, n) = 1$

```
int func(int m, int n)
{
    if (n == 1 || m == 0 || m == 1)
        return 1;
    if (m < n)
        return func(m, m);
    else
        return func(m - n, n) + func(m, n - 1);
}

int main()
{
    int m, n;
    while (~scanf("%d%d", &m, &n))
    {
        printf("%d\n", func(m, n));
    }
    return 0;
}
```

To go further, if it allows repeatable combinations, (1, 1, 2), (1, 2, 1) and (2, 1, 1) are repeatable combinations, they are regarded as three choices.

Same, we can use iterations to solve it.

```
int fun(int m, int n){
    if(m <= 0)
        return 1;
    if(n <= 0)
        return 0;
    int num=0;
    for(int i=0;i<=m; ++i){
        num += fun(m-i, n-1);
    }
    return num;
}
```