

## Inverse Polish Notation

Inverse Polish Notation is also named as “postfix expression”. Given a mathematic equation:

$$3+2*\{1+2*[-4/(8-6)+7]\}$$

Then we get its IPN as depicted below:

$$3\ 2\ 1\ 2\ -4\ 8\ 6\ -\ /\ 7\ +\ *\ +\ *\ +$$

From left side, search the arithmetic expression(+ - \* /), and use two numbers ahead to do the arithmetic. To be in detail,

-: $8 - 6 = 2$	==>	3 2 1 2 -4 2 / 7 + * + * +
/: $-4 / 2 = -2$	==>	3 2 1 2 -2 7 + * + * +
+: $-2 + 7 = 5$	==>	3 2 1 2 5 * + * +
*: $2 * 5 = 10$	==>	3 2 1 10 + * +
+: $1 + 10 = 11$	==>	3 2 11 * +
*: $2 * 11 = 22$	==>	3 22 +
+: $3 + 22 = 25$		

So the problem is transferred to two sub-problems:

- 1) How to derive the IPN?
- 2) How to do arithmetic from IPN?

For 1), we use a stack and a table to illustrate the procedure. The stack is used for storing the arithmetic expressions, the table is used for storing the numbers.

)
-
(
/
[
*
+
{
*
+

3	2	1	2	-4	8	6			
---	---	---	---	----	---	---	--	--	--

$$3+2*\{1+2*[-4/(8-6)+7]\}$$

3 2 1 2 -4 8 6 - / 7 + \* + \* +

/
[
*
+
{
*
+

3	2	1	2	-4	8	6	-		
---	---	---	---	----	---	---	---	--	--

$3+2*\{1+2*[-4/(8-6)+7]\}$

3 2 1 2 -4 8 6 - / 7 + \* + \* +

+
[
*
+
{
*
+

3	2	1	2	-4	8	6	-	/	
---	---	---	---	----	---	---	---	---	--

At this time, “+” comes, compare “/” with “+”, we need to do “/” before “+”. So we pop out “/” to the table and add “+” in stack.

$3+2*\{1+2*[-4/(8-6)+7]\}$

3 2 1 2 -4 8 6 - / 7 + \* + \* +

]
+
[
*
+
{
*
+

3	2	1	2	-4	8	6	-	/	7			
---	---	---	---	----	---	---	---	---	---	--	--	--

3+2\*{1+2\*[-4/(8-6)+7]}

3 2 1 2 -4 8 6 - / 7 + \* + \* +

}
*
+
{
*
+

3	2	1	2	-4	8	6	-	/	7	+		
---	---	---	---	----	---	---	---	---	---	---	--	--


3	2	1	2	-4	8	6	-	/	7	+	*	+	*	+
---	---	---	---	----	---	---	---	---	---	---	---	---	---	---

So we loop the original equations from left one by one:

- a) If it is a number, put it in the right table.
- b) If it is “)” or “]” or “}”, pop out the stack elements between last “(” or “[” or “{” to right table.
- c) If it is “(” or “[” or “{”, push them into stack.
- d) If it is “+ - \* /”, compare it with top element in stack, if the operation priority is higher (\*and / are higher than – and +), push it into the stack. Else, pop top element to the right table. Now the top element changes. Repeatly compare the priority with the new top element until it meets “(” or lower priority operator, then push it into stack. (Just give an example: 3 \* 4 \* 5 / 3 - 2)

For 2),

$$3+2*\{1+2*[-4/(8-6)+7]\}$$

3 2 1 2 -4 8 6 - / 7 + \* + \* +

-
6
8
-4
2
1
2
3

$$3+2*\{1+2*[-4/(8-6)+7]\}$$

3 2 1 2 -4 8 6 - / 7 + \* + \* +

2
-4
2
1
2
3

$$3+2*\{1+2*[-4/(8-6)+7]\}$$

3 2 1 2 -4 8 6 - / 7 + \* + \* +

/
2
-4
2
1
2
3

$$3+2*\{1+2*[-4/(8-6)+7]\}$$

$$3\ 2\ 1\ 2\ -4\ 8\ 6\ -\ /\ 7\ +\ *\ +\ *\ +$$

-2
2
1
2
3

$$3+2*\{1+2*[-4/(8-6)+7]\}$$

$$3\ 2\ 1\ 2\ -4\ 8\ 6\ -\ /\ 7\ +\ *\ +\ *\ +$$

+
7
-2
2
1
2
3

$$3+2*\{1+2*[-4/(8-6)+7]\}$$

$$3\ 2\ 1\ 2\ -4\ 8\ 6\ -\ /\ 7\ +\ *\ +\ *\ +$$

5
2
1
2
3

$$3+2*\{1+2*[-4/(8-6)+7]\}$$

$$3\ 2\ 1\ 2\ -4\ 8\ 6\ -\ /\ 7\ +\ *\ +\ *\ +$$

+
5
2
1
2
3

7
1
2
3

.....

Same, we loop the IPN from left one by one:

- 1) If it is a number, we push it into the stack.
- 2) If it is a "+ - \* /", we choose top 2 elements in the stack to do the calculation.