Particle Swarm Optimisation: Is velocity clamping necessary?

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***Abstract*—In particle swarm optimisation (PSO) algorithms, particle velocities can become very large which leads to particles leaving the boundaries of the search space. Velocity clamping is a common technique used to prevent velocities from exploding in PSO algorithms. This report tests the hypothesis that velocity clamping is not necessary given that the PSO control parameters are selected such that convergence conditions are satisfied. In order to derive an outcome for the stated hypothesis, a global best PSO algorithm with inertia weight is compared to a global best PSO algorithm with inertia weight and velocity clamping across four standard performance measures. These comparisons are made over different combinations of control parameter values and benchmark optimisation problems to obtain an understanding of whether or not velocity clamping provides a significant improvement in performance. It is observed that when the PSO convergence conditions are satisfied, adding a velocity clamping strategy to a standard PSO with inertia weight does not provide a significant improvement in performance and thus the hypothesis is accepted.**

I. Introduction

The standard PSO algorithm is prone to exploding velocities which leads to particles exhibiting roaming behaviour and hence wasting computational time and effort. This problem has led to the widespread use of velocity clamping as a mechanism to prevent velocities from exploding with the goal of preventing particles from leaving the search space. However, there are other techniques to prevent velocities from growing very large and this report will specifically test whether velocity clamping is necessary when certain convergence conditions are met.

For PSO algorithms with inertia weight, there exists convergence conditions such that, when satisfied, the particles are guaranteed to reach an equilibrium state, *i.e.*, the particles will stop moving. Given that the particles will stop moving at some point it seems intuitive that velocity clamping is not necessary, because if the velocities were to explode, the particles would not reach an equilibrium state. This report sets out to test the hypothesis that velocity clamping is not necessary given that the control parameters of the PSO algorithm satisfy the convergence conditions.

In order to derive an outcome for the stated hypothesis a standard global best PSO algorithm with inertia weight is compared to the same algorithm with a velocity clamping strategy across four standard performance measures. These measures are taken over time to create performance profiles for the different algorithms and control parameter assignments. These performance profiles are used to observe the behaviour of the associated algorithm from which it can be derived whether or not velocity clamping does significantly improve performance of the PSO algorithm.

The main observation obtained from this study is the similarity in performance between the PSO with a clamping strategy and the PSO without a clamping strategy when control parameter values are chosen such that convergence conditions are satisfied. Furthermore it is observed that although velocity clamping can to an extent prevent the explosion of velocities, it may fail to prevent particles from leaving the search space and thus is not a reliable mechanism when convergence conditions are not met.

The rest of the report proceeds as follows: Section II provides a high-level discussion of PSO, velocity clamping and the convergence conditions of PSO. Section III provides an explanation of the approach taken to derive an outcome for the stated hypothesis. Section IV introduces the procedure followed to obtain empirical results. Section V discusses the results obtained from the experiments and section VI draws a conclusion for the report.

II. Background

According to [1] the standard PSO algorithm is a nature-inspired, population-based, stochastic optimisation algorithm. The algorithm is based on the simulation of the social behaviour of birds within a flock. PSO algorithms use a swarm of particles, with each particle representing a potential solution to the optimization problem. Each particle moves through the search space and changes its position based on local and global information. This report uses the PSO with inertia weight algorithm developed by Shi and Eberhart [2]. According to this model described in [1], particle positions are determined according to equation (1)

$$\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \mathbf{v}_i^{t+1} \tag{1}$$

where $\mathbf{x}_i^{t+1}$ denotes the position of the i-th particle at iteration $t+1$ and $\mathbf{v}_i^{t+1}$ denotes the velocity of the i-th particle at iteration $t+1$. Particle velocities are determined according to equation (2)

$$\mathbf{v}_i^{t+1} = w\mathbf{v}_i^t + c_1\mathbf{r}_1(\mathbf{y}_i^t - \mathbf{x}_i^t) + c_2\mathbf{r}_2(\hat{\mathbf{y}}_i^t - \mathbf{x}_i^t) \quad (2)$$

where $\mathbf{y}_i^t$ denotes the personal best position of the i-th particle at iteration t, $\hat{\mathbf{y}}_i^t$ denotes the neighbourhood best position of the i-th particle at iteration t, w denotes the inertia weight which determines the effect of the momentum component of the movement of a particle, c_1 denotes the cognitive acceleration coefficient which determines the importance of local information, c_2 denotes the social acceleration coefficient which determines the importance of global information and $\mathbf{r}_1$ and $\mathbf{r}_2$ are vectors of uniform random numbers in the range $[0, 1]$. For the purpose of this report the global best PSO is used and thus the neighbourhood of a particle is the entire swarm.

Velocity clamping is used to prevent particle velocities from exploding. For the purposes of this report a clamping by dimension strategy will be used. For this strategy a maximum velocity $v_{max,j}$ is calculated for each dimension j according to a certain formula. Whenever the velocity is updated and the new velocity exceeds this maximum velocity, it is set to the maximum velocity. Velocities are thus updated according to equation (3) as stated in [1]

$$v_{i,j}^{t+1} = \begin{cases} v_{i,j}^{t+1} & \text{if } -v_{max,j} \leq v_{i,j}^{t+1} \leq v_{max,j} \\ v_{max,j} & \text{if } v_{max,j} < v_{i,j}^{t+1} \\ -v_{max,j} & \text{if } v_{i,j}^{t+1} < -v_{max,j} \end{cases} \quad (3)$$

where $v_{i,j}^{t+1}$ denotes the particle velocity of the i-th particle in dimension j at iteration $t+1$. Velocity clamping thus ensures that the velocity will never exceed this threshold with the goal of preventing particles from leaving the search space.

For a PSO with inertia weight algorithm there exists certain converge conditions, such that when satisfied it is guaranteed to reach an equilibrium state, *i.e.*, each particle will have a velocity of zero and stop moving. The PSO control parameter values, *i.e.*, w, c_1 and c_2, can be chosen such that these convergence conditions are satisfied.

III. Methodology

The objective of this report is to derive an outcome for the hypothesis that velocity clamping is not necessary provided that the PSO control parameters are selected such that convergence conditions are satisfied. To derive an outcome for the stated hypothesis a standard global best PSO algorithm with inertia weight is compared to the same algorithm with a velocity clamping strategy across four standard performance measures. The performance measures used to compare these two algorithms include the diversity of the swarm, quality of the global best position, percentage of particles that leave the search space and the average velocity magnitude. All of these performance measures are taken over time as the algorithm runs to create a performance profile for the associated algorithm. These performance profiles provide information about the behaviour of the algorithms over time.

Based on these performance profiles related to a specific algorithm and control parameter assignment, a comparison can be made between the performance of the different algorithms with respect to each performance measure. The diversity of the swarm gives an indication of how widely spread out the particles are and if the diversity grows very large in relation to the domain of the problem it can be inferred that velocities have grown large and particles have left the search space. The quality of the global best value gives an indication of the performance of the algorithm, because a global best that remains very large and far away from the associated global minimum indicates that the algorithm is not functioning properly. The percentage of particles outside of the search space is an efficient measure to indicate the roaming behaviour of the swarm. A high percentage of particles outside of the search space likely implies velocities have exploded. The average velocity magnitude is the base measure of whether or not the velocities grow large over time.

These performance profiles for the different algorithms and combination of control parameter values are created and compared for five different benchmark minimization problems. Five benchmark problems are used to obtain a better understanding of the general performance of these strategies across different problems and to avoid making conclusions based on problem-dependent results.

The formula used to calculate $v_{max,j}$ in this report is given by equation (4)

$$v_{max,j} = k*(x_{max,j} - x_{min,j}), \forall j = 1, .., n_x, k \in \{0.1, 0.3, 0.5\} \quad (4)$$

where $x_{max,j}$ is the upper boundary constraint in dimension j and $x_{min,j}$ is the lower boundary constraint in dimension j. In order to obtain an understanding of the velocity clamping strategy, a parameter k is used to alter the size of the maximum velocity and the performance between these different parameter assignments is compared. This gives an indication of whether smaller or larger maximum velocities are preferred according to the performance measures used.

Through the comparison of these performance profiles of the standard PSO and the standard PSO with velocity clamping an understanding of the necessity of velocity clamping can be obtained. First the performance of these two algorithms are compared under control parameter assignments that do not satisfy the convergence conditions. This comparison gives an indication of whether velocity magnitude is needed in general and provides insight into the performance of the algorithms when convergence conditions are not satisfied. The objective is to show whether a velocity clamping strategy alone can avoid the explosion of velocities and thus prevent the roaming behaviour of particles even if convergence conditions are not satisfied. However, the main observation of the report is derived through the comparison of the algorithms when control parameter values are assigned such that convergence conditions are satisfied.

Through the comparison of these algorithms it can be derived whether or not a velocity clamping strategy shows a significant performance improvement across the four performance measures under satisfied convergence conditions. If the standard PSO achieves similar performance to the PSO with velocity clamping, even for values of k that have shown better performance, it can be said that velocity clamping is not necessary when convergence conditions are met and thus the hypothesis is accepted. However, if the velocity clamping strategy shows significantly improved performance compared to the standard PSO across all four measures it can be said that velocity clamping is necessary and thus the hypothesis should be rejected.

IV. Empirical Procedure

This section describes the procedure that was followed to obtain the empirical results for this report.

Both the global best PSO with inertia weight and the global best PSO with inertia weight with clamping were evaluated over five benchmark minimization problems from [3]. The five problems included the Rosenbrock Function, Exponential Function, Qing Function, Step 3 Function and the Cosine Mixture Function. For all of the experiments a swarm of 30 particles and a dimension size of 30 were used.

The performance measurements used for the evaluation of the different algorithms included the diversity of the swarm over time, quality of the global best position over time, percentage of particles that are outside of the search space boundaries over time and the average velocity magnitude over time. The diversity of the swarm can be measured by taking the average Euclidean distance that particles are from the centre of mass of the swarm. The centre of mass of the swarm is the average particle.

In order to obtain the performance profile for a certain algorithm and combination of parameter values the following procedure was done: The algorithm was executed for 5000 iterations where at each iteration the results of all four performance measures were collected. The above step was done 20 times and the average and standard deviation of the values over the 20 independent runs were obtained at each iteration. The average is plotted on a graph to indicate how the algorithm performed over time according to the associated performance measure. The standard deviation was added to and subtracted from the average at each iteration and these points were plotted to indicate the standard deviation of the associated measures over time.

For each of the five benchmark problems the following sequence was applied: For the PSO without clamping the performance profiles were created as described in the procedure above for each combination of control value assignments. The same procedure is applied to the PSO with a clamping strategy for each value of $k \in \{0.1, 0.3, 0.5\}$. Thus for each value of k all of the control parameter assignments were applied and the performance profiles were generated.

TABLE I: Control parameter combinations

	w	c_1	c_2	Satisfy Convergence Conditions?
Combination 1	1.0	2.0	2.0	No
Combination 2	0.9	2.0	2.0	No
Combination 3	0.7	1.4	1.4	Yes
Combination 3	0.9	0.7	0.7	Yes

Convergence conditions used for this report is given by equation (5)

$$c_1 + c_2 < \frac{24(1 - w^2)}{7 - 5w} \tag{5}$$

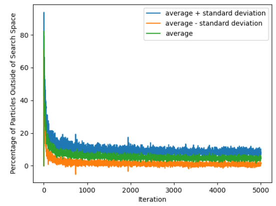
The combinations of control parameter values used and whether or not they satisfy the convergence conditions in equation (5) is given in Table I. The different combinations were included in the experiments to obtain insight into how the algorithms perform when the convergence conditions are satisfied as opposed to when they are not. Furthermore, Combination 1 and 2 were both included to observe whether similar trends would arise for control parameter assignments with different values where both do not satisfy the convergence conditions. Similarly both Combination 3 and 4 were included to observe whether similar trends arise for different control parameter assignments that satisfy the convergence conditions.

V. Research Results

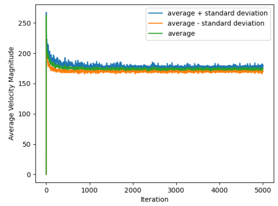
This section presents the empirical results and discusses the performance of the PSO without velocity clamping and the PSO with velocity clamping across different combinations of control parameter assignments. First, the performance for the different values of k in the velocity clamping strategy were compared. Next, the performance of the two algorithms were compared under conditions that do not satisfy the convergence conditions to show that velocity clamping alone does not ensure good performance for PSO algorithms. Finally, the algorithms were compared under conditions that do satisfy the convergence conditions which indicated that there is not a significant increase in performance when velocity clamping is applied.

A. Performance for different clamping parameters

The performance of velocity clamping is dependent on the clamping strategy, the formula used to calculate $v_{max,j}$ and parameters used for this formula. For the formula used in this report different values of k were applied to gauge the performance of the clamping strategy more accurately. The main objective of velocity clamping is to ensure that velocities do not explode and thus prevent particles from leaving the search space. Fig. 1 and Fig. 2 illustrate the performance profiles with respect to the percentage of particles that leave the search space and the average velocity magnitude over time for the Step 3 Function with parameter values of $k = 0.3$ and $k = 0.5$ respectively. These figures are representative of the behaviours observed for the other optimisation problems used in this report. Through the comparison of Fig. 1 and Fig. 2 it was observed that smaller k values, and thus smaller velocity maximums, led to improved performance.

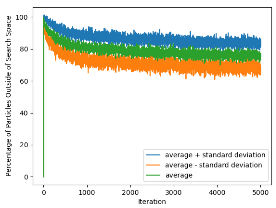


(a) Percentage of Particles Outside Search Space

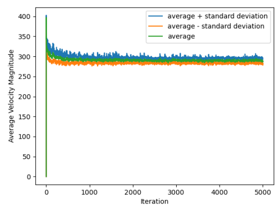


(b) Average Velocity Magnitude

Fig. 1: Performance profiles for Step 3 Function, $k = 0.3$



(a) Percentage of Particles Outside Search Space



(b) Average Velocity Magnitude

Fig. 2: Performance profiles for Step 3 Function, $k = 0.5$

Fig. 1 (a) showed that after a large number of particles left the search space in the first few iterations, the particles quickly returned and after 5000 iterations only 5% of particles remained outside of the search space. However, Fig. 2 (a) indicated that a large number of the particles stayed outside of the search space and after 5000 iterations 73% were outside of the search space. Similarly through the comparison of Fig. 1 (b) and Fig. 2 (b) it was observed that after the initial spike, the average velocity magnitude decreased to 175 and 288 after 5000 iterations for $k = 0.3$ and $k = 0.5$ respectively. This result is intuitive because the decreased velocity maximum will prevent the velocities from increasing in size beyond a certain threshold and thus reduce the average velocity magnitude. This reduced average velocity magnitude reduces the number of particles that leave the search space, because large step sizes may lead to particles moving outside of the search space. Furthermore it was observed that Fig. 1 obtained smaller standard deviations compared to Fig. 2 over the 20 independent runs which showed that the PSO algorithm executes more consistently for smaller values of k.

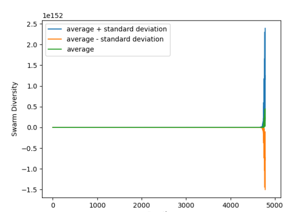
These trends that indicated the improved performance for lower values of k were observed throughout the different combinations of control parameter values and all of the values assigned to k. These results also showed that the performance of velocity clamping is highly dependent on parameter values and poor results may arise if these parameters are chosen incorrectly.

B. *Performance under unsatisfied convergence conditions*

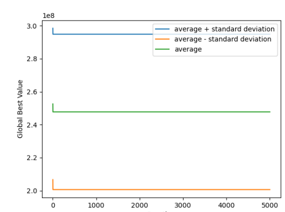
Although this report sets out to test the necessity of velocity clamping given that convergence conditions are satisfied, it is important to show the performance of these algorithms under conditions that do not satisfy the convergence conditions. This was done in order to see whether velocity clamping alone can ensure good performance for PSO algorithms. Fig. 3 and Fig. 4 show the swarm diversity, global best value, percentage of particles outside of the search space and the average velocity magnitude over time for the Rosenbrock function for the no clamping PSO and the clamping PSO with $k = 0.5$ respectively.

From Fig. 3 (a) it was observed that the diversity of the swarm exploded to extremely large values. This explosion in values indicated that particles are widely dispersed and that roaming behaviour occurred due to these diversity values being notably larger than the search space domain. This observation was substantiated by the high percentage of particles that left the search space which after 5000 iterations reached 98% as seen in Fig. 3 (c) and the extremely large average velocity magnitude as seen in Fig. 3 (d). Fig. 3 (b) indicated the bad performance with regards to the quality of the global best value, because the value remained very large and far away from the global minimum of zero. This poor performance with regards to the global best value can be attributed to the fact that most of the particles left and remained outside of the search space.

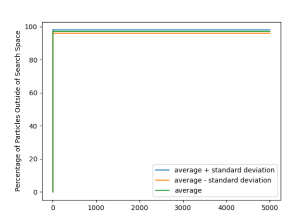
From Fig. 4 it was observed that after the initial increase in the diversity of the swarm and velocity magnitude, these measures settled at a certain level where they remained consistent with small variations. Fig. 4 (c) showed that a large number of the particles immediately left and remained outside of the search space. The percentage of particles outside of the search space reached 100% after 5000 iterations. From Fig. 4 (b) it was observed that the global best value does show a decreasing trend and outperformed the no-clamping strategy. However, the global best value still remained very large after 5000 iterations and far away from the global minimum of zero. Through the comparison of Fig. 3 and Fig. 4 it was observed that the standard deviation for Fig. 3 was notably



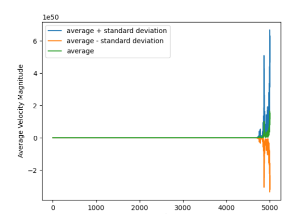
(a) Swarm Diversity



(b) Global Best Value

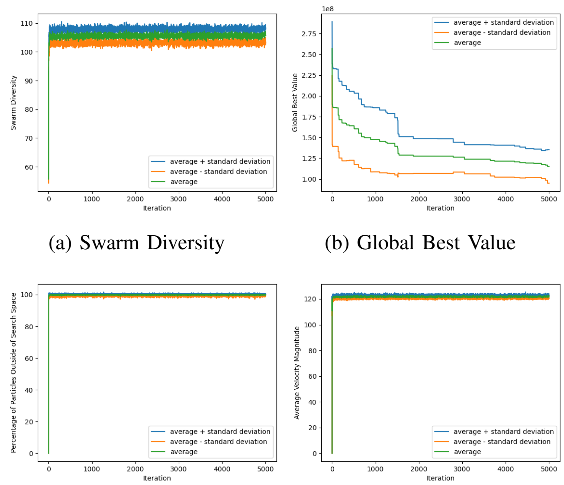


(c) Percentage of Particles Outside Search Space



(d) Average Velocity Magnitude

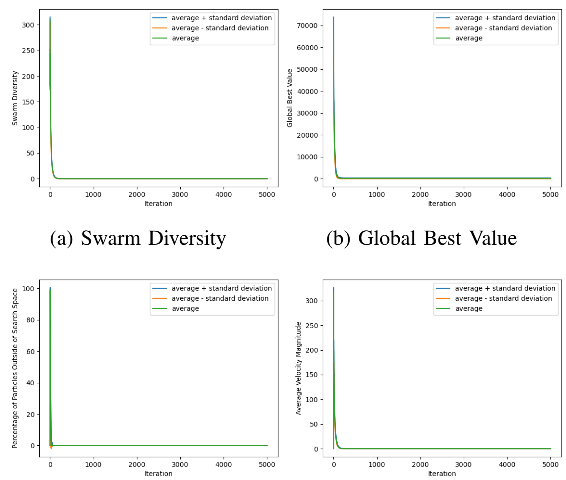
Fig. 3: Performance profiles for Rosenbrock Function under non-satisfying convergence conditions and no clamping



(a) Swarm Diversity (b) Global Best Value

(c) Percentage of Particles Outside Search Space (d) Average Velocity Magnitude

Fig. 4: Performance profiles for Rosenbrock Function under non-satisfying convergence conditions and $k = 0.5$



(a) Swarm Diversity (b) Global Best Value

(c) Percentage of Particles Outside Search Space (d) Average Velocity Magnitude

Fig. 5: Performance profiles for Step 3 Function under satisfying convergence conditions and no clamping

larger across the 20 independent runs which indicated that the clamping strategy used for Fig. 4 improved the stability of the PSO algorithm.

From these observations it was inferred that clamping does indeed improve the performance when convergence conditions are not satisfied but does not actually prevent particles from leaving the search space. Similar trends were observed across the different parameter assignments and optimisation problems used in this report. It should be noted that smaller values of k did perform better across the four performance measures, but the same problems persisted even though only at a smaller scale. It can thus be derived that in general velocity clamping on its own can not ensure good performance for a PSO algorithm. This statement becomes even more evident for higher dimension problems as stated in [3].

C. Performance under satisfied convergence conditions

The objective of this report is to test whether velocity clamping is necessary when control parameter values are chosen such that they satisfy the convergence conditions. Fig. 5 shows the performance profiles for the Step 3 Function under control parameter assignment combination 3, which satisfies convergence conditions, for the PSO algorithm with no clamping. This figure is representative of the trends shown for all of the benchmark problems used in this report and the trends shown under control parameter assignment combination 4.

Fig. 5 (a) indicated that the diversity of the swarm quickly decreased and converged to zero, which implies particles have all gathered to the same solution. From Fig. 5 (d) it was observed that the average velocity magnitude quickly decreased and converged to zero as well. This is due to the convergence conditions being satisfied and thus guaranteeing the particles will reach an equilibrium state. Furthermore, all of the particles quickly returned to the search space after the initial exodus of particles as seen in Fig. 5 (c).

It was observed that the performance of the PSO algorithm with clamping was very similar to that of the PSO algorithm with no clamping when control parameter assignment combination 3 was used. For the PSO with clamping and $k = 0.1$ the diversity of the swarm and the average velocity magnitude quickly decreased and converged to zero similarly to Fig. 5. Furthermore it was observed that both the no clamping and clamping PSO algorithms obtained small standard deviations which indicated that the satisfied convergence conditions led to PSO algorithm executing in a stable manner. Fig. 6 shows the percentage of particles outside of the search space over time for the PSO with a clamping parameter of $k = 0.1$ under control parameter assignment combination 3. Through the comparison of Fig. 5 (c) and Fig. 6 it was observed that far more particles left the search space for the no clamping strategy in the first few iterations. However, for both Fig. 5 (c) and Fig. 6 these particles returned to the search space quickly, and thus it did not have a marked impact on the performance of the algorithm. Furthermore for larger values of k it was observed that the behaviours among the two algorithms were practically the same with respect to the percentage of particles outside of the search over time.

Table II compares the global best values under combination 3 of parameters assignments for different values of k with clamping and no clamping. Although the clamping strategy with $k = 0.1$ outperformed no clamping with regards to the global best value after 5000 iterations, it was observed that no clamping outperformed the clamping strategy with $k = 0.3$ and $k = 0.5$. Similarly from Table II it was observed that the clamping strategy with $k = 0.1$ had a lower standard deviation compared to the no clamping strategy which indicated that the PSO performed more consistently. It should be noted that

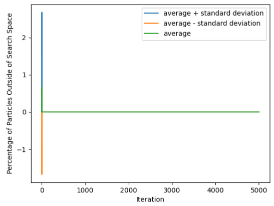


Fig. 6: Percentage of Particles Outside Search Space Step 3 Function under satisfying convergence conditions and $k = 0.1$

TABLE II: Global best value after 5000 iterations for Step Function 3 with convergence conditions satisfied

	Average	Standard Deviation
No Clamping	180.1	116.12
Clamping: k = 0.1	110	63.71
Clamping: k = 0.3	298.6	291.61
Clamping: k =0.5	438.85	414.61

from the results shown in Table II it can be inferred that all of the strategies led to premature convergence and thus relatively poor global best values were obtained.

These observations indicated that although clamping can improve the performance in terms of the quality of the global best value under convergence conditions, the improvements are not significant and are reliant on the parameters of the clamping strategy. These trends were observed for combination 3 and 4 of control parameter assignments which satisfy the convergence conditions for all of the benchmark problems used in this report. This observation that the clamping strategy performed very similarly to the PSO with no clamping across all four performance measures indicates that a clamping strategy does not have a significant impact on the performance of a PSO algorithm when convergence conditions are met.

VI. Conclusion

The objective of this report is to test the hypothesis that velocity clamping is not necessary provided that the PSO control parameters, *i.e.*, w, c_1 and c_2 are selected such that convergence conditions are satisfied. In order to derive an outcome for the stated hypothesis a standard particle swarm optimisation (PSO) algorithm with inertia weight was compared to the same algorithm with velocity clamping. These algorithms were compared according to four performance measures across different combinations of control parameters assignments where some combinations satisfy the convergence conditions and others do not.

From the results discussed in Section V it is derived that velocity clamping is not necessary when convergence conditions are satisfied due to velocity clamping strategy not significantly improving the standard PSO with inertia weight. The hypothesis is thus accepted. Furthermore it is concluded that although velocity clamping does improve the performance of a standard PSO with inertia weight it does not ensure good performance for PSO algorithms and may lead to large scale particle roaming behaviour if parameters are chosen incorrectly.

This report can be extended through the addition of experiments using higher dimensions and more benchmark optimisation problems.

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