Examples for Testing of the Indirect Estimation Prototype

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Introduction

This document contains a set of examples that demonstrates the estimation capabilities of the Indirect Estimation Prototype. These capabilities are demonstrated by a set of examples that are presented in increasing level of complexity.

Each example consists of a visual description of the model to be estimated. The states are numbered and transitions are noted by coefficient names. In case the model uses a coefficient table depending on a covariate, the table is presented after the graphic model. The coefficients are the unknowns to be deduced during estimation.

To estimate the model, each example provides a set of studies that provide sufficient information to estimate the model. Each study is presented graphically in model terminology, i.e. the start and end state of the study is provided using the states declared in the model. The information for each study consists of:

- T - Study length in time units
- Study related information of one of the following forms:
    - Studies that provide data
        - N – Initial population at start state
        - x – Incident count reaching the study end state by the end of study
    - Studies that provide regression information
        - $F(t,\lambda)$ – Regression formula
        - $\hat{\lambda}$ – Regression coefficients provided by the study
        - Σ – The covariance matrix associated with these coefficients
- Population – In some cases, population information is required to interpret the study or to support the system operation. The population is provided per study and may be presented in one of the following ways:
    - Default – theoretically not required, although the software may require some definition for proper functionality.
    - Dummy – Any population that describes one or more covariates to allow proper functionality of the software. Not required theoretically.
    - Distribution – a set of covariates and their marginal distribution function
    - Data – a table that its columns are covariates and each row contains the values for this column for each individual.

The expected outcomes that describe the optimal coefficient values are presented at the end of each example. In some cases, some comments or variations on the example are also provided as well as the expected partial likelihood related to a specific study.

For further information about the project, please visit the project web site:
http://www.med.umich.edu/mdrtc/cores/DiseaseModel/

Example 1: Simple Example with Trivial Outcome:

Model

$$\boxed{0} \xrightarrow{P_0} \boxed{1}$$

Study 1

$$\boxed{0} \longrightarrow \boxed{1}$$

Data: N=100 x=20 T=3
Population: Default - Theoretically none needed
Partial Likelihood:
[1-(1-p01)^T]^x [(1-p01)^T]^(N-x) [1-(1-p01)^3]^20 [(1-p01)^3]^(80)

Expected Outcome:

$$\hat{p}_0 = 1-(1-\frac{x}{N})^{1/t} = 0.0717$$

Example 2: Theoretical Example with previous Algorithms (based on Isaman, 2006, Table 1, model 1:1:3)

Model

$$\boxed{0} \xrightarrow{P_0} \boxed{1} \xrightarrow{P_1} \boxed{2}$$

Study 1

$$\boxed{0} \longrightarrow \boxed{1}$$

Data: N=500 x= 204.755 T=5
Population: Default - Theoretically none needed
Partial Likelihood: [1-(1-P0)^T]^x [(1-P0)^T]^(N-x)

Study 2

$$\boxed{1} \longrightarrow \boxed{2}$$

Data: N=500 x= 336.16 T=5
Population: Default - Theoretically none needed
Partial Likelihood: [1-(1-P1)^T]^x [(1-P1)^T]^(N-x)

Study 3-5

$$\boxed{0} \longrightarrow \boxed{2}$$

Data Study 3: N=500 x= 73.35 T=5
Data Study 4: N=400 x= 58.68 T=5
Data Study 5: N=300 x= 44.01 T=5
Population: Default - Theoretically none needed
Partial Likelihood: [1-(1-P)]^x [(1-P)]^(N-x) where
P = P0*P1+P0*P1*(1-P0)+P0*P1*(1-P1)+P0*P1*(1-P0)^2+P0*P1*(1-P1)^2+P0*P1*(1-P0)*(1-P1)+P0*P1*(1-P0)^3+P0*P1*(1-P1)^3+P0*P1*(1-P0)*(1-P1)^2+P0*P1*(1-P0)^2*(1-P1)

Expected Outcome:

$\hat{p}_0=0.1$ $\hat{p}_1=0.2$

Example 3: Theoretical Example - Missing Some Data- (based on Isaman, 2006, Table 1, model 0:1:3)

Model

$$\boxed{0} \xrightarrow{P_0} \boxed{1} \xrightarrow{P_1} \boxed{2}$$

Study 1

$$\boxed{1} \longrightarrow \boxed{2}$$

Data: N=500 x=336.16 T=5
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 2-4

$$\boxed{0} \longrightarrow \boxed{2}$$

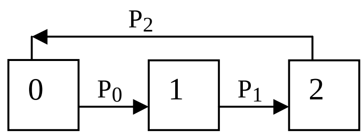
Data Study 3: N=500 x=73.35 T=5
Data Study 4: N=400 x=58.68 T=5
Data Study 5: N=300 x=44.01 T=5
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Expected Outcome:

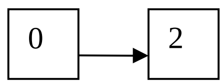
$\hat{p}_0$ =0.1 $\hat{p}_1$ =0.2

Example 4: Funny Loop Example – No Direct Data – (from Isaman, 2006, Table 2, model C)

Model

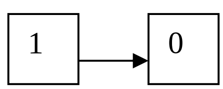


Study 1-3



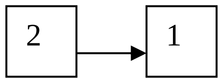
Data Study 1: N=500 x=73.35 T=5
Data Study 2: N=400 x=58.68 T=5
Data Study 3: N=300 x=44.01 T=5
Population: Default - Theoretically none needed
Partial Likelihood: [1-(1-P)]^x [(1-P)]^(N-x) where
P = P0*P1+P0*P1*(1-P0)+P0*P1*(1-P1)+P0*P1*(1-P0)^2+P0*P1*(1-P1)^2+P0*P1*(1-P0)*(1-P1)+P0*P1*(1-P0)^3+P0*P1*(1-P1)^3+P0*P1*(1-P0)*(1-P1)^2+P0*P1*(1-P0)^2*(1-P1)

Study 4-6



Data Study 4: N=500 x=176.55 T=5
Data Study 5: N=400 x=141.24 T=5
Data Study 6: N=300 x=105.93 T=5
Population: Default - Theoretically none needed
Partial Likelihood: [1-(1-P)]^x [(1-P)]^(N-x) where
P = P2*P1+P2*P1*(1-P2)+P2*P1*(1-P1)+P2*P1*(1-P2)^2+P2*P1*(1-P1)^2+P2*P1*(1-P2)*(1-P1)+P2*P1*(1-P2)^3+P2*P1*(1-P1)^3+P2*P1*(1-P2)*(1-P1)^2+P2*P1*(1-P2)^2*(1-P1)

Study 7-9

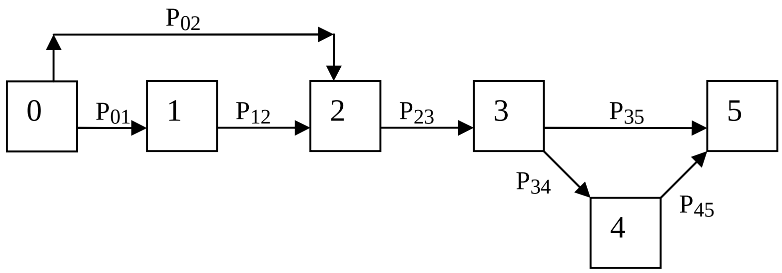


Data Study 7: N=500 x=99.15 T=5
Data Study 8: N=400 x=79.32 T=5
Data Study 9: N=300 x=59.49 T=5
Population: Default - Theoretically none needed
Partial Likelihood: [1-(1-P)]^x [(1-P)]^(N-x) where
P = P0*P2+P0*P2*(1-P0)+P0*P2*(1-P2)+P0*P2*(1-P0)^2+P0*P2*(1-P2)^2+P0*P2*(1-P0)*(1-P2)+P0*P2*(1-P0)^3+P0*P2*(1-P2)^3+P0*P2*(1-P0)*(1-P2)^2+P0*P2*(1-P0)^2*(1-P2)

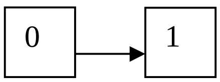
Expected Outcome:
$\hat{p}_0$=0.1 $\hat{p}_1$=0.2 $\hat{p}_2$=0.3

Example 5: Example with a Pooled State

Model

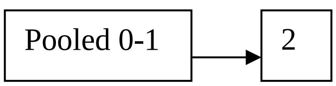


Study 1-2



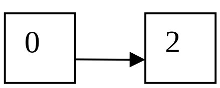
Data Study 1: N=79 x=12.871600960512 T=6
Data Study 2: N=90 x=20.7539552818476 T=9
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 3



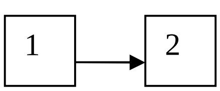
Data: N=398 x=79.18810184 T=4, Prevalence = 50% State 0
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 4



Data: N=176 x=16.118751520256 T=6
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 5-6



Data Study 5: N=49 x=22.959391 T=6
Data Study 6: N=45 x=18.42795 T=5
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 7

$\boxed{2} \rightarrow \boxed{3}$

Data: N=202 x=19.4079990464 T=5
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 8

$\boxed{3} \rightarrow \boxed{4}$

Data: N=1000 x=40 T=1
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 9-10

$\boxed{3} \rightarrow \boxed{5}$

Data: N=231 x=109.24914 T=3
Data: N=11929 x=7597.886603726 T=5
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 11

$\boxed{4} \rightarrow \boxed{5}$

Data: N=23 x=5.2030384375 T=5
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Expected Outcome:

$\hat{p}_{01}=0.03$, $\hat{p}_{02}=0.01$, $\hat{p}_{12}=0.1$, $\hat{p}_{23}=0.02$, $\hat{p}_{34}=0.04$, $\hat{p}_{35}=0.2$, $\hat{p}_{45}=0.05$

Example 6: Simple Example with Covariates:

Model

$$\boxed{0} \xrightarrow{P_{01}} \boxed{1}$$

Study 1

$$\boxed{0} \longrightarrow \boxed{1}$$

Data: According to the following table

Time	Gender = 0	Gender = 1
Initial count N	90	100
T= 3	30	20

Population: Any population defining Gender in the range [0,1]

Partial Likelihood: [1-(1-P01)^T]^xmale * [(1-P01)^T]^(Nmale-xmale)
[1-(1-P01)^T]^xfemale*[(1-P01)^T]^(Nfemale-xfemale)
=[1-(1-P01)^3]^20 [(1-P01)^3]^80 [1-(1-P01)^3]^30 [(1-P01)^3]^60

Expected Outcome:

$$\hat{p}_{01} = 1-\left(1-\frac{x_{male}+x_{female}}{N_{male}+N_{female}}\right)^{1/t} = 0.09678$$

Example 7: Non Unique Solution - Example with Covariates #2:

Model

$$\boxed{0} \xrightarrow[P_{01f}]{P_{01m}} \boxed{1}$$

Model coefficient table for the transition from 0 to 1:

Gender = 0	Gender = 1
P01f	P01m

Study 1

$$\boxed{0} \longrightarrow \boxed{1}$$

Data: N=100 x= 75.2250 T=3
Population: Gender ~ Bernoulli(0.75)
Partial Likelihood: [Wm*(1-(1- P01m)^T) + (1-Wm)*(1-(1- P01f)^T)]^x +
[Wm*((1- P01m)^T) + (1-Wm)*((1- P01f)^T)]^(N-x)
Where Wm is the prevalence of male, i.e. Wm = 0.75

Study 2

$$\boxed{0} \longrightarrow \boxed{1}$$

Data: N=100 x= 54.2500 T=2
Population: Gender ~ Bernoulli(0.25)
Partial Likelihood: [Wm*(1-(1- P01m)^T) + (1-Wm)*(1-(1- P01f)^T)]^x +
[Wm*((1- P01m)^T) + (1-Wm)*((1- P01f)^T)]^(N-x)
Where Wm is the prevalence of male, i.e. Wm = 0.25

Expected Outcome:

$\hat{p}_{01f}$ =0.3

$\hat{p}_{01m}$ =0.4

Simple variation to demonstrate the estimably:
If the same example is run without study 2, then the result is inestimable since the model requires more information than the study supplies. The likelihood graph will show a ridge, rather than a peak, meaning that there are multiple solutions. The system will return one such solution that may vary depending on the initial guess.

Example 8: Simple Example with Covariates #3:

Model

$$\boxed{0} \xrightarrow[P_{01f}]{P_{01m}} \boxed{1}$$

Model coefficient table for the transition from 0 to 1:

Gender = 0	Gender = 1
P01f	P01m

Study 1

$$\boxed{0} \longrightarrow \boxed{1}$$

Data: According to the following table

Time	Gender = 0	Gender = 1
Initial count N	90	100
T=3	30	20

Population: Any population that includes Gender in the range [0,1]
Partial Likelihood: [1-(1-P01m)^T]^xmale * [(1-P01m)^T]^(Nmale-xmale)
[1-(1-P01f)^T]^xfemale*[(1-P01f)^T]^(Nfemale-xfemale)
=[1-(1-P01m)^3]^20 [(1-P01m)^3]^80 [1-(1-P01f)^3]^30 [(1-P01f)^3]^60

Expected Outcome:

$$\hat{p}_{01m} = 1-(1-\frac{x}{N})^{1/t} = 0.07168$$

$$\hat{p}_{01f} = 1-(1-\frac{x}{N})^{1/t} = 0.12642$$

Example 9: Simple Example with Regression Outcome:

Model

$$\boxed{0} \xrightarrow{P_{01}} \boxed{1}$$

Study 1

$$\boxed{0} \longrightarrow \boxed{1}$$

Data: $F(t,\lambda) = e^{-\lambda t}$, $\hat{\lambda} = 0.1$, $\sigma^2 = 0.05$, T=5
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Expected Outcome:

$$\hat{p}_{01} = 1 - (1 - e^{-0.1*5})^{1/5} = 0.17018$$

Simple variation for authentication:
For the simple example where T=1 the result should be:

$$\hat{p}_{01} = e^{-0.1} = 0.90484$$

Example 10: Simple Example with Regression Outcome and Covariates:

Model

$$\boxed{0} \xrightarrow[P_{01f}]{P_{01m}} \boxed{1}$$

Model coefficient table for the transition from 0 to 1:

Gender = 0	Gender = 1
P01f	P01m

Study 1

$$\boxed{0} \longrightarrow \boxed{1}$$

Data: $F(t,\lambda) = e^{-(\lambda_0+\lambda_1 Male+\lambda_0 BP)t}$,

$$\hat{\lambda} = (0.1, 0.2, 0.05), \Sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \text{T=1,2,3}$$

Population: 4 individuals (Gender,BP) = (0,125), (0,115), (1,125), (1,115)
Where Gender =1 means male

Partial Likelihood: Not given

Expected Outcome:

$$\hat{p}_{01m} = 1 - \left(1-(1/n_{Male}) \sum_{i \ni Gender_i = Male = 1} e^{\left(-\hat{\lambda}_0-\hat{\lambda}_1 * Gender_i - \hat{\lambda}_2 * BP_i\right)T}\right)^{1/T} \approx$$

$$1-(1-e^{\left(-\hat{\lambda}_0-\hat{\lambda}_1 * Gender-\hat{\lambda}_2 * average(BP|Gender=Male)\right)T})^{1/T} = 1-(1-e^{(-0.1-0.2-0.05*120)T})^{1/T} = 1-(1-e^{-6.3T})^{1/T}$$

$$\hat{p}_{01f} = 1 - \left(1-(1/n_{Female}) \sum_{i \ni Gender_i = Female = 0} e^{\left(-\hat{\lambda}_0-\hat{\lambda}_1 * Gender_i - \hat{\lambda}_2 * BP_i\right)T}\right)^{1/T} \approx$$

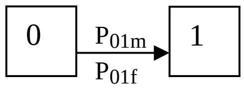
$$1-(1-e^{\left(-\hat{\lambda}_0-\hat{\lambda}_1 * Gender-\hat{\lambda}_2 * average(BP|Gender=Female)\right)T})^{1/T} = 1-(1-e^{(-0.1-0.05*120)T})^{1/T} = 1-(1-e^{-6.1T})^{1/T}$$

T	$P_{01\ m}$	$P_{01\ f}$
1	0.0018939888023991047	0.0023133231471746951
2	1.9011877737673544e-06	2.8362402018089483e-06
3	2.672246868229422e-09	4.8691513043763734e-09
1 Approx.	0.00183630477703	0.00224286771949
2 Approx.	1.6860090383818971e-06	2.5152309667264561e-06
3 Approx.	2.0640158471252334e-09	3.7608821612522547e-09

Note: The approximation gives an idea for results with similar populations

Example 11: Simple Example with Regression Outcome Consistent with Time:

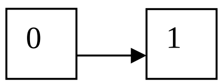
Model



Model coefficient table for the transition from 0 to 1:

Gender = 0	Gender = 1
P01f	P01m

Study 1



Data: $F(t,\lambda) = 1 - e^{-(\lambda_0+\lambda_1 Male+\lambda_0 BP)t}$,

$$\hat{\lambda} = (.1,.2,.005), \Sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \text{T=1,2,3}$$

Population: 4 individuals (Gender,BP) = (0,125), (0,115), (1,125), (1,115)
Where Gender =1 means male

Partial Likelihood: Not given

Expected Outcome:

$$\hat{p}_{01m} = 1 - \left((1/n_{Male}) \sum_{i \ni Gender_i = Male = 1} e^{\left(-\hat{\lambda}_0 - \hat{\lambda}_1 * Gender_i - \hat{\lambda}_2 * BP_i\right)T} \right)^{1/T} \approx$$

$$1 - e^{\left(-\hat{\lambda}_0 - \hat{\lambda}_1 * Gender - \hat{\lambda}_2 * average(BP|Gender=Male)\right)} = 1 - e^{(-0.1-0.2-0.005*120)} = 1 - e^{(-0.9)}$$

$$\hat{p}_{01f} = 1 - \left((1/n_{Female}) \sum_{i \ni Gender_i = Female = 0} e^{\left(-\hat{\lambda}_0 - \hat{\lambda}_1 * Gender_i - \hat{\lambda}_2 * BP_i\right)T} \right)^{1/T} \approx$$

$$1 - e^{\left(-\hat{\lambda}_0 - \hat{\lambda}_1 * Gender - \hat{\lambda}_2 * average(BP|Gender=Female)\right)} = 1 - e^{(-0.1-0.005*120)} = 1 - e^{(-0.7)}$$

T	$P_{01\ m}$	$P_{01\ f}$
1	0.59330328062324944	0.50325950521854423
2	0.59317626067050078	0.5031043626979157
3	0.59304935961526495	0.50294936539903712
Approx.	0.59343034025940089	0.50341469620859047

Note: The approximation gives an idea for results with similar populations

The Jacobian and the Covariance matrices for T=1 should be:

$$J = \begin{bmatrix} 0.40669672 & 0.40669672 & 48.75277982 \\ 0.49674049 & 0 & 59.54677974 \end{bmatrix}$$

$$\Sigma_{New} = J\Sigma J^T = \begin{bmatrix} 2377.16434498 & 2903.27306483 \\ 2903.27306483 & 3546.06572908 \end{bmatrix}$$

Note that row order in the Jacobian and diagonal elements in the covariance matrix may switch if the output vector is ordered by the program [female, male] rather than [male, female].

Example 12: Regression with UKPDS formula:

Model

$$\boxed{0} \xrightarrow[P_{01f}]{P_{01m}} \boxed{1}$$

Model coefficient table for the transition from 0 to 1:

Gender = 0	Gender = 1
P01f	P01m

Study 1

$$\boxed{0} \longrightarrow \boxed{1}$$

Data: $F(\lambda) = 1 - e^{-\lambda_1 \lambda_2^{(AGE-55)} \lambda_3^{(1-MALE)} \lambda_4^{(RACE)} \lambda_5^{(SMOKE)} \lambda_6^{(BP-135.7)/10} \left(\frac{1-\lambda_7^{T}}{1-\lambda_7}\right)}$

$\hat{\lambda} = [0.0112 \quad 1.0590 \quad 0.5250 \quad 0.3900 \quad 1.3500 \quad 1.0880 \quad 1.078]$

$\Sigma = Diag(2.0408 * 10^{-6} \quad 3.1497 * 10^{-5} \quad 0.0028699 \quad 0.010412 \quad 0.014994 \quad 0.00070387 \quad 0.00026656)$

Population: 4 individuals (Age, Gender, Race, Smoke, BP)

54,1,0,0,115
56,1,1,0,125
54,0,0,1,115
56,0,0,0,125

Where Gender =1 means male

Partial Likelihood: Not given

Expected Outcome:

$$\hat{p}_{01m} = 1 - \left((1/n_{Male}) \sum_{i \ni Gender_i = Male = 1} e^{-\lambda_1 \lambda_2^{(AGE-55)} \lambda_3^{(1-MALE)} \lambda_4^{(RACE)} \lambda_5^{(SMOKE)} \lambda_6^{(BP-135.7)/10} \left(\frac{1-\lambda_7^{T}}{1-\lambda_7}\right)} \right)^{1/T}$$

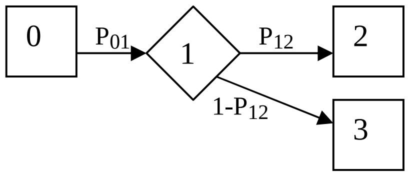
$$\hat{p}_{01f} = 1 - \left((1/n_{Female}) \sum_{i \ni Gender_i = Female = 0} e^{-\lambda_1 \lambda_2^{(AGE-55)} \lambda_3^{(1-MALE)} \lambda_4^{(RACE)} \lambda_5^{(SMOKE)} \lambda_6^{(BP-135.7)/10} \left(\frac{1-\lambda_7^{T}}{1-\lambda_7}\right)} \right)^{1/T}$$

T	$P_{01\ m}$	$P_{01\ f}$
1	0.00653004287191838	0.00597431015512395
2	0.00678083530428653	0.00620653272582317
3	0.007044275112843	0.00645076680543022

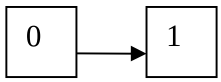
Note that for time = 1 the formula is simplified.

Example 13: Event States – Basic Test

Model

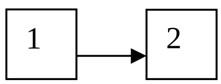


Study 1



Data: N=1000 x=200 T=1
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 2



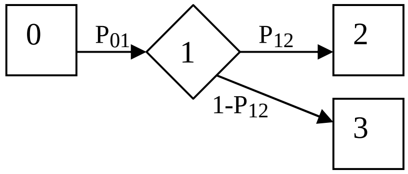
Data: N=1000 x=300 T=1
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Expected Outcome:

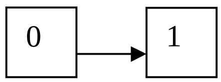
$\hat{p}_{01}$=0.2 $\hat{p}_{12}$=0.3

Example 14: Event States – Event Bypass Study

Model

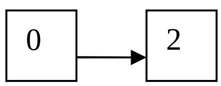


Study 1



Data: N=1000 x=200 T=1
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 2



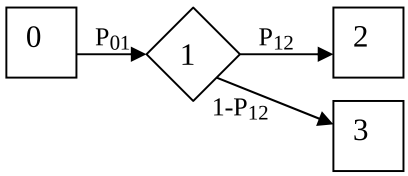
Data: N=1000 x=60 T=1
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Expected Outcome:

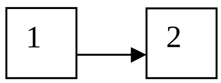
$\hat{p}_{01}$=0.2 $\hat{p}_{12}$=0.3

Example 15: Event States – Another Event Bypass

Model

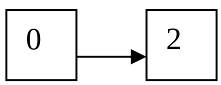


Study 1



Data: N=1000 x=300 T=1
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 2

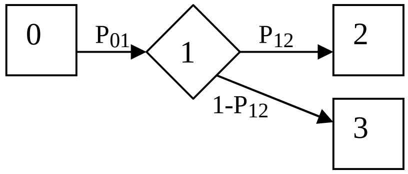


Data: N=1000 x=60 T=1
Population: Default - Theoretically none needed
Partial Likelihood: Not given

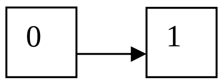
Expected Outcome:
$\hat{p}_{01}=0.2$ $\hat{p}_{12}=0.3$

Example 16: Event States – Two year study

Model

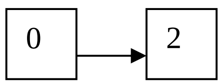


Study 1



Data: N=1000 x=360=(200+160) T=2
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 2



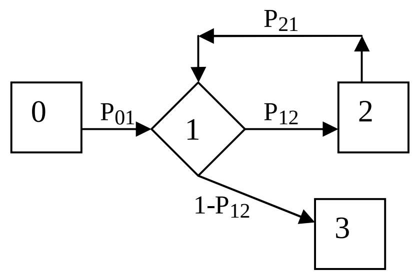
Data: N=1000 x=108=(60+48) T=2
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Expected Outcome:

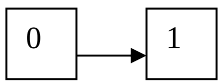
$\hat{p}_{01}$=0.2 $\hat{p}_{12}$=0.3

Example 17: Event States – Loop Back To Event

Model



Study 1



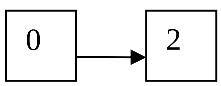
Data: N=1000 x=360 T=2

x= 1000*0.2 +1000*(1-0.2)* 0.2

Population: Default - Theoretically none needed

Partial Likelihood: Not given

Study 2



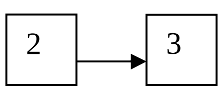
Data: N=10000 x=1080 T=2

x= 10000*0.2*0.3 +10000*(1-0.2)* 0.2 *0.3

Population: Default - Theoretically none needed

Partial Likelihood: Not given

Study 3



Data: N=10000 x=4816 T=2

x= 10000*(0.4*0.7) +10000*(1-0.4*0.7)*(0.4*0.7)

Population: Default - Theoretically none needed

Partial Likelihood: Not given

Expected Outcome:

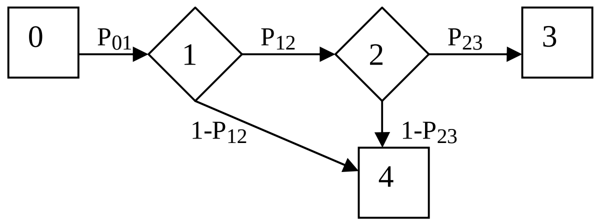
$\hat{p}_{01}$=0.2 $\hat{p}_{12}$=0.3 $\hat{p}_{21}$=0.4

Simple variation for authentication:

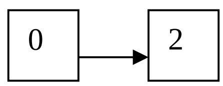
The same results should be reported for the same studies where T=1 and the study outcomes for studies 1-3 are 200, 600, 2800 respectively.

Example 18: Event States – Multiple Event States

Model

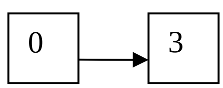


Study 1



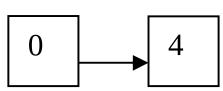
Data: N=1000 x=108 T=2
x=1000*0.2*0.3+1000*(1-0.2)*0.2*0.3
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 2



Data: N=10000 x=432 T=2
x=10000*0.2*0.3*0.4+10000*(1-0.2)*0.2*0.3*0.4
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 3



Data: N=10000 x=3168 T=2
x=10000*0.2*(0.7+0.3*0.6) +10000*(1-0.2)*0.2*(0.7+0.3*0.6)
Population: Default - Theoretically none needed
Partial Likelihood: Not given

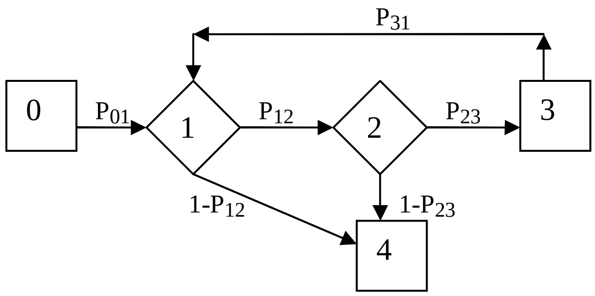
Expected Outcome:

$\hat{p}_{01}$=0.2 $\hat{p}_{12}$=0.3 $\hat{p}_{23}$=0.4

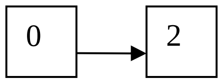
Simple variation for authentication:
The same results should be reported for the same studies where T=1 and the study outcomes for studies 1-3 are 60, 240, 1760 respectively.

Example 19: Event States – Extended Multiple Event States with a Loop

Model

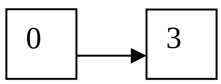


Study 1



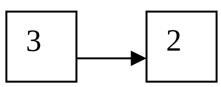
Data: N=10000 x=1080 T=2
x=10000*0.2*0.3+10000*(1-0.2)*0.2*0.3
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 2



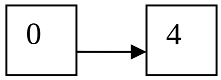
Data: N=10000 x=432 T=2
x=10000*0.2*0.3*0.4+10000*(1-0.2)*0.2*0.3*0.4
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 3



Data: N=1000 x=225 T=2
x=1000*0.5*0.3 + 1000 * (1-0.5) *0.5*0.3
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 4



Data: N=10000 x=3273.6 T=2
x=10000*0.2*(0.7+0.3*0.6) +10000*(1-0.2)*0.2*(0.7+0.3*0.6)
+10000*0.2*0.3*0.4*0.5*(0.7+0.3*0.6)
Population: Default - Theoretically none needed
Partial Likelihood: Not given

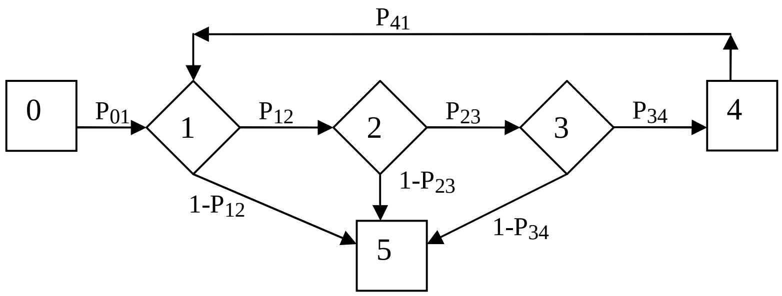
Expected Outcome:

$\hat{p}_{01}$=0.2 $\hat{p}_{12}$=0.3 $\hat{p}_{23}$=0.4 $\hat{p}_{31}$=0.5

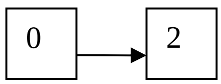
Simple variation for authentication:
The same results should be reported for the same studies where T=1 and the study outcomes for studies 1-4 are 600, 240, 150, 1760 respectively.

Example 20: Event States – Multiple Event States and a Loop

Model

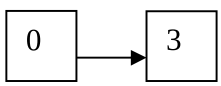


Study 1



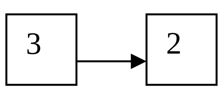
Data: N=10000 x=1080 T=2
x=10000*0.2*0.3+10000*(1-0.2)*0.2*0.3
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 2



Data: N=10000 x=432 T=2
x=10000*0.2*0.3*0.4+10000*(1-0.2)*0.2*0.3*0.4
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 3



Data: N=1000 x=126 T=3
x=1000*0.5*0.6*0.3 + 1000*0.5*(1-0.6)*0.6*0.3
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 4

$\boxed{4} \rightarrow \boxed{5}$

Data: N=10000 x=8099.04 T=2
x=10000*0.6*(0.7+0.3*(0.6+0.4*0.5)) + 10000*(1-0.6)*0.6*(0.7+0.3*(0.6+0.4*0.5)) + 10000*0.6*0.3*0.4*0.5*0.6*(0.7+0.3*(0.6+0.4*0.5))
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 5

$\boxed{0} \rightarrow \boxed{5}$

Data: N=10000 x=3451.68 T=2
x=10000*0.2*(0.7+0.3*(0.6+0.4*0.5)) + 10000*(1-0.2)*0.2*(0.7+0.3*(0.6+0.4*0.5)) +10000*0.2*0.3*0.4*0.5*0.6*(0.7+0.3*(0.6+0.4*0.5))
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Expected Outcome:

$\hat{p}_{01}$=0.2 $\hat{p}_{12}$=0.3 $\hat{p}_{23}$=0.4 $\hat{p}_{34}$=0.5 $\hat{p}_{41}$=0.6

Simple variation for authentication:
The same results should be reported for the same studies where T=1 and the study outcomes for studies 1-5 are 600, 240, 90, 5640, 1880 respectively.

Example 21: Example with study with multiple time outcomes

Model

$$\boxed{0} \xrightarrow{P_{01}} \boxed{1}$$

Study 1

$$\boxed{0} \longrightarrow \boxed{1}$$

Data: According to the following table

Time	Count
Initial count N	10000
T=1	1000
T=2	1900
T=5	4095.1

Population: Default - Theoretically none needed
Partial Likelihood: Not given

Expected Outcome:

$\hat{p}_{01}$=0.1

Example 22: Information with Different table Dimensions

Model

$$\boxed{0} \xrightarrow[P_{01m}]{P_{01f}} \boxed{1} \xrightarrow[P_{12H}]{P_{12L}} \boxed{2}$$

Model coefficient table for the transition from 0 to 1:

Gender = 0	Gender = 1
P01f	P01m

Model coefficient table for the transition from 1 to 2:

BMI<=25	BMI>25
P12L	P12H

Study 1:

$$\boxed{0} \longrightarrow \boxed{1}$$

Data: According to the following table

Time	Gender = 0	Gender = 1
Initial count N	50	100
T=2	9.5	36

Population: Any population defining:

Gender in the range [0,1]

BMI so that 25 is in the range. One option is [-inf, inf]

Partial Likelihood: Not given

Study 2:

$$\boxed{0} \longrightarrow \boxed{2}$$

Data: According to the following table

Time	BMI<=30	BMI>30
Initial count N	400	200
T=3	52	29.2

Population: 600 individuals = 100 times x 6 individuals (Gender, BMI)

0,31
0,26
0,19
1,31
1,26
1,19

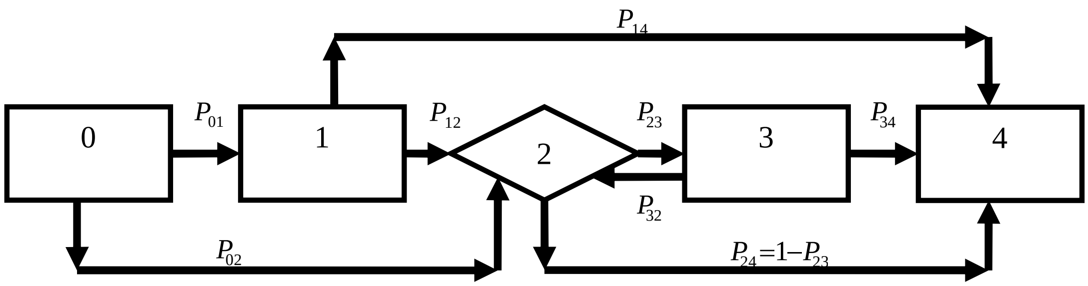
Partial Likelihood: Not given

Expected Outcome:

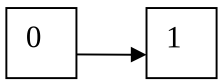
$\hat{p}_{01f} = 0.1$ $\hat{p}_{01m} = 0.2$ $\hat{p}_{12L} = 0.3$ $\hat{p}_{12H} = 0.4$

Example 23: Complex example with an event state

Model

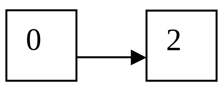


Study 1



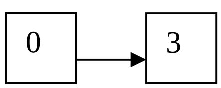
Data: N=1138 x=48.7255388591079 T=10
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 2



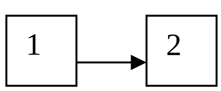
Data: N=890 x=172.110040233588 T=7
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 3



Data: N=4540 x=901.596644864313 T=10
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 4



Data: N=569 x=54.055 T=2
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 5

$\boxed{1} \rightarrow \boxed{4}$

Data: N=569 x=68.1021875 T=2
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 6

$\boxed{2} \rightarrow \boxed{4}$

Data: N=475 x=118.75 T=1
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 7

$\boxed{3} \rightarrow \boxed{2}$

Data: N=73 x=6.7388965376 T=5
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 8

$\boxed{3} \rightarrow \boxed{2}$

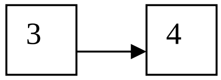
Data: N=169 x=21.0026880998605 T=7
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 9

$\boxed{3} \rightarrow \boxed{2}$

Data: N=78 x=7.2004647936 T=5
Population: none needed
Partial Likelihood: Not given

Study 10



Data: N=468 x=55.6472155078125 T=5

Population: Default - Theoretically none needed

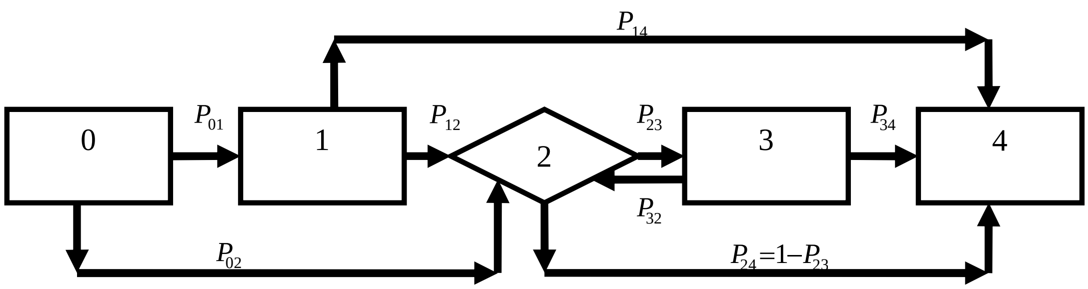
Partial Likelihood: Not given

Expected Outcome:

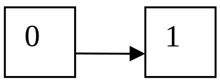
$\hat{p}_{01}$= 0.005 $\hat{p}_{02}$=0.03 $\hat{p}_{12}$=0.05 $\hat{p}_{14}$=0.05 $\hat{p}_{23}$=0.75 $\hat{p}_{32}$=0.02 $\hat{p}_{34}$=0.02

Example 24: Complex example with an event state and studies with tables and multiple times

Model

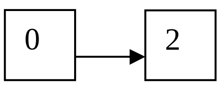


Study 1



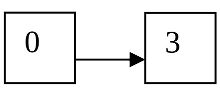
Data: N=1138 x=48.7255388591079 T=10
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 2



Data: N=890 x=172.110040233588 T=7
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 3



Data: According to the following table

Time	Gender = 0	Gender = 1
Initial count N	1897	2643
T=2	84.2268	117.3492
T=4	163.92128048625	228.38373448875
T=6	239.155679788621	333.204249700224
T=8	310.042767715336	431.9678624521
T=10	376.724413063348	524.872231800965

Population: Any population defining Gender in the range [0,1]
Partial Likelihood: Not given

Study 4

$\boxed{1} \rightarrow \boxed{2}$

Data: N=569 x=54.055 T=2
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 5

$\boxed{1} \rightarrow \boxed{4}$

Data: N=569 x=68.1021875 T=2
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 6

$\boxed{2} \rightarrow \boxed{4}$

Data: According to the following table

Time	Gender = 0	Gender = 1
Initial count N	163	312
T=1	40.75	78

Population: Any population defining Gender in the range [0,1]
Partial Likelihood: Not given

Study 7

$\boxed{3} \rightarrow \boxed{2}$

Data: According to the following table

Time	
Initial count N	73
T=1	1.46
T=2	2.8616
T=5	6.7388965376

Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 8

$$\boxed{3} \longrightarrow \boxed{2}$$

Data: N=169 x=21.0026880998605 T=7
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 9

$$\boxed{3} \longrightarrow \boxed{2}$$

Data: N=78 x=7.2004647936 T=5
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 10

$$\boxed{3} \longrightarrow \boxed{4}$$

Data: According to the following table

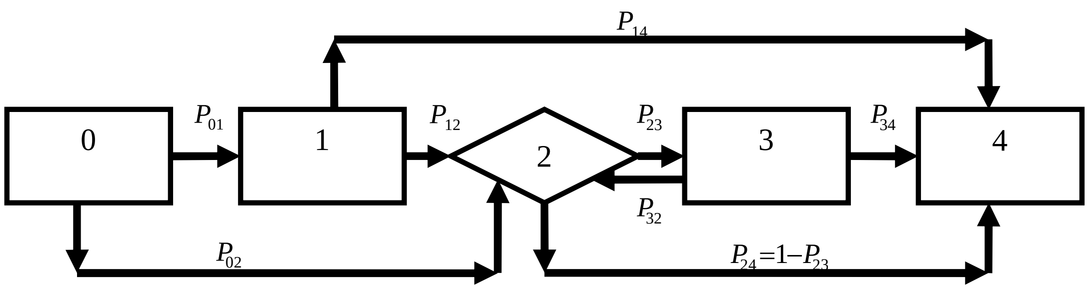
Time	Gender = 0	Gender = 1
Initial count N	181	287
T=5	21.5216795019531	34.1255360058594

Population: Any population defining Gender in the range [0,1]
Partial Likelihood: Not given

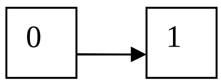
Expected Outcome:
$\hat{p}_{01}$ = 0.005 $\hat{p}_{02}$ =0.03 $\hat{p}_{12}$ =0.05 $\hat{p}_{14}$ =0.05 $\hat{p}_{23}$ =0.75 $\hat{p}_{32}$ =0.02 $\hat{p}_{34}$ =0.02

Example 25: Complex example with an event state, multiple outcomes and regression data using UKPDS form

Model

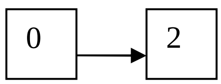


Study 1



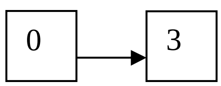
Data: N=1138 x=48.7255388591079 T=10
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 2



Data: N=890 x=172.110040233588 T=7
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 3



Data: $F(\lambda) = 1 - e^{-\lambda_1 \lambda_2^{(AGE-55)} \lambda_3^{(1-Male)} \lambda_4^{(1-WHITE)} \lambda_5^{(SMOKE)} \lambda_6^{(BP-135.7)/10} \left(\frac{1-\lambda_7^T}{1-\lambda_7} \right)}$

$\hat{\lambda} = [\mathbf{0.0399766995311838272211528166551450} \quad \mathbf{1.0590} \quad \mathbf{0.5250} \quad \mathbf{0.3900} \quad \mathbf{1.3500} \quad \mathbf{1.0880} \quad \mathbf{1.078}]$

$\Sigma = \mathrm{I} \quad , \quad \mathrm{T} = 10$

Population: 4 individuals (Age, Gender, Race, Smoke, BP)
54,1,0,0,115
56,1,1,0,125
54,0,0,1,115
56,0,0,0,125
Where Gender =1 means male
Partial Likelihood: Not given

Study 4

$\boxed{1} \rightarrow \boxed{2}$

Data: N=569 x=54.055 T=2
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 5

$\boxed{1} \rightarrow \boxed{4}$

Data: N=569 x=68.1021875 T=2
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 6

$\boxed{2} \rightarrow \boxed{4}$

Data: According to the following table

Time	Gender = 0	Gender = 1
Initial count N	163	312
T=1	40.75	78

Population: Any population defining Gender in the range [0,1]
Partial Likelihood: Not given

Study 7

$\boxed{3} \rightarrow \boxed{2}$

Data: According to the following table

Time	
Initial count N	73
T=1	1.46
T=2	2.8616
T=5	6.7388965376

Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 8

$\boxed{3} \rightarrow \boxed{2}$

Data: N=169 x=21.0026880998605 T=7
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 9

$\boxed{3} \rightarrow \boxed{2}$

Data: N=78 x=7.2004647936 T=5
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 10

$\boxed{3} \rightarrow \boxed{4}$

Data: According to the following table

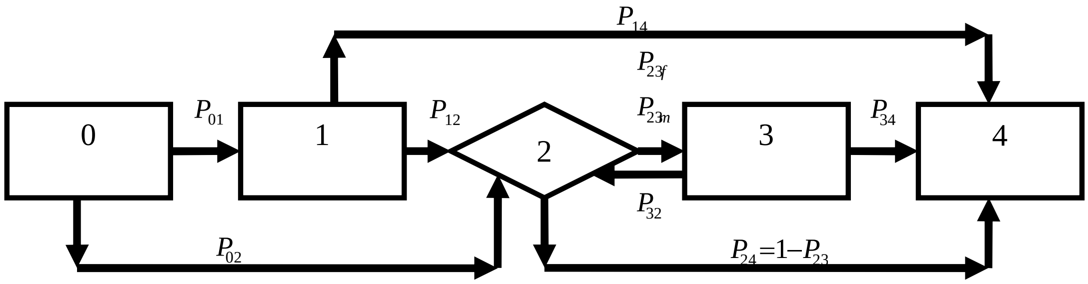
Time	Gender = 0	Gender = 1
Initial count N	181	287
T=5	21.5216795019531	34.1255360058594

Population: Any population defining Gender in the range [0,1]
Partial Likelihood: Not given

Expected Outcome:
$\hat{p}_{01}$ = 0.005 $\hat{p}_{02}$ =0.03 $\hat{p}_{12}$ =0.05 $\hat{p}_{14}$ =0.05 $\hat{p}_{23}$ =0.75 $\hat{p}_{32}$ =0.02 $\hat{p}_{34}$ =0.02

Example 26: Complex example with an event state, multiple outcomes and model coefficients table for one transition

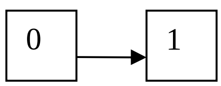
Model



Model coefficient table for the transition from 2 to 3:

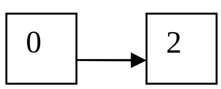
Gender = 0	Gender = 1
P23f	P23m

Study 1



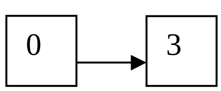
Data: N=1138 x=48.7255388591079 T=10
Population: Gender~Bernoulli(0.5)
Partial Likelihood: Not given

Study 2



Data: N=890 x=172.110040233589 T=7
Population: Gender~Bernoulli(0.5)
Partial Likelihood: Not given

Study 3



Data: According to the following table

Time	Gender = 0	Gender = 1
Initial count N	1897	2643
T=2	78.61168	125.17248
T=4	152.9931951205	243.609316788
T=6	223.211967802713	355.417866346905
T=8	289.373249867647	460.765719948907
T=10	351.609452192458	559.863713921029

Population: Any population defining Gender in the range [0,1]
Partial Likelihood: Not given

Study 4

$\boxed{1} \rightarrow \boxed{2}$

Data: N=569 x=54.055 T=2
Population: Gender~Bernoulli(0.5)
Partial Likelihood: Not given

Study 5

$\boxed{1} \rightarrow \boxed{4}$

Data: N=569 x=68.100765 T=2
Population: Gender~Bernoulli(0.5)
Partial Likelihood: Not given

Study 6

$\boxed{2} \rightarrow \boxed{4}$

Data: According to the following table

Time	Gender = 0	Gender = 1
Initial count N	163	312
T=1	48.9	62.4

Population: Any population defining Gender in the range [0,1]
Partial Likelihood: Not given

Study 7

$\boxed{3} \rightarrow \boxed{2}$

Data: According to the following table

Time	
Initial count N	73
T=1	1.46
T=2	2.8616
T=5	6.7388965376

Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 8

$$\boxed{3} \longrightarrow \boxed{2}$$

Data: N=169 x=21.0026880998605 T=7
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 9

$$\boxed{3} \longrightarrow \boxed{2}$$

Data: N=78 x=7.2004647936 T=5
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 10

$$\boxed{3} \longrightarrow \boxed{4}$$

Data: According to the following table

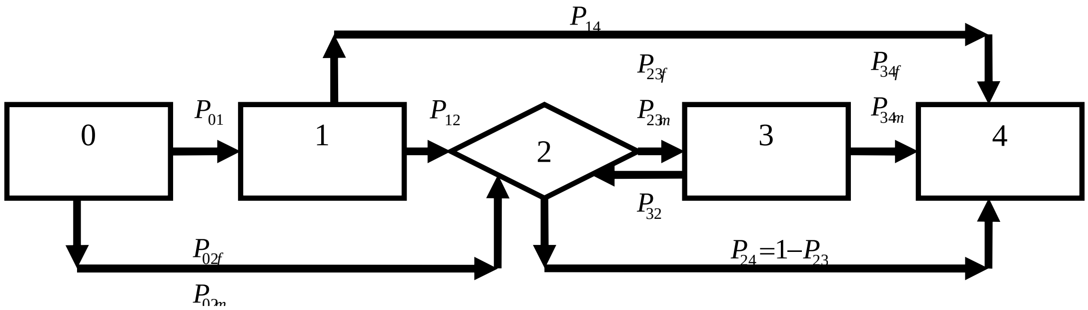
Time	Gender = 0	Gender = 1
Initial count N	181	287
T=5	22.3378411472491	32.8260810667131

Population: Any population defining Gender in the range [0,1]
Partial Likelihood: Not given

Expected Outcome:
$\hat{p}_{01}$= 0.005 $\hat{p}_{02}$=0.03 $\hat{p}_{12}$=0.05 $\hat{p}_{14}$=0.05 $\hat{p}_{23f}$=0.7 $\hat{p}_{23m}$=0.8 $\hat{p}_{32}$=0.02 $\hat{p}_{34}$=0.02

Example 27: Complex example with an event state, multiple outcomes and model coefficients table for several transitions

Model



Model coefficient table for the transition from 0 to 2:

Gender = 0	Gender = 1
P02f	P02m

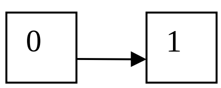
Model coefficient table for the transition from 2 to 3:

Gender = 0	Gender = 1
P23f	P23m

Model coefficient table for the transition from 3 to 4:

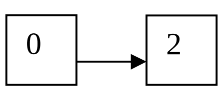
Gender = 0	Gender = 1
P34f	P34m

Study 1



Data: N=1138 x=48.7397240645688 T=10
Population: Gender~Bernoulli(0.5)
Partial Likelihood: Not given

Study 2



Data: N=890 x=171.715684534946 T=7
Population: Gender~Bernoulli(0.5)
Partial Likelihood: Not given

Study 3

$\boxed{0} \rightarrow \boxed{3}$

Data: According to the following table

Time	Gender = 0	Gender = 1
Initial count N	1897	2643
T=2	65.73105	145.57644
T=4	128.75690212	281.607759504
T=6	189.013327191458	408.487362202886
T=8	246.48247219871	526.64524635182
T=10	301.182029208623	636.532322647806

Population: Any population defining Gender in the range [0,1]
Partial Likelihood: Not given

Study 4

$\boxed{1} \rightarrow \boxed{2}$

Data: N=569 x=54.055 T=2
Population: Default - Theoretically none needed
Partial Likelihood: Not given

Study 5

$\boxed{1} \rightarrow \boxed{4}$

Data: N=569 x=68.1078775 T=2
Population: Gender~Bernoulli(0.5)
Partial Likelihood: Not given

Study 6

$\boxed{2} \rightarrow \boxed{4}$

Data: According to the following table

Time	Gender = 0	Gender = 1
Initial count N	163	312
T=1	48.9	62.4

Population: Any population defining Gender in the range [0,1]
Partial Likelihood: Not given

Study 7

$\boxed{3} \rightarrow \boxed{2}$

Data: According to the following table

Time	
Initial count N	73
T=1	1.46
T=2	2.8616
T=5	6.7392399889125

Population: Default - Gender~Bernoulli(0.5)
Partial Likelihood: Not given

Study 8

$\boxed{3} \rightarrow \boxed{2}$

Data: N=169 x=21.005307401201 T=7
Population: Default - Gender~Bernoulli(0.5)
Partial Likelihood: Not given

Study 9

$\boxed{3} \rightarrow \boxed{2}$

Data: N=78 x=7.200831768975 T=5
Population: Default - Gender~Bernoulli(0.5)
Partial Likelihood: Not given

Study 10

$\boxed{3} \rightarrow \boxed{4}$

Data: According to the following table

Time	Gender = 0	Gender = 1
Initial count N	181	287
T=5	18.2233771439173	39.2703173684648

Population: Any population defining Gender in the range [0,1]
Partial Likelihood: Not given

Expected Outcome:
$\hat{p}_{01}$ = 0.005 $\hat{p}_{02f}$ =0.025 $\hat{p}_{02m}$ =0.035 $\hat{p}_{12}$ =0.05 $\hat{p}_{14}$ =0.05 $\hat{p}_{23f}$ =0.7 $\hat{p}_{23m}$ =0.8 $\hat{p}_{32}$ =0.02
$\hat{p}_{34f}$ =0.015 $\hat{p}_{34m}$ =0.025