Crossbow Energy Transfer Analysis

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Abstract: The purpose of this experiment was to see how varying the mass of arrow's fired from a crossbow changed the energy transfer efficiency between the bow-limb and the arrow. In a perfect crossbow, the total potential energy stored from the action of cocking the crossbow would equal the kinetic energy of the arrow immediately after leaving the crossbow. An additional purpose of this experiment was to verify that a "130 lb Crossbow limb" means that at the full 7.5 inch draw length, 130 pounds of force is required to keep the crossbow in the drawn position.

Keywords: Crossbow, Energy Transfer

INTRODUCTION

This Project consisted of two experiments. The first was to determine the force vs. draw length curve of the crossbow, and to verify the 130 pound draw weight claim. The second experiment was to determine how varying the masses of arrows changed the efficiency of the energy transfer between the potential energy stored in the limb, and the kinetic energy of the arrows immediately after release. The second experiment required the force over distance curve of the crossbow limb from the first experiment to be integrated to acquire the potential energy stored in the crossbow.

CROSSBOW LIMB ANALYSIS

Method

A stand was constructed and reinforced to withstand 130 lbs of mass placed in the center. The crossbow limb was then placed on the stand along with a hook system. Buckets were then placed on the hook system, and the mass of the hook system plus the buckets was recorded, and the string displacement was measured. Water was added to the buckets in 1 liter increments and the additional displacement was measured with calipers. The density of the water was measured beforehand to determine the mass of the 1 Liter increments. A 0.125 inch dowel was taped to the string stopper to make the caliper measurements easier, and more accurate. After 40 Liters the dowel was replaced with 2 1.5 inch pieces of wood. This introduced slightly more error due to the softness of the wood compared to the calipers. The error in this part of the experiment came from the lack of a fixed distance to measure, due to the string's tendency to move slightly when being measured. This error was accounted for and recorded along with the data measurements.

Figure 1: Rig constructed for finding the crossbow's force over distance curve

Equations

Analyzing the force over distance curve shows that the crossbow does not quite follow Hooke's law. This might be explained by the limb bending backwards and having a tighter angle than at rest. It could also be explained by the string's elasticity. Denny[1] delves into much more mathematically rigorous models for a crossbow. Since a degree 2 polynomial fits the crossbow force over distance curve extremely well, the following equation will be used to approximate this crossbow's force over distance curve.

$$F = 10328x^2 + 1190.2x + 11.056 \quad (1)$$

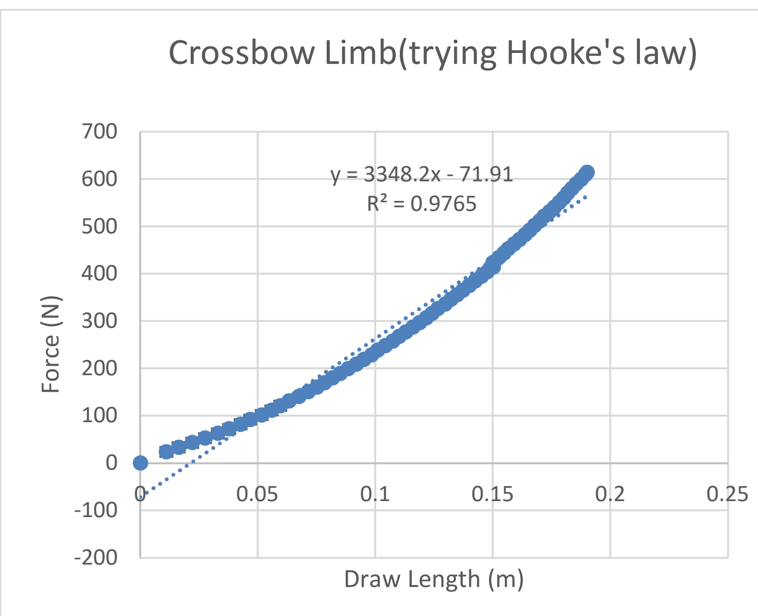
With an exception; the y-intercept will be zero for x = 0. This is because the "string stoppers" (seen in Figure 2) cause the string to be slightly under tension, and they prevent the force from being applied for a small distance. The potential energy of the crossbow is then the definite integral of (1) evaluated from 0 to 0.19 meters.

Results

At the full 7.5 inch draw length, the force of the crossbow is about 137.7 ± 0.9 pounds. This verifies the claim of the manufacturer, as the draw weight is within 10 pounds of their claim.

The total potential energy of the crossbow at full draw length was calculated to be 68.68 Joules.

Figure 2: Crossbow limb with mass hanging on string to measure string displacement. A dowel is taped to the string stoppers.



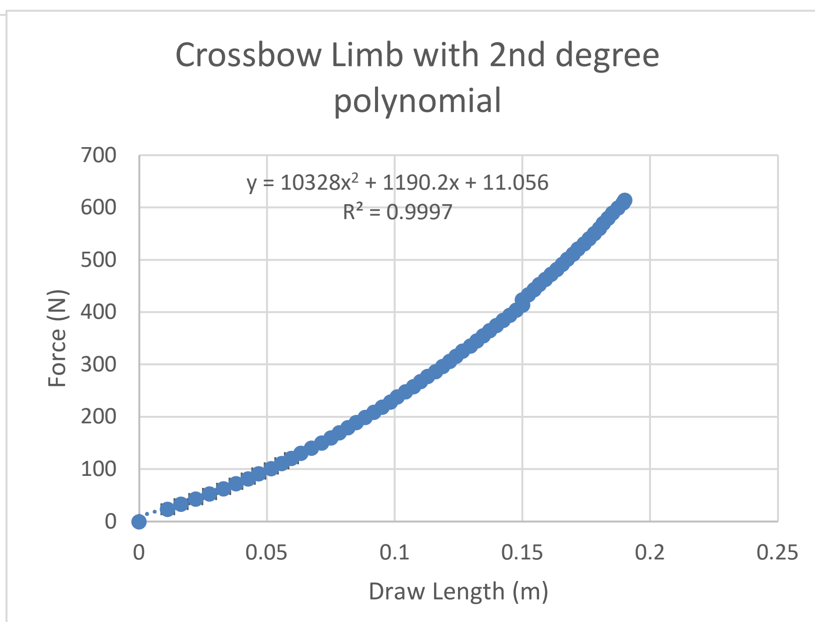


Figure 3: Force over distance curve of crossbow with 1st and 2nd order polynomial fitted

ARROW VELOCITY ANALYSIS

Method

Lead sheet metal was cut into strips and inserted inside 9 different arrows in order to change their mass. A measurement standard was created by graduating PVC pipe at 1-foot increments, for a total of 20 feet. A table with a crossbow mount on top, and a crossbow target were placed on either side of the PVC measure. The arrows' release velocity, the velocity immediately after losing contact with the crossbow string, were then measured using the slow-motion camera feature on a Google Pixel 4 and Tracker software[2]. Inside the tracker software, for each of the video analyses, the video had to be opened, the frames per second was set to 240 fps from 30.07, and a calibration stick was set from the beginning of the graduated PVC to the 20 ft mark and set as 6.096 meters. The arrow was then manually tracked frame-by-frame. There is an automatic tracker tool, but the background noise from the trees and forest was too much to be able to utilize this tool.

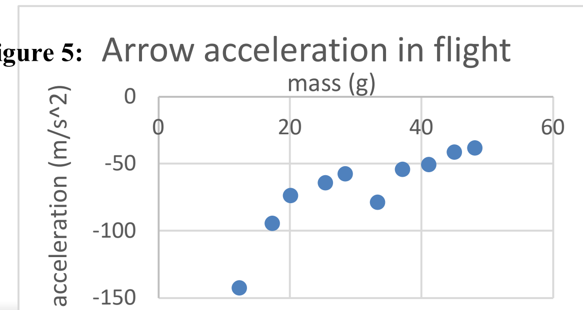
Equations

For each arrow, velocity over time measurements were created using the tracker software. These were exported as tab-delimited text documents, and imported into the "Arrow mass Modulation" excel document. A linear regression line was fitted to the data, the time of the beginning of the arrow's flight was set to 0, and the y-intercept of that line was used for the release velocity of the arrow.

Results

The kinetic energy of each arrow was calculated using the described process. An interesting (and expected) observation is that the decelerations of the arrows were decreasing in magnitude as the arrow mass increased.

Figure 5:



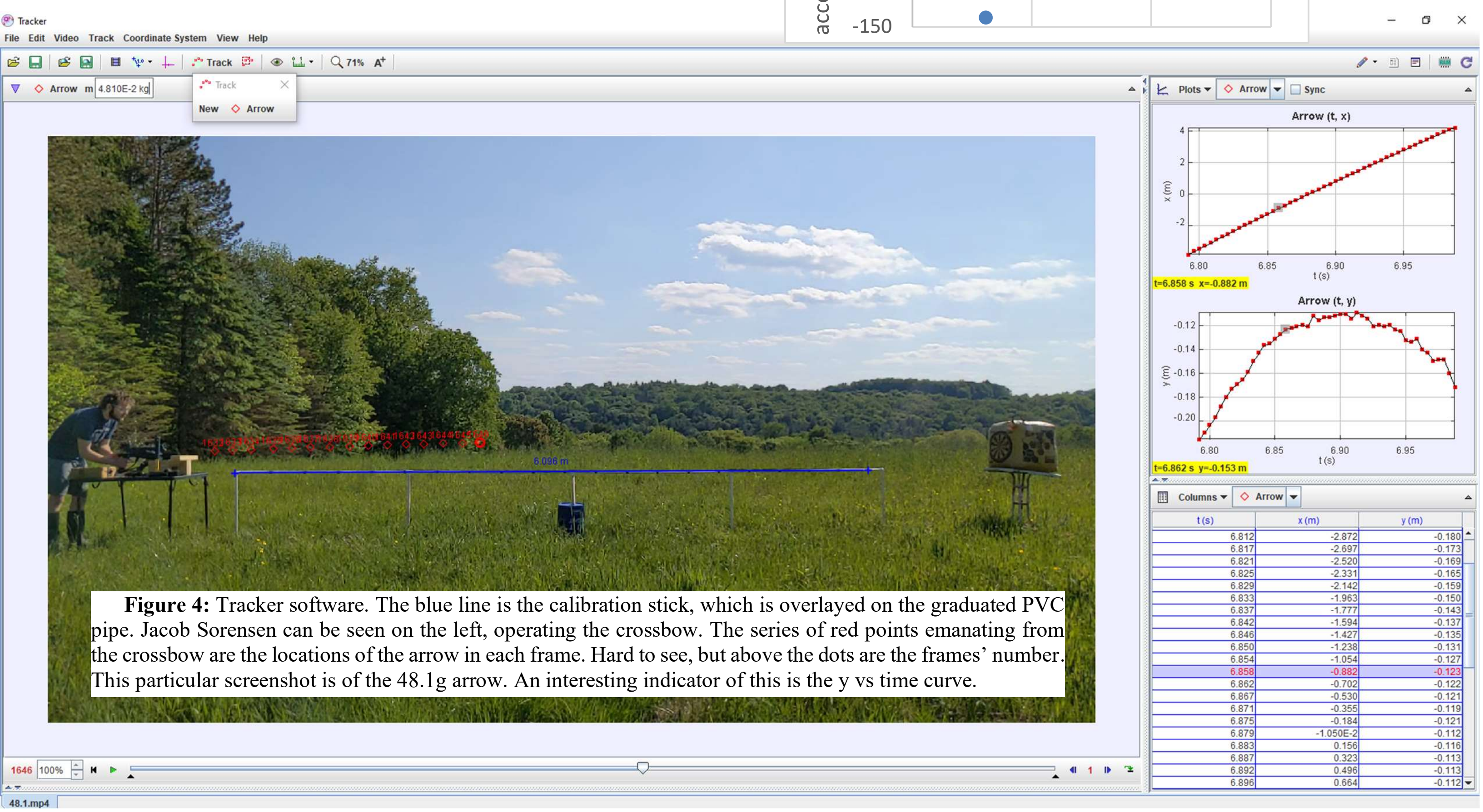


Figure 4: Tracker software. The blue line is the calibration stick, which is overlayed on the graduated PVC pipe. Jacob Sorensen can be seen on the left, operating the crossbow. The series of red points emanating from the crossbow are the locations of the arrow in each frame. Hard to see, but above the dots are the frames' number. This particular screenshot is of the 48.1g arrow. An interesting indicator of this is the y vs time curve.

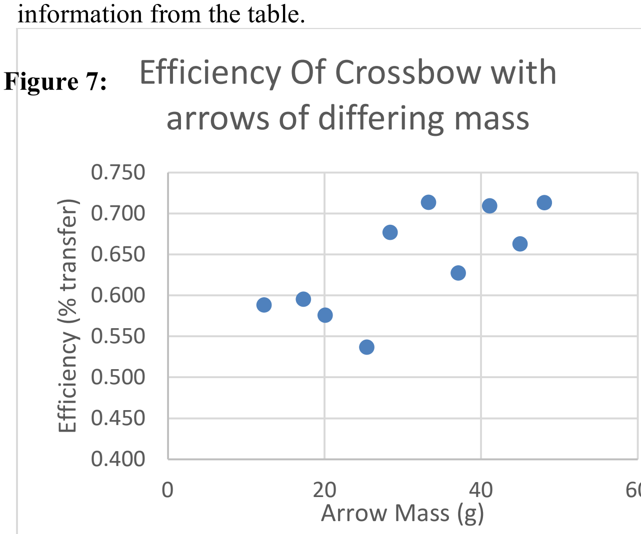
FINAL RESULTS

Figure 6: The following table was created from the data gathered during the second part of the project.

Arrows	grams	Velocity (m/s)	KE (J)	Efficiency
Control:	12.3	81.059	40.41	0.588
1	17.3	68.75	40.88	0.595
2	20.1	62.73	39.55	0.576
3	25.4	53.888	36.88	0.537
4	28.4	57.215	46.48	0.677
5	33.3	54.258	49.02	0.714
6	37.1	48.195	43.09	0.627
7	41.1	48.69	48.72	0.709
8	45	44.98	45.52	0.663
9	48.1	45.124	48.97	0.713

With the calculated PE of the crossbow of 68.68 Joules, the efficiency of the crossbow for each arrow was calculated. The following graph illustrates the information from the table.

Figure 7:



There seems to be some sort of positive relationship between arrow mass and efficiency. I am quite confident that these results have an error of at least 0.05.

CONCLUSION

The manufacturers claim of the 130lb draw-weight of the crossbow was confirmed. Additionally, the hypothesis that the efficiency of the energy transfer between the crossbow and an arrow depends on mass was confirmed. This relationship appears to be positive for some of the masses used in the experiment, although a simple thought experiment will demonstrate that this can't always be the case as arrow mass becomes arbitrarily large.

The error for the final result mostly comes from the video analysis. There were some problems encountered during the analysis. The main problem was that the background of the video was too noisy. The trees made it difficult to find the arrow during some frames. Another issue is that the Google video algorithm provided additional ambiguity when trying to identify exactly where the arrow was in a frame. A simple and easy fix to these problems would be to use a large sheet, or a long roll of paper to serve as a backdrop of the video. This would prevent the arrow from blending into the background.

For future experiments using this procedure, a noise-free background will be used to minimize tracking ambiguity. Additionally, it would be wise to shoot a single arrow more than once to reduce potential variance in how a specific shot goes. The reason this was not done in this experiment is that it is quite difficult to ensure an arrow will be retrieved easily and without damage for repeat experiments. In conclusion, Efficiency of a crossbow depends on its projectile's mass, among other factors.

REFERENCES

1. Denny, Mark, *Bow and catapult internal dynamics,* 2003 Eur. J. Phys. 24 367
2. (Open Source), *Tracker Video Analysis and Modeling Tool*, https://physlets.org/tracker/