Consequences in medieval logic: an introduction

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Abstract

This article introduces consequences in medieval logic, summarizing medieval definitions and divisions of consequences and explaining the import of the medieval development of the theory of consequence for logic today. I then introduce the various contributions to this special issue on consequences in medieval logic.

1 Consequences in medieval logic: a basic overview of the topic and its discussion

Logic is commonly said to be about what follows from what.[1] Within this common expression lies an understanding of logic with a concept of following, or consequence, at its center.

While consequences have existed in logic from its inception, one first finds consequences *thematized* as such in the later middle ages. From the time of Peter Abelard through the thirteenth century, consequence is discussed in textbooks, in treatises on *syncategoremata* and *sophismata*, and in commentaries on Aristotle's *Prior Analytics*, *Elenchi* and *Topics*.[2] Even still, the first treatises explicitly devoted to consequences do not appear until the turn of the fourteenth century.[3] Logic thus existed for more than fifteen hundred years before any extended treatment of this now-central concept. The understanding of logic mentioned above stands indebted to this uniquely medieval development.

[1] Cf. (William Kneale. "What Can Logic Do for Philosophy?" In: *Proceedings of the Aristotelian Society* 22 [1948], pp. 155–166, p. 156); (John Etchemendy. "Tarski on Truth and Logical Consequence". In: *Journal of Symbolic Logic* 53.1 [1988], pp. 51–79, p. 74); (Stephen Read. *Thinking about Logic*. Oxford: Oxford University Press, 1995, p. 99); (J. C. Beall and Greg Restall. *Logical Pluralism*. Oxford: Clarendon Press, 2006, p. 3).

[2] Cf. (Niels Jørgen Green-Pedersen. *The Tradition of the Topics in the Middle Ages. Commentaries on Aristotle's and Boethius' Topics*. Munich: Philosophia Verlag, 1984), (Eleonore Stump. *Dialectic and its Place in the Development of Medieval Logic*. Ithaca: Cornell University Press, 1989).

[3] Niels Jørgen Green-Pedersen. "Two early anonymous tracts on consequences". In: *Cahiers de L'Institut du Moyen-Âge Grec et Latin* 35 (1980), pp. 1–28; Niels Jørgen Green-Pedersen. "Walter Burley's *de consequentiis*. An edition". In: *Franciscan Studies* 40 (1980), pp. 102–166.

Conversely, much early research into medieval logic was motivated, then accelerated by the surprising ways in which it anticipated advances wrought by Tarski and his contemporaries.[4] Historical research into medieval consequences formed a core component in the recovery of medieval logic, and in the broader development of the history of logic as a field. Central to this development was an increased attention to the work of William of Ockham, and a near-wholesale revival of interest in the work of the Parisian Arts Master John Buridan.

This work has, in its turn, fundamentally changed the shape of medieval philosophy today. Examining earlier overviews and anthologies on medieval philosophy by Gilson, Copleston, Wolter, and others, one finds them focused largely on questions of natural theology and metaphysics. One could hardly then anticipate a future in which in the leading journals, encyclopedias, and companions devoted to medieval philosophy, logic would frequently claim greater attention than these traditional foci.[5]

A consequence is a relation between an antecedent and a consequent, variously described by medieval logicians as a habit (*habitudo*), argumentation (*argumentatio*), inference (*illatio*), clustering (*aggregatum*), or a following (*sequela*).[6] In earlier discussions of consequences, two divisions are preponderant: one, derived from Boethius' treatment of hypothetical syllogisms, between natural and accidental consequences; the other, between simple and as-of-now consequences.[7] In later discussions, the former distinction is supplanted by one between formal and material consequences. In the earlier division, natural and accidental consequences are types of simple consequences. In the later division, simple and as-of-now consequences are types of material consequences. Simplifying somewhat, a simple consequence is one where the antecedent cannot ever be true without the consequent; an as-of-now, one where the antecedent cannot be true at a specified time without the consequent being so; a natu-

[4]Cf. (E. A. Moody. *Truth and Consequence in Medieval Logic*. Amsterdam: North-Holland, 1952; Józef Maria Bocheński. *Formale Logik*. Freiburg: Karl Alber, 1956; Philotheus Boehner. "Does Ockham Know of Material Implication?" In: *Collected Articles on Ockham*. St. Bonaventure, NY: Franciscan Institute, 1958, pp. 319–351; William Kneale and Martha Kneale. *The Development of Logic*. Oxford: Oxford University Press, 1962).

[5]Cf. (Frederick Copleston. *A History of Philosophy Volume 2. Medieval Philosophy*. New York: Doubleday, 1948; Étienne Gilson. *History of Christian Philosophy in the Middle Ages*. New York: Random House, 1955; Norman Kretzmann, Anthony Kenny, and Jan Pinborg, eds. *The Cambridge History of Later Medieval Philosophy*. Cambridge: Cambridge University Press, 1982; Norman Kretzmann and Eleonore Stump, eds. *Cambridge Translations of Medieval Philosophical Texts, Vol. I: Logic and Philosophy of Language*. Cambridge: Cambridge University Press, 1988).

[6]Cf. (Green-Pedersen, "Two early anonymous tracts on consequences"; Niels Jørgen Green-Pedersen. "Bradwardine(?) on Ockham's Doctrine of Consequences. An Edition". In: *Cahiers de L'institut du Moyen Âge Grec et Latin* 42 [1982], pp. 85–150; W. K. Seaton. "An Edition and Translation of the Tractatus de consequentiis of Ralph Strode". PhD thesis. University of California at Berkeley, 1973); also the mss. mentioned in (Niels Jørgen Green-Pedersen. "Early British Treatises on Consequences". In: *The Rise of British Logic. Acts of the Sixth European Symposium on Medieval Logic and Semantics, Balliol College, Oxford, 19-24 June 1983*. Ed. by P. Osmund Lewry. Papers in Mediaeval Studies 7. Toronto: Pontifical Institute of Medieval Studies, 1983, pp. 285–307, pp. 300-302, 306-307).

[7]Boethius. *De Hypotheticis Syllogismis*. Ed. by L. Obertello. Brescia: Paideia, 1969, 835B.

ral consequence, one where there is a natural or causal relationship between what the antecedent and consequent express; an accidental, one where there is a mere relationship of temporal inseparability between what the antecedent and consequent express.[8] Accidental consequences are instead called temporal consequences by some authors, on account of the relationship they instantiate; non-natural or unnatural by others, on account of their contrast with natural consequences.[9]

After Buridan, most authors on the European continent define a formal consequence as one holding for all uniform substitutions on its categorematic terms,[10] and a material consequence as one which does not hold for every such substitution, but rather holds in virtue of its content.[11] Most British authors define a formal consequence somewhat more strictly, as one where the antecedent is said to virtually contain the consequent.[12] In both the earlier division of consequences into natural and accidental and the later one into formal and material, the former type is taken to be both more intensionally basic and more extensionally restricted, with the consequences in the latter somehow falling short of, or even being reducible to, those of the former type.[13]

One of the unacknowledged marvels of research into medieval logic generally, and consequences specifically, has been its near-inversion of the standard bell-curve history of medieval philosophy. According to a once-common historiography, medieval philosophy rapidly developed from the time of Saint Anselm, reached its height in the time of Aquinas and Bonaventure, and began its decline with Ockham (or, perhaps, Scotus).[14] The historiographical perspective implicit in much work in medieval logic, by contrast, hints that the age of the famous theological *summae* marked an ebb between the output of twelfth-century

[8](Walter Burleigh. *De Puritate Artis Logicae*. Ed. by Philotheus Boehner. St Bonaventure, NY: Franciscan Institute, 1955, pp. 60.28-61.25), (Richard Sophista. *Abstractiones*. Ed. by Mary Sirridge and Sten Ebbesen. Auctores Rerum Britannici Medii Aevi 25. Oxford: Oxford University Press, 2016, p. 141).

[9]Cf. (Garlandus Compotista. *Dialectica*. Ed. by Lambertus M. de Rijk. Assen: van Gorcum, 1959, p. 141), (Petrus Abaelardus. *Dialectica*. Ed. by Lambertus M. de Rijk. Assen: Van Gorcum, 1966, p. 172), (R. Kirchhoff. *Die Syncategoremata des Wilhelm von Sherwood*. Leiden: Brill, 2008, p. 141), (H. A. B. Braakhuis. *De 13de eeuwse Tractaten over syncategorematische Termen*. Meppel, 1979, vol. 2, pp. 199-200).

[10]Categorematic terms are called from their signifying beings in the ten Aristotelian categories; they are contrasted with syncategorematic terms, which function with categorematic terms to structure them into sentences.

[11](John Buridan. *Treatise on Consequences*. Trans. by Stephen Read. Bronx, NY: Fordham University Press, 2015, p. 68), (Pseudo-Scotus. *Quaestiones Super Libros II Priorum Analyticorum*. In: *Joannis Duns Scoti Doctoris Subtilis Ordinis Minorum Opera Omnia*. Ed. by Luke Wadding. Vol. 2. Paris: Wadding-Vives, 1891, pp. 81–197, p. 105).

[12](Catarina Dutilh Novaes. "Logic in the 14th Century after Ockham". In: *Handbook of the History of Logic. Mediaeval and Renaissance Logic*. Ed. by Dov. M. Gabbay and John Woods. Vol. 2. Amsterdam: Elsevier, 2008, pp. 433–504, p. 476).

[13](Kirchhoff, *Die Syncategoremata des Wilhelm von Sherwood*, p. 152), (John Buridan, *Treatise on Consequences*, p. 68), (Pseudo-Scotus, *Quaestiones Super Libros II Priorum Analyticorum*, p. 105). Cf. (Green-Pedersen, "Walter Burley's *de consequentiis*", p. 142).

[14]Cf. (Gilson, *History of Christian Philosophy in the Middle Ages*); more recently, (Catherine Pickstock. *After Writing. On the Liturgical Consummation of Philosophy*. Oxford: Blackwell, 1998).

schools and the great nominalists of the fourteenth century.[15] The latter perspective has at times brought with it an overemphasis on nominalist, reductive, and quasi-classical over realist, non-reductive pluralist, or more relevantistic approaches to consequence; magnified the influence of some logicians while diminishing that of others; exaggerated connections between like-minded thinkers while ignoring less straightforward, but more concrete connections, including those wrought in polemic. With this in mind, this collection aims to resituate medieval work on consequences into a broader perspective, providing greater attention to earlier work on consequences, a more nuanced account of the relations between well-known figures, and a more complete picture of the influences of each era on the next.

2 Introduction to the articles

In 'The Roots of the Notion of Containment in Theories of Consequence', [n.] addresses the question of whether and to what extent the containment criterion for consequence, common in both earlier discussions of natural consequence and later British discussions of formal consequence, is anticipated in the logical works of Boethius. [n.] argues that while the containment criterion does draw from Boethian source texts, those sources are different from those standardly assumed. [n.] shows that the later criterion draws much from Boethian texts *not* devoted to conditionals, including discussions of *per se* predication in his treatment of the Porphyrian predicables, and of the *locus from a genus* in his commentary on Cicero's *Topics*.

Next, [n.] provides an overview of consequences in the thirteenth century. Thirteenth century discussions of consequence have received less attention than those of later authors. Successively examining thirteenth-century discussions of syllogisms, *syncategoremata*, and *sophismata*, [n.] shows that across these genres, thirteenth-century work treated the relation of a consequent to its antecedent(s) on the model of an effect to its cause(s). Though the thirteenth-century logicians regarded these causes not necessarily as causes of being, but rather of following, the assimilation played an important role in thirteeth-century treatments of inferences involving impossible antecedents or necessary consequents. Many thirteenth-century logicians rejected such consequences for this reason, and those that admitted the consequences to be valid in some respect did not regard them as unqualifiedly so.

Moving into the fourteenth century, [n.] provides an overview of Walter Burley's understanding of consequence. With Ockham and John Buridan, Burley is often regarded as one of the most influential logicians later middle ages. One

[15] On consequences, cf. (Stephen Read. "The Medieval Theory of Consequence". In: *Synthese* 187.3 [2012], pp. 899–912; Catarina Dutilh Novaes. "Medieval Theories of Consequence". In: *The Stanford Encyclopedia of Philosophy*. 2016. URL: `https://plato.stanford.edu/archives/fall2016/entries/consequence-medieval`); on logic broadly, (Dov. M. Gabbay and John Woods, eds. *Handbook of the History of Logic. Mediaeval and Renaissance Logic*. Vol. 2. Amsterdam: Elsevier, 2008), roughly only three of whose thirteen essays touch on thirteenth-century developments.

of the earliest extant *consequentiae* treatises, and the earliest with a known author, belongs to Burley. In addition, Burley is one of the only early authors to discuss both the natural/accidental division and formal/material division of consequences at some length. Still, Burley has received less attention than Ockham or Buridan. After an overview of Burley's division of consequences [n.] compares Burley's work to the earliest anonymous treatises on consequences, then to Ockham and Buridan's treatments of the subject. [n.] highlights Burley's advances over the former treatises' treatment of existential import in consequences, his disagreements with Ockham and Buridan on rules governing consequences, and his influence on the broader place of the study of consequences in logic. The article closes with an explication of how Burley relates natural and formal consequence to formal and material consequence.

The best-known medieval accounts of consequences are those of William of Ockham and John Buridan. Still, the relation between these accounts remains obscure. In particular, Ockham classifies certain consequences as formal which Buridan admits only as material, and the exact reason for these differences has not been sufficiently explored. [n.] provides a classification of consequences both figures treat as formal, those both treat as material, and those which Ockham calls formal and Buridan calls material. [n.] then locates the difference between their classification of consequences in Buridan's more restricted application of the notion of propositional form to the realm of consequences. Where Ockham calls 'formal' a consequence following in virtue of its form, Buridan only admits as formal those consequences following *solely* in virtue of form. Conversely, where Buridan calls 'material' any consequence following in virtue of its matter, Ockham calls one so only if it follows solely in virtue if its matter.

One of the more outstanding continental authors writing on consequences after Buridan is Marsilius of Inghen, later founder and rector of the University of Heidelberg. Marsilius calls Buridan 'my teacher'[16] and with Albert of Saxony is traditionally regarded as a prominent member of a Buridanian school of logic. Though neither Marsilius nor Albert held such a relation to Buridan in any institutional sense,[17] their approaches to consequences share some broad similarities when compared to those of later British writers, particularly in their use of a substitutional criterion for formal consequence. In 'Marsilius of Inghen on the Definition of *consequentia*', [n.] compares Marsilius' account of consequences with those of Buridan and Albert, and finds that Marsilius diverges from Buridan and Albert in several important respects. Specifically, Buridan and Albert deny, while Marsilius affirms, that a consequence is a *propositio hypothetica*. Instead, Marsilius characterizes a consequence as an *oratio*. In addition, Marsilius rejects, where Buridan and Albert accept, *ut nunc*, or as-of-now consequence.[18]

[16]Marsilius of Inghen. *Quaestiones super libros De generatione et corruptione*. Venice, 1501, fol. 106va.

[17]William J. Courtenay. "The University of Paris at the Time of Jean Buridan and Nicole Oresme". In: *Vivarium* 42.1 (2004), pp. 1–17; J. M. M. H. Thijssen. "The Buridan School reassessed. John Buridan and Albert of Saxony". In: *Vivarium* 42.1 (2004), pp. 18–42.

[18]Cf. (Green-Pedersen, "Bradwardine(?) on Ockham's Doctrine of Consequences", pp. 92-93).

This issue closes with [n.]'s analysis of the development of the theory of natural consequence from Abelard to the eve of the fourteenth century. This development first begins in response to Alberic of Paris' proof of the internal inconsistency of Peter Abelard's account of inference, and draws to a close with Ockham's rejection of the notion of natural consequence as part of his broader attack on Scotistic natural theology and metaphysics.[19] [n.] argues that throughout the twelfth and thirteenth centuries, natural consequence is a relevant consequence relation, specifically one conforming to principles of modern connexive logic. The need for a stronger form of consequence than that holding merely in virtue of a standard semantic requirement - namely, the impossibility of the antecedent holding with the consequent not holding - is found in authors from Abelard through Kilwardby and other thirteenth century authors, and was especially acute in a species of disputational exercises, or *obligation*, involving the positing of an impossible proposition, called *positio impossibilis*. It is in this disputational context, and specifically in the different treatments of impossible *positio* in Scotus and Ockham, that the seeds of Ockham's alternative analysis, and the replacement of the earlier one, would be sown.

[19]Cf. (Christopher J. Martin. "Formal Consequence in Scotus and Ockham. Towards an Account of Scotus' Logic". In: *Duns Scotus in Paris, 1302-2002: Proceedings of the conference of Paris, 2-4 September 2002*. Ed. by O. Boulnois, E. Karger, and G. Sondag. Turnhout: Brepols, 2004, pp. 117–150).