Divided modality for Ockhamists

Jacob Archambault

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Introduction

- Standard discussions of the composite-divided distinction in Ockham's logic assimilate the difference between the one and the other to a contemporary scope distinction.
- In a composite modal, the modality takes wide scope; in a divided modal, narrow scope.
- (COMPOSITE$_1$) $\Box$(Some A is B)
- (COMPOSITE$_2$) $\Box(\exists x)(Ax \wedge Bx)$
- (DIVIDED$_1$) (Some A is $\Box$B)
- (DIVIDED$_2$) $(\exists x)(Ax \wedge \Box Bx)$

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The assimilation of divided to narrow-scope modality

(1) Nullum impossibile esse verum est necessarium, igitur nullum verum esse impossibile est necessarium. [3, p. 298]

(1') 'Nothing impossible is necessarily true' therefore 'Nothing true is necessarily impossible.' [2, p. 166].

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The assimilation of divided to narrow-scope modality

'For the antecedent is true, and the consequent is false. For nothing impossible is possibly true, and yet something true is possibly impossible. [...] For if I go to Rome, then it will be impossible afterwards [that I have not been to Rome]'. [2, p. 166]

The assimilation of divided to narrow-scope modality

'All that follows is that 'Something true is possibly impossible', but this is not what Ockham needs to show. He needs to show that 'Something true is necessarily impossible,' which appears to be a stronger proposition'. [1, p. 243]

The assimilation of divided to narrow-scope modality

- The choice to read Ockhamist divided modals according to the canonical reading invalidates some of the examples Ockham uses to illustrate his theory.
- While there is nothing inherently wrong with this, it has not sufficiently been tested whether there might be a better reading of Ockham in view.

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The reasons for this assimilation: conversion

- An important piece of evidence for the analysis of divided modals into their narrow scope formal counterparts is Ockham's account of their conversion.

Fallacious conversions

(1) 'That no impossibility be true is necessary, therefore that no truth be impossible is necessary'

(2) 'A being which creates is necessarily God' therefore 'God is necessarily a being who creates.'

(3) 'A man is necessarily understood by God' therefore 'Something understood by God is necessarily a man.' [3, II. 24, p. 298]

Their correct counterparts

(4) Every impossibility of necessity is not true, therefore something, which of necessity is not true, is impossible.

(5) The Creator of necessity is God, therefore something, which of necessity is God, is creating.

(6) Man of necessity is understood by God, therefore something, which of necessity is understood by God, is man. [3, II. 24, p. 298]

The reasons for the assimilation: conversion

Given the consequents of the above all conform to the canonical formalization of *de re* modals, it is assumed that the antecedents should as well.

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Divided modality in William of Ockham

'It should be known that for the truth of such a proposition [i.e. a modal proposition in the divided sense] we require that the predicate, under its proper form, belong to that for which the subject supposits, or to the pronoun referring to that for which the subject supposits; sc. such that the mode expressed in such a proposition may truly be predicated of an assertoric proposition in which the very same predicate is predicated of a pronoun referring to that for which the subject supposits, in a manner proportionate to that stated regarding propositions about the past and about the future.'

Divided modality in William of Ockham

'For example, for the truth of this: 'Every truth of necessity is true', we require that any given proposition be necessary in which this predicate 'true' is predicated of anything for which the subject [term] 'truth' supposits? that is, that any such [proposition] be necessary: 'This is true', 'that is true', indicating anything for which the subject supposits. And since not every such [proposition] is true, it follows that this is false simpliciter: 'Every truth of necessity is true' [3, II. 10, p. 249].

Divided modality in William of Ockham

We can think of this passage as giving the following procedure for determining the truth of a modal proposition.

1. Introduce a collection of terms we can treat as rigid; for Ockham, both demonstrative pronouns as well as proper names have this feature.
2. Replace the subject term in the sentence with the variable.
3. Check the truth-value of the sentence(s) predicating a mode, tense, etc. of the proposition indicated by the replacement sentence.

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3. Check the truth-value of the sentence(s) predicating a mode, tense, etc. of the proposition indicated by the replacement sentence.
   - When the subject term is a name, definite description, or quantified by a particular quantifier, then the truth of one such sentence suffices for the truth of the divided modal claim; if the subject term is universally quantified, then the number of true sentences must exhaust the variables designating objects in the subject class.
   - EXAMPLE: if the only human beings are Socrates and Plato, then for the truth of 'Every human of necessity is an animal', the sentences 'that x is an animal is necessary' and 'that y is an animal is necessary' must be true, where x and y designate Socrates and Plato, respectively.

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- To make this more conspicuous, let us introduce two-place modal and tense operators instead of their standard unary operators, where the first place is filled by a sentence, and the second by a term designating an object or collection of objects.
- For tense and possibility operators, we append subscripts to the operators to indicate whether they require their subject terms to supposit for present objects or past/future/possible objects.

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Divided modality in William of Ockham

(W_1) W_1('x is a playwright', The pope)

- This requires that something presently designated by the definite description 'the pope', and rigidly designated by x, was a playwright.

(W_2) W_2('x is a playwright', The pope)

- This is true in the case where x rigidly designates something that once answered to the definite description 'the pope', but perhaps no longer does.
- In neither case is it required that the sentence 'the pope is a playwright' have been true.

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In brief, using M_d as a placeholder for a binary divided modal operator; M_c, for a unary composite modal operator analogous to M_d; p_x, for a sentential function, with free x, denoting some state of affairs; Q, for a quantifier; and t, for a term denoting a class of objects, we can generalize Ockham's account as follows:

> *$M_d(p_x, Qt) \Leftrightarrow$ for Q formula(s) p', where p' is exactly like p_x except that each free occurrence of x in p_x is replaced by x', where x' rigidly designates some member of t: $v(M_c(p')) = True$.*

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Scope, negation, and equipollence in Ockham's modal theory

- Ockham's says sentences of the form 'S is possibly P' are equipollent to sentences of the form 'that S is P is possible' [3, II. 9, p. 246; 10, p. 248].
- But Ockham's theory is complicated by the interaction of divided modals with negation.
- According to Ockham, divided modals with unresolved negations fall into three types:
  1. Those where the quantity alone is negated;
  2. Those where both the quantity and the mode are negated;
  3. those where the mode alone is negated.

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	Example	**Resolution**
Quantity negated	It is necessary that not every animal be a man. It is possible that not every man be an animal.	It is necessary that some animal not be a man. It is possible that some man not be an animal.
Mode and quantity	It is not necessary that every animal be a man. It is not possible that every animal be a man.	It is possible that some animal not be a man. It is necessary that some animal not be a man.
Mode alone negated	No man of necessity is an animal. No man can be an animal.	Every man can not be an animal. Every man of necessity is not an animal.[1]

[1] SL III-3. 14.

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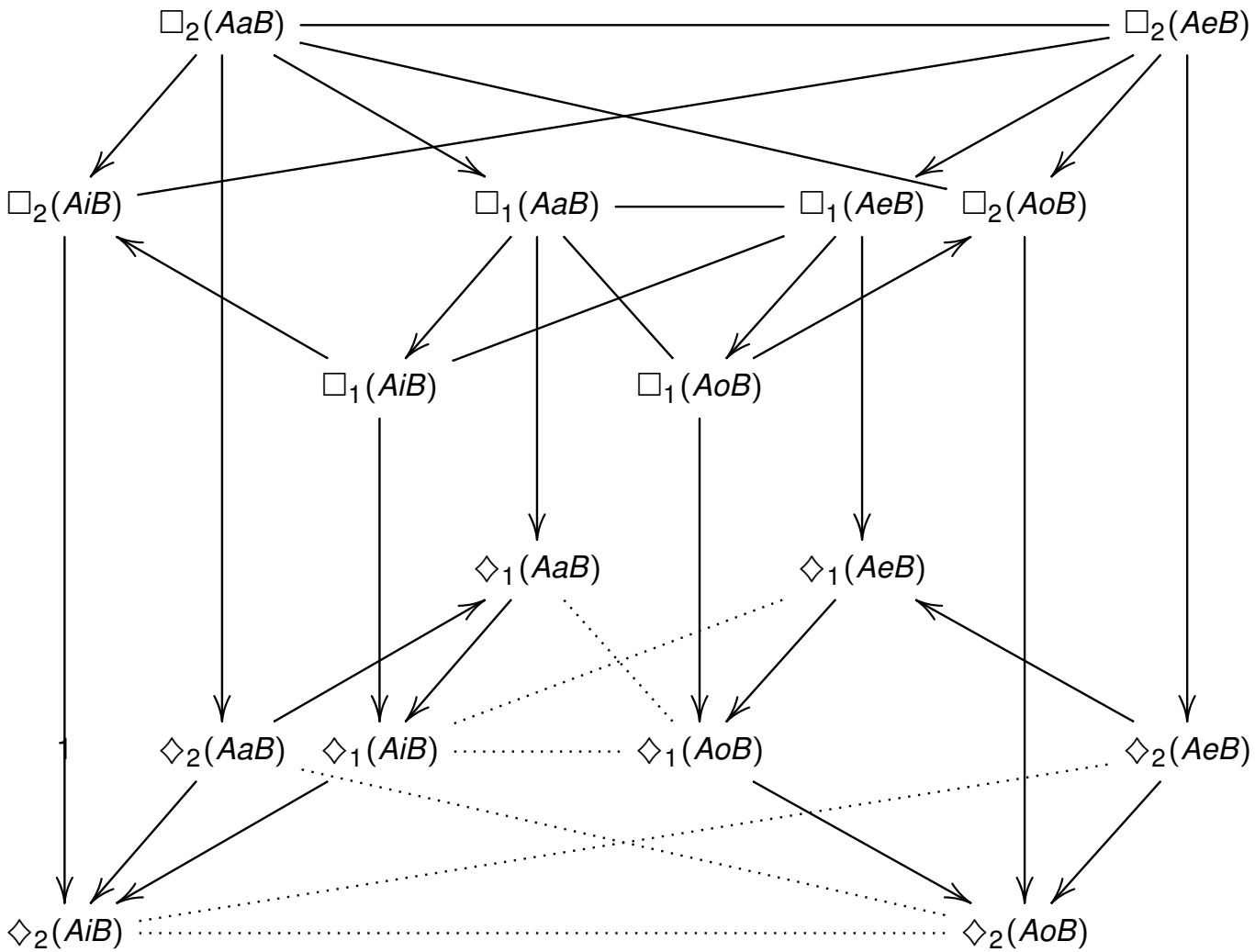
Equipollences	Contraries	Subcontraries	Contradictories	Subalterns
$\Box(AaB)$ $I(AoB)$	$\Box(AeB)$ $\Box(AoB)$ $\Diamond_1(AeB)$	None	$\Diamond_1(AoB)$	$\Box(AiB)$ $\Diamond_1(AaB)$
$\Box(AeB)$ $I(AiB)$	$\Box(AaB)$ $\Box(AiB)$ $\Diamond_1(AaB)$	None	$\Diamond_1(AiB)$	$\Box(AiB)$ $\Diamond_1(AeB)$
$\Box(AiB)$ $I(AeB)$	$\Box(AeB)$	$\Diamond_1(AoB)$	$\Diamond_1(AeB)$	$\Diamond_1(AiB)$
$\Box(AoB)$ $I(AaB)$	$\Box(AaB)$	$\Diamond_1(AiB)$	$\Diamond_1(AaB)$	$\Diamond_1(AoB)$
$\Diamond_1(AaB)$	$\Box(AeB)$	$\Diamond_1(AoB)$	$\Box(AoB)$	$\Diamond_1(AiB)$
$\Diamond_1(AeB)$	$\Box(AaB)$	$\Diamond_1(AiB)$	$\Box(AiB)$	$\Box_1(AoB)$
$\Diamond_1(AiB)$	None	$\Box(AoB)$ $\Diamond_1(AeB)$ $\Diamond_1(AoB)$	$\Box(AeB)$	None
$\Diamond_1(AoB)$	None	$\Box(AiB)$ $\Diamond_1(AaB)$ $\Diamond_1(AiB)$	$\Box(AaB)$	None

Ockham's necessity pairs with $\Diamond_1$. But $\Diamond_2$ has no dual. Given this, we can subscript Ockham's necessity operator as $\Box_1$, and add a second, $\Box_2$, to complement $\Diamond_2$. From here, we note the following entailments:

1. $\Box_2(AaB) \rightarrow \Box_1(AaB)$, $\Box_2(AeB) \rightarrow \Box_1(AeB)$
2. $\Diamond_2(AaB) \rightarrow \Diamond_1(AaB)$, $\Diamond_2(AeB) \rightarrow \Diamond_1(AeB)$
3. $\Box_1(AiB) \rightarrow \Box_2(AiB)$, $\Box_1(AoB) \rightarrow \Box_2(AoB)$
4. $\Diamond_1(AiB) \rightarrow \Diamond_2(AiB)$, $\Diamond_1(AoB) \rightarrow \Diamond_2 AoB$
5. $\Box_n\phi \rightarrow \Diamond_n\phi$
6. The subalternations of the assertoric square continue to hold when embedded under any mode.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- Rather, there is a sense in which even divided modals can take ‘wide scope’, though this sense will always be equivalent to a different sentence where the modal takes ‘narrow’ scope.
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- For negation, we must represent the quality of the formula as belonging to the *predicate*, rather than the subject, if we wish to obtain the correct truth conditions for a wide-scope divided modal.
- Ockhamist quantifiers do not ampliate. Ockham requires that the range of quantification be specified independently of its place in the sentence:
- Every sentence containing an intensional operator will thus be semantically ambiguous between different readings in accordance with the different possible readings of the scope of quantification.

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