The development of the medieval Parisian account of formal consequence

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-For Jim Garson, Gyula Klima, Graham Priest, Stephen Read, and anyone else who had their part in leading me down this rabbit hole.

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Chapter 1

Introduction

1.1 Formal consequence and formal logic

Logic is commonly singled out for special consideration among the sciences by the dictum 'logic is formal'. We speak about formal rigor, formalized languages, formal consequence, formal methods - indeed, only the other branches of mathematics come close to logic in the degree to which they are stamped with the language of formality. Other areas of inquiry are sometimes even considered formal to the degree that they incorporate logic into their methodology - formal epistemology, for instance, is an approach to epistemology heavily reliant on the use of logical apparatus.

The formality of logic is especially present in discussions of logical consequence. This is in part because the subject matter of logic itself is often taken to be *what follows from what* - not, of course, in any sense whatsoever, but as a matter of logical *form*; and in part because as modifiers of 'consequence',

'logical' and 'formal' are frequently taken to be synonymous[1] - a synonymy that, if applied consistently, would transform 'logical form' and 'formal logic' into the emphatically redundant 'formal form' and 'logical logic'. In short, the very parlance of logicians suggests the concept of form is at the center of logic.

This dissertation provides an account of how this came about, detailing the development of the notion of formal consequence at the time of the appearance of the first treatises on consequence - the first half of the 14th century - and culminating in the account advanced by John Buridan. Buridan's work provides a convenient focal point from both historical and theoretical perspectives: theoretically, because of its close resemblance to the model-theoretic accounts dominant today; historically, because of its lasting influence on treatments of formal consequence up to the advent of modernity.

1.2 Formal consequence from the early 20th century to present

In order to better understand the development of the notion of formal consequence, it will be useful to say something about the different shapes it takes today.

In [190], Alfred Tarski considers in turn two approaches to defining logical consequence: the syntactic approach, then represented by the Hilbert school; and the semantic approach, represented in then-recent work by Rudolph Car-

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[1]See [190, pp. 188, 193]. In what follows, I use 'formal' and 'logical' interchangeably - not to express agreement with this use, but to address the positions exposited on their own terms.

nap [36]. To a surprising degree, approaches to formal consequence continue to fall along these same lines. Semantic approaches, directly traceable to Tarski's own work, remain dominant in both mathematical and philosophical discussions of logical consequence; while through the influence of Gerhard Gentzen, Dag Prawitz and Michael Dummett, a broadly Hilbertian, syntactic approach has been pursued in the tradition of *Proof-Theoretic Semantics*.[2]

The following survey provides a general account of the major developments in logical consequence from the early twentieth century to today. We begin with a review of developments up to Tarski. After this, we consider more recent developments in the semantic approach, followed by an examination of approaches to logical consequence found in the proof-theoretic tradition.

1.2.1 Formal consequence prior to Tarski

> We say that the sentence *X follows logically* from the sentences
> of the class $\mathfrak{K}$ if and only if every model of the class **K** is at the
> same time a model of the sentence *X*. [190, p. 186]

The above quotes the definition of logical consequence offered by Tarski in his now classic 'The concept of following logically'. As others have pointed out,[3] the definition is not identical to its contemporary successor. And while Tarski's work represented a breakthrough in the development of formalized

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[2]For Gentzen's work, see [71]; cf. [66], [80], [139], [200]. For proof-theoretic semantics, see [147], [148]; also [87], [64], [63], [65], [180].

Proof-theoretic semantics is frequently coupled with an approach to meaning called *inferentialism*. But not all inferentialists are proof-theoretic semanticists. See [15], [69], [70], [84].

[3][86] [60] [59] [183]

notions of consequence, it stands not at the beginning, but in the middle of the notion's more recent development.

The syntactic approach

The ascendant tradition in mathematical logic at the turn of the twentieth century - the logicism of Frege, Russell and Whitehead, and the early Wittgenstein - viewed logic as a discipline whose main concern was the determination logical truth [60, pp. 74-77]. This conception of logic allowed the logicist programme to take the particular shape that it had: that of a reduction of mathematics to logic *as* a reduction of mathematical truth to logical truth.[4]

Today, a logic is more readily identified with its consequence relation than its logical truths, and, the focus on logical truth appears retrospectively out of place. This focus was in part a function of the dominance of the axiomatic method at the time, but it also had deeper theoretical roots in the program itself [198]. For Russell as for Frege, logic was grounded in reality and universal in scope: the realism of the program lent itself to characterizing logic in a way analogous to other disciplines, as a body of truths;[5] and the universality of the program did not facilitate the adoption of a metatheoretical perspective, *a fortiori* forestalling a metatheoretical investigation into formal consequence.

Around the same time, Hilbert and his school at Gottingen had succeeded

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[4]Without this background conception, Logicists could not take for granted that such a reduction would have counted as one of mathematics to logic.

[5]Hence, Russell's famous statement that 'Logic is concerned with the real world just as truly as zoology, though with its more abstract and general features.' [174, p. 169].

in reducing large parts of mathematics to just a few axioms, along with a few simple rules, such as substitution and detachment, for manipulating these axioms. Hilbert was the first to explicitly describe the project of metamathematics, and with this to detail the problem of whether the consequences of a given set of axioms and rules corresponded with the body of knowledge it was supposed to represent. Thus, though like Frege and Russell early formalism was liable to identify a formal consequence with the truth of its corresponding conditional, it explicitly recognized that identification was one in need of proof. Hence, one finds questions of soundness and completeness, as well as the need for a deduction theorem, coming to the fore in the Hilbert program.[6]

In accordance with the aims of the Hilbert school, a consequence is thought to *follow formally* from the axioms and rules of a system iff it is possible to obtain it from those axioms via applications of the permissible rules in a finite number of steps - ideally, in such a manner as to admit a decision procedure for any formula of the language;[7] and the formality of a formal consequence consists in its prescinding from any meaning the manipulated symbols might have.[8]

The semantic approach up to Tarski

In [36], Carnap proposed a definition of consequence according to which:

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[6]The first published formulation of the problem of the completeness of propositional logic appears to be in [83], itself based on Hilbert's lectures from the 1917-18 academic year, notes for which were prepared by Paul Bernays and published in [82]. Cf. [66].

[7]Cf. [66, p. 354].

[8][55] calls the notion of the formal involved in this project 'the formal as desemantification'.

> The sentence X *follows logically* from the class of sentences $\mathfrak{K}$ if and only if the class consisting of all sentences of the class $\mathfrak{K}$ and of the negation of the sentence X is contradictory.

Here, Carnap attempts to provide an explicit definition of following from, where the formalist school did not so much define a notion of consequence as presuppose one in its mathematical practice.[9] Tarski calls Carnap's definition 'The first attempt at the formulation of a precise definition' of following logically [190, p. 182], though this is incorrect: leaving aside the medieval accounts this dissertation discusses, the concept is defined by Bolzano in the 19th century.[10] It is probably more correct to say Carnap's notion is the immediate predecessor of Tarski's own, as well as the first attempt to define the notion in the wake of the explosion of interest in foundational research at the turn of the 20th century.

Tarski's definition differs from Carnap's in grounding notions Carnap leaves primitive, thereby widening the range of formalized languages the

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[9][79, p. 15] provides reason to think this failure was a deliberate part of the formalist enterprise:

> Let us observe in passing that there are far more axioms in Hilbert's scheme than in such a scheme as that of *Principia Mathematica*, and *no definitions* in the sense of *Principia Mathematica*. This is inevitable, since it is cardinal in Hilbert's logic that, however the formulae of the system may have been suggested, the 'meanings' which suggested them lie entirely outside the system ... The only conceivable sense of a definition in [Hilbert's] system is that of a symbolic convention which instructs us to replace a prolix formula by a more concise one.

[10]Bolzano's definition reads as follows:

> Propositions $M, N, O, \ldots$ *follow* from propositions $A, B, C, D, \ldots$ with respect to variable parts $i, j, \ldots$ if every class of ideas whose substitution for $i, j, \ldots$ makes each of $A, B, C, D, \ldots$ true also makes all of $M, N, O, \ldots$ true [27, p. 209].

[73] suggests Bolzano's definition is superior to standard post-Tarskian attempts, inasmuch as it demands the variable parts of a consequence be directly stipulated.

notion of 'following from' is applicable to. For instance, Carnap's definition only applies to languages containing a negation operator, where Tarski's definition is not so restricted; and Carnap's definition leaves 'contradictory' undefined, where Tarski's defines 'contradictory' in terms of the absence of any model.

One important respect in which the tradition has followed Carnap - and for that matter, Hilbert as well - as opposed to Tarski is on the interpretation of extra-logical symbols of a formal language. For Carnap, the non-logical constants of a language are uninterpreted until specified by a semantic interpretation. Tarski, by contrast, presupposes these symbols already carry a fixed interpretation, in accord with the formalized sphere of inquiry to which they apply [190, p. 167]. Accordingly for Tarski, it is not the *interpretation* of non-logical constants that is varied: rather, these constants are simply replaced. For instance, to evaluate whether the formula $\forall(x)(P(x) \wedge Q(x)) \supset P(x)$ is a logical truth, the standard approach varies the interpretation of P and Q, where Tarski's simply replaces these with second-order variables [60, pp. 68-69].

1.2.2 Formal consequence since Tarski

Semantic developments

Since Tarski's work, the semantic account of formal consequence has undergone several developments.

Tarski applied his definition not to *formal*, but to what he called *formalized* languages, like those for arithmetic and geometry, having a determinate

sphere of application. Though he made use of techniques having the effect of domain variation when applying his general definition to subordinate sciences [74], Tarski's general conception of a model does not include variations of the domain, while contemporary practice requires this.[11]

Indirectly through Gödel's completeness theorem for first-order logic and incompleteness theorems for higher-order logic; and more directly through the philosohpical influence of W. V. O. Quine, logical consequence became ever more associated with the more well-behaved, first-order consequence relation in the postwar period. Quine claimed Higher-order logic, on account of its expressive power, was not logic at all, but rather mathematics in disguise.[12] Another consequence of Quine's influence was the return of a fairly strong form of realism, like that of Russell, to philosophical logic. Though Quine did not exercise any major influence directly on the concept of logical consequence, his understanding of quantification as ranging over the entire universe of entities, along with the ontological use he put the quantifiers toward, ensured the domain of quantification continued to be held invariant in philosophical appropriations of mathematical logic for some time [155, pp. 31-32] [58] [124], even while domain variation garnered acceptance among those working more directly on mathematical logic [81] [104] [102] [188].

Quine influenced the understanding of logical consequence in a further way through his attack on the concept of analytic truth [158] [154].Both the earlier approach of Carnap and the proof-theoretic approach inspired by

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[11][60] [59]. A consequence of this is that for Tarski, the formality of a formalized language is not the same as its universality, since such a language may only be applicable to a restricted domain of objects.

[12][156, ch. 5]. For discussion of higher-order logic and Quine's criticism of it, see [181].

Gentzen base their notions of logical consequence on the notion of analyticity: For Carnap, a sentence A follows from a premise set Γ provided it follows strictly from the meaning of the logical terms contained in the sentences of Γ. In the limit case where Γ is empty, Carnap assumed A is a logical truth just in case it is true in virtue of the meaning of its logical terms. Likewise, Gentzen held the I-rules for the various propositional connectives were sound because they gave the meaning of the connective introduced in the conclusion. Now, if the notion of something holding in terms of meaning turns out to be imprecise, as Quine charged, then a notion of consequence presupposing it would be similarly inexact. Quine's attack on analyticity thereby helped bolster the dominance of model-theoretic approaches, where the fundamental notion is the extensional one of satisfaction, rather than that of meaning.

The next important development, implicit in the development of Kripke semantics for modal logic [121] [122], was the division of logical consequence into local and global varieties. In contemporary modal logic, a sentence K is said to follow *locally* from a premise set Γ for a class of frames C iff, for every frame in C, for every model on that frame, every world in that model modelling all members of Γ also models K; while K is said to follow *globally* from Γ iff, for every frame in C, for every model on that frame where all members of Γ are *valid* - i.e. where all members of Γ are modeled by *every* world in the model - K is also valid.[13] Unsurprisingly, this division brought

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[13]See [62, pp. 21-23] [19, pp. 31-32]. This does not mean no distinction was made between rules that preserve truth and those preserving *validity*, such as universal generalization in quantified systems and necessitation in modal logics, prior to the development of modal logic. But it does mean the formulation of different consequence relations in terms of local and global models was not yet present in this more basic point.

with it the question of which notion, if either, expresses the genuine notion of following logically.

The latter half of the 20th century through to today has witnessed the proliferation of a vast number of non-classical logics; of domain specific extensions of classical logic, including deontic logics, temporal logics, and epistemic logics; and even of logics with no intended 'logical' application, many of which are developed for use in computer science. Each of these developments brings with it new questions. The development of domain-specific logics poses the question of whether these - given once widespread views about the universality of logic - should be genuinely called *logics*. Individually, each non-classical logic brings with it the question of whether it, rather than classical logic, determines the correct class of formal consequences; collectively, this vast plurality prompts the question of whether the immediately preceding question is even a sensible one.[14]

Developments in the proof-theoretic tradition

At around the same time Tarski was developing his semantic account of formal consequence, Gerhard Gentzen and Stanisław Jaśkowski independently formulated the first systems of natural deduction. While Jaśkowski apparently did not apply this discovery to a consideration of consequence, Gentzen's formulation was accompanied by the claim that

> The introductions represent, as it were, the 'definitions' of the
> symbols concerned, and the eliminations are no more, in the final

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[14]This question is taken up in the debate between logical *pluralists*, on the one hand, and logical *monists*, on the other. See [13] [78].

analysis, than the consequences of these definitions [72, p. 80].

The approach to connective meaning present in this well-known quote thoroughly differs from that found in Tarski. In Gentzen's remark, the meaning of the logical symbols does not transcend their syntax, but is given immanently in the rules governing their use in the proof system. For Gentzen, it is the introduction rule that gives a connective its meaning, though alternative approaches, where the elimination rule is prioritized, have also been proposed [179, p. 186].

The broader principle invoked in Gentzen's approach is that meaning is given by *use*. On the supposition that correct use is a kind of rule following, this principle is itself usually equated with the idea that the meaning of a term is determined by rules. Both ideas are indebted to the philosophy of the later Wittgenstein, and probably achieved their widest audience in the work of Dummett. The claim in philosophy of language that meaning is determined by inferential role is the central tenet of *inferentialism*, and its advocates are called *inferentialists*. *Logical inferentialism* is the claim in philosophy of logic that the meaning of *logical constants* is determined by the rules governing them in some formal proof system. *Proof-Theoretic Semantics* is a project attempting to give the meaning of the connectives entirely within the language of proof-theory, and thus without recourse to model-theory.

Later inferentialists would build up an account of following formally from Gentzen's proof-theoretic account of meaning, just as Tarski's approach does for the semantic account.[15] In the wake of Gentzen and Jaśkowski's devel-

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[15][148, p. 159]. [66, pp. 366-376] suggests that already in [71], Gentzen regarded his

opment of natural deduction, two prominent proof-theoretic approaches to logical consequence have arisen. One, originating with Paul Lorenzen and championed today in the work of Peter Schroeder-Heister, regards logical consequence as a relation between rules [125] [178] [180] [179]. The other, present in the work of Prawitz, holds a sentence A is a logical consequence of a set of sentences Γ exactly when there is a logically valid argument for A from Γ [147] [148].

In the first of these two approaches, the meaning of a connective is straightforwardly identified with a rule.[16] In general,

$$A_1, ..., A_m \longrightarrow_{x_1...x_n} B$$

is taken to mean that for $m, n \geq 0$, there is a rule to move to B^σ from $A_1^\sigma, ..., A_m^\sigma$, where σ is any substitution for free occurrences of variables $x_1, ..., x_n$, taking standard precautions to preclude conflict between free and bound variables.

Since each formula (including atomic formulas) is identified with a rule, a corresponding notion of a consequence $\Gamma \overset{k}{\Rightarrow} A$ in k steps can be defined inductively as follows:

1. For atomic A:

   (a) $\Gamma \overset{k}{\Rightarrow} A$ iff $A \in \Gamma$; or

   (b) there is a rule of the form $(A_1, ..., A_m \longrightarrow_{x_1,...,x_n} B)$ in Γ and a

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proof of the admissibility of cut elimination as a proof of the completeness of the analytic fragment of his proof systems - i.e. the connective rules - with respect to the synthetic notion of following.

[16]In the following, we follow the exposition of [148, pp. 155-159].

substitution σ such that for each $i \leq m$, $\Gamma \overset{k'}{\Rightarrow} A_i^\sigma$ for some $k' < k$, and $\Gamma, B^\sigma \overset{k'}{\Rightarrow} A$ for some $k' < k$.

2. For nonatomic A of the form $A_1, ..., A_m \longrightarrow_{x_1,...,x_n} B$

   (a) $\Gamma \overset{k}{\Rightarrow} A$ iff $\Gamma, A'_1, ..., A'_m \overset{k}{\Rightarrow} B'$, where A'_i, B' are exactly like A_i, B except perhaps containing free $y_1, ...y_n$ (not occurring free in Γ) in place of $x_1, ..., x_n$.[17]

In a sufficiently expressive language, the meaning of different syntactic strings in propositional, first order, and higher-order logic may be identified with different rules. For instance, $A \vee B$ can be identified with the rule

$$(A \to X), (B \to X) \to_X X$$

.

The exact details outlined above need not concern us too much. The basic idea is that in this framework, B is a logical consequence of A just in case $A* \Rightarrow B*$, where $A*$ and $B*$ are translations of A and B into rule form.

The second of the above approaches, by contrast, does not *identify* formulas with rules, but advocates the weaker claim that the meaning of a connective is *given* by its rules. This claim has been interpreted in a number of ways. In some cases, it is the meaning of the connective that is said to be determined; in others, sentences in which the connective occurs.[18] In some cases, the meaning of the connective [sentence] is thought to be determined

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[17][148, p. 156].

[18]For instance, [164] makes use of the first formulation, while [148, p. 162] uses the second. The idea that it is the *sentences* that are given meaning is based on a claim attributed to Frege, that words only have their meaning in the context of a sentence. See [67]. [44, p. 308].

fully; in others, only a criterion for meaningfulness is given. When pairs of introduction and elimination rules meet the requirement for meaningfulness, the rules are said to be in harmony, and the system in which they occur is said to be harmonious.

The simplest and strongest account of connective meaning involves the idea that a language is meaningful provided it is *conservative*. More formally, a system S' for a language L' is conservative with respect to a system S for a language L provided that if a formula A of L is provable in S', then A is already provable in S. For instance, let L be the language for the implicational fragment of classical logic S, and L' the result of adding classical negation to L. Here, conservativity says adding negation to L shouldn't allow us to prove new formulas that do not themselves involve negation, i.e. any formula involving only implications should be provable without making use of negation. In point of fact, L' is *not* conservative with respect to L, since Pierce's law, $((A \rightarrow B) \rightarrow A) \rightarrow A$, can be proved in L' but not in L. Conservativity was proposed as a criterion for connective meaning in [14], and later advocated by [46], where the condition is called 'total harmony'. Because classical negation is not harmonious with respect to the other classical connectives, it is sometimes denied the status of a a genuine logical connective; and likewise because conservativity fails in classical logic, it is denied the status of a genuine logic.

The above account would suffice to rule out certain consequence relations as genuine species of logical consequence. It does not, however, give us a direct, positive proof-theoretic account of what it is for something to follow logically. For this, we turn to two additional criteria for connective meaning:

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i) normalization, and ii) inversion.

Harmony is a relation obtaining between introduction and elimination rules for a connective on account of which the connective is said to be meaningful. Total harmony is the requirement that each connective C of a proof system S' be conservative with respect to the fragment of S resulting from eliminating C from the language L for S. A different criterion, which Dummett calls 'intrinsic harmony', insists that proofs be *normalizable*. Normalization is a procedure performed on proofs, while inversion is a procedure that, given an introduction [elimination] rule, yields a corresponding elimination [introduction] rule for the same connective.

A proof $A \rightarrow B$ is *normalizable* when its *maximal formulae* - formulae occurring both as the conclusion of an I-rule and a major premise of an E-rule - are all eliminable. A proof resulting from the elimination of maximal formula is said to be in *normal form*, and is sometimes called a *normal proof*. In the majority of cases, a proof will be normalizable just when there is an inversion procedure for obtaining the E-rule for the connectives involved in it from their I-rules. There are cases, most of which involve the introduction of paradoxical connectives, where inversion does not guarantee normalization [164]. For Prawitz, the inversion procedure, what he calls 'the way in which the elimination rules are justified by the introduction rules', is 'what makes possible normalizations of proofs in natural deduction' [148, p. 160].

In Prawitz's proof-theoretical account of logical consequence, paradoxical connectives are left aside. For the moment, we do the same. For further simplication, we only detail the propositional case. For Prawitz, 'a sentence A is said to be a *logical consequence* of a finite set Γ of sentences when there

exists a logically valid argument for A from Γ' [148, p. 166]. Let's break this down.

For Prawitz, an argument is 'an arbitrary collection of linked inferences', where one sentence is asserted on the basis of other sentences [148, p. 166]. From here, we proceed in a manner analogous to the treatment of variables in quantification theory, with assumptions taking the place of variables, inferences the place of quantifiers, and arguments the place of formulas. An assumption occurring in an argument may be *bound* or *free*. When an inference discharges the dependency of an argument on an assumption, it is said to bind the assumption; and an assumption is free iff it is not bound, i.e. not discharged. An argument is open if it contains a free assumption, and closed if all its assumptions are bound.

Among arguments, certain ones are singled out as *canonical* arguments. The intuitive idea behind a canonical argument is that it is self-justifying, because it determines what it means for a sentence of that form to hold. For Prawitz, the archtypal examples of canonical arguments are the I-rules for the different connectives in a natural deduction system. He writes:

> if somebody asks why the rule for $\wedge$-introduction [...] is a correct
> inference rule, one can answer only that this is just part of the
> meaning of conjunction: the meaning is determined partly by lay-
> ing down that a conjunction is proved by proving both conjuncts,
> and partly by the understanding that a proof of conjunction could
> always be given in that way [148, p. 163].

While I-rules are regarded as laying down what it means for a sentence of a

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given form to be valid, E-rules are treated by Prawitz as *justifying procedures*. More specifically, they provide a way of transforming any non-canonical argument into a canonical one.[19]

An open argument is valid if its closure is valid. The validity of a closed argument is defined inductively as follows:

1. Every closed argument in canonical form whose immediate subarguments are valid is valid.

2. If an argument D is not in canonical form, but there is a set of justifying operations J that, when successively applied to D, transforms it into an argument where the previous condition holds.

Lastly, an argument is *logically valid* provided 'it is valid relative to each system of canonical arguments for atomic formulas' [148, p. 165].

So in its expanded form, Prawitz's account of logical consequence comes to the following: a sentence A is a logical consequence of Γ provided that in every system of canonical arguments for atomic formulas, i) if Γ/A is open, its closure is valid; ii) Γ/A is itself an argument in canonical form; or iii) if Γ/A is not in canonical form, there is a justifying procedure J whereby the argument may be transformed into a canonical one.

1.2.3 Common elements in the above accounts

The contemporary situation brings with it questions about the necessity of formal consequence, the correct class of formal consequences, the domain of formal consequence, and the purported universality of formal consequence.

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[19]Prawitz sees this claim as a generalization of Gentzen's proof that arguments of a Natural Deduction system can always be given in normal form.

In light of this, can we say much about what has remained common in the way the concept of logical or formal consequence has functioned from prior to Tarski to the present?

Yes. A surprising degree of unity underlies the developments and diversity outlined above. Partisans of all of the above accounts typically presuppose 1) that for a consequence to be logical and for it to be formal amount to the same thing.[20] 2) All of the above accounts accord a place of prominence to substitutionality, though in different ways. 2a) In the model-theoretic approach, valid consequences are determined by varying the interpretation of the non-logical components of a formalized language, or otherwise by varying those components themselves.[21] 2b) in the proof-theoretic approaches surveyed, substitution shows up in a less explicit manner, in the assumption that formally valid consequences hold schematically.[22] 3) In all model-

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[20]For an exception, see [163].

[21]This is strictly true for the accounts of consequence found in [190] and presupposed in the metaphysical projects of [155] and [124]. Later model-theoretic approaches take variability a step further, by allowing variations on the size and elements in the domain, on the set of possible worlds, etc. But when this is done, the invariance of consequence under permutation of non-logical terms becomes only a necessary condition for its holding formally.

The sense of 'substitutional' used above is wider than that used to distinguish substitutional from objectual semantics for first-order languages. A substitutional semantics in the more restricted sense is one on which the truth value of its quantified formulae in a model is determined by the truth value of instances of those formulae wherein the formerly bound variables have been replaced by new terms. An objectual interpretation, by contrast, is one on which it is not the terms, but the objects assigned to the variables that are varied. Typically, an objectual semantics is preferred on the grounds that substitutional semantics is not consistent with the intent to quantify over superdenumerable domains, e.g. the real numbers. But from a purely mathematical standpoint, the class of substitutional models can be represented as a subset of the objectual ones, i.e. those where the domain of the model is just the set of terms in the language. See [70, ch. 14].

[22]Cf. Dutilh Novaes (2011). Both Prawitz and Schroeder-Heister make this more explicit than usual: Schroeder-Heister by his use of propositional quantification in his interpretation of formulas as rules; Prawitz doubly so, by his distinction between open and closed arguments, and by his restricting logically valid arguments to those that hold in *every* system of canonical arguments.

theoretic accounts, substitutionality is taken not merely for a condition on consequence, but rather defines *what it is to be* a formal consequence.[23] And in the Tarski-Quine-Lewis tradition, one finds the assumptions that 4) that precisely those notions which are required to be invariant under all interpretations are the logical notions of a given language; 5) that a consequence is valid *in virtue of* these notions, and it is on account of these that a consequence has its logical form; 6) that a logic is individuated by its class of logical notions; and 7) that, accordingly, without a principled and sharp demarcation criterion for discriminating between the logical and non-logical components of a formal language, we also lack an adequate understanding of the scope and nature of logic [127].

To get a better grasp on some of the above points, it is worth reflecting on what formality in logic is most likely to be contrasted with. On the one hand, the formal is said to be the opposite of the *in*formal. In this sense, formality is typically associated with rigor on account of its use symbolization, itself in the service of obscuring the meaning of the matters to which it is applied for the sake of making these formulae more easily or even effectively calculable [55, pp. 321-325]. And so the spirit of informal logic would be typified by an approach to logic working in or otherwise heavily reliant on natural (as opposed to formalized) language, and one making use of the meanings of the

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[23]Cf. [60, p. 66]:

> ...as far as extensional adequacy goes, there are a multitude of equally correct (or equally incorrect) definitions of first-order consequence: when we specify any one of the many equivalent proof procedures for first-order languages, we have defined the consequence relation as adequately as when we define the relation model-theoretically. But from among these coextensive definitions, the model-theoretic account is account is typically afforded a special status, a status most clearly reflected in soundness and completeness theorems.

terms it treats in determining what follows from them. Such an approach is found, for instance, in the ordinary language tradition of Ryle.

On the other hand, the formal is contrasted with the *material*. In this hylomorphic contrast lifted from the framework of Aristotelian physics, form and matter are constitutive components of every material being (1); form is that which remains invariant in a material being throughout its existence (2, 3, 4); it makes a thing to be what it is, thereby determining its definition and quiddity (3, 5); and on some medieval accounts, serves as a principle of individuation (item 6).

Underlying the multitude of different positions and debates mentioned in the previous section is a common core of thinking about logic lifted from this hylomorphic framework, albeit manifested in different ways. This is surprising on several accounts: first, because logic is often thought to be formal precisely inasmuch as it demurs from either any particular content or metaphysical assumptions; second, because even among metaphysically minded logicians (or logically minded metaphysicians), *any* kind of hylomorphism remains a distinct minority opinion.

The most specific kind of hylomorphism present in the points outlined above - that of Tarski's early account - is what Catarina Dutilh Novaes, following Kathrin Koslicki, calls *mereological* hylomorphism [54] [118]. Mereological hylomorphism is characterized by the contention not merely that wholes are compounds of form and matter, but also that form and matter are themselves distinct integral parts of the hylomorphic compound. This is reflected in the partition of linguistic signs presupposed in Tarski's notion of formal consequence, where one part - the logical constants - corresponds to

the form, and the rest to the matter.

1.3   John Buridan's concept of formal consequence

The first known account of formal consequence in the western tradition directly defined in terms of a substitutional criterion is that of John Buridan, the 14th century Master of Arts at the University of Paris. Buridan was not the first of the medievals to distinguish between formal and material consequence - the distinction is first made explicit by Ockham, and implicit in Duns Scotus, Simon of Faversham, and others - but Buridan was the first to distinguish formal and material consequences by varying the categorematic terms of an argument.[24] Buridan's way of distinguishing material from formal consequences was especially influential on the European continent, having been adopted by Marsilius of Inghen, Albert of Saxony, and others [53]. According to Buridan,

> A consequence is called formal if it is valid in all terms retaining a
> similar form. Or, if you want to put it explicitly, a formal consequence is one where every proposition similar in form that might be formed would be a good consequence [..]. A material consequence, however, is one where not every proposition similar in form would be a good consequence, or, as it is commonly put, which does not

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[24]Note that for Buridan, in contrast with today's practice, the *terms themselves* are varied, rather than their interpretations. For instance, he says 'A man runs; therefore, an animal runs' is not a valid formal consequence, because 'A horse walks, therefore a tree walks' is not [94, TC 1.4.3].

hold in all terms retaining the same form.[25]

There are some important differences between the way formal conse-
quence is understood by Buridan, and the way it is understood today - to
name two, the basic units of Buridan's consequences are written or spoken
sentences, and therefore his semantics is token rather than type-based [109]
[48]; and Buridan doesn't assume a material consequence is thereby not a
logical one.[26]

But whatever one makes of these lesser differences, there is one difference
that makes studying Buridan and his contemporaries an especially fruitful
endeavor: the medieval application of hylomorphic language to consequences
could not but be a *conscious* one, taking place at the height of critical engage-
ment with both Aristotle's logic and his physics and metaphysics; whereas
the contemporary appropriation of hylomorphic language has been by and
large uncritical and at times unaware of this appropriation.[27] If, at times,
medieval treatments of consequence appear less sophisticated than their con-
temporary analogues, they're also somewhat less liable to the distractions
that accompany the development and long use of a technical vocabulary, and

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[25][94, I.4, p. 68]:

Consequentia 'formalis' vocatur quae in omnibus terminis valet retenta forma
consimili. Vel si vis expresse loqui de vi sermonis, consequentia formalis est cui
omnis propositio similis in forma quae formaretur esset bona consequentia [...]
Sed consequentia materialis est cui non omnis propositio consimilis in forma esset
bona consequentia, vel, sicut communiter dicitur, quae non tenet in omnibus
terminis forma consimili retenta.

[26]Much of the renaissance of Buridan scholarship in the past half century has been
motivated by *prima facie* similarities between Buridan's treatment of various logical topics
and contemporary treatments of what are recognizably the same topics and questions. Cf.
[137] [21] [119] [144].

[27]Important exceptions include [163] [169] [128] [55] [53] [54] [49].

thereby often closer to the matters themselves under discussion.[28]

1.4 Overview

The aim of this study, then, is to uncover the meanings implicit in our use of the notion of formal consequence by peeling back the layers of meaning imposed at the time when 'the main precursor of the modern concept of logical consequence' was first formulated [53]. The general plan of the work is to begin with Buridan's notion of formal consequence, and from there to move backwards in successive stages to its historically antecedent enabling conditions.

The questions surrounding the genesis of Buridan's notion, though not all answered, have at least reached a point where they are easily formulable and relatively tractable. The main questions are as follows:

1. What is Buridan's account?

2. How does Buridan's account relate to that of Ockham, the first to explicitly mention a distinction between formal and material consequence?

3. How does the division of consequences into formal and material relate back to the division between natural and accidental consequences, i.e. to the division it seems to have replaced?

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[28]This is an application of a broader point frequently ignored in both historical and systematic discussions today: our later standpoint on questions of philosophical importance is not wholly an advantageous one, inasmuch as the development of any body of knowledge brings with it a certain forgetfulness of its origins. To take an especially clear example, the vast proliferation of logics in the past century, while it has brought us a great many proofs, has not brought us a step closer to an understanding of what logic is or of what it is about. Aristotle, whatever one may think of his analytical and topical works, at least had some sense for what he was doing. Our current state regarding the sense of these questions, by contrast, is probably more bewildered than it has ever been.

There are, of course, more fine-grained questions ensconced within those mentioned, as well as questions that may be asked on either chronological side of these. One may ask, for instance, how the notion of formal consequence is developed by Buridan's followers, or about the development of earlier divisions of consequences. But answering the questions enumerated would yield a philosophically illuminating and relatively self-contained answer to the question of where formal consequence actually came from.

The immediately following chapter provides a more in-depth introduction to Buridan's concept of formal consequence in itself. I begin by reviewing classical formal consequence, then its differences from the account of formal consequence found in Tarski. This is followed by an analysis of Buridan's own approach, examining Buridan's definition and division of consequence into formal and material. The final part compares Buridan's account to Tarski's and its successors in current model-theory.

Chapter three compares the account of consequence found in Buridan to that of Pseudo-Scotus. Recently, Lagerlund and Read have both held that Pseudo-Scotus' treatment of formal consequence must antedate that of Buridan [123] [165]. Here, I show this is not the case. A detailed examination of parallels between Buridan's *Tractatus de consequentiis* and the relevant texts from Pseudo-Scotus' *Questions on the Prior Analytics* shows the Scotus text builds on that found in Buridan; while Buridan's own criterion for valid consequence was the likely target of the *Pseudo-Scotus* paradox, a paradox first found in Pseudo-Scotus' text, resembling Curry's paradox today.

Chapter four investigates modal formal consequences in the account of William of Ockham. Ockham distinguishes between two readings of modal

propositions: one called 'composite'; the other, 'divided'. Today, treatments of Ockham's distinction assimilate it to one between wide and narrow-scope modality in first-order classical modal logic. Doing so, however, renders certain consequences Ockham countenances invalid. I provide a formal reconstruction of Ockham's account which validates the arguments Ockham uses in his text, and gives a full account of the formal entailments and oppositions between two term categorical modal propositions according to their quality, quantity, and range of quantification.

Chapter five introduces the account of consequences found in Walter Burley's later *De Puritate Artis Logicae (On the Purity of the Art of Logic)*. Burley is perhaps best known for his 'realism' in metaphysics; and in part because of this, his own account of consequence has been understudied, and its relation to and influence on nominalists like Buridan has not been thoroughly considered. The later version of Burley's *De Puritate*[29] was completed while Burley was at Paris, and seems to have been widely read and circulated there. However, by the time the treatise appeared, Burley had a wealth of material from commentaries and short tracts stretching back to his time at Oxford, and Burley's treatise was itself conceived in part as a response to Ockham. Though Burley makes use of a distinction between formal and material consequence, the distinction does not hold the prominence it does in Buridan's account: in its place, we find a distinction between natural and accidental consequence. This chapter shows the relation of Burley's treatise to the earliest work on consequences, and explains the relation of Burley's

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[29]There is also an earlier version, sometimes called the *tractatus brevior*. Though sharing some material with its later counterpart, the earlier treatise is not an abbreviation of the later one, and includes material not included in the later treatise.

division of consequences to that which followed it.

The final chapter takes a synoptic view of the results detailed in the previous chapters, returning to their import for the ways in which logic is said to be formal today.

Chapter 2

The semantic account of formal consequence, from Alfred Tarski back to John Buridan

2.1 Introduction

The resemblance between Buridan's and Tarski's theories of formal consequence has long been remarked upon.[1] But while long noticed, it has not yet been subjected to sustained analysis.

In this paper, I provide just such an analysis. I begin by reviewing today's classical understanding of formal consequence, highlighting its differences from Tarski's 1936 account [190]. Following this, I introduce Buridan's account, detailing its philosophical underpinnings, content, and distinction from the semantic accounts of Tarski and his successors. This in turn, pro-

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[1]See [137] [116] [45] [53] [49] [144], and the editor introductions to [93], [119] and [106].

vides the conditions requisite for a partial genealogy of the modern concept, i.e. an account of the conditions that had to arise between Buridan and Tarski's account before Tarski's account became a real possibility.

2.2 From classical consequence back to Tarski

2.2.1 Classical consequence today

Today, classical consequence is constructed in the following way. One begins with an artificial language $L = (Trm, Pred, Con, F)$. $Trm = N \cup Var$ is a collection of *terms*, itself the union of a set of *names* $N = \{a, b, c, ...\}$, and *variables* $Var = \{x, y, z, w, x'...\}$. $Pred$ is an infinite set of n-ary relation symbols $\{P^1, Q^1, R^1, ..., P^2, ..., P^n\}$.[2] $Con = \{\neg, \supset, \vee, \wedge, \equiv \; \exists, \forall, (,)\}$ is a collection of logical constants. Occasionally, the two-place identity predicate $=$ is also classified as a constant. The class of formulas F is then determined recursively as follows:

1. If $t_1...t_n$ are terms, and P^n is an n-ary relation symbol, then $P^n(t_1...t_n)$ is a formula. These formulas are called *atomic formulas*. Any variables among the terms of $P^n(t_1...t_n)$ are said to be *unbound*.

2. If $A \in F$, then $\neg A \in F$. The unbound variables of $\neg A$ are the same as those of A.

3. If A, B are formulae, then so are $(A \supset B)$, $(A \vee B)$, $(A \wedge B)$, and

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[2]Here, the superscript indicates the *arity* of the relation symbol, i.e. the number of arguments it takes. Intuitively, a relation symbol of arity one is a one-place predicate, e.g. 'Red'; of arity two, a two-place relation, e.g. 'greater than'; of arity three, a three-place relation, e.g. 'the sum of', and so forth. In practice, the superscript is dropped, and the arity of a predicate is determined by context.

$(A \equiv B)$, their unbound variables being those of A, B.

4. If $A \in F$, and $v \in Var$, then $\forall v A$, $\exists v A$ are formulae. Here, the quantifiers $\forall v$ and $\exists v$ are said to *bind* the free occurrences in A of v, which thereafter is called a *bound variable*. A formula all of whose variables are bound is called a *sentence*.

5. Nothing else is a formula.

In practice, outermost parentheses are dropped, as are parentheses bracketing terms. Formulae involving the identity predicate are written $t_1 = t_2$, rather than $= (t_1, t_2)$; $\neg = (t_1, t_2)$ is written $t_1 \neq t_2$.

After giving conditions for what constitutes a formula, conditions are provided for determining when they are true or false. To do this, one defines a model $M = (D, I)$ as a pair of a *domain*, D, and an *interpretation I*. D is a collection of objects: cats and dogs, numbers, members of the Medici family, or whatever else one likes. The domain of a model may be finite, countably infinite, or even uncountably infinite, but is not allowed to be empty. The interpretation function I then assigns: each name a to an element in the domain D; and each n-ary relation symbol R^n to a subset of D^n.[3] Besides interpretations, one also has *valuations* $\{v, v', ...\}$ on M, each of which assigns values in D to all the variables of L. Valuations are normally not considered part of a model, since the truth or falsity of a *sentence* relies on no one valuation in particular, though the determination of the truth-value of various *formulae* is partially determined by valuations. One thus says that a formula A is true in I under a valuation v.

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[3]I.e. the nth Cartesian product of the domain, the result of multiplying the domain n times with itself.

For notational convenience, we let $(v \star I)$ be the operation such that $(v \star I)$ agrees with v on the assignment of variables, and with I on the assignment of terms, and write $(v \star I)(t_1, ...t_n)$ for $((v \star I)(t_1), ...(v \star I)(t_n))$. Lastly, call a valuation v' an *x-variant* (*y-variant*, etc.) of a valuation v iff v' agrees with v on all variable assignments except perhaps x. The truth value of different formulae in a model M - i.e. *truth in a model* - is then recursively determined as follows:

1. For atomic formulae of arity n, $(v \star I)(Rt_1...t_n) = T$ iff $(v \star I)(t_1, ...t_n) \in I(R)$

2. $(v \star I)(\neg\phi) = T$ iff $(v \star I)(\phi) = F$

3. $(v \star I)(\phi \wedge \psi) = T$ iff $(v \star I)(\phi) = (v \star I)(\psi) = T$

4. $(v \star I)(\phi \vee \psi) = T$ iff $(v \star I)(\phi) = T$ or $(v \star I)(\psi) = T$

5. $(v \star I)(\phi \supset \psi) = T$ iff $(v \star I)(\phi) = F$ or $(v \star I)(\psi) = T$

6. $(v \star I)(\phi \equiv \psi) = $ T iff $(v \star I)(\phi) = (v \star I)(\psi)$

7. $(v \star I)(\forall x\phi) = T$ iff, for every x-variant v' of v, $(v' \star I)(\phi) = T$

8. $(v \star I)(\exists x\phi) = T$ iff, for some x-variant v' of v, $(v' \star I)(\phi) = T$

A sentence ϕ is *satisfiable* if it is made true on some model M. When this happens, M is said to be a *model* of ϕ. Next, let Γ denote a string of sentences $\phi, \psi, ...$. We say Γ is satisfiable iff there is some model M on which every sentence in Γ is satisfiable. Similarly when this occurs, M is said to be a model of Γ. Lastly, a sentence ϕ is said to be a logical, or formal, consequence of a set Γ, written $\Gamma \models \phi$, iff every model of Γ is a model of ϕ.

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2.2.2 Tarskian consequence

On Tarski's account of formal consequence, $\Gamma \models \phi$ precisely when there is no uniform substitution on non-logical constants in Γ, ϕ that models Γ but not ϕ. This agrees with the modern classical approach inasmuch as formality is determined by variation of models, though it differs with respect to what precisely is varied.

In model-theory today, the languages one works with are left uninterpreted until given an interpretation by the interpretation function. The languages Tarski worked with, by contrast, were fully interpreted artificial languages, like those for Riemannian geometry or Peano arithmetic. Thus for Tarski, there is no question of assigning an interpretation to the non-logical constant '0', say, or varying its interpretation across models. Rather, where contemporary model theory varies the interpretation of logical constants across models to determine what follows logically from what, Tarski left these constants interpreted as they were, but replaced them uniformly with variables, whose valuations are then varied accordingly.[4] The same point also holds for predicate and relation symbols: where modern practice varies their interpretation, Tarski's replaces them with second-order variables, and varies the valuation of these.[5] While there is a conceptual difference between the two approaches, both lead to the same material results.

For Tarski, in contrast with modern practice, a model does not include an interpretation, but is simply a sequence of objects. As Tarski puts it:

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[4]According to [177, p. 433], the contemporary practice originates in the work of John Kemeny. See [102] [103].

[5]See e.g. [189, pp. 122-23]. Cf. [60, p. 69] [177, p. 448].

One of the concepts which can be defined with the help of the concept of satisfaction is the concept of *model*. Let us assume that, in the language which we are considering, to each extra-logical constant correspond certain variable symbols, and this in such a way that, by replacing in an arbitrary sentence a constant by a corresponding variable, we transform this sentence into a sentential function. Let us further consider an arbitrary class of sentences L, and let us replace all extra-logical constants occurring in the sentences of class L by corresponding variables (equiform constants by equiform variables, non-equiform by non-equiform); we shall obtain a class of sentential functions L'. An arbitrary sequence of objects which satisfies each sentential function of the class L' we shall call a *model of the class* L (in just this sense one usually speaks about a model of the system of axioms of a deductive theory); if in particular the class L consists of only one sentence X, we will simply speak about a *model of the sentence* X [190, pp. 185-186].

Some examples may be helpful. The sentence 'John and Peter are brothers' may be formalized by $B(j,p)$. Since both the relation B and the names j and p are non-logical constants, we replace them all to obtain a class of sentential functions $L' = \{X(x,y), X'(x,y')...\}$. On Tarski's definition, the models of $B(j,p)$ thus end up being those ordered pairs satisfying each sentential function of the form $X(x,y)$. This can be meant in two ways: in the first way, it can mean the models of $B(j,p)$ are those ordered pairs satisfying every binary function. If this were so, the resulting class would be empty, since,

e.g. no ordered pair satisfies both the identity function and its converse. And following this line of reasoning, anything would follow from it: since the class of models would empty, it would hold vacuously that every model of whatever else would be a model of $B(j, p)$. This isn't Tarski's intention. In the way Tarski did intend this, we consider $X(x, y)$ and $X(x', y')$ separately, as mere notational variants of each other, and take the class of models of $B(j, p)$ to be the class of ordered pairs satisfying some arbitrary binary relation which we let be designated by X. The class of models thus turns out to be just the class of ordered pairs.[6] This is counterintuitive to the degree one might expect the class of models of the *fully-interpreted* string $B(j, p)$ to simply be {<John, Peter>}. But notice that Tarski's method gives the same result as the modern one, on which B, j, and p are treated as uninterpreted symbols whose interpretations are varied across models.

The most debated question concerning Tarski's work from both pre- and post-war periods is whether he allowed for domain variation across different classical models, as modern practice does; or whether in accordance with then-current logicist assumptions, he assumed a single fixed domain of quantification, viz. the actual universe. Unlike the above, this difference *does* affect the extension of the class of formal consequences. For instance, if the domain of quantification is not varied, $\exists x \exists y x \neq y$ is a logical truth, and hence follows formally from, e.g. $\exists x x = x$. But intuitively (and on the variant-domain approach followed today), 'there are at least two objects' is

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[6]The proof of this is simple: both the identity and non-identity functions are candidate values for the second-order variable X in $X(x, y)$. Since these partition the class ordered pairs (i.e. every ordered pair satisfies one or the other of these), there is always some function the variable X may be mapped to to include an ordered pair as its arguments mapping to t.

not a truth *of logic*, nor does it follow from 'there is at least one object'.[7]

Today, all major parties to the debate agree Tarski employs a fixed-domain framework in his 1936 'on the concept of following logically', this being his broadest and most philosophical discussion of formal consequence in the period. The main points of disagreement concern 1) whether Tarski made use of variable domains where the range of variables was restricted to some more specified domain of discourse - e.g. numbers, points, or segments; and 2) whether Tarski ever changed his mind about domain variation. Readings starting with Etchemendy (1988) and including those of Sagüillo, Corcoran, and Mancosu, suggest Tarski changed from a fixed to variable domain approach in the late 40s or early 50s. Gómez-Torrente, however, holds Tarski's views did not change significantly throughout his life: rather, Tarski accepted a fixed-domain background theory for formal consequence, which permitted techniques for domain relativization when applied to more specified formal theories.

[74, pp. 251-267] copiously details cases where Tarski's actual mathematical practice from the same period as his 1936 essay allowed for domain variation. [188], often regarded as Tarski's first work to explicitly require domain variation in its approach to consequence [41, p. 367] [60, p. 65],

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[7]Alonzo Church had already read Tarski as varying the domain across models in 1956. The first paper to have suggested domain variation was *absent* in Tarski's original account appears to have been [40, p. 43]. That Tarski assumed a fixed-domain in his account of formal consequence was then defended at length by Etchemendy [60] [61] [59], and later taken up by Sagüillo [175] [176] [41], Bays [11], and Mancosu [130] [131]. A variable-domain reading of [190] was accepted by Sher [183] [182], Ray [160], and in Stroińska and Hitchcock's introduction to [190], each broadly on grounds of interpretive charity. The most sophisticated proponent of a variable-domain interpretation in Tarski's pre-WWII work is Gómez-Torrente - see his [75] and [74]. Mancosu has summarized the status of the current debate in [129].

unproblematically refers his reader to his 1936 paper for formal definitions of its semantical notions, including logical consequence [188, p. 8, n. 7] [74, p. 259]; and Tarski's *Introduction to Logic and to the Methodology of deductive Sciences*, first published in 1941 with subsequent editions in 1946 and 1965, does not undergo any significant change in its treatment of formal consequence. Lastly, as [41, pp. 365-366] notes, the framework for Tarski's broadest later discussion of the issues involved in his 1936 work, his 1966 lecture 'What are logical notions?' [191], is also domain-invariant.

In short, Tarski's pre-WWII mathematical work makes use of variable domains; where Tarski's use of domain variation is explicit in his post-WWII discussions of consequence, he refers his reader to his earlier work unproblematically; and his broadest later discussion of the issues of his 1936 paper also assumes an invariant domain. All of this suggests a high level of continuity between Tarski's earlier and later work on the subject.

To better understand the continuity as well as the differences between modern practice and Tarski's, it may be useful to reflect on the concerns and motivations behind Tarski's approach. Tarski's main concern was, positively, to secure the foundational unity of the deductive sciences; and negatively, to remove or overcome the main threats to that unity, found especially in Gödel's incompleteness results. Tarski's theory of formal consequence was not only meant to capture an intuitive notion of following from: it was also meant to provide a general theory for accommodating all then-known mathematical deductive practices.[8] To give a theory of formal consequence was thus

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[8]See [20] and esp. [88].

to be engaged in philosophy of science at the most general level.[9] Though its details are different, Tarski's aim, to secure logic as a unified framework for all mathematics, remains one with Russell and Whitehead's *Principia*.

Tarski locates the problem with both Carnap's approach and the proof-theoretic approach of the Hilbert school in a certain relation to *language*. For Tarski, both these approaches are too dependent on language, albeit in different ways: Carnap's is linguistically impoverished, since, e.g. is not sufficiently general to extend to languages lacking negation; it is the expressiveness embraced by the proof-theoretic approach to avoid ω-incompleteness, however, that allows Gödel's incompleteness results to go through.[10] The former failure justifies the shift to the model-theoretic approach; the latter, Tarski's limiting the concept of truth used in this approach so as to restrict self-reference.

Though both Tarski's and the modern approach call a consequence $\Gamma \models \phi$ formal when every model of the set Γ is at the same time a model of ϕ, they differ to the degree that their underlying concepts of 'model' differ. Tarski interprets 'model of Γ' to mean a sequence of objects satisfying the sentential functions obtained from Γ - i.e. constant-free versions of the sentences of Γ, where like constants are replaced by like variables. The modern approach takes a model of Γ to be an interpretation of the (uninterpreted) strings in

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[9]This same attitude persists in [191], where logic is regarded as the most general of the sciences, and logical notions are accordingly identified as those remaining invariant for 'all one-one transformations of the space, or universe of discourse, or 'world', onto itself' [191, p. 49].

[10]Gödel's original proof only applied to languages strong enough to formulate Peano Arithmetic, and hence including rules for mathematical induction (or their equivalent). Later, [173] and others extended Gödel's incompleteness results, showing they were replicable for all extensions of the the weaker system Q, not including induction rules.

Γ, mapping them to a domain of objects arbitrary both in content and in number. Tarski's approach corresponds more fully to the intuitive *meaning* of 'model'; while the modern approach, if idiosyncratic in its choice of terms, corresponds more fully to the intuitive *extension* of following formally.

Let us then see how these compare to Buridan's account.

2.3 Buridan's theory of formal consequence

2.3.1 Preliminaries

Buridan's *Treatise on Consequences* divides into four books: the first provides the theory of consequences, then discusses consequences holding between assertoric propositions; the second treats modal consequence; the third and fourth, respectively, provide detailed discussions of assertoric and modal syllogistic.

Buridan begins book 1 with a discussion of the truth and falsity of propositions (ch. 1), then the causes of the truth of propositions (ch. 2), before defining (ch. 3) then dividing consequence into its various kinds (ch. 4). After this, he treats the supposition (ch. 5) and ampliation of terms (ch. 6) and the matter and form of propositions (ch. 7) before providing a list of assertoric consequences in chapter 8.

Unlike Tarski, Buridan has no notion of a function, and unsurprisingly has even less the notion of a sentential function. However, the degree to which Buridan's remarks on propositions mimic the role played by the notion of sentential function in Tarski's is surprising. Earlier medieval logic

treatises invariably discussed the signification of terms prior to propositions. In doing so, they followed the order charted by Aristotle in the Boethian translations of the 'old logic'.[11] Because terms signified things belonging to the different Aristotelian categories of being, and terms were the principal parts of the proposition, an adequate grasp of propositions was thought to presuppose a correct account of the categories. This was all the more so in the case of syllogistic and other consequences, which themselves presupposed the theory of the proposition. In contrast with William of Ockham, whose approach was radical in proposing a different and more limited metaphysical basis for logical reasoning [170], Buridan's ordering of materials in both his *Tractatus de consequentiis* and his *Summulae* is radical in simply postponing the metaphysical question: what is important first and foremost for Buridan is the role terms play in a sentence.

Tarski both took the number of objects to be fixed and required constants referring to them to be stripped of any temporal or modal connotation.[12] For Buridan, by contrast, what exists changes from moment to moment; and with Prior and Kripke against Tarski and Quine, the propositional edifice upon which Buridan builds his account of consequence retains modality and tense.[13]

As a special case of the previous point, *propositions*, too, can come into and out of existence [109] [48]. Thus unlike Tarski, Buridan didn't require the elements of language to themselves lie outside the range of semantic referents.

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[11]I.e. those elements of the Aristotelian logical corpus that never disappeared from the Latin West: Porphyry's *Isagoge* and Aristotle's *Categories* and *On interpretation*. See [193, prologue], as well as the ordering of materials in [207].

[12]In this, Tarski is following Russell, Ramsey, and Wittgenstein. See [159, p. 59ff].

[13]For the influence of Buridan on Prior's work, See [196].

Buridan employs a version of truthmaker semantics, where the primary sense of 'exists' ranges over presently existing entities. But his 'presentism' leads to some modifications when compared to more common accounts of truthmakers, or what Buridan calls *causes of truth* [94, p. I. 1]. Typically, presentist approaches to truthmaking require that one give *presently-existing* truthmakers for past- and future-tense statements. These may be existing concrete entities [34] [35] [33], abstracta [42], or even the entire universe [18]. Buridan, by contrast, employs a version of what Sam Baron has recently called *tensed-truthmaker theory* [10]. According to Buridan,

> If Colin's horse, which cantered well, is dead, 'Colin's horse cantered well' ... is true because things were in reality as the proposition signifies they were. In the same way 'The Antichrist will preach' is true, not because things are in reality as the proposition signifies, but because things will be in reality as the proposition signifies they will be. Similarly, 'Something that never will be can be' is true, not because things are as the proposition signifies, but because things can be as it signifies them to be.' And so it is clear that it is necessary to assign causes of truth to different types of propositions in different ways [94, I. 1, p. 63].

Where Tarski prefaced his account of consequence by introducing the notion of a model, i.e. a sequence of objects satisfying a sentential function, Buridan uses the notion of the *causes of truth* or falsity of a proposition. A cause of truth of a present-tense assertoric proposition ϕ at time t is a state of affairs present at t making the proposition true, provided ϕ exists. If ϕ

does not exist at t, it is neither true nor false. In tensed and modal cases the same basic idea is applied, making appropriate adjustments. But where Tarski stratifies the concept of truth, Buridan jettisons it: his account of consequence makes no use of truth apart from that required to arrive at the concept of a cause of truth [112].

For Buridan, the causes of truth of a proposition may change over time. Some propositions have one cause of truth, while others have several. The proposition, 'Socrates is hanging from a basket' has no causes of truth at the time of my writing, since Socrates is dead, and not hanging from a basket. However, at a time t' where Socrates *is* hanging from a basket, Socrates' doing so is a cause of truth of 'Socrates is hanging from a basket'. If Socrates does so at t and t', then Socrates' hanging from a basket at t is a cause of truth of the proposition 'Socrates is hanging from a basket' at t, but not at t', and conversely. At present, 'some philosopher is concocting contrived examples for a paper', has my doing so as a cause of truth, though surely my doing so is not the only cause. The proposition 'everybody is hanging from baskets' only ever has at most one cause of truth at a time, namely all existing people hanging from baskets. Alas, this proposition has probably never had any causes of truth.

As we can see from the above, where Tarski's notion of a model is general from the start, countenancing e.g. every object satisfying a unary function from objects to truth as a model of 'Socrates runs', Buridan's remains particular at this point, countenancing only Socrates running as a cause of truth.

For Buridan, the relative number of causes of truth a typical quantified subject-predicate proposition can have is conditioned by the *supposition* of

its terms. In his *Summulae*, Buridan divides the most common type of supposition, *common material supposition*, into determinate and confused, the latter being subdivided into distributed and merely confused. The subject and predicate of a particular affirmative proposition, as well as the subject of a particular negative, have determinate supposition: each implies the disjunction of all sentences where the determinately suppositing term is replaced by one of its instances, and is implied by any of these. The subject of a universal affirmative has confused distributed supposition, as do both terms of a universal negative proposition: each implies any sentence replacing the confused distributed term with any object falling under it, and is implied by the conjunction of all such sentences. Merely confused supposition is that kind had by the predicate of a universal affirmative proposition: here, one cannot descend from the original sentence to either the conjunction or disjunction of all sentences resulting from replacing the merely confusedly suppositing term with an object falling under it, though one *may* descend from the original sentence to one where the merely confusedly suppositing term is replaced by the term disjunction of all its instances. Buridan explains the relations between these as follows:

> I say that both terms being undistributed but suppositing determinately, then there are more causes of truth than if one were distributed and the other confused without distribution. This is clear because every cause of truth enough to make 'Every B is A' true is enough to make 'B is A' true, but not vice versa. Therefore a proposition [1] has most causes of truth with each term undis-

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tributed [2] and fewer with one term distributed and the other confused without distribution, [3] and fewer still with one distributed and the other used determinately without distribution, [4] and fewest of all with both distributed [94, I. 2, p. 66].

What Buridan states here has a ready parallel in model theory, namely that every model of a sentence of the form $\forall x\phi$ is a model of '$\exists x\phi$'. Buridan, however extends the theory further, pointing out that whatever makes a proposition of the form 'Every A is *this* B' true also makes 'Every A is B' true (i.e. case 3), and that whatever makes 'Every A is every B' true also makes 'Every A is this B' true (case 4). Buridan is already operating on a highly abstract level in considering cases like 3 and 4, which hardly occur in natural language discussion. And by phrasing the point in terms of distribution rather than by mentioning specific determiners, he shows us he knows this holds across a wide-variety of sentential forms, rather than merely those mentioned above.

What is perhaps surprising is that for Buridan, a cause of truth does not suffice for the truth of a proposition, but only does so on the condition that the proposition itself exist. Buridan introduces this caveat as a way of dealing with certain propositions whose existence falsifies them, that nevertheless describe possible states of affairs. For instance, the proposition 'no proposition is negative' is self-falsifying whenever formed, but surely describes a possible state of affairs - indeed, it adequately captures each actual state of affairs before the advent of human speech [94, p. I. 3].

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2.3.2   Formal and material consequence

After laying out the above preliminaries, Buridan provides us first, with a definition, then a division of consequences.

Today, logicians distinguish consequence from hypothetical propositions, usually identifying the latter with conditionals. One then must provide a deduction theorem to establish that $A \models B$ iff $\models A \rightarrow B$. Buridan, by contrast, identifies these: 'A consequence is a hypothetical proposition; for it is constituted from several propositions conjoined by the expression 'if' or the expression 'therefore' or something equivalent' [94, I. 3, p. 66, alt.].

Just as modern practice reserves the term 'consequence' for formally valid consequence, Buridan reserves it for true hypotheticals, excluding false ones[94, I. 3, p. 66]. But where the classical emphasis on formal validity arises already in removing non-logical content from the models of a sentence, Buridanian consequences are not as such formal in this way. [14]

After considering definitions of consequence in terms of 1) the impossibility of the antecedent being true and the consequent not so, 2) the impossibility of the antecedent being true and the consequent false *when both are formed*, and 3) the impossibility of things being as the antecedent signifies without being as the consequent signifies, Buridan ultimately settles on the

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[14]This difference in definition hints at a much deeper one. For Buridan, consequences are always individual sentence tokens, i.e. actually written or spoken hypothetical expressions, which are evaluated as true or false by virtue of determining whether the connections they express hold in all possible situations (including those where the expressions themselves do not exist, and hence are neither true nor false). For Tarski and the modern approach, by contrast, consequences are *never* actual sentences, both because of the aforementioned abstraction at the level of the models of a sentential function, and because the antecedent Γ of a classical consequence $\Gamma \models \phi$ is always at least denumerably infinite, since it is closed under entailment.

following definition of consequence:

> A consequence is a hypothetical proposition composed of an antecedent and consequent, indicating the antecedent to be antecedent and the consequent to be consequent; this designation occurs by the word 'if' or by the word 'therefore' or an equivalent [94, I. 3, p. 67].

Unlike the model-theoretic definition, Buridan's is deflationary in spirit: Buridan takes 'antecedent' and 'consequent' to apply to the respective parts of a hypothetical proposition precisely when 'consequence' applies to the whole. Thus, whether the definition is materially adequate or not,[15] it does nothing to further determine either the extension or the intension of 'consequence' beyond what is already given in the name itself.

Buridan divides consequences into formal and material, dividing the latter into simple and as-of-now consequences. A simple consequence is one where things cannot be as the antecedent signifies and not as the consequent signifies; an as-of-now consequence, one where things cannot *now* be as the antecedent signifies, without also being as the consequent signifies. Thus, as-of-now consequence limits those cases considered in simple consequence to the present situation alone. Thus, every simple material consequence is an as-of-now material consequence, but not conversely.

Buridan explains the division between formal and material consequence as follows:

> A consequence is called 'formal' if it is valid in all terms retain-

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[15]It is not. See the following chapter.

ing a similar form. Or, if you want to put it explicitly, a formal consequence is one where every proposition similar in form that might be formed would be a good consequence [...]. A material consequence, however, is one where not every proposition similar in form would be a good consequence, or, as it is commonly put, which does not hold in all terms retaining the same form [94, I. 4, p. 68].

Buridan continues:

It seems to me that no material consequence is evident in inferring except by its reduction to a formal one. Now it is reduced to a formal one by the addition of some necessary proposition or propositions whose addition to the given antecedent produces a formal consequence [94, I. 4, p. 68].

Where the former passage shows formal and material consequence on equal footing with respect to their validity - both are true hypothetical propositions - the latter shows they are not so with respect to their evidential status: a material consequence is only evident in inference if it can be transformed into a formal one by appropriate additions to the antecedent. One might get the impression from this that for Buridan, material consequences are all enthymemes.[16] But Buridan lists enthymemes as only one kind of material consequence, alongside examples and inductions [94, III. 1, p. 113]; and in Buridan's treatment of dialectical topics, inductions may be proven not 'by virtue of being a formal consequence or by being reduced to

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[16]Cf. [202, p. 66].

a formal consequence, but by the natural inclination of the understanding towards truth' [92, p. 6.1.5]. In this way, the peculiar importance of formal consequence lies not in its preserving *truth*, but *evidence.*

But to know which consequences exactly are formal, one must know what the form of a proposition is. And just as in both Tarski's earlier and later work, this is determined by a partition. Buridan writes:

> I say that when we speak of matter and form, by the matter of a proposition or consequence we mean the purely categorematic terms, namely the subject and predicate, setting aside the syncategoremes attached to them by which they are [1] conjoined [2] or denied [3] or distributed [4] or given a certain kind of supposition; we say all the rest pertains to form [94, I. 7, p. 74].

There are two central differences between Buridan's division and its Tarskian counterpart. First, where Tarski's is a fixed division of terms in a *language* into two different *kinds*, Buridan's is one of terms in a *sentence* into two different *roles*. Thus, for instance, in the sentence 'if is a syncategoreme, therefore if is a word', Buridan's analysis correctly characterizes 'if' as a categorematic term, while Tarski's requires it be treated as a logical constant. Second, where Tarski's approach prioritizes determining the set of logical constants of a language, Buridan's begins by defining the *categorematic* terms of a sentence, i.e. the analogue of classical non-logical constants, and then defines the syncategoremata of a sentence as those not belonging among the categorical terms of the sentence. The categorematic terms are simply those

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operating as the subject and predicate of the sentence.[17] Buridan then lists four types of words as pertaining to form: 1) those conjoining the subject and predicate (e.g. 'is'); 2) those separating the subject and predicate (e.g. 'not'); 3) those giving the terms a certain distribution (e.g. quantifiers); 4) those giving terms a certain kind of supposition (e.g. modal, tense, and other intensional operators). These last two are the subjects to which Buridan's discussions of supposition and ampliation in chapters five and six pertain. Notably absent from this list are propositional connectives, which Buridan doesn't consider in the *Treatise on Consequences*.

Though Buridanian formal consequences may be represented schematically, these consequences are never themselves schematic,[18] but rather remain individual hypothetical propositions. As such, there is no problem on Buridan's account about whether the *ordering* and choice of schematic variables in a schematic consequence belongs to its matter or form.[19] For Buridan, while different good consequences may be representable by the same schema (e.g. different instances of *modus ponens*); while these same consequences may be representable by other schemata (i.e. by uniformly replacing the schematic variables used in the first schema with others); and while schemata for good and bad formal consequences may have the same ordering of their syncategorematic parts (e.g. *modus ponens* and affirming the consequent): because

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[17]A note on the language of 'syncategoremata': the phrase 'syncategorematic terms' does not occur in Buridan, nor to my knowledge in other medieval discussions of consequence. Terms are those words in which every sentence 'bottoms out' (hence the name 'term', i.e. end or limit), and so are just those words against which syncategoremes are divided.

[18]This is also true on Tarski's account, though it is not so on the received classical analysis. The basic reason for the latter is the decision to regard the constant symbols as uninterpreted.

[19]I thank Milo Crimi for bringing this problem to my attention.

schematic consequences are not properly consequences at all for Buridan, these problems disappear. Buridan's formal consequences are hypothetical propositions of a natural language; it is not because they belong to the same schema that they are formal consequences; rather, these formal consequences evidently belong to an equivalence class, and because of this can be represented schematically under the same form.

2.4 Formal consequence from Tarski back to Buridan

Having set Buridan's, Tarski's, and the standard classical accounts of formal consequence in order, we can now contrast them summarily. Doing so provides us with a condition requisite for an adequate genealogy of the Tarskian concept, i.e. an account of the conditions that had to arise between Buridan and Tarski's account before Tarski's account became a real conceptual possibility. With this in mind, we first summarize the differences between the Buridanian and Tarskian accounts of formal consequence; then, we list historical conditions necessary for arriving at the Tarskian account from the Buridanian one.

Both modern and Tarskian approaches begin with a partition of all terms of a *language* into logical and non-logical terms; Buridan's partition of terms into categorical and syncategorematic occurs not at the level of a language, but at that of the sentence.

Tarski's project prioritizes determining the logical terms of a language,

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the determination of the set of non-logical terms falling out of this. Buridan's partition begins by determining those terms pertaining to the matter of the sentence, fixing the set of formal terms in a sentence as the complement of those pertaining to the matter. Where Tarski's partition is one of different types of terms in a language, Buridan's is of different roles had by terms in a sentence. For Buridan, the terms pertaining to the matter of a sentence are its subjects and predicates.

Tarskian consequence was designed for recursively defined artificial languages, particularly those being developed in mathematics. Buridanian consequence was designed to capture Buridan's stilted fourteenth-century scholastic Latin. *Pace* humanist objections, the latter remained a natural language, albeit one making use of mild regimentation when doing so aided discussion. Buridan counts the copula, negation, modalities, tenses, quantifiers, intentional operators, as well as disjunction, conjunction and negation for terms among the formal parts of a sentence. But because he explains the formal parts of a sentence as those affecting the supposition of *terms*, he does not mention sentential connectives as pertaining to form. While admitting sentential connectives, Tarski discounted modalities, tenses, and intensional operators. While initially silent on the status of identity, Tarski later explicitly admitted it as a logical notion. Since then, modalities and other intensional operators have become standard in extensions of classical logic.

For Tarski, consequences are distinguished from hypothetical propositions, not least because the set of premises from which a consequent is derived is closed under entailment, hence countably infinite. Buridan, by contrast, explicitly identifies consequences with hypothetical propositions.

Where Tarski provides an informative definition of consequence in terms of models, Buridan provides a deflationary one in terms of the correlative notions of antecedent and consequent. However, the work done by the notion of a model on Tarski's account is mimicked by that done by the notion of a cause of truth on Buridan's.

On the received classical account, a model of a sentence ϕ in a language L consists of a domain D and an interpretation I, i.e. a mapping of the sentences of L to truth or falsity, recursively determined by a mapping of terms to elements in D and n-ary predicates to sets of n-tuples in D^n.

On Tarski's account, a model of a set of sentences Γ is a sequence of objects in a fixed domain satisfying the sentential functions obtained by uniformly replacing each non-logical constant in the sentences of Γ with variables of the appropriate order and arity.

The differences between the received classical and Tarskian understandings of a model thus lead to differences in their understanding of both the intension and extension of the concept of formal consequence. Both the Tarskian and received classical accounts of the models of a sentence, however, are general from the beginning: for instance, the classical models of an atomic sentence will not be the objects making it true on the intended interpretation of its non-logical constants, but those making it true on *any* interpretation; and the Tarskian models will be sequences satisfying any sentential function of the same form as the initial sentence.

Buridan's account of causes of truth, by contrast, maps hypothetical propositions to states sufficient to make them true on their *intended* interpretation. The determination of the relative number of causes of truth a

sentence has is given by the supposition of its terms, i.e. the manner in which one is permitted to descend from a general term modified by a determiner to a new sentence or sentences replacing the determined general term with a name (names) for an individual(s) falling under it. On Buridan's account, the causes of truth of a proposition are relative to a time of utterance; the models of a Tarskian sentence are not; and while not fixed, classical models are relativized in a way not based on external circumstances, but arbitrarily. On Buridan's account, formality is then achieved by the determination of the equivalence class of a hypothetical proposition: a Buridanian consequence is formal iff for every proposition equivalent in form to it that could be formed, it is impossible for things to be as the antecedent signifies without being as the consequent signifies (leaving aside problems Buridan recognizes with talk about things being as propositions signify).

Let us now say that for contingent concepts C, C', the concept C' is constitutive of C if the existence of C' is necessary for that of C, i.e. there is no possible situation in which C exists but C' does not. Let us further say an event e is a *necessary historical condition* with respect to the existence of C if for linearly ordered times t, t' where $t < t'$, at some t'' such that $t < t'' < t'$, e generates some concept C'' at t'' constitutive of C not present at t. In this way, we see that though C' is necessary for C, a necessary historical condition e generating C' is neither sufficient nor absolutely necessary for the existence of C, though it is sufficient for the existence of C'.

Those developments that *were* necessary historical conditions for the development of Tarski's account from the Buridanian one are comparatively few, but central. The first is the development of the concept of a model; the

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second, that of a sentential function; the third, accordingly, that of a function. Lastly, the decision to regard antecedents as premise sets closed under entailment, and hence infinite, presupposed the development of recursion and its application to logical entailment.

On the other hand, some elements of Tarski's account were straightforwardly available to Buridan, but not taken up by him. Tarski's rejection of modality is not historically dependent upon post-14th century developments. An understanding of the parts of consequence as non-linguistic was accepted in the 14th century by Walter Burley [32]. And the assumption of a fixed, eternalist domain is present in Buridan's own discussion of natural supposition [91, p. 4.3.4]. In assuming the same terms may supposit differently (e.g. for themselves) in different sentences, Buridan implicitly rejects Tarski's construal of different parts of a language as belonging to different kinds.[20] Other differences, including Buridan's decisions to identify consequences and hypothetical propositions, to determine causes of truth relative to a time, and to distinguish formal consequence from logical consequence as such, represent further philosophical disagreements.

As for motivations, one of the main ones behind Tarski's approach to languages - the complete elimination of equivocation from scientific language - was present in a different form in Ockham's adoption of *mental*, rather than spoken or written, language as his point of departure [195] [186] [37], but not in Buridan. Buridan does not require the elimination of equivocation, hence neither does he have the concern one finds in Tarski with the consistency of

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[20]Even if Tarski's division exacerbates a tendency, already found in Buridan, to prescind from treating the meaning of terms prior to their propositional role.

languages.[21]

Tarski's worries about Carnap's account of semantic consequence don't arise for Buridan, either: natural languages are sufficiently robust, and indefinitely extendable, thus sufficient to encounter most of Tarski's concerns with impoverished recursive languages.[22] The major exception to this is superdenumerable domains such as the real numbers, points of geometrical space, etc., which Buridan's theory was clearly not designed to handle. This discovery, while not a necessary historical condition for the development of the semantic approach to consequence, was and remains an important motivator for it.

2.5   Conclusion

The concept of formal consequence in classical logic today, in perfect verbal agreement with Tarski's 1930s definition, holds $\Gamma \models \phi$ iff every model of Γ is at the same time a model of ϕ. But behind this verbal agreement lies a substantive disagreement, grounded in different concepts of a model. Today's classical models interpret uninterpreted linguistic strings by mapping them to a domain of arbitrarily many objects. Tarskian models, by contrast, are sequences of objects, within the fixed domain of all objects in the world, satisfying sentential functions obtained from interpreted sentences.

Tarski's account represents a genuine development from the Buridanian account to the degree that it employs the concepts of model, sentential func-

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[21]This is part of what allows Buridan to treat the Liar paradox locally, rather than instituting a global ban on self-reference. See [91, p. 9.2.6]. Cf. [109] [112] [51] [17].

[22]Cf. [9].

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tion, and recursion, which were unavailable to Buridan. Other differences, however, represent more substantive disagreements. Buridan's presentism; his acceptance of modality, tense, and variable domains; his prioritization of the determination of the material parts of the sentence over the formal; his adoption of a token-based semantics grounded in natural languages, all were taken up *against* analogues of the contrary positions, found in Tarski, in Buridan's own time. In other ways, the difference between Buridan and Tarski's approach to consequence is not so wide as their chronological distance from each other would suggest. In contrast with modern practice, neither construes the *relata* of formal consequence schematically; Buridanian causes of truth form analogues to the Tarskian concept of models of a sentential function; and both Buridanian and Tarskian accounts of following formally are given in terms of substitution - a Buridanian formal consequence is good if all sentences that could be formed by uniform substitutions on its categorematic terms are good, a Tarskian one if it is invariant under satisfaction of sentential functions obtained from it by substituting its non-logical constants with variables. Given this closeness, it is perhaps unsurprising that may of the genuine developments in formal logic over the past sixty years have involved a reappropriation of the Buridanian standpoint on just those topics where he disagrees with Tarski. In this reappropriation of the best elements of Buridan's account into the context brought about by genuine developments since it, one might hope to find progress toward ... well, if not truth, at least how things are signified to be.

Chapter 3

Buridanian Consequence and the Pseudo-Scotus Paradox

3.1 Introduction

Among the works attributed to the Franciscan John Duns Scotus in Luke Wadding's early modern edition of his works, one finds a questions commentary on Aristotle's *Prior Analytics*. Critical work on Scotus' corpus has demonstrated the commentary is not authentic, though the true identity of the author remains unknown. Hence, the author is simply referred to as 'Pseudo-Scotus'. Pseudo-Scotus has, however, continued to attract the attention of scholars in the history of logic, particularly for his contributions to the theories of consequences, and the introduction of an eponymous paradox, ressembling Curry's paradox today.

A particularly important question, for both understanding the development of consequences and identifying the author of the commentary, concerns

its relation to the work of the 14th century Parisian arts master John Buridan. It is beginning to be more widely assumed that Pseudo-Scotus' account of consequence antedates that Buridan offers in his *Treatise on Consequences*. The principal evidence for this is that Buridan considers and rejects an analysis of divided modal propositions Pseudo-Scotus appears to advocate.[1] On Buridan's analysis, a modal of possibility is equivalent to one where the subject is ampliated to include possible instances of itself, e.g. 'A can be B' is analyzed as 'What is or can be A can be B'. On Pseudo-Scotus', the aforementioned proposition is said to be equivalent to 'What is A can be B or what can be A can be B'.

In this chapter, I show Pseudo-Scotus' commentary is dependent on Buridan's *Treatise on Consequences*, and therefore postdates it. First, I introduce Pseudo-Scotus' paradox by way of a comparison with that of Curry. I then survey the criteria for valid consequence in Buridan's text, and show that the paradox afflicts the criterion Buridan settles on. From there, I compare several passages from both texts, and show that in each case the Pseudo-Scotus text should be construed as the later one: Lastly, I diffuse the example used to support the priority of Scotus' text, and provide an alternative account of the target of Buridan's remarks on the ampliation of modal propositions. An appendix lists textual parallels discussed in the body of the paper.

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[1] [123, ch. 6] [100, pp. 252-253] [165, pp. 4-5]. Cf. [152, I, q. 26, pp. 143-144], [94, II. 4, p. 97].

3.2 The Curry and Pseudo-Scotus paradoxes

Curry's paradox is a paradox of self-reference that may be formed in any self-referential logical language L including a detachable conditional, substitution, and contraction. It is formed when a term (C) is defined in L as follows:

$$(C) \overset{def}{\equiv} (C) \to \phi$$

Here, (C) is a name for a proposition having itself as antecedent, and ϕ an arbitrary proposition. ϕ is then derived as follows:

1. C		
2. C $\to \phi$	Def C: 1	
3. ϕ	$\to$ **Elim:** 1, 2	
4. C $\to \phi$	$\to$ **Intro:** 1-3	
5. C	Def C: 4	
6. ϕ	$\to$ **Elim:** 4, 5	

Notoriously, Curry's paradox remains a problem even for logics which perform better than expected when treating other self-referential paradoxes like the Liar [171] [203].

In the above formulation, Curry's paradox involves an object-language conditional and substitution of co-referring terms. Curry's original formu-

lation of the paradox did not use substitution, but universal instantiation and the set-theoretic membership relation [43] [162]. More recently, another version of the paradox has been developed not from a detachable conditional, but from a binary validity predicate Val, defined as follows:

$$\vdash Val(\ulcorner \alpha \urcorner, \ulcorner \beta \urcorner) \text{ iff } \alpha \vdash \beta$$

Given the above schema, a deduction parallel to that above may be carried out with a sentence π, constructed as follows:

$$\pi \stackrel{def}{\equiv} Val(\ulcorner \pi \urcorner, \ulcorner \phi \urcorner)^2$$

In this last formulation, the paradox resembles another whose name the Curry paradox sometimes shares [151], the *Pseudo-Scotus paradox*. In question ten of Pseudo-Scotus' *Questions on the Prior Analytics*, the paradox has the following form:

God exists, therefore this consequence is not valid [153, p. 227].

If we let ϕ name the proposition 'God does not exist', and π the proposition 'This consequence is valid', then Pseudo-Scotus' paradox can be seen to contraposit the right-hand side of an instance of Curry's paradox. Prior to Pseudo-Scotus' formulation, a disjunctive equivalent to Curry's paradox for material implication is found in Thomas Bradwardine's *Insolubilia*.[3] And later, a conditional version of Pseudo-Scotus' paradox appears in Albert of Saxony's *Perutilis Logica*.[4]

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[2]See [12] [210] [140] [172].

[3]Bradwardine's example is 'A man is an ass or Socrates utters a false proposition', where this is all Socrates says. [28, 6.3, p. 97].

[4]Albert's example is 'If God exists, some conditional proposition is false', positing that

3.3 Pseudo-Scotus' paradox and Buridan's criterion for consequence

In his *Treatise on Consequences*, John Buridan defines consequence as follows:

> A consequence is a hypothetical proposition composed of an antecedent and consequent, indicating the antecedent to be antecedent and the consequent to be consequent; this designation occurs by the word 'if' or by the word 'therefore' or an equivalent [94, I. 3, p. 67, alt.].

Buridan offers this definition because of the material inadequacy of more common definitions on a token-based semantics, i.e. one where the bearers of truth are actual written or spoken sentences. The criterion according to which a consequence is good if it is impossible for the antecedent to be true and the consequent not fails in cases where the consequent may not exist, and hence be neither true nor false — e.g. 'Every man runs, therefore some man runs' is invalidated on this criterion in the case where the antecedent is uttered and the consequent is not. The same criterion amended to only consider cases where the antecedent and consequent are formed remains inadequate for propositions with self-falsifying antecedents (or self-verifying consequents) that nevertheless describe a possible situation — e.g. 'no proposition is negative, therefore no donkey runs'. A third criterion, on which 'one proposition is antecedent to another which is such that it is impossible for things to be altogether as it signifies unless they are altogether as the other signifies

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this is the only conditional proposition [1, pp. 359-360].

when they are proposed together' [94, I. 3, p. 67] is rejected for the same underlying reason as the prior two: it assumes things being as a proposition signifies suffice to make that proposition true.[5] Since Buridan's settled definition does not provide a criterion for valid consequence, Buridan frequently resorts to a variant on the third criterion, where the signification criterion shifts in accordance with the features of the proposition: a consequence with a past-tense affirmative antecedent, for instance, is good if it is impossible for things to *have been* as the antecedent signifies, etc.

Pseudo-Scotus' formulation of his paradox is given as a counterexample to an approach to consequence on which 'it is necessary and sufficient that it be impossible that when the antecedent and the consequent are formed simultaneously, the antecedent is true and the consequent is false' [153, p. 226] - that is, as a counterexample to the second criterion Buridan rejects above. Pseudo-Scotus considers: first, Buridan's first criterion, then his third, then his second, settling on a criterion of consequence that modifies Buridan's second [i.e. Pseudo-Scotus' third] considered criterion with an exceptive clause, excluding the case where the 'the signification of the consequent is incompatible with the signification of the sign of consequence' [153, p. 228].

What Pseudo-Scotus doesn't note, however, is that the paradox also provides a counterexample to Buridan's final, settled definition of consequence. Proof: Pseudo-Scotus' consequence is either valid or invalid. If it's valid, it's invalid, by *modus ponens*. And if it's invalid, then it's invalid by reitera-

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[5][94, I. 1, p. 63]. Cf. [91, pp. 854, 957-958]. Since for Buridan, a proposition signifies simply what its terms signify, signification does not vary with the quality, quantity, tense, or modality of a proposition - and hence, for instance, contradictory propositions with the same terms signify the same thing.

tion. So if it's either valid or invalid, it's invalid. Hence, if God exists, the consequence is invalid. God exists. Therefore, the consequence is invalid.

Continuing, whatever follows from a necessary truth is itself necessary. For both Buridan and Pseudo-Scotus, 'God exists' is a necessary truth. Hence, the consequence is necessarily invalid. I.e. it is necessary that it is possible that God exists and the consequence is not invalid. Hence, it is possible that the consequence is not invalid, i.e. it is not necessary that the consequence is invalid. Hence, it is both necessary and not necessary that the consequence is invalid. *Ergo, tu es asinus.*

3.4 The dependence of Pseudo-Scotus' *Quaestiones Super Libros II Priorum Analyticorum* on Buridan's *Tractatus de Consequentiis*

3.4.1 Buridan and Pseudo-Scotus on the definition and division of consequence

Let us now turn to the question of the relation between Buridan's and Pseudo-Scotus' texts.

Pseudo-Scotus' first criterion for consequence verbally parallels Buridan's first. But a minor change leaves it closer in content to Buridan's second criterion. Where Buridan's formulation has one proposition antecedent to the other when it's impossible for it to be true with the other *not being true,*

Pseudo-Scotus' replaces 'not being true' with 'false'. Since propositions are only true or false when they exist, Buridan's counterexample to the criterion does not apply to Pseudo-Scotus' formulation: he pays heed to the verbal expression while effectively speeding up the discussion by passing over it's content. Hence, the counterexample Pseudo-Scotus applies to the criterion is similar to Buridan's for the second: Buridan offers 'no proposition is negative, therefore no ass is running'; Pseudo-Scotus, 'Every proposition is affirmative, therefore no proposition is negative'. In both cases, the problem addressed is one where correspondence conditions for the truth of the consequence come apart from the conditions for its being satisfied, and so the consequent cannot itself exist in the situation wherein it obtains.

The parallel between Buridan's formulation of his second criterion and Pseudo-Scotus' of his third is also exact, excepting Pseudo-Scotus' substitution of 'false' for Buridan's 'not being true'. However, Pseudo-Scotus' use of his paradox as a counterexample to his third proffered definition, rather than a conditional with a self-falsifying antecedent, as Buridan uses, suggests he understands the criterion differently than Buridan does. Where Buridan's second definition is closer to Pseudo-Scotus' third in its formulation, albeit to Pseudo-Scotus' first in its content, Pseudo-Scotus' third formulation is actually closer to Buridan's settled definition. Today, we would say Pseudo-Scotus takes the phrase 'when the antecedent and consequent are formed simultaneously' to have wide scope, where Buridan construes the formation of the consequence as a condition within the scope of the phrase 'it is impossible that'.[6] Hence, Pseudo-Scotus' understanding of the criterion requires a

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[6]Formally, let Val be a validity predicate, ϕ and ψ be sentences, $Formed$ a predicate

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stronger counterexample than that Buridan offers against it.

The same economy is present in Pseudo-Scotus' dropping the phrase 'both being formed together' from the third definition Buridan offers. Since the definition is offered in terms of signification rather than truth, the actual formation of the consequent is irrelevant, and the phrase specifying its formation becomes otiose.

Other differences include the following. Pseudo-Scotus, mentions besides form, also the disposition of the terms in his definitions of formal and material consequence. Pseudo-Scotus subdivides formal consequence, where Buridan does not. And where Buridan defines simple and as-of-now consequence in terms of the impossibility of the antecedent being true and consequent being false together, Pseudo-Scotus defines it in terms of a criterion Buridan gives later, the ability to be reduced to a formal one by adding some necessary proposition to the antecedent. Pseudo-Scotus employs the same reduction criterion in his definition of as-of-now consequence, where Buridan simply states that an *ut nunc* consequence is one which is not good simply. In these last two cases, Pseudo-Scotus is actually *more* faithful to Buridan's account than Buridan himself is. Buridan's assumed definition of simple consequence conflicts with his division of consequences, the reason being

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meaning its argument is formed, T and F truth and falsity predicates, and S an 'are as signified' predicate. Pseudo-Scotus' third criterion for validity is:

$$Val(\phi, \psi) \equiv Formed(\phi) \wedge Formed(\psi) \wedge \neg \Diamond (T(\phi) \wedge F(\psi)).$$

Buridan's second, by contrast, is:

$$Val(\phi, \psi) \equiv \neg \Diamond (Formed(\phi) \wedge Formed(\psi) \wedge T(\phi) \wedge \neg T(\psi))$$

. Buridan's third formulation would then be identical to Pseudo-Scotus' third, albeit substituting S and $\neg S$ for Pseudo-Scotus' T and F.

that the definition does nothing to exclude formal consequences from being simple, though Buridan classifies simple under material consequence. And Buridan's definition of *ut nunc* consequence does nothing to distinguish *ut nunc* consequences from those which are in no way valid.

In short, close examination of parallels between the texts shows Pseudo-Scotus' formulations to be more complex than their Buridanian counterparts. This does not mean Pseudo-Scotus' formulations are more correct. But the addition of detail, whether clarifying or convoluting, suggests we are dealing with a later text.[7]

3.4.2 Pseudo-Scotus' analysis of divided modality

Buridan's remarks on the ampliation of divided modal propositions are central to the case for the priority of Pseudo-Scotus' text, as they are assumed to be the target of Buridan's critique. Buridan writes:

> A divided proposition of possibility has a subject ampliated by the mode following it to supposit not only for things that exist but also for what can exist even if they do not [...]. So the proposition 'B can be A' is equivalent to 'That which is or can be B can be A.'
>
> Some say that it is equivalent to a compound disjunction, namely, to 'That which is B can be A or that which can be B can be A.' But I do not accept this, because saying this and saying what I said earlier are very different. [94, II. 4, p. 97]

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[7]That Pseudo-Scotus' treatise is later is assumed, without being supported, by [26], [105], [52], and [117].

Buridan then goes on to discuss the example 'A creating God can not be God', and suggests that his disjunctive subject reading provides the correct analysis, while the disjunctive proposition analysis does not.

Turning to Pseudo-Scotus' analysis of divided modality in question 26 of his *Prior Analytics* commentary, we find that it is better analyzed as a hasty reading of Buridan's position, at worst, than as the target of Buridan's critique.

The structure of question 26 of the *Prior Analytics* commentary is that of a comparison between two approaches to modal conversions: one, an ampliation-based reading where the subject supposits disjunctively; the other, where divided modals of possibility are ambiguous. The first reading is evidently Buridan's; the second, Ockham's.[8] The question describes the manner of conducting conversions on both approaches, without deciding between the two. It is thus somewhat misleading to describe Pseudo-Scotus' description of Buridan's analysis as his own position, since he never gives it preference over the Ockhamist analysis.

That the Pseudo-Scotus analysis is intended as one of Buridan's position is clear from its invocation of the same language one finds in Buridan's *Tractatus de Consequentiis* to describe the position. Pseudo-Scotus' states that 'with respect to a verb of possibility in an indefinite or particular proposition, the *subject* supposits disjunctively for those which are or for those which can be'[152, I, q. 26, p. 143]. This is exactly what one finds in Buridan's own account [94, II. 4, p. 97]. This intention is further evident from Pseudo-Scotus' explicating the conversion of such a proposition as one with a disjunctive

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[8]For an analysis and extension of Ockham's approach, see the following chapter.

predicate [152, I, q. 26, p. 145].

Further analysis, however, shows the error in analysis is not Pseudo-Scotus', but *Buridan's*. On Buridan's analysis, 'Some A can be B' is analyzed as 'Something which is or can be A can be B'. This is formalized in turn as $\exists x((Ax \vee \diamond Ax) \wedge \diamond Bx)$, where quantification is assumed to be possibilist. Given that an assertoric proposition implies one of possibility, the disjunct from the left-hand side can be eliminated, and this simplifies to $\exists x(\diamond Ax \wedge \diamond Bx)$. Now, by disjunction introduction, this implies $\exists x(\diamond Ax \wedge \diamond Bx) \vee \exists x(Ax \wedge \diamond Bx)$. And that this is indeed equivalent to the first disjunct alone is shown by cases. If the first disjunct is true, then the conclusion is immediate. If the second, then something is both A and possibly B. But since truth entails possibility, it follows that $\exists x(\diamond Ax \wedge \diamond Bx)$, hence, in either case, the entailment follows. Thus, the equivalence holds just as Pseudo-Scotus says it does. The argument Buridan gives in *TC* II. 4 against the analysis is an *ignoratio elenchus*, since it changes the example to one where the modality is before a negation. This ensures the proposition is not one of the form Buridan says it is, since the example Buridan gives is equivalent to one of the form 'Some A is not necessarily B', which isn't at all an affirmative divided modal of possibility, but rather a negative one of necessity.

Further evidence of the later date of Pseudo-Scotus' analysis may be garnered from the following observations. First, it is already accepted that the Pseudo-Scotus knows Buridan's *Questions on the Posterior Analytics* [165, pp. 4-5]. Second, the author additionally appears to have knowledge of the *Sophismata*, since he uses a sophism Buridan admits as true - 'a man

will be a boy' - in an objection to the conversion of divided modals into assertoric propositions.[9] Third, one of the objections Buridan raises to the disjunctive-proposition analysis of particular divided modals of possibility is that its contradictory, would have to be not a disjunction, but a conjunction of two universal propositions [94, II. 4, p. 98]. But this cannot be used as an objection to Pseudo-Scotus' position, since he explicitly uses such an analysis [152, I, q. 26, p. 144]. Lastly, the analysis on which Pseudo-Scotus' text is earlier leaves us with an unsolved question, namely what Pseudo-Scotus' source is, if not Buridan's *Tractatus*, given that the Pseudo-Scotus presentation is clearly reporting the views of others. This problem doesn't arise, however, on the view that Buridan's text is the earlier one: we can accept the analysis Pseudo-Scotus reports is Buridan's own contribution.

3.4.3 The target of Buridan's critique

Now to dissolve the *causa apparentiae*: there is a simpler explanation for the target of Buridan's remarks. According to Buridan, ambiguous written or spoken propositions are mapped to a single mental proposition, i.e. the disjunction of its specified readings [90, sec. 3. 2]. But according to the early analysis found in Ockham and arguably even in Aristotle,[10] modals of possibility are ambiguous between just the readings one finds in Pseudo-Scotus listed as the proposition's disjuncts. Buridan is reading his own commitments concerning ambiguity back into the earlier analysis. As such, Buridan's rejection of the analysis in his *Treatise on Consequences* does nothing to suggest

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[9][152, I, q. 26, p. 143]. Cf. [91, pp. 878, 888].
[10][207, II. 25, pp. 330-332], [149], [8, I. 19, p. 32B.25-32], [100, pp. 243-245].

it postdates Pseudo-Scotus' text.

3.5   Conclusion

Buridan's is a logic without truth, inasmuch as the notion of truth plays no direct role in his account of consequence [112]. It is not thereby, however, a logic without paradox. The Pseudo-Scotus paradox appears to have arisen as a way of generating paradox precisely at a point where truth-based accounts of consequence were ceding to other alternatives — for instance, Buridan's in terms of signification. Pseudo-Scotus' paradox thus would have been understood to show paradox still lurks about even when it is no longer *truth* that we are concerned to save from the beast. The paradox thus represents a critical moment in the movement toward the proliferation of more *ad hoc* solutions to the paradoxes of self-reference in later medieval logic.[11]

3.6   Appendix: Parallel passages from Pseudo-Scotus and Buridan

The following provides parallel passages from Buridan's *Treatise on Consequences* and Pseudo-Scotus' *Questions on the Prior Analytics* relevant to the above analysis, all but the last concerning the notion of consequence. Buridan's passages are on the left, Pseudo-Scotus' on the right. Parallels are indicated by italics. Discrepancies within parallel passages are bolded.

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[11]See, for instance, the criticism of Swyneshed's solution in [167].

3.6.1 Conditions for consequence

Buridan and Pseudo-Scotus' first considered criterion:

Propositionum duarum illa est antecedens ad aliam quam *impossibile est esse veram illa alia **non existente vera*** et illa est consequens ad reliquam quam impossibile est non esse veram reliqua existente vera, ita quod omnis propositio ad omnem aliam propositionem est antecedens quam impossibile est esse veram illa alia non existente vera [93, I. 3, p. 21].

Primus modus est, quod ad bonitatem consequentiae requiritur, et sufficit, *quod impossibile est antecedens esse verum, et consequens **falsum*** [152, I, q. 10, p. 103].

Buridan's second, and Pseudo-Scotus' third considered criterion:

Illa propositio est antecedens ad aliam propositionem quam *impossibile est esse veram illa alia **non existente vera** illis simul formatis* [93, I. 3, p. 21].

Ad bonitatem consequentiae requiritur et sufficit quod *impossibile est antecedente et consequente simul formatis, antecedens esse verum, et consequens **falsum*** [152, I, q. 10, p. 104].

Buridan's third, and Pseudo-Scotus' second considered criterion:

Illa propositio est antecedens ad aliam quae sic se habet ad illam *quod impossibile est qualitercumque ipsa significat sic esse quin qualitercum-*

Ad bonitatem consequentiae requiritur, et sufficit, *quod impossibile est sic esse, sicut significatur per antecedens, quin sic sit, sicut signifi-*

que illa alia significat sic sit ipsis
simul propositis [93, I. 3, p. 22].

catur per consequens [152, I, q. 10,
p. 104]

3.6.2   Definition of consequence

Consequentia est propositio hypo-
thetica ex antecedente et consequente
composita, designans antecedens esse
antecedens et consequens esse conse-
quens [93, I. 3, p. 22].

Consequentia est propositio hypo-
thetica, composita ex antecedente, et
consequente, mediante conjunctione
conditionali, vel rationali, quae deno-
tat, quod impossibile est ipsis, scilicet
antecedente, et consequente simul
formatis, quod antecedens sit verum,
et consequens falsum [152, I, q. 10,
pp. 104-105].

3.6.3   Divisions of consequence

Definition of formal consequence:
Consequentia formalis vocatur
quae in omnibus terminis valet re-
tenta forma consimili [93, I. 4, p. 22]

Consequentia formalis est illa
quae tenet in omnibus terminis, stante
consimili **dispositione, et** *forma*
terminorum [152, I, q. 10, p. 105]

Subdivisions of formal consequence:

[No exact parallel. Cf. [93, III. 1,
p. 79].]

Consequentia formalis subdivid-
itur, quia quaedam est, cujus an-
tecedens est una propositio cate-
gorica, ut *conversio, aequipollentia,*

et hujusmodi. Alia est, cujus antecedens est propositio hypothetica, et quilibet istorum modorum potest subdividi in plures alios modos [152, I, q. 10, p. 105].

Material consequence:

Consequentia materialis est cui non omnis propositio consimilis in forma esset bona consequentia, vel sicut communiter dicitur, *quae non tenet in omnibus terminis forma consimili retenta* [93, I. 4, p. 23].

Consequentia materialis est illa *quae non tenet in omnibus terminis, retenta consimili* **dispositione, et** *forma* [152, I, q. 10, p. 105].

Simple consequence:

Quaedam vocantur consequentiae simplices, quia simpliciter loquendo sunt consequentiae bonae, cum non sit possibile antecedens esse verum consequente existente falso, vel esse ita etc.

... [Consequentia materialis] reducitur ... ad formalem *per additionem alicuius propositionis necessariae* vel aliquarum propositionum necessarium quarum appositio ad an-

Consequentia vera simpliciter est illa, quae potest reduci *ad formalem, per assumptionem alicuius propositionis necessariae* [152, I, q. 10, p. 105]

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tecedens assumptum reddit conse-
quentiam formalem [93, I. 4, p. 23]

Buridan and Pseudo-Scotus' definitions of as-of-now consequence:

Aliae vocantur consequentiae ut nunc, quae non sunt simpliciter loquendo bonae, quia possibile est antecedens esse verum sine consequente, sed sunt bonae ut nunc, quia impossibile est rebus omnino se habentibus ut nunc se habent antecedens esse verum sine consequente [93, I. 4, p. 23].

Consequentia materialis bona ut nunc, est illa, quae potest reduci ad formalem, *per assumptionem alicujus propositionis* **contingentis** *verae* [152, I, q. 10, p. 105]

3.6.4   Divided modality

On the supposition of divided particular modals:

Propositio divisa de possibili habet *subiectum* ampliatum per modum sequentem ipsum *ad supponendum* **non solum** *pro his quae sunt* **sed etiam** *pro his quae possunt esse* quamvis non sint [93, II. 4, p. 58]

Una ponit, quod *subjectum* respectu verbi de possibili, in propositione indefinita, vel particulari, *supponit* **disjunctive** *pro his quae sunt,* **vel** *pro his quae possunt esse* [152, I, q. 26, p. 143].

On the conversion of modal propositions:

Prima pars patet per syllogismum expositorium. Quia si *B potest esse A*, signetur illud B et sit hoc C. Tunc

Deinde dicendum, de modalibus de possibili, quod ipsae in sensu diviso similiter convertuntur illis de

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sic: *hoc C est vel potest esse B* et ipsum idem potest esse A; *ergo quod potest esse A est vel potest esse B* [93, II. 6, 5a conclusio, p. 66]

inesse, scilicet quantum ad hoc, quod universalis affirmativa convertitur in particularem affirmativam; et similiter particularis affirmativa, et universalis negativa in seipsam convertitur in universalem negativam, etc. sicut sequitur *Quoddam B potest esse A, igitur quoddam A est, vel potest esse B*, et debet praedicatum esse disjunctum, ex eo, quod subjectum in antecedente *supponit disjunctive, pro his quae sunt, vel pro his quae possunt esse* [152, I, q. 26, p. 145].

Chapter 4

Divided modality for Ockhamists

4.1   Introduction

The earliest treatise on consequences to discuss modal consequence in-depth is found in William of Ockham's *Summa Logicae*. Central to Ockham's account is his distinction between composite and divided modality. Standard discussions of the composite-divided distinction in Ockham's logic assimilate it to one of scope: in a composite modal, the modality takes wide scope; in a divided one, narrow scope.[1]

For example, the two senses of 'Some A is necessarily B' could be represented as follows. First the composite sense:

(COMPOSITE₁) □(Some A is B)

Or, more definitely:

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[1]Translations of Ockham's works, where not stated otherwise, are my own.

(COMPOSITE$_2$) $\Box(\exists x)(Ax \wedge Bx)$

The divided sense, by contrast, is represented thus:

(DIVIDED$_1$) (Some A is $\Box$B)

Or more definitely:

(DIVIDED$_2$) $(\exists x)(Ax \wedge \Box Bx)$

Today, this is the dominant reading of medieval discussions of the composite-divided distinction.[2] Call this the *canonical reading* of the composite/divided distinction.

According to this reading: 1) the medieval composite/divided distinction is represented by appealing to a contemporary distinction of scope; 2) inasmuch as divided modals are represented as a case of quantifying into an intensional context, the medieval composite/divided distinction is assimilated to the contemporary parsing of the de re/de dicto distinction; and 3) the modality in the divided case can be equally well thought of as a case of an object satisfying a complex modal predicate. Illustrating this last point, Normore writes:

> If I understand correctly Ockham's analysis of the syllogistic with
> all sentences in the divided sense, it can be obtained by treating
> 'a,' 'things that are necessarily a,' 'things that are contingently a,'
> and 'things that are possibly a' as distinct terms [...] and treating
> a modal sentence in the divided sense as an assertoric sentence
> with the appropriately modal predicate term and the appropriate

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[2]See [149], [170], [57, p. 350], [144, p. 298], and [100, pp. 237-238].

subject term. Once the sentences have been transformed in this way, the entire modal syllogistic in the divided sense reduces to the ordinary assertoric syllogistic. [141, p. 49]; cf. [47]

This assimilation is not without its consequences. For instance, Ockham offers the following as an example of an invalid conversion of a divided modal involving necessity:

(1) Nullum impossibile esse verum est necessarium, igitur nullum verum esse impossibile est necessarium. [207, II. 24, p. 329]

Freddoso and Schuurman translate (1) as follows:

(1') 'Nothing impossible is necessarily true' therefore 'Nothing true is necessarily impossible.' [205, p. 166].

In accordance with the canonical reading of divided modality, the translation moves the 'necessary' from the end of the phrase to immediately before the copula. Freddoso and Schuurman then provide the following as a translation of Ockham's argument for the invalidity of (1):

> For the antecedent is true, and the consequent is false. For nothing impossible is possibly true, and yet something true is possibly impossible. [...] For if I go to Rome, then it will be impossible afterwards [that I have not been to Rome]. [205, p. 166]

Echoing Freddoso and Schuurman,[3] [100] offers the following comment on the passage:

> All that follows is that 'Something true is possibly impossible', but

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[3][205, p. 203, fn. 4].

this is not what Ockham needs to show. He needs to show that 'Something true is necessarily impossible,' which appears to be a stronger proposition. [100, p. 243]

In short, the choice to read Ockham's divided modals according to the canonical reading invalidates some of the examples Ockham uses to illustrate his theory. While there is nothing inherently wrong with this - the history of logic is filled with great names putting forth false or inconsistent theories - it has not sufficiently been tested whether there might be a better reading of Ockham in view. Ockham's modal theory thus remains today where his supposition theory was prior to [150]: if it can be formalized, it appears it isn't workable.

In what follows, I provide a coherent and complete account of the relations between categorical modal propositions of subject-predicate form on Ockham's approach. I show the Ockhamist distinction between divided and composite modals is orthogonal to Russellian considerations of scope: for Ockham, a modal proposition with syntactically wide scope will always have both a composite and a divided reading. However, the difficulty in representing Ockham's views arises not so much from modality, as the ways in which modality interacts with quantification and negation. After describing these difficulties, the relations between divided Ockhamist modals are catalogued, then illustrated with an example from Ockham's text.

4.2 The reasons for assimilating divided to narrow-scope modality

An important piece of evidence for the analysis of divided modals into their narrow scope formal counterparts is Ockham's account of their conversion.[4] [100, p. 242] puts (1') alongside the following counterexamples Ockham gives to the general validity of conversion for divided necessary propositions:

(2) 'A being which creates is necessarily God' therefore 'God is necessarily a being who creates.'

(3) 'A man is necessarily understood by God' therefore 'Something understood by God is necessarily a man.' [207, II. 24, p. 329]

Ockham then gives (4), (5), and (6) as the appropriate conversions of (1), (2), and (3):

(4) Every impossibility of necessity is not true, therefore something, which of necessity is not true, is impossible.

(5) The Creator of necessity is God, therefore something, which of necessity is God, is creating.

(6) Man of necessity is understood by God, therefore something, which of necessity is understood by God, is man. [207, II. 24, p. 329]

Given the consequents of the above all conform to the canonical formalization of *de re* modals, it is assumed that the antecedents should as well.

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[4]For instance, [149, p. 276] uses conversion to show the account of divided modality in [137, sec. 12] inapplicable to Ockham.

4.3 Divided modality in William of Ockham: a formal reconstruction

Ockham himself describes the truth conditions for divided modals as follows:

> It should be known that for the truth of such a proposition [i.e. a modal proposition in the divided sense] we require that the predicate, under its proper form, belong to that for which the subject supposits, or to the pronoun referring to that for which the subject supposits; sc. such that the mode expressed in such a proposition may truly be predicated of an assertoric proposition in which the very same predicate is predicated of a pronoun referring to that for which the subject supposits, in a manner proportionate to that stated regarding propositions about the past and about the future.
>
> For example, for the truth of this: 'Every truth of necessity is true', we require that any given proposition be necessary in which this predicate 'true' is predicated of anything for which the subject [term] 'truth' supposits– that is, that any such [proposition] be necessary: 'This is true', 'that is true', indicating anything for which the subject supposits. And since not every such [proposition] is true, it follows that this is false simpliciter: 'Every truth of necessity is true'. [207, II. 10, p. 276]

We can think of this passage as giving the following procedure for determining the truth of a modal proposition. First, we introduce a collection of terms we can treat as rigid; for Ockham, both demonstrative pronouns

as well as proper names have this feature;[5] we can achieve the same effect by letting variables do the trick, choosing one variable to uniquely designate each object belonging to the subject class. Second, we replace the subject term in the sentence with the variable. Third, we check the truth-value of the sentence(s) predicating a mode, tense, etc. of the proposition indicated by the replacement sentence. When the subject term is a name, a definite description, or is quantified by a particular quantifier, then the truth of one such sentence suffices for the truth of the divided modal claim we started with; if the subject term is universally quantified, then it is required that the number of true sentences must exhaust the variables designating objects in the subject class. For example, if the only human beings are Socrates and Plato, then for the truth of 'Every human of necessity is an animal', the sentences 'that x is an animal is necessary' and 'that y is an animal is necessary' must be true, where x and y designate Socrates and Plato, respectively.

To make this more conspicuous, let us introduce two-place modal and tense operators instead of their standard unary operators, where the first place is filled by a sentence, and the second by a term designating an object or collection of objects. For tense and possibility operators, we append subscripts to the operators to indicate whether they require their subject terms to supposit for present objects or past/future/possible objects. For instance,

(W₁) W_1('x is a playwright', The pope)

requires that something presently designated by the definite description 'the pope', and rigidly designated by x, was a playwright; we let

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[5]Cf. his discussion of the expository syllogism and the Trinity in [207, II. 27, pp. 334-339].

(W_2) W_2('x is a playwright', The pope)

be true in the case where x rigidly designates something that once answered to the definite description 'the pope', but perhaps no longer does. In neither case is it required that the sentence 'the pope is a playwright' have been true. As it happens, the first is false at the time of writing (since Pope Francis was never a playwright), but the second is true (since John Paul II was).

For necessity, subscripts will be unnecessary, since for Ockham a true divided modal of necessity only requires the subject to supposit for its presently existing supposita. Let's take a familiar example:

$(\Box)$ $\Box$('x is greater than five', nine)

Both the example and the formulation should look familiar: this is the reconstruction of Quine's third grade of modal involvement given by David Kaplan in [101].[6] But where Quine abjured the 'essentialism' of the above formulation, Ockham takes it to be the only reasonable way to understand necessity. In his prologue to his exposition of Aristotle's Physics, Ockham writes:

> It does not depend on your consideration or mine whether a thing is mutable or immutable, necessary and incorruptible or contingent, any more than it does whether you are white or black, or whether you are inside or outside the house. [206, p. 14, alt.]

Here, Ockham addresses - and rejects - a medieval account of modality according to which whether a given modal proposition is true or false may

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[6][157].

depend on the manner in which its subject is designated.[7] Such a view would have Quine's cycling mathematician be necessarily rational on one designation, necessarily bipedal on another. But where Quine takes the puzzle as a reason to deny de re modality altogether, Ockham merely takes it to show one way of understanding de re modality inadequate, and that a stricter understanding is required.

In brief, using M_d as a placeholder for a binary divided modal operator; M_c, for a unary composite modal operator analogous to M_d; p_x, for a sentential function, with free x, denoting some state of affairs; Q, for a quantifier; and t, for a term denoting a class of objects, we can generalize Ockham's account as follows:

$M_d(p_x, Qt) \Leftrightarrow$ for Q formula(s) p', where p' is exactly like p_x except that each free occurrence of x in p_x is replaced by x', where x' rigidly designates some member of t: $v(M_c(p')) = $ True.

For instance:

$\Box_d($'x is greater than five', some number$) \Leftrightarrow$ for some formula 'x' is greater than five', where x' rigidly designates some number, $v(\Box_c($'x' is greater than five'$)) = $ True

This condition preserves several unique aspects of Ockham's account of divided modals. First, it shows how even Ockham's account of de re modality remains decidedly linguistic,[8] inasmuch as the de re modality is guaranteed indirectly through the verification of a particular sentence or sentences. Second, it shows how Ockham's account of divided modality presupposes his

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[7]Cf. [2, ch. 3, 12]; [192, ch. 12].
[8]Even if the language in question is mental. See [143].

account of composite modality: a divided modal sentence is true when an appropriate collection of composite modal sentences are true. Third, it allows us to see the indirect dependence of Ockham's theory of divided modality on a kind of truthmaker theory: a divided modal has as many truthmakers as there are distinct true sentences validating it, and the truthmakers are themselves the entities rigidly designated by the subject terms factoring into the composite modal sentences on the right-hand side of the above equation. For instance, if the universe consists of the human beings Socrates, Plato and Ockham, where Socrates and Ockham are necessarily rational, but Plato is not, then the sentences 'that this [pointing to Socrates] is rational is necessary' and 'that that [pointing to Ockham] is rational is necessary' are both true, and the divided modal 'some human is necessarily rational' has two truthmakers, i.e. Socrates and Ockham.

4.3.1   Scope and negation in Ockham's modal theory

Ockham's says sentences of the form 'S is possibly P' are equipollent to sentences of the form 'that S is P is possible'.[9] This comment, however, is not fully general, but is complicated by the interaction of divided modals with negation. Where Ockham's account would *prima facie* take 'no man can run' to be equipollent to 'it is possible that no man runs', Ockham's practice shows he takes its equipollent to be 'every man is necessarily not running', which Ockham assumes equivalent to 'necessarily, no man runs'.[10] Understood in the first way, Ockham's account would be running against

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[9][207, II. 9, p. 273; 10, p. 276].
[10]Cf. [207, III-3. 14, p. 645; 36, p. 720].

the most natural way to understand the Latin sentences he's working with. Rather, Ockham's discussion of equipollences in SL III-3 adds nuance to better reflect ordinary usage and Ockham's own practice. This discussion also provides the greatest amount of information in Ockham's work on what are today called scope distinctions.

According to Ockham, divided modals with unresolved negations fall into three types: 1) those where the quantity alone is negated, 2) those where both the quantity and the mode are negated, and 3) those where the mode alone is negated. Each type is resolved differently, as the following table demonstrates:

	Example	Resolution
Quantity alone negated	It is necessary that not every animal be a man. It is possible that not every man be an animal.	It is necessary that some animal not be a man. It is possible that some man not be an animal.
Mode and quantity negated	It is not necessary that every animal be a man. It is not possible that every animal be a man.	It is possible that some animal not be a man. It is necessary that some animal not be a man.
Mode alone negated	No man of necessity is an animal. No man can be an animal.	Every man can not be an animal. Every man of necessity is not an animal [207, III-3. 14, pp. 644-646].

In the first case, the modality is unaffected, and the resolution occurs solely in the assertoric portion of the sentence. In the second, the modality is changed to its dual and the assertoric portion of the sentence is replaced with its contradictory. In the third, the quantity of the quantifier remains unchanged, and the negation is 'pushed through' to the point where it only

affects the copula, forcing the mode to change to its dual. Once this is done, we get the equivalence Ockham assumes, i.e. when a modal proposition is taken in the divided sense, it can be expressed either by '[mode] that [quantified subject] [predicate]' or by '[quantified subject] is [mode] [predicate]': modalities and quantifiers commute. Hence for Ockham, the issue of modal scope remains orthogonal to that of whether a proposition is taken in the composite or divided sense.

4.3.2 Relations between Ockhamist divided modal propositions

In this section I list the relations between three of the four modalities treated explicitly by Ockham. Ockham explicitly addresses four modes – necessity, possibility, impossibility, and contingency. There are difficulties in Ockham's treatment of contingency, and so I leave it aside.[11] I use $\Diamond$ to symbolize possibility, $\Box$ for necessity, and I to symbolize impossibility. Two modes – possibility and contingency – each have two distinct divided readings: the first where the subject term supposits for present beings, the second, where it supposits for possible/contingent beings. Necessity and impossibility, by contrast, are not ambiguous in this way.[12] In the former case, we shall indicate

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[11]Ockham explicitly treats contingency only for universal affirmative and negative propositions. But rather than defining contingency as $\Diamond\phi \land \Diamond\neg\phi$, Ockham takes contingent universals to be equivalent to the conjunction of a universal and its *contrary*, each separately under a possibility operator. Letting Q stand for contingency, this entails that $Q(AaB) \equiv Q(AeB) \equiv \Diamond(AaB) \land \Diamond(AeB)$, a result Ockham explicitly embraces. However, given this, the appropriate analysis of contingent particulars remains unclear. See [207, III-3. 15, pp. 647-648].

[12]Ockham never gives a reason for this, though we might make the following conjecture. In his discussions of necessities and impossibilities, Ockham tends to focus on sentences with universal rather than particular quantity. Therefore, if a counterexample is found,

the present-supposition reading by subscripting the mode with a numeral 1; the alternative supposition, with a subscript numeral 2. We always list the mode first, followed by a categorical proposition within the scope of the mode, relying on standard symbolization[13] to indicate the different basic categorical propositions of Aristotelian syllogistic – e.g. $\Box(AaB)$ is read as 'it is necessary that all A is B', 'All A is necessarily B', 'that every A is B is necessary', all of which are taken as equivalent in the divided sense. Negations within the scope of the mode are always given by the lowercase letter of the categorical proposition. Since it is clear that one way of attaining the contradictory of a proposition is by prefixing it with a sentential negation operator, such sentences are not listed. I assume double negations behave classically, and I assume negating a mode does not change the modality of a divided modal (though Ockham is unsure about this).[14] The first column lists the main formula under consideration, while entries in the same row to the right of it give the contraries, contradictories, subcontraries, and immediate subalterns of that formula, with the understanding that subalternation is transitive.

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on the weaker, present supposition reading, one can infer that the reading ampliating to possible existents must fail as well. However, by mixing strong modes and weak quantifiers (or conversely), we can see that there were readings within Ockham's reach that he would have wanted to be able to assert. For instance, Ockham would likely want an Old Testament prophet's utterance of 'some man must be God', to be true of Jesus, prior to the Incarnation.

[13]See [39], [185], [197], [99].

[14]In a passage discussing the conversion of divided modals of impossibility, Ockham asserts that negations change the mode of the proposition itself. But Ockham himself apparently struck out the passage making the suggestion. [207, II. 26, p. 333].

Equipollences	Contraries	Subcontraries	Contradictories	Subalterns
$\Box(AaB)$ $I(AoB)$	$\Box(AeB)$ $\Box(AoB)$ $\Diamond_1(AeB)$	None	$\Diamond_1(AoB)$	$\Box(AiB)$ $\Diamond_1(AaB)$
$\Box(AeB)$ $I(AiB)$	$\Box(AaB)$ $\Box(AiB)$ $\Diamond_1(AaB)$	None	$\Diamond_1(AiB)$	$\Box(AiB)$ $\Diamond_1(AeB)$
$\Box(AiB)$ $I(AeB)$	$\Box(AeB)$	$\Diamond_1(AoB)$	$\Diamond_1(AeB)$	$\Diamond_1(AiB)$
$\Box(AoB)$ $I(AaB)$	$\Box(AaB)$	$\Diamond_1(AiB)$	$\Diamond_1(AaB)$	$\Diamond_1(AoB)$
$\Diamond_1(AaB)$	$\Box(AeB)$	$\Diamond_1(AoB)$	$\Box(AoB)$	$\Diamond_1(AiB)$
$\Diamond_1(AeB)$	$\Box(AaB)$	$\Diamond_1(AiB)$	$\Box(AiB)$	$\Box_1(AoB)$
$\Diamond_1(AiB)$	None	$\Box(AoB)$ $\Diamond_1(AeB)$ $\Diamond_1(AoB)$	$\Box(AeB)$	None
$\Diamond_1(AoB)$	None	$\Box(AiB)$ $\Diamond_1(AaB)$ $\Diamond_1(AiB)$	$\Box(AaB)$	None

In the above, we can see Ockham's necessity pairs with $\Diamond_1$. But $\Diamond_2$ has no dual. Given this, we can subscript Ockham's necessity operator as $\Box_1$, and add a second, $\Box_2$, to complement $\Diamond_2$. From here, we need only note the following entailments:

1. $\Box_2(AaB) \rightarrow \Box_1(AaB)$, $\Box_2(AeB) \rightarrow \Box_1(AeB)$

2. $\Diamond_2(AaB) \to \Diamond_1(AaB)$, $\Diamond_2(AeB) \to \Diamond_1(AeB)$

3. $\Box_1(AiB) \to \Box_2(AiB)$, $\Box_1(AoB) \to \Box_2(AoB)$

4. $\Diamond_1(AiB) \to \Diamond_2(AiB)$, $\Diamond_1(AoB) \to \Diamond_2 AoB$

5. $\Box_n \phi \to \Diamond_n \phi$

6. The subalternations of the assertoric square continue to hold when embedded under any mode.

Recognizing this allows us to construct all of the relations between both sets of necessity/possibility pairs as a pair of cubes, one embedded in the other. First, we let the front face give the relations of the modal square of opposition for $\Box_1/\Diamond_1$. Next, we add the traditional square as a depth dimension, placing universals in the back and particulars in the front. This gives us a cube for one of our pairs of modals. Lastly, we surround the whole figure in a similar square for $\Box_2/\Diamond_2$, connecting the squares by implication arrows connecting each corner of the outer square to its counterpart in the inner square. The resulting figure looks like this:
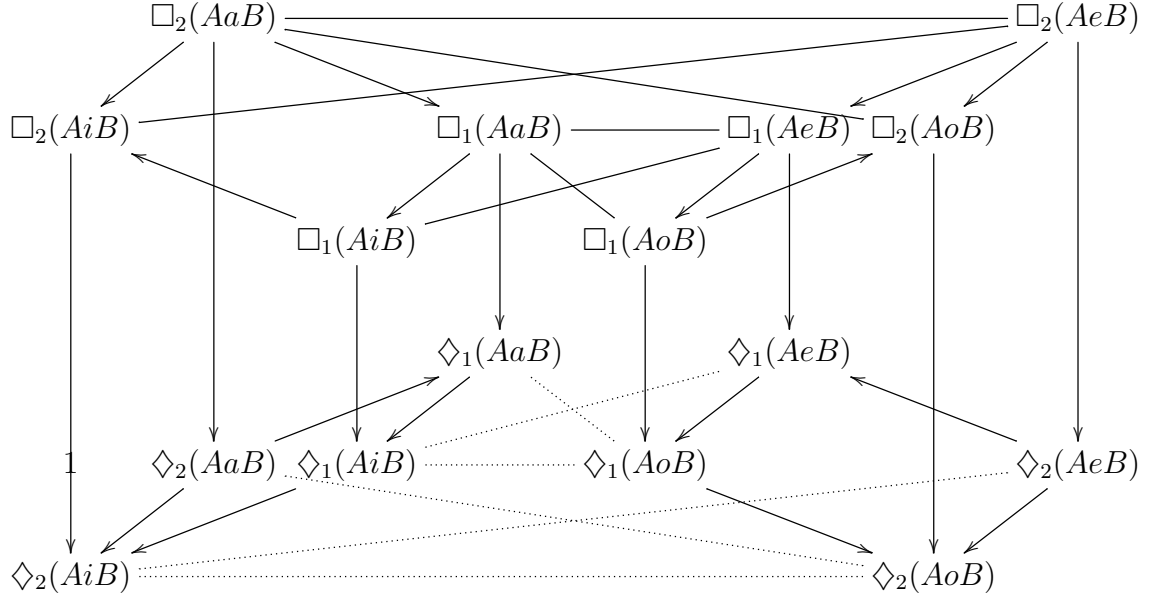
Here, arrows indicate entailments; solid lines, contraries; dotted lines, subcontraries. Note that some pairs, e.g. $\Diamond(AaB), \Diamond(AeB)$ and $\Box(AiB), \Box(AoB)$, are not logically related, either as contraries, contradictories, or subcontraries.

4.3.3 Ockham's theory in first-order modal logic

Up to now, we've been describing the content of Ockham's theory in a quasi-formal way, using formal notation to aid a natural language exposition. Here, we show how a first-order modal theory can be expanded to accommodate Ockhamist divided modality.[15] Let L be the language of a standard first-order modal logic with empty signature, expanded to include restricted quantifiers by the following syntactic rules:

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[15]The exposition given hereafter draws and expands on those of [107], [110].

1. All formulas of first-order modal logic with identity are formulas.

2. If A is a formula and x a variable, then $x.A$ is a restricted variable.

3. If $x.A$ is a restricted variable, then $(\forall x.A)$, $(\exists x.A)$ are restricted quantifiers.

4. If $(Qx.A)$ is a restricted quantifier and B a formula then $(Qx.A)(B)$ is a formula. $(Qx.A)$ is said to bind free occurrences of $x.A$ in B. The free variable occurrences of $(Qx.A)(B)$ are those of B less its free occurrences of $x.A$.

5 If B is a formula and $(Qx.A)$ a quantifier, then $\Box(B, (Qx.A))$ $\Diamond(B, (Qx.A))$ are formulas. Its free variable occurrences are those of B less its free occurrences of $x.A$.

In the above, $\Box$, $\Diamond$ are to be regarded as binary operators - distinct from (albeit analogous to) their unary counterparts. Restricted variables may occur in the same ways standard variables do. In a variable $x.A$, A is called the *matrix* of x. To reduce clutter, we write the full restricted variable only on its first occurrence, e.g. writing $(\forall x.Fx)(Gx.)$ to abbreviate $(\forall x.Fx)(G(x.Fx))$.

Now let $M = (W, R, D, 0, I)$ be a first-order model. W is a non-empty set of situations, R a reflexive, otherwise antisymmetric accessibility relation on W, D a domain function from each situation w in W to its non-empty domain, and I an interpretation mapping parameters in L to objects of the appropriate type, e.g. n-ary predicates from n-tuples on the domain of the model to truth values at situations, etc. 0 is the *zero-entity*, intended as the semantic value of empty terms, itself not an entity in the domain of

any situation. The semantics for sentential connectives and unary modal operators is as usual. Since predicates (including identity), only take objects in the domain of the model as their values, atomic sentences with empty terms must be false, hence making their negations true. To represent the different readings of the range of divided modals Ockham admits in his discussion, we allow both actualist and possibilist quantification. The default reading of quantifiers is actualist, and alternative quantifiers will be subscripted, e.g. a possibilist universal quantifier as $\forall x_\diamond$. The truth of formulas at worlds is determined recursively given an interpretation I of its parameters and valuation v of its free variables. We define the *range* of a restricted variable $x.A$ at w as those valuations of x such that A is true at w, provided this is non-empty. Otherwise, the range of $x.A = 0$. We say a valuation v' is an x-variant (y-variant, etc.) of a valuation v at w iff 1) v and v' agree on all variable assignments except perhaps x; 2) $v'(x) \in D(w)$; and 3) in the case of restricted variables $x.A$, $v'(x.A)$ is in the range of $x.A$ at w. Intuitively we can think of variables as demonstratives, taking different values with different interpretations. The semantics for restricted quantifiers is as follows:

1. $M, w \Vdash_v (\forall x.A)(B) \Leftrightarrow$ for every $x.A$-variant v' of v, $M, w \Vdash_{v'} B$.

2. $M, w \Vdash_v (\exists x.A)(B) \Leftrightarrow$ for some $x.A$-variant v' of v, $M, w \Vdash_{v'} B$.

and the semantics for binary modals is as follows:

1. $M, w \Vdash_v \Box(B, \forall x.A) \Leftrightarrow$ for every $x.A$-variant v' of v, $M, w \Vdash_{v'} \Box B$

2. $M, w \Vdash_v \Box(B, \exists x.A) \Leftrightarrow$ for some $x.A$-variant v' of v, $M, w \Vdash_{v'} \Box B$

3. $M, w \Vdash_v \Diamond(B, \forall x.A) \Leftrightarrow$ for every $x.A$-variant v' of v, $M, w \Vdash_{v'} \Diamond B$

4. $M, w \Vdash_v \Diamond(B, \exists x.A) \Leftrightarrow$ for some $x.A$-variant v' of v, $M, w \Vdash_{v'} \Diamond B$

To form negative quantifiers such as 'no' we place the quantity corresponding to it (i.e. universal or particular) in the quantifier place, prefixing the formula in its first place with a negation. As is clear, in affirmative cases, the truth conditions for a binary modal operator are exactly those for a sentence with its corresponding unary operator bound by the quantifier found in its second place, e.g. $M, w \Vdash_v \Box(B, \forall x.A)$ iff $M, w \Vdash_v \forall x.A \Box B$ iff for every x-variant v' of v, $M, w \Vdash_{v'} \Box B$. But in cases where negation is involved, the negation must prefix the formula (and not the quantifier or the modal) to ensure the semantics do not misrepresent the mode as within the scope of the negation.

4.4   Resolution of difficulties

Having outlined Ockham's account, let us now return to the example we began with.

Ockham mentions the illicit conversion in (1) at the beginning of his discussion of divided modals of necessity, after having discussed composite necessity. In the discussion of composite necessary propositions, Ockham has explained that if a conversion is licit assertorically, then by what we would now call general necessitation, the same conversion remains licit when the premise and conclusion, taken in the composite sense, are placed under necessity operators. In making his case, Ockham gives the following example:

> Since these may be converted, as was said before: no man is an
> ass, and no ass is a man; if this is necessary: no man is an ass;
> then it has to be that this is necessary: no ass is a man. [207, II.

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24, p. 328]

At the beginning of his discussion of divided necessary propositions, Ockham returns to this same case:

> It does not follow by the nature of conversion: no man of necessity is an ass, therefore no ass is of necessity a man, since it does not follow in the divided sense, accepting other propositions. [207, II. 24, p. 329]

Immediately following, Ockham continues with the passage we started with:

> Thus it does not follow: *that no impossibility is true is necessary, therefore that no truth is impossible is necessary*, since the antecedent is true and the consequent is false. For no impossibility can be true, and yet some truth can be impossible; for in this way, 'I have not been to Rome' is true, and yet it can be impossible; for if I go to Rome, it will be impossible afterward.' [207, II. 24, p. 329]

As is clear from the context, Ockham intends all of the following as equivalent:

1. That no impossibility is true is necessary

2. No impossibility can be true

3. Every impossibility of necessity is not true

'Impossible' and 'true' in the above are representable as simple monadic predicates, I and T: since the consequence is a formal one, their modal status

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has no bearing on the validity of the argument in question. The first proposition may now be formalized as $\Box(\neg(\exists y.Ty)x. = y., \forall x.Ix)$; the second, as $\neg(\exists x.Ix)\Diamond(\exists y.Ty)x = y$; the third, as $(\forall x.Ix)\Box\neg(\exists y.Ty)x. = y.$.[16] The first two propositions are found in the above passage, while the third is taken from (4) of section 2 above. As may be checked, the standard translations of the third and (one reading of) the second into narrow scope propositions of first-order modal logic are equivalent.[17] But there is no straightforward way to translate the first into a first-order guise, since wide-scope modality in first-order modal logic forces the medieval composite reading. To remedy this, we provide a formal countermodel to the invalid conversion following Ockham's text.

Let $M = (W, R, D, 0, I)$ be a model, where $W = \{w, w'\}$, wRw', and $D(w) = D(w) = \{o, p\}$, where o is the proposition that Ockham has not been to Rome and p some other arbitrary impossible proposition, e.g. 2+2=5.[18] Let $I(T, w) = \{o\}$, $I(I, w) = \{p\}$, $I(T, w') = \emptyset$, and $I(I, w') = \{o, p\}$.

Now since $I(T, w') = \emptyset$, it follows that $M, w' \not\Vdash_v x.Ix = y.Ty$, hence that it is not the case that for some $y.T$-variant v' of v, $M, w', \Vdash_{v'} (\exists y.Ty)x.Ix = y.Ty$, i.e. that $M, w' \not\Vdash_{v'} (\exists y.Ty)x.Ix = y.Ty$. By the semantics for nega-

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[16]For the reasons for representing Ockham's statements in terms of identity, see [113], [115].

[17]Proof:

1. $\neg\exists x(Ix \wedge \Diamond Tx)$ iff
2. $\forall x\neg(Ix \wedge \Diamond Tx)$ iff
3. $\forall x(\neg Ix \vee \neg\Diamond Tx)$ iff
4. $\forall x(Ix \rightarrow \neg\Diamond Tx)$ iff
5. $\forall x(Ix \rightarrow \Box\neg Tx)$

[18]For simplification, we let these be the only propositions, and ignore difficulties caused by entailments between propositions.

tion, this means that $M, w' \Vdash_{v'} \neg(\exists y.Ty)x.Ix = y.Ty$. Similarly, since $I(T, w) \cap I(I, w) = \emptyset$, it follows that $M, w \nVdash_v x.Ix = y.Ty$, and hence by reasoning parallel to the case at w', that $M, w \Vdash_{v'} \neg(\exists y.Ty)x.Ix = y.Ty$. Thus, at every world w' such that wRw', $M, w' \Vdash_{v'} \neg(\exists y.Ty)x.Ix = y.Ty$. Hence $M, w \Vdash_{v'} \Box\neg(\exists y.Ty)x.Ix = y.Ty$. Now since $I(I, w) = \{p\}$, and p is never in $I(T)$, it follows that for every $x.I$-variant v'' of v', that $M, w, \Vdash_{v''}$ $\Box\neg(\exists y.Ty)x.Ix = y.Ty$. Hence, both that $M, w \Vdash_{v''} \Box(\neg(\exists y.Ty)x. =$ $y., \forall x.Ix)$ and that $M, w \Vdash_{v''} (\forall x.Ix)\Box\neg(\exists y.Ty)x. = y$. And since there are no free variables in this formula, the valuation drops out. Thus both 'that no impossibility is true is necessary' and 'every impossibility is necessarily not true' come out true on the model at w.

However, since $I(T, w) = \{o\}$, $o \in I(I, w')$, and wRw', there is some $y.Ty$-variant v' at w, with $x.Ix$ variant v'' at w' such that $y.Ty = x.Ix$, namely that at which $v(y.Ty) = v(x.Tx) = o$. Hence for some w' such that wRw', it holds that $M, w' \Vdash (\exists x.)y.Ty = x.Ix$, hence that $M, w \Vdash \Diamond(\exists x.Ix)y.Ty = x.Ix$, hence that $M, w \Vdash \exists y.Ty\Diamond(\exists x.Ix)y.Ty = x.Ix$. Hence, some truth, namely 'Ockham has not been to Rome', can be impossible, just as Ockham held.

4.5 Conclusion

In the above, I've provided a way of construing Ockham's account of divided modality that decouples it from modern thinking about scope. On the above account, the question of the syntactic scope of the quantifier is orthogonal to that of whether a modal sentence is to be construed as composite or divided; rather, there is a sense in which even divided modals can take 'wide scope',

though this sense will always be equivalent to a different sentence where the modal takes 'narrow' scope. The real difficulties in representing Ockhamist divided modality do not so much concern the mode itself, but rather its relations to negation and quantification. With respect to the former, we must represent the quality of the formula as belonging to the *predicate*, rather than the subject, if we wish to obtain the correct truth conditions for a wide-scope divided modal. With respect to the latter, Ockhamist quantifiers do not ampliate: rather, Ockham requires that the range of quantification - e.g. possibilist, presentist, or restricted to a particular time - be specified independently of its place in the sentence: every sentence containing an intensional operator will thus be semantically ambiguous between different readings in accordance with the different possible readings of the scope of quantification.

With the above we can begin to restore Ockham to his place as a medieval logician whose theory provides a compelling alternative to that of his contemporary John Buridan, and from whom modern logicians doubtless have much to discover.

Chapter 5

Consequence and formality in the logic of Walter Burley

5.1 Introduction

In his 'The medieval theory of consequence', Stephen Read lists Walter Burley (or Burleigh) alongside William of Ockham and John Buridan as one of the most significant logicians of the medieval period .[1] However, Burley has received markedly less attention than either Ockham or Buridan.

Part of the reason for this, already noted by Boehner [22, p. VI], is historiographical. While Ockham and Buridan are nominalists, Burley is classified as a realist. To the degree that logic is regarded as one of the few philosophical subdisciplines where progress undisputedly occurs, it is natural that historians of logic first set their sights on those figures whose assumptions and methodology appear closest to ours.

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[1][168, p. 900] Translations throughout, unless stated otherwise, are my own.

A closer examination of Burley's thinking about consequences, including its relation to those of Buridan and Ockham, serves as a fitting counterweight to this inclination. The plan of the essay is as follows. I begin with Burley's divisions and organization of consequences. After this, I locate Burley's contribution to the theory of consequences within the context of the 14th century work on the subject: first detailing its relation to the earliest anonymous treatises on consequences; then to the work of Ockham and Buridan. From there, I discuss formal and material consequence in Burley's work, and relate this division back to that between natural and accidental consequences.

5.2 Burley's division and enumeration of consequences

5.2.1 Consequences in Burley's treatment of hypothetical propositions

In the longer version of his *De Puritate Artis Logicae*, Burley discusses consequences in the part of the treatise devoted to hypothetical propositions, specifically that devoted to *conditional* hypothetical propositions.

Conditionals form one kind of explicit hypothetical proposition, along with conjunctions, disjunctions, and temporal propositions; exceptive, exclusive, reduplicative, and others are counted as implicit hypotheticals [202, pp. 106-107]. By 'hypothetical proposition', Burley means what today one might call a compound or non-atomic proposition.[2] By 'implicit hypotheti-

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[2]Cf. [94, p. 66].

cal', he means a proposition analyzable into an explicit hypothetical, even if its surface grammar is categorical.[3] Besides conditionals and other kinds of compound propositions, Burley also refers to conditional *syllogisms*.

According to Burley, a consequence is an act performed in a conditional hypothetical proposition by words like 'if', 'thus', and 'therefore' - the same act signified by the term 'follows' in a categorical proposition. Consequences are not themselves conditionals. Neither are they distinguished from conditionals such that a conditional is indicated by the word 'if', while a consequence is indicated by 'follows' or 'therefore'. Rather, 'conditional' connotes the grammatical or syntactic structure wherein a consequence is shown; 'consequence', the semantic relation obtaining between a conditional's parts. Because of this close link between consequences and conditionals, Burley sometimes uses the terms interchangeably, and one finds the same vocabulary applied to both terms in Burley's work.[4]

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[3]For instance, the sentence 'Nothing besides Socrates runs' is called an exceptive proposition, from the function of the exceptive word 'besides'. Burley analyzes it into the conjunction 'Socrates runs and nothing other than Socrates runs', the conjuncts of which are called the *exponents* (*exponentes*) of the original proposition. [77, p. 121, par. 44], [202, pp. 164-165].

[4]Wilfrid Hodges claims Burley distinguishes consequences from conditionals by using 'if' for the latter, and 'therefore', 'follows', etc. to indicate the former, citing [202, p. 78.10-20] in support of this interpretation. Peter King locates the distinction in that between statements and arguments, arguing that i) the terminology used for conditionals and consequences is not interchangeable, and ii) specifically, conditionals 'are true or false whereas consequences are not'. [85]. [105, p. 120].

Hodges' interpretation of the passage he cites is incorrect. Burley's point in the passage is that from any conditional follows a *categorical* sentence in which nominalizations of the antecedent and consequent of the conditional are taken as terms, and vice versa - i.e. for any conditional $\phi \rightarrow \psi$, terms a, b respectively naming the conditional's antecedent and consequent, and a binary relation $Follows$: $Follows(a, b)$ if and only if $\phi \rightarrow \psi$. At [202, p. 141.26-30], Burley states that from a consequence in which the term 'therefore' (*ergo*) *performs* its semantic function, follows a categorical sentence where this function is *signified* by the word 'follows'. Burley makes the same point in the *Tractatus Brevior* with respect to the function performed by 'if' (*si*) at [202, p. 219.1-9]. Cf. [77, p. 143, par. 119].

Burley's consequences are located in a natural, not formal, language [85, pp. 4-5]. Because the linguistic limitations of Latin are taken seriously, certain kinds of consequences cannot be formed for Burley: for instance, syllogisms cannot be contraposited [202, pp. 65.3-17; 207.31-208.9]; one cannot move from sentential to term negation to negate a complex term, as in 'non-(white tree)';[5] and it is fair game to object to a consequence on the grounds that its premises are well-formed while its conclusion is not.[6]

5.2.2 Burley's enumeration of consequences

In both the shorter and longer versions of the *De Puritate*, Burley calls certain rules governing conditional hypothetical propositions 'principal', thereby distinguishing them from other rules which are said to depend on them. Burley gives ten principal rules in the *Tractatus Brevior*, reduced to five in the *Tractatus Longior*. Nearly all of the rules mentioned are also found in the earlier *de consequentiis*, though this last does not mention principal rules. The first, second, and fifth rules of the *Tractatus Longior* are the first three rules of the *Tractatus Brevior*. These are, respectively: (1) that 'in every good simple [as-of-now] consequence, the antecedent cannot [now] be true without

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The grounds for King's claims are false. Burley calls the conditional 'If a man is an ass, you are sitting' a good consequence at [202, p. 61]. At [77, p. 114, par. 8] and [202, p. 89.1-31], Burley refers to the conditionals linking a larger consequence as 'intermediate consequences (*consequentiae intermediae*)', with the latter passage also calling the conditionals making up a hypothetical syllogism 'consequences'. At [77, p. 128, par. 68], Burley divides conditionals into as-of-now and simple, and immediately thereafter refers to simple and as-of-now consequences without introducing these separately. The same division is found in [202, pp. 60.28-61.5] stated in terms of consequences. At [202, p. 78.27-30], Burley calls a certain consequence linking a categorical antecedent and hypothetical consequent with 'therefore' (*igitur*) a 'composite conditional'. Burley calls consequences true and false at [77, p. 113, par. 2-3]; cf. [76, p. 15, par. 19].

[5][77, p. 131, par. 80], [202, pp. 214.14-21, 215.6-21].
[6][77, p. 150, par. 135]; [202, pp. 211.31-33, 212.10-20].

the consequent' [202, p. 61.30-37, 199.26-27]; (2a) 'whatever follows from the consequent follows from the antecedent'; (2b) 'whatever entails (*antecedit*) the antecedent entails the consequent' [202, p. 62, 9-13]; and (5) 'whenever a consequent follows from an antecedent, the contradictory opposite of the antecedent follows from the contradictory opposite of the consequent' (i.e. contraposition) [202, p. 64.20-22]. Several other rules in the *Tractatus Brevior* are not straightforward rules about *consequence*, but rather govern supposition, negation, or denotation. The *Tractatus Longior* moves some of these to other sections of its text.[7] The rules governing consequences in the *Tractatus Longior*, by contrast, all explicitly concern relations of following, with the two new principal rules in the *Tractatus Longior*, rules three and four, relating following to compatibility and incompatibility. These rules, already present in the *de consequentiis* [77, p. 133, par. 88], are (3) whatever conflicts with the consequent conflicts with the antecedent, and (4) whatever stands with the antecedent stands with the consequent [202, pp. 63.7-10, 64.20-22]. William of Ockham lists them as general rules at SL III-3. 38 [207, III-3.38, pp. 727-731].

Burley gives no justification for principal rules one, two, or five. He does, however, provide arguments for principal rules three and four [202, p. 63.1-14]. That for the third relies on an instance of rule two. Burley restricts the fifth rule to non-syllogistic consequences, albeit on syntactic grounds. The fifth rule holds that from the opposite of the consequent the opposite of the

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[7]Rule four of the *Tractatus Brevior* [202, p. 208.12-13] is called a 'general rule' in the *Tractatus Longior* [202, p. 73.29]. Rule six of the earlier treatise [202, p. 210.11-12] is mentioned in the later treatise's section on supposition [202, p. 27.11]. Rule ten of the earlier treatise [202, p. 219.1-2] appears in the later treatise in connection with the solution to a sophism [202, p. 141.26-30].

antecedent follows. But for Burley, the premises of a syllogism do not form a proposition, either simple or complex. Hence neither does their negation.[8]

Principal and derivative rules

Besides these principal rules, Burley gives a small number of rules said to follow from these: from the first, that (1.1) in a simple consequence, the impossible does not follow from the necessary and (1.2) that the contingent does not follow from the necessary [202, p. 62.1-8]; from the second, that (2.1) whatever follows from the antecedent and consequent follows from the antecedent by itself (*per se*), and (2.2) that whatever follows from the consequent with something added follows from the antecedent with the same added [202, p. 62.22-38]; from the third and fourth, that (4.1) if the consequences of certain propositions conflict so do those propositions, (4.2) consequences with compossible antecedents have compossible consequences, and (4.3) that in a good consequence, the opposite of the consequent conflicts with the antecedent [202, pp. 63.15-64.7]; from the fifth, that (5.1) whatever follows from the opposite of the antecedent follows from the opposite of the consequent, and (5.2) whatever entails the opposite of the consequent entails the opposite of the antecedent.

Derivative rules are not restricted instances of the principal rules, nor are they derivable solely from those rules. Burley's expositions of derivative rules usually take the following structure: statement of the rule, reason supporting it, optional example, summary statement of the argument from the principal

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[8][202, pp. 65.3-17; 207.31-208.9]. Hence, Burley explicitly rejects the interpretation, found in Łukasiewicz, of the premises of a syllogistic inference as a conjunction. [126]. For criticism of this approach, see [38].

rule to the derivative one. Here, for instance, are Burley's remarks on rule 2.2:

> The reason for the second rule is this: the antecedent with something added implies the consequent with the same added; for 'Socrates runs and you are sitting, therefore a man runs and you are sitting' follows. Since, then, whatever follows from the consequent follows from the antecedent, it must be that whatever follows from the consequent with something added, it follows from the antecedent with the same added [202, p. 62.33-38].

Here and elsewhere, the reason gives information that must be used with the principal rule to obtain the derived one. In one place, Burley tells us 'from one nothing follows, neither enthymematically nor syllogistically' [77, p. 147, par. 130]. Burley's meaning here is that every good consequence relies on premises and/or connections other than those stated explicitly in the premise itself. This holds also for arguments deriving rules from other rules.

5.2.3 Burley's division of consequences

Burley divides consequences into simple and as-of-now, the former being divided into natural and accidental consequences.

According to Burley, a simple consequence is one 'which holds for every time, such that the antecedent can never be true unless the consequent is true'. An as-of-now consequence, by contrast, 'holds for a given time and not

always'.[9] The distinction is already present in the earlier *de consequentiis*, where it is also referred to as one between simple and as-of-now *conditionals* [77, p. 128, par. 69]. Burley's definitions for simple and as-of-now consequence do not substantially differ from those in Ockham or Buridan.[10]

Burley defines a natural consequence as one where 'the antecedent includes the consequent'. Such a consequence 'holds through an intrinsic topic', while an accidental consequence is one 'which holds by an extrinsic topic' [202, p. 61.6-10]. Surprisingly, this division is not found at all in the *Tractatus Brevior*. It is, however, in Burley's earlier *de consequentiis*, which further distinguishes two kinds of accidental consequences: one, which 'holds on account of the terms or matter', Burley's example being the proposition 'That God exists is true, therefore that God exists is necessary (since truth in God and necessity are the same)'; the other, for which Burley gives as examples propositions with an impossible antecedent or a necessary consequent [77, pp. 128-129, par. 70]. This division is not presented in the later *De Puritate*, though as we shall see, it shows up tacitly in the solution to a problem there.

Natural and accidental consequence are already distinguished, albeit in passing, by Boethius [24, 835B]. Later, their division is found in Duns Scotus, where as in Burley, we find it grounded in a division between intrinsic and extrinsic topics. Prior to Scotus, the notion of a natural consequence is found in William of Sherwood's *Syncategoremata*, where it is juxtaposed with non-natural consequence.[11] The same topical distinction Burley uses to

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[9][202, pp. 60.29-61.5]. Cf. [202, p. 199.19-25].
[10][207, III-3. 1, pp. 587-588]; [93, I. 4, p. 23].
[11][95, I, d. 11, q. 2, p. 136-137], [208, p. 80]; see [134].

distinguish natural from accidental consequence is employed by Ockham to distinguish two kinds of formal consequences [207, III-3. 1, p. 589].

In the *de consequentiis'* discussion of the *Tractatus Longior*'s rule (2.1), Burley makes clear that following can occur in different ways, each of varying strength: the rule is said to follow 'at least in the same way *ex impossibili quodlibet* follows' - i.e. accidentally, via an extrinsic topic, and not on account of the terms [77, p. 132, par. 85]. Burley's meaning here does not seem to be that the rule holds at least with the same strength as that consequence, but rather that instances of the rule may admit of varying levels of strength, the weakest of these being that associated with explosive consequences.

Burley's assumption of the weakness of explosion may come as a surprise, given classical and strict consequence, following Buridan, build explosion into the definition of consequence: a consequence is good just when the antecedent cannot be true with the consequent being false. Since an impossible antecedent cannot be true, neither can it be true with whatever is consequent to it being false. But for Burley, enthymematic consequences, such as 'if Socrates is a man, Socrates is an animal' are stronger than these.[12] The strength of a given consequence is thus not merely a syntactic matter, but dependent on real relations between things mentioned in it.

Following formally in Burley's logic

In the *Tractatus Longior*, Burley voices the following objection to rule 2.1:

> 'Brunellus is risible, therefore, Brunellus is a man' follows; and
> from these two, it follows formally that a man is risible. For

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[12]Cf. [108, pp. 333-337].

'Brunellus is risible, Brunellus is a man, therefore a man is risible' follows formally. And yet from the antecedent by itself the same does not follow formally. 'Brunellus is risible, therefore a man is risible' does not follow formally, because then distributing the consequent, the antecedent follows; and then 'Every man is risible, therefore Brunellus is risible' follows, which is false. And even if 'Brunellus is risible, therefore a man is risible' were to follow formally, it would follow in the same way placing the negation after. And then 'Brunellus is not risible, therefore a man is not risible' would follow, where the antecedent is true and the consequent false [202, p. 80.13-29].

In the earliest mss. of the *de consequentiis*, the rule is likewise paired with rule 2.2, albeit with their order the reverse of that in the later treatises. Burley says of the latter rule that it follows 'necessarily or formally', apparently intending to equate these notions [77, p. 132, par. 84]. In the revised texts of the same, one finds the words 'necessarily or formally' dropped, not to appear again in either version of the *De Puritate*. Thus, it seems that Burley early equated a formal consequence with a necessary one. This view, however, was dropped by 1302, the date given by the scribe of the London copy of Burley's *De Consequentiis*.

The objection in the *Tractatus Longior* is broken into two parts: the first assumes an undistributed antecedent that formally entails its undistributed consequent is itself entailed by the same consequent with term distributed; the second, that the presence of a negation should make no difference to whether an antecedent with an undistributed subject entails a consequent

with one. Burley accepts the first assumption already in the *de consequentiis*, and discusses it under the seventh principal rule[13] of the *Tractatus Brevior*[14] Both passages consider objections to the rule, in the light of which its domain of application is clarified. But neither mentions the notion of formality.

In the *de consequentiis*, Burley responds to the rejection of the rule 2.1 by distinguishing between two different ways a consequent can follow from its antecedent: some consequences hold 'by reason of its incomplex [parts]' - i.e. the *significata* of its terms; others, by reason of the whole complex [77, p. 118, par. 31]. The distribution rule applies to consequences of the former, but not the latter type. Furthermore, the distribution has to be applied to the terms on account of which the consequence holds.

The *Tractatus Longior* text repeats this, adding that the consequence 'Brunellus is risible, therefore a man is risible' holds formally 'by reason of three terms'; and that the distribution rule only holds for consequences that are formal 'by reason of two terms' [202, pp. 84.11-85.17]. Thus, while both texts distinguish consequences holding by virtue of their parts from those holding by virtue of their structure, the later text further distinguishes among the former type according to the number of terms it holds by, and only the later text considers these to be different ways of following *formally*.

For Burley, consequences which hold in virtue of their whole structure include conversions, syllogisms [202, p. 86.9-12], arguments from an exclusive to a universal transposing its terms and vice versa,[15] and presumably con-

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[13]Namely, that a consequence from a distributed superior to its inferior, taken either with or without distribution, holds. But not conversely.

[14][77, pp. 117-118, par. 26-31]; [202, pp. 211.21-212.28].

[15]Burley's example is 'every man is an animal, therefore only an animal is a man' [77, pp. 142-143, par. 118].

sequences such as obversion, contraposition, and the immediate inferences found in the square of opposition. These are contrasted with consequences holding formally in virtue of their terms. What Burley describes by 'formal consequence holding in virtue of its simple [parts]' consists chiefly of two things: the first, standard quantificational rules for descent to singulars from quantified common nouns, and for ascent from singulars to the nouns they fall under; the second, rules for replacing terms in an antecedent with terms containing or contained by them in the consequent, what Hodges calls Burley's 'calculus of monotonicity'. 'A man runs, therefore an animal runs' is said to hold by two terms; 'Brunellus is risible, therefore, a man is risible', by three; 'a man runs, therefore an animal moves', by four [202, p. 84.24-27].

One might think, then, that a consequence holding in virtue of two terms would be one where those terms are, with its syncategoremata, added to its list of invariant parts, with its other constants being among the variable parts. This approach is explicitly championed in the 19th century by Bernard Bolzano.[16] This does appear to capture Burley's practice with respect to consequences holding in virtue of two terms: when Burley wishes to show that a proposition fails to hold formally in two terms, he varies the third to find a counterexample. For instance, 'Brunellus is risible, therefore a man is risible' with 'risible' as variable part admits the counterexamples 'Brunellus is running, therefore a man is running', and 'Brunellus is a braying animal, therefore a man is a braying animal' [202, pp. 84.13-15].

But the problem with ascribing this approach to Burley becomes clear when one considers consequences said to hold in virtue of all of their cate-

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[16][73].

gorematic terms. As Tarski put it,

> The extreme would be the case in which we treated all terms of the
> language as logical: the concept of following formally would then
> coincide with the concept of *following materially*–the sentence X
> would follow from the sentences of the class $\mathfrak{K}$ if and only if either
> the sentence X were true or at least one sentence of the class $\mathfrak{K}$
> were false [190, pp. 188-189]

In the course of its reply, the *Tractatus Longior* records a third objection
making the above Tarskian assumption: a consequence where all terms are
held fixed must be a material one.[17] The objection, however, is ambiguous,
depending on what is meant by 'material consequence'. The first reading
assumes that such a consequence is material in the sense that its antecedent
is impossible, or consequent necessary. This is a modalized variant on Tarski's
claim above, and accords with the examples of material consequence given
in Ockham's *Summa Logicae*.[18] The second version assumes the consequence
is material in the sense that precisely because of the terms in it, application
of a rule that would not normally be admissible becomes so in that specific
context. This understanding of material consequence is that present in Simon
of Faversham [184, q. 36], and a clear statement of the view is found in
Ockham's late *Elementarium Logicae*:

> A material consequence is [one] which does not hold by virtue of

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[17][202, p. 86.4-9]. The main differences between Tarski's remark and the objection
to Burley are two: first, the determination of which parts are fixed takes place at the
level of the individual consequence for Burley's objector, but of the language for Tarski;
second, Burley's notion of following requires not merely the truth [falsity] of the antecedent
[consequent], but its being necessarily so.

[18][207, III-3. 1, p. 589]. Cf. [76, p. 7, par. 18;].

the mode of argument, but thanks to the terms it is composed from. In this way, 'an animal debates, therefore a man debates' follows, but because the predicate 'debates' cannot accord to any animal besides man.[19]

Ockham's example here is structurally analogous to the objection found in Burley, where the failure of the consequence with the predicate changed is used to press the non-formal character of the initial consequence. It seems likely to me that the objection as Burley construes it is working with an understanding of formal consequence closer to that of Faversham and the *Elementarium* than that of Tarski. Burley replies:

> For a consequence to hold in virtue of its terms is twofold: either because it holds materially by reason of its terms, or because it holds formally by reason of its terms, that is, from the formal measure (*ratione formali*) of the terms. I say then, that a consequence can be formal by reason of its terms, and this if it holds *per se* by reason of its terms. If, however, it holds by reason of its terms accidentally, then it is not formal [202, p. 173].

That Burley rejects Tarski's conclusion shows he rejects the reduction of formal consequence to a consequence's holding good under all substitutions for terms, even in the nuanced form one finds in Bolzano.[20]

More importantly, the above provides a way of relating the formal/material

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[19][204, VI. 4, p. 163]. The editors of the volume regard Ockham's authorship as doubtful. But the reasons for doubting its authenticity are not strong, mostly based on perceived discrepancies between the views in the text and those expressed in Ockham's *SL*. The external evidence in favor of authenticity is stronger and more concrete. For the case for Ockham's authorship of the work, see [23].

[20]*Pace* [85, pp. 16-17].

division to Burley's earlier natural/accidental division. By a consequence holding *per se*, Burley means the same as one where the meaning of the consequent is contained in the antecedent.[21] This is just the containment criterion found in his definition of natural consequence.[22] In contrast, the above response states a consequence holding in virtue of its terms accidentally is not formal. And as the context makes clear, Burley here intends that such a consequence be understood as a material one.

Thus, for Burley we have 1) formal consequences holding in virtue of their whole complex, including conversions and syllogisms; 2) formal consequences which hold in virtue of their terms, such as 'if a man is an animal, a man is a substance'; 3) accidental consequences holding in virtue of their terms, such as 'God exists is true, therefore God exists is necessary'; and another class of accidental consequences including *ex impossibili quodlibet* and *necessarium ad quodlibet* [77, pp. 128-129, par. 70]. The above types are listed in order of their strength, with consequences belonging to the first class being the strongest, and those of the last class, the weakest. The goodness of consequences holding by their structure is immediate and necessary. Consequences holding by virtue of their terms are enthymematic consequences, which 'have to be reduced to a syllogism' [77, p. 142, par. 117]. This class may be further divided into consequences holding by virtue of n terms, for any n. The third are consequences holding good by a restricted case of a rule that is not normally good, but is allowed in the given context: 'God exists is true, therefore God exists is necessary' holds as an accidental con-

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[21][77, p. 158, par. 160].

[22]Cf. [98, I, d. 11, q. 2].

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sequence because in the context of the antecedent, the term 'true' supposits for the same as what the term 'necessary' supposits for, i.e. God's being, thus permitting an otherwise impermissible substitution.[23] The final sort hold strictly by the extrinsic topic *from the less*. The first two of the above kinds are called natural consequences, and are said to hold by an intrinsic topic; the latter two, material consequences holding by an extrinsic topic.[24] Not all consequences holding in virtue of their terms are formal: there are accidental consequences which also hold in virtue of their terms.

5.3 Burley's place in the development of consequence

5.3.1 Burley and the Boethian tradition

Burley takes the language of topics (*loci*), used to ground his distinction between natural and accidental consequences, from Aristotle's *Topics*, as mediated by Boethius' *On Differential Topics*. Following the Aristotelian commentator Themistius, Boethius divides topics into intrinsic, extrinsic and middle topics. By the later 13th century, little emphasis is placed on middle topics: Radulphus Brito's commentary on the second book of Boethius' work (c. 1300), for instance, only devotes two of its twenty three questions to middle topics, with one of these concerning whether the category is a sensible

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[23]Burley's point is lost in the most straightforward English reading of *Deum esse est verum, ergo deum esse est necessarium*. i.e. that where *Deum esse* is taken for the sentence 'God exists'. To preserve Burley's point, one might better translate the infinitive clause as a gerund, as in 'God's being is true, therefore God's being is necessary'.

[24]Cf. [133, p. 130].

one [29].

According to the definition Boethius takes over from Cicero, a topic is the seat of an argument (*sedes argumenti*), and an argument an account making a doubtful matter sure (*ratio rei dubiae faciens fidem*) [25, p. 1048]. The term 'topic' can refer to either what Boethius calls a *maximal proposition*; or what such a proposition concerns, called the *difference* of such a proposition. A maximal proposition is one not in need of further justification, e.g. 'equals added to equals are equal'; the difference of such a proposition, the notion in the maximal proposition from which the deduction of the desired conclusion proceeds, e.g. equality.

The most important use Burley makes of extrinsic topics is in his justification of the rules *from the impossible anything follows (ex impossibili quodlibet)* and *the necessary follows from anything (necessarium ex quolibet)*. The former is justified by the extrinsic topic *from the lesser (a minore)*, on the grounds that if the impossible holds, then given it is less likely that the impossible holds than another proposition, that other proposition can be inferred.[25]

By the time of Burley, the topical apparatus grounding his distinction between natural and accidental consequences is both expanded and simplified in different ways. It is simplified by the straightforward identification of maximal propositions with rules: 'A maximal proposition is nothing but a rule whereby a consequence holds' [202, p. 76]. It is expanded by the claim that such a rule need not be a dialectical one - the traditional purview of

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[25][77, pp. 128-129, par. 70]; [202, p. 61]. Cf. [207, III-3. 38, pp. 727-731]. The *Tractatus Brevior* notes that this places some restrictions on the rule itself: for instance, the more impossible cannot be inferred from the less impossible [202, pp. 248.19-249.3].

topical argument - but may be more broadly logical [202, p. 76]; and by the admission of an indefinite number of differences of maximal propositions, rather than the traditional twenty-five or so admitted by Themistius and Boethius. For Burley,

> Not every maximal proposition arises from a difference of a maximal [proposition] known to us. For many maximal propositions are necessary, and still do not have names imposed on the differences of these maximal [propositions] [202, pp. 76-77].

Where today it is common to construct a logical calculus out of a minimal number of rules, defining familiar connectives in terms of others in order to achieve maximal economy,[26] Burley's theory admits an indefinitely large number of topical rules, with only some concern for their hierarchical organization.

5.3.2 Burley's work among the earliest treatises on consequences

Burley's *de consequentiis* is among the three earliest extant treatises on consequences. The other two are anonymous, and have been edited by N. J. Green Pedersen: one in a 1302 London manuscript, the other housed in Paris.[27]

A strong case can be made that Burley knew the text in London, BL, Royal 12 F XIX, ff. 111ra-112rb early on. The ms. containing the only ex-

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[26]E.g. disjunction in terms of conjunction and negation, or all classical connectives in terms of the Sheffer stroke. For discussion, see [145].

[27][76]. Translations of these and Burley's treatise are found in [3].

tant copy of the anonymous work also contains a copy of Burley's work at ff. 116ra-122rb. More conspicuous is Burley's treatment of an example found in the anonymous London treatise. In discussing the rule that in a good consequence, the opposite of the consequent cannot stand with the antecedent, the London treatise makes exception for consequences whose antecedents include opposites, and uses the proposition 'no time is' as an example of a statement including opposites. It then gives the following argument:

> If no time is, it is not night, and if it is not night, it is day; and if it is day, some time is. Therefore, if no time is, some time is [76, p. 7, par. 18].

Burley visits the same example in his discussion of the rule *from the first to the last* (i.e. transitivity). He writes:

> if one argues 'if no time is, it is not day; and if it is not day, and some time is, it is night; and if it is night, some time is; therefore *from the first to the last*: if no time is, some time is' - this consequence does not hold from the first to the last, since the consequent of the first conditional is 'it is not day', and the antecedent of the second conditional is the whole 'it is not day, and some time is' [77, pp. 114-115, par 8].

Where the anonymous treatise uses the example to prompt an exception to a standard rule, Burley diffuses the example by arguing that it rests on an equivocation.

Though Burley's *de consequentiis* has parallels with the treatise of Paris, BN lat. 16130, these parallels are mostly common rules, which are insufficient

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to establish any dependency of Burley's text on it. Comparing their style and content, the Parisian treatise is more terse, and has a marked preference for the active voice. Its proofs tend to be more detailed, and the connection to supposition theory is more pronounced. The Parisian treatise shows a clear grasp of the difference between downward monotonicity and descent to singulars [76, p. 12, par. 2]; whether the London author grasped the difference is less clear. In both works, the treatments of upward/doward monotonicity and ascent/descent from singulars are parallel.

The anonymous Parisian *de consequentiis* does, however, contain a substantial parallel to the objection to rule 2.1 from the *Tractatus Longior* discussed above. In the former, we find:

> One should know this rule is invalid: *whatever entails the consequent entails the antecedent*, as is shown if it is so argued: if a risible [thing] is an ass, then a man is an ass; for here by the rule *whatever entails the consequent entails the antecedent*, one argues that 'a risible [thing] is an ass' entails 'an animal is an ass', and thus 'a risible is an ass' also entails 'a man is an ass'. And it is so argued by this rule: *whatever entails the consequent entails the antecedent* since 'a risible [thing] is an ass' entails the consequent 'an animal is an ass' [76, p. 16, par. 21].

In Burley's treatise, we find the terms transposed and the common term '*asinus*' replaced with Brunellus, a proper name for a donkey. In both texts, what is resisted is the inference from 'an ass [Brunellus] is risible' to 'an ass [Brunellus] is a man'. Burley himself allows for a sense in which the inference

is formally good - namely, by reason of three terms. The anonymous text, with the objection, denies this, both resisting the inference from a proper accident to its bearer in contexts where *per impossibile*, the proper accident is predicated of something different from its standard bearer.[28] Neither text subsumes the inference under the rule *ex impossibili quodlibet*. From this, it seems likely that the author of the anonymous Parisian treatise was also a source for the objection as it appears in Burley.

The most fruitful comparison with both treatises comes in their approach to suppositional descent, and the corresponding difficulties brought about regarding existential import. None of these treatises adopt the approach to existential import - straightforwardly found in John Buridan, and attributed by Stephen Read to Aristotle himself[29] - according to which affirmative propositions have existential import while negative ones lack it. Rather, whether a proposition presupposes the existence of its supposita is dependent on the mode of predication, and hence on the mode of inherence exhibited by the things named by the terms. *Per se* predication is that wherein a constitutive property is predicated of what is constituted by it, as in 'a man is an animal'; *Per accidens* predication, either the predication of an accident of a substance, as in 'a man is white' or 'a man is risible'; or of some quality, be it essential or accidental, itself of an accident, as in 'to run is to move'.[30] In one medieval approach, found in both Simon of Faversham and Duns Scotus, *per se* predication does not presuppose existential import, but *per accidens*

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[28]Cf. [97, q. 11, par. 19].

[29][89, q. I. 38], [110], [161].

[30]This conflation of accidental predication with *per se* predication of an accident is present in both anonymous texts. See [76, pp. 10-11, par. 35-36; 25, par. 66].

does.[31] From these approaches to predication, corresponding notions of ascent and descent were developed. A descent from 'man' to 'white man', for instance, was called *from a superior to inferior per accidens*; from 'animal' to 'man', *from a superior to inferior per se.*

Both anonymous authors require all propositions where the same is predicated of itself to come out true, including predications involving accidents, such as 'a white man is a white man' and those involving impossible objects, such as 'a chimera is a chimera' [76, p. 8, par. 23; p. 11, par. 36-37; p. 18, par. 31-32]. Because these propositions are given separate treatment, many ascents and descents to and from such propositions are blocked: one cannot, for instance, ascend from 'a white man is a white man' to 'a white man is a man', because the latter proposition is existence implying while the former is not.

Relative to these treatises, Burley's principal contribution seems to have been the rejection of special treatment for statements predicating the same of itself, and with it a more uniform application of ascent and descent rules in *per accidens* predications.[32] On the resulting approach, *per se* ascent and descent work as follows: A universal affirmative categorical proposition is downwardly monotonic in its first term and upwardly monotonic in its second [202, p. 211.16-20]; the opposite is the case for a particular negative categorical; a particular affirmative is upwardly monotonic in both terms; a universal negative, downwardly monotonic in both. Descent *per accidens* (e.g. from 'no man is an animal' to 'no white man is an animal') holds where

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[31]See [184, q. I. 56]; [96, I. qq. 5-8. par. 49, 74]; [97, q. 11, par. 19]. More recently, this approach has been suggested by [16].

[32]See [77, pp. 116-117, par. 19-20; 134, par. 95; p. 158, par. 160.].

descent *per se* does in negatives [202, pp. 209.35-210.10]. Ascent from a *per accidens* inferior to its *per se* superior holds where *per se* ascent does in affirmatives [77, pp. 116-117, par. 20]. Excepting Burley's treatment of propositions predicating the same of itself, the above analysis appears to have been lifted from the London anonymous treatise.[33]

Lastly, Burley recognizes that where a class or object descended to can be empty, *per se* descent for affirmatives can fail. But Burley's practice on this point is inconsistent.[34] Instead, Burley generally employs these descents freely, ignoring existential complications [202, pp. 23.26, 26.26, 31.21, 67.19, 67.30, 85.16, 85.26, 211.27-28].

5.3.3   Relating Burley's work to that of Ockham and Buridan

The place of consequences within Burley's vision of logic

Unlike Burley's, Ockham's treatment of consequence does not begin with general rules, but with rules for suppositional ascent and descent for the various qualities and quantities of assertoric, then modal and other kinds of propositions. Ockham does not discuss general rules for consequences until the thirty-eighth chapter of his work,[35] and each of the rules mentioned therein is already explicit in Burley's much earlier *de consequentiis*. Thus, as with Burley's earlier *de consequentiis*, general consequences are not yet brought

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[33]Cf. [76, pp. 10-11, par. 35-37], [77, pp. 116-117, par. 19-20].
[34]See esp. [202, pp. 61.4, 85.16; 85.4, 211.27-28]. Cf. [138].
[35]This was originally the final chapter of the section on consequences. See [207, pp. 41*-43*].

to the fore;[36] and as with Burley's *Tractatus Brevior*, the differentiation of consequence from supposition theory is less pronounced in Ockham.

Furthermore, the divisions and order of Ockham's treatise follow those understood to govern Aristotle's *Organon*. The first two parts of Ockham's treatise, on terms and propositions, address simple concepts and judgment, the subject matters of Aristotle's *Categories* and *On Interpretation*. The third part, broadly concerned with reasoning, discusses the subjects of Aristotle's *logica nova* texts: the forms of reasoning (*Prior Analytics*) and their implementation in demonstrative (*Posterior Analytics*), dialectical (*Topics*), and fallacious argument (*On Sophistical Refutations*).[37] Within this setup, Ockham's treatise on consequences occupies the place of Aristotle's *Topics*. Thus, though the content of Ockham's thought is often radical, the structure wherein it is presented is a deliberately traditional one. In particular, Ockham does not yet conceive consequences as encompassing traditional syllogistic inference.

By contrast, Burley already includes syllogistic under consequence in his *de consequentiis*.[38] And both the intended structure of Burley's earlier and the actual structure of his later *De Puritate* betray a different understanding than Ockham's of the place of consequences in the Aristotelian curriculum.

According to the prologue of the *Tractatus Brevior*, Burley's plan was to treat in sequence: 1) common rules for the remainder of the work; 2) sophisms, 3) *obligationes*, and 4) demonstration [202, p. 199]. The preface

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[36]Burley does not discuss general rules until the last section of the treatise, beginning at par. 145.

[37]Cf. [194, prol.]

[38][77, pp. 131-132, par. 82-85]. Also [202, p. 219.19-32].

of the earlier treatise, and the intended ordering of its parts, betray a basic concern with the form of reasoning, which is then examined in the different contexts wherein it is utilized, e.g. demonstration corresponding to the traditional subject of the *Posterior Analytics*; sophistry, to that of the *Sophistical Refutations*.[39] As such, the assimilation found in Burley's earlier *De Puritate* is not to Aristotle's *Topics*, but to the *Prior Analytics*: its ambition, to provide an account of the forms of reasoning - one that, instead of reducing all reasoning to syllogistic,[40] takes up syllogistic as part of a broader work, one centered on consequence and expanded to address syncategorematic functions found in hypotheticals other than conditionals. Burley thus appears to be the first logician to attempt a unified account of consequence including syllogistic as a proper part. It is this vision in Burley's treatise, rather than Ockham's, which is adopted later in the structure of Buridan's *Treatise on Consequences.*

This vision remains in the later version of the treatise, albeit with an important revision. The intended inclusion of a treatise on supposition in the first part suggests Burley early regarded supposition theory as part of a broader theory of inference, perhaps as supplementing the kinds of inference-justifying rules one finds in topical treatises. This ordering agrees with that found in the *Summulae* of Peter of Spain.[41] In Burley's later treatise, by

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[39]Cf. [194, prol.]. The ordering of the *logica nova* materials suggested by the prologue of Burley's earlier treatise, with the *Prior* and *Posterior Analytics* separated by the *Elenchi* and *Topics* (under which Ockham discusses the *arte obligatoria*), is less common than that suggested by Ockham's. That it was nevertheless used is clear from some mss. recorded in the *Aristoteles Latinus* index, such as Metz, *Bib. Mun.* 508; Arras, Bib. Mun. 362 (451); Chambery, Bib. Mun. 27. The same separation is found in the ordering of the treatises of Buridan's *Summulae.*

[40]This reduction remains present in Buridan's *Summulae.* See [91, 6.1.5, pp. 398-400].

[41]Cf. [114].

contrast, the part on supposition begins with the words 'having laid down the signification of incomplex terms, in this tract I intend to examine certain properties of terms which only belong to them according to their being parts of a proposition' [202, p. 1.3-6]. Now, the signification of incomplex terms was the standard topic associated with the *Categories*.[42] The mention of the proposition makes clear the association of the treatise on supposition with the *On Interpretation*. Thus, it seems that by the time of the later treatise, Burley thought it more appropriate to treat supposition in closer connection with the content of the *On Interpretation*, in the broader context of subjects discussed under Aristotle's old logic, rather than in closer connection with discussions of inference traditionally treated under Aristotle's new logic. This relocation provides us with a reason why Burley may have abandoned the *Tractatus Brevior* independent of a need to respond to Ockham's logic: he sought to reorganize his materials so as to reflect this new understanding of the place of supposition theory within logic, a reorganization which then required a rethinking, and disentangling, of supposition theory from the theory of consequence, which in turn required rewriting much of his old material to reflect this new organization. This relocation of the treatise on supposition is the one later followed by the *Summulae* of John Buridan.

Disagreements over valid consequences

Burley's disagreements with Ockham are well-known, with the presence or absence of barbs towards Ockham playing a role in dating the works of Burley's corpus [142, 199]. Ockham borrowed liberally from Burley's logic [30],

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[42]See [146, p. 65].

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and the parallels between consequences countenanced in Ockham's *Summa Logicae* and Burley's earlier treatises are too extensive to be enumerated.[43] Though Burley and Ockham's underlying accounts of supposition differ [201], I've not found substantial disagreements in the consequences they accept or reject.[44]

Buridan's engagement with Burley is less known, but extensive. In his ontology, Buridan adopts Burley's reduction of the real Aristotelian categories to three - namely, substance, quality, and quantity - albeit understanding the reduction in a manner different from that in Burley.[45] In logic, Buridan had direct knowledge at least of the *Tractatus Brevior*: rules 3, 4, and 5 of Buridan's *Treatise on Consequences* are rules 3, 2, and 1.1/1.2 of the *Tractatus Brevior*; Buridan's sixth rule is a restricted instance of Burley's rule 2.1; and Buridan references the content of the eighth rule of Burley's shorter treatise in the eighth rule of the first book of his own treatise on consequences.[46]

With Buridan, however, one finds a more substantial disagreement, over consequences involving sentential and term negation. Unlike Buridan's, Burley's treatment of term negation, does not normally deviate from his treatment of sentential negation. 'Socrates is non-white' and 'Socrates is not white' have the same truth conditions, with both being true when Socrates does not exist [202, pp. 57.17-58.12; 215.6-21; 216.15-18]. Burley's thus accepts the standard rules for obversion and contraposition, both from af-

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[43]The consequences of [207, III-3.38, pp. 727-731], for instance, can be found at [77, par. 1, 4, 9, 14-15, 71-72, and 86-88].

[44]Thus, Read regards Ockham's SL as a radical ontological sheep in a traditional logic's clothing. [170, p. 276].

[45]Cf. [111, pp. 57-59, 204, 269-270]; [166, p. 13]; [56]. See also [136, p. 439], [132].

[46]Cf. [202, p. 212.29-31]; [93, p. I. 8].

firmatives to negatives and *vice versa*, albeit with restrictions on the kinds of terms that can occur [77, pp. 129-131, par. 73-81]. This contrasts with the approach one later finds in Buridan, on which only the inference from an affirmative to its contraposited or obverted negative is good, but not conversely.[47]

5.4   Conclusion

The above reveals a wide array of achievements attributable to the *Doctor Planus et Perspicuus*, which help us better understand his place in the developments in the theory of consequence during his lifetime.

Burley's *De consequentiis* provides: the most expansive known treatment of consequences prior to Ockham's *Summa Logicae*; one of the earliest explicit taxonomies of kinds of consequences; and the earliest general subsumption of syllogistic under the banner of consequences, an inclusion never adopted in Ockham's works. Burley streamlines the treatment of consequences found in the earliest treatises by leaving aside the rule that a proposition predicating the same of itself must always be true; and by providing a parallel treatment of sentential and term negation, he preserves both affirmative-to-negative and negative-to-affirmative directions of conversion and contraposition.

In the shorter *De Puritate*, Burley provides the earliest known organization of warrants governing consequences into principal and derivative rules. In the same treatise, he makes explicit that identity statements may fail in

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[47][94, p. 85]. The root difference consists in their different treatments of the range of an infinite negation: Burley takes an infinite negation to range over both beings and non-beings; Buridan assumes it only ranges over existent entities.

cases where the subject does not exist; qualifies the range of the rule *from the impossible anything follows*, since the less impossible does not entail what is more impossible; and provides a brief treatment of consequences involving modalities. In the movement from the shorter to the longer *De Puritate*, we see the relocation of supposition theory from being treated between topics and syllogisms, to the context of treatments of the proposition, a place it retains in the later work of Buridan.

By the time of the longer treatise, Burley has greatly streamlined the treatment of principal and derivative rules first attempted in the *Tractatus Brevior*; and expanded the range of dialectical maxims used in topical arguments to an indefinite number of *logical* maxims, to be used in consequences. In the same treatise, Burley relates the contrast between natural and accidental consequences found in the *De consequentiis* to that between formal and material consequences, and further subdivides enthymematic formal consequence according to the number of terms they hold by. Burley provides a plurality of different understandings of 'follows', with differing levels of strength, grounded in the strength of the relations involved in the *significata* in the consequences; natural structural consequences when grounded in the meaning of the syncategorematic terms; natural enthymematic consequences, grounded in an intrinsic containment relation between the significates of its categorematic terms; and accidental consequences grounded on weaker relations.

There are, of course, a few failings in Burley's theory. The ambiguity between the different senses of *per accidens* ascent and descent likely contributed to the distinction between natural and accidental consequence falling

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out of favor. Though Burley recognizes both downward monotonicity and descent to particulars may fail because of complications involving existence, his practice ignores this insight. The subdivision of enthymematic formal consequences by number is underdeveloped, and without further clarification it is hard to see how some among these consequences should be distinguished from material consequences.

However, today's more pluralist logical environment, aiming to give expression to different kinds of relevant containment while still allowing for classical consequence, is much friendlier to Burley's project than it is to the reductive monism that followed in Buridan's reduction of all consequences to formal ones - a reduction mirrored in the classical approach to formal consequence that dominated the last century. As such, now provides an opportune environment for a revival of interest in Burley's logical work, while in turn provides inspiration for a better, more fruitful grounding of pluralism than that available at present. At the same time, an examination of the logic of Burley the realist provides much of interest even for understanding the nominalists opposed by - and indebted to - him.

Chapter 6

Conclusion

6.1 Introduction

Now is the appropriate time to place the genesis of Buridan's theory in its proper context.

The renaissance of scholarship on medieval logic broadly and John Buridan specifically took place against the backdrop of two movements. The earlier movement involved a renewed look at the multiplicity of medieval Catholic thought after the cooling down of the modernist controversy. The later movement involved not a recovery of the middle ages as a source of Catholic thought, but rather its reappropriation for an increasingly non-religious academic sector.

The first Vatican council brought about a greater unification of Roman Catholicism than had existed at any time prior to it. This was secured not only institutionally via its proclamations concerning the authority and infallibility of the papacy, but also intellectually via a renewed focus on Catholic

philosophy, in particular on Augustine and Aquinas, later encapsulated by Pope Leo XIII's encyclical *Aeterni Patris*.[1] The intellectual aspect of this unification reached its apex in Pius X's condemnation of modernism in the 1907 encyclical *Pascendi Domini Gregis*, in whose wake one should locate the Neo-Thomistic revival of the work of Father Garrigou-Lagrange and others.

In the security brought about by the early 20th-century condemnation of modernism, one finds a reexamination of the diversity of Catholic thought. Part of this was the *nouvelle theologie* of Congar, De Lubac, and others who effected a return to patristic and early medieval theology and biblical exegesis. But another part was the recovery of non-Thomistic strands of scholasticism, first in the work of Thomists like Gilson, but later in its own right. To this movement we would owe the founding of the Pontifical Institute of Medieval Studies at the University of Toronto, the Franciscan Institute at St Bonaventure University, and the journal *Franciscan Studies*. Through the work of Fathers Philotheus Boehner, Allan Wolter, and others, the last-mentioned institution and journal played a critical role especially in the recovery of the logical contributions of Duns Scotus, Walter Burley, and William of Ockham.

The recovery of medieval logic generally, and that Buridan in particular, is in great part the work of the later movement. Through the early 20th century, anthologies and overviews of medieval intellectual history focused largely on natural theology and metaphysics. This focus was understandable, since it was contributions to these areas that medieval thought was best known for.

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[1]This renewal still governs the historiography of medieval philosophy as a whole, considering that there is really no good reason to consider Augustine a medieval thinker apart from the understanding of the medieval first wrought by this revival.

However, with the ill-repute into which metaphysics fell in the first half of the 20th century and in which natural theology largely remains, it became necessary, rather than leaving the medieval period as an intellectual vacuum to be abhorred, to find some other manner of redeeming it in the light of the values and achievements of the philosophical work of the time. This movement coalesced with an organic interest in the history of logic, instanced in the work of Łukasciewicz and Kneale and Kneale, to bring about an interest in medieval theories of consequence, with the task of reinforcing the image of medieval theories as an imperfect herald of modern approaches to the topic. To this directive, we owe the early work of Moody and Broadie on the topic; the founding of the *Corpus philosophorum Danicorum*, the editions and studies in the *Cahiers de L'institut du Moyen Âge Grec et Latin*, and the work of Pinborg, Ebbesen, Green-Pedersen, and others at the University of Copenhagen; the critical editions of De Rijk, his founding of the journal *Vivarium*, and the establishment of the *Artistarium* series exploring the work of secular medieval arts masters; and the work of Kretzmann, Stump and others at Cornell University, culminating in the 1982 *Cambridge History of Later Medieval Philosophy*, which devoted considerably more space to discussion of logic than any prior survey of the medieval period, deliberately sacrificing space devoted to metaphysics and philosophical theology. The recovery of Buridan's logic formed the centerpiece of work on secular masters, rather than those belonging to religious orders; and the assimilation of Buridan's theory of consequence to modern model-theoretic approaches to the topic, the centerpiece of the reappropriation of the medieval period *sans* its metaphysical and theological outlook. If you are reading this book, then chances

are you owe a great debt to the shape this recovery took, even if you may doubt its starting premises.

With a quarter century passed since the release of the aforementioned *Cambridge History*, we are now in a better place to more exactly assess both Buridan's relation to modern model-theory and to his contemporaries, and the veracity of the theories motivating his rediscovery as a major figure of medieval thought.

We begin by relating Buridan's theory to its modern model-theoretic counterparts. From there, we summarize Buridan's place in the development of formal consequence, and its relation to the broader development of consequence up to Buridan. Lastly, we offer some remarks on how the definition of formal consequence arrived at where it is today.

6.2 From semantic consequence to medieval formal consequence

6.2.1 Contrasting Tarskian and Buridanian consequence

Buridan's definition of consequence is a deflationary one: 'a consequence is a hypothetical proposition composed of an antecedent and consequent' [93, p. I.3]. In addition, he offers a criterion for consequence according for which a consequence is good when it is impossible for things to be as the antecedent signifies without being as the consequent signifies. For Buridan, this criterion needs to be reformulated to fit the tense, modality, and quality of propositions present in an antecedent and consequent - for instance, the proposition, 'if

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Socrates was running, Socrates was moving' is good if is impossible for things to *have been* as the antecedent signifies, etc.

Buridan divides consequence into formal and material, with the latter divided into simple and as-of-now consequence. A formal consequence is one belonging to an equivalence class, determined by its syntactic structure, such that all consequences in that class are good. A material consequence is a good consequence not of this sort. A formal consequence is said to be good in virtue of its formal parts, i.e. its syncategoremata. And though consequences may be valid without being formally valid, a materially valid consequence is only made evident by its reduction to a formally valid one. A simple consequence is one for which Buridan's criterion for a good consequence holds without qualification; an as-of-now consequence, one for which it holds for a given time.

Many of the better-known formal developments since Tarski have brought about a rehabilitation of Buridanian themes. Work on tense, modality, and other intensional operators, for instance, has been the norm since Prior and Kripke; the acceptance of domain variation, since Kemeny. There are also approaches to formal consequence which, with Buridan, take the time of utterance into account, though these remain non-standard.[2] In other ways, more recent approaches to consequence have moved away from aspects common to both Buridan and Tarski. For instance, in contrast with classical consequence today, neither Buridanian nor Tarskian consequence is schematic, and neither interprets the non-formal parts of the languages to which it applies arbitrarily.

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[2]See those systems drawn on in [48, 50].

Modern approaches to formal consequence depart from their medieval predecessors in their identification of formal and logical consequence, and their identification of consequence as the subject matter of logic as such. For Buridan and the medievals generally, there are logical consequences which are not formal, including induction and enthymematic consequences. And consequence forms only a small and relatively new part of what medieval logic studies. For Ockham 'consequence' is restricted to non-syllogistic argument. For Buridan, it encompasses both syllogistic and other kinds of argument.

6.2.2 Medieval consequence as a key to understanding semantic consequence today

Against the medieval backdrop, the most distinctive aspects of formal consequence today are the concepts of *language* and *function* it employs.

Formal *consequence* is now defined over a formal *language*: typically, a collection of syntactic strings divided into formal and non-formal primitives, with the referent of each formal element fixed to a semantic function, and that of each non-formal element varied arbitrarily; from which a countably infinite collection of formulas is recursively defined.

The concept of *function* herein employed is a mathematical one: an n-ary function f is a mapping from n-tuples in some collection of n-tuples, called the *domain* of the function to an element in some other, possibly distinct collection of elements, called its *range*. Such a mapping may be *total*, mapping each element in the domain to an element in the range; *partial*, i.e. only from some elements in the domain; *many-to-one*, mapping some distinct

elements in the domain to the same element in the range; or *one-to-one*, mapping each element in the domain to a distinct element in the range. But no function is one-to-many.

The formal consequence relations for classical logic, its extensions, and rivals all operate on a functional understanding of language. In practice, a phrase like 'the mother of' is treated as a function from the domain of individuals to itself, mapping each person to his or her mother. n-place predicate symbols are treated as functions from D^n, the nth Cartesian product of the domain of a model, to a set of truth values. Names in classical logic are assimilated to functions of arity 0 to the domain of a model; atomic propositions, to predicates of arity 0.

This functional understanding of language leaves a host of dualisms in its wake, which shape the landscape of philosophical logic today: between language and world; object language and metalanguage; names and predicates; use and mention, and between the logical and non-logical parts of a language. Each of these, in turn, brings with it one of modern logic's characteristic *insolubilia*: problems with self-referential terms; Tarski's hierarchy of languages, with its corresponding hierarchy of truth predicates; Frege's 'problem of the concept *horse*,' concerning the relation between concept and object; over the admissibility of intensional operators into formal languages; and the problem of demarcating the logical from non-logical constants.

Modern approaches to formal consequence thus build on a semantic base that assimilates meaning to function, and eliminates the medieval distinction between signification and supposition, i.e. that between the *meaning* of a lexical item, and its function within a given sentence. The medieval distinc-

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tion between the syncategorematic and categorematic is thereby transformed from a local one between sentential roles into a global one between linguistic types, generating the demarcation problem for logical constants. Because no function is one-to-many, formal languages are unequipped to distinguish between different kinds of supposition a term might have; nor to deal with problems caused by linguistic ambiguity, which were often at the root of medieval work on fallacies, *sophismata*, and *insolubilia*.

6.3 From formal and material to natural and accidental consequence

6.3.1 Summary

John Buridan provides the earliest account of formal consequence in terms of a substitution criterion. On Buridan's criterion, a consequence is formal if and only if it is good for all uniform substitutions on its categorematic terms. This is the ground for its claim to be a predecessor of Tarski's definition, which transforms Buridan's uniform substitution criterion on terms into one over models, i.e. over orderings of objects satisfying sentential functions obtained from one's initial sentences by substituting non-logical constants with like variables. Though the use of such substitution techniques to check validity goes back to Aristotle, Buridan appears to be the earliest to have seen in it both a necessary and sufficient condition for formal consequence.

Buridan's account is imperfect in several respects. Its notion of consequence undergirds a solution to the Liar paradox, but remains susceptible to

the Pseudo-Scotus paradox. Buridan subdivides material consequence into simple and as-of now consequence, but Buridan's stated criteria for these cannot distinguish simple material consequences from formal ones, or as-of-now consequences from purely invalid ones. Buridan's definitions of simple and as-of-now consequence are later improved upon by Pseudo-Scotus, who distinguishes them by the way they are reduced to a formal consequence: simple consequence, by adding a necessary proposition to the antecedent; as-of-now consequence, by adding a contingent one. The discovery of the Pseudo-Scotus paradox, however, seems to have led to despair over the possibility of providing a simple criterion for valid consequence, and the proliferation of more *ad hoc* criteria in later medieval logic.

Though Buridan admits good consequences that are not formal, any such consequence which is also *evident* is only made so by its reduction to a formal one. Not every material consequence is so reducible: examples and induction are not. For Buridan, the category 'consequence' is expansive enough to include both syllogistic and non-syllogistic consequences; formal consequence is the source from which any good consequence is evident; and material consequences are either reducible to formal ones, or lacking in evidence. The resulting picture is what one may call *reductive consequential monism*: there are no ontological differences present in the different ways of following, but only differences in evidence; as far as ontology is concerned, there is only one basic kind of consequence to which others are reducible. And those which are not so reducible have a secondary, imperfect epistemic status.

Buridan adopts the subsumption of syllogistic under consequence from Burley. This treatment, common to Burley and Buridan, differs from that of

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Ockham, who takes the domain of consequences to be that formerly allocated to topical argument.

The division of consequences into formal and material varieties goes back to Simon of Faversham, and is also present, albeit not as an explicit taxonomy, in the anonymous London *de consequentiis.* The earliest formal division between formal and material consequence is in William of Ockham's *Summa Logicae*, though Ockham's division differs from that found in Buridan. For Buridan, a consequence is formal if it is good for all uniform substitutions on categorematic terms. Ockham countenances as formal those which, in addition, are reducible to those which are formal in Buridan's sense.

In Ockham, there remains some distinction between different ways in which something may follow, albeit attenuated from that found earlier in Burley. Ockham calls 'material' those consequences which hold 'precisely by reason of the terms', and countenances the consequences *from the impossible anything follows* and *the necessary follows from anything* as of this type. In Buridan, by contrast, even this distinction is erased: Buridan appropriates the language of the formal-material division for his own purposes, and builds the validity of Ockham's material consequences into the criterion for consequence as such: since it is impossible, for instance, for things to be as an impossible antecedent signifies, it is also impossible for things to be as it signifies without things being as its consequence signifies. Buridan further takes the consequence to hold formally for explicit contradictions, justifying it by disjunctive syllogism, in the same manner C. I. Lewis would later in his account of strict implication.[3]

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[3]This method for proving anything from an explicit contradiction is first reported by

Ockham distinguishes Buridan's formal, structural consequences from those enthymematic formal consequences which Buridan classifies as material by appealing to topics: for Ockham, a structural consequence holds by an extrinsic topic; an enthymematic consequence holds by an intrinsic topic immediately, and mediately by an extrinsic topic. By 'topic', Ockham means what Boethius means by 'maximal proposition': a rule licensing an inference from a premise or premises to a conclusion.[4] An intrinsic topic is a premise added to an enthymematic argument to make it a formal one. It is intrinsic in that it governs things mentioned in the stated premise directly. An extrinsic topic for Ockham turns out to be a rule stated in terms of second intensions, under which the objects mentioned in the premises of a given argument fall accidentally, e.g. in virtue of their being given a certain supposition in a proposition, etc.

Prior to the distinction between formal and material consequence which gains currency in Ockham's work, one finds a distinction between natural and accidental consequence implicit in Boethius, and present later in William of Sherwood, Scotus, and in Burley's *de consequentiis*. Burley, following Scotus, grounds the distinction in one between intrinsic and extrinsic topics, though by these he means something different than what Ockham does. For Burley, a natural consequence, which holds through an intrinsic topic, is one which satisfies a containment criterion, where 'the antecedent includes the consequent' [202, p. 61.6-10]. An accidental consequence, which holds

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Alexander Neckham and attributed to William of Soissons. See [134].

[4]The straightforward identification of maximal propositions with rules is also found in Burley's longer version of the *De Puritate*, though Burley's identification likely antedates that of Ockham.

by an extrinsic topic, is one holding by some extrinsic relation between the things named therein. Examples include consequences from the positing of one contrary to the denial of the other, and from the positing of a species to its *proprium*, its inseparable attribute. Burley classifies the consequence *from the impossible anything follows* as an accidental consequence holding by the topic *from the less*.

Burley identifies his natural consequences with formal consequences in Ockham's sense. Against an objection resembling Buridan's position, Burley insists that not all formal consequences are structural consequences. In this much, he is in agreement with Ockham. But against Ockham, he does not classify *ex impossibili quodlibet* as a material/accidental consequence holding by virtue of its terms, but as one justified by appeal to the extrinsic topic *from the less*.

6.3.2 Analysis

From Burley and his predecessors to Ockham, and then from Ockham to Buridan, we see two important shifts, each brought about by the appropriation of the language of a prior position for alternative ends. The first terminological shift is in the understanding of a topic; the second, in the understanding of consequence.

In Boethius, a topic represents a real aspect of a thing from which something about it or another is inferred. That is, a topic is the difference of a maximal proposition. From this, an analogous use of the term arises, i.e. to refer to the maximal proposition itself.

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In Burley, the focus on the differences of such propositions is diminished, and a topic is more often simply identified with a rule licensing an inference. The distinction between intrinsic and extrinsic topics, however, continues to be made along traditional lines.

At the time of Burley's *de consequentiis* and the earliest anonymous treatises, the development of supposition theory brought about a great simplification in the number of topics actually appealed to, with a vast number of inferences justified by rules like *from an undistributed inferior to superior*, *from a distributed superior to a distributed inferior*, etc. That is, appeals to qualitative distinctions grounded in aspects of objects gives way to appeals to scope distinctions grounded in supposition of terms.

In Ockham, the distinction between intrinsic and extrinsic topic is no longer drawn along traditional lines: rather, an intrinsic topic is simply a premise added to an antedecent to make it a structurally formal one; and an extrinsic topic is a rule pertaining to second intensions, licensing the movement from antecedent to consequent in a particular kind of inference.

In Buridan, the language of topics disappears altogether from treatises on consequences, as does the distinction between intrinsic and extrinsic topics. In its stead, we see Buridan arguing that extrinsic topics are reducible to formal ones in exactly the same sense as arguments employing intrinsic topics:

> If by the topic *from contraries* we argue so: 'A is white; therefore A is not black', it is still the case that a syllogism or formal consequence is effected by the addition of the proposition 'no white is black'. And so the other dialectical topics, if they are not formalized, do not conclude on account of form, but can be reduced to a

form by additions [92, sec. 6.6].

This disappearance is closely related to the second shift, that from natural/accidental to formal/material consequence.

In Burley's work, the division of consequences into natural and accidental is a subdivision of simple consequence. The natural/accidental division is clearly intended as an ontological one, grounded in a distinction between intrinsic and extrinsic properties. However, the distinctions between *per se* and *per accidens* predication, on the one hand, and *per se* and *per accidens* ascent and descent, on the other, were poorly worked out in the earliest versions of the theory.

Ockham lists the two divisions, the one between formal and material, the other between simple and as-of-now, without subordinating either to the other. His example of a formal consequence holding by an intrinsic topic in the *Summa Logicae* - 'Socrates does not run, therefore a man does not run' - is deliberately chosen to provide an example of a formal consequence which is nevertheless as-of-now.[5] More importantly, the distinction between these in terms of topics gives way to a classification according to whether such consequences appeal to universal rules or terms: structural formal consequences hold by an extrinsic rule; enthymematic formal consequence, mediately by

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[5][207, III-3. 1, p. 588]. It is unlikely that Burley would have agreed with Ockham on this classification. For Burley, a simple consequence appears to be one where 1) there is a *per se* relation between the antecedent and the consequent, and 2) in every situation where the antecedent holds, the consequent holds. For Ockham, by contrast, such a consequence additionally requires that the predications in its antecedent and consequent themselves must be intrinsic; and 2) that the antecedent itself holds in every situation. Burley would accept the consequence as formal, given the containment criterion is satisfied for the terms 'Socrates' and 'man' (names of individuals were taken to imply their natures). Given this disagreement, the number of simple formal consequences seems to be greatly curtailed in Ockham's logic.

a structural rule and immediately by one pertaining to the terms, i.e. by an added premise; material consequences, by the terms alone. In Ockham, we thus see the denial of a distinction between different *ways* of following, relative to different ontological relations, and with it a co-opting of the topical vocabulary standardly used to express such a distinction. Given that Ockham identifies formal consequences with those reducible to formal consequences in Buridan's sense, it is only a natural step from there to Buridan's understanding of formal consequence; and with it, to the identification of the division between simple and as-of-now consequence as one subordinate to material consequence, between two different *ways* of reducing a material consequence to a formal one, i.e. by the addition of a necessary or contingent premise. Lastly, in Buridan, we see even the vestigial diminished status Ockham affords to *ex impossibili quodlibet* erased: for Buridan, the consequence is good not by appeal to an extrinsic topic, not by its terms, but simply by the definition of a good consequence.

6.4 There and back again

The triumph of Buridan's definition of formal consequence was the cumulative result of rapid changes in the theory of topical argument from the late thirteenth to mid-fourteenth century. At the same time, the unification initially wrought by the medieval subsumption of syllogistic and topical argument under the banner of consequence was intimately tied to the development of the notion of formal consequence as consequence *par excellence*, that to which any consequence worth its salt would be reducible.

What was this a triumph *for*? Doing logic without appealing to qualitative ontological distinctions, the norm in the topical framework consequences grew out of and eventually replaced. What was it a triumph *over*? Attempts to ground logic, however loosely, in ontology.

What was *sacrificed* in this achievement? Logic's relevance; its multifacetedness; its groundedness.

The loss of relevance occurs in two senses. The first, in the introduction of various irrelevant entailments such as *ex impossibili quodlibet*, and their gradual movement from the periphery to the center of the understanding of consequence.

The second was the loss of relevance to the process of *discovery*. The streamlining of the maximal propositions actually appealed to in logical disputation, wrought via the introduction and gradual improvement of the theory of supposition, was a remarkable achievement. But as with their modern analogues in metatheoretical appeals to set-theoretic containment, these appeals do a much poorer job of providing rules for the development of arguments than the qualitative topical arguments they replaced: for instance, the rule *from the first to the last*, i.e. transitivity, is simply much thinner, and accordingly less useful, than a rule like *from the positing of the effect of an efficient cause follows the positing of the cause*. In this way, the development of the medieval theory of consequence served as the immediate backdrop for the various early modern attempts, such as those of Bacon and Descartes, to develop a method for discovery; for the location of such endeavors, given the tarnished name of 'logic', under the banner of not logic, but epistemology; and for the development of rival approaches, from which

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arises the distinction principally between rationalists and empiricists, but many others as well. Without the transformation of the topical theory into the more supposition-oriented theory of consequences, characteristic early modern complaints about logic's uselessness and inapplicability for discovery would not have found footing.

The loss of multifacetedness occurs through the replacement of a distinction between two different ontologically grounded ways of following with a purely epistemic distinction. The earlier account is based on two different ways in which something may be predicated of another, i.e. intrinsically or accidentally, and thus has a basis in Aristotle's *Categories*, and is reflected in the distinction between analytics (which, as the name implies, is concerned with breaking down intrinsic components) and dialectic. The account of consequence Buridan advances is a reductively monistic one: all good consequences are reducible to formal ones. Here, one is not far from the identification of consequence with formal consequence as such, and from the identification of the latter with logical consequence. On the one hand, this brings with it a great unification of the previously disparate realms of analytics and dialectic. On the other, its doing so brought with it a forgetfulness of the question of what the different spheres of logic are concerned with, in favor of an understanding of logic as applicable to anything whatsoever.[6]

The loss of groundedness corresponds to this loss in multifacetedness.

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[6]Only with Kant's discovery of the synthetic *a priori* do we arrive at anything like a rehabilitation of the traditional domain of topical argument. However, Kant limits the domain of such arguments to bar them any role in metaphysics; and the distinction was strongly rejected by the logicist tradition at the start of early analytic philosophy. Though with [120], the analytic tradition has found a role even for *a posteriori* necessity, it has yet to appropriate, or even to grapple much with, Kant's earlier distinction, let alone to examine its affinity with the basic idea behind Boethian topical argument.

Where the earlier approach to consequences attempted a basic grasp of logic's nature, subject, and divisions, the later approach to consequences eschewed these concerns in favor of a more conventionalist approach to logical practice. Rules for consequences become those rules which are obvious to everybody, or which are derived from such rules in acceptable ways.

It is against the backdrop of this loss that the crisis in contemporary logic is given its sense. After the collapse of the formalist program and the fading out of the logicist program in the second quarter of the twentieth century, the ensuing widespread adoption of first-order classical logic was never provided with a solid, more than dogmatic foundation. From there, that foundation has been assailed with attacks on *reductio ad absurdum* proof by intuitionism; on *ex falso quodlibet* by the Lewis systems for strict implication; on non-relevant implication by the relevant logics of Belnap and others; on bivalence and excluded middle by many-valued systems and on non-contradiction by paraconsistent systems. Regardless of the strength of the objections, the foundation was weak enough that when the opposition circled about the walls and shouted their objections, the walls of the dogmatic consensus came crumbling down.

However, the loss of this consensus has not brought with it a serious attempt to understand what formal consequence is, what it is based on, or whether it constitutes a unified thing. Rather, the *de facto* successor to the dominance of the first-order, semantic approach to formal consequence has been the proliferation of a multitude of logics, employed seemingly at whim. As [135] foresaw, a thousand Carnapian flowers are now blooming in the field of mathematical logic. But the word sown by that Austrian logician is now

fallen on rocky ground. Immediately a thousand systems spring up; but since they have no root, they wither away in obscurity and disuse.

In this period of the proliferation of logics we have now entered, we see the continuation of trends present already in the logic of Buridan and his nominalist followers: on the one hand, the proliferation of a vast array of technical results; on the other, a loss of any real sense of what one is doing when one is doing logic. Where it falls in the literature of the field, work of philosophical depth requiring greater cultivation and care is often choked out by technical results.

Where does this leave us now?

With much work to do, yes. But with the field ripe for that work. A historical genealogy of the concept of formal consequence serves as a blueprint for a bridge to a better footing for formal logic. From here, the growing acceptance of relevant logics and other deviant logics provides an opportunity to revisit non-reductive approaches to 'following', as well as to examine its various kinds and senses. The growth of interest in logical pluralism provides an opportunity for revisiting and improving the real basis on which Burley and earlier logicians grounded early divisions of consequences. Metaphysical work on grounding provides an opportunity to enter into serious discussions concerning on the grounds of consequence.

Surely, let a thousand flowers bloom. But let the seed fall on good soil and bring forth good fruit. Thirty, sixty, a hundredfold.

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Appendix A

On consequences
London, BL, Royal 12 FXIX, ff.
111ra-112rb (*ante* 1302)

The following provides a translation of one of the earliest treaties on consequences, an anonymous treatise found in London, BL, Royal 12 FXIX, ff. 111ra-112rb and edited in [76, pp. 4-11]. Numbering follows the edition.

1. A consequence is a relation (*habitudo*) between an antecedent and a consequent. An antecedent is that from which another follows. A consequent is what follows from another. For instance, in 'if a man is, an animal is', 'an animal is' is the consequent, 'a man is' is the antecedent. And the antecedent is what immediately follows the sign [indicating] the connection.

2. One should know that in every good consequence *whatever follows from the consequent follows from the antecedent*. And in every good conse-

quence *from the opposite of the consequent the opposite of the antecedent follows.* For instance, this consequence is good: 'a man runs, therefore an animal runs'. Therefore from the opposite of the consequent the opposite of the antecedent follows so: 'no animal runs, therefore no man runs'.

3. If it is asked why 'a man runs, therefore an animal runs' is good, one should say because it argues *from an inferior to a superior without distribution*, since 'man' is inferior and 'animal' is superior.

4. Hence one should know that the consequence *from an inferior to a superior* is valid in two ways, and in two ways it isn't. Hence, the consequence holds without distribution and without a negation prefixed, as in 'a man runs, therefore an animal runs'.

5. And likewise, *from an inferior to a superior without distribution and with a negation placed after* is a good consequence, as in 'a man does not run, therefore an animal does not run'.

6. But the consequence *from an inferior to a superior with distribution* is invalid - as in 'every man runs, therefore every animal runs' - since the antecedent can be true without the consequent; when it is posited that every man runs, then the antecedent 'every man runs' is true, and the consequent 'every animal runs' is false, supposing in that case that an ass does not run. And in a good consequence the antecedent cannot be true without the consequent. Therefore, this consequence, 'every man runs, therefore every animal runs', is invalid.

7. Likewise, the consequence *from an inferior to a superior with a negation*

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prefixed is invalid - as in 'not a man runs, therefore not an animal runs' - since 'not a man runs' is equivalent to 'no man runs', and 'not an animal runs' is equivalent to 'no animal runs'. Now one should know that the consequence 'no man runs, therefore no animal runs' is invalid, since the consequence *from an inferior to a superior with distribution* is invalid, as was previously stated.[1] In the same way, the consequence 'not a man runs, therefore not an animal runs' is invalid.

8. But one should examine in what way the consequence *from a superior to an inferior* holds. For this, one should know that it holds in two ways, and in two ways it doesn't. Hence, the consequence *from a superior to an inferior with distribution* holds - for instance, 'every animal runs, therefore every man runs'; and likewise in the negative 'no animal runs, therefore no man runs'.

9. And one should know that the consequence *from a superior to an inferior with a negation prefixed* holds, as in 'not an animal runs, therefore not a man runs'. That this is good is shown: since 'not an animal runs' is equivalent to 'no animal runs', and 'not a man runs' is equivalent to 'no man runs', and the consequence 'no animal runs, therefore no man runs' is good, in the same way the consequence 'not an animal runs, therefore not a man runs' is good.

10. Note that the consequence *from a superior to an inferior* fails in two ways. One way is when one argues from a superior to an inferior without distribution /f.111rb/ or negation, as in 'an animal runs, therefore a

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[1]Par. 6.

man runs'. This consequence is invalid, since the antecedent can be true without the consequent, because positing that an ass runs and that no man runs, 'an animal runs' is true, but 'a man runs' is false, since its contradictory, namely 'no man runs', is true by the case.

11. In another way the consequence *from a superior to an inferior* fails, with a negation placed after as here: 'an animal does not run, therefore a man does not run'. This consequence is invalid, since the antecedent can be true without the consequent, because positing that an ass does not run and that every man runs, 'an animal does not run' is true, since an ass does not run, yet 'a man does not run' is false, since its contradictory, namely 'every man runs', is true by the case.

12. One should know that in every good consequence *if the antecedent is true, the consequent is true*, since the false does not follow from the true. But it is not necessary that if the antecedent is false, that the consequent is false, since what is true can well follow from what is false, as is clear from Aristotle's *Prior Analytics* I:[2] from falsehoods, truth; from truths, nothing but truth.

13. The second rule is: in a good consequence *if the consequent is false, it must be that the antecedent is false*, since the false does not follow except from falsehood or falsehoods. But it is not necessary in a good consequence that if the consequent is true that the antecedent is true, since the true can follow from the false.

14. Another rule is this: *whatever follows from the consequent follows from*

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[2][8, II. 2, p. 53b 7sq].

the antecedent. For instance, 'Socrates runs, therefore a man runs' follows; and 'a man runs, therefore an animal runs' follows: therefore, 'Socrates runs, therefore an animal runs' follows by the rule *whatever follows from the consequent follows from the antecedent.*

15. Likewise, in every good consequence *whatever entails (antecedit) the antecedent entails the consequent.* For instance, 'a substance runs, therefore something runs' follows; and 'a body runs, therefore a substance runs' follows: therefore, 'a body runs, therefore something runs' follows by the rule *whatever entails the antecedent entails the consequent.*

16. One should know that these two rules make a good argument:[3] *whatever follows from the consequent follows from the antecedent*; and *whatever entails the antecedent entails the consequent.* But these two rules make the fallacy of the consequent: *whatever follows from the antecedent follows from the consequent*; and *whatever entails the consequent entails the antecedent.*

17. One should know that when the opposite of the consequent cannot stand with the antecedent, then there is a good consequence, as is clear in the consequence, 'a man runs, therefore an animal runs' - since 'a man runs' and 'no animal runs' cannot stand together, but 'no animal runs, therefore no man runs' follows. What, then, stands with the consequent stands with the antecedent. Thus, if 'a man runs' and 'no animal runs', were to stand together, 'a man runs' and 'no man runs' would stand together; and so these two contradictories would stand together, which

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[3]or: 'make an argument good'.

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is impossible.

18. Hence there is this rule: in every good consequence *the opposite of the consequent cannot stand with the antecedent* (except where the antecedent includes opposites), since in every good consequence *from the opposite of the consequent the opposite of the antecedent follows*. Whatever, then, stands with the antecedent stands with the consequent. If then the opposite of the consequent stands with the antecedent, the antecedent and the opposite of the consequent can stand together. But the antecedent and the opposite of the antecedent[4] /f.111va/ are two contradictories. Therefore two contradictories would stand together. But this is impossible. Therefore, it is impossible for the opposite of the consequent to stand with the antecedent in a good consequence where the antecedent does not include opposites. If the antecedent does include opposites, then the opposite of the consequent can stand with the antecedent, as is clear in these consequences: 'if nothing is, something is', and 'if no proposition is true, some proposition is true' and 'if no time is, some time is'. In each of these consequences the opposite of the consequent can stand with the antecedent, since the antecedent in each of these includes opposites: 'if nothing is, something is'; 'if no proposition is true, some proposition is true'; and likewise 'if no time is, some time is'. Proof: if no time is, it is not night, and if it is not night, it is day; and if it is day, some time is. Therefore, if no time is, some time is. Hence [this] rule for seeing when a consequence is good and when not: one should see whether the opposite of the consequent can stand with

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[4]Folling the ms. *antecedentis*, rather than Green-Pedersen's *consequentis*.

the antecedent or not; if not, the consequence is good; if the opposite of the consequent can stand with the antecedent, the consequence is not valid from [its] form.

19. There is another rule [amounting] to the same: one should see whether the antecedent can be true without the consequent or not. If so, then the consequence is invalid; but if the antecedent cannot be true without the consequent, then the consequence is good.

20. It should be known as a rule that *if any two propositions are incompatible, one implies the opposite of the other*, as is clear in these: these two are incompatible - 'Socrates runs' and 'no man runs' - and so one implies the opposite of the other, since if Socrates runs, then a man runs, and 'a man runs' is opposed to 'no man runs'. Likewise, 'no man runs, therefore Socrates does not run' follows, and 'Socrates does not run' is opposed to 'Socrates runs'.

22. For giving a contradiction in singular propositions, placing the negation before or after doesn't matter, since for giving the contradictory of 'Socrates runs' it doesn't matter whether 'Socrates does not run' or 'not Socrates runs' is said, because either contradicts 'Socrates runs'. But in indefinite, particular, and universal propositions, placing the negation before matters much. Hence if it is asked what the contradictory of 'a man runs' is, one should say 'not a man runs' and not 'a man does not run'. In the same way you ought to place the negation before in universals and particulars. And if it is asked why 'a man runs' and 'a man does not run' do not contradict, one should say that they

can be true together, and contradictories can't; therefore they aren't contradictories. If it is asked how 'a man runs' and 'a man does not run' can be true together, one should say that positing that Plato runs and Socrates does not run, then each of them is true. Hence positing this case, 'a man runs' is true, since it has one true singular, namely 'that man runs', pointing to Plato.[5] And for the truth of an indefinite it suffices that one singular be true. And likewise, 'a man does not run' is true supposing this case, since it has one true singular, namely 'that man does not run', pointing to Socrates.[6] And for the particular to be true /f.111vb/, as well as the indefinite, it suffices that one singular be true.

23. One should know that for a universal to be true, it must be that each of its singulars is true, if it has singulars. But there is some universal which does not have singulars, e.g., 'every chimera is a chimera'. This is true, since it predicates the same of itself, and no proposition is more true than that in which the same is predicated of itself. And yet none of its singulars is true, since it doesn't have singulars. Likewise, positing that no man is white, then the proposition ['every white man is a white man'] is true and yet none of its singulars is true, since it does not have singulars in the case supposed. Hence in brief, for a universal that has singulars to be true, it must be that each of its singulars is true. And if it is asked what is required for a universal to be false, it must at least be that one singular is false, if it has a singular. Positing that no man

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[5]Correcting the ms. *Socrate.*
[6]Correcting the ms. *Platone.*

is white, 'no white man is a white man' is false, as is clear, and yet it does not have a false singular, since it does not have a singular.

24. What is required for a particular or indefinite to be true? That one singular be true, if it has a singular. What is required for a particular or indefinite to be false? It must be that each of its singulars is false, if it has singulars. Hence, though a particular or indefinite may have a false singular, an indefinite need not be false on account of this; but it must be that each of its singulars is false. But if an indefinite or particular has one true singular, such propositions are true.

25. This should be known as a rule: *whatever one convertible is predicated of, the other is also predicated of.* For instance, 'man' and 'risible' convert, so whatever the one is predicated of, the other is predicated of the same.

26. One should know that convertibility is twofold, namely convertibility between terms and between propositions. Convertibility between terms is when two terms convert, as with 'man' and 'risible'. And this rule, 'whatever one, etc.' has to be understood of this sort of convertibility. Convertibility between propositions is when two propositions convert, as with the two propositions 'a man runs' and 'a mortal rational animal runs'; and this rule is not to be understood of these. But rules concerning convertibility between propositions include: *if one among convertibles is true, the other will be true*; and *if one is false, the other will be false*; and *if one is necessary, the other will be necessary*; and *if one is contingent, the other will be contingent.*

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27. Hence to know when terms convert and when they don't, one should see whether each term is predicated universally of the other or not; if so, then the terms convert. As is clear, this term 'man' and this term 'risible' convert, since each is predicated universally of the other; for 'every man is risible' is universal and true, and likewise the converse 'every risible /f.112ra/ is a man'. And consequently, 'man' and 'risible' convert with respect to their supposita, since all the things that are supposita of the one are supposita of the other; and if this were not so, they wouldn't convert.

28. One should know that *from a universal to a singular* - both conjunctively and separately - is a good consequence, as 'every man runs, therefore Socrates runs' makes clear. And likewise this consequence is good: 'every man runs, therefore this man runs and that one...' and so on for each singular.

29. Likewise, *from all singulars taken conjunctively to a universal* is a good consequence. Thus, 'this man runs, and that one...', and so on for each singular, '...therefore every man runs'. But *from one singular to a universal* is not a good consequence. As is clear, 'this man runs, therefore every man runs' is not valid, but is the fallacy of the consequent *from the positing of the consequent*, since it follows conversely - 'every man runs, therefore this man runs' - and does not follow in this way.

30. One should know that *from one singular to an indefinite* is a good consequence, as here: 'this man runs, therefore a man runs'. Likewise, *from all singulars to an indefinite* is a good consequence, as here: 'that

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man runs, and that one...' and so on for each singular, '...therefore a man runs'. But the consequence *from an indefinite to a singular* is not valid, neither conjunctively nor separately, but [is valid] taking the singulars disjunctively; hence 'a man runs, therefore this one ... or that one runs' follows.

31. One should know that *from a universal to an indefinite* is a good consequence, as in 'every man runs, therefore a man runs'; but the converse consequence, as in 'a man runs, therefore every man runs', is invalid.

32. One should know that from the impossible anything follows, that is, from the impossible any proposition follows. For instance, 'a man is an ass, therefore a man is a she-goat', since the antecedent is impossible and from the impossible anything follows. Likewise: 'a man is an ass, therefore you are a bishop'. Likewise 'a man is an ass, therefore God does not exist' follows, and this is because the antecedent is impossible, and from the impossible anything follows.

33. Another rule is this, that the necessary follows from anything. For 'a man runs, therefore a man is an animal' follows in its terms, because the consequent is necessary, and the necessary follows from anything. Likewise 'a man is an ass, therefore God exists' follows, and this is because the consequent is necessary, and the necessary follows from anything - that is, from any proposition.

34. Against one stated before:[7] when it is said '*from a superior to an inferior with distribution* is a good consequence,' I prove that it is invalid. Since

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[7]Par. 8.

'every man runs, therefore every white man runs' argues from a superior to an inferior with distribution, and yet this consequence is invalid, because the antecedent can be true without the consequent; for positing that every man runs and that no man is white, then this antecedent, 'every man runs', is true, and this consequent, 'every white man runs', is false in the posited case, because 'every white man runs, therefore a man is white' follows. The consequent [is] false by the case; therefore, the antecedent is false by the case.

35. I respond to this argument and say that 'inferior' is twofold, namely inferior *per se* and inferior *per accidens*. /f.112rb/ An inferior *per se* is just as 'man' is *per se* inferior to 'animal', and 'animal' *per se* to 'substance'. An inferior *per accidens* is as 'white man' is inferior *per accidens* to 'man', and 'running' is inferior *per accidens* to 'moving'. And through this I respond to the argument, and say that the consequence *from a superior to an inferior per accidens with distribution*, and this affirmatively, is invalid, as here: 'every man runs, therefore every white man runs', since it argues from a superior to an inferior *per accidens*. But *from a superior to an inferior per se with distribution*[8] is a good consequence, as is clear here: 'every animal runs, therefore every man runs'. But *from a superior to an inferior per accidens with distribution* negatively is a good consequence, as here: 'no man runs, therefore no white man runs', since the negation negates for whichever, *per se* or *per accidens*.

36. Likewise, one should know that *from an inferior per accidens to a su-*

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[8]Correcting the ms. *ab inferiori ad superius.*

perior without distribution is not a good consequence *simpliciter*, as is shown: the consequence 'a white man is a white man, therefore a white man is a man' is invalid. Neither is 'a white man is a white man, therefore a white man is this white [thing]' valid, since the antecedent can be true without the consequent: for positing that no man is white, then 'a white man is a white man' is true, because it predicates the same of itself, and no proposition is more true than that in which the same is predicated of itself; therefore, etc. And yet 'a white man is a man' is false, and likewise 'a white man is this white [thing]', on account of a false implication, since it implies a man is white, which is false by the case.

37. Likewise, *from an inferior per accidens to a superior* is a good consequence, provided that one argues from an inferior to a superior where there is not a predication of the same of itself. This consequence is good, as is clear: 'a white man runs, therefore a man runs'. But where there is a predication of the same of itself the consequence is invalid. As is clear here, this consequence is invalid: 'a white man is a white man, therefore a man is a white man'.

Appendix B

On consequences

Paris, BN, lat. 16130, ff.

118va-120vb

The following translates an early anonymous treatise on consequences found in Paris, BN, lat. 16130, ff. 118va-120vb, and edited in [76, pp. 12-28]. Numbering follows the edition.

1. In every good consequence *whatever follows from the consequent follows from the antecedent*. For instance, 'Socrates runs, therefore an animal runs' follows, and 'an animal runs, therefore a substance runs' follows; therefore, *from the first to the last*, 'Socrates runs, therefore a substance runs' follows. Rule: *whatever follows from the consequent follows from the antecedent*.

2. I show the consequence 'Socrates runs, therefore an animal runs': the

opposite of the consequent implies the opposite of the antecedent; the opposite of the consequent is 'no animal runs'; the opposite of the antecedent is 'Socrates does not run'; now 'no animal runs, therefore Socrates does not run' follows. The proof that 'no animal runs, therefore no man runs' follows by this rule: *from the distribution of the superior follows the distribution of the inferior*; and 'animal' is superior to 'man'; therefore, from the distribution of 'animal' follows the distribution of 'man'. And 'no man runs, therefore Socrates does not run' follows by this rule: the consequence *from the universal to its singulars* holds. Therefore *from the first* [*to the last*], 'no animal runs, therefore Socrates does not run' follows. And consequently, from the opposite of the consequent follows the opposite of the antecedent.

3. Furthermore, I show the consequence 'Socrates runs, therefore an animal runs', since the opposite of the consequent does not stand with the antecedent, because these do not stand together: 'no animal runs', therefore 'Socrates runs' (because 'two do not stand together' is nothing other than that these two conflict). But these two - 'no animal runs' and 'Socrates runs' - conflict, because from the one the opposite of the other is implied, since 'no animal runs, therefore Socrates does not run' follows, as is shown above.[1] And 'Socrates runs' implies the opposite of 'no animal runs', since 'Socrates runs, therefore an animal runs' or 'some animal runs' (which are the same, since the judgment of an indefinite and a particular are the same) follows. But 'no animal runs' and 'some animal runs' contradict. Therefore, 'Socrates runs' implies

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[1]Par. 2.

the opposite of 'no animal runs'.

4. I show the consequence 'Socrates does not run, therefore a man does not run' by this rule: a consequence *from a inferior to a superior with the negation placed after* holds; but it is so argued here; therefore, the consequence 'Socrates does not run, therefore a man does not run' is good. And consequently, removing the negation from both, it follows affirmatively thus: 'Socrates runs, therefore a man runs'. And lastly 'Socrates runs, therefore an animal runs' follows, since from the positing of the inferior follows the positing of the superior. And consequently from the opposite of the consequent follows the opposite of the antecedent.

5. Furthermore, I show the consequence 'Socrates runs, therefore an animal runs', since the opposite of the consequent does not stand with the antecedent; therefore the first consequence [is] good. I show that they conflict by this rule: neither of 'no animal runs' and 'Socrates runs' is impossible; the impossible follows from them; therefore they conflict. I show that neither of them is impossible, for any singular of this universal is possible, therefore the universal is possible. For 'no animal runs' is not impossible, nor is 'Socrates runs' impossible; therefore neither of these is impossible; the antecedent is true; therefore the consequent. I show that the impossible follows from these, for I argue thus in the second figure: no animal runs; Socrates runs, therefore Socrates is not an animal. The conclusion is impossible, and neither premise is impossible; therefore they are incompossible,[2] and consequently conflict. I show that the conclusion is impossible, since the impossible follows from it: therefore

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[2]Ms. *impossibilia*; cf. [77, p. 155, par. 151].

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it is impossible (since from 'Socrates is not an animal' the impossible follows). I show it: for 'Socrates is not an animal, therefore Socrates is not Socrates' follows; that in which the same is removed from itself is impossible; such is 'Socrates is not Socrates'; therefore it is impossible. I show the consequence 'Socrates is not an animal, therefore Socrates is not Socrates', since 'Socrates is not an animal, therefore Socrates is not a man' follows by this rule: *from the denial (negatio) of a superior of something follows the denial of an inferior of the same.* And finally, 'Socrates is not a man, therefore Socrates is not Socrates' follows by the same rule.

6. One should know that this rule is good: *whatever entails (antecedit) the antecedent /f. 118vb/ entails the consequent.* For 'a man runs, therefore an animal runs' follows, since whatever entails 'a man runs' entails 'an animal runs'; and because 'Socrates runs, therefore a man runs' follows, 'Socrates runs, therefore an animal runs' follows.

7. Hence one should know that the consequence 'if a man runs, then an animal runs; therefore if Socrates runs, then an animal runs' is good, since it is argued by this rule: *whatever entails the antecedent entails the consequent.*

8. Hence one should see that one may always descend from a term standing under a condition with respect to the consequent, as is shown: 'if a man runs, then an animal runs; therefore if Socrates runs, then an animal runs' follows, for the consequent is always the same here, - that is, 'an animal runs' - and the descent occurs from a term placed under a

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condition, that is, from 'man'.

9. And one should know that a consequence of this sort always holds by this sort of rule: *whatever entails the antecedent entails the consequent.* One should know that the consequence 'if a man runs, an animal runs, therefore if a man does not run, an animal does not run' is good' [***]

10. [***] since here it is argued by this rule: *whatever follows from the consequent follows from the antecedent,* since 'a substance runs' follows from 'an animal runs', so it follows from 'a man runs'.

11. Hence one should know that when *from the inferior with respect to [something] following to the superior with respect to the same anteceding* is argued, the consequence is good, because it is argued by this rule: *whatever follows from the consequent follows from the antecedent* as is shown in the example above.[3]

12. One should know that this rule is good for disproving a consequence: *something entails the antecedent that does not entail the consequent*; therefore, the consequence is invalid, as is clear in the example 'an animal runs, therefore a man runs'.

13. I show that this consequence does not hold: something entails the antecedent that does not entail the consequent; therefore the consequence is invalid. Something entails 'an animal runs' that does not entail 'a man runs' since 'an ass runs, therefore an animal runs' follows, yet 'an ass runs, therefore a man runs' does not follow; and consequently, the consequence is invalid.

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[3]Par. 10.

14. Furthermore, I show that the consequence 'an animal runs, therefore a man runs' is invalid, because something follows from the consequent that does not follow from the antecedent; therefore, the consequence is invalid. For 'a man runs, therefore a rational animal runs' follows, yet 'an animal runs, therefore a rational animal runs' does not follow.

15. Furthermore, I show that 'an animal runs, therefore a man runs' does not follow, since the antecedent can be true without the consequent; therefore the consequence is invalid. I show that the antecedent can be true without the consequent, since 'an animal runs' can be true if an ass runs and if no other animal runs, since 'an ass runs, therefore an animal runs' follows, and this can be true even if no man runs. Therefore 'an animal runs' can be true with 'a man runs' being false.

16. Furthermore, I show that 'an animal runs, therefore a man runs' does not follow, since something stands with the antecedent that does not stand with the consequent; therefore the consequence is invalid. For 'no man runs' stands with the antecedent 'an animal runs', but[4] does not stand with the consequent 'a man runs'. I show that 'an animal runs' and 'no man runs' may stand together. For 'no man runs' and 'an ass runs' stand together; and 'an ass runs, therefore an animal runs' even follows. Therefore 'no man runs' and 'an animal runs' stand together, since what stands with the antecedent stands with the consequent.

17. Furthermore, I show that 'an animal runs' and 'no man runs' may stand together. Since if they won't stand together, then one implies the oppo-

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[4]Agreeing with Green-Pedersen's *sed* against the ms. *vel*.

site of the other; and consequently, 'no man runs, therefore no animal runs' follows. But this consequence is invalid, since it argues *from a distributed inferior to a distributed superior.*

18. Furthermore, the opposite of the consequent stands with the antecedent. Therefore 'an animal runs, therefore a man runs' does not follow. For 'an animal runs' and 'no man runs' stand together, since for some things to stand together is nothing other than for them to be able to be true together. But they can be true together - supposing, for instance, that no man runs and an ass runs.

19. One should know that the rule *whatever follows from the antecedent follows from the consequent* is invalid, but brings about a false consequence, as is shown if it is so argued: 'If a risible [thing] is an ass, then an animal is an ass'; but /f. 119ra/ by *whatever [follows from the antecedent] follows from the consequent* one argues that since 'a man is an ass' follows from the antecedent 'a risible [thing] is an ass', so 'a man is an ass' follows from the consequent 'an animal[5] is an ass'. This consequence is in no way valid.

20. Hence one should know that the consequence 'if a risible [thing][6] is an ass, an animal is an ass; therefore if an animal is an ass, a man is an ass' is invalid, because it argues by this rule: *whatever follows from the antecedent follows from the consequent.*

21. One should know this rule is invalid: *whatever entails the consequent entails the antecedent*, as is shown if it is so argued: if a risible [thing]

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[5]Agreeing with Green-Pedersen's '*animal*' against the ms. '*homo*'.

[6]Reading '*risible*' in place of '*homo*'.

is an ass, then a man is an ass; for here by the rule *whatever entails the consequent entails the antecedent*, one argues that 'a risible [thing] is an ass' entails 'an animal is an ass', and so 'a risible is an ass' also entails 'a man is an ass'. And it is so argued by this rule: *whatever entails the consequent entails the antecedent* since 'a risible [thing] is an ass' entails the consequent 'an animal is an ass'.[7]

22. Hence a descent under the consequent of a conditional with respect to the same antecedent, as occurs in the example above, is never valid, because[8] it is always argued by the rule *whatever entails the consequent entails the antecedent.*

23. Against the rule *whatever follows from the consequent follows from the antecedent*[9] one argues thus: the consequence 'the Antichrist is, therefore the Antichrist is right now' is good. Yet something follows from the consequent that does not follow from the antecedent: for 'The Antichrist is right now, therefore the false is true' follows; yet 'the Antichrist is, therefore the false is true' does not follow. Therefore not whatever follows from the consequent follows from the antecedent.

24. I show that the consequence 'the Antichrist is right now, therefore the false is true' follows. The antecedent is impossible, for 'the Antichrist is right now' is impossible; and from the impossible anything follows, whether it be impossible[10] or it be necessary. Therefore 'the Antichrist is right now, therefore the false is true' follows.

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[7]Cf. [202, pp. 80.13-29, 84.8-86.21].
[8]Reading '*quia*' for the ms. '*qui*', and instead of Green-Pedersen's '*ubi*'.
[9]Par. 1.
[10]Following Green-Pedersen's reading of '*impossible*' instead of '*possibile*'.

25. I show that the consequence 'the Antichrist is, therefore the false is true' is invalid, because the antecedent is possible (for 'the Antichrist is' is possible) and the consequent, namely 'the false is true',[11] is impossible; and from the possible the impossible does not follow; therefore 'the false is true' does not follow.

26. One should say that this rule is good: *whatever follows from the consequent follows from the antecedent.*

   To the argument one should say that 'the Antichrist is, therefore the Antichrist is right now', should be disambiguated for the reason that 'the Antichrist...' can denote an as-of-now (*ut nunc*) consequence or a simple consequence. If it denotes an as-of-now consequence, then 'The Antichrist is, therefore the Antichrist is right now' is good, because the antecedent, as of now, cannot be true without the consequent, for 'the Antichrist is' cannot be true as of now unless 'the Antichrist is right now' is true. And so it follows as of now, because an as-of-now consequence is nothing else than that the antecedent can[not] be true as of now unless the consequent is true. If 'the Antichrist...' denotes a simple consequence, then 'The Antichrist is right now' doesn't follow, since in a simple consequence it is required that the antecedent never can be true unless the consequent is true. But 'The Antichrist is, therefore the Antichrist is right now' is not so, because if right now is *a*, then it is the same to say 'The Antichrist is right now' and 'The Antichrist is at *a*', but 'the Antichrist is' can be true tomorrow with 'the Antichrist is at *a*' being false, because tomorrow it will be the past, and then 'The

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[11]Reversing the ms. '*verum est falsum*'.

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Antichrist is at a' is false. And so[12] it is clear that 'The Antichrist is, therefore the Antichrist is right now' does not follow as a simple consequence. Still, 'the Antichrist is, therefore the Antichrist is right now' does follow as of now. And so the consequence 'The Antichrist is, therefore the false is true' does follow as of now, because the antecedent cannot be true unless the consequent is true. Yet the consequence 'The Antichrist is, therefore the false is true' does not follow simply, because in a simple consequence it is required that the antecedent never is /f. 119rb/ true unless the consequent is true. But it is not so in the proposed, as it is clear that 'the Antichrist is' could be true and 'the false is true' never could be true; and consequently, the antecedent could be true without the consequent; and consequently the consequence is invalid spoken as a simple consequence.

27. Hence one should understand that never in a simple consequence does the impossible follow from what is contingent toward either. But 'The Antichrist is' is contingent toward either, and 'the false is true', impossible.

28. Then to the reasons for the opposite: when[13] it is argued the consequence 'the Antichrist is, therefore the false is true' is invalid because the antecedent can be true without the consequent, one should say that this consequence is good spoken as an as-of-now consequence, and 'the antecedent can be true without the consequent' is invalid spoken of an as-of-now consequence. But in a simple consequence the reason [given]

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[12] Agreeing with Green-Pedersen's '*sic*' against the ms. '*sicut*'.
[13] Par. 23.

proves hence that this consequence is not good spoken as a simple consequence.

29. Against the rule[14] *from the contingent toward either the impossible does not follow* spoken of a simple consequence, one argues so: from the necessary follows the contingent; therefore, from the opposite of the contingent follows the opposite of the necessary. But the opposite of the contingent is always contingent, and the opposite of the necessary is impossible. Therefore from the contingent the impossible follows.

30. To this one should say that from the necessary the contingent does not follow, because then from the opposite of the contingent - which is contingent - follows the opposite of the necessary, which is impossible. The consequent is false; therefore also the antecedent.

31. Against [this]: I prove that from the necessary the contingent follows, because 'a white man is a white man' is necessary, since it affirms the same of itself, and from this the contingent follows, for 'a white man is a white man, therefore a man is a white man' follows. The consequent is contingent, and the antecedent is necessary as is proven; therefore from the necessary the contingent follows. And consequently, from the opposite of the consequent follows the opposite of the antecedent, i.e. 'no man is a white man, therefore no white man is a white man'; and 'no man is a white man' is contingent, and 'no white man is a white man' is impossible, since the same is denied (*negatur*) of itself.

32. To this it is said that 'a white man is a white man, therefore a man is

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[14]Par. 27.

a white man' does not follow, because 'a white man is a white man' is necessary, since it predicates the same of itself, and 'a man is a white man' is contingent.

33. I show that 'a man is a white man' [is] contingent, since it can be true and can be false; therefore it is contingent. The consequence is clear from the definition of the contingent. It is certain that it can be true. And that it can be false, proof: for I suppose that no man is white, and then 'a man is a white man, therefore a man is white' is true.[15] The consequent is false in the case posited, and the case, possible. Therefore, 'a man is a white man' can be false.

34. I show the consequence 'a man is a white man, therefore a man is white' by this rule: *positing per accidens, also posits per se*. But 'white man' is a suppositum *per accidens* of something. With respect[16] to the same, 'white' is also posited.

35. Furthermore: from the opposite of the consequent follows the opposite of the antecedent. For 'no man is white, therefore no man is a white man' follows by this rule: *a suppositum distributed per se, is distributed per accidens*.

36. Against this I show the consequence 'a white man is a white man, therefore a man is a white man' by this rule: *positing per accidens posits per se*; but 'white man' is supposed *per accidens*, and so 'man' is supposed *per se*;[17] therefore positing 'white man' with respect to some predicate,

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[15]Substituting *verum* for the ms. *falsum.*
[16]Following Green-Pedersen's *respectu* in place of the ms. *regula.*
[17]Reading 'suppositum' instead of the ms. 'superius'.

'man' is posited with respect to the same.

37. Furthermore: from the opposite of the consequent follows the opposite of the antecedent; for 'no man is a white man, therefore no white man is a white man' follows by this rule: *distributing per se, distributes per accidens.*

38. One should say that the consequence 'a white man is a white man, therefore a man is a white man' is invalid, for /f. 119va/ the reasons given above.[18] On account of this, one should understand that any proposition in which a superior is predicated of its inferior *per accidens* is contingent. And [those] placed under *per accidens* are those that are combined out of two inhering in each other contingently, as *white man* is combined *per accidens* because it is put together out of *man* and *white*, which inhere in each other contingently. So 'a man is a white man', etc. is contingent. This is contingent, and its equivalents (*convertibilia*) are contingent, because in all of these a superior is assumed of an inferior *and* white[19] *per accidens* (since whatever is constituted by an addition with respect to another is inferior to it. Thus, 'white man' is inferior to 'man' and 'white').

39. Another thing that should be understood is that the consequence *from an inferior per accidens to its superior* is never valid when the same is compared to itself.

40. Because of this, to the reason[20] for accepting 'a white man is a white

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[18]Par. 32.
[19]Unbracketing *et* and *album.*
[20]Par. 36.

man, therefore a man is a white man' one should say that it doesn't follow for the reason given above.[21] To the proof: when it is accepted that 'positing *per accidens* the *per se* is also posited', one should say that it is true if that which is *per accidens* is not compared to itself, etc. 'Distributing *per se*,[22] the *per accidens* is also distributed', unless that which is *per accidens* is compared to itself, as in 'a white man is a white man' what supposits[23] *per accidens* is compared to itself. And thus it need not be that 'a white man is a white man, therefore man is a white man' follows, because *from the positing of something suppositing*[24] *per accidens with respect to itself* the positing of the superior does not follow. Nor *from the distribution of a superior* does the distribution of what is *per accidens* inferior with respect to itself follow. And thus 'no man is a white man, therefore no white man is a white man' does not follow'. Hence these two rules: *positing per accidens posits per se*, and *distributing per se, distributes per accidens* should be understood where the *per accidens* is not compared to itself.

41. One should know that this rule is good: *if one contradictory is true, the other will be false*, and conversely. And similarly, this rule will be good: *if one of conflicting [propositions] is true, the other will be false*, [but not conversely] because two conflicting [propositions] can be false together; for 'every animal is a man' and 'no animal is a man' conflict, and yet are false together.

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[21]Par. 38.
[22]Par. 37.
[23]Following the edition's *suppositum* against the ms. *sumptum*.
[24]Following the Green-Pedersen's *suppositi* against the ms. *sumpti*.

42. I show that 'every animal is a man' is false for this reason: *every propo-sition where something false follows from it and something true is false.* And this [proposition] is of this type, for I argue thus: every animal is a man, an ass is an animal, therefore an ass is a man. The conclusion is false and the minor isn't, so the major [is].

43. Furthermore, I show that it is false because every proposition from which something false follows is false. But it is so in the proposed, for 'every animal is a man, therefore an ass is a man' follows by this rule: the consequence *from a distributed superior to its per se inferior* holds.[25]

44. Furthermore, I show that 'Every animal is a man' is false because if one among contradictories is true, the other will be false. But 'some animal is not a man' is true'. I show that this is true because a proposition which follows from something true is [true]. 'An ass is not a man, therefore some animal is not a man' follows just from this rule: the consequence *from an inferior to a superior with the negation placed after* holds.[26]

45. Furthermore, I show that 'every animal is a man' is false, because what conflicts with it is true; therefore it is false. For 'an ass is not a man' is true; and 'an ass is not a man' and 'every animal is a man' conflict; therefore 'every animal is a man' is false.

46. I show that these conflict because one implies the opposite of the other, as is shown above,[27] so they /f. 119vb/ conflict.

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[25]Par. 2.
[26]Par. 4.
[27]Par. 43-44.

47. Against the second[28] rule[29] *an antecedent cannot be true without its consequent,* I argue thus: the consequence 'every man runs, therefore John the grammarian runs' is good, and yet the antecedent can be true without the consequent. I show that the antecedent can be true without the consequent, for supposing that every man runs, and that John is not a grammarian, then the antecedent is true and the consequent false. And consequently the antecedent can be true without the consequent, because the consequent is false, as is shown: for 'John the grammarian runs' is false because it implies a falsehood, that John is a grammarian. I prove the consequence both because it argues *from the universal to its singular* and because from the opposite of the consequent follows the opposite of the antecedent. For 'John the grammarian does not run, therefore some man does not run' follows, because the consequence *from an inferior to a superior with the negation placed after* holds.

48. To this one should say that the rule is good: the antecedent can be true without the consequent, therefore the consequence is invalid.

To the argument: when it is accepted that the consequence 'every man runs, therefore John the grammarian runs' is good, I say that this consequence is invalid, because the antecedent can be true without the consequent, as has been shown. To the proof: when it is accepted that the consequence *from the universal to its singular* holds, one should say that it is true spoken of a singular *per se,* but is not true spoken of a singular *per accidens;* but John the grammarian is a singular *per accidens*

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[28]Reading the ms. *secundam* rather than Green-Pedersen's *illam.*
[29]Par. 15; cf. par. 3.

of 'man'; therefore 'Every man runs, therefore John the grammarian runs' does not follow.

49. Hence one should know that spoken as a simple consequence, the consequence *from the universal affirmative to its singular per accidens* is never valid. But the consequence *from the universal negative to its singular per accidens* - and *to [its] singular per se* - always holds. For 'no man runs, therefore John the grammarian does not run' follows.

50. To the other proof: when 'John the grammarian does not run, therefore a[30] man does not run' is accepted, one should say that it does not follow. When it is argued that the consequence *from an inferior to its superior with the negation placed after* holds, one should say that this is true spoken of what is inferior *per se*, but is not true spoken of what is inferior *per accidens*. But 'John the grammarian does not run, therefore some man does not run' argues *from an inferior per accidens to a superior with the negation placed after*. and so the consequence is invalid.

51. Hence one should know that the consequence *from an inferior per accidens to a superior with the negation placed after* never holds.

52. One should know, then, that any proposition in which the sign of disjunction 'or' is placed should be disambiguated according to composition and division. E.g. in 'only a man runs or doesn't run', 'runs or doesn't run' should be disambiguated for the reason that consequences assert the 'or' [either] disjunctively (*disiunctive*) [or disjunctly (*disiunctim*).

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[30]Deleting Green-Pedersen's addition of *aliquis*.

It is asserted disjunctly] if it holds disjunctly in the consequent, since it distinguishes (*disiungit*) between terms. If it is asserted disjunctively, then it holds disjunctively [in the consequent], so it is understood as 'only a man runs or only a man does not run'. And so the 'or' holds disjunctively, since it distinguishes between propositions. But since the term 'or' holds disjunctively, it forms a disjunctive proposition.

53. Similarly, the word 'and' is sometimes held conjunctly (*copulatim*) and sometimes conjunctively (*copulative*). Conjunctly when it conjoins terms, and so forms a conjunction from conjoined extremes; but when it is held conjunctively it conjoins propositions, and so forms a conjunctive proposition.

54. I ask whether this consequence is good: 'every man besides Socrates runs, therefore every man besides Socrates /f. 120ra/ or Plato runs'. I say that the consequent should be disambiguated according to composition and division; and it should be disambiguated thus: in the composite sense, it is understood as 'every man besides Socrates or Plato [runs]'; in the divided sense, as 'Every man besides Socrates runs or every man besides Plato runs'; and in either sense 'every man besides Socrates runs, therefore every man besides Socrates or Plato runs' indeed follows.

I prove the consequence in the divided sense, since a disjunctive follows from either of its parts; so this consequence is good in that sense.

I prove the consequence in the composite sense: because a disjunction is superior to either part of a disjunction, and thus 'Socrates' is inferior to 'Socrates or Plato'; and because the exposition of the antecedent implies

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the exposition of the consequent. The affirmative implies an affirmative, since the middle is necessary; the negative implies a negative, because the consequence *from an inferior to a superior with a negation placed after* holds.

55. I prove that the middle is necessary, because its opposite is impossible. For 'every man other than Socrates or Plato is not a man other than Socrates' is impossible. And I prove that it is impossible, because from this the impossible follows, therefore it is impossible. Because 'a man other than Socrates or Plato is not a man other than Socrates; therefore some [man] other than Socrates or Plato is not [a man] other than Socrates or Plato' follows. But the consequent is impossible, therefore the antecedent [is, too].

56. I prove the consequence: from the opposite of the consequent follows the opposite of the antecedent, since 'every man other than Socrates or Plato is a man other than Socrates or Plato; therefore every man other than Socrates or Plato is a man other than Socrates' follows, because from the positing of the inferior follows the positing of the superior. And consequently 'every man other than Socrates or Plato is a man other than Socrates' is necessary.

57. I show that the consequence 'nothing besides a man is an animal, therefore nothing besides an animal is a man' is good. Because 'nothing besides a man is an animal, therefore only a man is an animal' follows (because a negative exceptive in which the exception is from a transcen-

dental[31] converts with an exclusive affirmative). And again 'therefore only a man is a man' follows, because the consequence *from a superior to an inferior on the part of the predicate, with an exclusive term occurring on the part of the subject* holds. And again 'Therefore only an animal is a man' follows [because] the consequence *from an inferior with an exclusive term immediately added* holds. And lastly 'Therefore nothing besides an animal is a man' is given, because an affirmative exclusive [is equivalent to] an exceptive negative in which the exception is taken from a transcendental.[32] Therefore *from the first [to the last]*, 'nothing besides a man is an animal, therefore nothing besides an animal is a man' follows.

58. It is asked whether the consequence 'if a man runs an animal runs; [therefore] if no substance runs, no man runs' is good or not. [I prove] that the consequence is good by this rule: *whatever entails (antecedit) the opposite of the consequent entails the opposite of the antecedent.* And because 'no substance runs' [entails] the opposite of the consequent, which is 'no animal runs', thus it entails the opposite of the antecedent, which is 'no man runs'.

59. I show that the consequence 'you are other than a man, therefore you are not a man' is good. Because the consequence *from an affirmative with an indefinite predicate to a negative with a definite predicate* holds.

60. It is asked whether the consequence 'nothing besides Socrates is an animal, therefore nothing besides Socrates is a white man' is good. I say

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[31]ms. *transmutatione* (?) Cf. [77, pp. 124-125, par. 158].
[32]Reading Green-Pedersen's *transcendente* instead of the ms. *transmutatione*, as above.

that is [in]valid, because something entails the antecedent that does not entail the consequent; therefore the consequence is invalid. Because 'Only Socrates is an animal, therefore nothing besides Socrates is an animal' follows; yet 'only Socrates is an animal, therefore nothing besides Socrates is a white man' does not follow; because something /f. 120rb/ follows from the consequent that does not follow from the antecedent. 'Nothing besides Socrates is a white man, therefore only Socrates is a white man' follows; yet 'Only Socrates is an animal, therefore only Socrates is a white man' does not follow, because something entails the antecedent that does not entail the consequent; therefore the consequence is invalid. Because 'Every animal is Socrates, therefore only Socrates is an animal' follows; but 'every animal is Socrates, therefore only Socrates is a white man' does not follow; because something follows from the consequent that does not follow from the antecedent. 'Only Socrates is a white man, therefore every white man is Socrates' follows; but 'every animal is Socrates, therefore every white man is Socrates' does not follow, since the consequence *from a universal affirmative to an inferior per accidens* does not hold.

61. Note that whatever follows from the consequent [with something added] follows from the antecedent with the same added. Because 'every man is black, therefore no man is white' follows, thus whatever follows from 'no man is white' with something added follows from 'every man is black' with the same added. But because from 'no man [is] white' with 'every animal is white' added, 'no animal is a man' follows, thus from 'every man is black' with the same added, 'therefore, no animal is a

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man' follows by this rule: *whatever follows from the consequent with [something] added follows from the antecedent with the same added.*

62. It is asked whether the consequence 'only every [man] is an animal, therefore only every man is white' is good. I say that 'only every man is white' and any [proposition] like it should be disambiguated, for the reason that the word 'only' can effect its exclusion either on account of matter or on account of form. If it effects [it] on account of form, then by effecting an affirmation it is said to remove the predicate from anything other than the subject in that very respect; and so it is understood as 'only every man is white, therefore nothing other than *man* is white'; so it includes opposites.[33] If it brings about its effect on account of matter, then it is given[34] by effecting the affirmation that the predicate inheres in the subject under the mode 'all', and is removed from anything other than man; hence these are exposited so: 'every man is white, and[35] nothing other than a man is white'.

63. It is asked whether the consequence 'only Socrates is moving, therefore only Socrates is running' is good. I say that this consequence is invalid.

    On the contrary: the consequence *from a superior to an inferior on the part of the predicate with an exclusive term occurring on the part of the subject* holds; therefore the previous consequence was good.

64. Furthermore: a universal in transposed terms implies a universal, because 'every moving is Socrates, therefore every running is Socrates'

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[33]Cf. [77, p. 120, par. 29-40].
[34]Following the ms. *detur* rather than Green-Pedersen's *dicitur*.
[35]Reading *et* instead of the ms. *ergo*.

follows; from the distribution of the superior the distribution of the inferior follows; therefore the consequence is good.

65. To the first I say that the consequence 'only Socrates is moving, therefore only Socrates is running' is invalid. To the proof *from a superior to an inferior*, etc. I concede it for an inferior *per se*, and not for an inferior *per accidens*.

66. To the other: when it is accepted that a universal implies a universal, I deny [it]. To the proposition[36] that 'every mover is Socrates, therefore every runner is Socrates follows, I say that [it is] not [so]. To the proof *from the distribution of a superior*, etc., I concede it for an inferior *per se*, and not *per accidens*. But 'runner' is inferior *per accidens*; therefore the consequence is not valid, nor others like it.

67. Concerning disjunctions and conjunctions: one should know that a disjunction either of whose parts is true is true, because a disjunction follows from either part, and if the antecedent is true, the consequent will be true. And so if part of a disjunction is true, the whole disjunction will be true, as 'a man is white or a man is an animal' is true because one part of the disjunction is true.

68. Hence one should understand that any disjunction has two causes of truth. And the proposition 'if a proposition has several causes of truth, it has one cause of truth'[37] is true. And if a disjunction has one true part and another false, that disjunction which has one /f. 120va/ cause[38] is

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[36]Reading the ms. *propositionem* rather than Green-Pedersen's *probationem*.
[37]Omitting Green-Pedersen's *si* from the consequent.
[38]Omitting Green-Pedersen's *veram*.

true, and any cause implies that of which it is the cause.

69. One should know that a disjunction implies neither of its parts, because a disjunction has many causes of truth. And thus arguing to one of them is the fallacy of the consequent. Therefore arguing *from a disjunction to either part* is the fallacy of the consequent.

70. One should know that for a conjunction to be true it is required that each part is true: because a conjunction implies each of its parts; and if the antecedent is true, the consequent will be true. Thus, if a conjunction is true it is required that each of its parts is true.

71. One should know that a conjunction follows from neither of its parts. For 'Socrates runs, therefore Socrates runs and Plato runs' does not follow, because something follows from the consequent that does not follow from the antecedent; therefore the consequence is invalid. For 'Socrates runs and Plato runs, therefore Plato runs' follows, but 'Socrates runs, therefore Plato runs' does not follow.

72. One should know that the opposite of a disjunction is equivalent to a conjunction having parts contradictory to the parts of the disjunction. For example, the opposite of the disjunction 'Socrates runs or Plato runs' is 'not Socrates runs or Plato runs', which is equivalent to the conjunction[39] 'Socrates does not run and Plato does not run', since the conjunction has parts contradictory to the parts of the disjunction 'Socrates runs or Plato runs'.

73. And one should know that the opposite of this disjunction implies the

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[39]Reading the ms. *copulative* rather than Green-Pedersen's *copulativa*.

negation of each part of the disjunction. For 'not Socrates runs or Plato runs, therefore Plato does not run' follows. And so it is clear how the negation of a disjunction implies the negation of each part.

74. One should know that the opposite of a conjunction is equivalent to a disjunction having parts contradictory to the parts of the conjunction. For instance, the opposite of the conjunction 'Socrates runs and Plato runs' is 'not Socrates runs and Plato runs'; but 'not Socrates runs and Plato runs' is equivalent to the disjunction 'Socrates does not run or Plato does not run'; because the disjunction has parts contradictory to the parts of the conjunction 'Socrates runs and Plato runs'.

75. Hence one should know that from the negation of a conjunction the negation of either part never follows, because the negation of a conjunction is equivalent to one disjunction, which implies neither of its parts. For instance, 'Socrates does not run or Plato does not run, therefore Socrates does not run' [does not follow]. All these consequences are clear for the reasons given above.

76. Hence one should understand that the opposite of a conjunction has as many causes of truth as a disjunction convertible with it. And just as the consequence *from the disjunction to either of its parts* is invalid, so the negation of a conjunction does not imply the negation of either conjunctive part.

77. Against where it is stated "a disjunction [implies] neither of its parts",[40] and against where it is stated "a conjunction does not follow from either

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[40]Par. 69.

of its parts"[41] I argue thus: 'Socrates runs or a man runs, therefore a man runs' follows, because the opposite of the consequent implies the opposite of the antecedent. For 'No man runs, therefore neither Socrates runs nor a man runs' follows. And lastly 'therefore not Socrates runs or a man runs' follows; the consequence is clear, because 'not Socrates runs or a man runs' and 'neither Socrates runs nor a man runs' are equivalent. 'Socrates runs, therefore a man runs'[42] follows. Therefore 'Socrates runs' implies each part of the conjunction[43] 'Socrates runs and a man runs', and consequently implies the whole conjunction.

78. To this it is said that the rule *a disjunction implies neither of its parts* and the rule *a conjunction does not follow from either of its parts* should be understood [of] when the parts of a disjunction and when the parts of a conjunction are not ordered as antecedent and consequent.

79. On account of this one should note that the consequence *from a disjunction to its other part* always holds when the parts of a disjunction are ordered as antecedent and consequent, when[44] one part follows from the other part; just as when the parts are ordered as antecedent and consequent, the consequence *from one part of a conjunction to the whole conjunction* holds.

80. One should know that the consequence does not hold when the parts are /f. 120vb/ so ordered indifferently, but only to the part that is the

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[41]Par. 71.

[42]Deleting Green-Pedersen's addition of *Socrates currit et* in the consequent.

[43]ms. *consequentiae*. If the ms. is followed, then in what follows the 'and' (*et*) should be changed to 'therefore' (*ergo*), and 'conjunction' (*copulativam*) should be changed to 'conjunctively' (*copulative*). But cf. par. 83.

[44]Following the ms. *quando*, rather than Green-Pedersen's *quoniam*.

consequent.

81. And one should understand that only when the parts are ordered from the part which is the antecedent to the conjunction does the consequence hold. And as it is with a conjunction and disjunction, so should it be understood of the opposite of a conjunction (which is equivalent to a disjunction) and the opposite of a disjunction (which is equivalent to a conjunction).

82. From this to the first argument:[45] when 'Socrates runs or a man runs, therefore a man runs' is accepted, I say that it indeed follows, because 'a man runs' is a consequent of 'Socrates runs'; therefore from the disjunction to 'a man runs' which is the consequent, the consequence holds. But 'Socrates runs or a man runs, therefore Socrates runs' does not follow, because something implies the antecedent that does not imply the consequent; for 'a man runs, therefore Socrates runs or a man runs' follows; but 'a man runs, therefore Socrates runs' does not follow.

83. To the other argument:[46] when 'Socrates runs, therefore Socrates runs and a man runs' is accepted, I say that it indeed follows, because in this consequence the parts are ordered as antecedent and consequent; for 'a man runs' is a consequent of 'Socrates runs'. Hence when the parts of a conjunction are ordered as antecedent and consequent, then the consequence from the part of the conjunction which is antecedent to the conjunction holds. Then 'Socrates runs, therefore Socrates runs and a man runs' does follow. But 'a man runs, therefore Socrates runs

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[45]Par. 77.
[46]Par. 77.

and a man runs' does not follow, because something follows from the consequent that does not follow from the antecedent; for 'Socrates runs and a man runs, therefore Socrates runs' follows; but 'a man runs, therefore Socrates runs' does not follow.

84. On the contrary: I show that 'Socrates runs, therefore Socrates runs and a man runs' does not follow, because from the opposite of the consequent the opposite of the antecedent does not follow; for 'not Socrates runs and a man [runs], therefore not Socrates runs' does not follow, because the negation of a conjunction, as stated previously,[47] does not imply the negation of either part.

I say that it indeed follows, and from the opposite the opposite follows. And when '*from a negation of a conjunction*', etc. is accepted, I say that it is true unless the conjunctive parts are ordered as antecedent and consequent. Therefore, etc.

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[47]Par. 75.

Appendix C

Walter Burley, on consequences

The following provides an English translation of Walter Burley's *de conse-quentiis*, made from Green-Pedersen's critical edition of the text [77]. I follow manuscripts B and P where these make sense, adopting their reading some-what more frequently than the edition.[1] Foliation for the Paris manuscript P is given between slashes. Where the underlying text for the translation differs from that of the edition, this is indicated via footnote. Longer variant readings are given in footnotes. Shorter variants are generally not indicated, except where 1) the text disagrees with the edition; or 2) the alternative read-ing affects a definite change to the meaning of the text and 3) the reading of BP is doubtful.

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[1]The reason for this is a disagreement over the stemma: Green-Pedersen assumes readings unique to B and P are deviations from a prior exemplar. I assume that B and P are the earliest versions of the text; that manuscripts C and L are later local revisions, perhaps from Burley or his circle; and that manuscript F represents an intermediate version of the work between these. The text of manuscript O is a clearly corrupt variant of the tradition represented by C.

C.1 Rules concerning consequences

1. Since we use consequences in testing and refuting sophisms, one ought to know many things about the nature of consequences. And so one should know that this rule is good: *whatever follows from the consequent follows from the antecedent*; and this likewise: *whatever entails (antecedit) the antecedent entails the consequent.*[2] So the consequences following from this rule are necessary.

2. And so this consequence is good: 'If I say you are an animal, I say something true; therefore, if I say you are an ass, I say something true'. For this is established by this rule: *whatever follows from the consequent follows from the antecedent.* Which is shown thus: for 'if I say you are an ass, I say you are an animal' follows; but by the rule, whatever follows from the consequent follows from the antecedent; but[3] 'to say something true' follows from the consequent 'I say you are an animal'; therefore 'to say something true' follows from the antecedent 'I say you are an ass', which is the antecedent. This, then, is a true or good consequence: 'if I say you are an animal, I say something true; therefore, if I say you are an ass, I say something true'. And it is established by this rule: *whatever follows from the consequent, follows from the antecedent.*

3. Solution: I say this must be distinguished, because the act of what is to be said can transfer to the said by reason of the thing or by reason

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[2][202, p. 62.9; p. 67.9; p. 200.11].

[3]but ... but] Therefore if something follows from the consequent 'I say you are an animal', it follows from the antecedent 'I say you are an ass', therefore if C, Since if the consequent 'I say you are an animal' follows from the antecedent 'I say you are an ass', then L.

of the said. In the first way it is false, /f. 19ra/ in the second it is true.

4. Likewise, this rule is good: *whatever entails the antecedent entails the consequent.* And consequences coming down from this rule are necessary, as 'if Socrates is, an animal is'. Being Socrates entails (*antecedit*) being a man, which is antecedent to being an animal; therefore, being Socrates entails entails being an animal, which is the consequent.

5. Again:[4] *whatever entails the opposite of the consequent, entails the opposite of the antecedent,*[5] since in every good consequence the opposite of the consequent implies the opposite of the antecedent, because always in every good consequence the opposite of the antecedent is implied by the opposite of the consequent. But then the opposite of the consequent becomes antecedent to the opposite of the antecedent; and by the rule *whatever entails the antecedent, entails the consequent,* therefore whatever entails the opposite of the consequent, entails the opposite of the antecedent.

6. Likewise: since whatever follows from the consequent, follows from the antecedent, and the opposite of the antecedent is consequent to the opposite of the consequent,[6] it is right that if something follows from the opposite of the antecedent, that it follows from the opposite of the consequent.

7. . Likewise, when one argues *from the first to the last,* one argues by this

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[4]Again] From these two rules follow two other rules. One is CFLO

[5]antecedent] Likewise: whatever follows from the opposite of the antecedent, follows from the opposite of the consequent. *add.* CFLO. Cf. [202, p. 65.18].

[6]consequent] and so the opposite of the consequent is antecedent *add.* CL

rule: *whatever follows from the consequent, follows from the antecedent.*[7] As when one argues so: 'If a man is, an animal is; and if an animal is, a body is; and if a body is, a substance is; therefore *from the first*, 'if a man is, a substance is'.[8]

8. One should know that when one argues *from the first to the last*, where the intermediate consequences are varied, then[9] the consequence *from the first*, etc. is not valid.[10] Note that consequences which are intermediate are varied when the antecedent in the second conditional is different from what the consequent was in the first conditional, as is clear if one argues: 'if no time is, it is not day; and if it is not day and some time is, then it is night; and if it is night, some time is; therefore *from the first to the last*: if no time is, some time is'. This consequence does not hold *from the first to the last*, since the consequent of the first conditional is 'it is not day', and the antecedent of the second conditional is the whole 'it is not day, and some time is', and so *from the first to the last* does not apply.[11]

9. Again: in a simple consequence, it is so, that *the antecedent cannot be true without the consequent.*[12] Hence, if in any simple consequence the antecedent can be true in some time without the consequent, whatever the possible case stipulated, the consequence is not valid.

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[7][202, p. 69.16; p. 200.20].

[8]is.] For here one argues that 'a substance is' follows from 'a body is', and 'a body is' follows from 'a man is' *add.* CO

[9]one argues ... then] there are many consequences, if the consequences are varied from the first antecedent to the last consequent CO, many varied consequences can be made L

[10][202, p. 70.1; p. 200.20].

[11]Cf. [76, p. 7, par. 18].

[12][202, p. 61.30; p. 199.26].

10. Note that from falsehoods truth can follow, but from truths nothing follows besides truth. Hence the saying: 'from falsehoods truth, from truths nothing but truth'.

11. Likewise: in every good consequence, *from the contradictory opposite of the consequent the contradictory opposite of the antecedent should be inferred.* Because the consequence 'a man is, therefore an animal is' is good, therefore from the opposite of the consequent the opposite of the antecedent should be inferred, and thus 'no animal is, therefore no man is' follows.

12. Likewise: from the contradictory of the consequent follows the contradictory of the antecedent, and then the consequence is good.[13][14]

13. And note that[15] for the goodness of a consequence it does not suffice that from the contrary opposite of the consequent the contrary opposite of the antecedent is inferred. For then 'every man runs, therefore every animal runs' follows; since from the contrary opposite of the consequent the contrary opposite of the antecedent is inferred; for 'no animal runs, therefore no man runs' follows.

14. Likewise, this follows from [what has been] said: *from the contingent the impossible does not follow*; for if so, the antecedent can be true without the consequent, because that which is contingent can sometimes be true, and that which is impossible will never be true. And so from

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[13][202, p. 64.1; p. 207.3].

[14]good] Hence to know whether a consequence is valid or not, one should see whether from the contradictory of the consequent the contradictory of the antecedent follows or not. If so, the consequence was good. If not, the consequence was not valid *add.* FPLO

[15]And note that] But F, Hence one should know that L

the contingent the impossible does not follow, since the antecedent can be true without the consequent.

15. Likewise: *from the necessary the contingent does not follow*, on account of the same cause, since the necessary will always be true and the contingent, sometimes false. Therefore, etc.

16. Likewise: *in no good consequence does the opposite of the consequent stand with the antecedent*; for if it stands, the consequence is not valid.[16] And to stand together with another is to be able to be true with it from the same form.

17. Likewise, *from the inferior to the superior without distribution* is a good consequence, if there is no extraneity, as with 'Socrates is an individual, therefore man is an individual'.[17]

18. Likewise if it is not argued with a negation, or with something having the force of negation or distribution, it is a good consequence.[18] For instance, 'only man runs, therefore only man moves' is not valid.[19]

19. Likewise: *from the inferior per se to the superior per se with a negation placed after* is a good consequence,[20] as here: 'a man is not, therefore an animal is not'. Yet *from the inferior per accidens to its superior per accidens with the negation placed after* is not a valid consequence. Hence, 'a white man is not, therefore a man is not' does not follow.

20. Likewise: *from the distribution of a superior to its inferior per accidens*

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[16][202, p. 63.30].
[17]individual] Likewise 'man is a species, therefore animal is a species' *add.* CLO
[18]or distribution... consequence] *om.* CLO
[19]is not valid] *om.* CLO.
[20][202, p. 210.17].

is not a valid consequence except *per accidens*; because if so, it holds by this extrinsic rule: *positing per se posits per accidens*, which rule is not valid. Yet negating the superior negates an inferior *per accidens*; hence 'no man runs, therefore a white man does not run' follows. And this consequence holds by this extrinsic rule: *negating per se, negates per accidens*; which is good, because that which is *per accidens* depends on that which is *per se.*

21. And one should know that these rules commit the fallacy of the consequent:[21] *whatever follows from the antecedent, follows from the consequent*, as with 'if a man is, a risible is; therefore, if an animal is, a risible is'. And one argues by that rule that risible, which is consequent, follows from man which is antecedent, so it follows from animal which is consequent.

22. Likewise this rule is not valid: *whatever entails (antecedit) the consequent entails the antecedent*, as 'if a man is, an animal is; therefore if a man is, an ass is'. And one argues by this rule that since being man entails being an animal, which is consequent, therefore it entails being an ass, which is antecedent.

23. Likewise: *from the inferior to the superior with a negation placed after* is the fallacy of the consequent.[22] Therefore the consequent 'no man is, therefore no animal is' is not valid.

24. Likewise: *from the inferior to the superior with distribution* is the fallacy of the consequent. Therefore 'every man runs, therefore every animal

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[21][202, p. 62.14; p. 200.16].
[22][202, p. 210.21].

runs' does not follow.

25. Likewise: *from the superior to the inferior without distribution* is the fallacy of the consequent. Therefore the consequent 'an animal is, therefore a man is' is not valid.

26. One should know that in every good consequence, distributing /f. 19rb/ the consequent, the distributed antecedent follows, if the consequent is distributable and the antecedent likewise.

27. Against [this]: 'Socrates runs, therefore a man runs' is good; and yet distributing the consequent, the distributed antecedent does not follow, because 'every man runs, therefore every Socrates runs' does not follow, since Socrates, because he is an individual, is not distributable.

28. Likewise, one can argue against the preceding rule so: the consequence 'a man is an ass, therefore a man is an animal' is a good consequence. And yet distributing the consequent, the distributed antecedent does not follow, because 'every man is an animal, therefore every man is an ass' does not follow.

29. Likewise, the consequence 'an animal is a man, therefore a man is an animal' is good. And yet distributing the consequent the distributed antecedent does not follow.

30. To the first argument: I say that this rule ought to be understood [so]: distribution is added to those on account of which the first consequence is good. And because the first consequence was good on account of the predicates, therefore the distribution ought to be added to the predicate, arguing so: 'man is every animal, therefore, man is every ass'.

31. To the other I say that the rule has to be understood [of consequences holding] on account of [its] incomplex [parts]. Now this consequence holds on account of the whole complex. Wherefore, etc.

C.2 On exclusives

32. Likewise, *from an exclusive to a universal with the terms transposed and conversely* is a good consequence in nominative terms. For 'only man is an animal, therefore every animal is man' follows, and conversely.

33. Likewise: *from an exclusive to its prejacent* is a good consequence: hence, 'only man runs, therefore man runs' is a good consequence. And a prejacent is what remains when the sign of exclusion is removed.

34. Likewise: *any exclusive has two expositions*; as 'only man runs' has 'man runs' and 'nothing other than man runs'. And if the exclusive is true, each exposition will be true; and if either exposition is false, the whole exclusive will be false. And they are called expositions, because they lay out (*exponunt*) the understanding of the exclusive. Hence, 'expositions' is like saying 'laying out'.

35. And one should note that the opposite of an exclusive proposition has two causes of truth,[23] either because no man runs, or because something other than man runs (it can have a third cause, namely [that] no man runs nor does [anything] other than man run). Hence, if one argues from an exclusive negative to either of these it is the fallacy of the consequent;

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[23]truth] as (for C) 'not only man runs' can (has to C) be verified (these C) *add.* CL; [202, p. 135.35].

as here 'not only man runs, therefore no man runs'; or '... an other than man runs'. And if the exclusive is true, each exponent will be true, since if either exponent is false, the whole exclusive will be false.[24]

36. And one should know that whenever one argues from a proposition having several causes of truth to one of them it is the fallacy of the consequent.[25]

37. Likewise one should know that a predicate in an exclusive affirmative stands confusedly and distributively, and the subject only confusedly.[26] Therefore the consequence 'only man is an animal, therefore only man is an ass' holds. Proof: 'only man is an animal, therefore every animal is a man' follows, and further 'therefore every ass is a man'. Therefore only man is an ass. Therefore from the first [to the last]. The first consequence holds by the rule posited above,[27] namely *from an exclusive*, etc. The second holds by this posited rule:[28] *from a distributed superior to its inferior per se distributed* is a good consequence. The third holds by this rule:[29] *from a universal to an exclusive*, etc. And the subject stands only confused, because it stands here in just the same was as the predicate in a universal proposition convertible with it.

38. And one should know that exclusive words are 'only' (*tantum*) and 'alone' (*solus*). And it is a rule that an exclusive word added to a subject excludes the predicate from the opposite of the subject; as when saying

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[24]And if ... false] *om.* CLO
[25][202, p. 212.29].
[26][202, p. 133.27].
[27]Par. 32.
[28]Cf. par. 20.
[29]Par. 32.

'only man runs', 'to run' is excluded from everything other than man, and attributed to man. And so it is exposited thus: 'man runs, and nothing other than man runs'.

39. One should know that when an exclusive word is added to a distributed subject, any such must be disambiguated on account of equivocation, because the exclusion can be made general or specific. If the exclusion is made general, then whatever is not included is excluded, and then any [proposition] like this includes opposites, and consequently is impossible.[30] Where the exclusion is made specific, on the other hand, such is possible, because then only species opposite in a certain respect are excluded; and it is excluded[31] so: 'only[32] man runs, and not every lion and not every ox ...'.

40. Likewise, if an exclusion is added to an integral whole the proposition is ambiguous, because the exclusion can be made on account of the form or on account of the matter, as in saying 'only Socrates is white'. If the exclusion is made on account of matter, then the proposition is true, and then it has to be exposited thus: 'Socrates is white, and nothing that is not Socrates or a part of Socrates is white'. If the exclusion is made on account of form, then the proposition is false, and is exposited thus: 'Socrates is white, and nothing that is not *Socrates* is white' - or

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[30]impossible] and so it is exposited so: 'every man runs and nothing other than every man runs'. But *add.* C, for instance 'only every man runs' is exposited so: 'every man runs and nothing which is not every man runs', and so it includes opposites, since 'every man runs' includes 'Socrates runs', and 'nothing that is not every man runs' includes 'Socrates does not run', since 'nothing that is not every man runs, Socrates is something which is not every man, therefore Socrates does not run' follows *add.* L

[31]excluded] exposited CLO

[32]only] every CO, only every L

no part of Socrates is white; and so these would be true together:[33] 'no part of Socrates is white' and 'some part of Socrates is white'.[34]

41. And one should note that any proposition where an exclusive word is added to a word of a proposition is ambiguous according to composition and division.[35] For instance, 'only man is man[36] is true' (*tantum hominem esse hominem est verum*)is ambiguous because the exclusion can be made in respect of the infinitive or the indicative verb; in the first way true, in the second false.

42. And one should know that *from the inferior to the superior with an exclusive word added to the subject* (that is, from the inferior to the superior on the part of the predicate) is not a valid consequence.[37] Hence 'only man runs, therefore only man moves' does not follow.

43. Likewise: *from the superior to the inferior with an exclusive word* is the fallacy of the consequent, such as arguing 'only an animal runs, therefore only a man runs'. Both among its affirmative expositions there is the fallacy of the consequent, as 'an animal runs, therefore a man runs'; and among its negative expositions there is the fallacy of the consequent from the destruction of the antecedent, as here: 'nothing other than an

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[33]or ... together] and then any such includes opposites, since then 'Socrates is white, therefore some part of Socrates is white' follows, and from the other part 'nothing that is not Socrates is white, therefore no part of Socrates is white' follows. Now these are opposites: 'no part of Socrates' C

[34]or ... white] and so it includes opposites, since 'Socrates is white' includes that part of Socrates is white, and the other exponent, namely 'nothing other than Socrates is white' includes that no part of Socrates is white, and so opposites are included L, from this it follows that no part of Socrates is white. This is so that these are opposites 'no part of Socrates is white' and 'some part of Socrates is white' O

[35][202, p. 140.7].

[36]man] animal BFOP

[37][202, p. 134.13].

animal runs, therefore nothing other than man runs'.[38]

C.3   On exceptives

44. What follows is on exceptives. Concerning these, one should know that any exceptive has two expositions,[39] as 'nothing besides Socrates runs' has two expositions, namely 'Socrates runs' and 'nothing other than Socrates runs' /f. 19va/

45. And one should know that exceptive words are 'besides' (*praeter*), 'besides that' (*praeterquam*), 'except' (*nisi*).[40] Yet there is a difference between these, because 'besides' and 'besides that' remove from an affirmative quantitative whole, but the word 'except' removes from a negative quantitative whole. Hence, 'no man except Socrates runs' is spoken properly, and 'every man runs except Socrates' is spoken improperly; yet 'every man besides Socrates runs' is spoken properly.

46. And an affirmative quantitative whole is a common term taken with an affirmative universal sign, and a negative quantitative whole is a common term taken with a negative universal sign.

47. One should know that every exceptive is made from an affirmative or negative quantitative whole, or from a term standing confusedly and distributively. Hence, if an exception were to be made from something other than these, the proposition would be improper; and thus these

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[38]runs] one should know (note C, hence O) that a consequence from an affirmative exclusive to a negative exceptive holds (is good CO). Hence (For instance O), 'only man runs, therefore nothing besides man runs' follows *add.* CLO

[39][202, p. 165.23].

[40][202, p. 164.19].

propositions and others like them are improper: 'some man besides Socrates runs' and 'this man besides Socrates runs'.

48. And one should note that every exception is on account of disparity[41] or conflict (*repugnantiae*), which are the same, since[42] in every exceptive the excepted part conflicts with the prejacent. And the prejacent is what remains, taking away the sign of exception with its case. For instance, the prejacent of 'every man besides Socrates runs' is 'every man runs', 'besides', then, is the exceptive word, 'Socrates' (*Socratem*) is its case. The excepted part is what is removed (*excipitur*) by the exceptive word. Hence 'every man runs' and 'Socrates does not run' conflict, since from this [i.e. the latter] its [i.e. the former's] contradictory would follow. Hence 'Socrates does not run, therefore some man does not run' follows.

49. One should note for a rule that *whenever some proposition implies the opposite of another, they conflict, and conversely*;[43] because conflict is of the nature of a contradictory,[44] and contradictories conflict most of all.

50. And one should note that a term following an exceptive word, if it is a common term, supposits merely confusedly, since it is neither possible to descend under it conjunctively nor disjunctively, as is clear enough, since the subject in an affirmative exclusive and in a negative exceptive proposition have the same supposition.

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[41]disparity] an instance CLO
[42]since] hence CLO
[43]and conversely] and whenever some two conflict, one implies the contradictory of the other O, *add.* C, since one implies the opposite of the other L
[44]contradictory] contradiction CLO

51. And one should note that the consequence *from an exclusive affirmative to a negative exceptive in the same terms* holds. Hence 'only Socrates runs, therefore nothing besides Socrates runs'.[45][46]

52. It is a rule in exceptives that every proposition true in part and false in part can be made true by excepting the false part;[47] as when every man other than Socrates runs, and Socrates does not run, then 'every man runs' is false on account of Socrates; and if Socrates is excepted so: 'every man besides Socrates runs', this proposition will be true.

53. Likewise, one should know that in a negative exceptive the predicate is attributed to the excepted part, and removed from everything else contained under the subject of the prejacent. For instance, if I say 'no man besides Socrates runs', running is attributed to Socrates and removed from everyone other than Socrates.

54. Likewise: in an affirmative acceptive the predicate is removed from the excepted part and attributed to all the others contained under the subject of the prejacent;[48] as in 'every man besides Socrates runs', running is removed from Socrates and attributed to all others contained, etc. And so it is exposited so: 'every man other than Socrates runs, and Socrates does not run'.

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[45]since it ... runs] Since a common term following an exceptive word stands in the same way as a subject in an affirmative exceptive; but a subject in an affirmative exceptive stands merely confusedly. Hence, all these convert: 'only man runs', 'every running is man' and nothing besides man runs' L

[46]And ... runs] and the predicate (*om.* O) in a universal (*om.* O) proposition with terms transposed, since all these convert in convertible terms, since they have the same suppositions. (Hence these say the same *add.* C) 'only man runs' and 'nothing besides man runs' and 'every running is man' CO

[47][202, p. 166.13].

[48][202, p. 166.34].

55. And one should note that every exceptive conflicts with its prejacent. Hence, the exceptive is in[49] an instance of the prejacent; for 'every man runs' and 'every man besides Socrates runs' conflict, because 'every man besides Socrates runs, therefore Socrates does not run' follows, and lastly 'therefore not every man runs', which contradicts 'every man runs'.

56. This rule holds generally, that *from an exceptive an exclusive [follows]*, just as 'nothing besides Socrates runs, therefore only Socrates runs' follows.[50]

57. And one should note that[51] in every exceptive four are required,[52] namely that from which the exception is made, the excepting word, the excepted part, and that in respect of which the exception is made; as is shown in 'Every man besides Socrates runs': man is that from which the exception is made, 'besides' is the exceptive word, Socrates is the excepted part, and running is that in respect of which the exception is made.

58. With respect to the supposition of the predicate and subject in an exceptive, one should know that that from which the exception is taken, or the subject of the exceptive (these are the same), always stands confusedly and[53] immovably with respect to the exception, and movably with respect to the predicate. With respect to the exception it stands im-

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[49]in] *om.* CLO
[50]an exceptive ... follows] every exclusive follows an exceptive, just as 'only man runs, therefore nothing besides man runs' follows CO, *mut.* L
[51]And ... that] since B(O)P
[52][202, p. 165.16].
[53]confusedly and] distributively *add.* CLO

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movably, because it is not possible to descend with respect to it; hence 'every man besides runs, therefore Plato besides Socrates runs' does not follow.[54] With respect to the predicate it is possible to descend, since every man besides Socrates runs, therefore Plato runs, and so on for singulars.[55] The predicate in an exceptive stands sometimes movably, confusedly, and distributively; sometimes immovably, confusedly, and distributively. It supposits movably if the exception is taken from a transcendental; e.g. 'no being besides an animal is a man'. Here, the exception is taken from a transcendental, and so it is possible to descend under the predicate. Hence, all these are transcendentals: being (*ens*), thing (*res*), something (*aliquid*), and one (*unum*). The predicate in an exceptive supposits in the second way when the exception is taken from something specific, as here: 'no animal besides man is running'; hence here it is not possible to descend under the predicate saying this: 'no animal is this running'.[56]

59. Likewise one should know that if the prejacent is true, the exceptive will be false, and if the exceptive is true, the prejacent will be false.[57] For instance, if 'no man runs' is true, then 'no man besides Socrates runs' will be false; and if /f. 19vb/ 'no man besides Socrates runs' is true', 'no man runs' will be false. But it need not be that if the prejacent is false the exceptive is true, because 'no animal is a man' is false, and likewise 'no animal besides an ass is a man'. Hence an exceptive and its

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[54]follow.] Yet *add.* CFL, *def.* O
[55]singulars] follows *add.* CFL, *def.* O
[56]this running] a running man C(L), *def.* O
[57][202, p. 169.30].

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prejacent can be false together, but never true together.

60. One should know that the opposite of any exceptive has two causes of truth, since the contradictory of an exception[58] 'not no man besides Socrates runs' has these: 'someone other than Socrates runs' and 'Socrates does not run'. And so if one argues so: 'not no man besides Socrates runs, therefore some man other than Socrates runs', it is the fallacy of the consequent from a proposition having several causes of truth to one of them.[59] Likewise 'not no man other than Socrates runs, therefore Socrates does not run' is the fallacy of the consequent, since it has several causes of truth.

C.4 On the words 'differs', 'not the same', etc.

61. One should know that this rule is general: *whatever makes an immovable movable makes a movable immovable, and conversely.* E.g. the negation 'not' makes a movable immovable in saying 'not every man runs'; here 'man' stands immovably, yet before adding the negation it stood movably. The negation 'not', then, makes a movable immovable and an immovable movable. For instance, in saying 'not man runs', 'man' stands movably, and before adding the negation it stood immovably.

62. Likewise one should know that when one argues from something stand-

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[58]since ... exception] as contradictory opposites, hence C, I call causes the contradictories of the exponents, hence the exceptive L

[59][202, p. 212.29].

ing immovably to it standing movably it is the fallacy of figure of speech, as here: 'you differ from anything (*a quolibet*), therefore anything differs from you', because in 'you differ from anything', 'anything' stands immovably, and in 'anything differs from you' 'anything' stands movably, since it is not immovable by virtue of the negation brought in by 'differs'. For this is valid,[60] '[you differ from anything, therefore] something you are not the same as', since 'anything' and 'something' are not equivalent.

63. And one should note that from a difference with respect to the prior follows a difference with respect to the posterior. Hence, 'you differ from man; therefore you differ from this man, and from that ...' follows. And[61] that is prior from which a consequence of subsistence is not converted. For instance, from man follows animal and not conversely, and so animal is prior.

64. From the first rule it follows that *whatever differ in genus differ in species*; and *whatever differ in species differ in number*. For instance, man and rock differ in species, and [thus] in number. Numerical difference is between two individuals of the same species.

65. One should know that these three have the same diffusive force (*vim confundendi*), namely 'differs' (*differt*), 'not the same' (*non idem*) and 'other' (*aliud*). Hence 'Socrates differs from man, therefore he differs

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[60]For ... valid] Hence 'you differ from anything' is equivalent (equipollent C) to 'you are not the same as any', and this is equivalent (equipollent C) to CL

[61]and ... And] and this is (*om.* C) because 'differs' (*om.* C) has the force (in L) of confusing a term (*om.* C), just as negation. (How we ought to know *add.* L) what is prior and what posterior, Aristotle teaches us (*om.* L) in the book of *Categories*, saying CL

from this man'[62] follows, and likewise 'Socrates is other than an ass, therefore he is other than this ass', and likewise 'Socrates is not the same as an ass, therefore he is not the same as this ass'. But there is a difference between 'distinct' (*alter*) and 'other' (*aliud*), because 'distinct' is aptly suited to differentiate an accidental term and 'other' a substantial term. Hence 'Socrates is not other than an ass, therefore from this ass' follows; and so 'Socrates is distinct from white, therefore distinct from this white' follows.

C.5   On conditionals

66. Concerning the suppositions of terms in a conditional, note that both the subject of the antecedent and the predicate supposit confusedly and distributively, and this with respect to the consequent. For 'if Socrates runs, a man runs' follows; and 'if a man runs, an animal runs'. Therefore *from the first*, 'if Socrates runs, an animal runs'. The consequence holds by this rule: *whatever follows from the consequent, follows from the antecedent.* E.g. Socrates running is the antecedent and man running is the consequent; therefore if from man running follows animal running, it should follow from its antecedent, which is Socrates running.

67. Likewise, on the part of the predicate both the subject and the predicate in a conditional in a consequent stand particularly, since it can be inferred from singulars. For 'if Socrates runs, a man runs; therefore if Socrates runs, an animal runs' follows. And it holds by this rule:

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[62]Reading 'man' instead of 'ass' in both places, with mss. BFP.

whatever entails the antecedent entails the consequent.

68. And one should know that nothing more is required for a good conditional than that if the antecedent is true, the consequent will be true. And so a conditional will be good though the antecedent would be false; as 'if man is a stone, man is not an animal'.

69. On should know that some conditionals hold *as of now*, and some *simply*.[63] An example of the first: 'If you are in Rome, then that which is false is true'; when you are not in Rome, the consequence is good, but when you are in Rome it is not valid. And an as-of-now consequence is when the antecedent as of now cannot be true without the consequent.

70. Simple consequence is divided so: some is natural and some is accidental.[64] A natural consequence is when the consequent is in the understanding of the antecedent, nor can the antecedent be true unless the consequent is true; as 'if a man is, an animal is'. An accidental consequence is twofold: some holds on account of the terms or on account of the matter, as 'God exists is true, therefore God exists is necessary'; and this holds on account of the terms or the matter, since truth in God and necessity are the same. Some accidental consequence is so: *from the impossible anything follows*; and *the necessary follows from anything*. An example of the first: 'You are an ass, therefore you are a goat, and a stone...', etc. An example of the second, as 'you are running, therefore God exists'. Again, an accidental consequence is when the consequent is not in the understanding of the antecedent.

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[63][202, p. 60.28].
[64][202, p. 61.6].

71. Again: from the impossible anything follows by the topic *from the less*.[65] Because if one argues so - 'man is an ass, therefore you are running' - this consequence holds by this rule, namely if that which seems less to inhere also[66] inheres, then that which seems to inhere more /f. 20a/ will inhere; but it seems less to inhere that ass inheres in man than running inheres in man.

72. Likewise: the necessary follows from anything by the topic *from the less*.[67] Hence 'you are running, therefore God exists' follows. And it holds by this rule, namely that given above, because it seems less that running inheres in man than that being inheres in God. But a consequence which is, namely *from the impossible anything follows*, holds by the topic *from the less*. Yet *from the opposite* [of the consequent the opposite of the antecedent follows] by the topic *from the greater*; for instance, if it is so - 'you are not running, therefore you are not an ass', it holds by this rule: if that which seems more to inhere also[68] does not inhere, neither will that which seems less to inhere inhere.

C.6 On oppositions

73. What follows concerns rules for correcting consequences, which certain rules[69] are handed down by Aristotle in the book *On interpretation* and in *Prior* [analytics] I. So now we must speak of these. The first rule

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[65][202, p. 61.12].
[66]also] *om.* CL, *def.* O
[67]less] greater B(C)P, *def.* O
[68]also] *om.* L, *mut.* C, *def.* O
[69][5, 6, p. 17a.25]; [8, I, 46, p. 51b.5].

is: *from the affirmation of an indefinite predicate follows the negation of a definite predicate*; for 'man is non-just, therefore man is not just' follows'.

74. And this rule should be understood so: if it is argued in nominative, absolute terms, with a present-tense substantive verb; for the consequence is not valid for a future-tense verb, nor for a past-tense verb. Hence, 'you were non-white, therefore you were not white' does not follow, since the antecedent can be true and the consequent false, positing that in time past you were white and non-white. Nor is it valid for a future-tense verb, since 'you will be non-sighted, therefore you will not be sighted' does not follow, since the antecedent can be true and the consequent false; for this is true 'you will be non-sighted' positing that a year from now you will be blind; and 'you will not be sighted' is false, because tomorrow you will be sighted positing that tomorrow you will see and not in a year.[70]

75. The rule does not hold in declined [terms], as 'the eye is a part of a non-man, therefore the eye is not a part of man', since the antecedent is true and the consequent false.[71] Likewise, neither does it hold formally in transitive verbs (*verbis adjectivis*); since 'you see a non-man, therefore you do not see a man' does not follow, since positing that you see a man and an ass, the antecedent is true and the consequent false.

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[70]and 'you ...' year] and not before, then the antecedent is true and the consequent false L and then the consequent is false O

[71]false] because the eye is part of a horse, which is not a man *add.* C, this is false 'the eye is not a part of man', and yet this is true 'the eye is part of a non-man', since it is part of a horse *add.* L, 'the eye is part of a non-man' is not true, since it is part of a horse, which is not a man *add.* O

76. Likewise it doesn't hold generally in respective and relative terms; since 'you are a non-father, therefore you are not a father' doesn't follow, since positing that you are a father and a son, the antecedent is true and the consequent is false; for 'you are a non-father' is true, since you are a son, and a son is a non-father.

77. A second rule is that *from the affirmation of a definite predicate follows the negation of an indefinite predicate.* For 'man is just, therefore man is not non-just' follows.

78. A third rule is that *from the affirmation of a privative predicate follows the negation of a definite predicate, and not conversely.* Hence, 'man is unjust, therefore man is not just' follows; and not conversely.

79. Note that privative terms are 'unjust', 'impossible', 'incongruent', etc.

80. A fourth rule is that *from the negation of a definite predicate follows the affirmation of an indefinite predicate* in simple terms both in utterance and understanding, such as are 'man' and 'animal' and 'white' taken formally and others like them. For 'Socrates is not an animal, therefore Socrates is a non-animal' follows. In composite terms, though, it is not valid; since 'man is not a white tree, therefore man is a non-white tree' does not follow, since the antecedent can be true and the consequent false. The falsity of the consequent is clear, since it implies a falsehood, namely 'man is a tree'. The consequent is false, therefore the antecedent by the rule posited earlier.[72] For the rule posited earlier is that if the consequent is false, then the antecedent is false, since falsehood does

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[72]Par. 9-10; 68.

not follow except from falsehoods.

81. A fifth rule is this: *from the negation of a privative predicate the affirmation of a definite predicate does not follow.* Hence 'man is not unjust, therefore man is just' does not follow, since the antecedent can be true and the consequent false, positing that no man is.[73]

C.7   On syllogistic consequences

82. The rules proving syllogistic consequence are two. The first is *if from the opposite of the conclusion of a syllogism with the other premise the opposite of the other premise follows, the first syllogism was good.*[74]

83. And the second rule is *what does not follow from the antecedent with something added,* either syllogistically or in another way, *does not follow from the consequent with the same added.*

84. And from this rule follows another, *whatever follows from the consequent with something added follows from the antecedent with the same added,*[75] namely necessarily or formally.[76] For since from 'every man is an animal' with 'Socrates is a man' follows 'Socrates is an animal' so from 'every substance[77] is an animal', (which entails 'every man is an animal') with 'Socrates is a man' follows 'Socrates is an animal'; since

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[73]is] Besides the mentioned (aforementioned O) rules, there are (still *add.* O) certain rules which prove consequences, and (*om.* O) these are in general of two types, because some (are those which *add.* O) prove syllogistic consequences, and (*om.* O) some enthymematic *add.* LO

[74]cf. [202, p. 65.10].

[75][202, p. 62.24].

[76]namely ... formally] *om.* CL

[77]every substance] man BF, Socrates P, *def.* C

the first consequence is syllogistic, the second is necessary.

85. Likewise: *whatever follows from the antecedent and the consequent follows from the antecedent per se.*[78] For since from 'this ass is only one man' with 'this ass is a man' which is consequent upon it, the conclusion 'a man is only one man' follows formally, thus from 'this ass is only one man' follows the same conclusion; at least just as from the impossible anything follows. Yet the way something follows from an antecedent and consequent need not be wholly the same as how it follows from the antecedent *per se.*

C.8   On enthymematic consequences

86. The rules proving an enthymematic consequence are these: *the opposite of the consequent does not stand with the antecedent, and*[79] *therefore the consequence is good.*[80] Since 'no animal is' does not stand with 'a man is', thus 'a man is, therefore an animal is' follows. What the antecedent stands with the consequent stands with; for what 'a man is' stands with, 'an animal is' stands with.

87. From this rule follows this: *what conflicts with the consequent also*[81] *conflicts with* /f. 20rb/ *the antecedent.*[82] For since 'no animal is' conflicts with 'an animal is', so it conflicts with the antecedent, which is 'a man is'.

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[78][202, p. 62.22].
[79]and] *om.* CLO
[80][202, p. 64.1]; cf. par. 16.
[81]also] *om.* CFLO
[82][202, p. 63.1].

88. And this rule follows from the prior: *what stands with the antecedent, stands with the consequent.*[83] Likewise *from the opposite*: what does not stand with the consequent does not stand with the antecedent. And to not stand with another is to conflict with it. Therefore, what conflicts with the consequent conflicts with the antecedent.

89. Another rule is: *if the antecedent is, the consequent is.* The consequence is clear, since being and being true convert.

90. Another rule is: *if the antecedent is possible, the consequent is possible,* following from what is proven in the book of the *Prior Analytics.*[84]

91. Another rule is: positing the possible in being, the impossible does not occur. Which is proven so, since if the impossible were to occur from positing the possible in being, since the antecedent to it is possible, from the possible the impossible could follow. And so positing this in being 'you being in Rome is possible, therefore you are in Rome', [would follow]. And if it happens to be false, it is still not impossible.

92. Another rule is this: From the otherness (*alietas*) of the consequent follows the otherness of the antecedent. Hence, 'Socrates is other than animal, therefore Socrates is other than man' follows. Against this rule it is objected so: 'man is man, therefore man is an animal'; yet 'an other than man is an animal, therefore an other than man is man' does not follow, since the antecedent is true and the consequent false. One should say to this that the rule has to be understood of a consequent in following and predicating (*consequens in consequendo et praedicando*),

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[83][202, p. 63.7].

[84][8, I, 13, p. 32b.26].

because animal is consequent upon man, for it is predicated of man, and so is consequent. And it is not so with the consequent 'man is an animal', since it is only consequent in following.

93. Another rule is: from any proposition follows its statement to be true, just as 'man is, therefore that man is is true' follows. And the statement of a proposition is when the nominative is turned into an accusative, and a verb in the third-person into the infinitive mode, as 'man is' (*homo est*) is converted into 'that man is' (*hominem esse*).

94. One should note that these three are really the same: the utterance (*enuntiabile*), the statement (*dictum*), and the infinitive phrase (*infinitiva oratio*).

95. Another rule is that *everything having itself by addition with respect to another is inferior to*[85] *it*. And so 'white man' and the like are inferior to 'man' only[86] accidentally. And so in such consequences as 'a white man is, therefore a man is' here one proceeds *from the inferior to the superior*.

96. Another rule is: *subjects being under*[87] *the same predicates varied according to below and above, the consequence holds.* For instance, 'Socrates was white in *a*, therefore Socrates was white'; and let *a* be yesterday.

97. Another rule: *A disjunctive follows from either of its parts*;[88] as 'Socrates

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[85]to] with BF

[86]only] at least O, *def.* CL

[87]under (*in*)] *om.* CLO

[88]parts] likewise a disjunct[ion] signifies nothing other than [what is] determinate[ly] implied by either part *add.* L, likewise a disjunct not signified or not demonstrated is implied by either part *add.* O; [202, p. 116.12].

runs, therefore Socrates or Plato runs'.[89]

98. Another rule is: *the negation of a disjunct is equivalent to a conjunction.*[90] And so this consequence will be good: 'not Socrates or Plato runs, therefore neither Socrates nor Plato runs'; since these two propositions are equivalent.

99. Another rule is: *a negation of a conjunction*[91] *has three causes of truth.* For instance, 'not Socrates and Plato runs', either that Socrates does not run, or that Plato does not run, or that neither Socrates nor Plato runs. And if one argues from the negation of this conjunction to either of its [disjuncts] it is the fallacy of the consequent.

100. Another rule is: *the opposite of a conditional should be given by placing a negation before the whole.*[92] Hence the opposite of 'if a man is, an animal is' is 'not if a man is, an animal is', which is equivalent to a man being and no animal being. And these do not stand together.

101. Another rule is that *in singulars, placing the negation before or after makes no difference.* Hence, the proposition 'Socrates runs' is contradicted by each of 'Socrates does not run' and 'not Socrates runs'. The reason for this rule is that singulars do not have many supposita under them, so placing the negation before or after makes no difference.

102. Another rule is that *in necessities speaking universally or particularly makes no difference.* Hence it makes no difference saying 'man is an

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[89]runs] an example of the second (does not follow *add.* O) 'man is man, therefore man (is *add.* O) or non-man is man' *add.* LO

[90][202, p. 113.5].

[91]of a conjunction] *om.* BFPL, *def.* C

[92][202, p. 209.16].

animal' and 'every man is an animal'. Against this rule it is objected so: 'an animal is a man' is necessary', and yet saying 'an animal is man' and 'every animal is man' makes a difference with respect to truth and falsehood, since the first is true and the second false. One should say that the rule has to be understood where the predicate is direct and proper, and it is not so in the proposed, since in 'an animal is a man' the inferior is predicated of the superior.

103. Another rule is this: *if one of opposites is ambiguous, the other will be ambiguous.* And the rule is general for all complex opposites. And if one of opposites is ambiguous for the reason that in one proposition it is ambiguous, for the same reason in the other, and consequently in both. Hence one should know that 'every dog runs' is ambiguous, so what appears opposite to it will be ambiguous, namely 'some dog does not run'. And the cause is that the reason for ambiguity remains in each under each[93] of the terms, i.e. in the term[94] 'dog'. And note that this rule ought to be understood of complex opposites. If we wish to extend this rule to the opposition that is between complexes,[95], and likewise of privative and relative opposites, then one objects to this rule so: this must be disambiguated 'none running, you are an ass',[96] yet its opposite need not be disambiguated, namely 'something running you are an ass'. One should say that the cause of ambiguity does not remain the same in

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[93]under each] *om.* CF

[94]in each ... term] the same in each, namely the term LO

[95]complexes] *ita* FOP '*complexa*' simples (*incomplexa*) B, these complexes L, *def.* C

[96]ass] because the negation can negate the concomitance of these accidents or the formal (*om.* O) composition (of forms O) implied by the participle 'running' but (and O) *add.* LO

each, since the first must be disambiguated by reason of the negation, so that the negation is the whole cause of the distinction. Since this does not remain in its opposite, therefore etc.

104. Another rule is: in accidents, /f. 20va/ the consequence *from inherence to being* does not hold. Hence 'whiteness is in Socrates, therefore Socrates is white' does not follow, since positing that it is only in Socrates' teeth, then the antecedent will be true, the consequent being false.[97] And note that only in accidents does the consequence *from inherence to being* fail.

105. Another rule is: in contradictories the consequence *from the contrary* holds.[98] And a consequence holds from the contrary when from the opposite of the consequent follows the opposite of the antecedent, since[99] from the antecedent follows the consequent. For instance, from a man being follows an animal being, so from no animal being follows no man being.

106. Another rule is: *in contraries and privations and relatives, a consequence holds in these themselves.* And for a consequence to hold in these themselves is when from the opposite of the antecedent follows the opposite of the consequent, just as from the antecedent follows the consequent. So[100] from 'Every man runs' follows 'Socrates runs', so that from 'no man runs' follows 'Socrates does not run', which is opposed to

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[97]positing ... false] the antecedent is true for whiteness in the teeth and (*om.* C) the consequent (*om.* C) false (*om.* C) CLO

[98][202, p. 64.20].

[99]since] just as L, *def.* CO

[100]So] Just as LO, *def.* C

the previous consequence. And this rule is good in complex contraries, when that which is inferred from one of contraries is inferred from it formally and on account of distribution. Hence, though 'no man is one man alone, therefore Socrates is not Plato' follows, nevertheless 'Every man is one man only, therefore Socrates is Plato', does not follow, since 'no man is one man only' implies 'Socrates is not Plato' by reason of negation and by reason of distribution.

107. Another rule is: *if the abstract from the abstract, then the concrete from the concrete.* Hence if 'whiteness is a color' is true, 'the white is[101] colored' will be true. Likewise, *if the concrete from the concrete, then the abstract from the abstract*; for instance, 'if the white is colored, white is a color' as is clear. Against this rule one argues so: 'the white is musical' is true, yet 'whiteness is music' is false; therefore, etc. One should say that this rule has to be understood so: if one concrete is posited *per se* on account of form and in concrete things pertaining to matter and form. Hence, the second conditional is lacking here, namely 'man is an animal, therefore humanity is animality'. And one should note that a concrete thing pertaining to a subject[102] is as 'man' and 'animal', a concrete pertaining to matter and suppositum[103] is as 'white' and 'black'.

108. Another rule is: *the middle being a particular object (hoc aliquid), the extremes must be conjoined.* Therefore this syllogism is good 'this man

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[101]is] will be BP, *mut.* C
[102]subject] suppositum L, substance B, *mut.* C, *def.* O
[103]and suppositum] and form *ed.*, *om.* L, *mut.* C, *def.* O

is an animal, this[104] man is a man, therefore a man is an animal'. And so a middle is a particular object when the middle is some singular, as 'this' or 'that' and so on for singulars. Against this rule it is argued thus: this syllogism is not valid: 'this man is an individual, this[105] man is a man, therefore man is an individual', since the premises are true and the conclusion false. One should say that the rule has to be understood where the middle is a particular object and not varied; but the middle in the proposed is varied, wherefore it is not valid.

109. Another rule is: *when the extremes convert, it is necessary for the middle to convert with the extremes.* For instance, if one argues 'every b is a, every c is b, therefore every c is a'; and so[106] c and a, which are the extremes, convert, it is necessary that b, which is the middle, converts with them. Against this rule one argues thus: 'every man is capable of braying, every ass is a man, therefore every ass is capable of braying'.; the extremes, namely 'ass' and 'capable of braying' convert, and yet that which is the middle does not convert with them, that is with 'ass' and 'capable of braying'. One should say that this rule has to be understood of unqualifiedly probative syllogisms (*syllogismo simpliciter ostensivo*), where the premises are true, hence if the premises are true, falsehood cannot arise.[107]

110. Another rule is this: *a proposition having one cause of truth is true simply, notwithstanding that it have several causes of falsehood.* And

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[104]this] and that CO, *om.* P
[105]this] that F, and that O
[106]so] BFP, if O, here L, *def.* C
[107]hence ... arise] but in the proposed the premises are false, hence the truth of the premises standing, an exception (*instantia*) cannot be found (there *add.* O) LO

the reason for this is that a cause of truth entails (*antecedit*) what it is the cause of. But if the antecedent is true, then the consequent will be true. But it does not follow if the antecedent is false that the consequent is false. And so notwithstanding that a proposition have several causes of falsehood, if it yet have one cause of truth, it will be called true.

111. Another rule is this: *when something inheres in many things, and it does not inhere in one by another, it inheres first in something that is common to them.* For instance: risibility inheres in Socrates and Plato, and not in Socrates through Plato or the contrary; therefore, it inheres first in man. Against this rule one argues thus: the property which is 'being primarily generated' is in Socrates and Plato, and not in one through the other, for 'Socrates is primarily generated' and 'Plato is primarily generated' is true; Therefore, it inheres primarily in man. Which is false, since 'man is primarily generated' is false, because generation is a real act primarily of singulars. One can say that the rule has to be understood in things ordered essentially. But Socrates and Plato are not of this type. Therefore, etc. Or one can say that the passion 'being generated' inheres in Socrates and Plato through something common in them, and inheres in that common primarily. And that common can be called a concrete universal (*universale concernens*).

112. Another rule is: *if the cause is a particular in its proper act, its effect will be proper.* For instance, if there is building, then a house is being made'; since for a house to be made is the proper effect of building.[108]

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[108]building] LO, a builder BFP, *def.* C

113. Another rule is: *if [one] opposite is said of [another] opposite, then the other of the other (propositum de proposito).* For instance, man and non-man are opposed, ass and non-ass are opposed; if man, then, is a non-ass, an ass will be a non-man. Against this rule one argues so: 'every man' and 'not /f. 20vb/ every man' are opposed; and Socrates and non-Socrates are opposed; if, then, not every man is non-Socrates, every man will be Socrates; therefore Socrates will be every man; which is impossible. One should say that this rule has to be understood where one of the opposites of one order does not extend itself to two of the other order. But 'every man' in the proposed extends itself to Socrates and non-Socrates, for instance, to Plato. So this consequence is not valid.

114. Another rule is: *of anything, one of contradictories is said; of none both.* As 'man' and 'non man', which are incomplex contradictories, both one or the other of them is said of anything, and both of nothing. For 'an ass is a non man' is true. Therefore, 'non-man' is said of anything distinct from man, and 'man' is said of any man. Since everything that is is either man or non-man, it follows that one or the other of these is said of anything. Against this rule one argues so: running and non-running contradict; and yet neither of these is true of Socrates alone, since neither 'Socrates alone runs' nor 'Socrates alone does not run' is true; therefore the rule is not valid. One should say that the rule has to be understood thus: one of contradictories is said of any simple, and both of none. Now 'Socrates alone' is not simple, but rather composite.

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115. Another rule is this: *A good consequence holds by a necessary middle.* Hence, the consequence is[109] good 'a man is, therefore an animal is'footnoteis'] holds *add.* L by the middle 'every man is an animal'. The reason for this rule is this, that the middle is the cause of the consequence, but if the effect is, it must be that the cause of that effect is. And if the consequence is also necessary,[110] it must be that the middle is true at every time, and consequently necessary, since whatever is true and unable to be false is necessary. And this must be understood of simple consequence, since it holds for every time.

116. One should know that a consequence is always good when it holds through a true middle. But an as-of-now consequence holds through a middle true as of now, just as 'if the Antichrist exists is so, the false is true' holds by the middle 'that the Antichrist exists is false'. But a simple good consequence holds through an intrinsic necessary middle; but an accidental consequence holds through an extrinsic middle, such as 'an accident[111] is, therefore a substance is'.[112][113]

117. Another rule is this: *every good consequence has to be reduced to a syllogism.* In different ways, though: for a consequence holding by a descent made under the subject has to be reduced to a syllogism by the

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[109]is] *om.* LP, *def.* CO

[110]also necessary] necessary F, good CLO

[111]accident] L, animal BFP

[112]such ... is] the cause (*principio*) of an accident presupposes the being of a consequence; a consequence by an accidental suppositum, that its conclusion holds by a hypothetical middle, for the consequence 'a white man is, therefore a man' holds by the middle 'a white man, if he is or when he is, is a man' O, *om.* C

[113]is] holds by this extrinsic middle: *the being of an accident presupposes the being of substance.* The consequence *from an inferior per accidens to the superior per se* holds by a composite middle, for the consequence 'a white man is, therefore a man is' holds by the middle 'a white man, if he is or when he is, is a man' *add.* L

assumption of a minor; as the consequence 'every man runs, therefore Socrates runs' has to be reduced to a syllogism by the assumption of the minor 'Socrates is a man'. If an ascent is made above the predicate, it has to be reduced to a syllogism by the assumption of a major; as this,[114] 'every man is a man, therefore every man is an animal' has to be reduced into a syllogism by the assumption of the major 'everything that is a man is an animal, every man is a man, therefore every man is animal'.

118. Against this: the consequence 'every man is an animal, therefore only an animal is a man' is good; and yet it does not have to be reduced to a syllogism. Just as the consequence 'a man runs, therefore an animal moves' is also not reduced to a syllogism, since there are four terms. One should say that consequence is twofold, since some is which holds with respect to the whole structure (*totius complexionis*), and some holds with respect to the terms.[115] The first sort does not need to be reduced to a syllogism. All conversions are of this sort. Likewise, when one argues *from an exclusive to a universal with the terms transposed and conversely*, it does not need to be reduced to a syllogism, since it holds by reason of the whole structure. To the other, one should say that it has to be reduced to two syllogisms. Hence the understanding of the rule is: every consequence holding in respect of incomplex [parts], or formally of an incomplexity, has to be reduced to a syllogism or syllogisms. The reason for this rule is that in every genus there is one

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[114]this] consequence is good *add.* BFP, *mut.* C
[115][202, p. 84.20].

first, which is the ruler and measure of all existing in that genus, just as the ounce in the genus of weights and whiteness in the genus of colors. In this way, the syllogism is one first in the genus *habit*, from which all habits receive evidence of necessity. So all other habits have to be reduced to a syllogism, as the posterior to what is prior to it.[116]

119. Another rule is this: *from every exercised act follows the act signified*;[117] as 'every man besides Socrates runs, therefore Socrates is excepted from man' follows; likewise, 'no man is an animal, therefore man is denied of animal'. And one should note that the word 'excepted' signifies an exception, which the verb[118] 'besides' and 'alone' exercises; and 'none' and 'not' exercise negation, which the verb 'is denied' (*negatur*) signifies; 'if' and[119] 'therefore'[120] exercise consequence, which the word 'follows' or 'implies' signifies. But since these do not seem especially useful, the converse of these rules should be noted; this is valid as a general rule: *from any signified act follows the exercised act.* Hence, 'man implies animal, therefore if a man is, an animal is' follows. Likewise, 'man is denied of animal, therefore no man is an animal'. Likewise, 'man is denied of white, therefore no man is white', etc.

120. Another rule is: *from the inferior to the superior with a sign of otherness is not a valid consequence.* Hence, 'Socrates is other than an ass,

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[116]In this way ... it] in the genus of habit there is one (right O) principle, namely that from which all (other O) habits receive (have O) evidence (and necessity *add.* O) and (so *add.* O) all other (*om.* O) habits have to be reduced to this, as the posterior has to be reduced to its prior LO

[117][202, p. 219.1].

[118]verb] adverb BFO

[119]'if' and] likewise if B, Just as F, if LO, *def.* C

[120]therefore] *om.* BFP, *def.* C

therefore he is other than an animal' does not follow. Nor is such a consequence with a sign of difference valid, just as 'Socrates differs from a stone, therefore he differs from substance' does not follow, since the antecedent is true and the consequent is false. The falsity of the consequent is clear, since 'you differ from substance, therefore, you differ from this substance', pointing to your substance, follows, since 'differs' has the force of diffusing a common term to which it is immediately /f. 21ra/ joined. The truth of the antecedent is clear, since 'Socrates is, and a stone is, and Socrates is not a stone, therefore he differs from a stone' follows; since from the denial of something of another, assuming the being of the extremes, the otherness, that is the privation, of the one from the other follows.

121. Another rule is this: in relatives, a consequence *from qualifiedly to unqualifiedly (a quo ad simpliciter)* holds; since 'a mountain is small with respect to a greater one, therefore a mountain is small' follows;[121] since great and small are in the genus of relation. Likewise, 'two cubits is not double[122] with respect to three cubits, therefore two cubits is not double'.[123] Likewise 'Two cubits is double with respect to [its] half, therefore two cubits is double and not double',[124] since 'double' and 'not double' are relatives.

122. Another rule is this: *positing the possible, the necessary should not be denied, nor the impossible conceded.* For instance, if I posit that you are

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[121]follows] likewise (and O) 'a mountain is (*om.* L) small (large O) with respect to a large (small O), therefore a mountain is small (large O) follows *add.* LO
[122]not double] non-double CFLP
[123]not double] non-double CO
[124]and not double] *om.* CLO

in Rome, one ought not therefore concede that you are not an animal, nor something else impossible. Neither this statement nor some other necessary one should be denied, since to deny the necessary is to concede the opposite of the necessary; But the opposite of the necessary is the impossible; therefore, to deny the necessary is to concede the impossible.

123. Another rule is this: *from the attribution of some predicate to some subject universally follows the removal of the same predicate from the opposite of the subject particularly*; since 'every man is an animal, therefore some non man is not an animal' follows. This consequence can be proven thus: since 'every man is an animal, therefore every non-animal is a non-man' follows by conversion by contraposition; and further 'therefore a non-man is not an animal'[125] by this rule: *from an affirmative with an indefinite predicate follows a negative with a definite predicate.* Against this one argues: if this is given, falsehood follows from truth, because then the consequence 'Every non-man is imaginable, therefore some man is not imaginable'. One should say that the *from the attribution of some [indefinite] predicate*, etc. has to be understood in simples according to utterance and understanding, when it is such that the predicate in a universal does not apply to the opposite of the subject, as is[126] true in the proposed, since 'imaginable' is the same as 'imaginable thing'. And so it is not valid.

124. Another rule is: the consequence *from being predicated as a third adjacent to being predicated as a second adjacent* holds,[127] where the op-

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[125]not an animal'] BF, is a non-animal CLOP
[126]is] not *add.* LO, *def.* C
[127][202, p. 57.36].

posite is not in what is added, as in 'a man is dead, therefore a man is'; and where the predicate[128] is not[129] second adjacent,[130] such as this 'Homer is something, namely a poet, therefore Homer is'. The consequence does not hold. Against this: 'a man is running, therefore a man is' surely follows; likewise, 'a man is white, therefore a man is'; and yet 'a man is dead, therefore a man is' does not follow, nor does 'Homer is something, namely a poet, therefore Homer is' follow. And the reason for the disparity is that in the first the opposite is in what is added, and in the second, the predication is second adjacent.[131]

125. And one should note that Aristotle understands by 'second adjacent[132] predication when the predicate is what does not require the subject to exist, of which sort are generally all substantial predicates.

126. Another rule is this: *no accidental destroys its substantial.* For instance: the significate of the term 'man' is essential to it, and so by no accidental added to it is its significate taken away, so that whether it is posited in speech or outside it or whatever it is matched with, it signifies 'rational animal'. Against this rule one argues so: 'man' naturally looks to its present, past, and future supposita. Yet if it is matched with a present-tense verb, it only supposits for present things. But being matched with a present-tense verb is accidental to it. Wherefore, etc. One should say that though[133] an accidental does not destroy its substantial, it can still

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[128]predicate] predication BO, *def.* C, *mut.* L
[129]not] BFP, *def.* CO, *mut.* L
[130]second adjacent] according to accident O, *ed.*; cf. [31, pp. 119-120; 2.216].
[131]second adjacent] according to accident O, *ed.*
[132]second adjacent] LO, second adjacent or according to accident BP, second adjacent or second adjacent F, *def.* C
[133]that though] that any BFP, that the O, *def.* C

destroy its essential, as is so in the proposed. And hence, if 'man' in general (*in communi*) is placed in a phrase, it supposits only for present [things]; yet its significate is not destroyed with respect to past or future.

127. Another rule is this: *whatever 'being' and 'one' are added to do not bring about a distinction.*[134] For instance, 'being man' and 'one man' do not differ, since they convert. Against this one objects thus: that then from truth falsehood would follow, for 'every man is an animal, therefore being every man is an animal'; and yet the antecedent is true and consequent false. The falsity of the consequent is clear because it implies being every man is something. Likewise, it implies that just as 'every man runs' is universal, so 'being every man runs' would be universal; the antecedent is true and the consequent false, because the a particular sign can be added to the subject of the consequent, saying 'some being every man runs'. One should say that the rule has to be understood, that 'being' and 'one' added to whatever simple do not bring about a distinction.

128. Another rule is this: *when two functions are included in one word, the one pertains to nothing without the other pertaining to the same.* Against this rule one objects so: the word 'not' includes two functions, namely distribution and negation, and yet negation attaches to a predicate and distribution doesn't, as in the phrase 'no man is an animal'. One should say that the rule has to be understood of a word having two functions, neither of which is negation. But this is not so in the proposed. Wherefore, etc.

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[134][4, IV, c. 2, p. 1003.24].

129. Another rule is this: *whenever some are the same as one and the same thing numerically, they are the same as each other.* For instance, man and animal are the same as Socrates, /f. 21rb/ and so man and animal are the same as each other. For 'man is an animal' is true. Against this rule: man and ass are the same as animal, therefore they are the same as each other. The consequent is false, therefore also the antecedent.[135][136] One should say that the rule has to be understood of those which are the same as one and the same [thing] numerically. In this way, man and ass are not the same as animal in number, but in genus. Wherefore, etc.[137]

130. Another rule is that *from one nothing follows*, neither enthymematically nor syllogistically. Hence, from 'a man runs', removing whatever other proposition following its understanding and utterance, neither 'an animal runs' nor anything else follows. Against this: 'Socrates runs, therefore a man runs', and yet only one is put in the place of the antecedent. Likewise, from every good enthymeme from one follows something. One should say that the rule has to be understood of one as one, prescinding from whatever is expressed and understood.[138] To the argument

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[135]therefore they ... antecedent] and yet (man and ass *add.* C) are not the same as each other, since this is false (*om.* C) 'man is an ass' CL

[136]The consequent ... antecedent] The previous is true and the same and yet man and ass are not the same as each other, for this is false 'man is an ass' O

[137]of ... etc.] so: whatever are essentially one, [etc.]. But it is not so in the proposed, because animal generally is not one in itself C whatever [are] one and the same in number [and] the same essentially, these are essentially the same. It is not so in the proposed, since animal is not one in number but in genus L whatever are the same [are] essentially one and the same, and yet man and ass are not one and the same as each other, for 'man is an ass' is false, but it is not so in the proposed, because animal is not one in number but one in genus O

[138]prescinding ... understood] prescinding from whatever else is expressed in the thing and in understanding L, hence prescinding (?) whenever from things implicitly expressed,

to the contrary one should say that circumscribing 'Socrates runs'[139] in reality and in utterance, 'Socrates runs, therefore a man runs' no more follows than 'therefore a rock runs' or 'a rock runs, therefore a man runs'. Hence, in every good enthymeme, even if only one is expressed, another still is understood in virtue of which the consequence holds; just as in 'Socrates runs, therefore a man runs', 'Socrates is a man', which is really part of the antecedent, is understood.

131. Another rule is: *a negative proposition has multiple causes of truth*, if it is a negative implication. Hence, 'not a man who is white runs' can be verified, either because no man is white, or because no man runs, or because neither some man is white nor does some man run.

C.9 On propositions with declined [terms]

132. Concerning propositions with declined [terms] certain rules should be known. One is: *when a nominative and declined are placed in some proposition in the same part of a sentence, one should consider whether they are placed in the part of the subject or in the part of the predicate.* For instance, 'this horse is a man's horse', the whole 'a man's horse' is the predicate. Nor does it much matter whether the nominative is placed before the declined, or the declined before the nominative in the part of the predicate. Hence these two do not really differ: 'this horse is the horse of a man' and 'this horse is a man's horse'.

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and understood (?) or follows, and this is true O

[139]runs] is a man *CLO*

If the nominative and declined are placed in the part of the subject, then one should consider whether the nominative precedes the declined or conversely.

If the nominative precedes the declined, the whole is held to be in the part of the subject, as when saying 'any ass of a man runs', the whole 'ass of a man' is the subject, and under this one should take up the minor 'this ass of a man is an ass'. And note that the whole 'ass of a man' is the subject of distribution, and since the taking up ought to be made under the subject of distribution, therefore etc.

But if the declined precedes the nominative, then the nominative is held to be in the part of the predicate, and the declined in the part of the subject. For in saying 'of any man an ass runs' 'ass' is held in the part of the predicate, and 'of man' is the subject of distribution, and so under man one should make the assumption, if one should syllogize from this proposition, so: 'of any man an ass runs, this man is a man, therefore of this man an ass runs'. One should note that any declined can be the subject of distribution, but not any can be the subject of a locution grammatically[140] speaking, since a declined is not able to supposit with respect to a verb, since it is dependent on a nominative.

133. Another rule is this: *any possessive construction, of the sort as 'of any man an ass runs', has to be resolved into a nominative of the possessor and an accusative of possessing*[141] *with the verb 'possesses'.* Hence the

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[140]but ... grammatically] yet cannot be the subject or suppositum significatively LO
[141]into ... possessing] by changing the nominative into an accusative case of possessing something, and the genitive of the possessor in[to] a nominative L into an accusative or genitive of the possessor nominative (?) O

proposition 'of any man an ass runs' has to be resolved thus: 'any man possesses a running ass'. And one should see that the mode of supposition in the resolving and the declined resolved always remains the same.

134. Concerning the mode of syllogizing with declined propositions there are certain rules. The first is this: *the major occurring in declension, and the minor in the nominative, the conclusion follows in declension.* Thus: 'of any ass a man runs, this man is a man, therefore of this man an ass runs'.

135. A second rule is: *from a major occurring in the nominative and a minor in declension follows a conclusion in declension,* if the major will have been affirmative,[142] so: 'every wisdom is a discipline, of the good is wisdom, therefore of the good is discipline'. If the major were negative, then it is not valid. Thus, 'no head is a hand, of Socrates is a head, therefore of Socrates is not a hand'. Against this rule one argues so:[143] the syllogism 'every ass runs, of Socrates is an ass, therefore of Socrates runs' is not valid, because the premises are well-formed (*congruae*) and the conclusion badly-formed (*incongrua*), positing the case to be possible. One should say that the conclusion follows provided that the 'running'[144] is taken substantively, and then the conclusion is true just as the premises.

136. A third rule is that *from both premises occurring in declension a con-*

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[142]if the major ... affirmative] *om.* CLO
[143]Against ... so] *om.* BFP, *def.* C
[144][207, III-1. 9, p. 386.44].

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clusion follows both in the nominative and in declension. Thus, 'What the discipline is of, the genus is of. The discipline is of the good. Therefore the genus is of the good'; and in the nominative, thus: 'What the discipline is of, the genus is of, the discipline is of the good, therefore the genus is good'. One should know, though, that these syllogisms are not purely categorical, but hypothetical in a way, being equivalent to conditional syllogisms.

137. In the second figure these rules are given: *a negative major occurring in the nominative* /f. 21va/ *and affirmative minor in declension, a conclusion follows neither in the nominative nor in declension.* That it does not follow with terms in the nominative is shown thus: 'no animal[145] is a head,[146] of any man is a head,[147] therefore no man is an animal';[148] That the conclusion does not follow in declension for the same cause is shown thus: 'no ass is a head, of any man is a head, therefore of no man is an ass'; The premises can be true with the conclusion being false, positing that each man has an ass.

138. Another rule: *a negative premise occurring in declension, and an affirmative in the nominative, a conclusion follows both in the nominative and in declension.* Thus, every ass is an animal, of no man is an animal, therefore of no man is an ass.

139. Another rule: *each premise occurring in declension, a conclusion can follow in the nominative,* whether the major is affirmative or the minor,

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[145]animal] ass BFP, *def.* C
[146]head] goat BP, *def.* C
[147]head] goat BP, *def.* CF
[148]animal] ass BFP, *def.* C

so: 'of no ass is a horse, of any man is a horse, therefore no man is an ass'. Yet the conclusion does not follow in declension, since the premises can be true and the conclusion false.

140. In the third figure are rules, the first of which is this: *from each in declension and each affirmatively a conclusion never follows.* Hence 'of any man is an ass, of any man is a horse, therefore some horse is an ass' does not follow.

141. Another rule is: *from each occurring in declension and one negatively, a good syllogism with respect to a conclusion in the nominative can be made*, so: 'of no man is an ass, of any man is a horse, therefore no horse is an ass'.

142. Another rule: *if one is in the nominative and the other in declension, if the syllogism is negative, then for such a syllogism to be valid it must be that the premise in declension is negative and the affirmative in the nominative*; but if conversely, the syllogism is not valid, since the premises can be true with the conclusion being false, so: 'no man is an ass, of each man is this ass, therefore this ass is not an ass'.

143. Another rule is: *one premise occurring in the nominative and another in declension in an affirmative syllogism, for the syllogism to be good it must be that the proposition which is the major after reducing it to the first figure would be in declension*; then it must be that the major extremity in the conclusion is taken in declension. Otherwise the premises can be true, the conclusion being false, arguing so: 'every man is an animal, of each man is a hand, therefore a hand is an animal'.

144. Hence, to see generally how one ought to syllogize from declined [terms] in the third and second figure, one should see whether the syllogism brought about in each of these after reducing it to the first figure is useful. If so, the syllogism was good; if not, it was not good.

C.10 General rules

145. Besides the rules already mentioned, there are other general rules, the first of which is this: *for any possible proposition, one or the other of contradictories is compossible with it.* Against this: this is a possible proposition, 'Socrates sees only every man not seeing himself', since it is possible that Socrates sees only every blind man, and since every blind man is not seeing himself; and yet this is compossible with neither of these, namely with 'Socrates sees himself' nor 'Socrates does not see himself'. One should say that the proposition 'Socrates sees only every man not seeing himself' is not possible, since it includes opposites, since from this it follows that Socrates neither sees himself, nor does he not see himself. To the proof, one should say that the consequence 'Socrates sees every blind man, therefore Socrates sees every man not seeing himself' is not valid.

146. Another rule is: *every proposition including opposites is impossible*; such as this, 'You know you are a stone'. For it includes these two 'you are a stone' and 'you are not a stone', since[149] from the fact that it posits

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[149]since] and yet by reason of the disparity it posits the one and the other, for it posits this (*om.* O) 'you are a stone', (you know a stone O) LO

you to know, it posits you not to be a stone, since a stone does not know; and from the fact that it posits you to know you are a stone, since nothing is known unless true, it posits you to be a stone.[150] And just as it is with this proposition including opposites, so it is with any other.

147. Another rule is: *every true proposition of the present leaves another true of the past.* For instance: if 'you are keeping vigil' is true of the present, tomorrow 'you kept vigil' will be true of the past. Against this rule one argues so: this was at some time true of the present, 'every animal is in the ark of Noah', namely in the time of the flood; and yet its past [tense] is false, namely 'every animal was in the ark of Noah', on account of the animals passed away. One should say that this is not its past [tense], since its past [tense] should be given for the same supposita. Thus, this its proposition of the past corresponding to it, 'every animal that was in *a* was in the ark of Noah'; and let *a* be the time of the flood, and this is true, just as the other.

148. Another rule is this: *every true proposition of the past is necessary.* And if this is true of the past 'you were a boy or white', it cannot not be true, and consequently is necessary, since everything which is truth not able not to be true is necessary. Against this: this is true of the past 'you've not been to Rome', and yet it is not necessary, since a year from now it could be /f. 21vb/ false. One should say that this rule ought to be understood: *every proposition true of the past and affirmative is*

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[150]posits you to be a stone] it posits this (the other O) 'you are not a stone' from the fact that it posits you to know, and (because L) knowledge is not in a stone (not a stone O) LO

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necessary. But this is not so in the proposed. Wherefore, etc.

149. Another rule is this: *every false negative proposition of the past is impossible.* And this follows from the the other, because if every affirmative and true proposition of the past is necessary, the opposite of any such proposition is impossible, since the opposite of the necessary is impossible. But the opposite of such is some false negative proposition of the past, and consequently any such is impossible. But the rule does not extend itself to false affirmative propositions of the past, since the proposition 'you've been to Rome' is false of the past, and yet a year from now can be true.

150. Another rule is that *from the contingent the necessary can follow*, for 'a man is white, therefore a man is a such (*qualis*)'; the antecedent is contingent, and the consequent is necessary. The necessity of the consequent is clear, since it is implied by a necessity: 'a man is risible, therefore man is a such' follows; the antecedent is necessary, therefore the consequent.

151. Another rule is: *from the possible the impossible does not follow.* Against this: 'every man runs, every man is sitting, therefore something sitting is running' follows; the conclusion is impossible, but the premises are possible. One should say that this rule has to be understood so: *from possible propositions, if they are compossible with each other, the impossible never follows.* So in the proposed, though each of the premises are possible, still they are incompossible with each other.

152. Another rule is this: *when two propositions are incompossible, from one*

follows the opposite of the other, just as is clear in the argument made above.[151] For these two - 'every man runs' and 'every man is sitting' - are incompossible, and so from one of them follows the opposite of the other: for 'every man runs, therefore every man is not sitting' follows.

153. Another rule is this: *for any proposition of inherence, a proposition of necessity conflicts with it*: of the same necessity, the power of that of inherence will conflict with it.[152] For instance, the proposition 'of necessity every man is an animal' conflicts with the proposition of inherence 'some man is not an animal' so it will conflict with 'it is possible that some man is not an animal'. And the reason for this rule is this, that necessity bring with it perpetual truth, and so what conflicts with the act conflicts with the power of the same act.

154. One should note that a proposition of inherence as-of-now can be true and false, as 'man is running'; and a proposition of inherence simply is always true, as 'man is an animal'.

155. Another rule is this: *from a reflexive inferior to its reflexive superior* is not a good consequence, but is the fallacy *from qualifiedly to unqualifiedly so (secundum quid et simpliciter)*. For instance, 'Socrates knows himself to know nothing, therefore Socrates knows a knowable'. Here, 'knowing oneself to know nothing' is inferior to 'knowable'. And one should know that where one argues *from a reflexive inferior*, etc. one of opposites is determined by the other, just as 'Socrates knowing nothing'

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[151]Par. 151.
[152]of ... with it] the possibility (possible O) of the same (of it O) proposition of inherence is in conflict with the same proposition of necessity LO

is determined by what I call 'knowable', which is something to know.[153]

156. And one should note that when the same is reflected upon its own privation, if one argues *from the privation* it is the fallacy of the consequent or *from qualifiedly to unqualifiedly so*. Likewise, when something is reflected upon its own opposite, as 'I see myself not to see',[154] 'I know myself not to know, or to know nothing'. And generally in all such there is reflection, and in all such the consequence *from the inferior to the superior* is not valid. Against this rule it is argued so: 'Plato knows Socrates knows nothing, therefore Plato knows a knowable'; therefore by the same reason, 'Socrates knows himself to know nothing, therefore Socrates knows a knowable'; and yet here there is reflection. On should say that 'Plato knows Socrates knows nothing, therefore Plato knows a knowable' indeed follows; 'Socrates knows nothing', with respect to the knowledge of Plato, is one knowable unqualifiedly. Yet it does not follow of Socrates, since 'Socrates knows nothing', with respect to that of Socrates, is one knowable in a certain respect. And so proceeding thus from that to its common is the fallacy *from qualifiedly to unqualifiedly so*.

157. Another rule is: *every true proposition of the past has another true proposition of the present.* For instance. 'you ran' is true of the past, and so implies the proposition of the present 'you are running' (*tu es currens*). Against this one argues so: 'you existed yesterday' is true

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[153]And one ... to know] yet the inferior is reflexive, because for Socrates to know nothing is determined by its opposite, which is for Socrates to know something (nothing O) LO

[154]to see] in the dark, 'therefore I see' does not follow but is the fallacy *from qualifiedly to unqualifiedly so.* (It is O) Likewise when something is reflected upon its opposite, as here 'I see myself not to see' *add.* LO

of the past, and yet 'you are yesterday' is never true of the present. And likewise, 'the day was past' is true of the past, and yet 'the day is past' is never true of the present. One should say that this rule has to be understood speaking of a thing *per se* and not *per accidens*, not accidentally.

158. And one should know that composition is twofold, namely substantial and accidental. Substantial is where the predicate is from the intrinsic account of the thing, as 'man is an animal'; accidental, where the predicate is not intrinsic etc., but brings in some accident, as 'man is white' and the like.

159. Another rule is this: *each for a reason, and that more so.* For instance, 'if water is hot because of fire, fire is more hot'. Likewise, if a boy is loved on account of his nurse, it follows that the nurse is loved more. Against this one argues: a man is drunk on account of wine, therefore it follows that[155] the wine is more drunk. Likewise, 'the iron is hot on account of burning wood,[156] therefore the burning wood[157] is hotter' does not follow. One should say that /f. 22ra/ the rule has to be understood formally in principal causes, in causes both *per se* and univocal. But wine is not a univocal cause of drunkenness in this way. To the other objection, I say that burning wood[158] is not *per se* the cause of heat, but with respect to a part; wherefore the objection fails.

160. Another rule is this: *what inhere per se inhere of necessity.* Since animal

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[155]it follows that] *om.* FLO, *def.* C
[156]burning wood] fire BFP, *def.* C
[157]burning wood] wood L, iron BFP, *def.* C
[158]wood] iron BFP, *def.* CO

inheres *per se* in man, thus 'man is an animal' is true of necessity. Against this rule it is argued thus: being mortal inheres *per se* in man, and so 'man is mortal' is necessary; the consequent is false, therefore etc. The falsity of the consequent is shown, since a man, for instance Socrates, is able to not be mortal; therefore it is not necessary. Likewise, 'a surface is white' is *per se*, and yet 'a surface is necessarily white' is false, since a surface is able to be black. One should say that this rule has to be understood thus: 'what inheres *per se* in something inheres of necessity' is true, taking 'per se' after how the Philosopher takes it in the book of the *Posterior [Analytics]*;[159] for there he says that those that inhere *per se* inhere of necessity.

On account of this, one should know that a proposition is called 'per se' in three ways: in one way when the predicate falls in the definition of the subject, as 'man is an animal'; in a second way, when the subject falls in the definition of the predicate, and in this way 'man is risible' is true;[160] in a third way, when the subject is the efficient cause of the predicate, as 'a killer kills',[161], from killing.

To the reasons: to the first I say that a surface is not *per se* white, taking *per se* as it is taken in the book of the *Posterior [Analytics]*; yet this can indeed be *per se*, because immediate. To the other I say that 'man is mortal' is not *per se*, since 'mortal' is a passion resulting in a defect, and such is not predicated *per se* of the species, nor of the individual.

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[159][7, I. 4, p. 73a.35].
[160]true] *per se* L, *def.* CO
[161]a killer kills] the killed is killed L, *def.* CO

161. Another rule is this: *names and verbs transposed signify the same.* Hence, 'man is white' and 'white is man' signify the same. Against this rule it is argued: 'man every is' and 'every man is' signify the same, yet one is proper and the other improper. Likewise, this: 'of all opposites, the same is the discipline, therefore the same is the discipline of all opposites'. One should say that this rule has to be understood so: where each of the transposed signifies a thing. By this, to the first argument, that 'every' does not signify some thing as 'man' does. Likewise, the rule has to be understood where the transposition is made from the same part of the extremes. By this to the second argument, since in the second argument the transposition is of the whole proposition.

162. Another rule is this, that *only one is opposed to one* - as 'no man is an animal' is opposed to the proposition 'every man is an animal' - *in the same genus of opposition.* Against: the proposition 'every man runs' is one, and yet these two propositions, 'no man runs' and 'some man does not run' are opposed to it. One should say that the rule has to be understood in the same genus of opposition, as was already stated. And by this the argument is resolved, since though the two 'no man runs' and 'some man does not run' are opposed to the proposition 'every man runs', yet this is in different genera of opposition, since the one is opposed as contrary and the other as contradictory.

163. Another rule is this: *when something is predicated of another, whatever is predicated of the predicate is predicated of the subject.* For instance, since 'animal' is predicated of 'man', and 'substance' of 'animal', there-

fore 'substance' is predicated of 'man'. Against this it is argued so: 'proposition or its contradictory' is predicated of this, that is 'you are an ass', since this is true, "you are an ass' is a proposition, or its contradictory'. But 'true' is predicated of this proposition or its contradictory, and yet 'true' is not predicated of this proposition 'you are an ass',[162] since 'you are an ass'[163] is false. One should say that this rule has to be understood of those which are ordered above and below in a categorical series, in the way 'man' and 'animal' are; but it is not so in the proposed.

164. Another rule is: *of different genera, one of which is not placed under the other nor both under a third, the species and differences are diverse.* For instance, 'animal' and 'science' are diverse genera not placed under each other, and so of these different genera are different species and differences. For the differences of science are like this: 'natural' and 'moral'; the differences of animal, like this 'rational' and 'irrational', etc. Likewise the species of these are different, since the species of animal are as ox and lion etc, and the species of science are as grammar and dialectic, etc. Against this: 'rational' is a difference[164] of science and animal, and yet science and animal are different genera. One should say that 'rational' is one difference, inasmuch as it is a difference of science, and another, of animal, for a science to be rational is nothing other than

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[162]But ... ass] but 'true' is predicated of the predicate 'proposition or its contradictory', and a proposition or its contradictory is true and yet C (*here C's text is missing*), and true is predicated of this predicate so: 'a proposition or its contradictory is true', and yet 'true' is not predicated of the subject, because 'you are an ass is true' is false L

[163]ass] is true *add.* L, *def.* CO

[164]is a difference] are differences BFP, *def.* CO

that it be under the act of reason, but 'being rational' according as it is a difference of animal is 'apt by nature to use reason'. And so 'rational' is said equivocally there. Wherefore, etc.

165. Now we must speak of distributive signs. Note that some are signs distributive of substantial terms.[165] Of this sort are 'every' (*omnis*), 'whole' (*totus*), 'each [of two]' (*uterque*), and the like; likewise 'whatever' (*quicquid*), 'any' (*quilibet*), 'whoever' (*quicumque*), 'on both sides' (*utrumque*), 'however many' (*quotienscumque*). And of these signs of distribution, /f. 22rb/ some are distributive[166] with respect to one verb, whose function (*respectu*) is fixed to the[167] verb. 'Every', 'whole', 'every single' (*unusquisque*), 'each', and the like are of this sort. Hence I say 'every man is', 'each man is', 'every single man is', and 'the whole man is'.

Others of these are distributed[168] so, with respect to diverse verbs, and of this sort are nearly all distributive signs which take to themselves the suffix '-ever' (*-cumque*), as 'whosesoever' (*cuiuscumque*), 'wherever' (*ubicumque*), 'however many' (*quotienscumque*), 'whenever' (*quandocumque*)'; for 'whenever man runs' is not spoken well unless some other verb is added, so: 'whenever a man runs, a man moves' or some such thing; and so for the others.

Moreover, among the number of these which are distributed[169] with

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[165] [202, p. 256.29].

[166] likewise ... distributive] signs which are properly distributive of accidentals, as however (*qualecumque*), however much (quantumcumque), wherever (*ubicumque*) however often (*quotienscumque*) and the like. Still there are signs which distribute L

[167] the] one L, *def.* CO

[168] are distributed] distribute L, *def.* CO

[169] are distributed] distribute L, *def* CO

respect to one verb, some distribute for several subjects, as 'every', 'each', and the like; some for several integral [parts] only, as 'whole' and the like. For if I say 'the whole Socrates is less than Socrates', so that 'whole' is taken significatively there, it will be equivalent to 'any part of Socrates is less than Socrates'.

Moreover, among the number of those which distribute for subjective parts, some are distributed only for two,[170] of such sort being 'each' and 'both' (*ambo*), etc.; others negatively, as 'neither' and the like. Hence 'this man runs, and this', pointing to two, 'therefore each of them runs' follows, which would not be true unless 'each' distributed for two. And likewise with the word 'both'.

Some distribute not only for two but for all contained under a term. Of this sort are 'every', 'whichever' (*quaecumque*), which distribute a term according to the possibility[171] found in it. Hence they can be added to a term having unactualized supposita (*supposita infinita*), of such sort as 'man' and 'lion' etc. And according to the Philosopher,[172] they can be added to a term having only one suppositum in act; of this sort are 'sun' and 'moon';[173] likewise to terms having only one suppositum in act and potency, of this sort is 'phoenix'.[174] For we say 'every lion', 'every ox', 'every phoenix', 'every sun', and 'every moon'. For even if 'sun' has one suppositum, it can still have several. That is, it is such that there is no

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[170][202, p. 255.8].
[171]possibility] plurality L, *def.* CO
[172][6, I. 1, p. 268a.15-19].
[173]'sun' and 'moon'] 'phoenix', 'sun', etc. BFP, *def.* CO
[174]of ... 'phoenix'] L, as sun and moon BFP, *def.* CO

conflict in its having several, as much as is on its part.[175]

Moreover, among the number of signs, some distribute a substantial term for subjective parts, and this in two ways. Some distribute a common term for supposita, inasmuch as any is distinguished from another;. The first sign is 'every'; A sign of the second type is 'every single' and the like.[176] Thus, Aristotle says in *Posterior [Analytics]* I[177] that one who says 'a triangle has three angles', etc. and [says] 'every single triangle' speaks according to number, but one who says 'every' speaks according to species.

Moreover, of the number of signs which distribute a term, some distribute in the nominative, others in declension. Hence, all signs distributing in the nominative distribute a term in the nominative, signs in declension distribute in declension. For instance, saying 'of any man, an ass runs', 'of any', which is a sign distributive in declension, namely 'of man' in the genitive case.

166. All those names distributive in some way are called syncategorematic words; besides these, there are some syncategorematic[178] words which effect or imply consequence, of this sort are 'if' (*si*), 'or if' (*sive*), 'inasmuch as' (*inquantum*), and 'unless' (*nisi*). Among these, 'if' implies consequence absolutely. Sometimes it effects an as-of-now consequence,

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[175]it can ... part] in act, one cannot have another as much as is on the part of matter. Since, though, it is not repugnant to sun to be multiplied in several supposita, and a sign is added to a term by reason of its form, thus 'every sun is' is well-spoken L

[176]any ... the like] these supposita are among themselves and distribute. Of this sort are 'every', no' L

[177][7, I. 4, pp. 73b.25-74a.3].

[178]syncategorematic] categorematic BFP

sometimes a simple one. Hence, 'if' and 'while' effect[179] the same thing. 'Or if' implies an absolute or as-of-now consequence. And *sive* is put together from *si* and from *-ve*; the meaning of the utterance *-ve* implies disjunction, and the meaning of the utterance *si* implies consequence. 'Inasmuch as' states not only a consequence, but the cause of a consequent or consequence. 'Unless' implies a consequence with negation. Hence it is put together from *non* and from *si*. The meaning of the utterance *non* implies negation, the meaning of the utterance *si* implies consequence.

167. Another rule is that *to a necessary proposition corresponds a necessary act, and to a proposition of contingency corresponds an act true as-of-now.* Hence, to the proposition contingent toward either 'a man runs', corresponds an act true as-of-now, namely 'a man runs', which is true for now,[180] supposing that some man runs. But to a necessary proposition corresponds a necessary act. For instance, to the proposition of necessity 'every man is an animal' corresponds a necessarily true act, namely 'every man is an animal', which is necessary.

168. Note that some indefinite proposition is equivalent to a universal, just as 'man is an animal' is equivalent to 'every man is an animal', since the predicate in each inheres in any contained under the subject. Against this: this chain of reasoning is good: 'every man is an animal, Socrates is a man, therefore Socrates is an animal'; therefore this as well, namely

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[179]'if' ... effect] 'if' and 'therefore' effect what the verb 'follows' signifies, hence it implies a consequence with distribution L

[180]namely ... now] which is true for now, namely 'man runs' for now BFP, 'man runs' which is a proposition of inherence as-of-now O, *def.* CL

'man is an animal, Socrates is a man, therefore Socrates is an animal', since according to you it converts. The consequent is false, therefore the antecedent, too. One should say that for a chain of reasoning to be good can occur in two ways: either materially or formally. Hence the syllogism 'man is an animal' etc. is indeed valid on account of its matter, namely by reason of such propositions, but not formally, since it does not hold in every matter with the premises disposed in this way. Wherefore the objection is not valid.

Abstract

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    The development of the medieval Parisian account of formal consequence

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The concept of formal consequence is at the heart of logic today, and by extension, plays an important role in such diverse areas as mathematics, computing, philosophy, and linguistics. In this dissertation, I trace the roots of this concept in medieval logic from Pseudo-Scotus and John Buridan back to the earliest treatises on consequences, and provide translations of the three earliest known treatises on consequences.

Chapter one introduces the reader to the dominant philosophical approaches to formal consequence from the turn of the twentieth century to today. After this, I introduce the account of formal consequence advanced by John Buridan, the medieval predecessor to the semantic account advocated by Tarski and his followers.

Chapter two provides a detailed contrast of Buridan's account of formal consequence with those of Tarski, on the one hand, and later classical logic,

on the other.

Chapter three examines the account of formal consequence in Pseudo-Scotus. I show that Pseudo-Scotus' account is dependent on that of Buridan, and therefore most post-date it.

Chapter four examines the account of divided modal consequence in William of Ockham. I show that Ockham's divided modalities are not fully assimilable to narrow-scope propositions of classical modal logic; formalize Ockham's account in an extension of first-order modal logic with restricted quantification; and provide a complete account of relations between two-term divided modal propositions on Ockham's account.

Chapter five introduces Walter Burley's thinking about consequences, examining: Burley's division and enumeration of consequences; his distinction between principal and derivative rules licensing good consequences; the relation of the division of consequences into formal and material varieties to Burley's preferred division between natural and accidental consequences; the relation Burley's work bears to Buridan, to the Boethian reception of Aristotle's Topics, and to the earliest treatises on consequences.

The final chapter concludes: highlighting the characteristic marks of medieval and modern approaches to consequences relative to each other; summarizing the various developments that led to the adoption of the account of formal consequence epitomized in Buridan's work; and suggesting prospects for recovering the most promising aspects of the medieval treatments of the topic.

Vita

Jacob Wesley Archambault was born on 9 January 1986 to parents Gerald Renaud and Aline Archambault in Woonsocket, Rhode Island. He graduated from Woonsocket High School in 2004.

Jacob received his BA in philosophy from Franciscan University of Steubenville in 2009, and his MA in philosophy in 2011 from the University of Houston. He was a visiting postgraduate at the University of St Andrews'Arché Research Centre in 2014; and received his Ph.D in 2017 from Fordham University.

Jacob specializes in the history and philosophy of logic, medieval philosophy, and the philosophy of religion. He has interdisciplinary competence in mathematical logic, medieval theology, and Latin paleography, and has taught courses including Ethics and Philosophy of Human Nature. He reads Latin, French, and German.

Jacob's current projects include: a monograph on consequences in medieval philosophy; several articles on categoricity and natural semantics for various axiomatic, natural deduction, and sequent calculi in mathematical logic; and a book on the concept of time in religious worship. He has presented his research at dozens of venues spanning three continents, and is the author of multiple journal articles and book chapters from publishers

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After completing his Doctorate, Jacob began teaching at Highlands Latin School in Louisville, Kentucky, where he lives with his wife, Amanda, and children, Elijah and Estelle.