Grounding medieval consequence

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Abstract

1 Introduction

Though elements that would enter into the theory may be found in earlier work on Aristotelian syllogistic, fallacies, and topical argument, the theory of consequence proper first arises in the later medieval period. In what follows, I explain how consequence was grounded in the medieval period. I begin with a basic introduction to consequences in medieval philosophy. I then show how first medieval theories of topical argument, then of supposition served to ground medieval accounts of inference, detailing two important shifts: the first from a non-unified, topics-based grounding of consequence to one based on the theory of supposition; the second, within the supposition-based theory, from an understanding of personal supposition licensing appeal to concepts to one based wholly on a sparse ontology of individuals. Over time, these changes in the foundations of the medieval theory of inference accelerate a shift in its focus, one away from a characteristically medieval concern with sound and demonstrative arguments towards a more modern focus on formal validity.

2 Formal consequence in modern and later medieval logic

To begin, it will be useful to consider what theories of consequence are meant to be theories *of*. In its broadest sense, a consequence is a relation obtaining between an antecedent and a consequent, signified by a sign of consequence.[1] An antecedent is a premise or set of premises from which a consequent

[1] Wallace Knight Seaton. "An Edition and Translation of the Tractatus de consequentiis of Ralph Strode". PhD thesis. University of California at Berkeley, 1973; Niels Jørgen Green-Pedersen. "Two early anonymous tracts on consequences". In: *Cahiers de L'Institut du Moyen-Âge Grec et Latin* 35 (1980), 1–28.

follows; a consequent, a conclusion following from an antecedent; a sign of consequence, a word, symbol, or phrase signifying a consequent's following from an antecedent, e.g. 'if', 'therefore', 'because' and their analogues in other natural languages, the $\vDash$ of model theory, or the proof-theoretic $\vdash$.

Since the early 20th century, discussion of consequence has centered on formal consequence, commonly identified with logical consequence.[2] Such a consequences is usually defined either proof-theoretically, e.g. as one where the consequent follows from the antecedent via the strict application of the rules of a given proof-system,[3] or model-theoretically as one where every model of the antecedent is a model of the consequent.[4] Because of their historical significance and continued dominance over rival semantic approaches, model-theoretic accounts of logical consequence are sometimes simply referred to as semantic accounts.

Today's model-theoretic approaches to formal consequence derive from Alfred Tarski, whose definition is itself a modification of condition (F), which he presents in his 'On the concept of Following Logically':

> If in sentences of the class K and in the sentence X we replace the constant terms which are not general-logical terms correspondingly by arbitrary other constant terms (where we replace equiform constants everywhere by equiform constants) and in this way we obtain a new class of sentences K' and a new sentence X' , then the sentence X' must be true if only all sentences of the class K' are true.[5]

(F), in turn, is markedly similar to a medieval account of consequence common at the University of Paris and other continental European universities from the fourteenth century. John Buridan, the most influential of the early fourteenth century Arts Masters at the University of Paris, gives his version as follows:

> A consequence is called formal if it is valid in all terms retaining a similar form. Or if you want to put it explicitly, a formal consequence is one where every proposition similar in form that

[2] Alfred Tarski. "On the Concept of Following Logically". Trans. by Magda Stroińska and David Hitchcock. In: *History and Philosophy of Logic* 23.3 (2002), 155–196; Mario Gómez-Torrente. "A Note on Formality and Logical Consequence". In: *Journal of Philosophical Logic* 29.5 (2000), 529–539.

[3] Cf. (Dag Prawitz. "Remarks on Some Approaches to the Concept of Logical Consequence". In: *Synthese* 62.2 [1985], 153–171; Peter Schroeder-Heister. "Validity Concepts in Proof-theoretic Semantics". In: *Synthese* 148.3 [2006], 525–571; Ole T. Hjortland. "The Structure of Logical Consequence: Proof-Theoretic Conceptions". PhD thesis. University of St. Andrews, 2009; Curtis Franks. "Cut as Consequence". In: *History and Philosophy of Logic* 31.4 [2010], 349–79)

[4] Tarski, "On the Concept of Following Logically".

[5] Tarski, "On the Concept of Following Logically", pp. 183–184.

> might be formed would be a good consequence, e.g. 'That which is A is B, so that which is B is A.'[6]

Buridan goes on to hash out propositions 'retaining a similar form' to another proposition as those that uniformly substitute different *categorematic* terms - those terms whose primary function is to signify some being, rather than to provide structure to other elements in a proposition - for those of the original. He writes:

> I say that when we speak of matter and form, by the matter of a proposition or consequence we mean the purely categorematic terms, namely, the subject and predicate, setting aside the syncategoremes attached to them by which they are conjoined or denied or distributed or given a certain kind of supposition; we say all the rest pertains to the form. So we say that the copulas of both simple subject-predicate and compound propositions pertain to the form, as do negations, [other] signs, the number of propositions and terms and the mutual relation of all these, and relations of anaphoric terms and modes of signifying pertaining to the quantity of the proposition, for example, whether discrete or general, and many other things that the attentive reader can recognize if they occur.[7]

[6](John Buridan. *Tractatus de Consequentiis*. Ed. by Hubert Hubien. Philosophes médiévaux 16. Louvain: Publications universitaires, 1976, pp. 23–24)'Consequentia 'formalis' vocatur quae in omnibus terminis valet retenta forma consimili. Vel si vis expresse loqui de vi sermonis, consequentia formalis est cui omnis propositio similis in forma quae formaretur esset bona consequentia, ut 'quod est A est B; ergo quod est B est A'. Translation taken from (John Buridan. *Treatise on Consequences*. Trans. by Stephen Read. Bronx, NY: Fordham University Press, 2015, p. 68). Cf. (Pseudo-Scotus. *Quaestiones Super Libros II Priorum Analyticorum*. Joannis Duns Scoti Doctoris Subtilis Ordinis Minorum Opera Omnia vol. 2. Paris: Wadding-Vives, 1891, 81–197, p. 105).

Since Buridan assumes the truth of a proposition entails its existence, his definition is actually a bit more careful than Tarski's to avoid paradoxes of self-reference. See (Gyula Klima. "Consequences of a closed, token-based semantics: the case of John Buridan". In: *History and Philosophy of Logic* 25.2 [2004], 95–110; Catarina Dutilh Novaes. "Buridan's *Consequentia*. Consequence and Inference Within a Token-Based Semantics". In: *History and Philosophy of Logic* 26.4 [2005], 277–297).

[7](John Buridan, *Treatise on Consequences*, p. 74) = (John Buridan, *Tractatus de Consequentiis*, pp. 30.7–18): 'Et dico quod in proposito, prout de materia et forma hic loquimur, per 'materiam' propositionis aut consequentiae intelligimus terminos pure categorematicos, scilicet subiecta et praedicata, circumscriptis syncategorematicis sibi appositis, per quae ipsa coniunguntur aut negantur aut distribuuntur vel ad certum modum suppositionis trahuntur; sed ad formam pertinere dicimus totum residuum. Unde copulas tam categoricarum quam hypotheticarum propositionum dicimus ad formam pertinere, et negationes, et signa, et numerum tam propositionum quam terminorum, et ordinem omnium praedictorum ad invicem, et relationes terminorum relativorum, et modos significandi pertinentes ad quantitatem propositionis, ut est discretio et communitas, et multa quae diligentes possunt videre si occurrant.

Here, both Buridanian and broadly post-Tarskian traditions define formal consequence by partitioning the syntactic elements of a consequence into logical and non-logical, varying or leaving the interpretation of the non-logical elements of a consequence indeterminate so as to focus on what is demanded by the interpretation of its logical elements.[8] Though formal consequence is less central to later medieval theories of consequence in general than to their modern counterparts, one can see, given the achievement that Buridan's theory of formal consequence effected in particular, how the later modern focus could have developed out of it.[9] Furthermore, the centrality of formal consequence to modern thinking about the notion of consequence generally, witness to its dominant aim as that of providing a theory of *validity*, i.e. of which inference forms hold universally, regardless of their material content[10]

It may come as a surprise, then, that this focus on formal validity only arises fairly late in medieval logic - instead, from its infancy in Boethius, through its first mature formulation in the time of Peter Abelard, to the early fourteenth century work of Walter Burley, the theory of consequences largely revolves around a division of consequences into natural and accidental, with its pedagogical focus less on formal validity than on high-level categorization of the sources of materially correct arguments.[11] One can better understand the centrality of formally valid consequence in logic today by understanding the focus it replaced. And to do this, it'll be useful to further inspect its font, the logic of Boethius.

3 The roots of medieval *consequentiae* in hypothetical syllogistic and topical argument

Among the earliest work which we might classify under the banner of consequence,[12] one finds work on categorical syllogisms, hypothetical syllogisms,

[8] Catarina Dutilh Novaes. "The Different Ways in which Logic is (said to be) Formal". In: *History and Philosophy of Logic* 32.4 (2011), 303–332, pp. 314–321.

[9] Jacob Archambault. "Introduction: Consequences in Medieval Logic". In: *Vivarium* 56.3-4 (2018), 201–221.

[10] Cf. (John Etchemendy. "Reflections on Consequence". In: *New Essays on Tarski and Philosophy*. Ed. by Douglas Patterson. Oxford: Clarendon Press, 2008, 263–299; John MacFarlane. "What does it mean to say that logic is formal?" PhD thesis. University of Pittsburgh, 2000).

[11] For discussion of the earlier division and its relation to the later one, see (Christopher J. Martin. "The Theory of Natural Consequence". In: *Vivarium* 56.3-4 [2018], 340–366; Christopher J. Martin. "Formal Consequence in Scotus and Ockham. Towards an Account of Scotus' Logic". In: *Duns Scotus in Paris, 1302-2002: Proceedings of the conference of Paris, 2-4 September 2002*. Ed. by O. Boulnois, E. Karger, and G. Sondag. Turnhout: Brepols, 2004, 117–150; Archambault, "Introduction: Consequences in Medieval Logic").

[12] Early medieval logicians themselves did not: the earliest treaties dedicated to the concept of consequence as such don't appear until turn of the fourteenth century. See (Jacob

topical argument, and fallacies. Though neither '*consequentia*' nor its variants occur in Boethius' *On the Categorical Syllogism*, the terms occur frequently in Boethius' *On Differential Topics* and especially his *On Hypothetical Syllogisms.*

3.1 An overview of Boethian topical argument

3.1.1 The Boethian concept of an argument

Following Themistius, Boethius defines a topic as 'the seat of an argument', and an argument as 'a reason granting credence to a doubtful matter'.[13] Boethius' use of several key terms here differs from modern usage. In his discussion of the term 'argument', Boethius distinguishes argument in the sense of what is put forth (*argumentum*) from argument in the sense of a speech putting forth an argument in the previous sense (*argumentatio*).[14] He also means something different by the term than what we might mean by it in the first of his senses: he states, for instance, that a conclusion is a proposition established by arguments (note the plural);[15] that 'if something is added to any thing, the whole is made greater' is an example of an argument that is both necessary and plausible (*probabilis*), and argues, against the claim that all arguments must be plausible, that more advanced theorems of geometry may still serve as *argumenta* for conclusions that follow from them even to one who has failed to grasp their necessity.[16]

3.1.2 Maximal propositions as grounds of consequence

Boethius goes on to say that the term *argumentum* is ambiguous between two meanings: that of a maximal proposition, and that of the difference of such a proposition. By the former, he means a general proposition whose truth is grasped immediately which serves to derive more specific claims less sure than it, e.g. 'equals subtracted from equals are equal' - or, in the absence of a proposition whose truth is immediately known, one whose plausibility is sufficiently vouched for, e.g. by an artisan within his proper domain. In its second sense, an argument is that feature, thing, or relation principally appealed to in the formulation of a maximal proposition, e.g. equality.[17] Since a topic, then is the seat of an argument, it is understood

Archambault. "The development of the medieval Parisian account of formal consequence". PhD thesis. Fordham University, 2017)

[13][1174C-D](Boethius. *De Differentiis Topicis.* In: *Patrologia Cursus Completus. Series Latina.* Vol. 64. Paris: J. P. Migne, 1841-1855, 1173B–1216D): 'Argumentum est ratio rei dubiae faciens fidem ... Locus autem est sedes argumenti.'

[14]Boethius, *De Differentiis Topicis*, p. 1174C.

[15]Boethius, *De Differentiis Topicis*, p. 1180C.

[16]Boethius, *De Differentiis Topicis*, 1181A-C.

[17](Boethius, *De Differentiis Topicis*, 1185A-B). Cf. (Toivo J. Holopainen. "Anselm's *argumentum* and the early medieval theory of argument". In: *Vivarium* 45.1 [2007], 1–29;

as the seat of a maximal proposition, and thereby of the difference of such a proposition.

Understood as such, the Boethian theory of topics aims to provide an account of the fundamental propositions to be appealed to in an argument, and the real features on account of which these hold in whatever way they do. It is in this sense that the earlier medieval theory of topics should be understood as a theory for grounding consequence.

3.1.3 The different kinds of topical argument

Following the Aristotelian commentator Themistius, Boethius divides topics into intrinsic, extrinsic and middle topics, in accordance with the kinds of media, or *middles*, they employ to reach a given conclusion. In an intrinsic topic, the argument proceeds by eliciting some property, description, or relation belonging to what is signified in the minor term to confirm or remove what is suggested of it by a major term. Types of intrinsic topics include arguments from the predication of a species to the predication of a genus (e.g. 'Adam is a man, therefore he is an animal'), from a predication of a description to what follows from it (e.g. 'God is that than which nothing greater can be thought, therefore God exists') or from the existence of a whole to that of its parts (e.g. 'If there's a house, there's a roof, walls, and a foundation'). In an extrinsic topic, by contrast, one exploits a relation that the subject term of the desired conclusion bears to some other concept, constructs an argument about the latter, and thereafter leverages the original relation to infer something about the original subject one was inquiring about. Types of extrinsic topics include arguments from authority, from analogy, from various kinds of opposites, and *a fortiori*/*a minori* arguments.

3.2 Boethian topical theory as ground of hypothetical syllogistic

With this laid down, we can begin to look inspect Boethius main source of his remarks on consequence, his *On Hypothetical Syllogisms*. In that work, Boethius takes the Latin '*conditionalis*' to mean the same as the Greek '*hypothetica*'.[18] He distinguishes conditional from categorical propositions, stating that 'in a conditional, the nature of a consequence' rather than that

Jacob Archambault. "Monotonic and Non-monotonic Embeddings of Anselm's Proof". In: *Logica Universalis* 11 [2017], 121–138)

[18](Boethius. *De Hypotheticis Syllogismis*. Ed. by L. Obertello. Brescia: Paideia, 1969, p. 1.3.2). Since Boethius counts not only conditionals in the modern sense, but also disjunctions among hypothetical propositions (on the grounds that 'A or B', taking 'or' to exclude the possibility that both disjuncts are true, is equivalent to 'If A, then not B'), his understanding of 'conditional' must differ from the syntactic definition employed today.

of a predication, 'is assumed from the condition',[19] and further distinguishes consequences that follow only accidentally - his example is 'if fire is hot, the heavens are round' - from those, such as 'if it's a man, it's an animal' that follow naturally.[20]

In both the *On Hypothetical Syllogisms* and the *On Differential Topics*, Boethius divides the hypothetical proposition into four types according to the quality of its antecedent and consequent: a positive proposition following from a positive, a negative from a negative, a negative from a positive, and a positive from a negative.[21] since it applies solely to hypothetical propositions whose immediate parts are atomic, the division is incomplete[22] and even when restricted to these appears trivial at best. It's not. We can better understand the intent of this fourfold division by examining Boethius' use of this division in light of his theory of topical argument.

Boethius first discusses affirmations following from affirmations:

> Now in order for one thing to precede and another to follow, in just these things it usually comes about as I recounted a little earlier. For a [1] genus, or [2] difference, or [3] definition, or [4] property, or [5] an inseparable accident follows from the species. Again, a species follows from the [6] property or [7] definition, the [8] difference and [9] definition follow from a property, and a [10] property or [11] difference follows from the definition in this way: for example, [1] if there's a man, there's an animal; and [2] if there's a man, it's rational; and [3] if there's a man, there's a mortal rational animal; and [4] if there's a man, it's risible; and [5] if there's an Ethiopian, he's black. [6] If [something] is risible, it's a man; [7] if there's a mortal rational animal, there's a man. [8] If [something] is risible, it's rational; [9] if [something] is risible, it's a mortal rational animal; if there's a mortal rational animal, it's [10] risible or [11] two-footed.[23]

[19](Boethius, *De Hypotheticis Syllogismis*, p. 1.1.6): 'Primum igitur dicendum est quod praedicatiua propositio uim suam non in conditione sed in sola praedicatione constituit, in conditionali uero consequentiae ratio ex conditione suscipitur'.

[20]Boethius, *De Hypotheticis Syllogismis*, pp. 1.3.6–1.3.7.

[21](Boethius, *De Hypotheticis Syllogismis*, p. 1.3.5) (Boethius, *De Differentiis Topicis*, 1176B-C)

[22]The division has also been taken to suggest that Boethius conflates the denial of a consequent with the denial of a consequence as a whole (Christopher J. Martin. "Denying Conditionals: Abaelard and the Failure of Boethius' Account of the Hypothetical Syllogism". In: *Vivarium* 45 [2007], 153–168, pp. 157–158).

[23](Boethius, *De Differentiis Topicis*, 1179A-B): Nam ut praecedat aliquid et aliud consequatur, in his fere rebus euenire solet quas paulo superius commemoraui. Speciem quippe sequitur genus, uel differentia, uel definitio, uel proprium, uel inseparabile accidens. Item proprium ac definitionem sequitur species, proprium uero sequitur differentia et definitio, et definitionem sequitur proprium uel differentia, hoc modo: nam si homo est, animal est; et si homo est, rationale est; et si homo est, animal rationale mortale est; et si homo est,

We'll consider two further passages before expounding. Moving to the case of arguments with both an affirmative and a negative, Boethius writes 'of those cases that consist of an affirmation and a denial, that division is usually that they are comprised either of diverse genera, or diverse species, or of contraries, or of habit and privation.'[24] And considering arguments from a negative to an affirmative, Boethius writes:

> It cannot happen that an affirmation follows a denial ... except in those contraries which lack a middle ground and where it is always necessary for one or the other of them to inhere, in this manner: if it is not day, it is night; if it is not dark, it is light.[25]

We can now begin to see what Boethius is attempting: the division of hypothetical propositions according to the quality of their parts lays the groundwork for an inquiry into what kind of real relations things signified by terms must stand in to each other in order for there to be relations of following among consequences they factor into. For instance, in an affirmative-to-affirmative consequent, the positing of a species, property, and difference follow reciprocally from each other, and one can move from the positing of a species to positing the corresponding genus, but not conversely. In arguments from an affirmation to a negation, different relations predominate, i.e. diversity of genus or species, contrariety, or that of privation and habit. The conditions under which an argument from a negative antecedent to an affirmative consequent hold are a subset of those which hold for the converse case.

3.3 The limits of Boethius' grounding of consequence

The examples Boethius gives above provide the theory's paradigmatic use cases, but also reveal its limitations. When Boethius discusses consequences in both the *On Differential Topics* and the *On Hypothetical Syllogisms*, the majority of cases he considers are arguments where the antecedent and consequent affirm or deny existence claims (e.g. 'if it is day, it is not night', 'if there's a man, there's an animal'), substitute a predicate term in the consequent for a different one in the antecedent (e.g. 'if it's white, it's colored'), or provide a consequent where the implied subject for a predicate posited or

risibile est; si Aethiops est, niger est. Si risibile est, homo est; si animal rationale mortale est, homo est. Si risible, rationale est; si risible est, animal rationale mortale est; si animal rationale mortale est, risibile uel bipes est.

[24](Boethius, *De Differentiis Topicis*, p. 1179D): 'Earum uero quaestionum quae ex affirmatione et negatione consistunt, illa fere diuisio est, quod uel in diuersis generibus, uel in diuersis speciebus, uel in contrariis, uel in priuatione atque habitu continentur'

[25](Boethius, *De Differentiis Topicis*, 1180A) Ut autem negationem affirmatio consequatur, quae erat quarta conditionalis propositionis differentia, fieri non potest, nisi in his contrariis quae medio carent, et quorum alterum semper inesse necesse est, hoc modo: si dies non est, nox est; si tenebrae non sunt, lux est.

denied in the consequent is an anaphoric referent back to what is posited or denied in the antecedent (e.g. 'if there's an Ethiopian, he's black'). Now in Latin, these different kinds of inferences have the same syntactic form, i.e. 'if *a* is (not), *b* is (not)' (*si a (non) est, b (non) est*). From the preponderance of his examples, it is clear that Boethius primarily conceives of the *relata* of consequence not as propositions or their contents, but as the referents of terms.[26] The advantage of such a theory is that it provides a grounding for why consequences with antecedents and consequents of a given quality hold: because the things signified by the terms posited or denied therein stand in relations like contrariety, that of a habit to its privation, that of a genus to a species, etc. The disadvantage is that the theory does not generalize to cover more complex cases, e.g. arguments with modal or non-atomic antecedents or consequents, those involving quantified or oblique terms, or those where the subject of the antecedent and consequent differ, e.g. 'if the pilot of a ship should not be chosen by lot, neither should the governor of a city' - Boethius' own example of an argument by analogy.[27] Crucially, though Boethius discusses a wide variety of middle and extrinsic topics with illuminating examples, the less cohesive treatment he provides of these does not fit naturally into the division of inferences he fleshes out in the *On Hypothetical Syllogisms*. Furthermore, to the degree that the theory grounds consequences in the varied relations that it does, it would seem to presuppose a fairly robust metaphysical framework in which these relations may obtain. It's perhaps for these reasons that we begin to see more streamlined ways of grounding inference arise in the later medieval period.

4 Medieval consequence and the theory of supposition

4.1 Canonical supposition theory: Ockham and Buridan

By the time the earliest treatises on consequences appear at the turn of the fourteenth century, the use of topics in discussions of consequence has been altered by the development of *supposition theory* - roughly analogous to modern theories of reference, but better understood as a theory governing the interpretation of terms in propositional contexts..[28] This shift brought

[26] See (Martin, "Denying Conditionals: Abaelard and the Failure of Boethius' Account of the Hypothetical Syllogism"; Bianca Bosman. "The Roots of the Notion of Containment in Theories of Consequence: Boethius on Topics, Containment, and Consequences". In: *Vivarium* 56.3-4 [2018], 222–240).

[27] Boethius, *De Differentiis Topicis*, 1191A-B.

[28] Catarina Dutilh Novaes. "Theory of Supposition vs. Theory of Fallacies in Ockham". In: *Vivarium* 45.2-3 (2007), 343–359; Catarina Dutilh Novaes. "An Intensional Interpretation of Ockham's Theory of Supposition". In: *Journal of the History of Philosophy* 46.3 (2008), 365–393.

about a simplification in the number of topics actually appealed to, with a vast number of inferences justified by rules like *from an undistributed inferior to superior*, and *from a distributed superior to a distributed inferior*, while also simplifying the account of what a maximal proposition is: a maximal proposition is simply a rule in virtue of which an inference holds.[29]

On one of the best known accounts, that of William of Ockham, supposition divides into three types: simple, material, and personal.[30] Personal supposition occurs when a term is taken to refer to what it signifies; simple, when a term refers to its concept (understood as an intention of the soul); material, when one refers to itself as a spoken utterance or written word. Other accounts, e.g. that of John Buridan, view simple supposition as a variety of material supposition. But on both views, Personal supposition is taken to be the most typical, and is interpreted as the term's supposition for *individuals*.[31] On these accounts, determinate supposition of a term is interpreted in terms of the ability to descend to a disjunctive sentence where in each disjunction, the class term is replaced by the name of a different individual member of the class, with the number of disjuncts equal to the number of members. The case is similar for confused and distributed supposition, albeit employing conjunction.

This standard view of how supposition works and the presumed priority it ascribes to personal supposition seem to be confirmed in an anonymous London consequence treatise - perhaps the earliest still extant - where the notion is only found in a part of the treatise asserting that convertible terms, e.g. 'man' and 'risible', share the same supposita.[32] The same pattern is found in Burley's *de consequentiis*. Consider, for instance, the following passage on exceptive propositions:

> With respect to the supposition of the predicate and subject in an exceptive, one should know that that from which the exception is taken, or the subject of the exceptive ... always stands confusedly and immovably with respect to the exception, and movably with respect to the predicate. With respect to the ex-

[29](Walter Burleigh. *De Puritate Artis Logicae*. Ed. by Philotheus Boehner. St Bonaventure, NY: Franciscan Institute, 1955, pp. 76.5–7): 'Nam propositio maxima non est nisi regula, per quam consequentia tenet.' Cf. (William of Ockham. *Elementarium Logicae*. In: *Opera Philosophica. Opera Dubia et Spuria*. Ed. by Eligius M. Buytaert. Vol. 7. St. Bonaventure, NY: Franciscan Institute, 1988, 59–304, pp. 31.3–6).

[30]William of Ockham. *Summa Logicae*. In: *Opera Philosophica*. Ed. by Philotheus Boehner, Gedeon Gàl, and Stephen Brown. Vol. 1. St. Bonaventure, NY: Franciscan Institute, 1974, pp. 193–197.

[31]This nominalist reading is reflected in modern formalizations of the notion, e.g. those of Klima and Parsons. (Gyula Klima. *Ars Artium: Essays in Philosophical Semantics, Medieval and Modern*. Budapest: Institute of Philosophy of the Hungarian Academy of Sciences, 1988; Terence Parsons. *Articulating Medieval Logic*. Oxford: Oxford University Press, 2014).

[32]Green-Pedersen, "Two early anonymous tracts on consequences", 9, par. 27.

> ception it stands immovably, because it is not possible to descend with respect to it; hence 'every man besides Socrates runs, therefore Plato besides Socrates runs' does not follow. With respect to the predicate it is possible to descend, since every man besides Socrates runs, therefore Plato runs, and so on for singulars.'[33].

This passage also makes clear that the supposition of a term is given relative to another term or terms which are held fixed. Thus, a term's supposition may differ based on what terms it is being considered with respect to.

4.2 The Parisian *De consequentiis*

Given the close connection between the rise of theories of supposition and those of consequence, it is surprising to find that the approaches to supposition in early discussions of consequence can diverge from the standard Ockham-Buridan model, sometimes significantly. In an anonymous work on consequences in Paris, BN lat. 16130, in the majority of cases where the term 'supposition' is mentioned one finds that what is taken to supposit is a term or concept. Consider, for instance, the following:

> I show the consequence 'a man is a white man, therefore a man is white' by this rule: positing *per accidens*, also posits per se. But 'white man' is a suppositum *per accidens* of something. With respect to the same, 'white' is also posited.[34]

As we can see, what is taken to supposit here is not individual white men, but the term or concept 'white man', which is 'placed under' the concept 'man'.

Furthermore, the dominant division of supposition in the treatise is not that between material, simple, and personal, but that between *per se* and accidental. The text explains the notion of *per accidens* supposition as follows:

> And [those] placed under *per accidens* are those that are combined out of two inhering in each other contingently, as white man is combined *per accidens* because it is put together out of man and white, which inhere in each other contingently. So 'a

[33](Niels Jørgen Green-Pedersen. "Walter Burley's *de consequentiis*. An edition". In: *Franciscan Studies* 40 [1980], 102–166, 124, par. 58): . Translations for (Green-Pedersen, "Two early anonymous tracts on consequences"; Green-Pedersen, "Walter Burley's *de consequentiis*") taken from (Archambault, "The development of the medieval Parisian account of formal consequence", pp. 171–273).

[34](Green-Pedersen, "Two early anonymous tracts on consequences", 18, par. 34): 'Ostendo istam consequentiam 'homo est homo albus, ergo homo est [homo] albus' per istam regulam: posito per accidens ponitur et per se. Sed 'homo albus' est suppositum per accidens alicuius respectu eiusdem ponitur et 'album'.

> man is a white man', etc. is contingent. This is contingent, and its equivalents (*convertibilia*) are contingent, because in all of these a superior is assumed of a *per accidens* inferior (since whatever is constituted by an addition with respect to another is inferior to it. Thus, 'white man' is inferior to 'man' and 'white').[35]

Here, it's clear that the supposita themselves are not individuals, but forms or concepts.

4.3 Burley's *On the Core of the Art of Logic*

Walter Burley's *On the Core of the Art of Logic* exists in two versions, a shorter, earlier version and a later, longer one more engaged with Ockham's work. Burley's *Shorter Treatise* contains two different approaches to personal supposition.[36] Burley gives an example the first where, in his fifth principal rule for consequences, he describes suppositional descent as licensing a descent from genus to species:

> From the negation of the superior follows the negation of any inferior. And this rule must be understood [for] when the negated superior supposits personally. For it follows: Socrates is not an animal, therefore Socrates is not a man nor an ass, and so on of others.[37]

The key to this understanding of personal supposition is found in Burley's introduction of the notion in the *Longer Treatise*, where he treats it as a division of *suppositio formalis*.[38] Burley writes

[35](Green-Pedersen, "Two early anonymous tracts on consequences", 19, par. 38): 'Et sunt supposita per accidens ista quae aggregantur ex duobus contingenter sibi invicem inhaerentibus, ut illud 'homo albus' est aggregatum per accidens, quia aggregatum ex 'homine' et 'albo', quae contingenter sibi invicem inhaerent. Ideo haec est contingens 'homo est homo albus' etc. Illa est contingens, et eorum convertibilia sunt contingentia, quia in omnibus his praedicatur superius de inferiori [et] per accidens [album], quia unumquodque se habens per appositionem respectu alterius inferius est eo. Ideo 'homo albus' inferius est 'homine' et 'albo".

[36]For my translation of the Title of Burley's work, see (Paul Vincent Spade and Stephen Menn. "A Note on the Title of Walter Burley's *On the Purity of the Art of Logic*". 2003. URL: `http://pvspade.com/Logic/docs/BurlNote.pdf`).

[37](Walter Burleigh, *De Puritate Artis Logicae*, pp. 209.35–210.2): 'Quinta regula principalis est ista: Ad negationem superioris sequitur negatio cuiuslibet inferioris. Et haec regula est intelligenda, quando superius negatum supponit personaliter; sequitur enim: Sortes non est animal, ergo Sortes non est homo nec asinus et sic de aliis.'

[38]According to Dutilh Novaes, Burley appears to revive the notion of *suppositio formalis* unmentioned by Peter of Spain, Roger Bacon, or the *logica Lamberti*, from William of Sherwood (Catarina Dutilh Novaes. "Form and Matter in Later Latin Medieval Logic. the Cases of *Suppositio* and *Consequentia*". In: *Journal of the History of Philosophy* 50.3 [2012], 339–354, p. 360).

> Formal supposition is twofold, for sometimes a term supposits for its significate, sometimes for its suppositum or for some singular of which it is truly predicated. And thus formal supposition is divided into personal supposition and simple supposition.[39]

Burley continues:

> Personal supposition is when a common term supposits for its inferiors, *whether those inferiors be singular or common, whether they be things or words,* or when a concrete accidental term or a composite term supposits for that of which it is predicated accidentally.[40]

Burley's account here is more varied than the canonical account based on Buridan and Ockham, including both individuals and common natures as supposita in his account of personal supposition. When Burley states that personal supposition also captures that of composite and concrete accidental terms, he appears to have in mind those cases we found discussed earlier in the Paris treatise, like 'a man is white, therefore a man is a white man'. The account thus appears to better capture uses of personal supposition in ascents and descents in the earliest treatises on consequences.

One might ask whether the account of supposition found in Burley's *On the Core of the Art of Logic* and the anonymous Paris treatise demands a more extensive ontological foundation than that of Ockham or Buridan. Burley himself provides an emphatic 'no'. Discussing the supposition of 'man' in the sentence 'Man is a species', Burley remarks, '*I don't care at present* whether it [i.e. 'man'] is a common thing outside of the soul or it is a concept; it suffices merely that that which the name primarily signifies is a species'.[41] While committed to the existence of species in a non-foundational sense, Burley emphasizes that his account of supposition should hold regardless of what the ultimate foundations of the concepts a theory of inference employs may be. In this intention, his account contrasts with Ockham's contemporary theory, and perhaps also with Boethius' earlier grounding of consequence in the theory of topical argument.

[39](Walter Burleigh, *De Puritate Artis Logicae*, pp. 3.1–5): 'Suppositio formalis est duplex, quoniam terminus quandoque supponit pro suo significato, quandoque pro suo supposito vel pro aliquo singulari, de quo ipsum vere praedicatur Et ideo suppositio formalis dividitur in suppositionem personalem et in suppositionem simplicem'.

[40](Walter Burleigh, *De Puritate Artis Logicae*, pp. 3.19–24): 'Suppositio personalis est, quando terminus communis supponit pro suis inferioribus, sive illa inferiora sint singularia sive communia, sive sint res sive voces, vel quando terminus concretus accidentalis vel terminus compositus supponit pro illo, de quo accidentaliter praedicatur'.

[41](Walter Burleigh, *De Puritate Artis Logicae*, pp. 8.8–10): 'Sed sive illud commune sit res extra animam sive sit conceptus in anima, non curo quantum ad praesens; sed tantum sufficit, quod illud, quod hoc nomen primo significat, est species'. Cf. (William of Ockham, *Summa Logicae*, pp. 196.33–37)

5 Conclusion

In the preceding, we've detailed two change in the foundations of logic that ultimately affects our understanding of the nature of the discipline as such.

In earlier frameworks found, for instance, in Boethius, good arguments are discussed under a variety of types each with different grounds: some arguments, for instance, argue from topics concerning language, others from metaphysical relations between things, others move from effects to cause, and still others relate premises to conclusion as cause to effect.

At the time of the earliest *consequentiae*, the majority of consequences discussed are grounded in the theory of supposition. In this theory, personal supposition is the most frequently found, but consequences involving simple supposition are not uncommon. More importantly, terms in personal supposition may be taken to refer not only to individuals, but also to types. Both are permissible readings of the anonymous London *De consequentiis*, both readings are explicitly found in Burley's logic, and the latter is the only variety explicitly invoked in an anonymous Parisian treatise on consequences.

We can now see, then, that Ockham's restriction of personal supposition to exclusively refer to individuals, which today remains the dominant interpretation of *all* theories of personal supposition in the secondary literature, was more novel than hitherto recognized. In adopting this change, Ockham not only changed the understanding of personal supposition. He also changed that of simple supposition - now taken to refer to a term's referring to itself as a second intention, but before taken merely to refer to what a term signifies, leaving open whether this be an intention of the soul, a separate universal, or something else.

More importantly, in changing the account of supposition, Ockham also necessarily modified that of the theory of consequence which depended on it. While the earlier account of supposition, with its inclusion of forms or concepts, did better at explaining inferential practice, Ockham's sparse focus on individuals would grant the theory of consequence a surer foundation in reality at the price of leaving our grasp of it more opaque. In this way, Ockham's anti-metaphysical grounding for his logic may be regarded as *more* metaphysical than that of Burley and his predecessors: where Burley's was consistent with, but did not require realism, Ockham's built a decision on the metaphysical foundations of logic directly into his work.[42] And it seems to be this decision, if any, that sets the stage not only for the defects of nominalism, but also for the rise of 'extreme realism' to counter it.

[42]Cf. (Stephen Read. "William of Ockham's *The Sum of Logic*". In: *Topoi* 26.2 [2007], 271–277).