

On the Genesis of the Medieval Parisian Notion of Formal Consequence

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Abstract

This chapter provides an outline of the dissertation as a whole. The chapter begins with a discussion of notion of form in modern logic, providing an overview of more recent attempts to define the notion of formal consequence, and showing that each such attempt brings with it the general problem of determining exactly what it means to call a consequence relation formal, as well as the more specific problem of providing a demarcation criterion for the logical constants of a language. This motivates the primary question of the work: what does it mean to call a consequence relation ‘formal’ in the first place? and what did it mean at the time that the language of formal consequence was first adopted? From here, I provide a brief description of John Buridan’s theory of formal consequence, followed by a plan of the treatise, beginning with a description of John Buridan’s theory of formal consequence, and moving chronologically backwards to its predecessors in Ockham, Burley, and others. The final two sections respectively explain the methodology of the work, and offer several objections to the project along with their replies.

1 Formal consequence and formal logic

One of the common ways in which logic is separated out for special consideration when compared to other areas of inquiry is by the dictum ‘logic is formal’. We speak about formal rigor, formalized languages, formal consequence, formal methods - indeed, only the other branches of mathematics come close to logic in the degree to which they are stamped with the language of formality. Apart from this family of cases, it even seems that other areas of inquiry are considered formal exactly to the degree that they incorporate *logic* into their methodology - formal epistemology, for instance, is merely an approach to epistemology heavily reliant on the use of logical techniques.

One area in which the formality of logic is especially present is in discussions of logical consequence. This is in part because the subject matter of logic itself is often taken to be *what follows from what* - not, of course, in any sense whatsoever, but as a matter of logical *form*; and in part because as modifiers of

‘consequence’, ‘logical’ and ‘formal’ are frequently taken to be synonymous¹ - a fact that, assuming fixity of meaning for these terms, would transform ‘logical form’ and ‘formal logic’ into the emphatically redundant ‘formal form’ and ‘logical logic’. In short, our unconscious and reflexive patterns of speaking about logic and consequence suggest that the concept of form is somehow at the heart of the very logicity of logic.

The aim of this dissertation is to provide an account of how this came about. As such, the plan of inquiry is to detail the development of the notion of formal consequence at the time of the appearance of the first treatises on consequence - the first half of the 14th century - culminating in the account advanced by John Buridan. Buridan’s work will provide a convenient focal point in part because it represents the last major change in the notion prior to the advent of the modern period; and in part because the account is rather similar to the model-theoretic accounts dominant today through the influence of Tarski and others. Before moving on to Buridan, though, we ought to say something about the notion as it operates today.

2 A summary of developments in the notion of formal consequence from the early 20th century to present

We say that the sentence X *follows logically* from the sentences of the class \mathbf{K} if and only if every model of the class \mathbf{K} is at the same time a model of the sentence X .

Tarski 1936/2002, 186

The above quote is, of course, the definition of logical consequence first offered by Tarski in his classic 1936 paper *On the concept of following logically*. As others have pointed out² the definition is not identical to its contemporary successor. And while the work represents a breakthrough in the development of formalized notions of consequence, it stands not at the beginning, but in the middle of the notion’s more recent development.

The most important advances prior to Tarski 1936, both mentioned by Tarski himself, are those of the syntactic approach of the Hilbert school, on the one hand, and those made by Carnap and the Vienna Circle, on the other.

¹Tarski 1936/2002, pp. 188, 193; cf. Read 1995, pp. 36-7. In what follows, I will use these terms interchangeably - not because I myself agree with this use, but rather for the sake of addressing the traditions I aim to exposit on their own terms.

²Hodges 1986, Etchemendy 1988, Sher 1991

2.1 Formal consequence prior to Tarski

2.1.1 The syntactic approach to formal consequence - Hilbert

The main advances made by the Hilbert school with respect to the notion of following were the recognition that bodies of knowledge could be represented by just a few axioms, along with a few simple rules, such as substitution and detachment, for manipulating these axioms; and the corresponding reduction of large parts of mathematics to these formalisms. In accordance with the aims of the Hilbert school, a consequence is thought to *follow formally* from the axioms and rules of a system iff it is possible to obtain it from those axioms and applications of the permissible rules in a finite number of steps - ideally, in such a manner as to admit a decision procedure for any formula of the formalized language; and the formality of a formal consequence consists in its prescinding from any meaning the manipulated symbols might have - a notion of the formal Dutilh Novaes calls “the formal as de-semantification.”³

Relatively early on, the discovery of the ω -incompleteness of arithmetic thus formulated, without the addition of the rule of infinite induction, showed that this notion of formal consequence was materially inadequate to the intuitive notion it aimed at capturing. And later, the discoveries of Gödel showed that the notion remained inadequate even after relaxing the finitistic character of the system itself. Lastly, the theorem of Church showed that the close connection between formality and computability desired by the formalists⁴ fails even in the limited case of first-order logic.

2.1.2 The semantic approach - Carnap

In his *Logical Syntax of Language*, Carnap proposed a definition of consequence according to which

the sentence X *follows logically* from the class of sentences K if and only if the class consisting of all sentences of the class K and of the negation of the sentence X is contradictory.

Compared to the formalist school, what is new in Carnap’s approach is the mere fact *that* he attempts to provide a definition of following from, whereas the formalist school did not so much define a notion of consequence as presuppose one in its mathematical practice.⁵ Tarski himself calls Carnap’s definition “The

³Dutilh Novaes 2011, 318

⁴See Dutilh Novaes 2011, pp. 321-25

⁵Hardy gives us reason to suggest that this failure was, in fact, a deliberate part of the formalist enterprise:

Let us observe in passing that there are far more axioms in Hilbert’s scheme than in such a scheme as that of *Principia Mathematica*, and *no definitions* in the sense of *Principia Mathematica*. This is inevitable, since it is cardinal in Hilbert’s logic that, however the formulae of the system may have been suggested, the ‘meanings’ which suggested them lie entirely outside the system ... The only conceivable sense of a definition in [Hilbert’s] system is that of a symbolic convention which instructs us to replace a prolix formula by a more concise one. [Hardy 1929, 15]

first attempt at the formulation of a precise definition” of ‘following logically’,⁶ though this is certainly incorrect: even leaving aside the medieval accounts with which this dissertation shall be concerned, the concept is explicitly defined by Bolzano in the 19th century.⁷ What is probably more correct to say is that Carnap’s notion is the immediate predecessor of Tarski’s own, as well as the first attempt to define the notion in the wake of the explosion of interest in foundational research of the late 19th/early 20th century.

Tarski’s definition differs from that of Carnap inasmuch as it attempts to provide ground for notions Carnap must take as primitive, thereby in principle widening the range of formalized languages the notion of ‘following from’ is applicable to. For instance, Carnap’s definition makes use of the notion of negation in such a way as to limit its scope to languages that themselves contain a negation operator. Furthermore, Carnap’s definition takes the notion of contradictoriness as primitive, whereas Tarski’s notion allows contradiction to be defined in terms of the absence of any model.

One important respect in which the tradition has followed Carnap - and for that matter, Hilbert as well - as opposed to Tarski is on the question of the interpretation of the extra-logical symbols of a formal language. For Carnap, the non-logical constants of a language are regarded as *uninterpreted* until specified by a semantic interpretation. Tarski, by contrast, implicitly presupposes these symbols already carry a fixed interpretation, in accord with the formalized sphere of inquiry to which they apply.⁸ An important corollary of this is that for Tarski, it is not the *interpretation* of the non-logical constants that is varied: rather, these constants themselves are simply replaced. For instance, to evaluate whether the formula $\forall(x)(P(x) \wedge Q(x)) \supset P(x)$ holds, the standard approach today would vary the interpretation of P and Q, while Tarski’s approach simply replaces them with second-order variables, whose interpretations are then varied.

2.2 Formal consequence since Tarski

Since Tarski’s work on the subject, the notion has undergone several additional developments.

First, Tarski envisioned his definition would be applied not to *formal*, but to what he would call *formalized* languages, languages having a determinate sphere of application. Given this, Tarski’s conception of a model does not include variations of the domain of the model in addition to its assignments, whereas contemporary practice does require variation of the domain. And so,

⁶Tarski 1936/2002, 182

⁷Bolzano 1837/1972, p. 199/209. Bolzano’s definition reads as follows:

Propositions *M, N, O, ... follow* from propositions *A, B, C, D, ..* with respect to variable parts *i, j, ...* if every class of ideas whose substitution for *i, j, ...* makes each of *A, B, C, D, ...* true also makes all of *M, N, O, ...* true.

George [1988] suggests that, inasmuch as it demands that the variable parts of a consequence be stated directly, Bolzano’s definition is superior to standard post-Tarskian attempts.

⁸Stroińska and Hitchcock 2002, 167.

for instance, on Tarski's account, it would come out as a logical truth, for a collection of models having a domain of n objects, that there are exactly n objects. A consequence of this is that for Tarski, the formality of a formalized language - or, for that matter, of a formal consequence relation - has nothing to do with its purported universality, since the language or relation in question may be intended for a specified domain of objects.

The next important development, implicit in the development of Kripke semantics for modal logic but not explicitly addressed until later, was the division of logical consequence into local and global varieties. In contemporary modal logic, a sentence K is said to follow *locally* from a premise set Γ for a class of frames C iff, for every frame in C , for every model on that frame, every world in that model modelling all members of Γ also models K ; while K is said to follow *globally* from Γ iff, for every frame in C , for every model on that frame where all members of Γ are *valid* - i.e. where all members of Γ are modeled by *every* world in the model - K is also valid.⁹ Unsurprisingly, this division, made possible by the advent of modal logic, has brought with it around the question of which, if, either, expresses the genuine notion of following logically.

Lastly, apart from the above developments, the last quarter of the 20th century through to today has witnessed the proliferation of vast number of non-classical logics; domain specific extensions of classical logic, including deontic logics, temporal logics, and epistemic logics; and even logics with no intended 'logical' application, many of which are developed and intended for applications in computer science. Each of these developments brings with it new kinds of questions to be asked. Of itself, each non-classical logic brings with it the question of whether it, rather than classical logic, determines the correct class of formal consequences for our world; while collectively, this vast plurality prompts the question of whether the immediately preceding question is even a sensible one, a question taken up in the debate between logical *pluralists*, on the one hand, and logical *monists*, on the other. The development of domain-specific logics poses the question of whether these - given once widespread views about the universality of logic - should be genuinely called *logics*, or whether the appearance of the word in their titles rests on an equivocation.

In short, the contemporary situation brings with it questions about the necessity of formal consequence, the correct class of formal consequences, the domain of formal consequence, and the purported universality of formal consequence.¹⁰ In the light of this, can we say much at all about what has remained common in the way the concept of logical or formal consequence has functioned from prior to Tarski to now?

⁹Cf. Fitting and Mendelsohn 1998, pp. 21-23; Blackburn et al. 2002, pp. 31-32

¹⁰note that these last two are not exactly the same: the universality of formal consequence would be satisfied by a collection of distinct but disjointly exhaustive and otherwise stable consequence relations on their respective domains. What I have closer to mind by 'deniers of the universality of logic' are those who would take a pragmatic or contextual approach to even domain-specific logics, as if one could accept the epistemic logician's principle of positive introspection Monday through Friday and deny it on weekends. Cf. Mehlberg 1960, pp. 410-414

3 Common elements in the above accounts

The short answer to the above question is ‘yes’. A surprising degree of unity underlies the developments and diversity outlined above. All of the above positions, save for that of the early formalists, presuppose the following:

1. that for a consequence to be logical and for it to be formal amount to the same thing;
2. that the appropriate criterion for determining whether a given consequence is a logical one is substitutional: consequences are determined by varying the interpretation of the non-logical components of the formalized language, (or, otherwise, varying those components themselves)¹¹
3. that precisely those notions which are required to be invariant under all interpretations are the logical notions of a given language;
4. that a consequence is valid *in virtue of* these notions, and it is on account of these that a consequence has its logical form.
5. that substitutionality amounts not merely a condition on consequence, but rather defines *what it is to be* a formal consequence.¹²
6. that a logic is individuated by its class of logical notions. Hence, for instance, many extensions of classical logic are developed by adding to its stock of logical notions.

¹¹note that the sense of ‘substitutional’ used above is meant to be wider than that used to distinguish substitutional from objectual semantics for first-order languages. In brief, a substitutional semantics in this more restricted sense is one on which the truth value of its quantified formulae in a model is determined by the truth value of instances of those formulae wherein the formerly bound variables have been replaced by new terms, i.e. either constants or variables. An objectual interpretation, by contrast, is one on which it is not the terms, but the objects assigned to the variables that are varied. Typically, an objectual semantics is preferred on the grounds that substitutional semantics is not consistent with the intent to quantify over superdenumerable domains such as, e.g. the real numbers. But from a purely mathematical standpoint, the class of substitutional models can be represented as a subset of the objectual ones, i.e. those where the domain of the model is just the set of terms in the language. See Garson 2014, ch. 14

¹²Cf. Etchemendy 1988, 66, 67:

...as far as extensional adequacy goes, there are a multitude of equally correct (or equally incorrect) definitions of first-order consequence: when we specify any one of the many equivalent proof procedures for first-order languages, we have defined the consequence relation as adequately as when we define the relation model-theoretically. But from among these coextensive definitions, the model-theoretic account is typically afforded a special status, a status most clearly reflected in soundness and completeness theorems.

...We do not prove the “soundness” and “completeness” of the model-theoretic account of consequence; indeed the very idea would strike most of us as deeply confused. Our attitude here is characteristic of our attitude toward an analysis: extensional adequacy is guaranteed on a *conceptual* level, by our close adherence to the intuitive notion we aim to characterize. It is in this sense that the model-theoretic account is treated as a genuine analysis of the intuitive notions of logical truth and logical consequence.

7. that, accordingly, without a principled and sharp demarcation criterion for discriminating between the logical and non-logical components of a formal language, we also lack an adequate understanding of the scope and nature of logic¹³

To get a better grasp on some of the above points, it is worth reflecting on that with which formality in logic is most likely to be contrasted. On the one hand, the formal is said to be the opposite of the *informal*. In this sense, the notion of formality is typically associated with rigor on account of its use symbolization, itself in the service of obscuring the meaning of the matters to which it is applied for the sake of making these formulae more easily or even effectively calculable.¹⁴ And so the spirit of informal logic would be typified by an approach to logic working in or otherwise heavily reliant on natural (as opposed to formalized) language, and one that makes use of the meanings of the terms it treats in determining what follows from them. Such an approach is found, for instance, in the ordinary language tradition of Ryle and others.

On the other hand, the formal is contrasted with the *material*. And it is clearly this contrast that is at work in the above bullets. In this hylomorphic contrast lifted from the framework of Aristotelian physics, form and matter are constitutive components of every material being (item 1); it is that which remains invariant in a material being throughout its existence (items 2, 3); it makes a thing to be what it is, thereby determining its definition and quiddity (items 4, 5); and on some medieval accounts (though not most), it also serves as a principle of individuation (item 6).

In short, underlying the multitude of different positions and debates mentioned in the previous section is a common core of thinking about logic that is essentially hylomorphic in structure. This in itself is surprising on several accounts: first, because logic is often thought to be formal precisely inasmuch as it demurs from an association with either any particular content or metaphysical assumptions - unmixed, like Aristotle's agent intellect;¹⁵ second, because even among metaphysically minded logicians (or logically minded metaphysicians), *any* kind of hylomorphism remains a minority opinion; third, because, the kind of hylomorphism specified in the above points is itself highly specific - particularly as is evident in the problem of providing a demarcation criterion for the logical constants of a language, the kind of hylomorphism present in the points outlined above seems to be what Dutilh Novaes called *mereological* hylomorphism.¹⁶ According to Dutilh Novaes, mereological hylomorphism is characterized by the contention not merely that wholes are compounds of form and matter, but also that form and matter are themselves, strictly speaking,

¹³see MacFarlane 2009, Introduction

¹⁴Cf. Gödel 1995, p. 45

¹⁵This analogy is itself in some respects telling: Aristotle himself does not identify the form of the human body with the intellect, though, of course, Descartes and others did, thereby bequeathing from the old problems of hylomorphism the still-discussed mind-body problem. Buridan himself identifies the soul with the intellect, by holding the soul to be identical to its powers.

¹⁶Dutilh Novaes 2012, pp. 396-97.

distinct *parts* of the hylomorphic compound. This is reflected in the notion of formal consequence inasmuch as it presupposes a partition of the logical vocabulary, with one part - the logical connectives - corresponding to the form, and the rest corresponding to the matter, with these sets providing a disjoint and exhaustive partition of all the signs of the language under discussion.

4 Buridan's concept of formal consequence

The first known account of formal consequence in the western tradition defined in terms of such a substitutional criterion is that of John Buridan, the 14th century Master of Arts at the University of Paris. Buridan's was not the first of the medievals to distinguish between formal and material consequence - the distinction is first explicitly mentioned by Ockham, but seems to have been implicit in Duns Scotus, Simon of Faversham, and others - but he was the first to distinguish formal and material consequences in much the way we do today, by varying the categorematic terms of an argument.¹⁷ According to Buridan,

A consequence is called 'formal' if it is valid in all terms retaining a similar form. Or, if you want to put it explicitly, a formal consequence is one where every proposition similar in form that might be formed would be a good consequence

...A material consequence, however, is one where not every proposition similar in form would be a good consequence, or, as it is commonly put, which does not hold in all terms retaining the same form¹⁸

Buridan's way of distinguishing material from formal consequences was also, especially on the continent, by far the most influential, having been adopted by Marsilius of Inghen, and Albert of Saxony, among others.¹⁹

There are some important differences between the way formal consequence is understood by Buridan, and the way it is understood today - to name two, The basic units of Buridan's consequences are written or spoken sentences, and therefore his semantics is token rather than type-based; and Buridan doesn't assume that a material consequence is therefore not a logical one. But these differences appear to be of an accidental rather than essential character.²⁰

¹⁷note though, that for Buridan, it is the *terms themselves* that are varied, rather than their interpretations. For instance, he says that 'A man runs; therefore, an animal runs' is not a valid consequence, because 'A horse walks, therefore a tree walks' is not (*TC* 1.4.3).

¹⁸*TC* I. 4.2-3:

Consequentia 'formalis' vocatur quae in omnibus terminis valet retenta forma consimili. Vel si vis expresse loqui de vi sermonis, consequentia formalis est cui omnis propositio similis in forma quae formaretur esset bona consequentia [...] Sed consequentia materialis est cui non omnis propositio consimilis in forma esset bona consequentia, vel, sicut communiter dicitur, quae non tenet in omnibus terminis forma consimili retenta

Translations taken from Read 2015, unless otherwise noted.

¹⁹Dutilh Novaes 2012b, sec. 3.3

²⁰Indeed, much of the renaissance of Buridan scholarship in the past half century or so has been explicitly motivated by the *prima facie* similarities between Buridan's treatment of

Whatever one may make of these more minor differences between the Buridanian notion of a formal consequence and its present-day counterpart, there is one difference that makes studying Buridan and his contemporaries an especially fruitful endeavor: the medieval application of hylomorphic language to consequences could not but be a *conscious* one, taking place at the height of critical engagement with both Aristotle’s logic and his physics and metaphysics; whereas the contemporary appropriation of hylomorphic language has been by and large uncritical and even at times unaware of this appropriation.²¹ If, at times, medieval treatments of consequence appear less sophisticated than their contemporary analogues, they’re also somewhat less liable to the distractions that invariably accompany the development and long use of a technical vocabulary, and thereby often closer to the matters themselves under discussion.²²

The aim of this study, then, is to uncover the meanings implicit in our use of the notion of formal consequence by peeling back the layers of meaning imposed at the time when “the main precursor of the modern concept of logical consequence”²³ was first formulated. Its plan is detailed in the following section.

5 Overview

The general plan of the work is to begin with Buridan’s notion of formal consequence, and from there to move backwards in successive stages to the discussion of its historically antecedent enabling conditions. The questions surrounding the genesis of Buridan’s notion, though not all of them have been answered, have at least reached a point where they are easily formulable and relatively tractable. The main questions are as follows:

1. Most basically, what is Buridan’s account?
2. How does Buridan’s account relate to that of Ockham, the first to explicitly mention a distinction between formal and material consequence?
3. How does the division of consequences into formal and material relate back to the division between natural and accidental consequences, i.e. to the division it seems to have replaced?

various logical topics and contemporary treatments of what are recognizably, at least on some level, the *same* topics and questions. Cf. Parsons 2014, esp. pp. 164-76

²¹Important exceptions to this include Read 1994 and 1995; MacFarlane 2000; and Dutilh Novaes 2011, 2012a, 2012b, 2012c.

²²This is really an application of a much broader point frequently ignored, or merely paid lip service to, in both historical and specialized systematic discussions today: our later standpoint on questions of philosophical importance is not wholly an advantageous one, inasmuch as the development of any body of knowledge brings with it a certain forgetfulness of its origins. To take an especially clear example of this, the vast proliferation of logics in the past century, while it has brought us a great many proofs, has not brought us a step closer to an understanding of what logic is or of what it is about. Aristotle, whatever one may think of his analytical and topical works, at least had some sense for what he was doing. Our current state regarding the sense of these questions, by contrast, is probably more bewildered than it has ever been.

²³Dutilh Novaes 2012b

There are, of course, more fine grained questions ensconced within those mentioned, as well as questions that may be asked on either chronological side of these. One may ask, for instance, how the notion of formal consequence is developed by Buridan's followers, or about the development of earlier divisions of consequences. But it seems to me that these questions are the main ones, inasmuch as an answer to them would yield a philosophically illuminating and relatively self-contained answer to the question of where the language of formal consequence actually came from.

The immediately following chapter divides into two parts. The first part provides a more in-depth introduction to Buridan's concept of formal consequence in itself. The second part treats Buridan's notion as it relates to modern accounts. I begin the chapter with a fairly gentle introduction to some of the consequences that Buridan enumerates as valid in the first book of his *Tractatus de Consequentibus*. A survey of these results then provides us with the opportunity to examine Buridan's use of hylomorphic language in his descriptions more closely; and so the following section determines the precise function and connotations of Buridan's use of this language. The last section of the first part discusses an unfortunate inconsistency in Buridan's account, involving his simultaneous adoption of the rule *ab impossibili quodlibet* and the rule that a positive consequent cannot be entailed by an antecedent that is wholly negative. The final part situates Buridan's account with respect to current model-theory.

Chapter three compares Buridan's account of formal and material consequence to that of Ockham. The first part of the chapter provides a systematic comparison between the two accounts, while the second part investigates the question of influence. Recent literature has been apt to distinguish Ockham's account from Buridan's - and indeed, British from Continental medieval approaches to formal and material consequence more generally - by saying that while the tradition on the continent formulated the distinction between formal and material consequences substitutionally, the British tradition formulated the distinction in epistemic terms.²⁴ I show that in the case of Ockham, this isn't quite right. Rather, the formality of Ockham's formal consequence essentially consists in its holding by virtue of an extrinsic rule that is normatively binding on the thought patterns of actual reasoners. In this way, Ockham's account of formal consequence is thus of a piece with his ethics. The second part answers the question of whether either the language or the content of Buridan's distinction is in fact derived from Ockham. In short, I argue that though Buridan had read Ockham by the time he composed his commentary on the *Elenchi*, nothing in the content of Buridan's notion of formal consequence in the *TC* gives us reason to believe that he had read Ockham up to that point. Furthermore, I present evidence that the notion of formal consequence had existed at Paris prior to Ockham's writing the *Summa Logicae*. Hence, it is at least as likely that Buridan's development of the notion and adoption of the language is an independent development as it is that it is a reaction to Ockham.

²⁴See, for instance, Dutilh Novaes 2012b

Chapter four introduces the distinction between natural and accidental consequence, as it appears in Burley. I show that Burley's distinction between natural and accidental consequences almost exactly parallels Ockham's between formal and material consequence. But while Burley accepts the content of Ockham's distinction, he also rejects the language Ockham uses to express it. Burley himself introduces a distinction between consequences that hold by virtue of form and those that hold by their matter. But he is insistent that a formal consequence is one that holds by virtue of a form in the metaphysical sense of the term.

The penultimate chapter explores the relatively infrequent uses of '*consequentia formalis*' in Duns Scotus, Radulphus Brito and Simon of Faversham. Here, I show that the notion of formal consequence found in these figures is roughly identical to that found in Burley. But unlike Burley, they seem to use the distinction between natural and accidental consequence and its hylomorphic counterpart synonymously.

The final chapter takes a synoptic view of the results detailed in the previous chapters, and returns them to the question of their import for the ways in which logic is said to be formal today.

6 A note on aims and methodology

My reader is welcome to regard this dissertation simply as a contribution to the history of philosophy (to the history of medieval logic in particular), interesting in its own right. But in doing so, she would be missing much of the impetus behind my taking up the project in the first place. The aim of this dissertation is the clarification of a concept - that of formal consequence - operant in logic today. The means chosen for attaining that aim is that of a historical exposition of the genesis of this concept in medieval logic. While the study of the history of philosophy is in a better place now than it was a few generations ago, it has become so by clearing a space for itself, as it were, wherein it is regarded as valuable for its own sake. And so, to continue with the metaphor, the history of philosophy has attained its current respectability as a subfield precisely by remaining a *subfield* - a marginal and marginalized area of interest that, while it may offer accidental helps to theoretical philosophy, is not strictly necessary for its operation.²⁵ And so even for those who have no particular antipathy to historical study, the aim of explicating the content of a concept *through* its

²⁵Isn't it harsh, and perhaps even a bit ludicrous, to call the history of philosophy 'marginalized', both given its current status and in the face of more obviously marginalized fields (e.g. critical race theory)? No. In short, a discipline may be called 'marginalized' exactly when its domain of inquiry is regarded as more restricted than it actually is. In this essential sense, the critical race theorist, for instance, is marginalized even when he is viewed as doing highly respectable work in the 'area' of race. The problem with this sort of praise is that the critical race theorist does not study race as one subject among others, but rather views race as a formal content whose effects are practically ubiquitous. Similarly, the history of philosophy remains marginalized to the degree that we treat its content as *separable*, and thereby fail to recognize its full weight as impinging on the practice of philosophy itself.

history may seem to be a case of adopting a means intrinsically inadequate to the desired end. This problem is compounded by the fact that the topic under discussion is one of *logic*, which, on one conception, is (or should be) ontologically ‘neutral’, i.e. completely free from assumptions about the way the world actually is, and *a fortiori* uninformed by that world’s history.

Though this is not the place to provide a complete account of the relationship the content of a concept bears towards its contingent historical origins, some such account must be at least outlined if the above project is to be grounded. For similar reasons, the issue of what could be meant by the ontological neutrality of logic must be addressed, if only to ward off the impression that no metaphysical concept could have *absolutely any* bearing on a properly purified logical concept. This will be done in the following section.

7 Objections

As described in the previous section, we have two fundamental objections to the shape of our inquiry as it stands. I will call the first *the theorist’s objection to history*; the second, *the logician’s objection to metaphysics*. Though seldom formulated explicitly, I take it that these objections are extremely widespread. Their content is as follows:

Objection 1: the theorist’s objection to history - project p aims to clarify the content of a concept C with domain D via the history of C . But the content of no concept is clarified by its history. Proof: The content of any concept necessarily belongs to it. But the history of a concept belongs to it contingently, and nothing belonging to another in a contingent manner can clarify what belongs to the same necessarily. Hence, history cannot illuminate conceptual content. And so nothing can show that it does.

Objection 2: the logician’s objection to metaphysics - project p aims to show how a concept lifted from a metaphysical framework bears on a logical one. But since logic is ontologically neutral, no concept belonging to metaphysics has any bearing on another belonging to logic. And so no inquiry can show that a metaphysical concept bears on a logical one.

While the latter objection raises a problem for the relation between the concepts of one kind of inquiry and those of another, the former raises the problem of the relation of history to any properly theoretical endeavor. Since I take the former problem to be both more entrenched and more difficult in itself, I will save it until after the latter difficulty has been addressed.

8 Reply to objection 2: ontological neutrality and the ontology of logic

In order to determine what is involved in the ontological neutrality of logic, one must first have unified accounts of what ontology and metaphysics respectively consist in. In the case of metaphysics, the current state of the discipline suggests no such account; rather, metaphysics comes across as something of a grab-bag umbrella discipline encompassing all the traditionally philosophical topics and questions that don't seem to nicely fit anywhere else.²⁶ Ontology is slightly better off, inasmuch as its object is pithily contained in the slogan 'ontology studies what exists'. But the content of this statement is itself not sufficiently clear. Accordingly, different accounts of the meaning of this slogan provide different accounts of what the ontological neutrality of logic consists in. We will examine each of these meanings in turn.

8.1 Ontology as the determination of types of existents through first-order quantification

The slogan 'ontology studies what exists' is most often understood to mean that ontology provides an exhaustive classification of which kinds of beings there are in the universe: If ontology asks the question 'what exists?', the sort of answer it envisions could be given in the form of a universal quantifier ranging over a disjunction, with each disjunct attributing a monadic predicate to the value of a variable bound by the universal quantifier, such that the empty set is the value of no predicate, the intersection of the range of any two predicates is the empty set, and the union of all of them yields the domain of quantification. More succinctly, ontology ultimately aims to provide a disjoint and exhaustive classification of the most basic kinds of beings. If the requirement that the intersection of different predicates be empty is dropped, the result is a mediate aim of ontology, i.e. determining what kinds of beings exist. If the ontological enterprise aimed at is eliminativist in character, then the beings in the latter group would be regarded as not 'really' existing, and a translation schema for reducing these sentences to others quantifying over only primitive existents would likely have to be given; if the enterprise is merely reductionist, then the beings of this latter group would probably be regarded as existing in some diminished sense.

On such an account, for logic to be ontologically neutral would be for it to be in principle unfit for the use the ontologist makes of it, i.e. determining kinds

²⁶For instance, the most recent edition of Blackwell's *Metaphysics: An Anthology* includes the following headings: causation; identity; modality; objects; ontology; persistence; persons; properties. Some of these (identity, modality) have connections at least as strong with other disciplines (logic, in this case) as with metaphysics. Others (ontology, objects) are not clearly differentiated from each other. Still others (persistence, persons) seem to have domains too restricted to qualify as metaphysical, and only bear faint relations to the discipline as traditionally conceived. This is not so much a fault of this particular volume (or any of the many others encoding the same *prima facie* arbitrariness) as a testimony to the state of the discipline itself, i.e. one wherein most of its practitioners are content to work on smaller problems without ever addressing what it means to say that this work is work in *metaphysics*.

of existing entities through the use of first-order quantification.

Without determining whether logic(s) really is (are) ontologically neutral in the above sense (and I suspect it is), it is enough to make three minor remarks here: first, that in practice, the ontological enterprise has been little more than intuition pumping in a mathematical guise, and so is not yet at a point where it can be taken seriously as a threat to the ontological neutrality of logic; second, that the sort of basicness aimed at in the grounding of some beings by others is often ambiguous between that of basic *types* and that of basic *parts*;²⁷ third, that even a completed ontology in the above sense wouldn't be able to ward off rival paradigms whose sentences were intertranslatable with those of the desired ontology's language.²⁸

To these minor complaints, however, a much more basic complaint must be added: that ontology in the above sense simply isn't terribly interesting or even especially philosophical. It is essentially an exercise in the conferral of status. All of its energies are focused on determining which beings do and do not 'count' in some relevant sense (where relevance is usually determined by the utility of the beings quantified over for a predetermined set of sufficiently canonical tasks, e.g. the hard sciences), and the stance embodied in such an enterprise is, frankly, banal. There is just too little of the theoretical stance here, encapsulated in Aristotle's dictum 'philosophy begins in wonder', for me to think there is anything especially philosophical going on in this activity. And so, the above described project has no interest in engaging in ontology in this sense.

8.2 The object of ontology as ways of being

A second notion of ontology can take as our aim not the enumeration of types of entities, but the determination of their properties.²⁹ Ontology in this sense would not be concerned so much with beings as the *being* of beings; and, especially if considered as the determination of *most basic* ways of being, would seem to have its predecessor, if not its first instantiation, in Aristotle's *Categories*.

On this account, the ontological neutrality of logic would consist in the following: that logic does not determine or otherwise reveal the way things are, and so one cannot 'read off' the structures or properties of things from logic.

Typically, this concept of ontological neutrality is invoked against those who would take the syntax of a *language* - whether natural, formal, mental, or otherwise - to reveal the structure of reality. Even so, the claim can be understood in several ways.

Perhaps most innocently, the claim can be taken to mean that the syntactical structure of the logical language one uses cannot as such be relied on to reveal the

²⁷This is the reason mereology forms such a large part (*sic*) of the sort of ontology described above

²⁸see Jared Warren, "Quantifier Variance and the Collapse Argument" *Philosophical Quarterly* (forthcoming).

²⁹Here, the term 'property' should be understood in the broad sense of whatever can be predicated of an individual, rather than as, e.g. Lewisian classes, or as a translation of the more restricted medieval '*proprium*'. See Lewis (1983), p. 344

structure of reality. In this sense, it amounts to the claim that one cannot assume that the logic one is using for the inquiry and domain at hand is the right one. This claim is primarily methodological in character, and seems unobjectionable.

In another sense, the above phrasing could be taken to mean that a logic is not as such in the business of determining the way things are; and *soqua* logic, the question of a correct application is one that simply does not come up. Some logic might, for all we know, hold the key to the hitherto hidden structure of reality. But as logicians, the question of a correct or incorrect application simply doesn't concern us. By analogy, one might call Euclidean geometry 'false' and Riemannian geometry 'true' inasmuch as the latter seems to provide a correct account of the nature of space, where the former does not. But strictly speaking, neither the one nor the other has as its aim any possible application, but both have as their immediate object certain eidetic structures to which each is perfectly adequate. In this sense, too, the ontological neutrality of logic seems unobjectionable, since actually doing logic does require prescinding from ontological questions and concerns.

Lastly, the claim can be taken in a stronger sense to mean that *even if* one has the correct logical language, this does not of itself determine or otherwise presuppose a specification of the (necessary, actual, possible) structure of reality.

The most important thing to note about the above modal parsings is that *all* of them have corollaries that are metaphysical in character; and so as defenses of ontological neutrality, they are self-undermining. For instance, if a language doesn't determine or otherwise specify the ways things actually are, then *things are such that* language doesn't specify their actual structure. So rather than enforcing ontological neutrality, these kinds of defenses (and I think they are extremely common, albeit implicit) themselves presuppose an answer to the metaphysical question of how a logical language relates to the world. To state the broader principle, the negation of a metaphysical claim remains a metaphysical claim: if, e.g. the claim that modal logic presupposes modal realism is metaphysical in character, then so, too, is the claim that it *doesn't*. Only diamonds cut diamonds.

I suspect that the appeal of ontological neutrality in the sense described derives from a broader picture of the relation between logic and metaphysics. Roughly, the relation is taken to be analogous to that present in Aquinas' understanding of the relation between philosophy and theology.³⁰ For Aquinas, the truths of philosophy do not provide a decision for or against the truths of sacred doctrine. Rather, philosophy takes one to a certain point, and then if one wishes to go farther, the gaps remaining must be filled in by theology. In this way, philosophy provides a solid foundation for theological study without replacing it. Similarly, on the account envisioned by partisans of ontological neutrality, the study of logic (whether conceived strictly as the study of the formal properties of formal systems, or broadly as more or less synonymous with philosophy of language) is taken to provide a sure and unbiased foundation for the study of higher philosophical sciences, such as metaphysics. It is given the

³⁰See *ST* Ia, q. 1.

first place pedagogically because it provides a method universally applicable in later inquiries without in any way predetermining the results of any of these later inquiries. As such, appeals to ontological neutrality in the described sense accord with and contribute to a vision of philosophy that is, broadly speaking, both foundationalist and progressivist in character. It is foundationalist inasmuch as it in principle accepts the paradigmatically Cartesian aim of finding a space free of presuppositions from which philosophical inquiry should begin, whether this space is found in epistemology, phenomenology, philosophy of language, semantics, etc.³¹ It is progressivist inasmuch as it sees this space as one ultimately to be filled. It accepts a *terminus a quo* distinct from its *terminus ad quem*, and views the philosophical endeavor as one of moving from *A* to *B*.³²

The basic enterprise encapsulated in works like Aristotle's *Metaphysics* is not that of determining which kinds of beings there are, but more basically that of determining *what it is* for a being *to be* a being. If one takes 'being' to be said univocally of all beings, then one way of answering this question is to give the kind of universally quantified disjunction mentioned in the previous subsection.³³ But it is the multiplicity of ways of answering this original question that undergirds the unity within the multiplicity of traditionally metaphysical disciplines: metaphysics discusses causality because causes are beings in a higher sense than their effects; identity, because being a being as such implies being self-identical; modality, because the extension of being to being-potential coincides with the discovery of a notion of being extending even to what are, strictly speaking, non-beings, etc.

In keeping with this theme, while it is unlikely that a) the presence of a metaphysical notion in our logician's toolkit would lead to the failure of ontological neutrality in the most common sense, or even that b) the presence of such a notion would provide us with a clue to the way logic ideally should be, c) it *does* uncover a structural analogy conditioning what it presently means for something to be logical, thereby providing a clue to the being of logicity as such.

My project is interested in ontology in the aforementioned way, albeit in a limited sense. The project is not an inquiry into the being of beings exactly insofar as they are beings, and so it does not belong to ontology as such. But it is an inquiry into the being of formality as it appears in logic, and so, in this sense, forms part of an inquiry into the ontology of logic³⁴. The inquiry maintains a

³¹Note that the inquiry remains foundationalist in principle even if it admits that in practice this space can never be completely secured.

³²I take it that this is all that is essentially involved in progressivism. And so progressivism, properly speaking, is only at best derivatively associated with the view that things 'keep getting better'.

If the reader is hard-pressed to think of a way of doing philosophy that *isn't* progressive in this broad sense, he might take the Platonic view of philosophy as recollection, introduced as a solution to Meno's Paradox, as an example. I take it to be significant that Aristotle's *Posterior Analytics* begins with a discussion of this very problem. Cf. *Post. An.* Bk. 1, ch. 1

³³Hence, the modern understanding of ontology relates to the more traditional one as a genuine attenuation of it.

³⁴Note, again, that by this I do *not* mean a determination of types belonging to the domain

further relation to metaphysics inasmuch as the sense of formality considered is at least analogously related to that found in Aristotelian metaphysics, the latter concept clearly providing the source material for the former. If I thought the domain of logic were uniquely circumscribable, I would describe this project as a regional ontology, in Husserl's sense. Since, however, I think the universality of logic precludes its specificity, it would be better to regard the project as contributing to a critique of formal logic - not in the sense of a criticism, but rather in the sense of 'critique' invoked by Kant and Husserl.

8.3 A response to the question

At this point, we can return to the objection as phrased. The crux of the objection is that to distinct sciences belong distinct concepts. While this might not at all seem self-evident, I think it is true. Most basically, every science will be what it is as directed towards a more or less restricted domain of beings, understood in a certain respect. The concepts proper to a science will be those that must be added to one's common stock of concepts in order to understand the specified domain in the sense required, all of which will typically belong to that science inasmuch as they relate to some primary sense. When these senses are encapsulated in a language, the resulting body of words and phrases will constitute the *jargon* of that science.

Now, a condition for a term t 's belonging to a language L is that it have some use in L , and so must meaningfully relate to at least one other term in L .³⁵ For parallel reasons, the senses indicated by these terms themselves must similarly relate.³⁶ In some cases, the concepts proper to one science will be a proper subset of those belonging to another one. In others, the concepts proper to different sciences will be disparate, etc. Lastly, because these senses are at least mediately relatable, so, too, will be the sciences studying them.³⁷

Roughly, the number of concepts³⁸ necessary for understanding a given domain of objects will be inversely proportional to the size of that domain, the

of a limited region of ontological space. Rather, I mean an inquiry into what it means for something to be logical, i.e. what it is to be logical

³⁵The following argument is a variation on a point first made by Sommers, 1962

³⁶It should be evident to my reader that I am using 'sense' and 'concept' in a fairly interchangeable way. The reason for this is a duality inherent in the concept of a concept as such. On the one hand, 'concept' can refer to that by which or in which a given sense is captured. On the other, it can mean the sense itself as understood. I think both uses are legitimate - much as, for instance, the word 'judgment' is used to indicate both the act of judging and what is judged. But I also think that too heavy a reliance on the first meaning of 'concept' has left many of us thinking that concepts are a kind of third entity mediating between the mind and ordinary external objects. This is unfortunate in two senses: the first, in that it invariably treats concepts as reified; the second, in that it treats concepts as primarily suppositive rather than signifying.

For both of these reasons, I also avoid calling this nexus of concepts a 'conceptual scheme', laden as that phrase is with the connotation that such a scheme is just an object or collection of objects applicable to others.

³⁷It is this point which grounds the unity of the sciences, and thereby also provides the occasion for the actualization of this unity in the structure of the university.

³⁸I. e. both the common and proper concepts of a science

reason for this being that stricter concepts implicitly contain in their sense more general ones. Inasmuch as these more general senses are implicitly presupposed by sciences focusing on the more specific senses, it is possible for there to be a science that explicitly thematizes these more general concepts. This dependence of senses grounds a corresponding dependence between the sciences aiming at the clarification of those senses, and so the science explicitly thematizing the more general sense will accordingly be called *higher* than the other.

Now, to say that one science does not presuppose another is to say that the sense or senses explicitly thematized by the former do not themselves implicitly contain those explicated by the latter. And so, it is also to say that the latter is not higher than the former. Therefore, to say that logic does not presuppose metaphysics is to say both 1) that the concepts studied in logic do not in the aforementioned way presuppose those explicated in metaphysics, and 2) that metaphysics is not a higher science than logic. But the primary sense studied in metaphysics is that of *being*. If, then logic does not presuppose metaphysics, then the concepts it studies do not presuppose or otherwise include that of being. This does not mean that logic does not study being *qua* being: it means that logic does not study being in *any* sense. This, in turn, can only occur in one of two ways. If logic is not a higher science, then either a) logic is higher than metaphysics, or b) no subordinating relation obtains between them - they are disparate.³⁹ If the former, then the primary sense studied in logic - call it *N* - will itself be contained in that of being, and so being will be a derivative mode of *N*-ing. If the latter, then the notions of the one and the other will be in principle relatable by some higher concept, itself in principle thematizable by a higher science.⁴⁰ But neither of these can occur: to be logical, after all, is *to be* logical. And so, far from needing to be free from metaphysics, logic must presuppose it for its very sense. The appearance to the contrary only occurs in the presence of a more attenuated understanding of metaphysics or ontology.

³⁹My reader will have noticed that these are, logically, not the only ways for sets of concepts to relate to each other: identity and partial overlap are also options. I don't mention identity because I think that two purportedly distinct sciences sharing the *exact* same concepts are not two sciences, but one, just as two objects sharing the exact same properties are the same object (Call this principle of the identity of indiscernibles for sciences, if you like). I don't mention the case of partial overlap because I think that whatever can be said of that case also applies to sciences with completely distinct conceptual apparatus.

⁴⁰In this way, the former approach actually undergirds the Quine-Goodman program of 'reading off' a metaphysics from one's logic. Contrary to what one might assume, this approach is not arrogant because it reads metaphysical considerations into logic, but precisely because it *doesn't*: the logic dictates the metaphysics because the logic itself is treated in practice as untouchable by it.

The latter approach, by contrast, is what has motivated the rise of semantics from the 20th century to today, which exactly parallels the rise of epistemology in the 17th: in the 17th century, one began by separating out the mental realm from that of being as if they were members of a common genus, only to take epistemology as a *tertium quid* required to connect the two; now, instead of cutting our heads off, it is *language* that is amputated from reality, and we bring in semantics to bridge the gap.

9 Reply to objection 1: the place of history in theoretical philosophy

The theorist's objection to history contends that a concept's contingent history is not relevant to the determination of its necessary internal content, relying on the basic assumption that contingent matters do not shed light on necessary ones.

The force of this basic assumption, however, rests on an equivocation. The imagery of shedding light on, illuminating, clarifying, straightforwardly implies a hierarchical ordering, wherein one matter is benefited by another (i.e. it is illuminated), but not conversely: to assume that the necessary clarifies the contingent is to assume that the necessary is that on account of which the contingent is able to be made manifest. Necessity grounds contingency. Obviously, these sorts of judgments are taking the kind of clarification at hand in thoroughly ontological terms, wherein one kind of object is made capable of appearing as it does on account of its subordinate relation to another. Understood thus, the judgment is true, but not relevant. On the other hand, the verb 'to clarify' and its synonyms can be taken in the above judgment to express not an ontological, but an evidentiary grounding. In accordance with this, the judgment would mean: the contingent does not disclose the necessary. Understood in this way, the judgment is relevant, but false. On the contrary, far from being impossible, evidence of necessity is hardly grounded in any other way. Even if, as Aristotle put it, all knowledge is taken from the senses, the immediate objects of which are contingent, this would not imply that all knowledge is *about* contingents. Thus, most basically, history serves theoretical philosophy in an 'exemplary' manner: it provides it with examples and data from which to adduce deeper claims.

The sort of dependence involved here, however, is relatively weak. It does not belong to history as such, but is nothing more than the manner in which theorizing presupposes experience more generally. In this sense, theorizing maintains at least a possible relation to history.

But one might wonder whether theoretical philosophy presupposes history in a stronger way. And the answer to this can be given simply by considering the conditions for the possibility of the objection itself. We shall divide these conditions into constitutive content conditions, on the one hand, and existential enabling conditions, on the other.

It should be clear even to the most virulently anti-historical philosopher that the theorist's objection to history would not arise in every epoch with equal ease; and that it would not, even if raised, carry equal force in every age. It would carry little weight, for instance, in an epoch where deference to ancient authorities was regarded as the surest pathway to truth. And as far as this point goes, the presence of an objection or viewpoint is no different than the presence of anything else. At the extremes of this phenomenon are cases where the structural conditions in being either necessitate certain developments, or make them impossible. In the case at hand, the objection would not arise if

there did not seem to be a distinction between the necessary and contingent as though of two distinct realms, a notion historically indebted to the separation of the contingent from the eidetic first carried out in the ontology of Plato. To give another example, equity in the workplace, either as an ideal or as a reality, presupposes the workplace - that is, it presupposes a world in which there locus of work is distinct from that of the home, and thereby presupposes much of the development that historically came to pass in the industrial revolution. Thirdly, the push for marriage equality begun in the late 2000s presupposes the presence of marriage as a public good,⁴¹ as well as the assumed normality of lifelong monogamous relationships.⁴² Again, the development of Euclidean plane geometry in ancient Greece presupposed: the prior development of writing, as a way of carrying out long chains of reasoning; the existence of relatively flat surfaces, as objects from which the concept of two-dimensional Euclidean space might be constructed; and even the development of skills like carpentry, the problems of which provide the occasion for the concern with purely theoretical structures to arise in the first place. Lastly, the distinction between objectual and substitutional quantification only garnered its full force in the presence of the notion that there are more things in being than can be named even in principle, a notion that only came to fruition in the real analysis of Dedekind, Cantor, and others.

In short the coming into existence of any concept presupposes certain conditions without which it would not be enabled, and others without which it would be unlikely to flourish. These conditions always take a certain historical shape, not all of the parts of which are strictly necessary for the existence or flourishing of the notion itself. When the enabling conditions for a structure are replaced with others, or drastically changed, this often brings about a major shift in the structure itself: think for instance, of the difference underwent by the British Monarchy as it has moved from having its main support in the notion the king as appointed by god, to its current state, where it is in great part propped up by tabloid culture.

⁴¹Otherwise, it would not be a matter for governmental regulation

⁴²Without this, the push for legal recognition of same sex relationships would be unlikely to take the shape of a push for *marriage* rights