Buridanian consequence and Curry's paradox

October 2, 2016

Abstract

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In his *Treatise on Consequences*, the medieval logician John Buridan defines consequence as follows:

> A consequence is a hypothetical proposition composed of an antecedent and consequent, indicating the antecedent to be antecedent and the consequent to be consequent; this designation occurs by the word 'if' or by the word 'therefore' or an equivalent, as was previously stated [**?**, TC I, 3, 67, alt.].

Buridan offers this definition because of the material inadequacy of more common definitions on a token-based semantics, i.e. one where the truth-bearers are actual written or spoken sentences. The definition according to which a consequence is good if it is impossible for the antecedent to be true and the consequent false falsifies good consequences in cases where the consequent may not exist, and hence be neither true nor false - e.g. 'Every man runs, therefore some man runs' is invalidated by the case where the antecedent is uttered and the consequent is not. The same definition amended to only consider cases where the antecedent and consequent are formed remains inadequate for propositions with self-falsifying antecedents (or self-affirming consequents) that nevertheless describe a possible situation - e.g. 'no proposition is negative, therefore no donkey runs'. A third definition, on which 'one proposition is antecedent to another which is such that it is impossible for things to be altogether as it signifies unless they are altogether as the other signifies when they are proposed together' is rejected for the same underlying reason as the prior two: it assumes things being as a proposition signifies suffice to make that proposition true.

Curry's paradox is a paradox of self-reference that may be formed in any self-referential logical language L including a detachable conditional, substitution, and contraction. It is formed when a term (C) is defined in L as follows:

$$(C) \overset{def}{\equiv} (C) \to \phi$$

Here, (C) is a name for a proposition having itself as antecedent, and ϕ an arbitrary proposition. ϕ is then derived as follows:

1. C	
2. $\mathsf{C} \rightarrow \phi$	Def C: 1
3. ϕ	$\rightarrow$ **Elim:** 1, 2
4. $\mathsf{C} \rightarrow \phi$	$\rightarrow$ **Intro:** 1-3
5. C	Def C: 4
6. ϕ	$\rightarrow$ **Elim:** 4, 5

Notoriously, Curry's paradox remains a problem for paraconsistent logics, which perform better than expected when treating other self-referential paradoxes like the Liar.

Buridan's TC is a logic without truth,[1] inasmuch as the notion of truth plays no direct role in his account of consequence. It is not thereby, however, a logic without paradox. Happily, not *all* instances of paradoxical hypothetical propositions in his language are false: some are even good material consequences. Let the following proposition be formed within a Buridanian account of logical consequence

If this is the antecedent, this article will be published.

Therefore, etc.

References

[1] Cf. Klima 2008