Grounding medieval consequence

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Abstract

1 Introduction

Grounding is an irreflexive, assymetric, and transitive relation between different elements within an ontology, according to which one or several elements serve as grounds for another [24, p. 364]. Grounding elements that are themselves ungrounded are said to be fundamental. The notion of grounding is meant to capture that expressed in non-causal uses of phrases like 'because' or 'in virtue of' [25, pp. 6–7], and commonly invoked examples of grounding include that of true statements in facts, of true non-atomic propositions in their atomic constituents, of qualities in substances, and of sets in their members. More controversial examples include the grounding of the mental in the physical, of word meaning in linguistic use, and of normative features of the world in natural ones. On one currently widespread understanding of metaphysics, the basic problem of metaphysics is to determine what beings are fundamental, and how other beings are grounded in them [24, p. 376]. The contemporary resurgence of interest in grounding can be traced back to Jonathan Schaffer's 2009 'On What Grounds What', though Schaffer's article has important antecedents in Kit Fine's work on essence [9, 10] and Saul Kripke's on truth [18, 19].

In its broadest sense, a consequence is a relation obtaining between an antecedent and a consequent, signified by a sign of consequence [27, 14]. An antecedent is a premise or set of premises from which a consequent follows; a consequent, a conclusion which follows from an antecedent; a sign of consequence, a word or symbol signifying a consequent's following from an antecedent, e.g. 'if', 'therefore', 'because' and their analogues in other natural languages, the $\vDash$ of model theory, or the proof-theoretic $\vdash$. The theory of consequence proper first arises in the later medieval period, though elements that would enter into the theory may be found in earlier work on Aristotelian syllogistic, fallacies, and topical argument.

Contemporary inquiries into the relation between grounding and consequence include the employment of consequence relations to define ground [12], appeals to a notion of grounding to define logical consequence [25], and the use of consequence relations to determine the relation grounding holds to other important concepts [6, 11, 20]. In what follows, I'll be focusing on how medievals ground the consequence relation itself.

The order of the paper is as follows: I begin by providing a basic introduction to consequences in medieval philosophy. I then argue for the treatment of the medieval theory of supposition as a theory whose main use is that of grounding consequence. From there, I survey the changes in medieval groundings of consequence focusing on the later thirteenth to the fourteenth century. What we find are two important shifts: the first from a non-unified, topics-and analytics-based grounding of consequence to one based on the theory of supposition; the second, within the supposition-based theory, from a shift in the understanding of personal supposition - specifically, from an interpretation of it licensing appeal to concepts to one based wholly on a sparse ontology of individuals.

2 The medieval theory of consequence

To begin, it will be useful to consider what theories of consequence are fundamentally meant to be theories *of*. Since the early 20th century, discussion of consequence has centered on formal consequence, commonly identified with logical consequence [28, 15]. Such a consequence may be defined model-theoretically as one where every model of the antecedent is a model of the consequent [28], or proof-theoretically, e.g. as one where the consequent follows from the antecedent via the strict application of the rules of a given proof-system [22, 26] or in other ways [16, 13]. In these ways of defining the concept, the syntactic elements of a consequence are partitioned into logical and non-logical, and the exact interpretation of the non-logical elements of a consequence is varied or left indeterminate so as to focus on what is demanded by the interpretation of its logical elements [7, pp. 314–321]. Given this, I think it's no stretch to say that the dominant intention among modern theories of consequence has been to provide a theory of validity.[1]

3 The roots of medieval *consequentiae* in hypothetical syllogistic and topical argument

It may come as a surprise, then, that this focus on validity only arises fairly late in medieval logic. Among the earliest work which we might classify under the banner of consequence (but which the medievals employing it

[1] Cf. [8, 23].

did not), one finds work on categorical syllogisms, hypothetical syllogisms, topical argument, fallacies, and other topics.

Boethius' work *On the categorical syllogism* follows Aristotle in defining the syllogism as 'discourse in which, certain things having been put forth and conceded, something other than those which have been put forth and conceded, comes about through those same which have been conceded.'[2]. In expounding Aristotle's definition, Boethius goes on to exclude one-premise arguments, reflexive arguments where the conclusion is identical with one of the argument's premises, and arguments including any premises irrelevant to its conclusion [4, 821A-822C]. Though there is some debate concerning whether Aristotle meant his *Analytics* to cover deductive argument generally or only the restricted set of arguments we today would call 'syllogisms', Boethius' discussion of the categorical syllogism works with the more restricted understanding.

Neither the term '*consequentia*' nor its variants occur in the aforementioned work. But these terms occur frequently in Boethius' *On Differential Topics* and especially his *On hypothetical syllogisms*, and it is in these works that we begin to find the rudiments of a theory of consequence.

3.1 Boethius' *On hypothetical syllogisms*

In the *On hypothetical syllogisms*, Boethius takes the Latin '*conditionalis*' to mean the same as the Greek '*hypothetica*'. But since he counts not only conditionals in the modern sense, but also disjunctions among hypothetical propositions (on the grounds that 'A or B' is equivalent to 'If A, then not B'), Boethius' understanding of what a conditional proposition is must differ from the syntactic definition employed today.[3] Boethius distinguishes conditional from predicative propositions by saying that 'in a conditional, the nature of a consequence is assumed from the condition'.[4] He also states that some consequences follow only accidentally - his example is 'if fire is hot, the heavens are round' - while others, such as 'if it's a man, it's an animal' follow from nature [3, pp. 1.3.6–1.3.7]. Given his inclusion of irrelevant inferences in the scope of consequences, his manner of contrasting hypothetical and predicative propositions, and the absence of the term 'consequence' in Boethius' work on categorical syllogism, one can infer that though Boethius didn't *identify* conditionals with consequences, he does appear to have associated

[2] [821A][4]: 'Syllogismus est oratio in qua positis quibusdam atque concessis, aliud quiddam quam sint ea quae posita et concessa sunt, necessaria contingit per ipsa quae concessa sunt.'

[3] [3, p. 1.3.2]. The above example makes clear that Boethius construes the logical 'or' in the exclusive, rather than inclusive sense.

[4] [3, p. 1.1.6]: 'Primum igitur dicendum est quod praedicatiua propositio uim suam non in conditione sed in sola praedicatione constituit, in conditionali uero consequentiae ratio ex conditione suscipitur'.

the two in such a way as to exclude categorical syllogisms and consequences from each others' respective domains.[5]

In both the *On Hypothetical Syllogisms* and the *On Differential Topics*, Boethius divides the hypothetical proposition into four types according to the quality of its antecedent and consequent: A positive proposition following from a positive, a negative from a negative, a negative from a positive, and a positive from a negative [3, p. 1.3.5] [2, 1176B-C]. At first glance, this appears to be both arbitrary and incomplete: Boethius applies the division solely to hypothetical propositions whose immediate parts are atomic, and the division has also been taken to suggest that Boethius conflates the denial of a consequent with the denial of a consequence as a whole [21, pp. 157–158]. These points aren't wrong, but dwelling on them now can obfuscate what is going on in the theory. Instead the fourfold division will be better understood by reading Boethius' account of hypothetical syllogisms in light of his theory of topical argument.

3.2 Boethius' *On Differential Topics*

In Boethius account, a topic is 'the seat of an argument', and an argument is defined as 'a reason granting credence to a doubtful matter'.[6] Now Boethius' use of several key terms here differs substantially from modern usage. In his discussion of the term 'argument' Boethius distinguishes argument in the sense of what is put forth (*argumentum*)from argument in the sense of a speech putting forth an argument in the previous sense (*argumentatio*) [2, p. 1174C]. But what he means by 'argument' also differs from what we mean by the term: he states, for instance, that a conclusion is a proposition established by arguments (note the plural) [2, p. 1180C]; that 'if something is added to any thing, the whole is made greater' is an example of an argument that is both necessary and plausible (*probabilis*), and argues, against the claim that all arguments must be plausible, that more advanced theorems of geometry may still serve as *argumenta* for conclusions that follow from them even to one who has failed to grasp their necessity [2, 1181A-C]. Later, Boethius will say that the term *argumentum* is ambiguous between two meanings: that of a maximal proposition, and that of the difference of such a proposition. By the former, he means a general proposition whose truth is grasped immediately which serves to derive more specific claims less sure than it, e.g. 'equals subtracted from equals are equal' - or, in the absence of a proposition whose truth is immediately known, one whose plausibility is sufficiently vouched for, e.g. by an artisan within his proper domain. In

[5]The association of consequences with non-syllogistic argument remains the norm to the time of Ockham's *Summa Logicae*, and is only broken in Buridan's *Treatise on Consequences*. Cf. [1, p. 295], [29, pp. 141.26–30, 219.1–9].

[6][1174C-D][2]: 'Argument est ratio rei dubiae faciens fidem . . . Locus autem est sedes argumenti.'

its second sense, an argument is that feature, thing, or relation principally appealed to in the formulation of a maximal proposition, e.g. equality [2, 1185A-B]. Since a topic is the seat of an argument, it is then understood as the seat of a maximal proposition, and thereby of the difference of such a proposition. Understood as such, the theory aims to provide an account of the fundamental propositions to be appealed to in an argument, and the real features on account of which these hold in whatever way they do.

Following the Aristotelian commentator Themistius, Boethius divides topics into intrinsic, extrinsic and middle topics, in accordance with the kinds of media, or *middles*, they employ to reach a given conclusion. In an intrinsic topic, the argument proceeds by eliciting some property, description, or relation belonging to what is signified in the minor term to confirm or remove what is suggested of it by a major term. Types of intrinsic topics include arguments from the predication of a species to the predication of a genus (e.g. 'Adam is a man, therefore he is an animal'), from a predication of a description to what follows from it (e.g. 'God is that than which nothing greater can be thought, therefore God exists') or from the existence of a whole to that of its parts (e.g. 'If there's a house, there's a roof, walls, and a foundation'). In an extrinsic topic, by contrast, one exploits a relation that the subject term of the desired conclusion bears to some other concept, constructs an argument about the latter, and thereafter leverages the original relation to infer something about the original subject one was inquiring about. Types of extrinsic topics include arguments from authority, from analogy, from various kinds of opposites, and *a fortiori*/*a minori* arguments.

Now let's return to the division of types of conditionals mentioned above. Boethius first discusses affirmations following from affirmations.

> Now in order for one thing to precede and another to follow, in these things it is accustomed to come about as a recounted a little earlier. For a [1] genus, or [2] difference, or [3] definition, or [4] property, or [5] an inseparable accident follows from the species. Again, a species follows from the [6] property or [7] definition, the [8] difference and [9] definition follow from a property, and a [10] property or [11] difference follows from the definition in this way: for [1] if there's a man, there's an animal; and [2] if there's a man, it's rational; and [3] if there's a man, there's a mortal rational animal; and [4] if there's a man, it's risible; and [5] if there's an Ethiopian, he's black. [6] If [something's] risible, it's a man; [7] if there's a mortal rational animal, there's a man. [8] If [something] is risible, it's rational; [9] if [something] is risible, it's a mortal rational animal; if there's a mortal rational animal, it's [10] risible or [11] two-footed.[7]

[7] [2, 1179A-B]: Nam ut praecedat aliquid et aliud consequatur, in his fere rebus euenire

We'll consider two further passages before expounding. Moving to the case of arguments with both an affirmative and a negative, Boethius writes 'of those cases that consist of an affirmation and a denial, there is a division, that they are contained either in diverse genera, or diverse species, or in contraries, or in habit and privation.'[8] And considering arguments specifically from a negative to an affirmative, Boethius writes

> It cannot happen that an affirmation follows a denial, except in contraries which lack a middle ground and it is necessary for one or the other of them to always inhere, as in this way: if it is not day, it is night; if it is not dark, it is light.[9]

Taken on its own, Boethius division of simple hypotheticals according to the quality of their antecedent and consequent holds, but comes across as trivial. But when combined with remarks such as those above, we can begin to see what Boethius is attempting. That division is useful for Boethius because it lays the groundwork for an inquiry into what kind of real relations things signified by terms must stand in to each other in order for there to be relations of following among consequences they factor into. For instance, in an affirmative-to-affirmative consequent, the positing of a species, property, and difference follow reciprocally from each other, and one can move from the positing of a species to positing the corresponding genus, but not conversely. In arguments from an affirmation to a negation, different relations predominate, i.e. diversity of genus or species, contrariety, or that of privation and habit. The conditions under which an argument from a negative antecedent to an affirmative consequent hold are a subset of those which hold for the converse case.

The examples Boethius gives provide the theory's paradigmatic use cases, but also reveal its limitations. When Boethius discusses consequences in both the *On Differential Topics* and the *On Hypothetical Syllogisms*, the majority of cases he considers are arguments where the antecedent and consequent affirm or deny existence claims (e.g. 'if it is day, it is not night', 'if

solet quas paulo superius commemoraui. Speciem quippe sequitur genus, uel differentia, uel definitio, uel proprium, uel inseparabile accidens. Item proprium ac definitionem sequitur species, proprium uero sequitur differentia et definitio, et definitionem sequitur proprium uel differentia, hoc modo: nam si homo est, animal est; et si homo est, rationale est; et si homo est, animal rationale mortale est; et si homo est, risibile est; si Aethiops est, niger est. Si risibile est, homo est; si animal rationale mortale est, homo est. Si risible, rationale est; si risible est, animal rationale mortale est; si animal rationale mortale est, risibile uel bipes est.

[8] [2, p. 1179D]: 'Earum uero quaestionum quae ex affirmatione et negatione consistunt, illa fere diuisio est, quod uel in diuersis generibus, uel in diuersis speciebus, uel in contrariis, uel in priuatione atque habitu continentur'

[9] [2, 1180A] Ut autem negationem affirmatio consequatur, quae erat quarta conditionalis propositionis differentia, fieri non potest, nisi in his contrariis quae medio carent, et quorum alterum semper inesse necesse est, hoc modo: si dies non est, nox est; si tenebrae non sunt, lux est.

there's a man, there's an animal'), substitute a predicate term in the consequent for a different one in the antecedent (e.g. 'if it's white, it's colored'), or provide a consequent where the implied subject for a predicate posited or denied in the consequent is an anaphoric referent back to what is posited or denied in the antecedent (e.g. 'if there's an Ethiopian, he's black'). Now in Latin, these different kinds of inferences frequently have the same syntactic form, i.e. 'si *a* [non] est, *b* [non] est' ('if *a* is/isn't, *b* is/isn't'). What is clear from the preponderance of Boethius' examples, and what others have pointed out, is that Boethius primarily conceives of the *relata* of consequence not as propositions or their contents, but as the referents of terms [21, 5]. The advantage of such a theory is that it provides a grounding for why consequences with antecedents and consequents of a given quality hold: because the things signified by the terms posited or denied therein stand in relations like contrariety, that of a habit to its privation, that of a genus to a species, etc. The disadvantage is that the theory does not readily generalize to cover more complex cases, e.g. arguments with modal or non-atomic antecedents or consequents, those involving quantified or oblique terms, or those where the subject of the antecedent and consequent differ (e.g. 'if the pilot of a ship should not be chosen by lot, neither should the governor of a city'). More specifically, though Boethius discusses a wide variety of extrinsic topics with illuminating examples, the less cohesive treatment he provides of middle and extrinsic topics (excluding those from opposites) does not fit so easily into the inferential framework he fleshes out in the *On hypothetical syllogisms*. Lastly, to the degree that the theory grounds consequences in the varied relations that it does, it would seem to presuppose a fairly robust metaphysical framework.

It's perhaps for these reasons that we begin to see alternative ways of grounding inference arise in the later medieval period.

The basic reason for the lack of unified account of consequence is a holistic tendency to think of arguments not in terms of validity but more strictly in terms of whether they are *good*. This notion of the goodness of an argument also includes elements such as the truth of its premises, whether the premises are better known than their conclusion, and whether the conclusion genuinely follows *from* the premises. It also allows for different standards of goodness materially sensitive to the intended application of an argument form. In his *Posterior Analytics* commentary, for instance, Thomas Aquinas distinguishes the subject matters of Aristotle's *Topics* and *Posterior Analytics* by saying that the former treats arguments applied to necessary matter, while the latter only applies to arguments *in materia probabili*.

The medieval invention of the theory of consequence differs from its modern successor in important respects. Medieval logicians do not distinguish between object and metalanguages, and consequently do not distinguish con-

sequences from conditionals in the modern way.[10] When they do make the distinction, medievals treat the former as a propositional medium whereby a following relation is asserted to hold. Furthermore, though the term 'formal consequence' first arises in medieval logic of the later 13th century, its discussion does not dominate medieval logic to the degree that its modern counterpart dominates logic today. Thirdly, where the breadth of modern discussions arises from the sheer variety of semantics and proof systems invoked to model logical consequence, that of medieval discussions arises more from its inclusion of types of consequences that we would classify as non-logical.

4 Grounding consequence from the 12th to the early 14th century

The modern semantic notion of formal consequence derived from Tarski is a modification of condition (F) he presents in his 'On the concept of Following Logically':

> If in sentences of the class K and in the sentence X we replace the constant terms which are not general-logical terms correspondingly by arbitrary other constant terms (where we replace equiform constants everywhere by equiform constants) and in this way we obtain a new class of sentences K' and a new sentence X' , then the sentence X' must be true if only all sentences of the class K' are true (Tarski 2002, pp. 183-184).

(F) is substantially identical with a medieval definition found in the work of John Buridan, and common at the University of Paris and other continental universities in the fourteenth century, though Buridan's definition is actually more careful to avoid paradoxes involving the notion of truth. On this more basic medieval notion, a consequence is formal if every consequence like it in form is also a good consequence. Here 'like it in form' is cashed out in terms of uniform substitution over categorematic terms - those terms whose primary function is to signify some being, rather than to provide structure to other elements in a proposition - and a consequence is good if it is impossible for things to be as its antecedent signifies without being as its consequent signifies.

The medieval theory differs from its modern counterpart in that it presupposes a notion of 'good consequence' broader than that involved in logical validity, also including material consequence. Good material consequences include those, like 'Socrates is Athenian. Therefore he is Greek' where

[10] For an argument explaining why some later medievals would not have needed the object/meta-language distinction, see [17].

the conclusion is inferred from premises with the aid of a further unstated premise or premises. In this theory, both formal and material consequences are taken to be kinds of simple consequence - i.e. consequences which hold good irrespective of their time of assertion. Simple consequence is contrasted with *ut nunc* consequence, whose holding is relative to a specified time.

Prior to the development of the dichotomy between formal and material consequence, earlier thinkers divide consequences into those that are natural and those that are accidental. This division, derived from Boethius, reaches its canonical form in the 14th century *De Puritate Artis Logicae* of Walter Burley.

5 Grounding medieval consequence

5.1 Maximal propositions and the theory of supposition

5.2 Grounding and token-based semantics

5.3 Early British treatises on consequence

By the time the earliest treatises on consequences appear at the turn of the fourteenth century, the use of topics in discussions of consequence has been fundamentally altered by the development of *supposition theory* - roughly analogous to modern theories of reference, but better understood as a theory governing the interpretation of terms in propositional contexts.[11]. This shift brought about a simplification in the number of topics actually appealed to, with a vast number of inferences justified by rules like *from an undistributed inferior to superior*, and *from a distributed superior to a distributed inferior*.

On one of the best known accounts, that of William of Ockham, supposition divides into three types: simple, material, and personal. Personal supposition occurs when a term is taken to refer to what it signifies; simple, when a term refers to its concept (understood as an intention of the soul); material, when one refers to itself as a spoken utterance or written word. Other accounts, e.g. that of John Buridan, view simple supposition as a variety of material supposition. But on both views, Personal supposition is taken to be dominant, and is interpreted as the term's supposition for *individuals*. This nominalist reading is reflected in modern formalizations of the notion, e.g. those of Klima and Parsons. On these accounts, determinate supposition of a term is interpreted in terms of the ability to descend to a disjunctive sentence where in each disjunction, the class term is replaced by the name of a different individual member of the class, with the number of disjunctions equal to the number of members. The case is similar for confused and distributed supposition, albeit employing a conjunction rather than a disjunction.

[11] DutilhNovaes2007,DutilhNovaes2008b

Given this, and given the close connection between the rise of theories of supposition and those of consequence, it is surprising to find that the approaches to supposition in early discussions of consequence diverge from this, sometimes significantly. In the anonymous London consequence treatise, the notion is only found in a part of the treatise asserting that convertible terms, e.g. 'man' and 'risible', share the same supposita. This seems to accord well with both the standard view of how supposition works and the presumed priority ascribed to personal supposition.

The same pattern is found in Burley's *de consequentiis* consider, for instance, the following passage on exceptive propositions:

> With respect to the supposition of the predicate and subject in an exceptive, one should know that that from which the exception is taken, or the subject of the exceptive ... always stands confusedly and immovably with respect to the exception, and movably with respect to the predicate. With respect to the exception it stands immovably, because it is not possible to descend with respect to it; hence 'every man besides Socrates runs, therefore Plato besides Socrates runs' does not follow. With respect to the predicate it is possible to descend, since every man besides Socrates runs, therefore Plato runs, and so on for singulars.' (par. 58).

Another important point made clear here is that the supposition of a term is given relative to another term or terms which are held fixed. Thus, supposition may differ based on what fixed term or terms are taken to be the comparison.

5.4 The Parisian *De consequentiis*

But in the anonymous work on consequences in Paris, BN lat. 16130 one finds in the majority of cases where the term 'supposition' is mentioned that the term itself or concept itself is taken to be what supposits. Consider, for instance, the following:

> I show the consequence 'a man is a white man, therefore a man is white' by this rule: positing per accidens, also posits per se. But 'white man' is a suppositum per accidens of something. With respect to the same, 'white' is also posited.

As we can see, what is taken to supposit here is not individual white men, but the term or concept 'white man', which is 'placed under' the concept 'man'.

Furthermore, the dominant division of supposition in the treatise is not that between material, simple, and personal, but that between *per se* and

accidental. The text explains the notion of *per accidens* supposition as follows:

> And [those] placed under per accidens are those that are combined out of two inhering in each other contingently, as white man is combined per accidens because it is put together out of man and white, which inhere in each other contingently. So 'a man is a white man', etc. is contingent. This is contingent, and its equivalents (convertibilia) are contingent, because in all of these a superior is assumed of an inferior and white per accidens (since whatever is constituted by an addition with respect to another is inferior to it. Thus, 'white man' is inferior to 'man' and 'white').

Here, it's clear that the supposita themselves are not individuals, but forms or concepts.

5.5 Burley's *De puritate*

Walter Burley's *Tractatus Brevior* contains two different approaches to personal supposition. On one, Burley gives an example where suppositional descent is described as licensing a descent from genus to species in his fifth principal rule for consequences:

> From the negation of the superior follows the negation of any inferior. And this rule must be understood [for] when the negated superior supposits personally. For it follows: Socrates is not an animal, therefore Socrates is not a man nor an ass, and so on of others. (p. 209.35-210.2)

The key to this understanding of personal supposition is found in Burley's introduction of the notion in the *Tractatus Longior*, where he treats it as a division of *suppositio formalis*.[12] Burley writes

> Formal supposition is twofold, for sometimes a term supposits for its significate, sometimes for its suppositum or for some singular of which it is truly predicated. And thus formal supposition is divided into personal supposition and simple supposition. (DPAL 3.5)

Burley continues:

[12] According to Dutilh Novaes, Burley appears to revive the notion of *suppositio formalis* unmentioned by Peter of Spain, Roger Bacon, or the *logica Lamberti*, from William of Sherwood (Dutilh Novaes 2012, 360)

> Personal supposition is when a common term supposits for its inferiors, *whether those inferiors be singular or common, whether they be things or words,* or when a concrete accidental term or a composite term supposits for that of which it is predicated accidentally (DPAL 3.19-24).

Burley's account here is considerably more varied than the canonical account based on Buridan and Ockham. Here, Burley includes both individuals and common natures as supposita in his account of personal supposition. When Burley states that personal supposition also captures that of composite and concrete accidental terms, he appears to have in mind those cases we found discussed earlier in the Paris treatise, like 'a man is white, therefore a man is a white man'. The account thus appears to better capture uses of personal supposition in ascents and descents in the earliest treatises on consequences.

5.6 Ockham

6 Conclusion

In the preceding, we've detailed a change in the foundations of logic that ultimately affects our understanding of the nature of the discipline as such.

In earlier frameworks found, for instance, in Boethius and Aquinas, good arguments are discussed under a variety of types each with different grounds: Some arguments argue from topics concerning language, others from metaphysical relations between things, others move from effects to cause, and still others relate premises to conclusion as cause to effect.

At the time of the earliest *consequentiae*, the vast majority of consequences discussed are based in the logic of supposition, that is, a theory of the interpretation of terms in the context of sentences. In this theory, Personal supposition is the most common interpretation of a term, but consequences involving simple supposition are not uncommon. But importantly, terms in personal supposition may be taken to refer not only to individuals, but also to types. Both readings are permissible for the anonymous London treatise, both readings are explicitly found in Burley's logic, and the latter is the only variety explicitly invoked in an anonymous Parisian treatise on consequences.

We can now see, then, that Ockham's restriction of personal supposition to exclusively refer to individuals, which today remains the dominant interpretation of *all* theories of personal supposition in the secondary literature, was more novel than hitherto recognized. In adopting this change, Ockham not only changed the understanding of personal supposition. He also changed that of simple supposition - now taken to refer to a term's referring to itself as a second intention, but before taken merely to refer to what a term signifies for, leaving open whether this be an intention of the

soul, a separate universal, or something else. More importantly, in changing the account of supposition, Ockham also necessarily modified that of consequence. While the earlier account of supposition, with its inclusion of forms or concepts, did better at explaining how we *use* logic, Ockham's sparse focus on individuals grants logic's realism, but leaves our mind's grasp of it opaque. In this way, Ockham's anti-metaphysical foundation for his logic is actually *more* metaphysical than that of Burley and his predecessors: where Burley's was consistent with, but did not require realism, Ockham's built a decision on the metaphysical foundations of logic directly into his work. And it seems to be this decision, if any, that sets the stage not only for the defects of nominalism, but also for the rise of 'extreme realism' to counter it.

References

[1] Jacob Archambault. "Consequence and formality in the logic of Walter Burley". In: *Vivarium* 56.3-4 (2018), 292–319.

[2] Boethius. *De Differentiis Topicis*. In: *Patrologia Cursus Completus. Series Latina*. Vol. 64. Paris: J. P. Migne, 1841-1855, 1173B–1216D.

[3] Boethius. *De Hypotheticis Syllogismis*. Ed. by L. Obertello. Brescia: Paideia, 1969.

[4] Boethius. *De Syllogismo Categorico*. Ed. by J. P. Migne. Vol. 64. Patrologia Latina, 793C–832A.

[5] Bianca Bosman. "The Roots of the Notion of Containment in Theories of Consequence: Boethius on Topics, Containment, and Consequences". In: *Vivarium* 56.3-4 (2018), 222–240.

[6] Samuele Chilovi. "Grounding entails supervenience". In: *Synthese* (2018).

[7] Catarina Dutilh Novaes. "The Different Ways in which Logic is (said to be) Formal". In: *History and Philosophy of Logic* 32.4 (2011), 303–332.

[8] John Etchemendy. "Reflections on Consequence". In: *New Essays on Tarski and Philosophy*. Ed. by Douglas Patterson. Oxford: Clarendon Press, 2008, 263–299.

[9] Kit Fine. "Essence and Modality". In: *Philosophical Perspectives* 8 (1994), 1–16.

[10] Kit Fine. "Guide to Ground". In: *Metaphysical Grounding: Understanding the Structure of Reality*. Ed. by Fabrice Correia and Benjamin Schnieder. Cambridge: Cambridge University Press, 2012, 37–80.

[11] Kit Fine. "Identity criteria and ground". In: *Philosophical Studies* 173 (2016), pp. 1–19.

[12] Kit Fine. “The Pure Logic of Ground”. In: *Review of Symbolic Logic* 5.1 (2012), 1–25.

[13] Curtis Franks. “Cut as Consequence”. In: *History and Philosophy of Logic* 31.4 (2010), 349–79.

[14] Niels Jørgen Green-Pedersen. “Two early anonymous tracts on consequences”. In: *Cahiers de L’Institut du Moyen-Âge Grec et Latin* 35 (1980), 1–28.

[15] Mario Gómez-Torrente. “A Note on Formality and Logical Consequence”. In: *Journal of Philosophical Logic* 29.5 (2000), 529–539.

[16] Ole T. Hjortland. “The Structure of Logical Consequence: Proof-Theoretic Conceptions”. PhD thesis. University of St. Andrews, 2009.

[17] Gyula Klima. “Consequences of a closed, token-based semantics: the case of John Buridan”. In: *History and Philosophy of Logic* 25.2 (2004), 95–110.

[18] Saul A. Kripke. “Outline of a Theory of Truth”. In: *Journal of Philosophy* 72.19 (1975), 690–716.

[19] Saul A. Kripke. “Ungroundedness in Tarskian Languages”. In: *Journal of Philosophical Logic* (2018).

[20] Michele Lubrano. “The emergence of ground: some limitative results”. In: *Synthese* (2019).

[21] Christopher J. Martin. “Denying Conditionals: Abaelard and the Failure of Boethius’ Account of the Hypothetical Syllogism”. In: *Vivarium* 45 (2007), pp. 153–168.

[22] Dag Prawitz. “Remarks on Some Approaches to the Concept of Logical Consequence”. In: *Synthese* 62.2 (1985), 153–171.

[23] Stephen Read. “Proof-Theoretic Validity”. In: ().

[24] Jonathan Schaffer. “On What Grounds What”. In: *Metametaphysics: New Essays on the Foundations of Ontology*. Ed. by David J. Chalmers, David Manley, and Ryan Wasserman. 2009, 347–383.

[25] Benjamin Schnieder. “On ground and consequence”. In: *Synthese* (2019).

[26] Peter Schroeder-Heister. “Validity Concepts in Proof-theoretic Semantics”. In: *Synthese* 148.3 (2006), 525–571.

[27] Wallace Knight Seaton. “An Edition and Translation of the Tractatus de consequentiis of Ralph Strode”. PhD thesis. University of California at Berkeley, 1973.

[28] Alfred Tarski. “On the Concept of Following Logically”. Trans. by Magda Stroińska and David Hitchcock. In: *History and Philosophy of Logic* 23.3 (2002), 155–196.

[29] Walter Burleigh. *De Puritate Artis Logicae*. Ed. by Philotheus Boehner. St Bonaventure, NY: Franciscan Institute, 1955.