Deflationism, liars, and the ontic use of ‘true’

Abstract: This paper has three goals. The first is to identify the common ground of the inflationist/deflationist debate – what I call the *substantiality thesis* – and show that the failure of both inflationist and deflationist responses to the Liar are attributable to this commonly held thesis. The second is to show how a notion of truth for sentences can and should be anchored in the pre-sentential meaning of ‘true’, one for which paradoxes like the Liar cannot occur. In the third part, I develop a semantics for a theory of truth that avoids treating truth as a substantial property, without thereby being deflationary.

Keywords: inflationism; deflationism; truth; falsity; liar paradox; negation; truth-value gaps; lambda calculus.

1 Demarcating the Inflationist/Deflationist debate

This paper brings together two claims surrounding the notion of truth in order to show their bearing on our treatment of Liar paradoxes and their kin. The first, frequently debated claim, is that truth is *substantive*. The second, put forth by Frege [1956] and hardly discussed since, is that the use of ‘true’ as applied to sentences is essentially disparate from its non-sentential uses, - the use it has when predicated of friends, love, stories, geniuses, etc.

Let us collectively refer to those theses according to which truth is significant, important, etc. only if it is substantive as the *substantiality thesis* [ST]. I intend ST to capture a wide variety of different forms: one version might say that truth is a property only if it is substantive; another might take it to have a distinct domain only if it is substantive; others might take it to be meaningful, informative, metaphysically robust, etc. Note that ST is not the categorical claim that truth is substantial. Rather, it is the hypothetical claim that truth is substantial only if meaningful, etc.

To the degree that deflationist/inflationist debates are well-defined, [1] ST is the fixed point around which they revolve, being the common ground on which inflationists and deflationists agree. Kevin Sharp describes deflationism as holding that “truth is not a substantial notion and has

[1] They may not be. See Edwards [2013].

no analysis. Instead, truth predicates play an important expressive role in our linguistic practice,"[2] with the usual connotation that relegating the importance of truth predicates to the realm of language serves as a way of emptying them of any *metaphysical* baggage associated with substantiality. Asay [2014] characterizes deflationism in terms of two different kinds of substantiality claims: 1) that truth is not a (robust, metaphysically substantive, etc.) property; and 2) "that our concept of truth is explanatorily impotent."[3] On the inflationist side, this same language of substantiality has been adopted by Putnam, Wright, Lynch, and others.[4]

If we might think of deflationism as roughly characterizable by the dictum that "there is no substantial property of truth, and so there is nothing – no domain of objects, or properties, or phenomena – which the theory of truth describes",[5] we might characterize the inflationist as an adherent of the following thesis:

> **Inflationism**: There is something – some domain of objects, properties, or phenomena – that the theory of truth describes, and so there *is* some substantial property of truth.

Doing this allows us to see that inflationism and deflationism as two sides of the same coin. Both concede the theory of truth is about something only if there is some substantial property of truth. From here, the only difference is that of the deflationist's *modus tollens* to the inflationist's *modus ponens*.

In what follows, I show that the substantiality thesis itself is mistaken. In the section immediately following, I distinguish between two different kinds of predicates, the former of which I call '*real*', the latter '*ontological*'. From here, I show: i) that 'true' in its common *non-sentential* application is an ontological, as opposed to a real, predicate; ii) that the notion of truth

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[2] Sharp [2013], 14.
[3] Asay [2014], 148.
[4] See Putnam [1994], Wright [2001], Lynch [2009].
[5] Glanzberg [2003], 13. Glanzberg intends this as a description of minimalism, but it seems to me to apply to deflationism in general.

applied to sentences can be defined in terms of the corresponding pre-sentential notion; and iii) that grounding sentential truth in a pre-predicative, ontological notion of truth suffices to provide principled support to what is likely still the most intuitive response to liar-like sentences: that they are syntactically ill-formed. I then provide a syntax and semantics for a logical language capable, on this assumption, of: i) expressing truth, falsity, *and* indeterminacy for sentences; ii) accommodating circularity for real predicates, while providing a principled rejection of it for ontological ones; and iii) accommodating and expressing certain kinds of *presupposition failure* that themselves trigger indeterminacy. In the penultimate section, I raise an issue about the relation between negation and falsity.

2 Real and Ontological Predicates

Let's begin with a look at two problematic inferences.

(1) Pegasus is thought about. Therefore, Pegasus is.

(2) This person is a good thief. Therefore, this person is good.

The relation of these fallacies to the Liar is not immediately apparent. But Medieval logicians treated both under the same heading, that of the fallacy *secundum quid et simpliciter*. In each of the above cases, we have a shift from calling something something else with respect to some contextual parameter to calling it that thing absolutely.[6] To be with respect to thought is not the same as being *tout court*; and to be a good thief is not the same as being good.

The terms "to be" and "to be good" are among the most basic terms of our language, both with respect to their importance and with respect to their range of applicability. And it is just this range of applicability that suggests that the terms are not being used in the way we use predicates

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[6] The Medievals grouped these together as examples of the fallacy *secundum quid et simpliciter*. All such fallacies involve this move from a predication in a certain respect (*secundum quid*) to one without qualification (*simpliciter*). Cf. Aquinas, *De Fallaciis*, 13

like “is green,” “is five feet tall,” and “is the sister of.”[7] Taking a cue from the tradition, I will call these latter, unproblematic types of predicates *real* predicates, while I will call the predicates 'good,' 'is,' and 'true,' *ontological*.[8]

The mark of a real predicate is that it adds some content to the concept of the thing it is predicated of. [9] This does not mean that the thing itself needs to sometimes be capable of lacking a property that the predicate refers to in order for the predicate to be real. For instance 'is an animal' and 'is even' are real predicates, even though they apply to the objects that they are truly predicated of necessarily. What it does mean, though, is that the predicate is in some respect *informative*[10] of the notion of the object we have in mind.

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[7] Cf. Frege [1956], 290: “One finds truth affirmed of pictures, ideas, statements, and thoughts. It is striking that visible and audible things occur here alongside things which cannot be perceived with the senses. This hints that shifts of meaning have taken place.”

[8] The phrase 'real predicate' is chosen here because of certain connotations that the word ‘real’ itself has, though we do not often pay attention to them. 'Real' comes from the Latin *res*, while 'ontological' refers back to the Greek τὸ ὂν, rendered into Latin as *ens*, from which we also obtain the English ‘entity’. Aquinas expresses the difference between the terms as follows:

> [R]es…in hoc differt ab ente…quod ens sumitur ab actu essendi, sed nomen rei exprimit quidditatem sive essentiam entis.
>
> *[R]es* differs from *ens* in this respect, that the name *ens* is taken from the act of being, but the name *res* expresses the quiddity or essence of a thing [DV q. 1, art. 1, co.]

A real predicate, then, is one taken from the essence or nature of the thing as it presents itself to the mind, whereas an ontological predicate is one that the thing has on account not of the essence, but the actualization of that essence (*actus essendi*). The most straightforward case of the latter is the predicate “exists” itself.. Cf. Kant [1998], 567.

All translations from non-English texts are my own, unless noted otherwise.

[9] And so, by these lights, the meaning of a term will be related to the concept it refers to. This remark clearly needs to be taken in a way different from that in which Quine [1948] criticizes it. The ground of Quine's criticism of Wyman is the belief that our concepts are a different *kind* of thing than the thing that they refer to, e.g. the Parthenon is different from my idea of the Parthenon because one is physical, the other is not. This standpoint is simply a hangover from a kind of dualism about the physical world and mental content. Concepts are not a kind at all: they are manifestations of the thing from which they are drawn, and therefore belong – albeit in a derivative manner – to the kind of that thing, just as a scholastic would take rational souls to belong to the species *human*, and yet deny that such souls are themselves humans. The being of my concept of a thing is nothing other than the being that the thing itself has *qua* understood, caused by the object that it is about. What prompts the Quinean standpoint is the tendency to regard concepts as spontaneous acts of the mind itself, thereby creating a rift between the realm of thinking and that of the real world. This is completely unnecessary.

[10] Notice the Latin root of this word: *in forma*. The meaning intended here is faithful to this root. If a predicate is informative, then it in-forms our notion of the thing in the scholastic sense – it 1) *abstracts* from specific spatio-temporal conditions, 2a) *determines* the thing *as* one subjected to the form, aspect, or *eidos*, indicated by the predicate, 2b) correlatively, *posits* the property indicated by the predicate as inhering in the subject, and 3) thereby *clarifies* the being had by the subject itself. This does not entail, e.g. that things can only be to the left of other things by some to-the-leftness property. But this is not the place to enter into that discussion.

Let's give a few examples. One common way a term is informative with respect to another is by specification: my idea of a given triangle is made clearer by the information that it is equilateral. The converse case of placing something in a given genus is also informative: being told that a whale is a mammal tells me that whales give birth to live young. The same holds for the specification of certain qualities: being told that a person is tall, what color her hair is, her approximate age, are all informative predicates.

When a predicate is informative in this manner, it is so because it posits the *same* attribute in whatever it is attributed to: and so the predicate is used univocally.[11]

Now, take the use of 'good' as in the phrase, 'good thief.' The good thief is the one who has successfully instantiated the properties most conducive to thievery. The good tennis player is, similarly, the person in possession of all of the skills, powers, and technique necessary to win at tennis. The same point can be made for the comparative degree: "Venus is better than Serena," said of the tennis players, compares respects in which the quality in question – namely tennis playing ability – has been actualized. The same sentence could be uttered with a different intent – for instance, not by an avid tennis fan, but by the mother of the sisters – to indicate that one is a better *person* than the other, i.e. that the former instantiates the ideal qualities of a human being to a greater degree than her sister. By contrast, "Venus is better than Serena," where the first term refers not to the sister, but to the planet, is opaque until the respect in which 'better' is said is specified.[12]

In other words, the meaning of 'good' piggybacks on the meaning of some other concept that it modifies. This suffices for it to lack the univocity had by terms like 'to the left of,' 'two-legged,' and 'toxic.' The result of this is that there are, in effect, as many different ways of saying

[11] That we know, on some level, that we are working with univocal predicates in these cases is the precondition for our ability to notice obvious violations of the use of the predicates themselves, e.g. "His heart and sweatshirt were black." Cf. Ryle [1949], 23.

[12] The point has been previously made by Geach with respect to the notion of sameness. Geach [1962], 39.

'good' as there are kinds of things the term can connote in the language. [13]

3 'True' in its pre-sentential meaning

Now let's apply this same point to the question of the meaning of 'true.' Since the question was first pushed aside by Frege,[14] little attention has been given to the relation of the sentential use of 'true' to what I shall call its pre-sentential uses. Part of what the claim that 'true' is an ontological, and not a real, predicate will do is help us to reforge this link.

We speak of true friendship, true heroes, true love, true judgments, true beliefs, true testimonies, and a host of other cases. The last three of the above have more in common with sentential truth than with the former three: what they all *prima facie* suggest is that the belief, testimony, or judgment in question is adequate to what it happens to be about. A true belief, for instance, is one that believes of what is, that it is – a one-word modification of the Aristotelian definition of truth known to most of us through Tarski.[15] On the other hand, the former three cases are markedly different.

A true friendship is one fully instantiating the ideals of friendship. Accordingly, a friend is regarded as less than true to the degree that he fails to live up to the ideals and aims of friendship, whatever these may be. A true hero is one that exemplifies heroism fully. Therefore, her actions instantiate the ideal of heroism. Similarly, true love is love fully actualized, just as true heroism is heroism actualized.[16]

The common ingredient in each of these is the actualization of some ideal. This is not the same as the actualization of an *idea*: one can be surprised and struck by an ideal's 'coming onto

[13] The same point could be made with respect to 'is,' though I do not think detailing this case is necessary for our purposes. Cf. Ryle [1943].
[14] Frege [1956], 290-91.
[15] See Tarski [1944], 342-43.
[16] In both cases, 'true' further tends to connote *stability* or *perdurance*: a love which is true, one says, is a love that lasts.

the scene', while one is hardly surprised when things go exactly as one plans them.

More importantly, though, the actualization of this ideal must be prior to our adaptation of the ideal itself: it is on account of a prior actualization of the ideal that we can later go back and judge whether a given reality "conforms" to it. One's idea of an ideal may be inadequate: the adage, 'a friend is someone who supports you' represents a concept of friendship that, while it captures something of what it means to be a friend, is enabling and unhealthy if taken for complete. Hence, we say indifferently of both the idea and sentence expressing it, that they are 'not entirely true'. Truth as actualization gives rise to truth as correspondence.

In short, in its pre-sentential use, 'true,' like 'good,' signifies the instantiation or actualization of what it is that the term modifies– thievery, love, friendship, heroism, etc.[17]

From here, the step to a two-place 'true of' predicate is fairly simple: To say that *P* is true of an object *d* is to say that P is actualized 'at' *d*.[18] And from this, as a convenient abbreviation

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[17] One might here raise the question of what exactly accounts for the difference between our uses of 'true' and 'good,' if both, at bottom, signify nothing other than the instantiation of the properties signified by the terms they modify. On the one hand, we should affirm that the distinction, especially if it is used as the basis for drawing a sharp dichotomy between the realm of fact and that of value, is greatly exaggerated. On my reading, the distinction can't be cashed out in extensionally, even in a modal context: both, of necessity, signify the same thing that is signified by 'being,' but in different ways.

But there clearly is some difference in meaning between the two. Aquinas suggests the following distinction:

> Si autem modus entis accipiatur … secundum ordinem unius ad alterum, hoc potest esse dupliciter: uno modo secundum *divisionem* unius ab altero;… alio modo, secundum *convenientiam* unius entis ad aliud. Et hoc quidem non potest esse nisi accipiatur aliquid quod natum sit convenire cum omni ente. Hoc autem est anima, quae quodammodo est omnia, sicut dicitur in III *de anima*.
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> In anima autem est vis cognitiva et appetitiva. Convenientiam ergo entis ad appetitum exprimit hoc nomen *bonum* …Convenientiam vero entis ad intellectum exprimit hoc nomen *verum* (DV q. 1, art. 1, co.)
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> If, however, the mode of a being is taken with respect to the order of one to another, this can be done in two ways: the first, with respect to the difference of one from the other;[…]the second, with respect to the accord of one toward another. And this can only be if we accept [that there is] some being apt to accord with any being. But this is the soul, which "in a way, is all things," as is said in Bk. III of Aristotle's *De Anima*.
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> In the soul, there is a cognitive and an appetitive power. The name 'good,' therefore, expresses the accord, of a being to appetite […] while the name 'true' expresses the accord of a being to an intellect.

The suggestion here – one I think substantially correct – is that both 'good' and 'true' signify the same thing signified by 'is' – namely, the actuality of what it is that the term it modifies refers to. The difference is that the former denotes this in its aspect as desirable, whereas the latter denotes it *qua* knowable.

[18] To do justice to the difference between the different ontological predicates we have, it would be appropriate to add a third parameter, like this: '*P* is true at *a* with respect to an intellect *I*.' However, we leave this aside for now. Cf.

(though one that glosses over important details) we come to the T-schema:

(T) $\textbf{True}(\alpha)\leftrightarrow p$, where what replaces 'α' is a name of a sentence replacing '*p*'.

This last point deserves a bit more elaboration. I'm not saying that the T-schema itself is mistaken. Rather, I'm claiming that paradoxes like the Liar arise in part because the T-Schema is taken to be fundamental in a way that it isn't. Judgments of truth do not *at bottom* predicate being-true of a propositional content, however this content is understood. Rather, they are a shorthand for predicating truth in the pre-sentential sense of a predicate-subject pair, thereby signifying the actualization of a property in a subject.[19]

ST I. 16. 6.

[19] This helps make historical sense of the fact that the earliest formulations of the principle of non-contradiction are in terms of what a subject can or cannot simultaneously undergo, rather than in terms of the meanings of sentences. Cf. Plato, *Rep*. IV 436b8-9.

Even Aristotle's formulation, conforms to this pattern, though Tarski's rendering of the passage obscures this point. I give Aristotle's passage, with my own rendering of it, as follows:

> τὸ μὲν γὰρ λέγειν τὸ ὂν μὴ εἶναι ἢ τὸ μὴ ὂν εἶναι ψεῦδος, τὸ δὲ τὸ ὂν εἶναι καὶ τὸ μὴ ὂν μὴ εἶναι ἀληθές.
> For to ascribe non-being to a being, or being to a non-being, [is] false; whereas [to ascribe] being to a being, and non-being to a non-being, [is] true. [*Metaph.* IV. 7. 1011^{b}26-28]

The key point is that Tarski's rendering of εἶναι and μὴ εἶναι as 'that it is' and 'that it is not', respectively, itself suggests the interpretation of τὸ ὂν and τὸ μὴ ὂν – 'what is' and 'what is not' – in terms of something like facts, states of affairs, or propositional content, thereby leading to the paraphrase Tarski provides: "A sentence is true if it designates an existing state of affairs" [Tarski 1944, 343]. Tarski's own formalization, in turn, while more precise, remains a precification of *this latter* definition, rather than the Aristotelian one Tarski prefers to it.

Aristotle's passage, by contrast, analyzes truth in terms of the ascription of being to *a* being – literally, he says that "to speak being of a being" is true. This differs from Tarski's definition in the following respects.

First, the being designated by τὸ ὂν, on the most straight forward reading, is nothing especially complex, and certainly nothing like a state of affairs. Rather, it likely refers to an ordinary object, though the being accorded to that object will differ in accordance with the category to which it belongs.

Second, the being (εἶναι) ascribed to the being in question is not itself *a* being, i.e. it is not an object, even in the most straightforward cases. 'Pegasus is Pegasus', for instance, is not an identity statement, and the reason for this has nothing to do with the non-existence of Pegasus: rather, the statement posits the actuality of a mode of being – being Pegasus ('Pegasizing') – in the thing, Pegasus.

Third, since Aristotle leaves the being of what is actualized (εἶναι) in the being addressed (τὸ ὂν) unspecified, the kind of correspondence covered in Aristotle's definition is broader than that covered by Tarski. Where Tarski's schema maps truth as predicated of the names of statements to statements, Aristotle's maps the actuality of properties named by predicates to subjects. While in Tarski's schema the function from names of statements to statements is typically surjective, (a statement may have more than one name), truth predication conceived as the predication of the actuality of property in a subject is more complex, since not only may more than one property be actualized in a single subject, but also more than one subject typically actualizes a single property.

Lastly, whereas Tarski's rendering of τὸ μὴ ὂν and μὴ εἶναι as 'what is not' and 'that it is not' suggests, in accordance with the thinking of τὸ ὂν and εἶναι as a correspondence between states of affairs, that the μὴ in both cases is to be understood in terms of sentential negation, a logical operation; Aristotle's original more readily suggests *two* different *metaphysical* distinctions. I shall call the first of these *ontic*, and the second, *ontological.* The *ontic* pair ὂν - μὴ ὂν contrasts two 'kinds' of *beings*: beings and non-beings – for example, light and darkness, heat and cold, sight

4 Syntax and Semantics for Bridging the Truth Gap

4.1 Syntax

With this in mind, we can give precision to what likely remains the most intuitive response to Liar-like sentences: that they aren't actually saying anything at all. To do so, we construct a first-order language $\mathcal{L} = (C, Q, \Lambda, \boldsymbol{Real}, \boldsymbol{Trm}, \boldsymbol{Frm})$, where $C = \{\sim, \&, \text{v}, =, \Rightarrow, \Box, \Diamond\}$, $Q = \{\forall, \exists\}$, ***Real*** is a set of real predicates (i.e. our set of unproblematic, 1st order predicates – not including a truth predicate), $\boldsymbol{Trm} = \boldsymbol{Con} \cup \boldsymbol{Var}$ is a set of terms, where $\boldsymbol{Con} = \{a, b, c, d\ldots\}$ is our collection of constants and $\boldsymbol{Var} = \{x, y, z, w, x'\ldots\}$ is a set of variables. The set $\Lambda = \text{T} \cup \text{F}$ is a set of functions, intended as variants of lambda abstraction. The syntax here parallels that in Fitting and Mendelsohn [1998]. ***Frm*** is the set of formulas of the language, membership in which is recursively defined as follows:

(1) If $F^n \in \boldsymbol{Real}$ and $(x_1 \ldots x_n) \in \boldsymbol{Var}^n$, then $F^n(x_1 \ldots x_n) \in \boldsymbol{Frm}$, with $x_1 \ldots x_n$ as free variable occurrences.[20]

(2) If $\phi \in \boldsymbol{Frm}$, then each of $\sim\phi$, $\Box\phi$, and $\Diamond\phi \in \boldsymbol{Frm}$, the free variable occurrences of the resulting formula being those of ϕ.

(3) If $\phi, \psi \in \boldsymbol{Frm}$, then so are $(\phi \,\&\, \psi)$, $(\phi \text{ v } \psi)$, and $(\phi \Rightarrow \psi)$, the free variable occurrences of which being those of ϕ together with those of ψ.

(4) If $\phi \in \boldsymbol{Frm}$ and $x \in \boldsymbol{Var}$, then $(\forall x)\phi$, $(\exists x)\phi \in \boldsymbol{Frm}$, with free variable occurrences being

and blindness. The second, *ontological* pair, εἶναι - μὴ εἶναι contrasts two different ways of *being*, e.g. being hot or cold. One consequence of this is that the quality of a statement cannot be simply read off of its syntax. A predication of non-being of a non-being, for instance, need not involve a negation at all: 'darkness is the absence of light' predicates non-being of a non-being. Similarly, the presence of an ὂν - μὴ εἶναι pair (or, conversely μὴ ὂν - εἶναι) is not sufficient for falsity, since an ὂν in one respect may be a μὴ ὂν in another: for instance, "A crow is non-white" is true, because though the crow *is* with respect to crowness (and is therefore *a* crow), it is also a non-being with respect to whiteness (and therefore a non-white thing). See Figure 1.

Elsewhere, e.g. *Physics II.1*, Aristotle will equate the being of a being with its form. The connotation, one we often do not notice, is that correspondence, as the accord between *a* being and *its* being, is originally hylomorphic.

[20] Note that atomic formulae involving terms, e.g. $F(a)$, are *not* part of our language, the reason for this being that the combination of non-rigid terms and modal formulae brings with it syntactical ambiguity. See Fitting and Mendelsohn [1998], 187-190. We *do*, however, count open sentences in the language as formulae.

those of ϕ less its free occurrences of x.

Membership in T is defined as follows:

(T) If $\phi \in$ ***Frm*** and $x \in$ ***Var***, then $\langle \mathbf{T}x.\phi \rangle$ is a *truth abstractor,* the free variable occurrences of which are those of φ less its free occurrences of v.

The syntax for functions in F is exactly parallel, and we accordingly call a function of the form $\langle \mathbf{F}x.\varphi \rangle$ a *falsity abstractor*.

Lastly, we give the syntax for formulae involving members of Λ.

(5) If $\langle \lambda x.\varphi \rangle \in \Lambda$ and $t \in$ ***Trm***, then $\langle \lambda x.\varphi \rangle(t) \in$ ***Frm*** , with free variable occurrences being those of $\langle \lambda x.\varphi \rangle$ as well as those of t.

$\langle \mathbf{T}x.\phi \rangle(t)$ may be read in any of the following ways:

(1) 'The truth of ϕ is with respect to t.'

(2) 't's ϕ-ness is true.'

(3) 'ϕ is true of t.'

(4) 't is truly ϕ.'

Note that within this syntax, it isn't possible to formulate Liars for the simple reason that all formulae involving functions in Λ ultimately have to "bottom out" in some formula ϕ *not* involving **T** or **F** that grounds the whole chain of functions performed on ϕ. For instance

$\langle \mathbf{F}x.\langle \mathbf{T}y.Rxy \rangle(c) \rangle(d)$.

is well-formed, but

$\langle \mathbf{F}x. \rangle(b)$

is not. Thereby, we concede to the deflationist that 'True' and 'False' are not substantial predicates.

4.2 Semantics

From here, we move to the task of explaining what truth *is* on such an account. Let $\mathcal{M} = (\mathcal{W}, \mathcal{R}, \mathcal{D}, \mathcal{I})$ be a first-order varying-domain model. $\mathcal{W}$ is our set of worlds. $\mathcal{R}$ is a reflexive, transitive, symmetric binary relation on $\mathcal{W}$. $\mathcal{D}$ is a domain function, taking us from each world w in $\mathcal{W}$ to the domain of that world (such that $\bigcup\{\mathcal{D}(w): w \in \mathcal{W}\}$ gives us the domain of the model, written $\mathcal{D}(\mathcal{M})$).[21] $\mathcal{I}$ is an interpretation function, mapping constants at each world to objects in $\mathcal{D}(\mathcal{M})$, and *n-ary* predicates P^n at each world w in $\mathcal{W}$ to a *pair*, $(\mathcal{E}, \mathcal{A})$ of subsets of $\mathcal{D}(\mathcal{M})^n$, where, intuitively, $\mathcal{E}$ is the *extension* of the predicate P^n (i.e. the set of n-tuples of which P^n is true at w – written $\mathcal{I}_w{}^{\mathcal{E}}(P^n)$), and $\mathcal{A}$ is its *anti-extension* there (i.e. the set of n-tuples for which it is false – written $\mathcal{I}_w{}^{\mathcal{A}}(P^n)$). The extension and anti-extension of a predicate must be disjoint, but need not be exhaustive.

Given such a model, we define a valuation function v assigning variables to objects in the domain of the model; and a relation ρ relating each formula ϕ at world w on valuation v to the values $\{0, 1\}$ (written $\phi\rho_w{}^v$). We allow both interpretations and valuations to be partial. Note that while interpretations of constants are world-indexed, thereby allowing for non-rigid designation, valuations are not.

Semantics for atomic formulae are as follows:

(α_1) $P^n(x_1 \ldots x_n)\rho_w{}^v 1$ iff $\langle v(x_1), \ldots v(x_n)\rangle \in \mathcal{I}_w{}^{\mathcal{E}}(P^n)$

(α_2) $P^n(x_1 \ldots x_n)\rho_w{}^v 0$ iff $\langle v(x_1), \ldots v(x_n)\rangle \in \mathcal{I}_w{}^{\mathcal{A}}(P^n)$[22]

And for connectives:

($\&_1$) $(\phi \,\&\, \psi)\rho_w{}^v 1$ iff $\phi\rho_w{}^v 1$ and $\psi\rho_w{}^v 1$

($\&_2$) $(\phi \,\&\, \psi)\rho_w{}^v 0$ iff $\phi\rho_w{}^v 0$ or $\psi\rho_w{}^v 0$

(v_1) $(\phi \vee \psi)\rho_w{}^v 1$ iff $\phi\rho_w{}^v 1$ or $\psi\rho_w{}^v 1$

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[21] Though (w) is a function and not a set, it will be occasionally be convenient to treat it as one; and so we will occasionally write e.g. $\{d: d \in (w)\}$ to denote the set of objects the function outputs given an input w.

[22] Note that the semantics for binary predicates gives the semantics for '=' as a special case, adding the usual conditions that $\mathcal{I}_w{}^{\mathcal{E}}(=)$ be symmetric, reflexive, and transitive for elements existing at w; while $\mathcal{I}_w{}^{\mathcal{A}}(=)$ must be irreflexive and symmetric at w.

($\vee_2$) $(\phi \vee \psi)\rho_w{}^v0$ iff $\phi\rho_w{}^v0$ and $\psi\rho_w{}^v0$

($\sim_1$) $(\sim\phi)\rho_w{}^v1$ iff it is not the case that $\phi\rho_w{}^v1$

($\sim_2$) $(\sim\sim\phi)\rho_w{}^v0$ iff $\phi\rho_w{}^v0$

($\Box_1$) $(\Box\phi)\rho_w{}^v1$ iff for all w' s. t. $w\mathcal{R}w'$, $\phi\rho_{w'}{}^v1$

($\Box_2$) $(\Box\phi)\rho_w{}^v0$ iff for some w' s. t. $w\mathcal{R}w'$, $\phi\rho_{w'}{}^v0$

($\Diamond_1$) $(\Diamond\phi)\rho_w{}^v1$ iff for some w' s. t. $w\mathcal{R}w'$, $\phi\rho_{w'}{}^v1$

($\Diamond_2$) $(\Diamond\phi)\rho_w{}^v0$ iff for all w' s. t. $w\mathcal{R}w'$, $\phi\rho_{w'}{}^v0$

($\Rightarrow_1$) $(\phi \Rightarrow \psi)\rho_w{}^v1$ iff for all w' s. t. $w\mathcal{R}w'$ and $\phi\rho_{w'}{}^v1$, $\psi\rho_{w'}{}^v1$

($\Rightarrow_2$) $(\phi \Rightarrow \psi)\rho_w{}^v0$ iff for some w' s. t. $w\mathcal{R}w'$, $\phi\rho_{w'}{}^v1$ and $\psi\rho_{w'}{}^v0$

Next, we give the semantics for quantifiers and abstractors. We begin by defining the notion of an x-variant.

(x-variant) – for any variable x in ***Var***, world w and valuations v, v', v' is an x-variant of v at w iff: 1) v and v' agree on all variables except perhaps x; and 2) $v'(x) \in w$.

And now the semantics for quantifiers:

($\forall_1$) $(\forall x)\phi\rho_w{}^v1$ iff for every x-variant v' of v, $\phi\rho_w{}^{v'}1$

($\forall_2$) $(\forall x)\phi\rho_w{}^v0$ iff for some x-variant v' of v, $\phi\rho_w{}^{v'}0$

($\exists_1$) $(\exists x)\phi\rho_w{}^v1$ iff for some x-variant v' of v, $\phi\rho_w{}^{v'}1$

($\exists_2$) $(\exists x)\phi\rho_w{}^v0$ iff for every x-variant v' of v, $\phi\rho_w{}^{v'}0$

Semantics for abstractors is as follows:

(**T**$_1$) $\langle \mathbf{T}x.\phi\rangle(t)\rho_w{}^v1$ iff $\phi\rho_w{}^{v'}1$, where v' is the x-variant of v assigning: $\mathcal{I}_w(t)$ to x if t is a constant; and $v(t)$ to x if t is a variable.

(**T**$_2$) $\langle \mathbf{T}x.\phi\rangle(t)\rho_w{}^v0$ iff $\phi\rho_w{}^{v'}0$, where v' is the x-variant of v assigning: $\mathcal{I}_w(t)$ to x if t is a constant; and $v(t)$ to x if t is a variable.

($\mathbf{F}_1$) $\langle \mathbf{F}x.\phi\rangle(t)\rho_w{}^v 1$ iff $\phi\rho_w{}^{v'}0$, where v' is the x-variant of v assigning: $\mathcal{I}_w(t)$ to x if t is a constant; and $v(t)$ to x if t is a variable.

($\mathbf{F}_2$) $\langle \mathbf{F}x.\phi\rangle(t)\rho_w{}^v 0$ iff $\phi\rho_w{}^{v'}1$, where v' is the x-variant of v assigning: $\mathcal{I}_w(t)$ to x if t is a constant; and $v(t)$ to x if t is a variable.

In order to provide a test case for models that enforce the failure of bivalence, we adopt the following *neutrality constraint*:

(n) For any $w \in \mathcal{W}$, $\langle d_1 \ldots d_n\rangle \in (\mathcal{M})^n$ and $P^n \in Real$: if $\langle d_1 \ldots d_n\rangle \in \mathcal{I}_w{}^{\mathcal{E}}(P^n)$ or $\langle d_1 \ldots d_n\rangle \in \mathcal{I}_w{}^{\mathcal{A}}(P^n)$, then $d_i \in \mathcal{D}(w)$ for all $1 \leq i \leq n$.

This condition ensures that if a term fails to designate an object existing at a given world, then the truth value of an atomic formula involving that term is undefined there.[23]

Given this semantics, we can then *define* one-place truth and falsity operators as follows:

($\boldsymbol{T}$) $\boldsymbol{T}(y) \stackrel{\text{def}}{=} \langle \mathbf{T}x.\phi\rangle(t)$, where y denotes the proposition resulting from assigning the value of t to each free occurrence of x in ϕ.

($\boldsymbol{F}$) $\boldsymbol{F}(y) \stackrel{\text{def}}{=} \langle \mathbf{F}x.\phi\rangle(t)$, where y denotes the proposition resulting from assigning the value of t to each free occurrence of x in ϕ.

Lastly, we note that in the bilateral context we've been setting out, rather than having a single notion of logical consequence, there are several such notions that can be set out. We detail four.

(1) We say that a premise set S is a *positive ground of truth*[24] w. r. t. a formula ψ iff for every

[23] The formulation is modified from Priest [2008], 483, though the condition is assumed and defended much earlier. See Frege [1948], Strawson [1950].

[24] The language of grounds of truth combines that of grounds for assertion/denial, taken from Francez [2014], (though his approach is proof-theoretic as opposed to the model-theoretic approach taken here), with that of *causes of truth*, taken from the Medieval logician John Buridan, though the resulting notion is only analogous to, and not identical with, that used by Buridan. For Buridan, the causes of truth of an atomic proposition are set of *entities* (or n-tuples), rather than premise sets. See *TC* I. 2.

model $\mathcal{M} = (\mathcal{W}, \mathcal{R}, \mathcal{D}, \mathcal{I})$, for every world w in $\mathcal{W}$ and valuation v in $\mathcal{M}$ s. t. for each formula ϕ in S $\phi\rho_w{}^v1$, $\psi\rho_w{}^v1$. This corresponds to the standard case where ψ is a logical consequence of all the members of S taken as local assumptions.

(2) We call a premise set S a *privative ground of truth* w. r. t. a formula ψ iff for every model $\mathcal{M} = (\mathcal{W}, \mathcal{R}, \mathcal{D}, \mathcal{I})$, for every world w in $\mathcal{W}$ and valuation v in $\mathcal{M}$ s. t. for each formula ϕ in S, $\phi\rho_w{}^v0$, it is the case that $\psi\rho_w{}^v1$.

(3) A premise set S is a *positive ground of falsity* w. r. t. a formula ψ iff for every model $\mathcal{M} = (\mathcal{W}, \mathcal{R}, \mathcal{D}, \mathcal{I})$, for every world w in $\mathcal{W}$ and valuation v in $\mathcal{M}$ where for each formula ϕ in S, $\phi\rho_w{}^v1$, it is also the case that $\psi\rho_w{}^v0$.

(4) A premise set S is a *privative ground of falsity* w. r. t. a formula ψ iff for every model $\mathcal{M} = (\mathcal{W}, \mathcal{R}, \mathcal{D}, \mathcal{I})$, for every world w in $\mathcal{W}$ and valuation v in $\mathcal{M}$ where for each formula ϕ in S, $\phi\rho_w{}^v0$, $\psi\rho_w{}^v0$.

5 Falsity, Negation, and Contrariety

In what follows, I'd like to say a few things about the aims, presuppositions and the import of the above semantics.

As regards its *aims*, there is no pretense that the above semantics is capable of providing a notion of truth for languages sufficiently strong to express their own syntax. As has been pointed out by Field,[25] this cannot be done *even* in theories weaker than classical logic, provided that that portion of the language within which the syntax is expressible remains classical.

Rather, my philosophical aim has been to disambiguate the genuine question 'what is truth?' from the question "what kind of *property* must truth be in order for it to do what it does?"[26] or

[25] Field [2008], 27.

[26] Answer: This is a loaded question. not every predicate is a property. Even less does every 'genuine' predicate signify an *intrinsic* property, as the history surrounding terms like 'substantive' would suggest.

from the even more attenuated "what, if anything, do I have to assume about truth in order to get it to do what it does in language?",[27] by providing a model-theoretic aid to the question of what truth *is*; while my technical aim has been, given that the syntactic expressiveness of a language containing a truth predicate must be limited, to construct one that, for a first-order language, can still say a startling amount.

It should be clear, given the definition of the one-place truth (falsity) predicate above, that the language must forbid circularity for formulae that themselves involve this predicate;[28] but it need not forbid circularity in other cases.

The main *presuppositions* whereby the semantics differs better known many-valued logics are those informing its treatment of negation. The best known many-valued logics, such as K_3 and *LP*, take for granted that negating a formula with a classical truth value leads to another classical truth value – to negate that something is true is to assume it false, and conversely – while negating a non-classical value simply spits that same value back. By doing so, these logics turn negation into a *contrariety* forming device. The downside to this is that the semantics is thereby left without the most obvious means of expressing *contradiction*, since a formula and its negation may take the same truth value. For example, the 'neither' and 'nor' used in common English to say that a given expression is neither true nor false cannot be expressed by this kind of negation.

Thomas Aquinas expressed the difference in the following way:

> It must be said that the true and the false are opposed as *contraries*, and not like affirmation and negation, as some have said. In evidence of this, we must recognize that a negation neither posits anything, nor determines anything as subject to itself. And on account of this, [a negation] can be said both of a being and a non-being; for instance, 'not seeing' and 'not sitting.' … Now a contrary posits something and determines a subject: black is a species of color. But 'false' posits something. Something is false, as the Philosopher says in Metaphysics IV, from this, that 'it is said or seems to be that

[27] Answer: nothing. But this only shows that inquiry characterized by this attitude allows for ignorance to the answers of questions the answers to which presuppose a different kind of attitude.
[28] Given that a well-formed formula of finite length must terminate upon definitional expansion.

> something is what it is not, or not to be what it is.' Just as 'true' posits an acceptance adequate to the thing, so 'false' [posits] an acceptance not adequate [to the thing]. From whence it is manifest that the true and the false are contraries.[29]

Though Aquinas here, as is clear from his examples, is likely thinking of negation as an operation on terms as opposed to propositions, his general point sticks: a negation does not posit anything, and *a fortiori* does not posit falsity. To call something false, however, *does* posit something (figure 1). Aquinas calls this an 'acceptance' (*acceptatio*); and it is clear that, in linking this *acceptatio* to seeming as much as saying, he intends it to refer to the same phenomenon that stands behind the English phrase 'false impression'. This acceptance is, in turn, a two-place relation that takes the subject of the statement as one term, and what is predicated of the thing as the other. For instance, 'green' is accepted adequately with respect to grass, but not with respect to snow, so it is true of grass and false of snow. But it is neither true nor false of Julius Caesar, centaurs, or round squares, since none of these things are around for the predicate to be true or false *of*: to continue with our above comparison, they make no impression one way or another. To say that a *statement* is false is, in turn, just to say that it predicates some attribute of an object that the object does not have, or that it denies an object an attribute that it does have.

For this reason, the semantics obeys the relations between truth, falsity, and negation outlined in the square of opposition of figure 3.

The *import* of the semantics is that it gives us a way of expressing all of the following:

(1) That a given proposition is true at a world in a model;[30]

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[29] ST I. 17. 3, co:

> Respondeo dicendum quod verum et falsum opponuntur ut contraria, et non sicut affirmatio et negatio, ut quidam dixerunt. Ad cuius evidentiam sciendum est quod negatio neque ponit aliquid, neque determinat sibi aliquod subiectum. Et propter hoc potest dici tam de ente quam de non ente; sicut non videns, et non sedens…Contrarium vero et aliquid ponit, et subiectum determinat: nigrum enim est aliqua species coloris. Falsum autem aliquid ponit. Est enim falsum, ut dicit Philosophus IV *Metaph*., ex eo quod "dicitur vel videtur aliquid esse quod non est, vel non esse quod est." Sicut enim verum ponit acceptionem adaequatam rei, ita falsum acceptionem rei non adaequatam. Unde manifestum est quod verum et falsum sunt contraria.

[30] I will not repeat this qualification, but it is to be understood to hold in what follows.

(2) That it is false;

(3) That it is neither true nor false;

(4) That a term denotes;

(5) That it *fails* to denote.

The first two cases were shown in the previous section. To show the third, we can define an indeterminacy operator $\boldsymbol{I}$ as follows:

$$(\boldsymbol{I}) \qquad \boldsymbol{I}(y) \stackrel{\text{def}}{=} \sim\boldsymbol{T}(y) \;\&\; \sim\boldsymbol{F}(y)$$

Where the same preconditions hold on $\boldsymbol{T}(y)$ and $\boldsymbol{F}(y)$ as before.

Denotation and denotation failure are easily expressed: though the semantics has changed from the classical case, the syntactical means of expressing these two remains the same as in Fitting and Mendelsohn [1998], in part because of the difference in the way negation operates. For denoting, we simply let $\mathbf{D}(t)$ abbreviate $\langle \mathbf{T}x.x=x\rangle(t)$; while denotation failure can be expressed by $\sim\mathbf{D}(t)$.

In short, what we have here is not merely a philosophically grounded logic for truth and falsity, but also one capable of expressing when certain *preconditions* on the expressiveness of a sentence hold or fail to hold.

6 Conclusion

This paper doesn't so much solve the Liar[31] as unmask the particular conceptual confusion that the problem rests upon, one evident in the standpoint of inflationists and the deflationists alike. Both the deflationist and the inflationist accept the claim that 'true' being about something entails

[31] That is, on the assumption that a solution must or should involve giving a revenge-immune semantics capable of expressing truth and any other apparently problematic notions in its vicinity. Especially when coupled with a strong form of deflationism identifying substantiality and meaningfulness, this demand effectively amounts to requiring semantics to do the job of syntax: truth would then be a meaningless notion that, instead of being barred from the syntax by virtue of this status, would be required to behave in accord with a semantics that, *qua* semantics, presupposes the meaningfulness of its subject matter.

that it must refer to some substantial property. We agreed with the deflationist that truth does not refer to some substantial property, but disagreed in holding that 'true' is nevertheless a meaningful term. From an analysis of the pre-sentential meaning of truth, we concluded that 'true' is not a real, but an ontological predicate, and the role it plays in sentences is fundamentally a modification of its pre-sentential meaning: 'true,' like 'good' and ‘the same as', is a piggy-back term that relies for its meaning on the significate of the term that it primarily modifies. From here, we developed a syntax and semantics adequate for these ideas based on Church's λ-calculus, followed by a more in-depth discussion of the relation between negation, truth, and falsity embodied therein. Hopefully, this has given proof to the following leading ideas.

(1) Truth is a robust notion. But it does not thereby signify a substantial or absolute property.

(2) Therefore, the semantics for representing truth in a language are somewhat more nuanced than the T-schema suggests.

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