

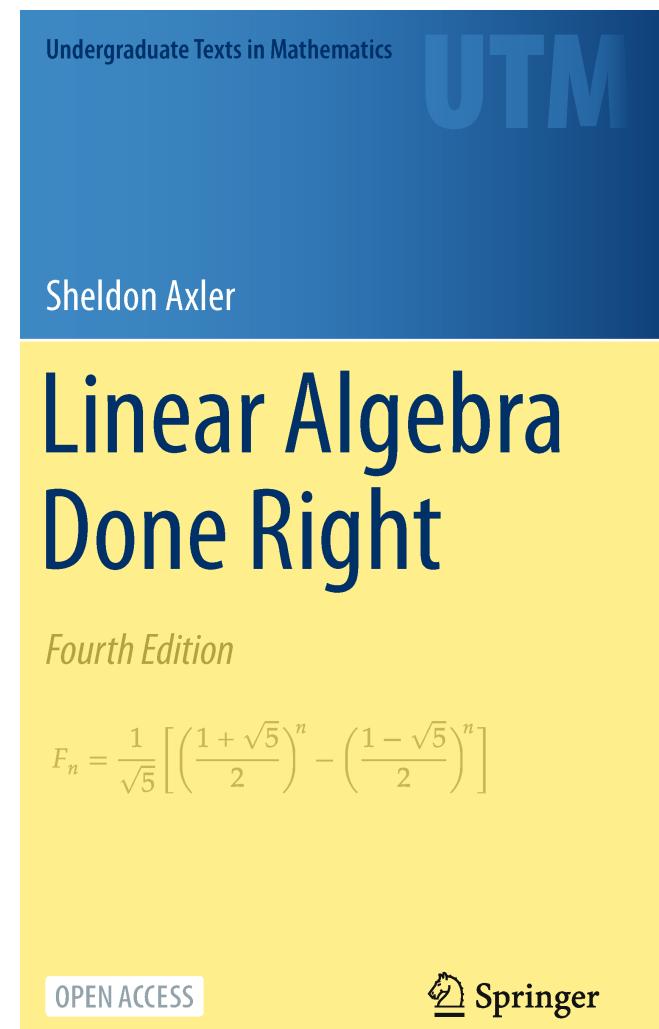
# Welcome to

# Math 416: Abstract Linear Algebra

Fall 2025

Instructor: Jacob Bekey

Textbook



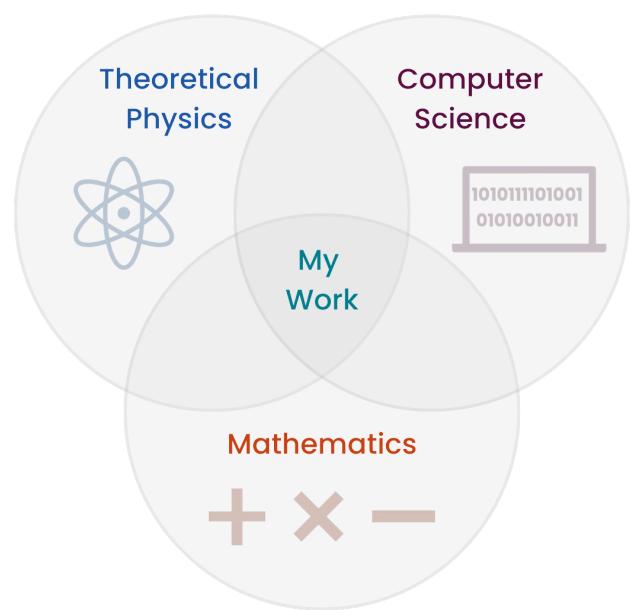
# A bit about me

Name: Jacob Beckey

Role: NSF & IQUIT postdoc  
in the math department

## Research interests:

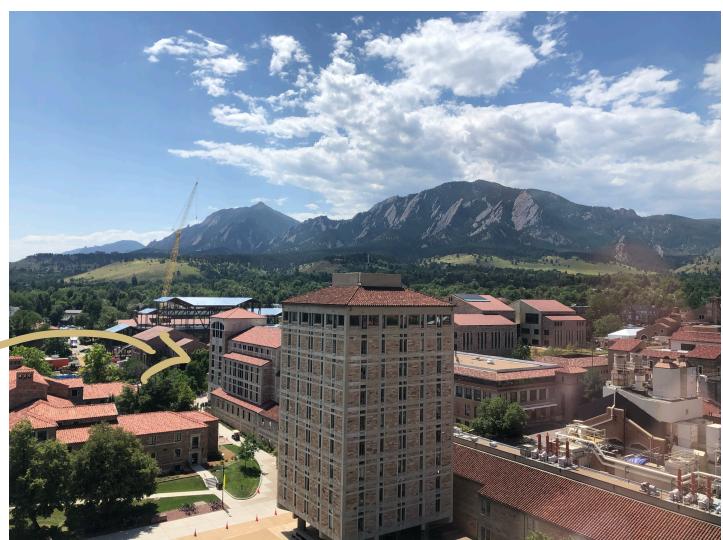
- quantum learning theory
- entanglement theory
- representation theoretic methods in quantum info



## Background:

- Ph.D. Physics from Univ. of Colorado, Boulder

my office



# A bit about the course

Lets review the syllabus together.

\* If you missed the first lecture, please review the syllabus on Canvas or the course website

[www.jacobbekey.com/teaching](http://www.jacobbekey.com/teaching)

Discussion: what is linear algebra useful for?

Form small groups, introduce yourselves, and discuss the above question.

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# 1. Quantum mechanics

$V(x) = \begin{cases} 0, & x < 0, \\ V_0, & x \geq 0. \end{cases}$   $\sigma_x \sigma_p \geq \frac{\hbar^2}{2}$   $|E = \hbar\nu\rangle$   $E = \frac{\hbar^2 k^2}{2m}$   
 $\Psi_1(x) = \frac{1}{\sqrt{k_1}} (A_+ e^{ik_1 x} + A_- e^{-ik_1 x})$   $x < 0$    
 $\Psi_2(x) = \frac{1}{\sqrt{k_2}} (B_+ e^{ik_2 x} + B_- e^{-ik_2 x})$   $x > 0$   $T|j,m\rangle \equiv |T(j,m)\rangle = (-1)^{j-m} |j,-m\rangle$   
 $i\hbar \frac{\partial}{\partial t} \Psi(r,t) = \hat{H} \Psi(r,t)$   $|\Psi\rangle_{AB} = \sum_{i,j} c_{ij} |\psi_i\rangle A \otimes |j\rangle B$   
 $P[a \leq X \leq b] = \int_a^b \rho(x) dx$   $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} (e^{-x^2})$    
 $-\frac{\hbar^2}{2m} \frac{d^2 \Psi}{dx^2} = E \Psi$    
 $\Psi(x) = A e^{ikx} + B e^{-ikx}$   $U(t) = \exp(-iHt/\hbar)$    
 $i\hbar \frac{d}{dt} |\Psi(t)\rangle = H |\Psi(t)\rangle$   $A[x] = \exp\left(\frac{i}{\hbar} \int X(t) dt\right)$   
 $P(a,b) = \int d\lambda \cdot \rho(\lambda) \cdot p_A(a,\lambda) \cdot p_B(b,\lambda)$

Exactly 100 years ago, physicists began to uncover the rules of quantum mechanics...

To their surprise, a natural language to describe quantum systems was the language of linear algebra!

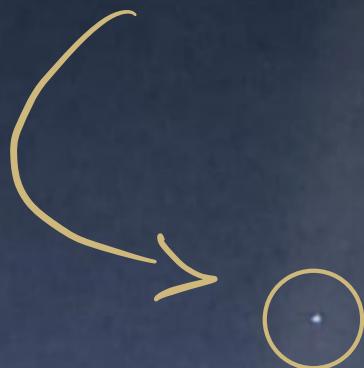
States  $\longleftrightarrow$  Unit vectors

Measurements  $\longleftrightarrow$  linear operators

Measurement outcomes  $\longleftrightarrow$  eigenvalues

## 2. Error correction

Earth from 4 billion miles away



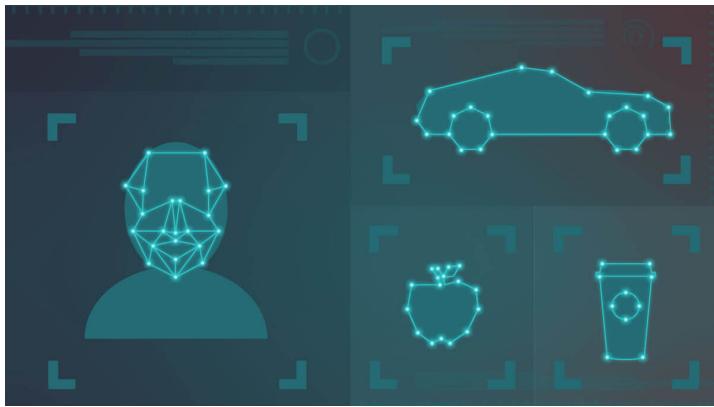
- In 1990, as Voyager 1 was leaving our solar system, it snapped the famous "pale blue dot" photo
- Without error correction, this data never would have made it back to earth!

codeword  $\longleftrightarrow$  vector in codespace

error  $\longleftrightarrow$  vector

syndrome measurement  $\longleftrightarrow$  linear operator

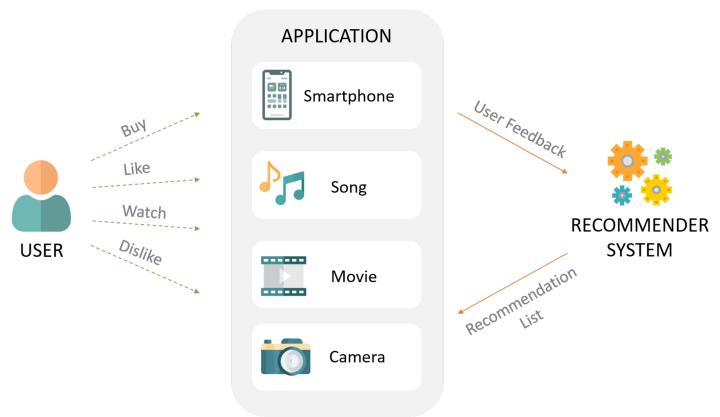
### 3. Machine learning



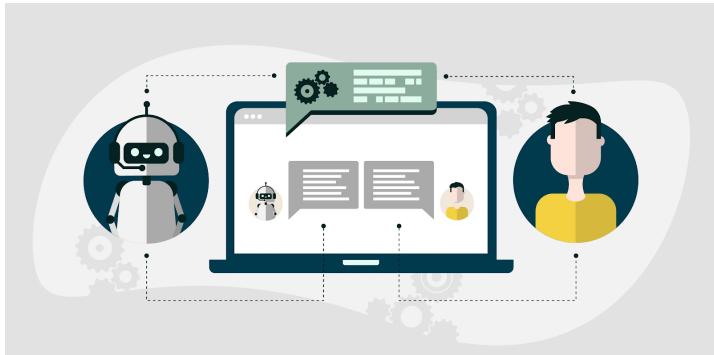
recommendation  
algorithms



Image recognition



natural language  
processing

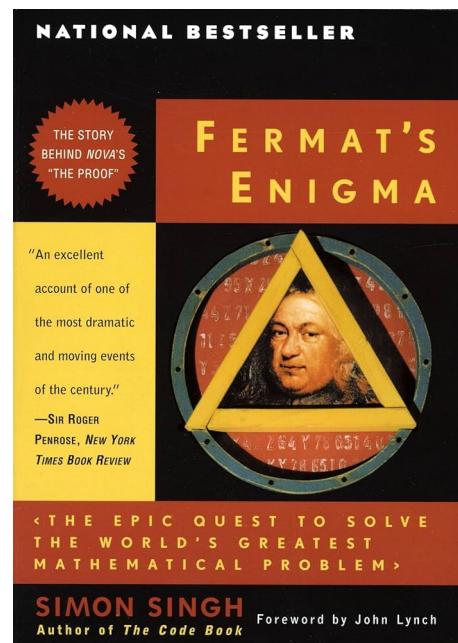
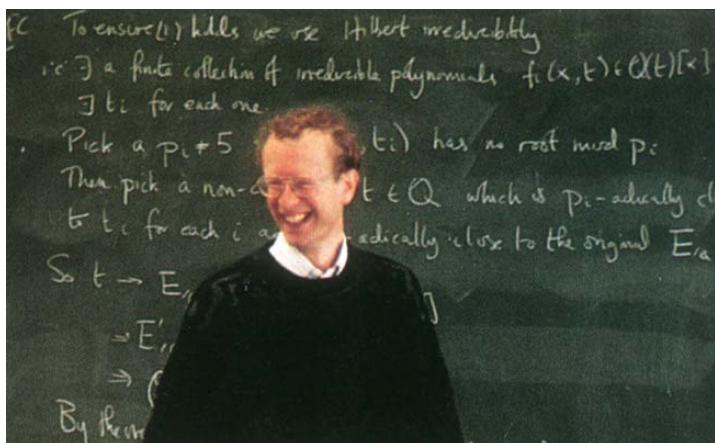


- All of these applications utilize vectors, matrices, and tools from linear algebra

# 4. Modern Mathematics

$$a^n + b^n = c^n$$

has no integer solutions when  $n > 2$ .



- Linear algebra is essential for representation theory, without which Andrew Wiles could not have proved Fermat's last theorem — a problem that stumped mathematicians for 350 years!

# Clicker Question

Linear algebra as a field didn't appear overnight. When do you think the first seeds of linear algebra were planted?

A) ~200 BCE

B) ~800 CE

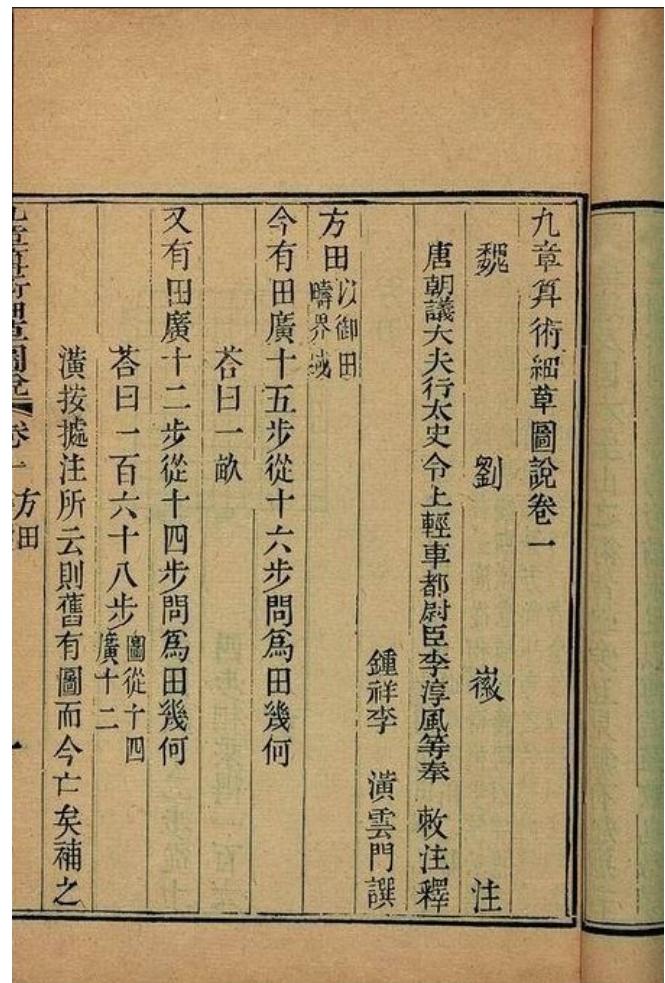
C) ~1600 CE

D) ~1900 CE

"Nine chapters  
on the  
mathematical art"

from ~200 BCE

is a book written  
by many Chinese  
scholars



- Contains a method for solving linear equations that is nearly identical to Gaussian elimination (~1700s)

We will pick up here Wed.