Matrix Multiplication

Recall Def. 3.31 (matrix of a linear map)

- $T \in \mathcal{L}(V, W)$
- $v_1, \dots, v_n$ basis of V
- $w_1, \dots, v_m$ basis of W

Then, $\mathcal{M}(T)$ w.r.t these bases is defined by

$$T v_k = \sum_{i=1}^{m} A_{ik} \, w_i$$

element of V basis

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element of W basis

See page 70 for helpful diagram:

$$\mathcal{M}(T) = \begin{array}{c c} & \begin{array}{ccccc} v_1 & \cdots & v_k & \cdots & v_n \end{array} \\ \begin{array}{c} w_1 \\ \vdots \\ w_m \end{array} & \left(\begin{array}{ccccc} & & \boxed{\begin{array}{c} A_{1,k} \\ \vdots \\ A_{m,k} \end{array}} & & \end{array} \right) \end{array}$$

Then, matrix mult. is defined as

$$(AB)_{jk} = \sum_{r=1}^{n} A_{jr} B_{rk}$$

So that $\forall \ T \in \mathcal{L}(U,V)$ & $S \in \mathcal{L}(V,W)$, we have

$$\mathcal{M}(ST) = \mathcal{M}(S)\mathcal{M}(T).$$

Further, via Axler 3.73, we define

$$\mathcal{M}(v) = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$$

to be the matrix of v w.r.t $v_1, \ldots, v_n$ basis of V. This ensures

$$\mathcal{M}(Tv) = \mathcal{M}(T)\mathcal{M}(v)$$

(see Axler 3.76)