Solution-Guide

1) Heuristic and Tabu Search methods

a. [8 marks]

i. *Explain the A* algorithm and conditions for its optimality.*
A* is a best first search (it expands best available unexpanded node) that uses an evaluation function f(n)=g(n)+h(n)
Where g(n) is cost from start node to n and h(n) is estimate cost from n to the goal
A* uses admissible heuristic h(n)<= h*(n) (the actual cost from n to the goal, and h(n)>=0). Using admissible heuristics is a condition for A* optimality.

ii. *What is meant by completeness and optimality of a search strategy?*
Completeness: strategy guaranteed to find a solution if it exists.
Optimality: strategy finds the best solution (shortest path)

iii. *Explain approaches that have been proposed to make Tabu Search adaptive.*
One of the adaptive approaches incorporated into TS is to allow the length of the short term memory (tabu list) to vary dynamically. This can be achieved by computing in advance a range of tabu list length and a new length is randomly selected from the range every predetermined number of iterations.
Another approach is to restrict the tabu length between Lmin and Lmax. If solution is improved we decrease the length by 1 if it is not improving we increase by 1. The values of Lmain and Lmax are changed every a set number of iterations.

b. [12 marks] Consider using TS to solve the problem of assigning numbers from 1 to 8 to the nodes of the following graph such that each number appears exactly once and no connected nodes have consecutive numbers

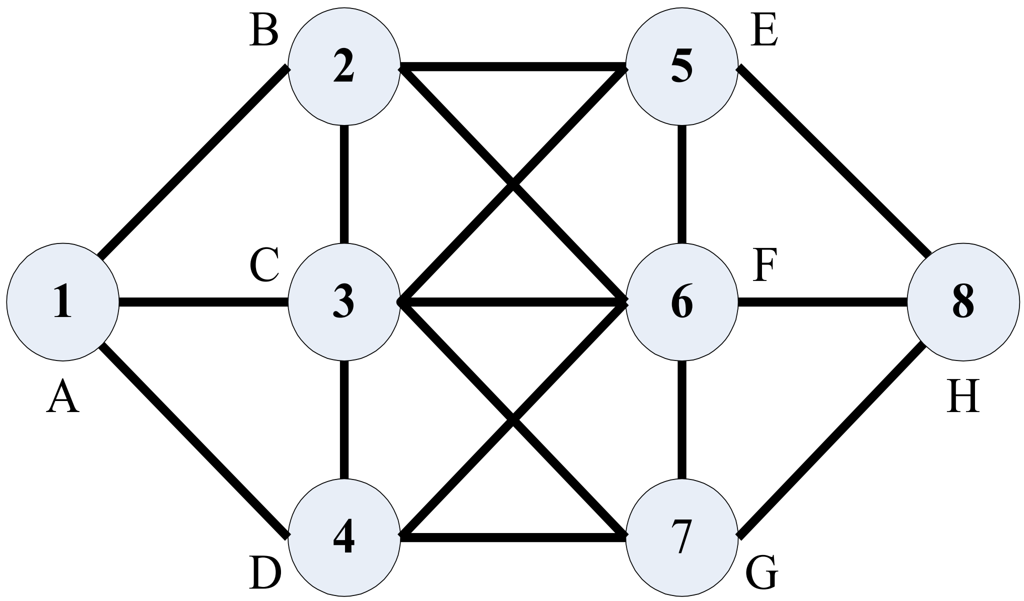
Starting from a random initial assignment such as (1 to A, 2 to B, 3 to C, 4 to D, 5 to E, 6 to F, 7 to G, 8 to H)

i. *Define a cost function to specify the quality of an assignment*,
A cost function can be the number of edges that have nodes with consecutive numbers (i.e number of edges violating the constraint). The objective is to minimize this cost.

ii. *Define a suitable neighborhood operator,*
Swap the labels of two nodes that do not satisfy the constraint and not connected to the same edge

iii. *Define Tabu moves and a suitable matrix-structured Tabu memory showing how this memory is updated after a certain move is selected,*
Tabu moves: when two labels are swapped between two nodes, they should not be swapped back again.
Tabu memory: m-by-m matrix where m is the number of nodes. The element A(i, j) is set to tabu tenure T when the two nodes are swapped. Non-zero elements are decremented every iteration.

iv. *Perform TWO iterations of TS and show how the tabu memory structure is updated after each selected move. Restrict the neighborhood to 2 applications of your operator.*



Initial solution, cost=6

Iteration 1: explore 2 neighboring solutions, select the best one, update tabu list, e.g swap values in A and B, cost=6, swap values in A and D cost=5 this is best of the two neighbors then select this move.

In the tabu list in entry A(1,4)=T

Iteration 2: explore 2 neighboring solutions avoiding tabu moves, select the best one, update tabu list,

2 neighbors: Swap values in C and F, cost =4, Swap values in B and E, cost=5. Select first one. Add in A(3,6)=T and A(1,4)=T-1

2) Simulated Annealing (SA)

a. [8 marks]

i. *What should be observed in selecting the initial and final temperatures of the SA cooling schedule? Provide guidelines to select these temperatures.*

The initial temperature should be high enough to allow a move to possibly any state in the search space. Must not be too hot otherwise SA would behave as random search for a number of iterations,

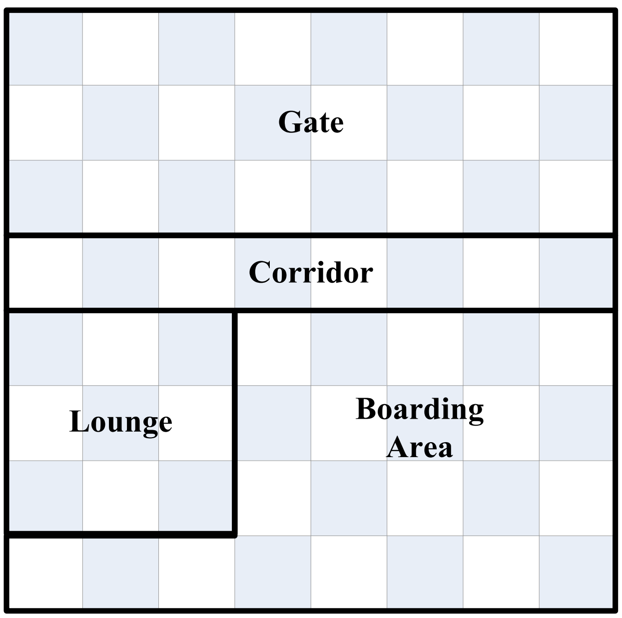
The maximum change of the cost function could be used as a guide to set this value. A good initial temperature is a one that accepts about 60% of the worse moves.

The final temperature doesn't have to reach zero, as it might take a very long time to reach zero when some decrement approaches are used. The algorithm should be stopped if it reaches a reasonable final temperature and no better moves are generated nor worse moves are accepted (i.e frozen).

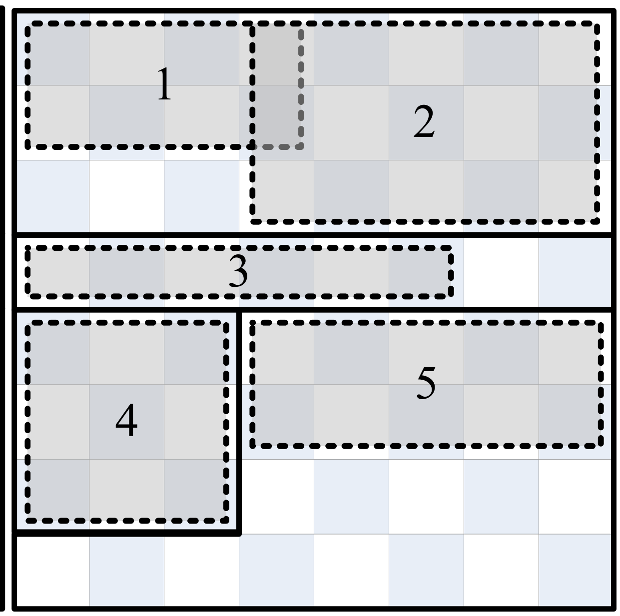
ii. *Mention some of the approaches that have been proposed to improve the computational cost of the basic SA method.*

1-Using lookup table to do the exponential calculation.
2-Using non-exponential formulas to calculate acceptance probability.
3-Using update functions for calculating the cost.

b. [12 marks] In a simple sensor placement problem, it is required to place 5 pan-tilt cameras in such a way that maximizes the coverage of an airport. The airport consists of 8x8 cells divided into four areas of interest, namely: gate, corridor, lounge and boarding area as illustrated in Figure 1-a. The gate area is considered as airport perimeter sensitive area, and accordingly uncovered cells in this area are assigned double the cost of uncovered cells in other areas. The sensing range of each camera is given in Table 1. This sensing range represents the union of all fields of view that a pan-tilt camera can view. Figure 1-b shows a possible placement for the cameras that is obtained by placing the cameras in the areas in sequence.



(a) Airport Plan



(b) Possible Placement of Cameras

Figure 1 – Camera Placement Problem

Camera	Sensing Range	
	L1 (Cells)	L2 (Cells)
1	2	4
2	3	5
3	2	6
4	3	4
5	2	7

Table 1 – Cameras Sensing Ranges

- Any camera can be placed to get a sensing range of L1-by-L2 or L2-by-L1
- The sensing ranges of two cameras may intersect.

- The camera can be placed in an area smaller than the sensing range.

Consider solving this problem using SA

i. Define a suitable problem representation,

The solution can be represented as an m-by-3 array where m is the number of cameras. The first two elements of row i represent the (x, y) coordinates of the top-left corner of camera i sensing range. The third element represents the orientation of the camera (horizontal or vertical). For example, the given solution can be represented as ((1, 1, H), (1, 3, H), (4, 1, H), (5, 1, H), (5, 4, H)).

ii. Define suitable neighborhood operators,

One possible operator is to move a camera from its current location to any other location in the airport (such that at least one cell of the area is covered by this camera). Another possible operator is to swap the locations of two cameras. A third operator is to rotate a camera by 90 degrees.

iii. Define the objective function for calculating the cost of a solution,

The objective function can be defined as: minimize (2 * the number of uncovered cells in the gate area + the number of uncovered cells in other areas). We can multiply the objective function by a constant a > 1 to prevent obtaining very high values for the acceptance probability

iv. Starting from the solution in Figure 1-b, apply simulated annealing for 2 iterations using an initial temperature of 500, a linear decrement rate of 50, and one iteration at each temperature. Assume the following random numbers in sequence [0.0071, 0.6799, 0.9027].

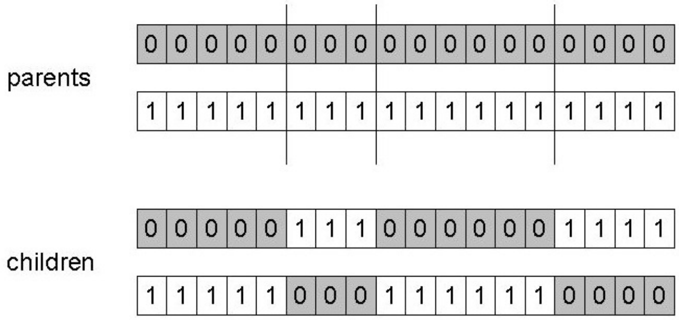
Initialize the temperature to 500
Calculate the objective function for the initial solution
Repeat 2 times:
Select one of the suggested operators at random and apply it on the current solution to get a neighboring solution
Calculate the objective function for this solution
If it's better than the current solution (lower objective function), accept it
Otherwise, calculate the acceptance probability p = exp (-diff / temp) and accept the solution if p > the next given random number
Update the temperature: temperature = old temperature – 50
(students should use actual values)

3) Genetic Algorithms (GA)

a. [7 marks]

i. Explain the n-point crossover operator and illustrate your answer with an example.

Choose n random crossover points,
Split along those points,
Glue parts, alternating between parents,
Example



ii. Explain the difference between Generational GA and Steady State GA

Generational model: each individual survives for exactly one generation, the entire set of parents is replaced by the offspring.

Steady-State model: Only part of the population is replaced by the offspring, Usually one member of population is replaced, the proportion of the population replaced is called the Generational Gap: 1.0 for GGA, 1/pop_size for SSGA

iii. Describe the Baker's Stochastic Universal Sampling. What does it guarantee?

Construct a roulette wheel with n evenly spaced arms and spin once. Guarantees floor(E(ni)) ≤ ni ≤ ceil(E(ni)). That is the samples are not repeated as what may happen in roulette wheel with one arm.

b. [13 marks] A printer paper company has received orders for four different groups of companies as follows:

1) 2 rolls of 2-foot paper at $2.5 per roll.
2) 6 rolls of 2.5-foot paper at $3.10 per roll.
3) 2 rolls of 4-foot paper at $5.25 per roll.
4) 2 rolls of 3-foot paper at $4.40 per roll.

Due to heavy demands on printer paper, the company has only 20 feet of paper from which to fill these orders. Consider using GA to solve this problem to maximize the company's income assuming that partial orders are not allowed (i.e., the customers only accept orders with the requested number of rolls):

i. Develop a representation for the solution and the coding you will use to represent the genes.

As partial orders are not accepted, a suitable representation for the solution is a string of 4 binary variables (e.g., 1100). Each variable represents one order. The variable will be true if the corresponding order is satisfied. Otherwise, it will be false. The solution is feasible if the total number of feet in all orders is less than 20.

ii. Starting with a population of 4 feasible solutions, generated randomly, perform fitness based parent selection.

Generate any four feasible solutions with total feet < 20

Calculate the fitness of each solution as the number per rolls * the price per roll

Select 2 pairs of parents using any fitness-based parent selection like fitness proportionate selection, roulette-wheel selection, rank-based selection or tournament selection. The following table shows fitness-proportionate selection:

Solution	Feet	Fitness	Expected count	Actual count
1011	18	24.3	1.81	2
1010	12	15.5	1.15	1
1000	4	5	0.37	0
0001	6	8.8	0.65	1
Average		13.4		

The selected parents are: {1011, 1011, 1010, 0001}

iii. *Define a crossover operation and apply it to the selected parents to generate off-springs.*

Suitable crossover operators are 1-point, n-point and uniform crossover
The following shows the application of a 1-point crossover operator
{101|1, 101|0} -> {1010, 1011}, {10|11, 00|01} ->{1001, 0011}
All children are feasible.

iv. *Define a survival strategy and apply it to the generated solutions.*

Using the generational age-based strategy the new generations replaces the old, i.e the new generations will be {1010,1011,1001,0011}
Other strategies can be steady state model, Elitism and GENITOR

v. *If the customers can accept partial orders (i.e., some of the rolls they requested), develop a representation for the solution and the coding you will use to represent the genes.*

A suitable representation for the solution is a string of 4 integers (e.g., 2410). Each integer represent the number of rolls provided a customer. The ranges of these variables are [0, 2] [0, 2] [0, 6] [0, 2] respectively. The solution is feasible if the total number of feet in all orders is less than 20.

vi. *Suggest a suitable crossover operation for the case in part v.*

Suitable crossover operators are the single, simple and whole arithmetic operators. The variables are rounded to the nearest integer after the operator is applied.

4) Ant Colony Optimization (ACO)

a. [10 marks]

i. *Mention the different parameters that need to be set to run ACO algorithm. Discuss the implication of selecting certain values (large, small, etc) for these parameters.*

Number of ants: more ants more computations, but also more exploration.
Max number of iterations: has to be enough to allow convergence.
Initial pheromone: constant, random, max value, small value.
Pheromone decay parameter ρ
Parameters alpha and beta

ii. *What does the Max-Min Ant System (MMAS) algorithm introduced over the basic AS algorithm and for what purpose?*

The update is done using the best solution (best ant) in the current iteration or the best solution over all
The values of the pheromone are restricted between τ_{min} and τ_{max} , allows high exploration in the beginning (τ_{max}) and more intensification later.

Purpose is to prevent stagnation

iii. *Describe the near-parameter-free ACS approach*

- The parameters of ACS were adapted using the same approach for optimizing the problem in hand (an ant approach),
- Each ant is allowed to select the suitable values of its parameters and to select the next solution component.
- Each ant has its own parameter values adaptively selected using an ant approach:
  - A separate pheromone matrix is kept for learning these values,
  - For each parameter, the interval is discretized with a step P
  - The parameter division is chosen using transition rule based on the pheromone values v(i, j).

b. [10 marks] The Quadratic Assignment Problem (QAP) is defined as the assignment of a number of ***n*** facilities to ***n*** different locations while minimizing the product of the flow cost between every two facilities and the distances between the two locations that these facilities where assigned to. The flow costs between every two facilities is given in a matrix ***F*** while the distances between every two locations is given in a matrix ***D***.

Ant Colony Optimization could be applied to solve this problem:

*i. Define mathematically the objective function to be used in the search if the solution is defined as a permutation **P**, where the index is the location and the element value is the facility,*

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} F(P(i), P(j)) D(i, j)$$

*ii. What are the decisions to be taken by any ant **m** in every iteration?*

At each step, the ant selects an assignment for one of the unassigned facilities to a location. This can be represented as a fully connected graph of (facility, location) pairs and the ant moves over this graph avoiding repeated assignments.

Or

At each step, the ant selects an unassigned facility to consider and then select a location for this facility from the list of locations.

iii. How the pheromone trails τ_{ij} are applied and updated for such a problem?

Each facility is assigned a pheromone value and each location assignment for this facility is assigned a pheromone value. The pheromones are initially assigned to the same value. Each ant selects the next move based on a probability calculated based on the pheromones values (give equation).

The pheromone values are updated at the end of iterations by doing evaporations and then update based on the best solution (give equations)

iv. What heuristic information η_{ij} could be used for such a problem?

The heuristic information for a facility-location assignment could be the product of the flow from this facility to other facilities times with the distance between the current facility (in the new assigned locations) and all other facilities (in the previous assigned locations).

5) Particle Swarm Optimization (PSO)

a. [10 marks]

i. Discuss difference between physical neighbors and social neighbors in neighborhood selection and their impact on the performance of PSO algorithm.

- The most obvious way to select the neighbours for a certain particle m is to choose the particles that are closest to it in the search space (physical neighbours). The notion of closest is based on the distance in the Cartesian space. This approach might be computationally expensive as distances have to be computed each time the particle changes its position.
- If the particles are kept in a matrix data structure for example, the most commonly used approach is to pick the particles that are stored next to m in the matrix (social neighbours). The performance is affected by the size of the neighbourhood selected (2, 4, 6, …).

ii. Describe the binary version of PSO and how the particle velocity and position are defined and updated.

- In binary PSO, each particle represents a position in the binary space Each element can take the value of 0 or 1.
- Velocities are defined as probabilities that one element will be in one state or the other, Since velocities represent probabilities, the values of the velocity elements need to be restricted in the range [0,1] using the sigmoid function.
- The velocity components could remain as real-valued numbers using the original equation, but fed to the sigmoid function before updating the position vector.

iii. Explain how to solve the clustering problem using PSO with multiple cooperating swarms.

One approach is to divide the problem search space and assign regions to swarms. Each swarm may be designed to maximize the intra-clutser distance and maximize the distance with other clusters.

Another approach is to have each swarm solve the complete clustering but with different initialization and parameters and the swarms exchange information between them (such as gbest) every pre-determined number of iterations.

b. [10 marks] Robot motion planning involves finding the path a robot needs to traverse from a given start point to a given end point while avoiding obstacles along the way. This problem can be solved using PSO. Using the following figure to illustrate your answer:

i. With the objective of minimizing the path length show how the problem is formulated explaining what the particles will represent giving the equations of motion and update.

Vertical lines are drawn for each of the vertices of the obstacles. The best path can be determined as the y values along these vertical lines. Each particle is assigned a set of values (one for each vertical lines). The quality of the solution is inverse the length of the path. PSO updates the locations of particles based on the inertia, global and local best.

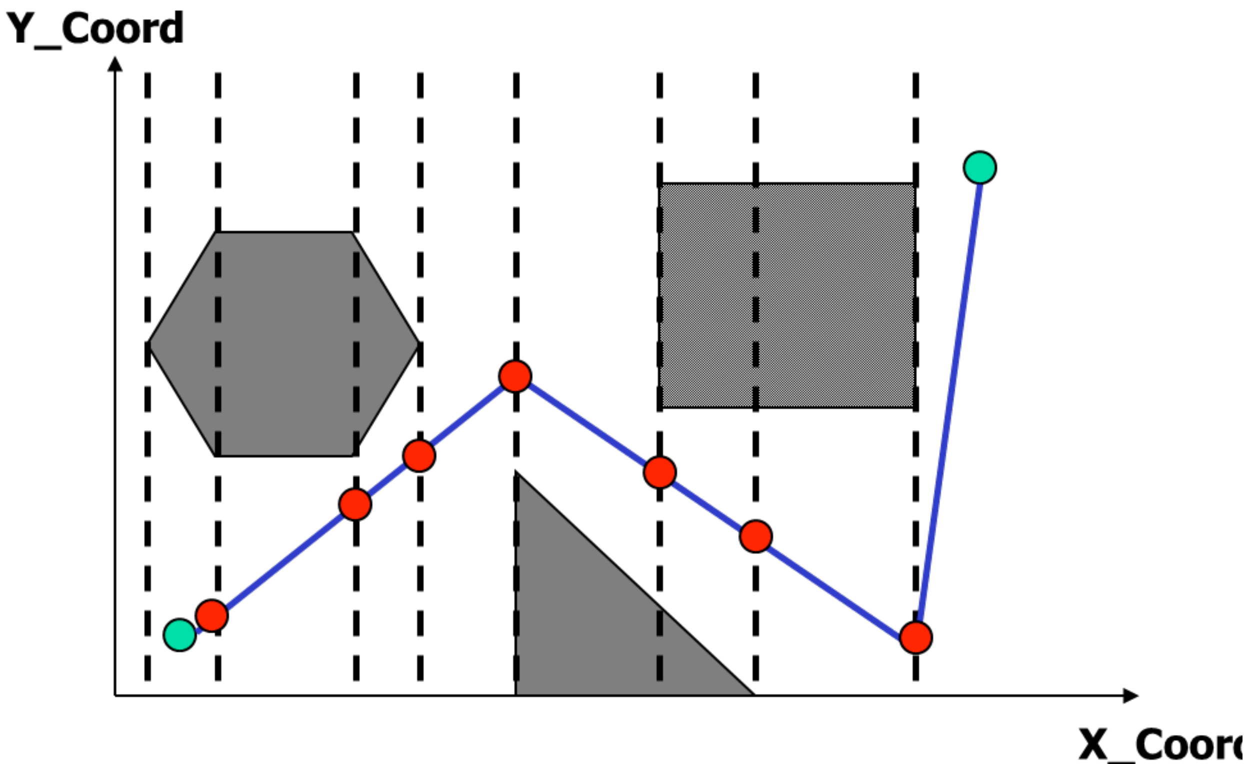
ii. *If the objective is to maximize the distance to obstacles, discuss how this can be achieved and its implication on the PSO algorithm*

The quality of the solution is modified to include the distance of the robot at each vertical line from the obstacles.

iii. *What will need to change if the number of obstacles and their geometries change?*

The number and positions of the vertical lines will be changed

The idea is to optimize the y-coordinates at which the path intersects with the vertical lines



Vertical lines are drawn at the obstacles vertices