Optimal Energy/Time Routing in Battery-powered Vehicles

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***Abstract*—The limited battery capacity of Electric vehicles (EVs), and consequently limited cruising range, hinders their widespread adoption. This paper proposes a solution to the problem of optimal energy/time routing under battery constraints. A multi-criteria path-finding algorithm, A^*, is proposed to function in two modes. The first is an energy mode to solve the problem of energy-optimal routing under battery constraints. This mode computes the most energy-efficient route from a source to a destination, thus extending the cruising range of the battery. The second is a travel-time mode to compute the time-optimal route under the battery constraints. The multi-criteria model aims to use the modes to strike a balance between energy consumption and travel time, so as to satisfy the user constraints and needs. This research reports simulation work conducted to test and validate the proposed model under various driving conditions.**

***Index Terms*—Electic vehicles, Energy optimal routing, Travel-time optimal routing**

I. Introduction

The limited cruising range of their battery is a critically significant issue of EVs. Cruising range of the battery is defined as the distance that a vehicle can travel over time until the battery runs out of energy. Energy-optimal routing has been proven to effectively prolong the driving-range by reducing energy consumption [2, 3, and 4]. Since there are constraints associated with the battery of the EV, the energy-optimal routing technique for EVs should not be limited only to finding the cheapest route in terms of energy cost but should also take the battery constraints into account. Travel time of a road trip is an important attribute of traffic. For electrically-powered vehicles with battery constraints, it is not efficient for drivers to use a route that is inexpensive in travel time cost but violates the constraints of the battery, since the battery-constraint issue is the overriding factor in reaching the final destination.

Artmeier et al. [2], proposed certain shortest path techniques that tackle energy-optimal routing. They presented a shortest path algorithm that takes into account the battery constraints and solves the problem in a running time of $O(n^3)$. Jochen et al. [4] employed a result by Johnson [5] and some significant observations about Dijkstra [6] under non-constant edge costs to obtain an $O(nlogn + m)$ query time after an $O(mn)$ pre-processing phase for any road network weighted with energy edge costs. Martin et al. [3] have proposed a solution to the problem of energy-optimal routing taking into account the battery constraints using a framework of the A^* search algorithm, thus solving the problem in a running time of $O(n^2)$. The recent studies that have been conducted to tackle optimal routing of EVs [2, 3, and 4] are still incomplete since all of them have focused on optimal routing that is based only on the energy cost aspect. Thus, optimal routing based on travel-time costs under battery constraints has not been addressed yet. In addition, the solution proposed by Artmeier et al. in [2] has a worst case time complexity of $O(n^3)$, which makes this solution not a preferred choice of navigation system designers and route planners. While the solution introduced by Martin et al. in [3] has a time complexity of $O(n^2)$, it is still incomplete. First, the energy cost function used to model road segment costs is not complete. The energy cost function does not consider other energy forms that may occur along a certain path, such as the energies dissipated and recuperated during acceleration and deceleration phases or the dissipation of energy by on-board electric devices, such as air conditioners, radios, etc. Second, the solution computed by the A^* algorithm is not verified in its optimality.

The objective of this research is to extend the work conducted in [3] by proposing a multi-criteria path-finding technique that considers the battery constraints as an overriding issue while finding the most efficient path in terms of energy or travel-time cost. This paper is organized into four sections: Section II describes theoretical information and introduces the proposed solution to the problem of optimal energy/time routing for EVs. Section III reports the simulation work performed to validate the proposed algorithmic technique. Finally, concluding remarks and future research directions are presented in Section IV.

II. Optimal Path Finding: A Multi-criteria Model

A multi-criteria model within a framework of the A^* search technique that relies on a heuristic function during its search is proposed. This model functions in two modes, namely, energy mode and travel-time mode. For the energy mode, the A^* algorithm is modified to take the battery constraints into account and exclude the optimality of travel time, thus computing the most energy-efficient path among all possible paths. For the travel-time mode, the A^* algorithm is modified to take the battery constraints into account and exclude the optimality of energy, computing the most travel-time-efficient path among all possible paths. The problem of battery constraints is solved by dynamically adjusting the energy cost function in the algorithms during the search process; consequently, drivers are assured of reaching their destinations if they use the optimal solution.

A. Energy Mode

The first battery constraint is that a path can no longer be used if it has an energy cost that is greater than the battery charge level. We propose that this constraint problem be solved by turning the path energy cost value into infinity, thus excluding the path from the search process. The second battery constraint, which matters only with edges having negative energy costs, is that recuperation, that is, gaining energy from downhill edges and during deceleration phases, is not possible if the battery is already fully charged. We propose dynamically adjusting the energy cost function in the A^* algorithm such that the energy gained is stored in the available free capacity of the battery until the battery is full. The rest of any energy gained is lost.

Assume a directed graph is given, $G = (V, E)$ having $|V| = n$ and $|E| = m$, to represent a road network, where vertices $v \in V$ represent points, and edges $e \in E$ represent connections between these points corresponding to road segments. Assume for each vertex, an elevation $u : v \rightarrow R_0^+$ is given, and for each edge, a length $l : E \rightarrow R^+$ and a speed limit $S : E \rightarrow N$ are given. A path P can be defined as a sequence of k vertices $(v_1, v_2, ..., v_k)$ and the edge is two vertices $(v_i, v_{i+1}) \in E$ with $i = 1, 2, ..., k-1$. In addition, the following parameters are defined: C_{max} is the maximum capacity of the battery; J is the charge level of the battery, where $J \leq C_{max}$; U is the remaining free capacity of the battery, where $U = C_{max} - J$; and Δ^k is the amount of energy consumed or gained along a path. We consider different forms of energy costs that can occur from taking a path $P^k = (v_1, v_2, ..., v_k)$ as follows:

Potential Consumed/Gained Energy:

When the EV travels over an edge (a, b), the potential energy $E_P(a, b) = u(b) - u(a)$ is consumed or drawn from the battery only if the EV is going uphill $(i.e., u(b) - u(a))$. The energy cost function of the potential consumed energy is defined to be

$$C_{PC}(a, b) = \frac{1}{\eta_c}[mg(u(b) - u(a))] \tag{1}$$

where, m is the mass of the vehicle, including payload, g is the gravitational acceleration factor, and η_c is the efficiency factor. This potential consumed energy takes the following values:

$$C_{PC}(a, b) = \begin{cases} 0 & \text{if } k = 1 \\ 0 & \text{if } k > 1, \quad \Delta^k < 0 \\ \Delta^k & \text{if } k > 1, \quad 0 \leq \Delta^k \leq J \\ \infty & \text{if } k > 1, \quad \Delta^k > J \end{cases}$$

Moreover, the gained energy $E_G(a, b) = u(a) - u(b)$ is regenerated and stored in the battery only if the EV is going downhill $(i.e., u(a) - u(b))$. The energy cost function of the potential gained energy is defined to be

$$C_{PG}(a, b) = \eta_c[mg(u(a) - u(b))] \tag{2}$$

This potential gained energy is stored in the battery and is lost only if the battery is fully charged. It takes the following values:

$$C_{PG}(a, b) = \begin{cases} 0 & \text{if } k = 1 \\ 0 & \text{if } k > 1, \quad 0 \leq \Delta^k \leq J \\ -\Delta^k & \text{if } k > 1, \quad 0 > \Delta^k \leq U \\ \Delta^k & \text{if } k > 1, \quad 0 > \Delta^k > U \end{cases}$$

Loss of Energy:

The loss of energy, due to the aerodynamic and rolling resistances, occurs even if the vehicle is going downhill, which means this energy cost value cannot be recuperated. However, the aerodynamic resistance component, which is the right hand side of Equation 3, is not dissipated if and only if two conditions are satisfied: first, the wind is in the same direction as the vehicle, and second, the wind has a speed that is greater than or equal to the speed of the vehicle.

$$C_{LE}(a, b) = \frac{1}{\eta_c}[f_r mgl(a, b) + \frac{1}{2}\rho A c_w S(a, b)^2 l(a, b)] \tag{3}$$

where f_r is the friction coefficient, ρ is the air density coefficient, A is the vehicle's cross sectional area, c_w is the air drag coefficient, S is the average speed on the edge (a, b), and l is the length of the edge (a, b). This loss of energy on the road segment (a, b) takes the following values:

$$C_{PG}(a, b) = \begin{cases} 0 & \text{if } k = 1 \\ \Delta^k & \text{if } k > 1, \quad \Delta^k > 0 \\ \Delta^k & \text{if } k > 1, \quad \Delta^k < 0 \\ \infty & \text{if } k > 1, \quad \Delta^k > J \end{cases}$$

Acceleration and Deceleration Energy:

We define a function $E_{a,d}(P, t_{a,d}(e))$ that models energy consumption due to mechanical loss, that is, the acceleration energy required to bring the EV back up to the average speed, and then deceleration energy used to bring the vehicle to a stop. An EV dissipates zero energy during idling. The energies dissipated and recuperated during acceleration and deceleration phases on the road segment (a, b) are

$$C_{AE}(a, b) = \frac{1}{\eta_c} P t_A \tag{4}$$

$$C_{DE}(a, b) = \eta_r P t_D \tag{5}$$

where t_A and t_D are the times taken by the vehicle during acceleration and deceleration respectively, and P is the power of the motor in $watts$.

Driving Style:

After calculating the four energy forms of a road segment (a, b), the total energy cost function is multiplied by the driving style coefficient, DS_{coeff}. For normal driving, this coefficient may take a value of 1, and for aggressive driving, it may take the value 1.2.

On-Board Electric Devices:

We define an energy form for the energy consumed by EV's on-board electric devices, such as air-conditioners, windshield wipers, etc. This type of consumed energy is determined by the power drawn by the electric device, which is a static value provided by the vehicle maker and the time that the electric device is in use. This type of energy is defined by the following form:

$$C_{ED} = \sum_{i=1}^{n} (P_{ED(i)} t_i) * Status_{(i)} \tag{6}$$

where $Status_{(i)}$ has a value of 0 if the electric device i is off; otherwise, it has a value of 1; t_i is the time that the electric device takes in the status on; $P_{ED(i)}$ is the power drawn by the electric device i.

Total Energy Cost:

The complete form of the total energy cost function on the edge (a, b) is as follows:

$$\begin{aligned} C_E(a,b) = [&C_{AE}(a,b) + C_{LE}(a,b) + C_{PC}(a,b) \\ &+ C_{PG}(a,b) + C_{DE}(a,b)] * DS_{coeff} \end{aligned} \tag{7}$$

and the total energy cost of a path $P^k = (v_1, v_2, ..., v_k)$ is

$$C_E(P^k) = \sum_{j=1}^{j=k-1} C_E(v_j, v_{j+1}) \tag{8}$$

According to [14], one strategy for solving the problem of energy-optimal routing in EVs is to transform the weight function C_E into a positive reduced weight function C_Π. Assuming no negative cycles exist, it is proven that whenever a potential function Π satisfies the fact that $\Pi(b) - \Pi(a) \le C_E(a,b)$ and also C_Π is determined as $C_\Pi(a,b) = C_E(a,b) + \Pi(a) - \Pi(b)$, then the optimal routes in the weighted graph (V, E, C_Π) are also the optimal in the weighted graph (V, E, C_E).

Lemma 1: Π *implies a positive reduced weighted function* C_Π.

Proof.

$$\begin{aligned} C_\Pi &= C_E(a,b) + \Pi(a) - \Pi(b) \\ &= C_{AE}(a,b) + C_{LE}(a,b) + C_{DE}(a,b) \\ &\quad + \Pi(b) - \Pi(a) + \Pi(a) - \Pi(b) \\ &= C_{AE}(a,b) + C_{LE}(a,b) + C_{DE}(a,b) \ge 0 \end{aligned}$$

□

where energies during acceleration and deceleration phases on the road segment (a, b) take the following values:

$$C_{AE}(a,b) = \begin{cases} 0 & \text{if } k = 1 \\ \Delta^k & \text{if } k > 1, \quad \Delta^k > 0 \\ \Delta^k & \text{if } k > 1, \quad \Delta^k < 0 \\ \infty & \text{if } k > 1, \quad \Delta^k > J \end{cases}$$

$$C_{DE}(a,b) = \begin{cases} 0 & \text{if } k = 1 \\ -\Delta^k & \text{if } k > 1, \quad 0 > \Delta^k \le U \\ \Delta^k & \text{if } k > 1, \quad 0 > \Delta^k > U \\ -\Delta^k & \text{if } k > 1, \quad \Delta_\Pi \ge 0 \\ \Delta^k & \text{if } k > 1, \quad \Delta_\Pi < 0 \end{cases}$$

where Δ_Π is the amount of energy consumed or gained on the road segment (a, b) in the weighted graph (V, E, C_Π). The A^* algorithm for the energy mode, adopted from [16], is modified such that the energy-optimal path is determined in the weighted graph (V, E, C_Π). The battery constraints for the optimal path, however, are dynamically adjusted and resolved based on the energy costs in the weighted graph (V, E, C_E).

1) Energy Mode Heuristic Function: The air-line distance and the minimum speed over all speed limits are used. Let us define two vertices, u and v, and a destination, t. Obviously, the air-line distance l' is a consistent heuristic for a road length.

$$l'(u,t) \le l(u,v) + l'(v,t) \tag{9}$$

Therefore, the following heuristics are defined:

$$h_L(u,t) = f_r mgl'(u,t) + \frac{1}{2}\rho A c_w S_{min}^2 l'(u,t) \tag{10}$$

$$h_A(u,t) = T\frac{S_{min}}{r} g_r t_A \tag{11}$$

$$h_D(u,t) = T\frac{S_{min}}{r} g_r t_D \tag{12}$$

Thus,

$$h(u,t) = h_L(u,t) + h_A(u,t) + h_D(u,t) \le C_\Pi(u,t) \tag{13}$$

Lemma 2: The heuristic $h(u,t)$ *is consistent in the weighted graph* (V, E, C_Π).

Proof. Since $h(u,t)$ is linearly increasing in l, $h(u,t) \le h(u,v) + h(v,t)$. As $h(u,t)$ is monotonic in S and l, $h(u,v) \le C(u,v)$ for all u and v for which $C(u,v)$ is defined, and therefore,

$$h(u,t) \le C(u,v) + h(v,t)$$

$$\begin{aligned} V(u,v) = &f_r mg + \frac{1}{2}\rho A c_w S^2(u,v) \\ &+ T\frac{S(u,v)}{r} g_r t_A + T\frac{S(u,v)}{r} g_r t_D \end{aligned}$$

$$\begin{aligned} V' = &f_r mg + \frac{1}{2}\rho A c_w S_{min}^2 \\ &+ T\frac{S_{min}}{r} g_r t_A + T\frac{S_{min}}{r} g_r t_D \end{aligned}$$

Since S_{min} is a lower bound of $S_{(u,v)}$, $V^{'}$ is also a lower bound of $V(u,v)$ and therefore consistency follows down from

$$h(u,t) \leq V(u,v)l(u,v) + V^{'}l^{'}(u,v)$$

$$h(u,t) \leq C(u,v) + h(v,t)$$

□

B. Time Mode

In this mode, the A^* algorithm, adopted from [16], is used to compute the most efficient path in terms of travel-time cost among all possible paths. During the search for the most efficient travel-time path, the battery constraints are taken into account. Even when drivers care only about time in their traveling, any path with an energy cost that is greater than the battery charge level is excluded from the search process by turning its travel-time cost value into infinity. The travel-time cost, adopted from [15], includes time and safety costs as shown in Equation 14.

$$C_T = \frac{t}{S(tr)} + b(tr)S^2 \tag{14}$$

According to Jan Rouwendal [15], $b(tr)$ must be increased in traffic density, and it can be determined as

$$b(tr) = \frac{t}{2S^3(tr)} \tag{15}$$

By substituting Equation 15 into Equation 14, the travel-time cost function of a road segment (a,b) takes the following form:

$$C_T(a,b) = \frac{3t}{2S(tr)} \tag{16}$$

Equation 16 states that when accident risk is involved in the travelling, the true value of travel-time cost is about 50 percent higher than that of time cost [15]. Thus, the travel-time cost function of a path $P^k = (v_1, v_2, ..., v_k)$ is

$$C_T(P^k) = \sum_{j=1}^{j=k-1} C_T(v_j, v_{j+1}) \tag{17}$$

1) Time Mode Heuristic Function: The air-line distance and maximum speed over all speed limits are used. Let us define two vertices u and v and a destination t. The air-line distance $l^{'}$ is a consistent heuristic for a road length, $l^{'}(u,t) \leq l(u,v) + l^{'}(v,t)$. Therefore, the travel-time heuristic function is

$$h_t(u,t) = \frac{3}{2}\frac{l^{'}(u,t)}{S^2_{max}} \leq C_t(u,t) = \frac{3}{2}\frac{l(u,t)}{S^2(u,t)} \tag{18}$$

Lemma 3: The heuristic $h_t(u,t)$ is consistent in the weighted graph (V,E,C_T).

Proof. Since $h_t(u,t)$ is linearly increasing in l, $h_t(u,t) \leq h_t(u,v) + h_t(v,t)$. As $h_t(u,t)$ is monotonic in l and S, then

$$h_t(u,t) \leq C_t(u,v) + h_t(v,t)$$

$$V(u,v) = \frac{3}{2S^2_{u,v}} \quad \text{and} \quad V^{'} = \frac{3}{2S^2_{max}}$$

Since S_{max} is a higher bound of $S(u,v)$, $V^{'}$ is a lower bound of $V(u,v)$ and therefore,

$$h_t(u,t) \leq V(u,v)l(u,v) + V^{'}l^{'}(v,t)$$
$$h_t(u,t) \leq C_t(u,v) + h_t(v,t)$$

□

III. Simulation Work

This section reports simulation work performed to test and validate the proposed multi-criteria routing model. The model can operate in either of two modes, namely, time mode, to compute the travel-time-optimal path from source to destination, or energy mode, to compute the energy-optimal path from source to destination.

A. Energy Mode Results

The computed optimal path here may be longer in travel distance and in travel time, but optimal in energy. Figure 1 depicts the constructed road network with the source node marked in red, the destination node marked in blue, and the energy-optimal path, computed by the technique, marked in green. To make certain that the computed paths are optimal, we verify the optimality conditions of the A^* search algorithm stated in [13]. The results are provided in two tables (Tables I and II) in which the admissibility and consistency conditions along the computed path are satisfied. Table I illustrates the admissibility condition satisfaction. The fourth column of Table I shows the values of $f(n)$, which is equal to $g(n) + h(n)$, where $g(n)$ is the real cost and $h(n)$ is the estimated cost (i.e., heuristic). The fifth column demonstrates the total path cost, which is greater than the values of $f(n)$, proving that heuristic $h(n)$ is admissible. Table II proves that heuristic $h(n)$ is consistent along the path by satisfying the consistency condition stated in Equation 19. In Table II, $h(n,t)$ is the estimated energy cost to arrive at the destination from node n, $c(n,v)$ is the real energy cost from node n to node v, the successor of n, and $h(v,t)$ is the estimated energy cost to arrive at the destination from the successor node v.

$$h(n,t) \leq c(n,v) + h(n,v) \tag{19}$$

Figure 2 illustrates the battery charge level and the effects of negative energy costs on the battery charge along the optimal path in the weighted graph (V,E,C_E). It can be seen from the graph that during this run, the first road segment of the travel path has a negative energy cost value. Due to the dynamic adjustment of the travel energy cost, the negative energy cost

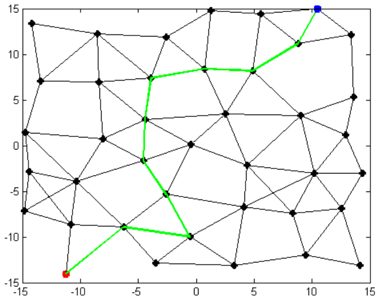

Fig. 1: Optimal Path Energy-based Mode

TABLE I: Admissibility of Energy-optimal Path in kwh

Path Nodes	$g(n)$	$h(n)$	$f(n)$	Total Path Cost
18	0	0.1348	0.1348	0.2699
2	0.0373	0.1086	0.1460	0.2699
10	0.0667	0.1018	0.1686	0.2699
27	0.0874	0.0902	0.1776	0.2699
35	0.1067	0.0837	0.1905	0.2699
36	0.1347	0.0719	0.2066	0.2699
13	0.1561	0.0611	0.2173	0.2699
26	0.1889	0.0446	0.2336	0.2699
6	0.2112	0.0336	0.2448	0.2699
39	0.2374	0.0165	0.2539	0.2699
7	0.2699	-0.0016	0.2682	0.2699

incurred by taking the first road segment was lost since the battery was fully charged when the EV first started traveling. All the other negative energy costs incurred by taking this path were stored in the battery until the destination was reached. The total path energy cost in the weighted graph (V, E, C_E) is depicted in Figure 3. The total path energy cost here refers to the total energy that resulted from taking the optimal path and not that consumed from the battery.

B. Time Mode Results

The computed path here may be more energy consuming and longer in travel distance but optimal in travel time. Although this mode is used by drivers when the concern is travel time and not energy, the battery constraints are considered and not violated. Figure 4 depicts the constructed road network with the source node marked in red, the destination node marked in blue, and the travel-time-optimal path, computed by the technique, marked in green. The results of this operation mode are provided in two tables (Tables III and IV) in which the admissibility and consistency conditions along the computed path are satisfied. Following the same argument as in Section III-A, Tables III and IV prove the satisfaction of the admissibility and consistency conditions along the computed path, respectively. Figure 5 depicts the battery charge level and the effects of negative energy costs on it along the computed path. As can be seen from the graph, the negative energy costs incurred by taking this travel-time-optimal path were stored in the battery due to the adjustment of battery constraints. Figure 6 illustrates the total energy cost of the travel-time-optimal path. Although this path is optimal in travel-time cost, it is not necessarily an optimal path in energy cost. The total energy cost here is the total energy incurred by taking the travel-time-optimal path and not that consumed from the battery. This path was verified by the proposed model to be feasible in the sense that its total energy cost is less than the battery charge level, so the path can be used by the EV.

TABLE II: Consistency of Energy-optimal Path in kwh

$h(n,t)$	$c(n,v)$	$h(v,t)$
0.1348	0.0373	0.1086
0.1086	0.0294	0.1018
0.1018	0.0206	0.0902
0.0902	0.0193	0.0837
0.0837	0.0280	0.0719
0.0719	0.0214	0.0611
0.0611	0.0327	0.0446
0.0446	0.0223	0.0336
0.0336	0.0261	0.0165
0.0165	0.0324	-0.0016
-0.0016	0	0

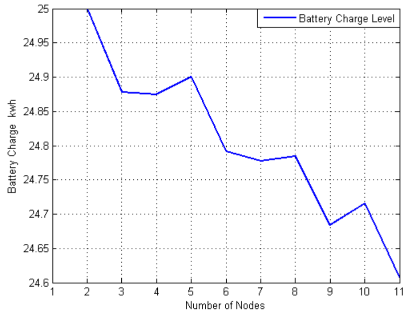

Fig. 2: Battery Charge Level along the Energy-optimal Path

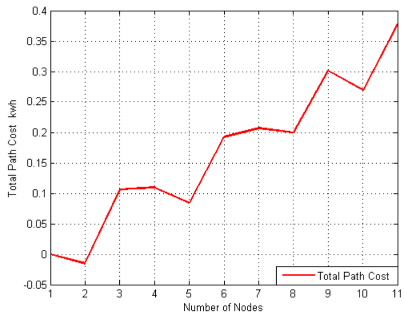

Fig. 3: Total Path Energy Cost in (V, E, C_E)

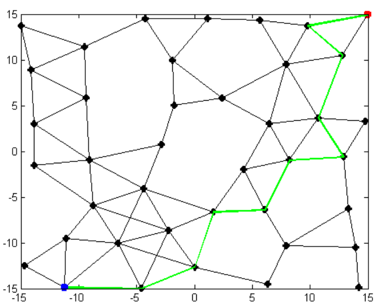

Fig. 4: Optimal Path Travel-time-based Mode

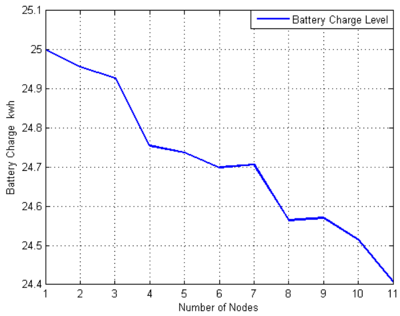

Fig. 5: Battery Charge Level along the Travel-time-optimal Path

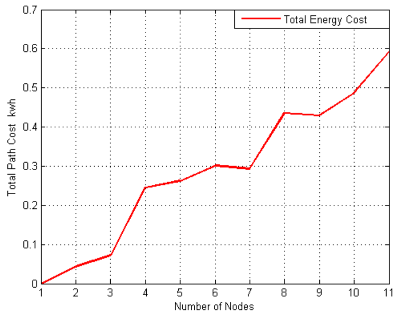

Fig. 6: Total Energy Cost of the Travel-time-optimal Path

TABLE III: Admissibility of Travel-time-optimal Path in h^2/km

Path Nodes	$g(n)$	$h(n)$	$f(n)$	Total Path Cost
39	0	0.0165	0.0165	0.0615
6	0.0048	0.0147	0.0196	0.0615
20	0.0090	0.0145	0.0235	0.0615
29	0.0169	0.0119	0.0288	0.0615
11	0.0205	0.0116	0.0321	0.0615
35	0.0258	0.0099	0.0358	0.0615
25	0.0328	0.0080	0.0409	0.0615
1	0.0382	0.0063	0.0445	0.0615
10	0.0467	0.0047	0.0514	0.0615
40	0.0543	0.0027	0.0571	0.0615
21	0.0615	0	0.0615	0.0615

TABLE IV: Consistency of Travel-time-optimal Path in h^2/km

$h(n,t)$	$c(n,v)$	$h(v,t)$
0.0165	0.0048	0.0147
0.0147	0.0041	0.0145
0.0145	0.0078	0.0119
0.0119	0.0036	0.0116
0.0116	0.0053	0.0099
0.0099	0.0070	0.0080
0.0080	0.0053	0.0063
0.0063	0.0084	0.0047
0.0047	0.0076	0.0027
0.0027	0.0071	0
0	0	0

IV. Conclusions

Optimal energy/time-based routing for EVs under constrained batteries will become significantly important in the near future since the global trend now is to introduce the technology of EVs as a strategy to help reduce greenhouse gas emissions. This research has formalized the problem of optimal energy/time routing in EVs within a framework of a multi-criteria routing technique, as a solution to finding optimal energy/time routes. The routing technique relies on using the A^* search algorithm and can be run on two modes based on driver needs: an energy mode to compute the optimal path in energy among all possible paths or a time mode to compute the optimal path in travel time among all possible paths. The battery constraints were taken into account and resolved in both modes of operation. The results reported in this research prove that the computed paths are optimal by satisfying the optimality conditions of the A^* algorithm. Further research should be conducted for designing a routing technique that considers the battery constraints and minimizes the tradeoff between the optimality of energy and travel time in EVs.

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