



ECE 457A ADAPTIVE COOPERATIVE ALGORITHMS



LOCAL SEARCH Definitions and Strategies

Recall

- Which of the following are examples of ill-structured problems?
 - ☐ Converting temperature from Celsius to Fahrenheit
 - ☐ Increasing water supply for a growing community
 - ☐ Calculating speed given distance and time
 - ☐ Designing a new underground subway system that spans multiple countries
 - ☐ Maximizing the efficiency of a manufacturing process
 - ☐ Scheduling a NHL season to maximize viewing
- Which of the following aspects are **not** well defined in an ill-structured problem?
 - ☐ Goals
 - ☐ Beginning state
 - ☐ Actions
 - ☐ End state
 - ☐ Constraints

SEARCH TREE - TERMINOLOGY

We search for the best state possible:

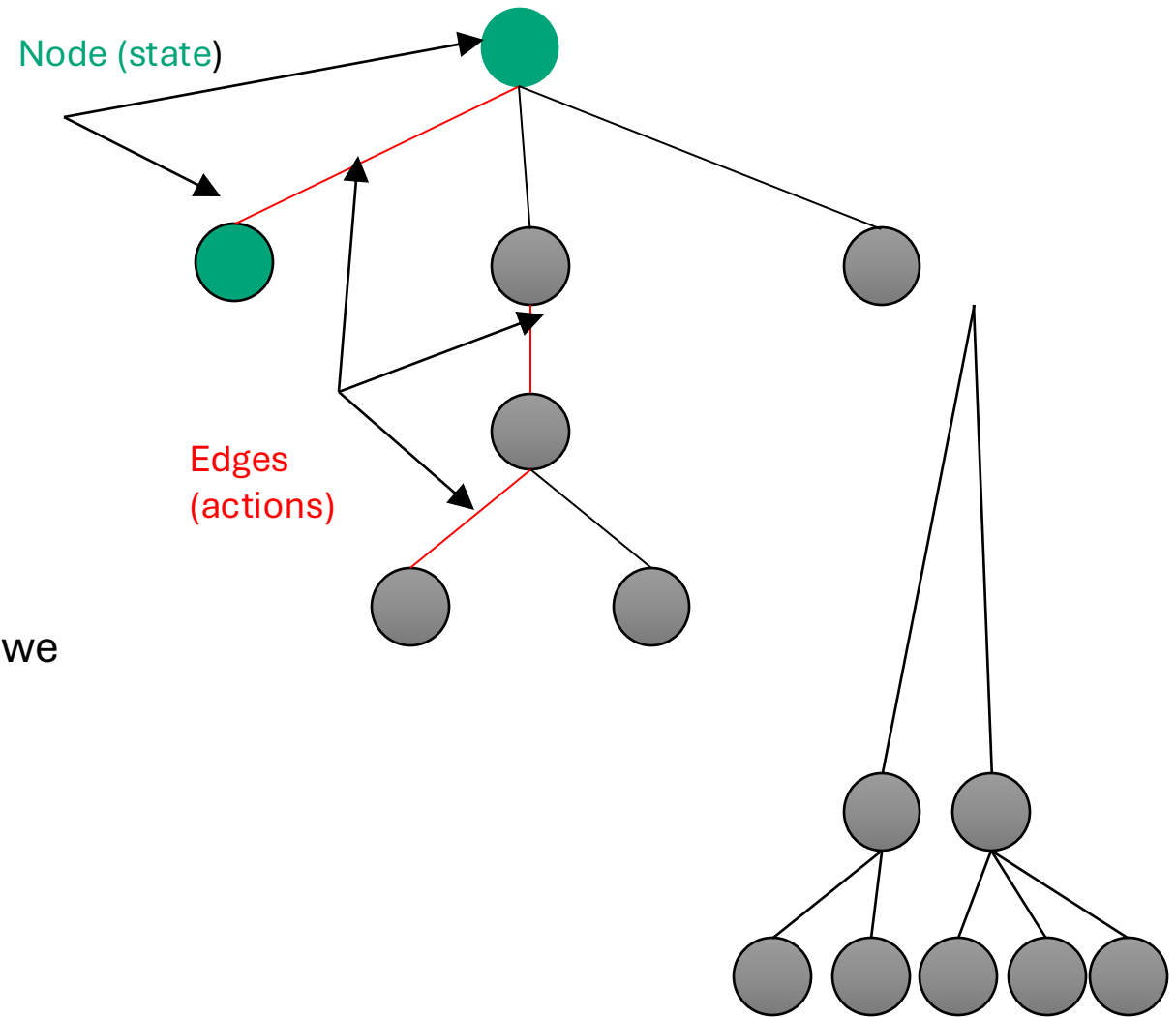
Problem graph representation

Node=State

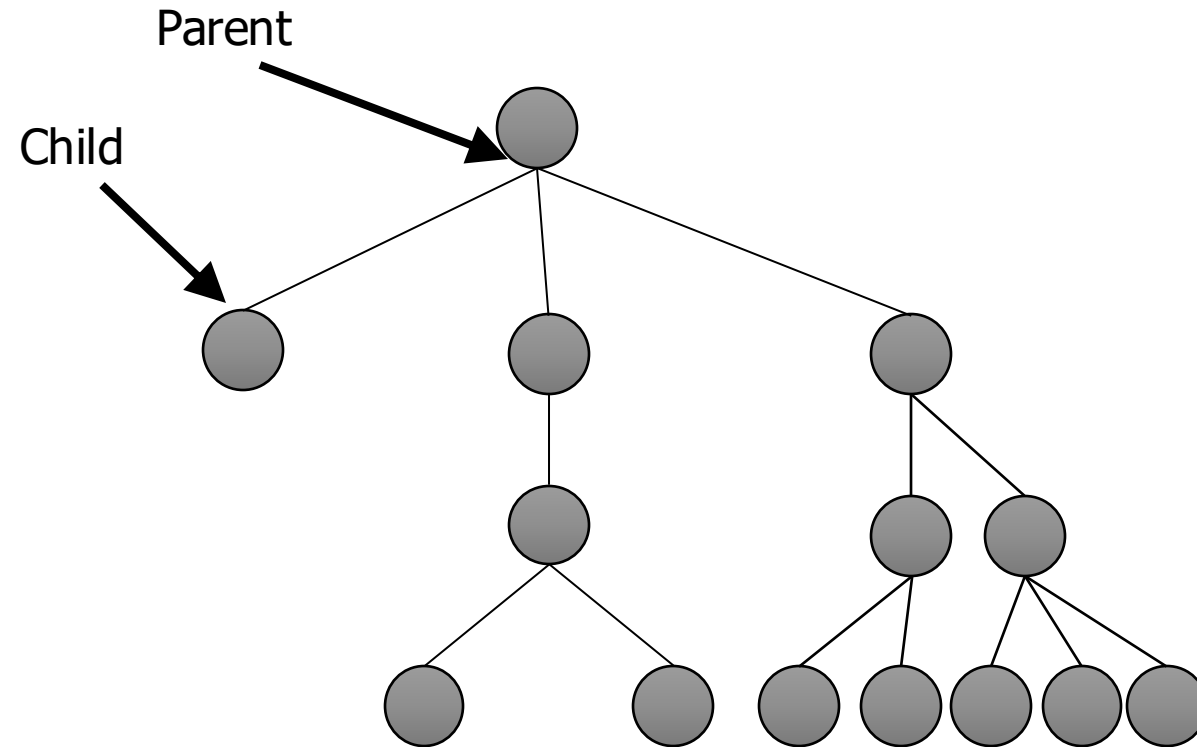
- Optimal state: nothing better.
- Suboptimal state: Good enough
- When at a state, we evaluate its merit wrt to the goal we are searching for: as a goal state itself, or as a step forward towards our goal.

Edge=Action

- An Edge tells us two things:
 1. The states that we can reach/materialize in one move from the state we are at- by taking the action associated with edge.
 2. A labeled edge signifies a cost/gain associated with the action. No label implies unit cost/gain.

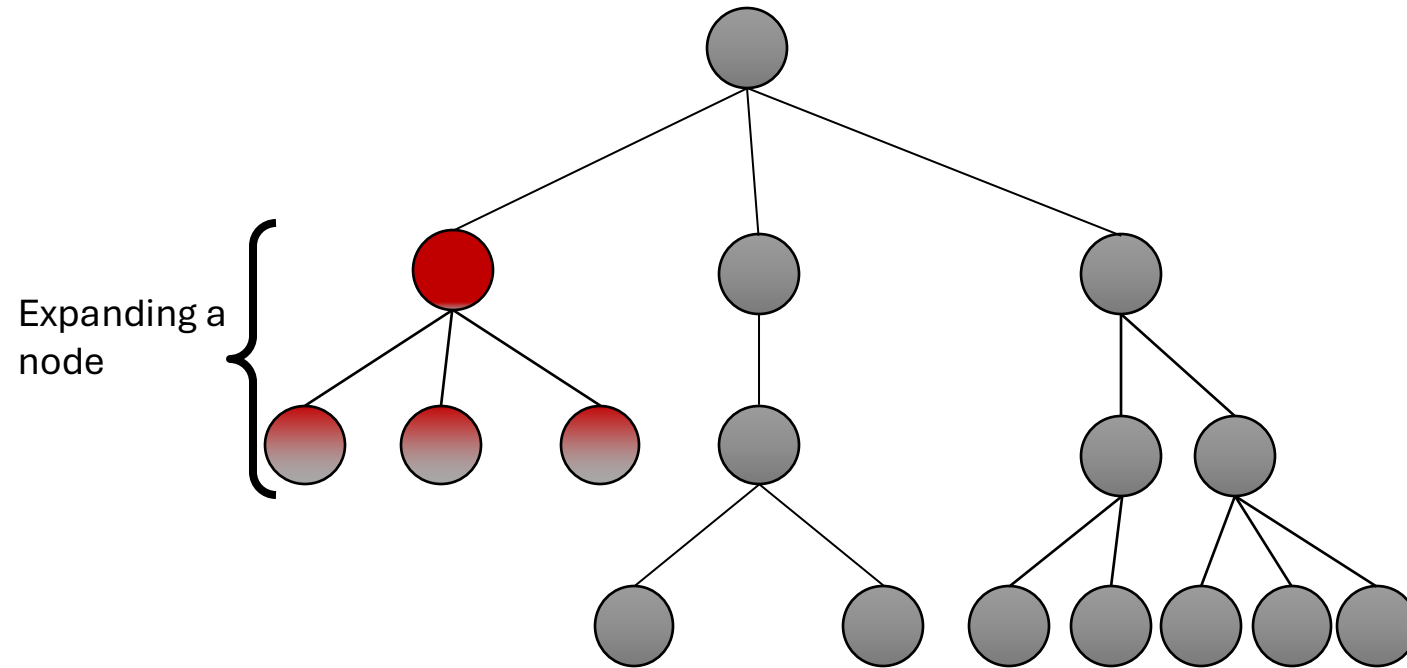


SEARCH TREE - TERMINOLOGY

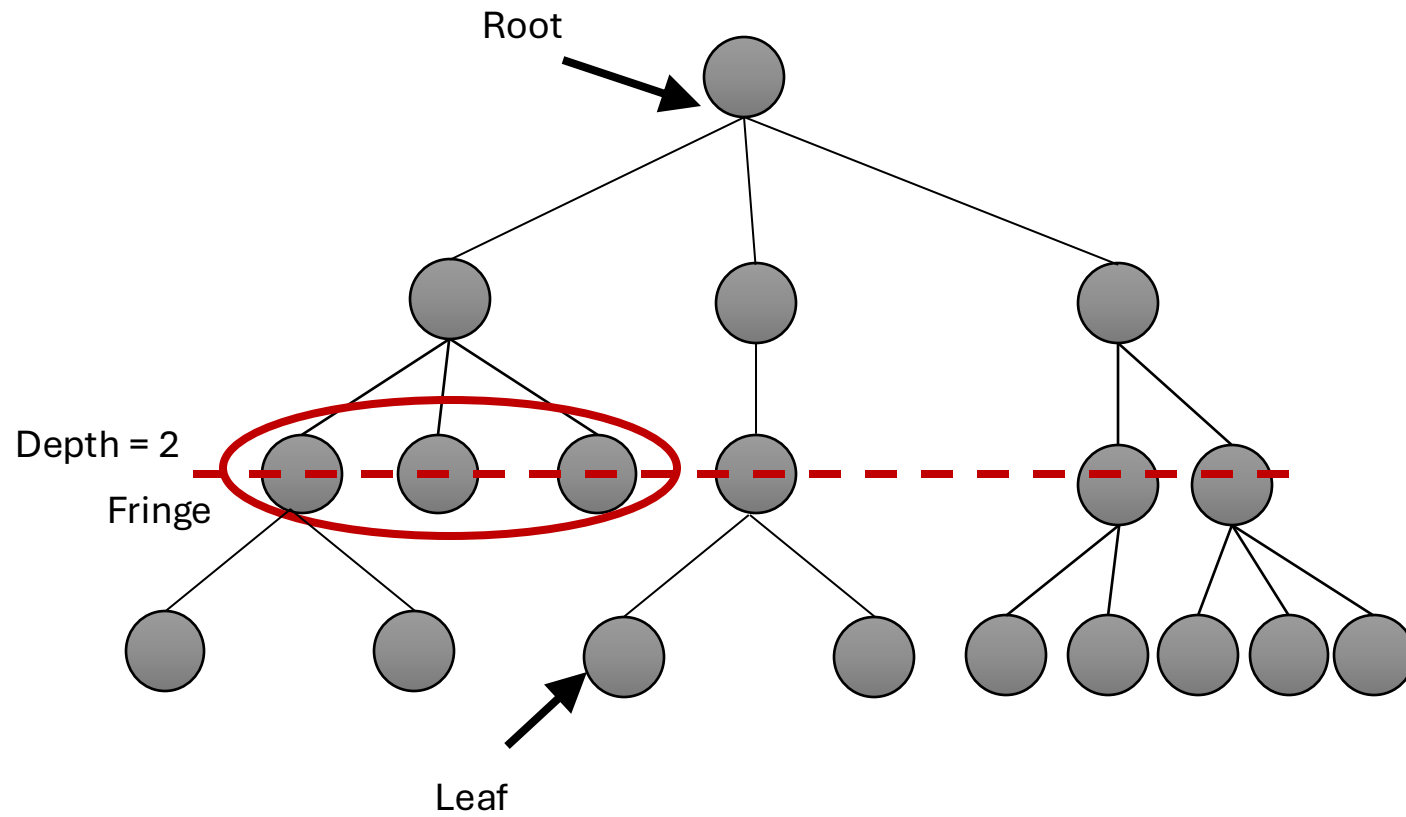


SEARCH TREE - TERMINOLOGY

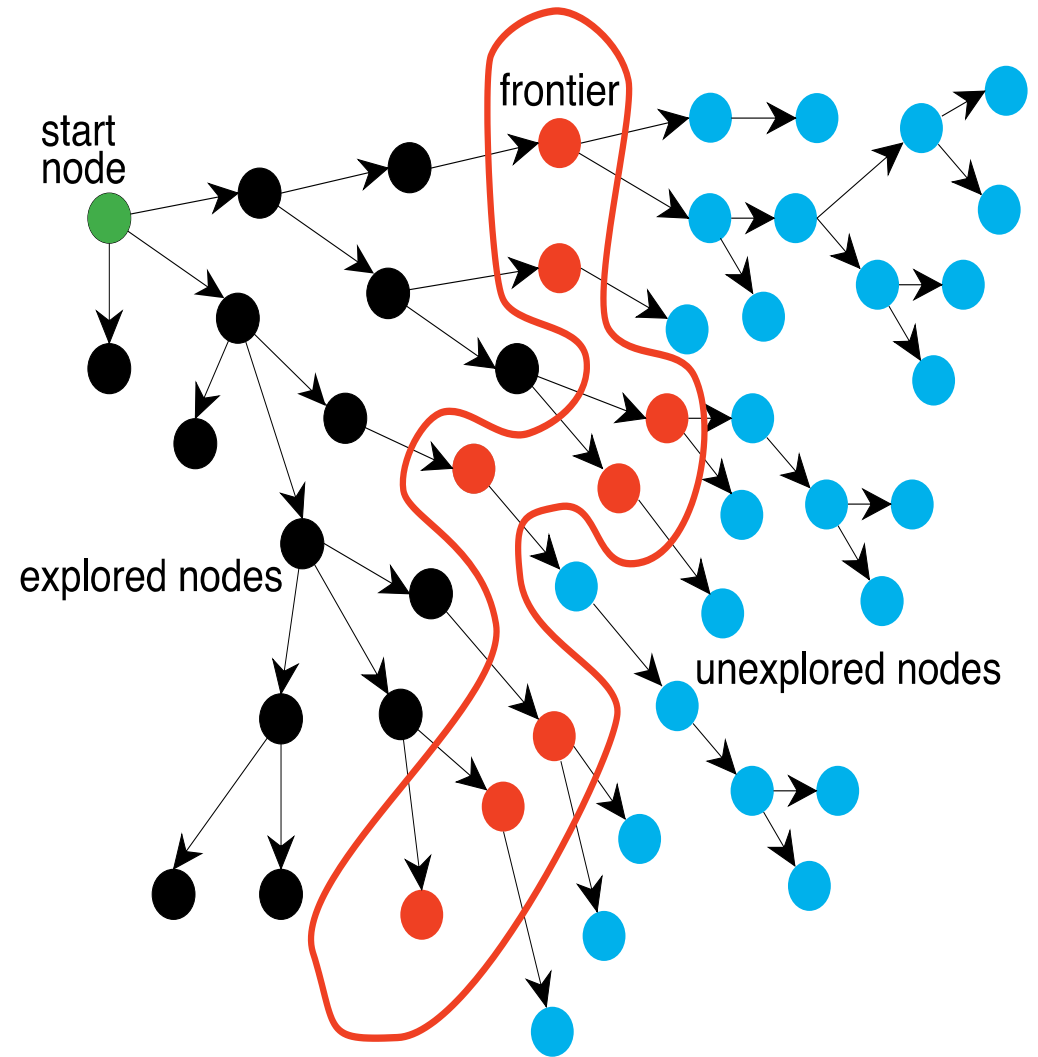
We perform our search as a series of node (state) expansions



SEARCH TREE - TERMINOLOGY



States: ARE EITHER
EXPLORED, AT THE
FRONTIER, OR
UNEXPLORED



LEAVES

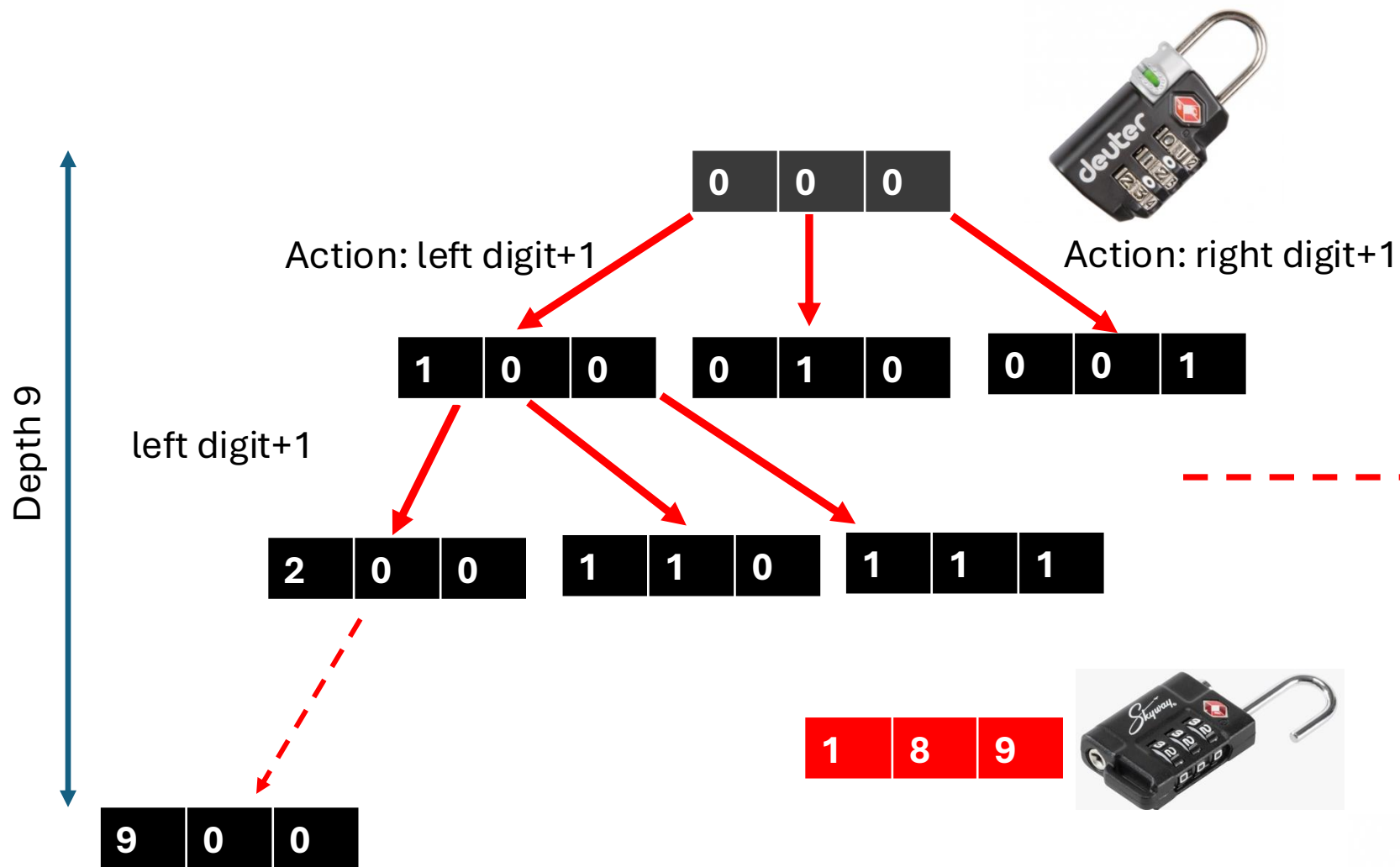
VS

FRINGE

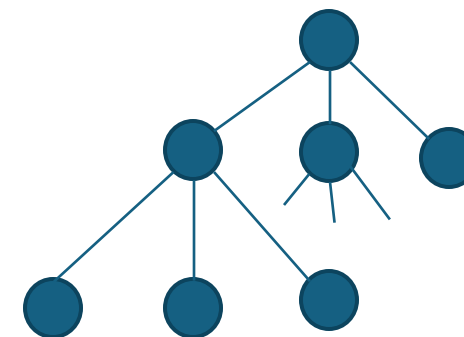


leaves

SIMPLE EXAMPLE



Branching factor:3
Depth: 9
Search space:



The Locker unlocking problem.



Soln Search



Search strategy performance attributes

Complete search strategy

Definition (complete)

If a solution exists, a **complete** algorithm is guaranteed to find a solution within a finite amount of time.

Optimal search strategy

Definition (optimal)

If a solution exists and an algorithm finds a solution, then the first solution found by an **optimal** algorithm is the solution with the lowest cost.

SEARCH STRATEGY PERFORMANCE ATTRIBUTES

Memory requirement growth

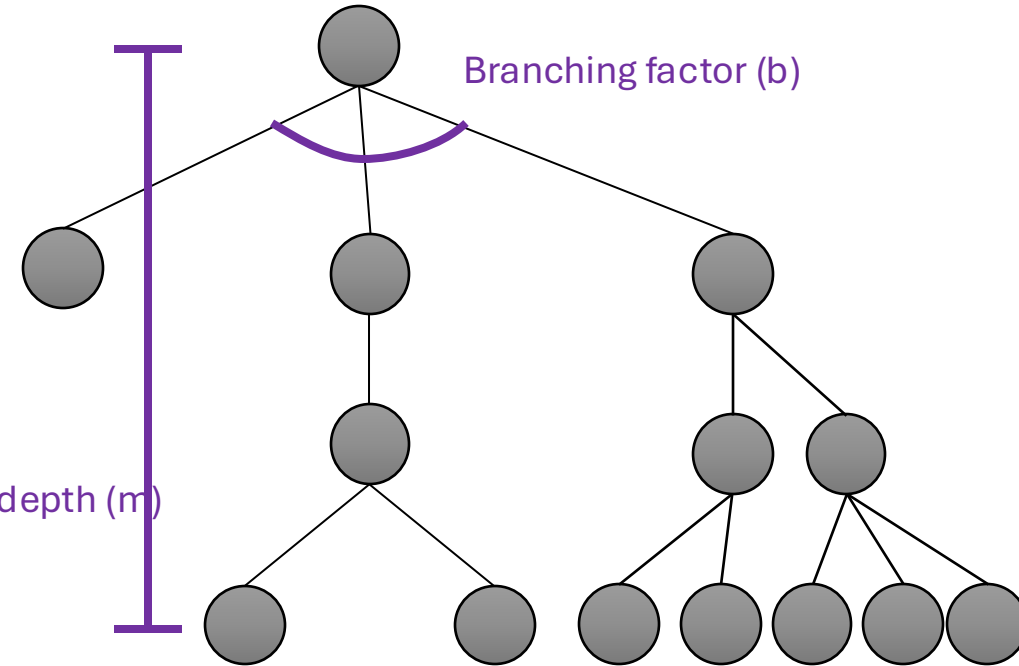
Definition (space complexity)

The **space complexity** of a search algorithm is an expression for the worst-case amount of memory that the algorithm will use, expressed in terms of b , d , and m .

Useful definitions:

- ▶ b : the maximum branching factor (may be infinite).
- ▶ d : the depth of the shallowest goal node (finite).
- ▶ m : the maximum path length (may be infinite).

Maximum depth (m)



Time requirement growth

Definition (time complexity)

The **time complexity** of a search algorithm is an expression for the worst-case amount of time it will take to run, expressed in terms of b , d , and m .



Local Search strategies

How they operate and cost implications.

Local Search Strategies

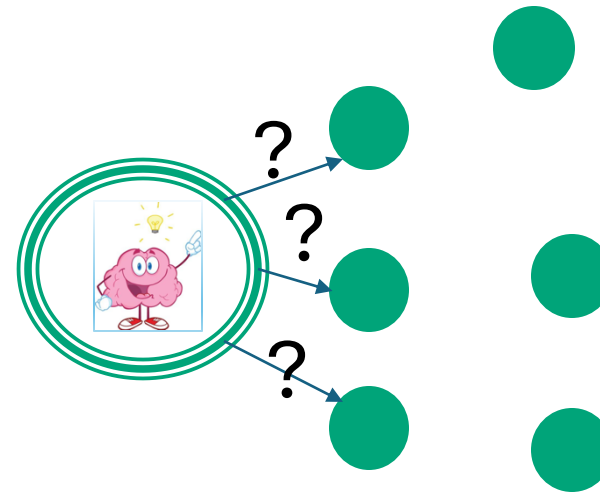
```
graph TD; A[Local Search Strategies] --> B[Uninformed Strategies]; A --> C[Informed Strategies];
```

Uninformed Strategies

Informed Strategies

Types of Local Search strategies

Template of Generic Search



Given a graph representation of the problem, we construct a search tree,

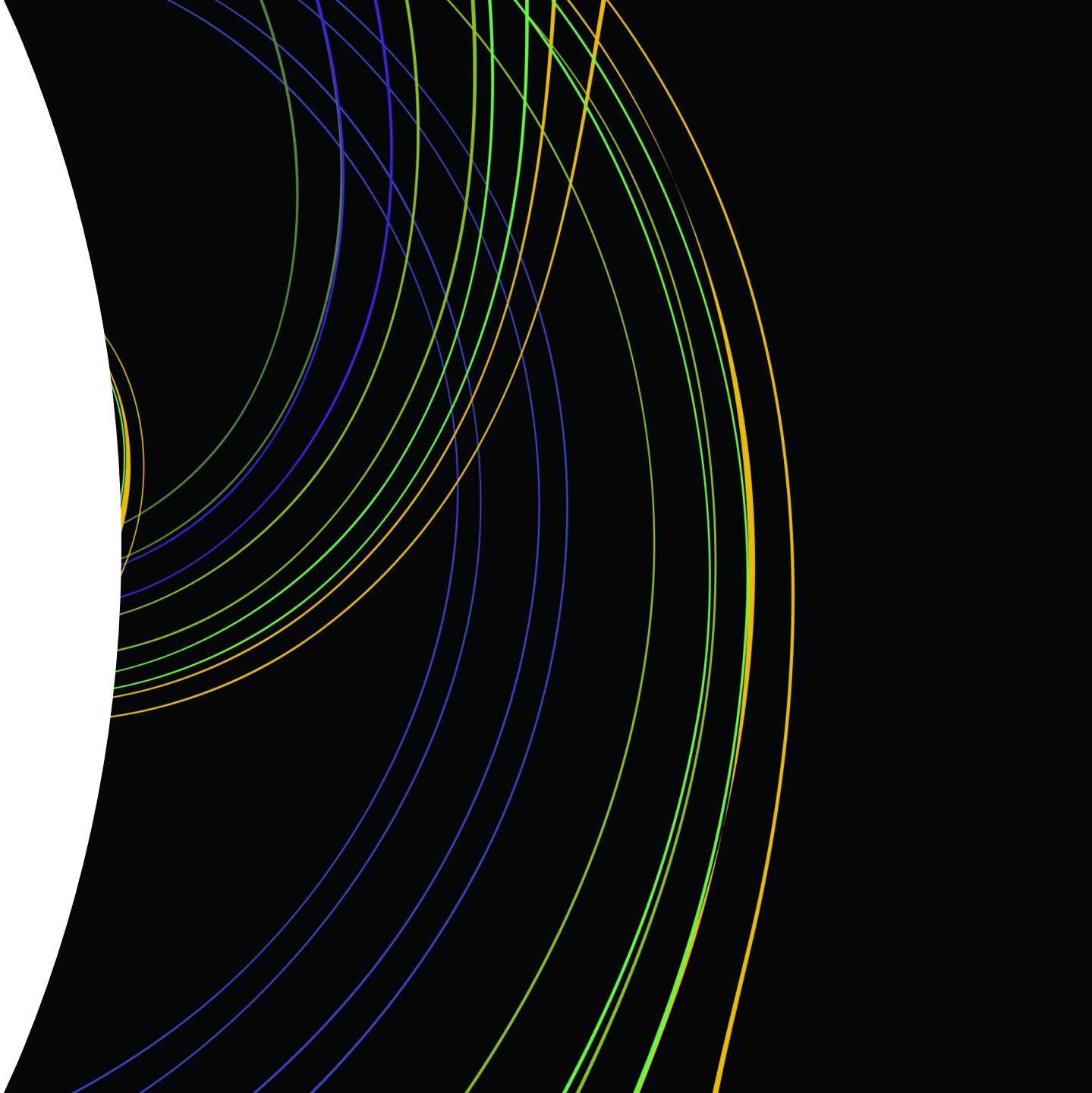
A generic search on the solution tree is a repetition of **choose a state, test the state, and expand the state**.

A particular search strategy influences how we choose the **next node (state) to consider** in the search! (this is where search strategies differ).

Generally, a **queue** structure is used to store nodes on the **fringe** to be expanded.

Different search strategies use different data structures.

Uniformed/blind Search



Uninformed/blind Search Strategies

- Also known as “**blind search**,” uninformed search strategies use no information about the likely “direction” of the goal node(s)
- Only has the information provided by the problem formulation (initial state, set of actions, goal test, cost)
- Uninformed search methods:
 - breadth-first,
 - depth-first,
 - depth-limited,
 - uniform-cost,
 - depth-first iterative deepening,
 - bidirectional

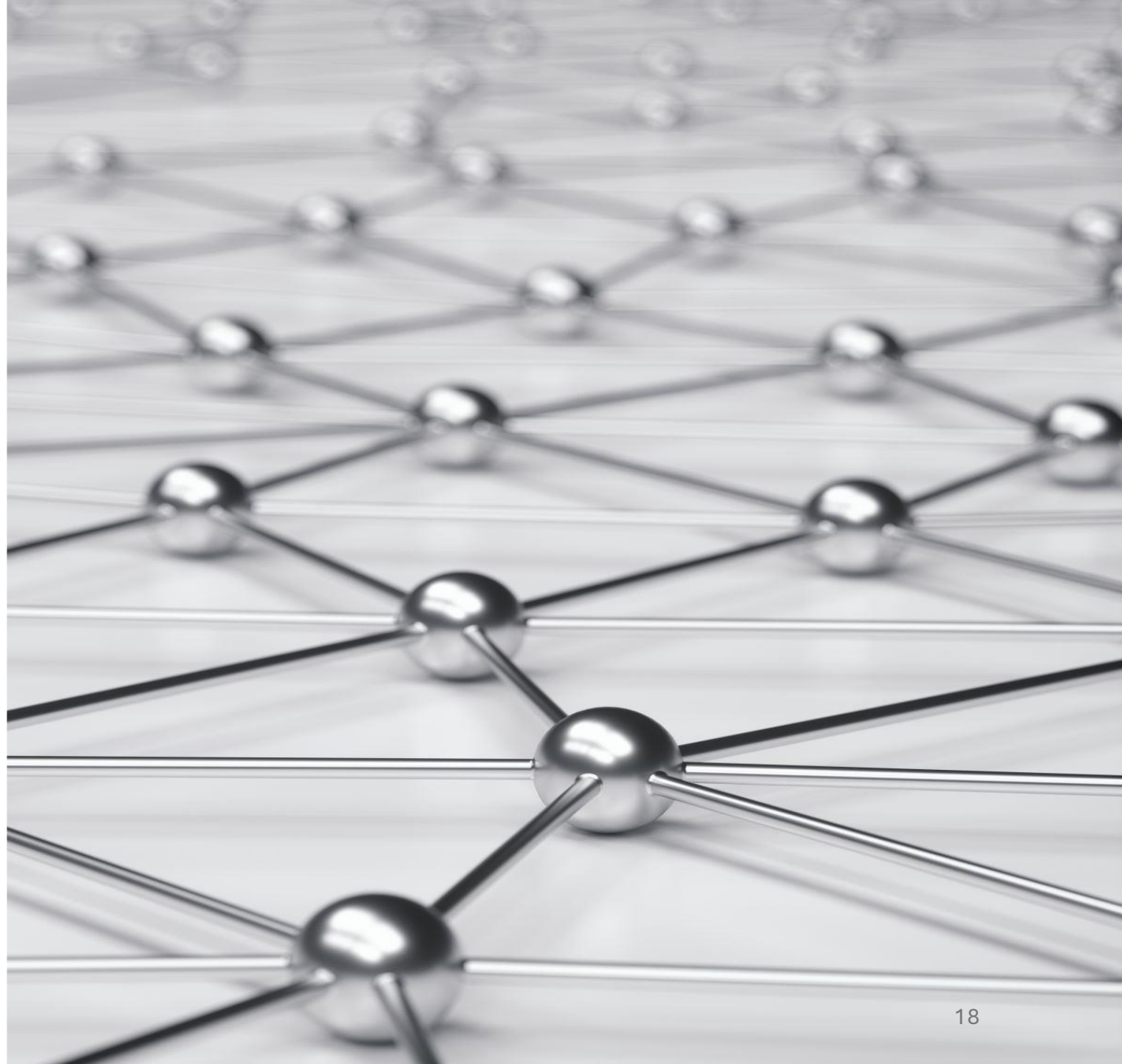
RECAL: Types of Search

• Informed Search

- Also known as “heuristic search,” informed search strategies use information about the domain to head in the general direction of the goal node(s)
- Has additional information that allows it to judge the promise of an action, i.e. the estimated cost from a state to a goal
- Informed search methods:
 - Hill climbing,
 - best-first,
 - greedy search,
 - beam search, A, A*

Breadth-First Search

Uninformed search



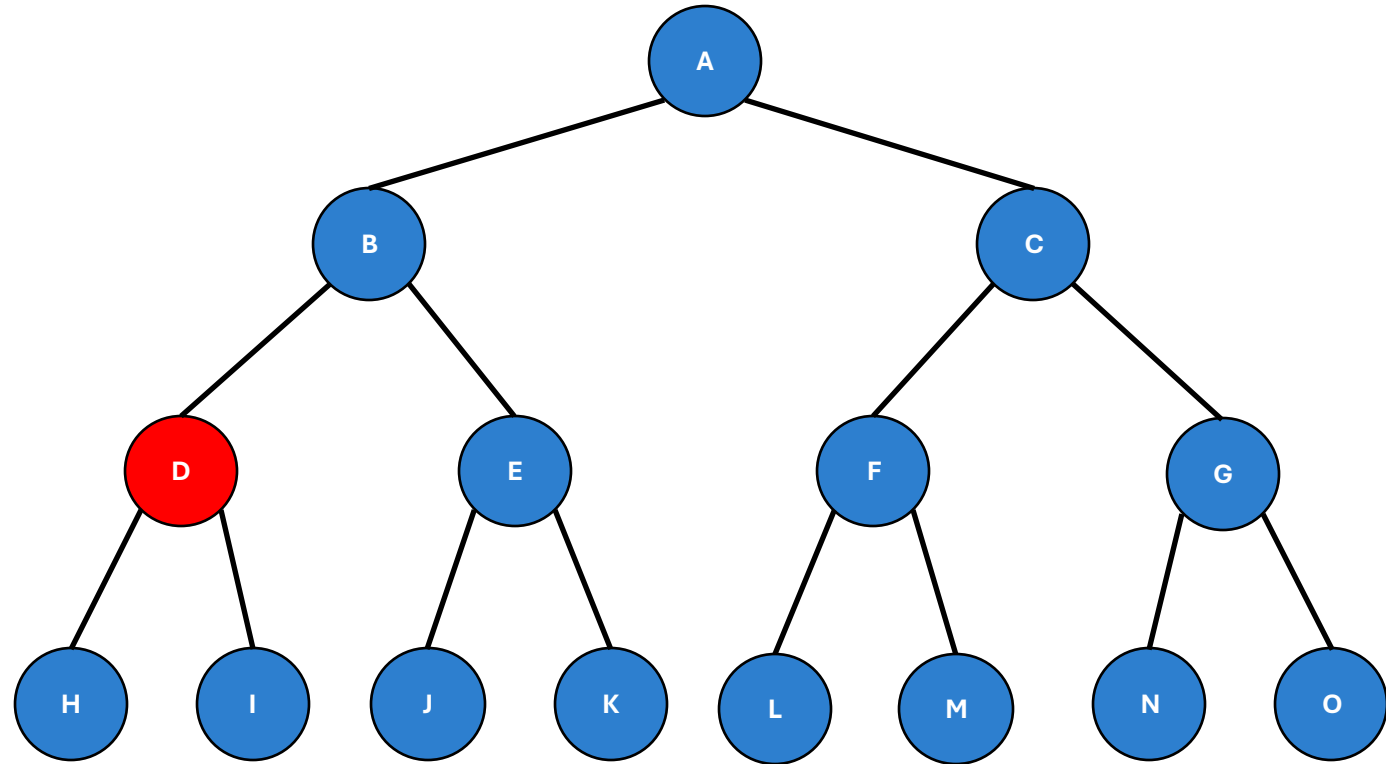
Breadth-First Search

Problem: Search for state D

Search Strategy: Breadth-first search

Search through a tree one level at a time.

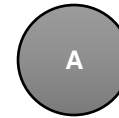
- We traverse through **one entire level of children nodes first**, before moving on to traverse through the grandchildren nodes.
- And we traverse through an **entire level of grandchildren nodes** before going on to traverse through **great-grandchildren** nodes.



Breadth-First Search

- Mechanics:
- Expand the ***shallowest unexpanded node first***
- Fringe: nodes waiting in the queue to be expanded
 - Fringe is a ***FIFO*** queue (first-in, first-out)
 - New successors go in the end of the queue

Initial State

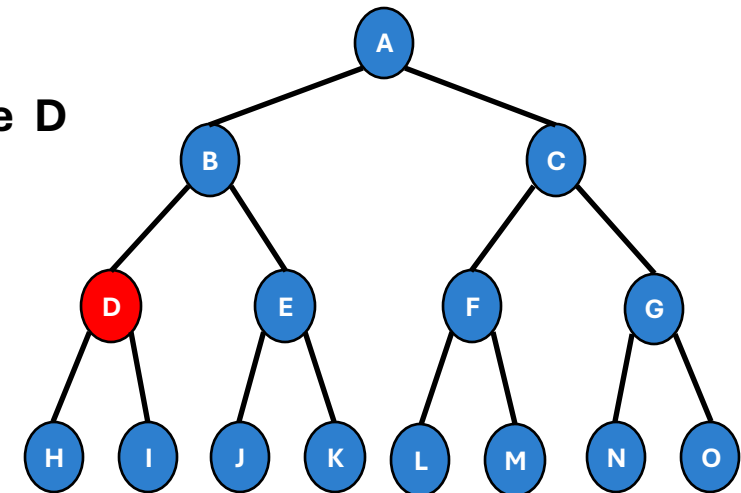


<< Is this the goal state?

Problem: Search for state D



Fringe queue

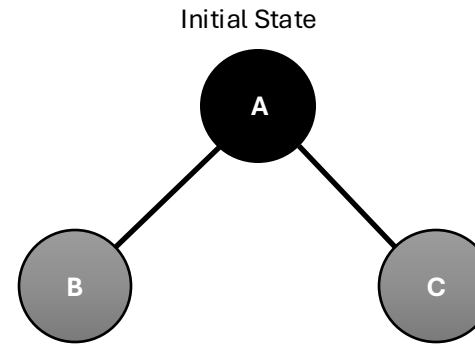


Breadth-First Search

Breadth First strategy:

- Expand the shallowest unexpanded node first
- New successors go in the end of the queue

Is this the goal state? >>

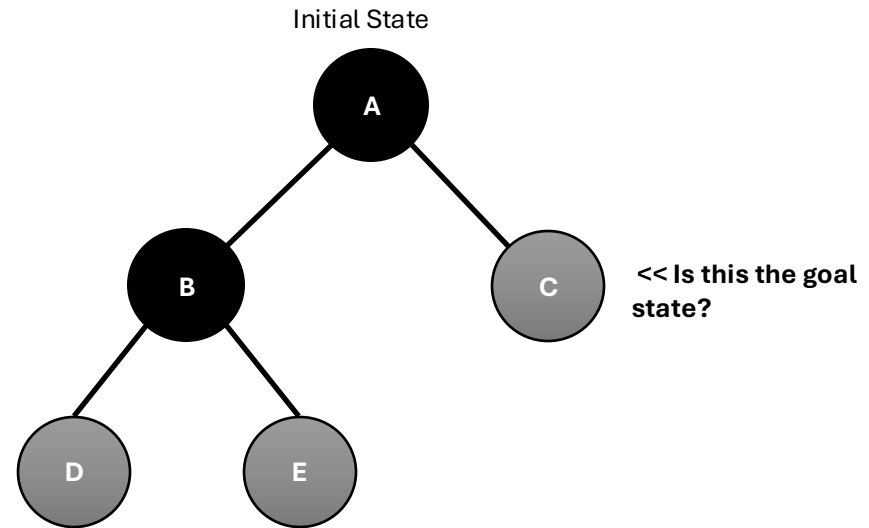
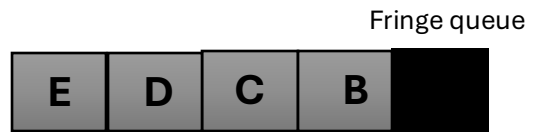


Fringe queue

Breadth-First Search

Breadth First strategy:

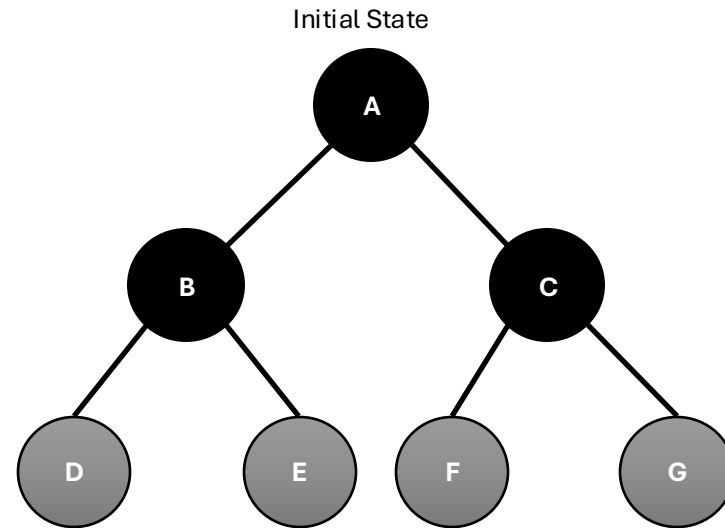
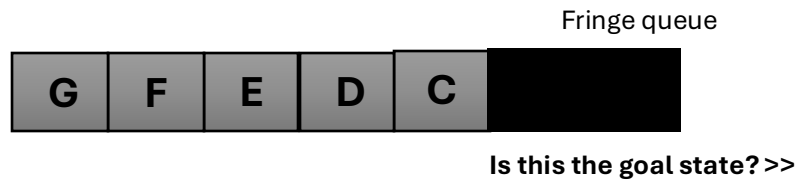
- Expand the shallowest unexpanded node first
- New successors go in the end of the queue

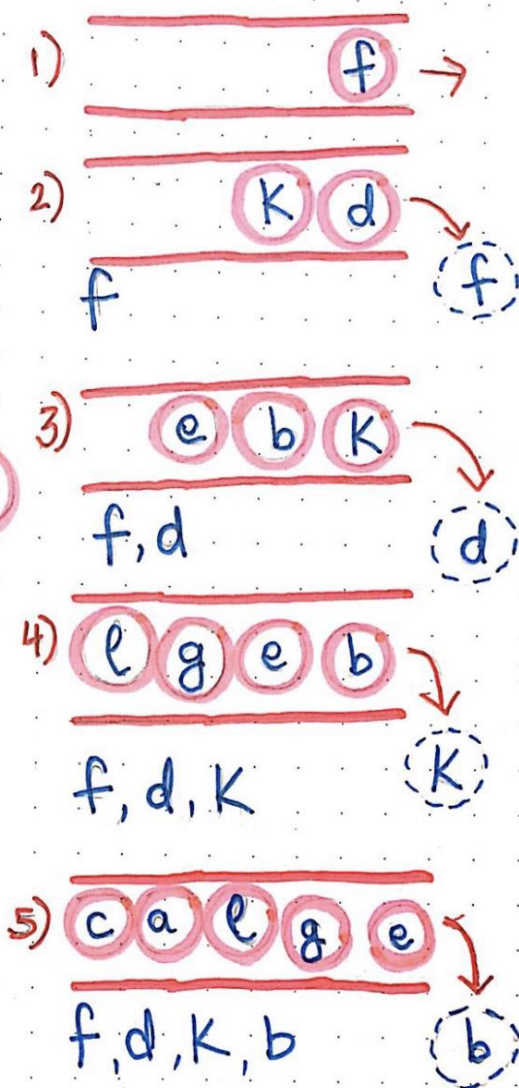
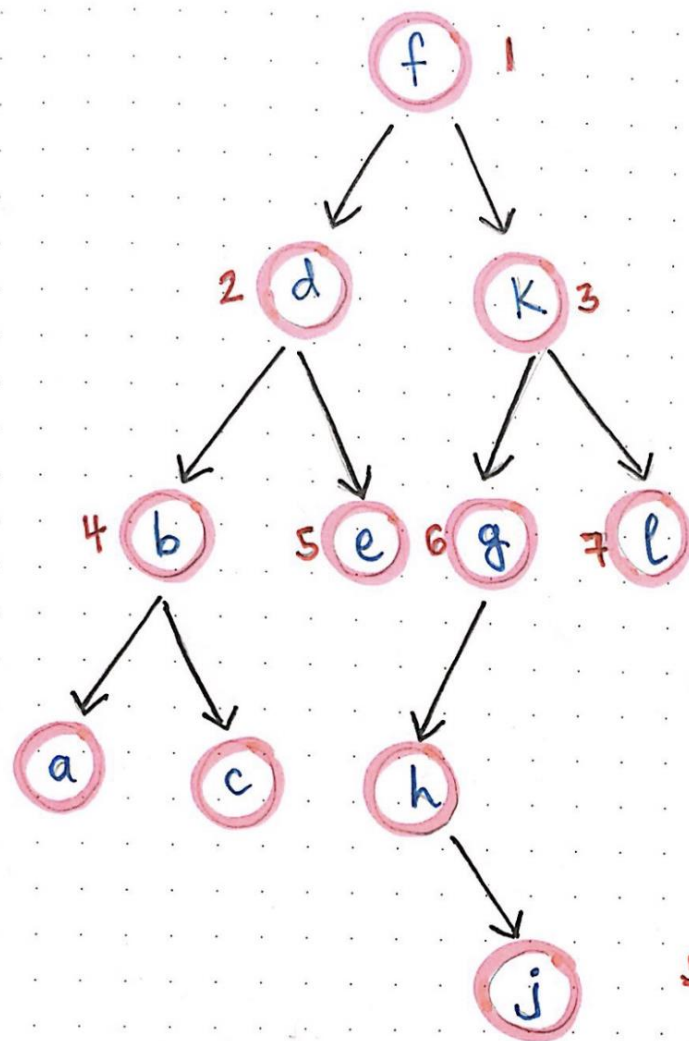


Breadth-First Search

Breadth First strategy:

- Expand the shallowest unexpanded node first
- New successors go in the end of the queue





BREADTH-FIRST-SEARCH

Algorithm 2 Breadth-First Search

```
1: put the start state in the frontier
2: while frontier is not empty do
3:   remove the oldest node added to the frontier
4:   if the node contains a goal state then
5:     return the solution
6:   end if
7:   generate all the successors of the node
8:   add every successor of the node to the frontier
9: end while
10: return failure
```

Breadth-First Search

- Upper-bound case: goal is last node of depth d
 - Number of generated nodes:
 - Space & time complexity: all generated nodes

We generate

at $d=0$) $b^0 = 1 +$

at $d=1$) $b=2+$

at $d=2$) $b^2=4+$

at $d=3$) $b^3=8+$

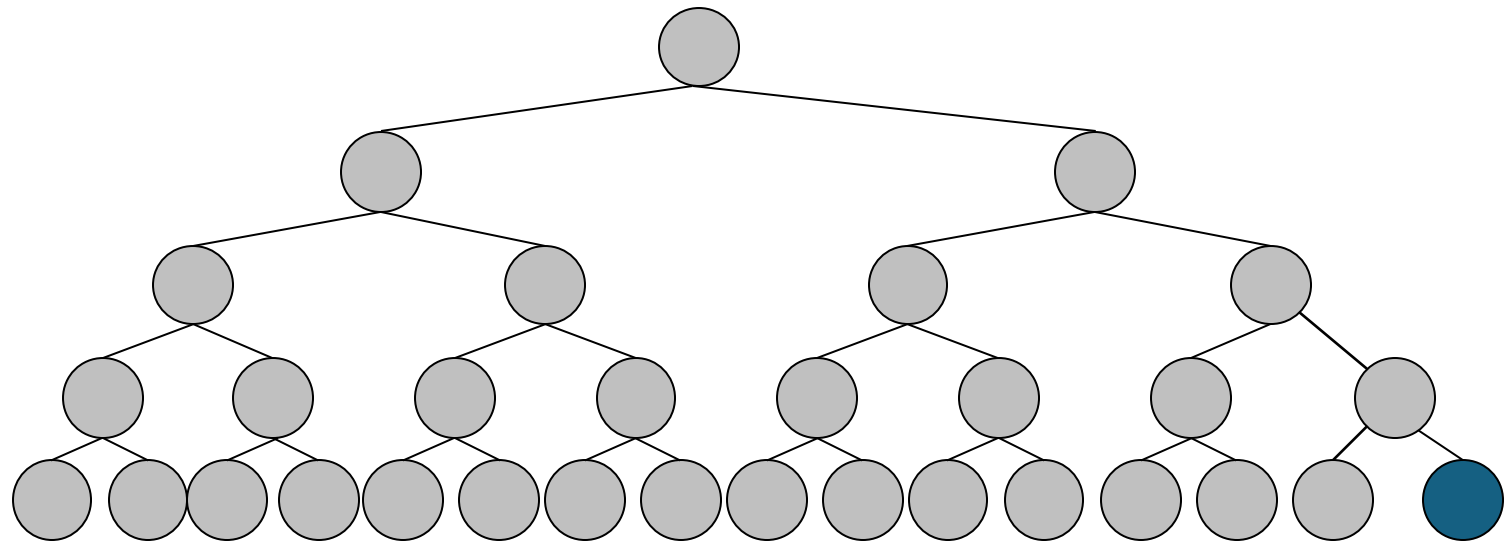
at $d=4$) $b^4 = 16$

at d+1) $b^5 = 32 = (16 + 8 + 4 + 2 + 1) + 1$

- Total of states generated

4

$$\text{No of states} = 1 + b + b^2 + b^3 + \dots + b^d = b^{d+1} - 1$$

$$= O(b^{d+1})$$


Breadth-First Search

Another formula:

We generate

at $d=0$) $1+$

at $d=1$) $b=2+$

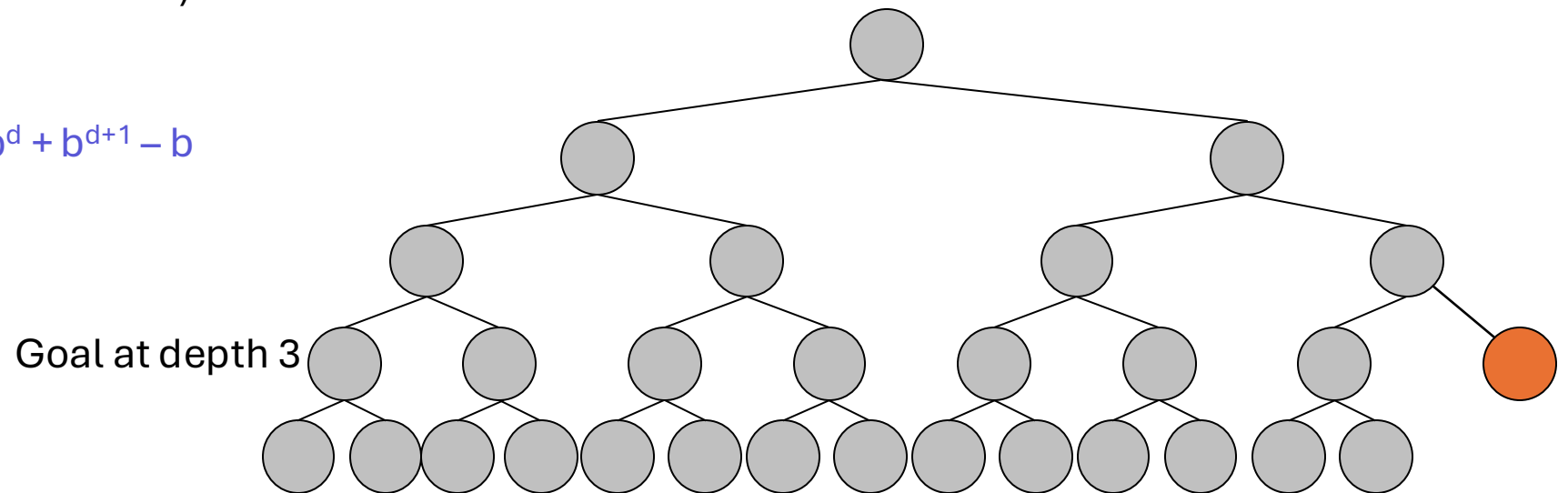
at $d=2$) $b^2=4+$

at $d=3$) $b^3=8+$

plus we would have generated states at $d=4$) $b^4=16-b$

at $d+1$) $b^5=32=(16+8+4+2+1)+1$

Total No of states = $1+b+b^2+b^3+...+b^d+b^{d+1}-b$
 $=O(b^{d+1})$



Breadth-First Search

- **Complete**, if b is finite
 - **Optimal**, if path *cost is equal to depth* (if all operators have the same cost)
 - Guaranteed to return the shallowest goal (depth d)
 - **Exponential time and space complexity**
 - Time complexity = $O(b^{d+1})$
 - Space complexity = $O(b^{d+1})$
- where d is the depth of the solution and b is the branching factor (i.e., number of children) at each node

The costs, however, are very high. Let b be the maximal branching factor and d the depth of a solution path. Then the maximal number of nodes expanded is

$$b + b^2 + b^3 + \dots + b^d + (b^{d+1} - b) \in O(b^{d+1})$$

Example: $b = 10$, 10,000 nodes/second, 1,000 bytes/node:

Depth	Nodes	Time	Memory
2	1,100	.11 seconds	1 megabyte
4	111,100	11 seconds	106 megabytes
6	10^7	19 minutes	10 gigabytes
8	10^9	31 hours	1 terabyte
10	10^{11}	129 days	101 terabytes
12	10^{13}	35 years	10 petabytes
14	10^{15}	3,523 years	1 exabyte

COMPLEXITY TIME AND MEMORY