

Range Anxiety Reduction in Battery-Powered Vehicles

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Abstract—The limited cruising range of Electric vehicles (EVs) and the lack of recharging stations have contributed to what is viewed by potential adopters as range anxiety- contributing to a reserved attitude towards EVs. Within the context of this paper, range anxiety is defined as the concern of the vehicle operator that the EV is running out of energy. Thus, there is a critical need that the vehicle path is planned such that the operator is assured of access to energy recharging and such that the EV must possess a path rejection capability to prevent the potential of an out-of-energy state in-between recharging stations. This paper addresses this issue and proposes a technique to minimize range anxiety. This technique analyzes the battery capacity the EV needs to reach a charging station so that drivers are guaranteed not to be stranded. It also computes a robust estimate of driving range on a specific path. The paper reports simulation work conducted to test and validate the proposed techniques under various driving conditions.

Index Terms—Electric vehicles, Range anxiety, Remaining driving Range

I. INTRODUCTION

Due to the rechargeable batteries' limited energy supply and the difficulty of improving their lifetime, EVs have stringent power-consumption requirements. The electric energy they consume depends on driving and environmental conditions, and the use of energy-consuming technology, and it is often higher than the amount expected by manufacturers. EVs can recuperate some of their consumed energy (i.e., they can regenerate energy during downhill and deceleration phases), thus extending their cruising range by approximately 20 percent in urban areas [1]. The battery of the EV has two main constraints: (1) a route cannot be used by the vehicle if it has an energy cost that is greater than the battery charge level and, (2) when the battery is already fully charged, then a route cannot be used if it has a negative energy cost (gaining energy from downhill or deceleration phases). While the single battery charge of EVs may, depending on conditions, support a driving range that is roughly just about 200 km, the full tank of conventional vehicles can support a driving range of around 600 km or even further [2]. For operators, a shorter driving range translates to higher range anxiety [3].

Yuhe et al. [2] proposed a telemetric basic service for EVs that is designed to provide an estimate of the remaining driving distance, classifying the process into rough range and precise range estimation. The rough range estimation is based on the

maximum driving distance determined by the EV maker as well as the battery charge level and its maximum capacity. The approach starts by performing the rough range estimation until the battery charge level reaches a preset threshold value that is determined in advance. Then, the precise range estimation, which is based on computing the energy cost values of road segments, is performed and displayed to the user. However, this approach is still incomplete for many reasons. First, the most important problem for EVs, battery constraints, is not addressed. Second, the energy cost function used in the precise range estimation does not integrate the acceleration and deceleration energies at traffic lights as well as the loss of energy due to rolling resistance. Third, the study shows that the precise range estimation is very expensive in terms of time and computing resources, as the estimation process includes a map-matching approach and shortest distance path-finding approach, which also address inaccuracy in the process.

This research reports that a guarantee that operators can always reach at least one charging station to recharge their drained batteries would effectively reduce their range anxiety. This paper is organized into four sections: Section II describes theoretical information and introduces the proposed solution to overcome the concern of range anxiety. Section III reports the validation and testing of the proposed algorithmic technique, including the simulation environment created and results. Finally, concluding remarks are presented in Section IV.

II. RANGE ANXIETY REDUCTION MODEL

This section presents a model posed for reducing range anxiety. The model is designed to analyze the battery charge an EV requires to reach at least one charging station before the battery is completely drained. It thus provides a guarantee to drivers that, wherever they travel, they will not be stranded en route. The two battery constraints are resolved by dynamically adjusting the path energy costs. We propose that the first constraint of the battery be solved by turning the path energy cost value into infinity, thus excluding the path during travel. For the second constraint problem, we propose dynamically adjusting the energy cost function such that the energy gained from downhill edges and during deceleration phases is stored in the available free capacity of the battery until the battery is full. The rest of any energy gained is lost. In addition, a circle around the vehicle's current location is formed to establish

a boundary that helps determine charging stations that may be reachable with the current battery charge. The formula in Equation 1, adopted from [2], is used to determine the radius of the boundary circle.

$$BR = d_{max} \frac{J}{C_{max}} \quad (1)$$

where d_{max} , the maximum driving distance determined by the EV maker (e.g., 200 km for a single battery charge) is the radius of the circle, C_{max} is the maximum battery capacity, and J is the battery charge level. The division of battery charge level by maximum capacity of the battery represents the remaining battery charge.

In the beginning, charging stations within the circular area are localized, and the path energy cost to each charging station is computed. A charging station is designated reachable or not based on the travel energy cost, not the travel distance. The routing technique used to compute the most energy-efficient path among all possible paths to each charging station is the A^* algorithm. The routing technique here uses the vehicle's current location as the source node and the charging stations as destinations in its computations. The routing technique and the energy cost function used to represent weights in a road network, which are adopted from [11], are presented in the next sections.

After computing the energy-optimal paths to all reachable charging stations, the charging station that has the minimum path energy cost is compared to the battery charge level. If the battery charge level is within a preset threshold value, which is about twice the cost to the cheapest charging station, then a warning is displayed alerting drivers that they have insufficient energy to travel anywhere and to follow the energy-optimal path to the cheapest charging station. The battery charge level is updated and the process is repeated every time the vehicle enters a new road segment. The update of the battery charge level and repetition of the process can be timer specified (e.g., drivers may want the update to be made every three or four minutes). During the process, if a charging station within the circular area, determined in the beginning, is not reachable, its path energy cost is turned into infinity and thus excluded from the search process.

A. A^* Search Technique

A^* Algorithm involves using a heuristic function to search for nodes that are likely to have the cheapest cost. For node evaluation, A^* algorithm combines $g(n)$, the real cost value to arrive at node n , and $h(n)$, the heuristic (i.e., the estimated cost value) from node n to the destination.

$$f(n) = g(n) + h(n) \quad (2)$$

A^* Search depends on two conditions for optimality. First, $h(n)$ must be an admissible heuristic—one that does not overvalue the true path cost. For $h(n)$ to be an admissible heuristic, $f(n)$, stated in Equation 2, must not overestimate

the true cost of a path to arrive at node n . Second, and more importantly, heuristic $h(n)$ must be consistent, that is, the condition stated in Equation 3 is satisfied. In Equation 3, $h(n)$ is the heuristic from node n to arrive at the destination, $c(n, n')$ is the true cost between node n and the following node n' , and $h(n')$ is the heuristic from n' to arrive at the destination.

$$h(n) \leq c(n, n') + h(n') \quad (3)$$

In addition, it is claimed that “if $h(n)$ is consistent, then the values of $f(n)$ along any path are non-decreasing”[4]. For convenience, the proof is presented as follows: let us assume that n' is a following node of n , then

Proof.

$$g(n') = g(n) + c(n, n')$$

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, n') + h(n') \\ &\geq g(n) + h(n) = f(n) \end{aligned}$$

□

B. Energy Cost Function

Assume a directed graph is given, $G = (V, E)$ having $|V| = n$ and $|E| = m$, to represent a road network, where vertices $v \in V$ represent points, and edges $e \in E$ represent connections between these points corresponding to road segments. Assume for each vertex, an elevation $u : v \rightarrow R_0^+$ is given, and for each edge, a length $l : E \rightarrow R^+$ and a speed limit $S : E \rightarrow N$ are given. A path P can be defined as a sequence of k vertices (v_1, v_2, \dots, v_k) and the edge is two vertices $(v_i, v_{i+1}) \in E$ with $i = 1, 2, \dots, k-1$. In addition, the following parameters are defined: C_{max} is the maximum capacity of the battery; J is the charge level of the battery, where $J \leq C_{max}$; U is the remaining free capacity of the battery, where $U = C_{max} - J$; and Δ^k is the amount of energy consumed or gained along a path. We consider different forms of energy costs that can occur from taking a path $P^k = (v_1, v_2, \dots, v_k)$ as follows:

Potential Consumed/Gained Energy:

A function $E_{P,G}(u(a))$ is defined to represent an elevation of a vertex (a) . When the EV travels over an edge (a, b) , the potential energy $E_P(a, b) = u(b) - u(a)$ is consumed or drawn from the battery only if the EV is going uphill (i.e., $u(b) - u(a)$). Then, the energy cost of the edge (a, b) induced by the potential consumed energy is defined to be

$$C_{PC}(a, b) = \frac{1}{\eta_c} [mg(u(b) - u(a))] \quad (4)$$

where, m is the mass of the vehicle including payload, g is the gravitational acceleration factor, and η_c is the efficiency factor. This potential consumed energy on the road segment (a, b) takes the following values:

$$C_{PC}(a, b) = \begin{cases} 0 & \text{if } k = 1 \\ 0 & \text{if } k > 1, \quad \Delta^k < 0 \\ \Delta^k & \text{if } k > 1, \quad 0 \leq \Delta^k \leq J \\ \infty & \text{if } k > 1, \quad \Delta^k > J \end{cases}$$

When the EV travels over an edge (a, b) , the energy gained $E_G(a, b) = u(a) - u(b)$ is regenerated and stored in the battery only during downhill travel (i.e., $u(a) - u(b)$). The energy cost of the edge (a, b) induced by the potential gained energy is defined to be

$$C_{PG}(a, b) = \eta_c [mg(u(a) - u(b))] \quad (5)$$

This potential gained energy is stored in the battery and is lost only if the battery is fully charged. It takes the following values:

$$C_{PG}(a, b) = \begin{cases} 0 & \text{if } k = 1 \\ 0 & \text{if } k > 1, \quad 0 \leq \Delta^k \leq J \\ -\Delta^k & \text{if } k > 1, \quad 0 > \Delta^k \geq U \\ \Delta^k & \text{if } k > 1, \quad 0 > \Delta^k > U \end{cases}$$

Loss of Energy:

We define a function $E_L(l(e), s(e))$ that models energy loss due to the aerodynamic and rolling resistances. The aerodynamic resistance component, which is the right hand side of Equation 6, is not dissipated and has a value of zero if and only if two conditions are satisfied: first, the wind is in the same direction as the vehicle, and second, the wind has a speed that is greater than or equal to the speed of the vehicle.

$$C_{LE}(a, b) = \frac{1}{\eta_c} [f_r mgl(a, b) + \frac{1}{2} \rho A c_w S(a, b)^2 l(a, b)] \quad (6)$$

where f_r is the friction coefficient, ρ is the air density coefficient, A is the vehicle's cross sectional area, c_w is the air drag coefficient, S is the average speed on the edge (a, b) , and l is the length of the edge (a, b) . This loss of energy on the road segment (a, b) takes the following values:

$$C_{PG}(a, b) = \begin{cases} 0 & \text{if } k = 1 \\ \Delta^k & \text{if } k > 1, \quad \Delta^k > 0 \\ \Delta^k & \text{if } k > 1, \quad \Delta^k < 0 \\ \infty & \text{if } k > 1, \quad \Delta^k > J \end{cases}$$

Acceleration and Deceleration Energy:

We define a function $E_{a,d}(P, t_{a,d}(e))$ that models energy consumption due to mechanical loss, that is, acceleration energy required to return the EV to the average speed, and then deceleration energy used to stop the vehicle. An EV expends no energy while idling. If the EV has a tire with diameter r , then the angular velocity of the tire is

$$w_r = \frac{S}{r} \quad (7)$$

where S is the linear speed (i.e., average speed on the edge in

m/s), r is the diameter of the tire in m , and w_r is the angular velocity of the tire in rad/s . If the EV has a gear ratio of g_r , then the angular velocity and power of the motor are as follows:

$$w_m = w_r g_r \quad (8)$$

$$P = T w_m \quad (9)$$

where T is the torque in $N.m$, w_m is the angular velocity of the motor in rad/s , and P is the power of the motor in $watts$. Thus, the energies dissipated and recuperated during acceleration and deceleration phases on the road segment (a, b) are

$$C_{AE}(a, b) = \frac{1}{\eta_c} P t_A \quad (10)$$

$$C_{DE}(a, b) = \eta_r P t_D \quad (11)$$

where t_A and t_D are the acceleration and deceleration times, respectively.

Driving Style:

Once the four energy forms of road segment (a, b) have been calculated, the total energy cost function on the road segment (a, b) is multiplied by the driving style coefficient, DS_{coeff} . For normal driving, this coefficient may take a value of 1, and for aggressive driving, it may take the value 1.2 or it may be set by the EV maker to take other values based on different driving behaviours.

On-Board Electric Devices:

This energy consumption is determined using the power drawn by the electric device (i.e., air-conditioner, windshield wipers, etc.), which is a static value provided by the vehicle maker and the duration of use. This energy is considered to be drawn directly from the battery and is not treated as part of the energy expended by following a path. Consequently, periodic updating of the battery charge level is necessary. This type of energy is defined by the following form:

$$C_{ED} = \sum_{i=1}^n (P_{ED(i)} t_i) * Status_{(i)} \quad (12)$$

where $Status_{(i)}$ has a value of 0 if the electric device i is off; otherwise, it has a value of 1; t_i is the time that the electric device i takes in the status on; $P_{ED(i)}$ is the power drawn by the electric device i , and n is the EV's number of on-board electric devices.

Total Energy Cost:

The complete form of the total energy cost function on the edge (a, b) is as follows:

$$C_E(a, b) = [C_{AE}(a, b) + C_{LE}(a, b) + C_{PC}(a, b) + C_{PG}(a, b) + C_{DE}(a, b)] * DS_{coeff} \quad (13)$$

and the total energy cost of a path $P^k = (v_1, v_2, \dots, v_k)$ is

$$C_E(P^k) = \sum_{j=1}^{j=k-1} C_E(v_j, v_{j+1}) \quad (14)$$

Assuming no negative cycles exist, it is proven in [5] that whenever a potential function Π satisfies the fact that $\Pi(b) - \Pi(a) \leq C_E(a, b)$ and also C_Π is determined as $C_\Pi(a, b) = C_E(a, b) + \Pi(a) - \Pi(b)$, the optimal routes in the weighted graph (V, E, C_Π) are also the optimal in the weighted graph (V, E, C_E) .

Lemma 1: Π implies a positive reduced weighted function C_Π .

Proof.

$$\begin{aligned} C_\Pi &= C_E(a, b) + \Pi(a) - \Pi(b) \\ &= C_{AE}(a, b) + C_{LE}(a, b) + C_{DE}(a, b) \\ &\quad + \Pi(b) - \Pi(a) + \Pi(a) - \Pi(b) \\ &= C_{AE}(a, b) + C_{LE}(a, b) + C_{DE}(a, b) \geq 0 \end{aligned}$$

□

where energies during acceleration and deceleration phases on the road segment (a, b) take the following values:

$$C_{AE}(a, b) = \begin{cases} 0 & \text{if } k = 1 \\ \Delta^k & \text{if } k > 1, \quad \Delta^k > 0 \\ \Delta^k & \text{if } k > 1, \quad \Delta^k < 0 \\ \infty & \text{if } k > 1, \quad \Delta^k > J \end{cases}$$

$$C_{DE}(a, b) = \begin{cases} 0 & \text{if } k = 1 \\ -\Delta^k & \text{if } k > 1, \quad 0 > \Delta^k \leq U \\ \Delta^k & \text{if } k > 1, \quad 0 > \Delta^k > U \\ -\Delta & \text{if } k > 1, \quad \Delta_\Pi \geq 0 \\ \Delta & \text{if } k > 1, \quad \Delta_\Pi < 0 \end{cases}$$

where Δ_Π is the amount of energy consumed or gained on the road segment (a, b) in the weighted graph (V, E, C_Π) . The A^* algorithm, which is adopted from [11], is modified such that the energy-optimal path is determined in the weighted graph (V, E, C_Π) . The battery constraints for the optimal path, however, are dynamically adjusted and resolved based on the energy costs in the weighted graph (V, E, C_E) .

1) Heuristic Function: For the heuristic function of the energy cost in the weighted graph (V, E, C_Π) , the air-line distance and the minimum speed over all speed limits are used. Let us define two vertices, u and v , and a destination, t . Obviously, the air-line distance l' is a consistent heuristic for a road length.

$$l'(u, t) \leq l(u, v) + l'(v, t) \quad (15)$$

Therefore, the following heuristics are defined:

$$h_L(u, t) = f_r m g l'(u, t) + \frac{1}{2} \rho A c_w S_{min}^2 l'(u, t) \quad (16)$$

$$h_A(u, t) = T \frac{S_{min}}{r} g_r t_A \quad (17)$$

$$h_D(u, t) = T \frac{S_{min}}{r} g_r t_D \quad (18)$$

where

$$h(u, t) = h_L(u, t) + h_A(u, t) + h_D(u, t)$$

$$\begin{aligned} h(u, t) &= f_r m g l'(u, t) + \frac{1}{2} \rho A c_w S_{min}^2 l'(u, t) \\ &\quad + T \frac{S_{min}}{r} g_r t_A \\ &\quad + T \frac{S_{min}}{r} g_r t_D \leq C_\Pi(u, t) \end{aligned} \quad (19)$$

Lemma 2: The heuristic $h(u, t)$ is consistent in the weighted graph (V, E, C_Π) .

Proof. Since $h(u, t)$ is linearly increasing in l , $h(u, t) \leq h(u, v) + h(v, t)$. As $h(u, t)$ is monotonic in S and l , $h(u, v) \leq C(u, v)$ for all u and v for which $C(u, v)$ is defined, and therefore,

$$\begin{aligned} V(u, v) &= f_r m g + \frac{1}{2} \rho A c_w S^2(u, v) \\ &\quad + T \frac{S(u, v)}{r} g_r t_A + T \frac{S(u, v)}{r} g_r t_D \end{aligned}$$

$$\begin{aligned} V' &= f_r m g + \frac{1}{2} \rho A c_w S_{min}^2 \\ &\quad + T \frac{S_{min}}{r} g_r t_A + T \frac{S_{min}}{r} g_r t_D \end{aligned}$$

Since S_{min} is a lower bound of $S(u, v)$, V' is also a lower bound of $V(u, v)$ and therefore consistency follows down from

$$h(u, t) \leq V(u, v) l(u, v) + V' l'(u, v)$$

$$h(u, t) \leq C(u, v) + h(v, t)$$

□

C. Remaining Driving Range Estimation of a Specific Path

The driving range of a specific path cannot be estimated based only on the maximum driving distance determined by the EV maker along with the remaining battery charge, for two main reasons: (1) the battery constraints of limited capacity and the ability to recuperate energy during downhill and deceleration phases, and (2) the differences in road conditions, environmental conditions, and EV features. Both factors must be considered in any remaining-driving-range estimation approach, in order to accurately provide drivers with the exact remaining range on a specific path. Therefore, the estimation here is based on using the path energy cost defined in Section II-B with the remaining battery charge, which together can provide accurate driving-range estimation.

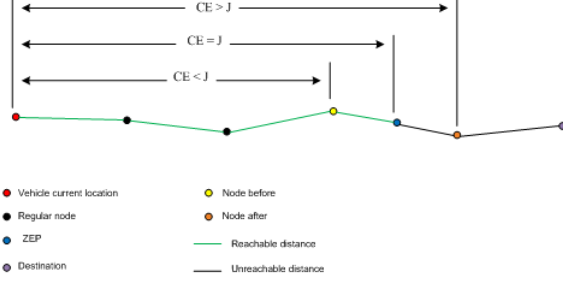


Fig. 1: Zero Energy Point Determination

Figure 1 provides a simple theoretical example to explain the estimation approach. If the battery charge is not sufficient to supply the EV along the whole distance of a specified path, then from the vehicle's current location to the specified destination there must be a point on the path at which the battery charge level, J , equals the path energy cost, $C_E(P^k)$. We term this point the Zero Energy Point (ZEP), meaning that traveling beyond this point is no longer possible. The objective of the approach is to use the battery charge level and path energy cost to compute the ZEP. The strategy is to use the next node (i.e., the node following the vehicle's starting point) where the edge energy cost for the vehicle to travel between its initial location and next node is compared to the battery charge level. If the battery charge level is greater than the first edge energy cost on the path, then the edge energy cost is subtracted from the battery charge, and the next node is used for the next step. This process is repeated along the specified path until a road segment with an energy cost exceeding the battery charge level is found. It is then known that the ZEP occurs on this road segment.

For example, in Figure 1, at the node marked in yellow the battery charge is greater than the path energy cost. However, at the node marked in orange the battery charge is less than the path energy cost. Therefore, it is known that the ZEP occurs on the road segment connecting those two nodes. The difference between the battery charge level and the path energy cost at the node marked in yellow will be used in the path energy cost, represented in Equation 13, to compute the maximum distance the EV can reach on this road segment. Then, this distance is added to the combined lengths of previous road segments to compute the maximum distance of driving the entire path.

III. SIMULATION WORK

This section reports simulation experiments performed to test and validate the proposed techniques. Matlab was used to construct the test and validation environment. Figure 2 depicts the constructed road network, including four charging stations marked in red and an EV marked in blue. The charging stations are assumed to be positioned at intersections. The EV is assumed to travel around the road network in a random manner but with the consideration of

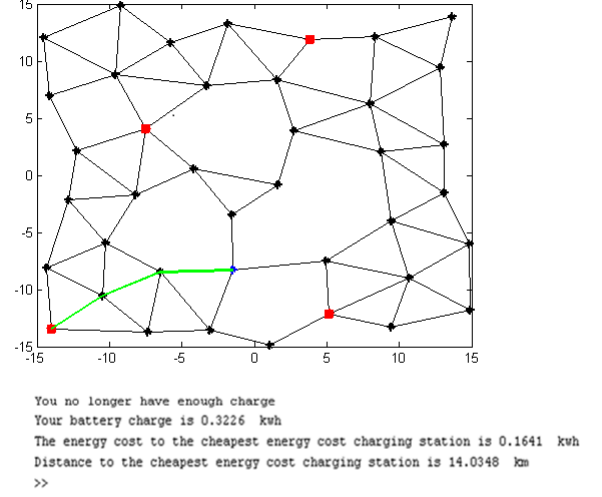


Fig. 2: Road Network with Four Charging Stations

battery constraints. In this operation scenario, the battery charge level was assumed to be $5kwh$ when the EV started travelling. The energy-optimal path to the charging station having the cheapest energy cost is marked in green. The first important observation in Figure 2 is that the nearest charging station to the EV's current location, which is about two road segments in length, was not selected by the routing technique. Apparently, this occurred because the optimal path to the charging station is energy cost-based and not distance-based. The second important observation proves the inaccuracy of estimating driving range based on the formula represented in Equation 1. The distance to the cheapest charging station is about 14 km, while the maximum driving distance according to the formula of Equation 1 is 3.226 km, with $d_{max} = 250km$, $C_{max} = 25kwh$, and $J = 0.3226kwh$. This observation also demonstrates how the driving range can be extended by recuperated energy from downhill and deceleration phases. The message displayed to the user is also depicted in Figure 2; it shows the current battery charge, energy-optimal path cost to the cheapest charging station, and distance to the cheapest charging station.

Table I provides the total path energy cost to each charging station in the weighted graph (V, E, C_E) along with the indices of charging stations. Noticeably, the charging station with an index of 29 has the minimum path energy cost over all charging stations, and hence, it was chosen by the search technique as the best charging station. Table II proves that the path marked in green, Figure 2, is the optimal path to the charging station in the weighted graph (V, E, C_{Π}) by satisfying the admissibility and consistency conditions of the A^* search technique. The values of $f(n)$, which never overestimate the path cost and are also non-decreasing along the path, prove that heuristic $h(n)$ is both admissible and consistent.

Finally, Figure 3 demonstrates the result gathered from

TABLE I: Total Path Energy Costs to Charging Stations

Charging station indices	14	29	11	5
Path cost <i>kwh</i>	0.16708	0.16407	0.17306	0.21157

TABLE II: Satisfaction of Admissibility and Consistency Conditions

Path Nodes	$g(n)$ <i>kwh</i>	$h(n)$ <i>kwh</i>	$f(n)$ <i>kwh</i>	Total Path Cost <i>kwh</i>
24	0	0.051306	0.051306	0.095815
12	0.033869	0.035744	0.069613	0.095815
37	0.061435	0.02001	0.081445	0.095815
29	0.095815	-0.0061339	0.089681	0.095815

testing the model for the driving-range estimation on a specific path. Simply, a sequence of nodes, chosen randomly to represent a path, is used to validate the model. The path starts with the source node (marked in red) and ends with the destination node (marked in blue). A low battery charge that is unlikely to supply the vehicle along the entire path is used to verify the success of the approach. The model marks the reachable distance on the path in yellow and the ZEP in purple. The maximum distance that the EV can reach in this operation scenario, 25.379 km, is returned by the model.

IV. CONCLUSIONS

A new model to reduce drivers' range anxiety has been proposed in this research work. The underlying idea was to compare an EV's battery charge with the energy-optimal path costs that would be incurred driving to charging stations within reach. If the battery charge level is about twice the path energy cost to the charging station having the cheapest energy cost over all charging stations, then a warning as well as directions to the cheapest charging station are displayed to the driver, prompting him/her to recharge the drained battery. Additionally, the model includes a driving-range estimation approach to provide an accurate estimate of how far drivers can travel on a specific path. This estimation is based on travel energy cost and not distance. The simulation results reported in this research demonstrate that the proposed model can help reduce range anxiety and therefore, help remove one barrier to the widespread use of EVs.

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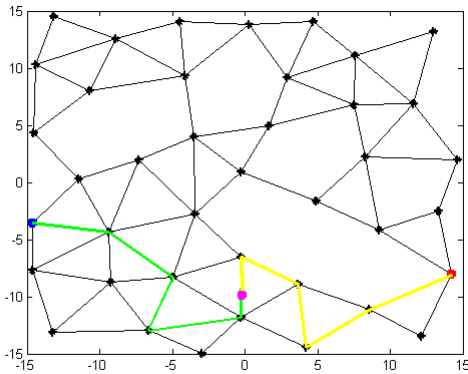


Fig. 3: Driving Range Estimation on a Specific Path