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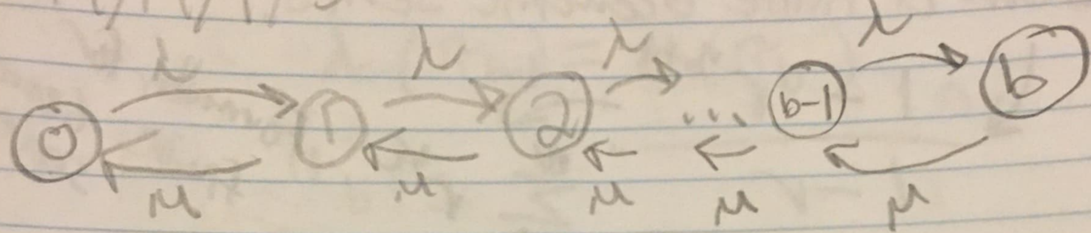
# Markov Chain Challenge Project

- M/M/1/b




Global Balance Equations

- $\pi_0 \rightarrow \lambda\pi_0 = \mu\pi_1$

$$\pi_1 = \frac{\lambda}{\mu}\pi_0$$

- $\pi_1 \rightarrow (\lambda+\mu)(\pi_1) = \mu\pi_2 + \lambda\pi_0$

$$\left(\frac{\lambda}{u}\right)\pi_0$$

$$\frac{1}{N}\cdot\frac{\lambda^2}{\mu}\pi_0 + \cancel{\lambda\pi_0} = \frac{\mu\pi_2}{\mu} + \cancel{\lambda\pi_0}$$

$$\left(\frac{\lambda}{\mu}\right)^2\pi_0 = \pi_2$$

limit

- $\pi_k = \pi_0\left(\frac{\lambda}{\mu}\right)^k = [0, b]$

$\sum_{k=0}^{b}\left(\frac{\lambda}{\mu}\right)^k\pi_0 = 1$ by total probability →

$$\pi_0 \sum_{k=0}^{b} \left(\frac{\lambda}{\mu}\right)^k = 1$$

Sum of finite Geometric series:

$$\frac{1 - r^{n+1}}{1-r} \rightarrow \sum_{k=0}^{n} r^k$$ Formula for finite Geometric series

$$\sum_{k=0}^{b} \left(\frac{\lambda}{\mu}\right)^k = \frac{1 - \left(\frac{\lambda}{\mu}\right)^{b+1}}{1 - \frac{\lambda}{\mu}}$$

$$1, \frac{\lambda}{\mu}, \left(\frac{\lambda}{\mu}\right)^2 \ldots \left(\frac{\lambda}{\mu}\right)^b = 1, \frac{\lambda}{\upsilon} \ldots \left(\frac{\lambda}{\mu}\right)$$

$$= \frac{\left(1 - \left(\frac{\lambda}{\mu}\right)^{b+1}\right)}{1 - \frac{\lambda}{\mu}}$$

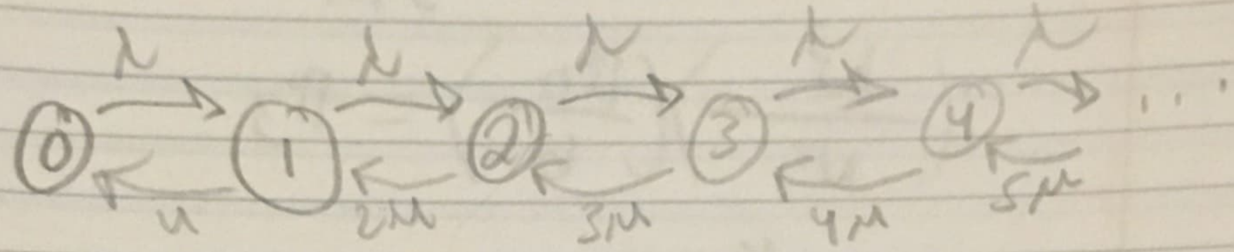
$$= \frac{1 - (\upsilon)^{b+1}}{1 - \upsilon}$$

$$\frac{1}{\phantom{x}} = \pi_0$$

$$\pi_k = \left(\frac{1-\upsilon}{1-(\upsilon)^{b+1}}\right) \cdot (\upsilon)^k$$

$$\pi_k = \upsilon^k \left(\frac{1-\upsilon}{1-(\upsilon)^{b+1}}\right)$$

- $M/M/\infty$




Balance Equations

$\pi_0$ — $\lambda \pi_0 = \mu \pi_1$

$$\pi_1 = \frac{\lambda}{\mu} \pi_0$$

$\pi_1$ — $(\lambda + \mu)\pi_1 = 2\mu \pi_2 + \lambda \pi_0$

$$(\lambda + \mu)\frac{\lambda}{\mu}\pi_0$$

$$\frac{\lambda^2}{\mu}\pi_0 + \cancel{\lambda \pi_0} = 2\mu\pi_2 + \cancel{\lambda \pi_0}$$

$$\frac{\lambda^2}{\mu}\pi_0 = \frac{2\mu \pi_2}{2\mu}$$

$$\frac{\lambda^2}{2\mu^2}\pi_0 = \pi_2$$

$$\pi_2 = \frac{\pi_0}{2}\left(\frac{\lambda}{\mu}\right)^2$$

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$\pi_2$ -

$$(\lambda + 2\mu)\pi_2 = 3\mu\pi_3 + \lambda\pi_1$$

$$(\lambda + 2\mu)\left(\frac{\pi_0}{2}\left(\frac{\lambda}{\mu}\right)^2\right) = 3\mu\pi_3 + \lambda\left(\frac{\lambda}{\upsilon}\pi_0\right)$$

$$\frac{\lambda^3}{\mu^2}\cdot\frac{\pi_0}{2} + \cancel{\frac{\lambda^2}{\upsilon}\cdot\frac{\pi_0}{2}} = 3\mu\pi_3 + \cancel{\frac{\lambda^2}{\mu}\pi_0}$$

$$\frac{\left(\frac{\lambda^3}{\mu^2}\right)\left(\frac{\pi_0}{2}\right)}{3\mu} = \frac{3\mu\pi_3}{3\mu}$$

$$\left(\frac{\lambda^3}{\mu^3}\right)\left(\frac{\pi_0}{6}\right) = \pi_3$$

$$\pi_k = \upsilon^k \frac{\pi_0}{k!}$$

$$\pi_0 \sum_{k=0}^{\infty} \frac{\upsilon^k}{k!} = 1$$

exponential series $\sum_{k=0}^{\infty} \frac{z^k}{k!} = e^z$

$$\pi_0 e^{\upsilon} = 1$$

$$\pi_0 = \frac{1}{e^{\upsilon}}$$

$$\pi_k = \frac{\upsilon^k\left(\frac{1}{e^{\upsilon}}\right)}{k!}$$