Sprint 3 Deliverables

- Newton-Pepys Problem

- Reason about # of successes out of n simultaneous trials - Binomial Distribution?

1. 6 fair dice, success = six appears

   P(rolling 6) = $p = 1/6$

   P(not rolling 6) = $1-p = 5/6$

   $n=6$

   X = number of 6's that apper

   $X=1$ so 1 6 appears

   ... $X=6$ since we want at least 1

   $1-(X=0) = (X>0)$

   $k=0$

$$P(X=0) = \binom{6}{0}(1-p)^{6-0}(p)^{0}$$

$$P(X=0) = \binom{6}{0}\left(1-\frac{1}{6}\right)^{6}(1)$$

$$\frac{6!}{6!\,0!} = 1 \cdot \left(1-\frac{1}{6}\right)^{6} = .33489 = 33.5\%$$

$$P(X>0) = 1-.33489 = .6651 = 66.5\%$$

Second Scenario:

$X = \#$ of sixes
$X = 2 \ldots X = 12$ $\quad X > 1$

$1 - (X=0) - (X=1) = (X>1)$

$$P(X=0) = \binom{12}{0}\left(1-\frac{1}{6}\right)^{12}\left(\frac{1}{6}\right)^{0}$$
$$= 1 \cdot \left(1-\frac{1}{6}\right)^{12} = .112156654$$

$$P(X=1) = \binom{12}{1}\left(1-\frac{1}{6}\right)^{11}\left(\frac{1}{6}\right)^{1}$$

$$\frac{12!}{(11!)(1)!} \qquad \frac{12 \cdot \cancel{11}}{\cancel{11} \cdot 10} = 12$$

$$(12)\left(\frac{5}{6}\right)^{11}(1/6) = .269175$$

$P(X=1) = .2691$

$1 - .269175 - .1121565 = .618667$

61.87%

Third Scenario:

$K = 3$ $\quad x > 3$

$P(x>3) = 1 - P(x=0) - P(x=1) - P(x=2)$

$P(x=0) = \binom{18}{0}(1-1/6)^{18}(1/6)^{0}$

$= (1-1/6)^{18} = .037561$

$P(x=1) = \binom{18}{1}(1-1/6)^{17}(1/6)^{1}$

$(18)(5/6)^{17}(1/6) = .1352197$

$P(x=2) = \binom{18}{2}(1-1/6)^{16}(1/6)^{2}$

$\frac{18!}{16!\,2!} = (153)$

$(153)(5/6)^{16}(1/6)^{2} = .22987$

$1 - .037561 - .1352197 - .22987 = .5973$

59.73%

☆ - Scenario 1 has the greatest chance of success

Geometric Urn

$E[X] = \frac{1}{p}$ in geometric distribution

$p \cdot \frac{1}{p} = 20 \cdot p \qquad p = \frac{1}{20}$

$R+B=100 \qquad \frac{R}{R+B} = p$

$p =$ red ball
$1-p =$ black ball

$\frac{1}{20} = \frac{R}{100} = \frac{20R}{20} = \frac{100}{20} \qquad R = 5$ red balls

$$E[X] = \sum_{k=1}^{\infty} k(1-p)^{k-1} p = 20$$

$$\sum_{k=1}^{\infty}$$

95 black, 5 red

Dragon Dice $p = 1/6$ ← success

$X =$ outcome after all 3 trials (how many galleons she gets or loses)

$E[X] = \sum_x x\, f(x)$

little x

$P(X=k) = \binom{n}{k}(1-p)^{n-k}(p)^k$

big X

4 cases in summation:

$x = -1, +1, +2, +3$

little $x = -1$ → $(P(X=0))$ ← $k=0$ since her # doesn't come up at all

$P(X=0) = \binom{3}{0}(1-1/6)^3(1/6)^0 = .5787$

little $x = 1$

$P(X=1) = \binom{3}{1}(1-1/6)^2(1/6)^1 = .3472$

little $x = 2$

$P(X=2) = \binom{3}{2}(1-1/6)^1(1/6)^2 = .06944$

little $x = 3$

$P(X=3) = \binom{3}{3}(1-1/6)^0(1/6)^3 = .0046296$

$$E[X] = \sum_{x} (-1)(.5787) + (1)(.3472) + (2)(.06944) + (3)(.004629)$$

$$= -.0787 \text{ galleons}$$