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Sprint 2 Questions Deliverables
Wizard People, Dear Reader?

1. P(she's a witch) = .75

P(Not receiving a letter | she's a witch) = .03
(B = Not receiving a letter, A = she's a witch)

P(Not receiving a letter | she's not a witch) = .99

P(she's a witch | Not receiving a letter)?
(A = she's a witch, B = Not receiving a letter)

$$P(A|B) = \frac{P(A)\cdot P(B|A)}{P(B)}$$

$$P(A|B) = \frac{(.75 \cdot .03)}{.27} = .08333$$

P(B)?

$$P(B) = P(B|A)P(A) + P(B|\neg A)P(\neg A)$$

$$(.03)(.75) \quad (.99)(.25)$$

$$.0225 + .2475 = .27$$

Chocolate Frogs

X = Number of frogs needed to open to get every unique card

$$E[X] = \sum_{i=1}^{30} E[X_i]$$ where X_i is number of frogs needed to open to get the ith unique card

Add expected values

$$\sum_{i=1}^{30} E[X_i] = \sum_{i=1}^{30} \left(\sum x_1 p(x_1) + \sum x_2 p(x_2) \ldots \sum x_{30} p(x_{30}) \right)$$

$$p(x_i) = \frac{n - i + 1}{n}$$ where n is the upper limit 30. This is probability of getting any ith card

Geometric distribution means $E[X] = \frac{1}{p}$

So $\frac{1}{p(x_i)} = \frac{n}{n-i+1}$ for each expected value in the total summation.

$$\sum_{i=1}^{30} \frac{n}{n-i+1} = E[X]$$

$$\frac{30}{30} + \frac{30}{29} + \frac{30}{28} + \frac{30}{27} \ldots + \frac{30}{1}$$

$E[X] \approx 120$ frogs

$$P(B)P(A|B) = \frac{P(A \cap B)}{P(B)} \cdot P(B)$$

$$P(A \cap B) = P(A) \cdot P(B)$$

Hat Problem

$P(\underset{A}{Evil} | \underset{B}{Slytherin})$ $P(\underset{B}{Slytherin} | \underset{A}{Evil})$

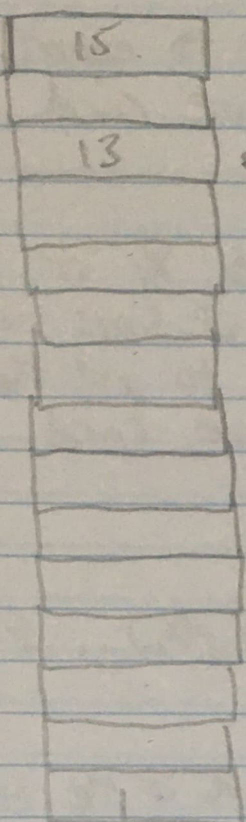
$$P(A|B) = \frac{P(A) \cdot P(B|A)}{P(B)}$$

$$P(A|B) = \frac{.10 \cdot 1.00}{P(B)} \qquad \frac{.10}{.28} = .357$$

35.7%

$$P(Slytherin) = \underset{P(B|A)P(A)}{.10} + \underset{P(B|\neg A)P(\neg A)}{.18} = .28$$

Elevator Problem



← Hermione

Logic: If Hermione is on 13th floor, then she must consider the elevator coming down 2 floors or traveling up 12 floors to get to her (continuous elevator).

However, if the elevator was already at 13, it's ambiguous as to which way the elevator was traveling.

2 Floors above her: $\frac{2}{14}$

12 Floors below her: $\frac{12}{14}$

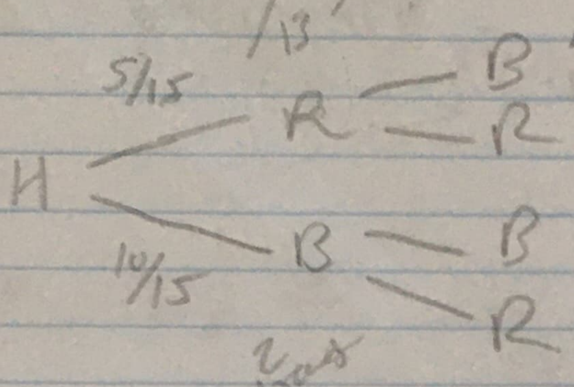
Disregard 13th floor because of ambiguity

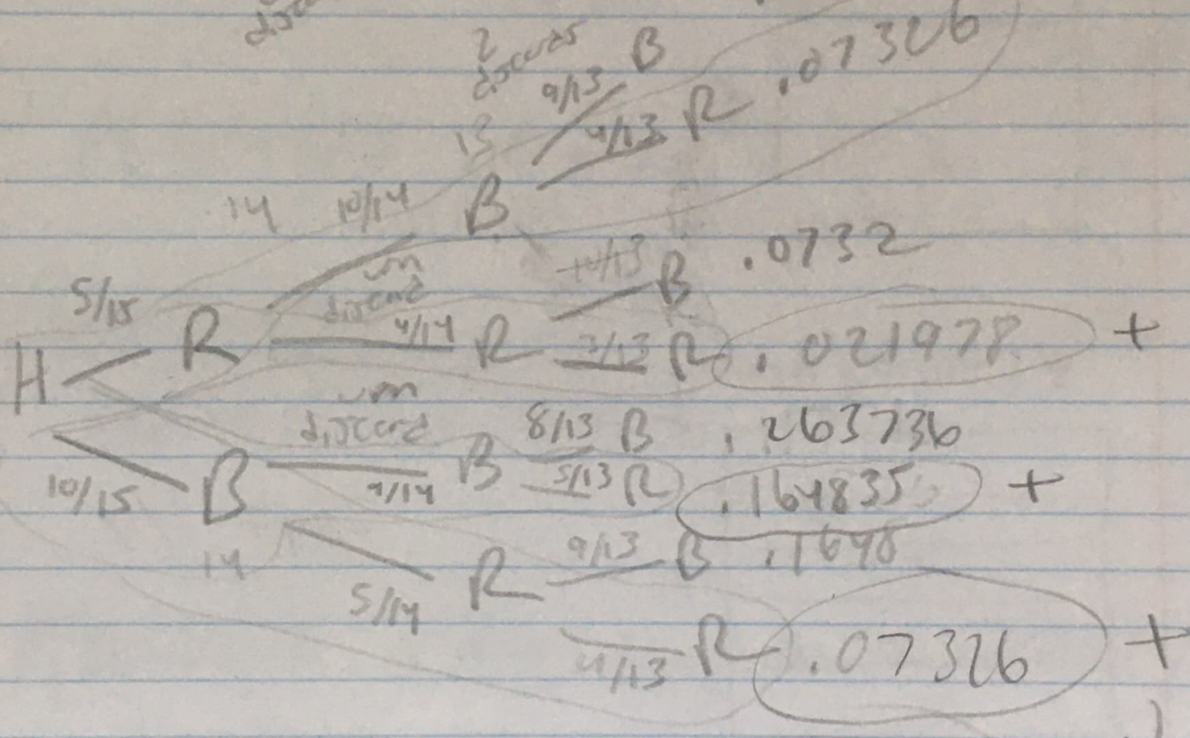
Thus, The elevator could've been coming down from either the 15th or 14th floor and that would be 2/14 total possible floors.

Probability that elevator moved down = 2/14

Urn While You Learn

10 black, 5 red 15 total



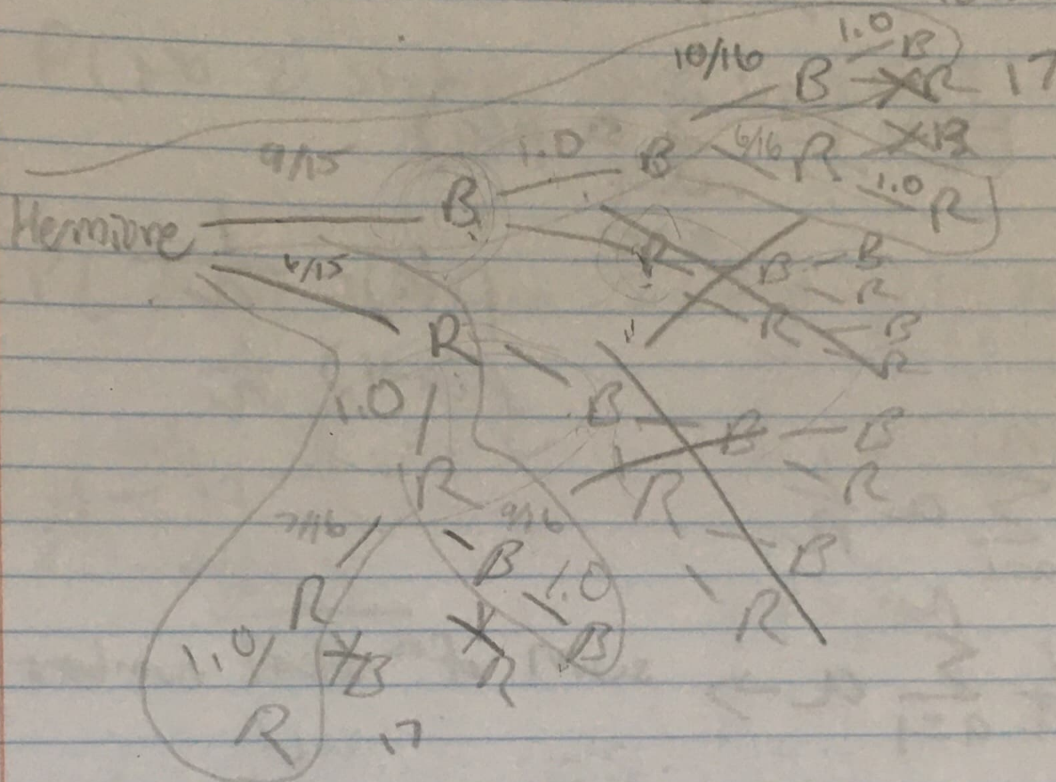


P(2nd ball is red) = .333 or 1/3

P(2nd ball is red) = Σ probabilities of each case when 2nd ball is red

Polya's urn

9 black 6 red 15 total



9, 10, 11

BR
RR
BB
RB

X = # red balls in urn

$E[X]$

$\sum_x x\,p(x)$

$\overset{6 \to 17}{X = 6, 7, 8}$

$6 \cdot .375 + 7 \cdot .45 + 8 \cdot .175$

$E[X] = 6.8$ red balls ≈ 7 red balls

$E[Y]$ Y = # of black balls in urn

$\sum_y y\,p(y)$

$9 \cdot .175 + 10 \cdot .45 + 11 \cdot .375$

1.575 + 4.5 4.125

$E[Y] = 10.2$ black balls

Arithmancy

$$E[x] = \sum_{x=1}^{n} x \; p(x)$$

Because it is a discrete uniform distribution:

$$p(\text{any } x) = \frac{1}{n}$$

so

$$\sum_{x=1}^{n} x \frac{1}{n}$$

$$\frac{1}{n} \sum_{x=1}^{n} x = x+(x+1)+(x+2)\ldots \text{ or } 1+2+3+4,\ldots+n$$

sum of natural numbers

$$1+2+3+4\ldots+(n-2)+(n-1)+n$$

2. reverse → $n+(n-1)+(n-2)+\ldots 4+3+2+1$

2. add the two sequences

$$(1+n)+(1+n)+(1+n)\ldots(1+n)+(1+n)+(1+n)$$

in other words $n(n+1)$ but you only want 1 half of this sequence so $\frac{n(n+1)}{2}$

$$\frac{1}{n} \sum_{x=1}^{n} x = \frac{1}{\cancel{n}} \cdot \frac{\cancel{n}(n+1)}{2} = \frac{n+1}{2}$$

$$E[x] = \frac{n+1}{2}$$

Birthday attack

40 students

P(No 2 students share the same birthday)

$\left(\frac{365}{365}\right) \cdot \left(\frac{364}{365}\right)$ if there are 2 students

if there are 3:

$\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365}$

so for 40 students

$\left(\frac{365}{365}\right) \cdot \frac{364}{365} \cdot \ldots \cdot \frac{326}{365}$

1 student   2 student   40 student

Thus $\frac{365!}{365^n(365-n)!}$ = P(No 2 students sharing same birthday) where n is number of students

so $\frac{365!}{365^{40} \cdot (325)!} = .108768$