HW2

Wednesday, January 29, 2025 7:57 PM

1.1 Calculate by hand the gradient and the Hessian for each of the three functions at the specified point. You must show your work for credit (scan or include an image in your report).

(a) $x_0 = [-2, 2]^T$, $f(x_1, x_2) = x_1^2 + x_2^2$

(b) $x_0 = [4, 2]^T$, $f(x_1, x_2) = (x_1 - 4)^2 + (x_2 - 2)^3$

(c) $x_0 = [1, 2, -1]^T$, $f(x_1, x_2, x_3) = sin(x_1) + x_2 x_3^2$

a. $f(x_1, x_2) = x_1^2 + x_2^2 \qquad \frac{d}{dx_1} = 2x_1 \qquad \frac{d}{dx_2} = 2x_2$

$df = [2x_1, 2x_2]$

$x_0 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

$\boxed{d^2f = [-2, 2]}$

b. $x_0 = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \quad f(x_1, x_2) = (x_1 - 4)^2 + (x_2 - 2)^3$

$df = 2(x_1 - 4) + 3(x_2 - 2)^2$

$d^2f = 2_{(x_1)} + 6(x_2 - 2)$

$@\, x_0 = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \qquad d^2f = [2_{x_1} + \cancel{6(2-2)}^{0}]$

$\boxed{d^2f = [2_{x_1}, 0]}$

c. $x_0 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

$f(x_1, x_2, x_3) = \sin(x_1) + x_2 x_3^2$

$df = [\cos(x_1), x_3^2, 2x_2 x_3]$

$d^2f = [-\sin(x_1), 0, 2x_2]$

$\boxed{d^2f\, @ x_0 = [-\sin(1), 0, 4]}$

1.2 Calculate the directional derivative AND the directional Hessian, i.e. curvature, along the direction $p = [1, -2]^T$ for each of the above three functions. (Use $p = [1, -2, 0]^T$ for question 1.1(c) above). Show your work.

a.) $df = [-4, 4]$ $\quad d^2f = [-2, 2]$

$$[-4 \;\; 4]\begin{bmatrix}1\\-2\end{bmatrix} = -4 + -8 = \boxed{-12} \text{ deriv}$$

along P @ x_0

$$d^2f = [-2 \;\; 2]\begin{bmatrix}1\\-2\end{bmatrix} = -2 + -2 = \boxed{-4} \text{ Hessian}$$

b.) $df = [0, 0]$

$$\cdots = \boxed{0} \rightarrow \text{deriv}$$

$$d^2f = [2, 0]\begin{bmatrix}1\\-2\end{bmatrix} = \boxed{2} \rightarrow \text{hessian}$$

c.) $$df = [\cos(1), 4, 2]\begin{bmatrix}1\\-2\\0\end{bmatrix} = \boxed{\cos(1) - 8} = \text{deriv}$$

$$d^2f = [-\sin(1), 0, 4]\begin{bmatrix}1\\-2\\0\end{bmatrix} = \boxed{-\sin(1)} = \text{hessian}$$

* If these weren't supposed to be numerical answers, you just dot the derivative and direction with the Hessian

HW 2: Bracketing and Pinpoint Algorithms

1.3

See Attached Code

1.4

- **Slanted Quadratic Function:**
  - Optimal Step Length: 4.0
  - Point: (-2,2)
  - Function Calls: 6
- **Rosenbrock Function:**
  - Optimal Step Length: approximately 0; 1.57e-30
  - Point: (apx 0, 2)
  - Function Calls: 209
- **Jones Function:**
  - Optimal Step Length: 0.59375
  - Point: (1.59375, 2.1875)
  - Function Calls: 24

What I learned: It was pretty difficult to figure out initially, especially pinpointing as I had a bit of a harder time figuring out what it was supposed to be doing. Switching from attempting the cubic fit to a simple bisection method helped things work smoother, although I still had some errors jumping wrong directions etc. The bisection method will cause more function calls as it isn't fitting a line or using the derivatives, so I will need to fix that. The Rosenbrock function seemed to start at the minimum, and then my first step jumped away and it was really difficult to work its way back. This shows me that direction is really important as guessing the wrong way can add significant difficulty.