HW2

Wednesday, January 29, 2025 7:57 PM

1.1 Calculate by hand the gradient and the Hessian for each of the three functions at the specified point. You must show your work for credit (scan or include an image in your report).

(a) $x_0 = [-2, 2]^T$, $f(x_1, x_2) = x_1^2 + x_2^2$

(b) $x_0 = [4, 2]^T$, $f(x_1, x_2) = (x_1 - 4)^2 + (x_2 - 2)^3$

(c) $x_0 = [1, 2, -1]^T$, $f(x_1, x_2, x_3) = sin(x_1) + x_2 x_3^2$

a. $f(x_1, x_2) = x_1^2 + x_2^2 \qquad \frac{d}{dx_1} = 2x_1 \qquad \frac{d}{dx_2} = 2x_2$

$$df = [2x_1, 2x_2]$$

$$x_0 = \begin{bmatrix} -2 \\ 2 \end{bmatrix} \qquad \boxed{d^2f = [-2, 2]}$$

b. $x_0 = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \quad f(x_1, x_2) = (x_1 - 4)^2 + (x_2 - 2)^3$

$$df = 2(x_1 - 4) + 3(x_2 - 2)^2$$

$$d^2f = 2_{(x_1)} + 6(x_2 - 2)$$

$$@\, x_0 = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \qquad d^2f = [2_{x_1} + \cancel{6(2-2)}^{\,0}]$$

$$\boxed{d^2f = [2_{x_1}, 0]}$$

c. $x_0 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

$$f(x_1, x_2, x_3) = \sin(x_1) + x_2 x_3^2$$

$$df = [\cos(x_1), x_3^2, 2x_2 x_3]$$

$$d^2f = [-\sin(x_1), 0, 2x_2]$$

$$\boxed{d^2f\ @\, x_0 = [-\sin(1), 0, 4]}$$

1.2 Calculate the directional derivative AND the directional Hessian, i.e. curvature, along the direction $p = [1, -2]^T$ for each of the above three functions. (Use $p = [1, -2, 0]^T$ for question 1.1(c) above). Show your work.

a.) $df = [-4, 4]$ $d^2f = [-2, 2]$

$$[-4 \;\; 4]\begin{bmatrix}1\\-2\end{bmatrix} = -4 + -8 = \boxed{-12} \text{ deriv}$$

along P @ x_0

$$d^2f = [-2 \;\; 2]\begin{bmatrix}1\\-2\end{bmatrix} = -2 + -2 = \boxed{-4} \text{ Hessian}$$

b.) $df = [0, 0]$

$$\ldots = \boxed{0} \rightarrow \text{deriv}$$

$$d^2f = [2, 0]\begin{bmatrix}1\\-2\end{bmatrix} = \boxed{2} \rightarrow \text{hessian}$$

c.) $$df = [\cos(1), 4, 2]\begin{bmatrix}1\\-2\\0\end{bmatrix} = \boxed{\cos(1) - 8} = \text{deriv}$$

$$d^2f = [-\sin(1), 0, 4]\begin{bmatrix}1\\-2\\0\end{bmatrix} = \boxed{-\sin(1)} = \text{hessian}$$

* If these weren't supposed to be numerical answers, you just dot the derivative and direction with the Hessian