ME 575 - Assignment #2 Gradients, Hessians, and a Line Search Algorithm

due January 29th, 2025, 11:59pm via Learning Suite

Overview: Practise solving for gradients and Hessians and write/develop part of an unconstrained gradient-based optimization algorithm.

1.1 Calculate by hand the gradient and the Hessian for each of the three functions at the specified point. You must show your work for credit (scan or include an image in your report).

(a) $x_0 = [-2, 2]^T, f(x_1, x_2) = x_1^2 + x_2^2$

(b) $x_0 = [4, 2]^T, f(x_1, x_2) = (x_1 - 4)^2 + (x_2 - 2)^3$

(c) $x_0 = [1, 2, -1]^T, f(x_1, x_2, x_3) = sin(x_1) + x_2 x_3^2$

1.2 Calculate the directional derivative AND the directional Hessian, i.e. curvature, along the direction $p = [1, -2]^T$ for each of the above three functions. (Use $p = [1, -2, 0]^T$ for question 1.1(c) above). Show your work.

1.3 Develop a line search algorithm (i.e. Algorithm 4.3 and 4.4) that given a direction p, your algorithm returns the optimal point along that line (i.e. the optimal step length along p). Include your code in a zip folder.

1.4 Test your line search algorithm with the following conditions. Report the optimal step length and point found ALONG p (i.e. report both α^* and $x_{(k=1)} = x_{(k=0)} + \alpha^* p$) and the number of function calls required to obtain $x_{(k=1)}$. (Remember this isn't the global minimum, just the minimum along the current p direction.) Discuss what you learned with these tests in 200 words or less.

(a) $p = [-1, 1]^T, x_0 = [2, -6]^T$, Slanted Quadratic Function with $\beta = 1.5$ (See D.1.1 in Text Book)

(b) $p = [1, -3]^T, x_0 = [0, 2]^T$, Rosenbrock Function (See D.1.2 in Text Book)

(c) $p = [1, 2]^T, x_0 = [1, 1]^T$, Jones Function (See D.1.4 in Text Book)