H filters for PCA 2

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1 Computation

We need K convolutions with filters h_k without subsampling by a factor 2, then $J-K$ convolutions with filters $h_k, K+ \leq k \leq J$ with subsampling by factor 2.

This is equivalent to a convolution with a single filter ϕ_J and a subsampling by a factor 2^{J-K} whose Fourier transform is:

$$\hat{h}(\omega) \approx \hat{h}_1(\omega)\hat{h}_2(\omega)\dots\hat{h}_K(\omega)\hat{h}_{K+1}(2\omega)\hat{h}_{K+2}(2^2\omega)\dots\hat{h}_J(2^{J-K}\omega). \tag{1}$$

To compute the filters, we need to have:

$$\hat{h}_1 \approx \frac{\hat{\phi}_0(\omega)}{\hat{\phi}_1(\omega)} \tag{2}$$

and for all $k \leq K$:

$$\hat{h}_k \approx \frac{\hat{\phi}_{k-1}(\omega)}{\hat{\phi}_k(\omega)}. \tag{3}$$

Since $\hat{\phi}_k(\omega) \approx \hat{\phi}(2^k\omega)$, we have that:

$$\hat{h}_k \approx \frac{\hat{\phi}_(2^{k-1}\omega)}{\hat{\phi}_(2^k\omega)} \tag{4}$$

then in subsampling range for $k > K$:

$$\hat{h}_k(2^{k-K}\omega) \approx \frac{\hat{\phi}_{k-1}(\omega)}{\hat{\phi}_k(\omega)} \tag{5}$$

so finally:

$$\hat{h}_k(\omega) \approx \frac{\hat{\phi}_k(2^{-K+1}\omega)}{\hat{\phi}_(2^{-K}\omega)} = \hat{h}(\omega). \tag{6}$$

So, the h filters do not depend on k.