CS 6001 Homework 3

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1 Problem 1

$$(9x^2 + 3x + 5)/(7x + 3)$$

$$\begin{aligned} 9/7 &= 9 * 7^{-1} \\ 1 &= 7 * 8 \ mod\ 11 \\ 7^{-1} &= 8 \\ 9 * 8 \ mod\ 11 &= 6 \end{aligned}$$

The first term is $6x$.

$$\begin{aligned} In\ GF(11): & \\ (7x + 3) * 6x &= 9x^2 + 7x \\ -4^{-1} &= 3 \\ (9x^2 + 3x + 5) - (9x^2 + 7x) &= 3x + 5 \end{aligned}$$

$$\begin{aligned} 3/7 &= 3 * 7^{-1} \\ 3/7 &= 3 * 8 \ mod\ 11 \\ 3/7 &= 2 \end{aligned}$$

$(9x^2 + 3x + 5)/(7x + 3) = 6x + 2$

2 Problem 2

2.1 Addition

$(x^5 + x^3 + x^2 + x + 1)\ +\ (x^2 + x + 1)$
$= x^5 + x^3$

2.2 Subtraction

$$(x^5 + x^3 + x^2 + x + 1) \; - \; (x^2 + x + 1)$$
$$= x^5 + x^3$$

2.3 Multiplication

$$(x^5 + x^3 + x^2 + x + 1) \; * \; (x^2 + x + 1)$$

$$x^5 + x^3 + x^2 + x + 1 * x^2 = x^7 + x^5 + x^4 + x^3 + x^2$$
$$x^5 + x^3 + x^2 + x + 1 * x^2 = x^6 + x^4 + x^3 + x^2 + x$$
$$x^5 + x^3 + x^2 + x + 1 * 1 = x^5 + x^3 + x^2 + x + 1$$

$$\begin{array}{rrrrrrr} x^7 & + x^5 & + x^4 & + x^3 & + x^2 & & \\ + x^6 & & + x^4 & + x^3 & + x^2 & + x & \\ & + x^5 & & + x^3 & + x^2 & + x & + 1 \end{array}$$
$$= x^7 + x^6 + x^3 + x^2 + 1$$

2.4 Division

$$(x^5 + x^3 + x^2 + x + 1) \; / \; (x^2 + x + 1)$$

$$\begin{array}{rrrrrrr} & & & x^3 & - x^2 & + x & + 1 \\ \cline{4-7} x^2 + x + 1 \big) & x^5 & & + x^3 & + x^2 & + x & + 1 \\ & - x^5 & - x^4 & - x^3 & & & \\ \cline{2-4} & & - x^4 & & + x^2 & & \\ & & x^4 & + x^3 & + x^2 & & \\ \cline{3-5} & & & x^3 & + 2x^2 & + x & \\ & & & - x^3 & - x^2 & - x & \\ \cline{4-6} & & & & x^2 & & + 1 \\ & & & & - x^2 & - x & - 1 \\ \cline{5-7} & & & & & - x & \end{array}$$
$$= x^3 - x^2 + x + 1, \; R \; - x$$