

Question 1: (15 points) Find all the Nash equilibria (pure and mixed) of the following two-player simultaneous-move game:

	<i>L</i>	<i>R</i>
<i>U</i>	300, 200	10, 100
<i>D</i>	100, 10	100, 100

Question 2: Suppose that there are two firms in the soft drink market, Coke and Pepsi, who sell the same carbonated soft drink. There are 100 customers in the soft drink market, each of whom is willing to buy only one bottle and pay \$10 for it. Of these 100 customers, 40 are loyal to Coke and 20 are loyal to Pepsi. The remaining 40 customers will decide to buy from the firm with the cheaper product. If the firms charge the same price, the firms split the residual business. All consumers will buy only if the price per bottle is less than \$10. Assume that both firms have zero costs.

- (a) (5 points): Is $P_{Coke} = P_{Pepsi} = 10$ a Nash equilibrium?
- (b) (5 points): Is $P_{Coke} = P_{Pepsi} = 0$ a Nash equilibrium?
- (c) (10 points): What is the pure-strategy Nash equilibrium in this pricing game?

Question 3: (15 points) Consider the following simultaneous-quantity setting game between two firms. The market price is given by $p = 12 - q_1 - q_2$ where $q_i \geq 0$ is the quantity produced by firm i . Firm 1's marginal cost equals 2 while firm 2's marginal cost is 4. Find the Cournot-Nash equilibrium output and profit for each firm.

Question 4: A major factor that affects earnings in movie production is the timing of the movie release. Suppose that two major production houses are thinking of releasing their movies on either the Christmas week or later. The production houses are Universal Pictures (Universal) and Warner Bros Pictures (WB). If both movies open on Christmas, both movies will split the market and earn \$115 million each. If one opens on Christmas and other releases later, then the former will earn \$120 million, while the latter will earn \$105 million. If both open on the post-Christmas week, both firms will earn \$95 million each. Suppose Universal

is the first-mover and decides between releasing the movie on either the Christmas week or later. After observing Universal's action at the initial node, WB chooses between releasing its movie on either the Christmas week or later.

- (a) (5 points): Draw the extensive-form game (that is, the game tree). How many proper subgames are there?
- (b) (5 points): In class, we learnt that any sequential game with finite moves (or finite number of nodes) can be written in the corresponding normal-form. Represent the extensive-form game in normal form and identify the pure-strategy Nash equilibrium outcomes.
- (c) (6 points): Find the subgame-perfect Nash equilibrium of the game.
- (d) (4 points): Is there a first-mover advantage for Universal Pictures?

Question 5 (15 points): Consider the following simultaneous-move game of incomplete information among two players. Player 2 (column player) can be of two types and has complete information about its two types: type H and type L. Player 1 (row player) has only one type and is uncertain about player 2's type. Player 1 believes that player 2 is of H type with probability 0.75 and of L type with probability 0.25.

If player 2 is of type H, then the following payoff matrix is being played:

	x	y
a	1, 3	1, 2
b	3, 1	2, 5

If player 2 is of type L, then the following payoff matrix is being played:

	x	y
a	3, 2	1, 3
b	2, 1	0, 4

What is the Bayes-Nash equilibrium of the game?

Question 6 (*15 points*): Consider the infinitely repeated game of the following stage game with discount rate δ :

	C	D
C	3, 4	0, 7
D	5, 0	1, 2

Use a “trigger strategy” (similar to the one discussed in class) to find the condition on δ under which mutual cooperation $((C, C))$ every period can be supported as a subgame-perfect equilibrium of the infinitely repeated game.