CITY UNIVERSITY OF HONG KONG

Course code & title: MA2185 Discrete Mathematics

Session: Semester A, 2023/2024

Time allowed: 2 hours, December 21, 2023

This exam has THREE pages (including this cover page).

1.   This exam contains **5** questions. The total number of points is **70**.
2.   Please answer all questions and justify all your steps.

The exam is **closed book**. You may use a calculator.

No materials or aids are allowed in the examination. If any unauthorized materials or aids are found on a student during the examination, the student will be subject to disciplinary action.

1. (a) (6 points) There are two types of people in an island: Goodies who always tell the truth and baddies who always lie. You visit the island and are approached by two natives A and B. Only A addresses you as: "Both of us are baddies." What are A and B? Give reasons for your answer.

   (b) (8 points) Let Ω = the set of human beings; $S(x)$: x is a student; $P(x)$: x is a professor; $L(x)$: x is a Liar; $T(x, y)$: x trusts y.

   Express the following argument using predicates and quantifiers, and determine whether it is valid.

   "Some students trust every professor, and no student trusts any liar. Therefore, no professor is a liar."

2. (a) (8 points) Let A, B, and C be sets. Show that

   $$(A - B) - C = (A - C) - (B - C)$$

   without using Venn diagram.

   (b) (8 points) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

   $$f(x) = \begin{cases} 4x - 3, & x < 0 \text{ or } x > 2 \\ -7x + 6, & 0 \le x \le 2. \end{cases}$$

   Find $f^{-1}([-7, 5])$.

3. (10 points) A certain disease has an incidence rate of $x\%$ (that is, it afflicts $x\%$ of the population). A test has been devised to detect this disease. The test has 1% rate of false negatives (that is, about 1% of people who do have the disease will test negative); the false positive rate is 2% (that is, about 2% of people who do not have the disease will test). Suppose if a person takes the test and tests positive, then the probability that this person actually has the disease is 20%. What is the value of x?

4. (a) (7 points) Consider the following weighted graph of 9 vertices.
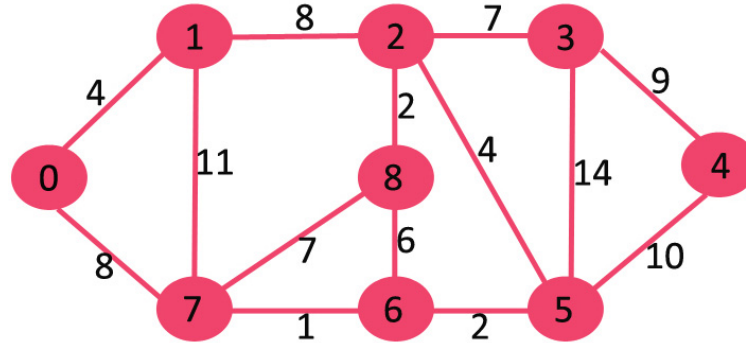
Draw the subgraph containing vertices 0,1,2,6,7,8 with all the induced edges (i.e. all edges have both end points in {0,1,2,6,7,8}). Does this subgraph has an Euler path or Euler circuit?

(b) (7 points) For the original graph in (a), use Dijkstra's Algorithm to find the shortest path between 0 and 4.

5. (a) (10 points) Suppose we want to send the message "ATTACK TONIGHT", knowing that it has been enciphered using the affine cipher. In the system we used, it has the 26 letters (A through Z) and 4 notations (total number of letters and notations $n = 30$), and will have the corresponding values found in the following table.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

P	Q	R	S	T	U	V	W	X	Y	Z	!	&	?	~
15	16	17	18	19	20	21	22	23	24	25	26	27	28	29

If the **deciphering function** is

$$D(c) = 7c + 9 \bmod 30,$$

write down the **enciphering function** for this cipher, and write down the ciphertext for the message "ATTACK TONIGHT". (Please present the function with the form $ap+b$ where a and b are non-negative integers less than 30.)

(b) (6 points) Find gcd(385, 880), and express it as a linear combination of 385 and 880 using the Euclidean algorithm.

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