GE2256: Game Theory Applications to Business
Lecture 13
Sample Question Paper and Solutions

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Q1

(15 points) Find all the Nash equilibria (pure and mixed) of the following two-player simultaneous-move game:

	L	R
U	300, 200	10, 100
D	100, 10	100, 100

Q1

- There are two PSNE: (U, L) and (D, R).
- Let p denote the probability that the row player plays U and q denote the probability that the column player plays L.
- Row player's expected payoff from

$$U : 300q + 10(1 - q)$$

$$D : 100$$

- Equating both, we have:

$$300q + 10(1 - q) = 100$$

$$\implies q = \frac{90}{290} = 0.31$$

Q1

- Column player's expected payoff from

$$L : 200p + 10(1 - p)$$

$$R : 100$$

- Equating both, we have:

$$200p + 10(1 - p) = 100$$

$$\implies p = \frac{90}{190} = 0.47$$

- Thus, the MSNE is $((0.47, 0.53), (0.31, 0.69))$.

Q2

Suppose that there are two firms in the soft drink market, Coke and Pepsi, who sell the same carbonated soft drink. There are 100 customers in the soft drink market, each of whom is willing to buy only one bottle and pay $10 for it. Of these 100 customers, 40 are loyal to Coke and 20 are loyal to Pepsi. The remaining 40 customers will decide to buy from the firm with the cheaper product. If the firms charge the same price, the firms split the residual business. All consumers will buy only if the price per bottle is less than $10. Assume that both firms have zero costs.

(a) *(5 points)*: Is $P_{Coke} = P_{Pepsi} = 10$ a Nash equilibrium?

(b) *(5 points)*: Is $P_{Coke} = P_{Pepsi} = 0$ a Nash equilibrium?

(c) *(10 points)*: What is the pure-strategy Nash equilibrium in this pricing game?

Q2

- (a) If $P_{Coke} = P_{Pepsi} = 10$, Coke gets $40 + 20 = 60$ customers with a profit of $\$600$.
- If Coke undercuts Pepsi and charges a slightly lower price, it can increase its customers to 80 (that is, 100 minus the Pepsi loyalists). Then, Coke's profits would be higher than $\$600$. For example, if Coke charges $\$9.99$, the profits are $\$799.20$.
- Thus, Coke will have an unilateral deviation that is profitable. $P_{Coke} = P_{Pepsi} = 10$ is not a Nash equilibrium.
- Note that you can argue from the Pepsi's side as well, and show that there is an unilateral deviation for Pepsi. Providing argument for any one firm is enough as in this case we are showing that the profile is not a Nash equilibrium.

Q2

- (b) If $P_{Coke} = P_{Pepsi} = 0$, each firm gets zero profits. Each firm can charge $\$10$ and sell to its loyal customers and get positive profits.
- Thus, unilateral deviation to higher prices is profitable (for each firm).
- (c) There is no PSNE in this game. Regardless of whether $P_{Coke} < P_{Pepsi}$ or $P_{Pepsi} < P_{Coke}$ or $P_{Coke} = P_{Pepsi}$, there is always an incentive to deivate unilaterally.

Q3

(15 points) Consider the following simultaneous-quantity setting game between two firms. The market price is given by $p = 12 - q_1 - q_2$ where $q_i \geqslant 0$ is the quantity produced by firm i. Firm 1's marginal cost equals 2 while firm 2's marginal cost is 4. Find the Cournot-Nash equilibrium output and profit for each firm.

Q3

- Firm 1's profit is given by:

$$u_1(q_1, q_2) = (12 - q_1 - q_2)q_1 - 2q_1 = (10 - q_2)q_1 - q_1^2$$

- Thus, firm 1's best-response function is:

$$q_1 = \frac{10 - q_2}{2}$$

Q3

- Firm 2's profit is given by:

$$u_2(q_1, q_2) = (12 - q_1 - q_2)q_2 - 4q_2 = (8 - q_1)q_2 - q_2^2$$

- Thus, firm 2's best-response function is:

$$q_2 = \frac{8 - q_1}{2}$$

Q3

- The two best-response functions are:

$$2q_1 + q_2 = 10$$

$$q_1 + 2q_2 = 8$$

- We need to simultaneously solve the two equations:

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \end{bmatrix}$$

Q3

- Thus,

$$D_{q_1} = \begin{vmatrix} 10 & 1 \\ 8 & 2 \end{vmatrix} = 12$$

$$D_{q_2} = \begin{vmatrix} 2 & 10 \\ 1 & 8 \end{vmatrix} = 6$$

$$D = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$$

$$q_1 = \frac{D_{q_1}}{D} = \frac{12}{3} = 4 \qquad q_2 = \frac{D_{q_2}}{D} = \frac{6}{3} = 2$$

Q3

- Nash equilibrium profits are:

$$u_1 = 16$$

$$u_2 = 8$$

Q4

A major factor that affects earnings in movie production is the timing of the movie release. Suppose that two major production houses are thinking of releasing their movies on either the Christmas week or later. The production houses are Universal Pictures (Universal) and Warner Bros Pictures (WB). If both movies open on Christmas, both movies will split the market and earn \$115 million each. If one opens on Christmas and other releases later, then the former will earn \$120 million, while the latter will earn \$105 million. If both open on the post-Christmas week, both firms will earn \$95 million each. Suppose Universal is the first-mover and decides between releasing the movie on either the Christmas week or later. After observing Universal's action at the initial node, WB chooses between releasing its movie on either the Christmas week or later.

Q4

(a) *(5 points)*: Draw the extensive-form game (that is, the game tree). How many proper subgames are there?

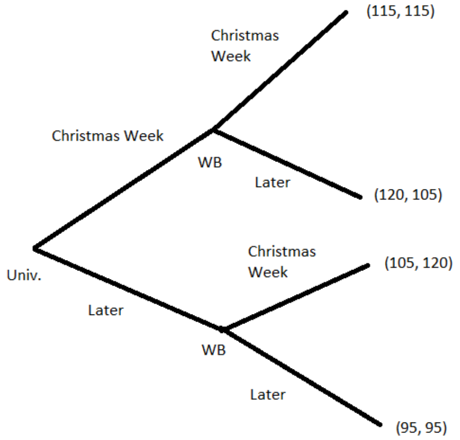
(b) *(5 points)*: In class, we learnt that any sequential game with finite moves (or finite number of nodes) can be written in the corresponding normal-form. Represent the extensive-form game in normal form and identify the pure-strategy Nash equilibrium outcomes.

(c) *(6 points)*: Find the subgame-perfect Nash equilibrium of the game.

(d) *(4 points)*: Is there a first-mover advantage for Universal Pictures?

Q4

(a) The game tree is as follows. There are two proper subgames.



Q4

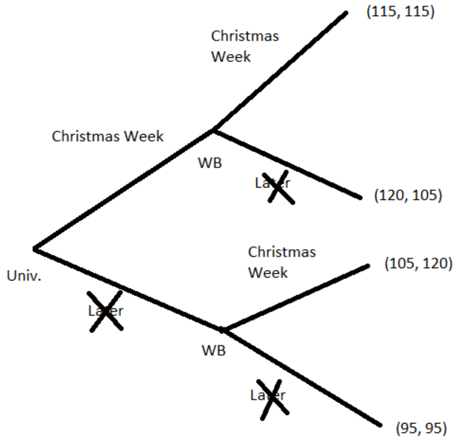
(b)

		(Xmas Wk., Xmas Wk.)	(Xmas Wk., Later)	(Later, Xmas Wk.)	(Later, Later)
Universal	Xmas Wk.	$\underline{115}$, $\underline{115}$	$\underline{115}$, $\underline{115}$	$\underline{120}$, 105	$\underline{120}$, 105
	Later	105, $\underline{120}$	95, 95	105, $\underline{120}$	95, 95

There are two PSNE: (Christmas week, (Christmas week, Christmas week)); (Christmas week, (Christmas week, Later))

Q4

(c)


(115, 115)
Christmas
Week
Christmas Week
WB
Later
(120, 105)
Christmas
Week
Univ.
(105, 120)
Later
WB
Later
(95, 95)

Q4

(c) SPE strategies: Universal Studios: Christmas week WB: (Christmas week, Christmas week))
SPE payoffs: $(115, 115)$
SPE outcome path: Universal decides to release its movie on Christmas week, after observing which, WB also decides to release its movie on Christmas week.

Q4

(d) No, there is no first-mover advantage for Universal Pictures.

Q5

If P2 is of type H :

	x	y
a	1, $\underline{3}$	1, 2
b	3, 1	2, $\underline{5}$

If P2 is of type L :

	x	y
a	3, 2	1, $\underline{3}$
b	2, 1	0, $\underline{4}$

P2's BRs are underlined.

Q5

P1's expected payoff from

a : $(1)(0.75) + (1)(0.25) = 1$

b : $(2)(0.75) + (0)(0.25) = 1.5$

Q5

Since expected payoff from b is greater than that from a,

P1's BNE strategy is b.

P2's BNE is : y if of type H OR y regardless of type.
y if of type L

Q6

(15 points): Consider the infinitely repeated game of the following stage game with discount rate δ:

	C	D
C	3, 4	0, 7
D	5, 0	1, 2

Use a "trigger strategy" (similar to the one discussed in class) to find the condition on δ under which mutual cooperation ((C, C)) every period can be supported as a subgame-perfect equilibrium of the infinitely repeated game.

Q6

- Consider the following trigger strategy:
  start the game by playing C; play C every period unless someone has ever played D in the past. Play D forever if someone has played D in the past.
- If player 2 follows the trigger strategy, player 1's payoff (intertermporal discounted) from playing C is:

$$3 + 3\delta + 3\delta^2 + ... = \frac{3}{1-\delta}$$

Q6

- Player 1's payoff from deviating:

$$5 + \delta + \delta^2 + ... = 5 + \frac{\delta}{1-\delta}$$

- P1 will not deviate if:

$$\frac{3}{1-\delta} \geqslant 5 + \frac{\delta}{1-\delta}$$

$$\implies 3 \geqslant 5(1-\delta) + \delta \implies \delta \geqslant \frac{1}{2}$$

Q6

- If player 1 follows the trigger strategy, player 2's payoff (intertermporal discounted) from playing C is:

$$4 + 4\delta + 4\delta^2 + ... = \frac{4}{1-\delta}$$

- Player 2's payoff from deviating:

$$7 + 2\delta + 2\delta^2 + ... = 7 + \frac{2\delta}{1-\delta}$$

Q6

- P2 will not deviate if:

$$\frac{4}{1-\delta} \geqslant 7 + \frac{2\delta}{1-\delta}$$

$$\implies 4 \geqslant 7(1-\delta) + 2\delta \implies \delta \geqslant \frac{3}{5}$$

- Thus, we need $\delta \geqslant \frac{3}{5}$.