Academic Honesty Pledge

I pledge that the answers in this exam/quiz are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,

- I will not plagiarize (copy without citation) from any source;
- I will not communicate or attempt to communicate with any other person during the exam/quiz; neither will I give or attempt to give assistance to another student taking the exam/quiz; and
- I will use only approved devices (e.g., calculators) and/or approved device models.

I understand that any act of academic dishonesty can lead to disciplinary action.

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Date: 2020.12.14

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评分：90分
Q3: 6 6 5
Q4: 2 5 4 1
Q6 3 2 2 2 2

Question 1:

$\begin{cases} \text{Player 1: } s_1 \\ \text{player 2: } s_2. \end{cases}$

If $s_1 + s_2 > 1$, only when $s_1 = s_2 = 1$. where neither of them have the incentive to deviate the number,

so $s_1 + s_2 \le 1$. Other wise one of them could change

If $s_1 > s_2$. and $s_1 + s_2 \le 1$ the share so that he can get some share > 0

$\Rightarrow$ if $s_1 + s_2 < 1$

both of the player have an incentive to increase the number

$\Rightarrow$ so $s_1 + s_2 = 1$

Similarly if $s_2 > s_1$ $\Rightarrow$ $s_1 + s_2 = 1$

If $s_1 = s_2$. $\Rightarrow$ $s_1 + s_2 = 1$ $s_1 = s_2 = 0.5$.

So the pure strategy Nash equilibria. is.

$s_1 + s_2 = 1$ where $\begin{cases} 0 \le s_1 \le 1 \\ 0 \le s_2 \le 1 \end{cases}$.

or $s_1 = s_2 = 1$

Question 2:

(a)

	~~W~~ W	~~X~~	Y	~~Z~~
A	3,3	2,1	0,2	2,1
~~B~~	~~1,1~~	~~1,2~~	~~1,0~~	~~1,4~~
C	0,0	1,0	3,2	1,1
~~D~~	~~0,0~~	~~0,5~~	~~0,2~~	~~3,1~~

step 1: for Row player. $(\frac{1}{2}, 0, \frac{1}{2}, 0)$ dominates B.

$\frac{1}{2}\times 3 > 1$ $\quad \frac{1}{2}\times 2 + \frac{1}{2}\times 1 > 1$ $\quad \frac{1}{2}\times 3 > 1$ $\quad \frac{1}{2}\times 2 + \frac{1}{2}\times 1 > 1$.

step 2: for Column player Y dominates Z.

step 3: for Row player $\frac{1}{2}A + \frac{1}{2}C$ dominates D.

$\frac{1}{2}\times 3 > 0$ $\quad \frac{1}{2}\times 2 + \frac{1}{2}\times 1 > 0$ $\quad \frac{1}{2}\times 0 + \frac{1}{2}\times 3 > 0$.

step 4: Y dominates X

~~step 5~~: so the rationalizable strategies are.

{A, C} for Row player.

{W, Y} for Column player

(b).

	W	X	Y	Z.
A	3,3	2,1	0,2	2,1
B	1,1	1,2	1,0	1,4
C	0,0	1,0	3,2	1,1
D.	0,0	0,5	0,2	3,1

so the pure strategy NE is {A, W}.

Mixed Strategy NE:

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		q_1 $\frac{1}{2}$ W	q_2 0 X	q_3 $\frac{1}{3}$ Y	0 Z
P_1 $\frac{2}{3}$	A	3,3	2,1	0,2	2,1
P_2	B	1,1	1,2	1,0	1,4
P_3	C	0.0	1.0	3,2	1,1
	D	0.0	0.5	0,2	3,1

$\Rightarrow$

		q W	$1-q$ Y
p	A	3,3	0,2
$1-p$	C	0,0	3,2

A: p C: $1-p$ W: q Y: $1-q$

A: $3q$ C: $3-3q$ $3q = 3-3q \Rightarrow q = \frac{1}{2}$

W: $3p$ Y: $2p + 2 - 2p = 2 \Rightarrow p = \frac{2}{3}$

$\therefore$ Row Player: $(\frac{2}{3}, 0, \frac{1}{3}, 0)$

Column Player: $(\frac{1}{2}, 0, \frac{1}{2}, 0)$.

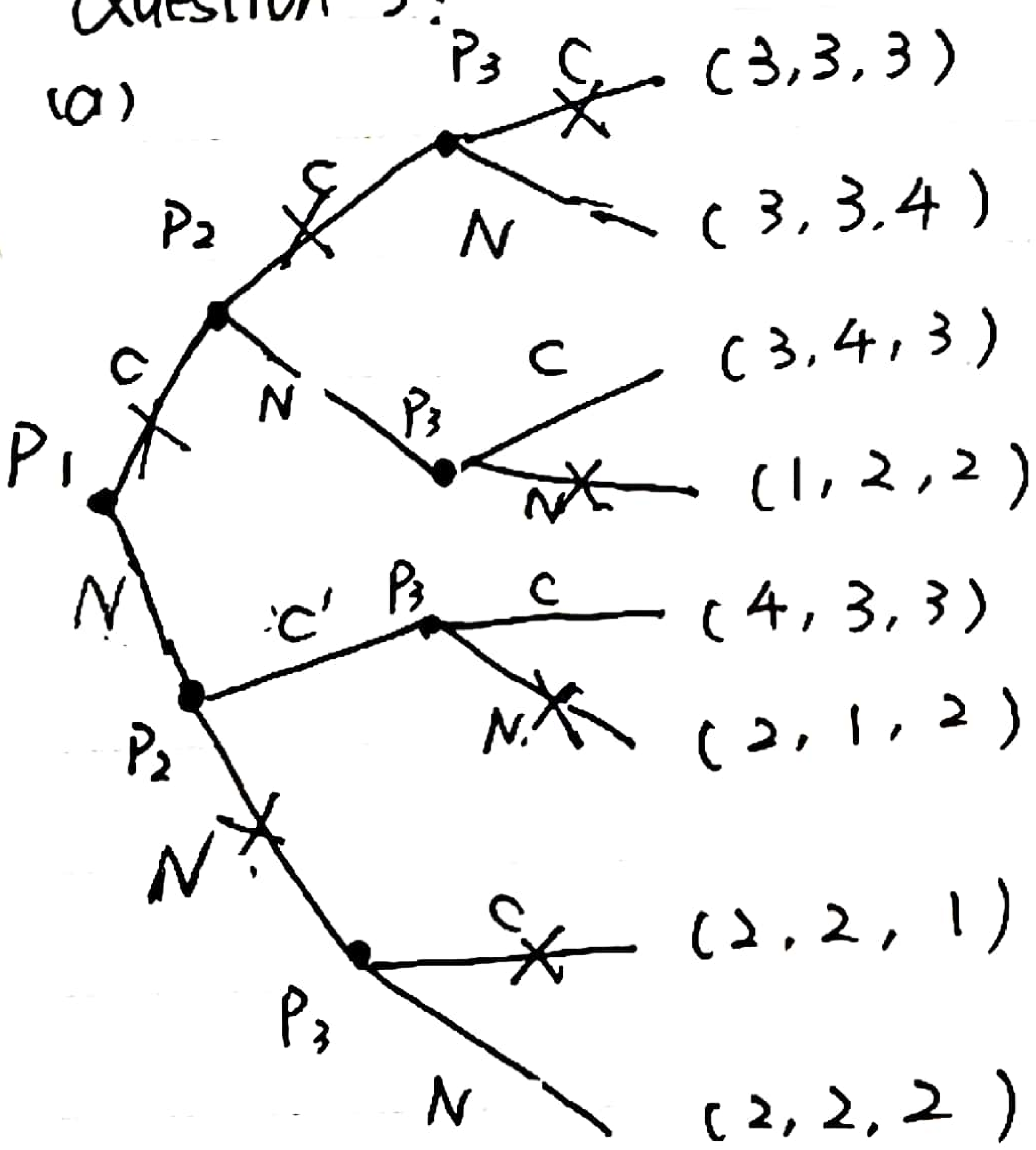
Answer: the pure strategy NE is (A, W) with a payoff (3,3)

the Mixed Strategy NE is $(\frac{2}{3}, 0, \frac{1}{3}, 0)$ for Row Player

$(\frac{1}{2}, 0, \frac{1}{2}, 0)$ for Column Player.

Question 3:

(a)



Ans:

there are 6 proper subgames in addition to the initial one

(b) player 1 has the following strategies:

$\{C, N\}$

player 2 has the following strategies:

$\{CC', CN', NC', NN'\}$

player 3 has $2^4 = 16$ strategies.

$\Rightarrow$ Player 3 has 16 strategies.

(c) As is shown in the game tree of (a).

so the SPNE is:

N for Player 1

NC' for Player 2.

NCCN for Player 3.

Question 4:

Payoff function:

Firm 1 $= (1000 - 2q_1 - 2q_2) \cdot q_1 - 200q_1$

$= -2q_1^2 + (800 - 2q_2) \cdot q_1$.

Firm 2. $= -2q_2^2 + (800 - 2q_1) \cdot q_2$.

After noticing Firm 1's quantity:

Firm 2' FOC: $\frac{\partial \text{Profit}}{\partial q_2} = -4q_2 + (800 - 2q_1) = 0$.

$q_2 = 200 - \frac{1}{2}q_1$

Since Firm 2 is rational.

Profit 1 $= -2q_1^2 + (800 - 400 + q_1) \cdot q_1$

$= -q_1^2 + 400q_1$ $\quad \frac{\partial \text{Profit}}{\partial q_1} = -2q_1 + 400 = 0$.

when $q_1 = 200$ Firm 1 has a maximum profit.

$\Rightarrow$ The SPNE is $\begin{cases} q_1 = 200 \\ q_2 = 100 \end{cases}$.

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Question 5:

Person 2 strong: α

Person 2 weak: $1-\alpha$

Strong:

$P_1 \backslash P_2$	F	Y
F	$\underline{(-1,1)}$	(1,0)
Y	$\underline{(0,1)}$	(0,0)

Weak

$P_1 \backslash P_2$	F	Y
F	(1,-1)	$\underline{(1,0)}$
Y	$\underline{(0,1)}$	(0,0).

Player 1 chooses F: Player 2. F if strong. Y if weak.

Player 1 chooses Y. Player 2: Y if strong F if weak.

Expected payoff for Player 1:

F: $-\alpha + 1-\alpha = 1-2\alpha$

Y: $0+0=0$

if $\alpha < \frac{1}{2}$ $F > Y$ Player 1 chooses F, player 2 F if strong. Y if weak

if $\alpha > \frac{1}{2}$ $F < Y$ player 1 Y. Player 2 F.

Answer: BNE.

if $\alpha < \frac{1}{2}$. player1 chooses F

player 2 chooses F if strong. Y if weak.

if $\alpha > \frac{1}{2}$. player 1 chooses Y.

player 2 chooses F

No.

Date

Question 6

	Collude	Compete
Collude	2, 2	0, πd
Compete	πd, 0	1, 1

(a) Compete is a strictly dominant strategy.

$\Rightarrow \pi d > 2$.

(b)

if chooses Collude infinitely.

Expected payoff is:

$$2(1+\delta+\delta^2+\delta^3+\cdots).$$

$$=\frac{2}{1-\delta}$$

if has a trigger strategy:

payoff is: $\pi d + (\delta+\delta^2+\delta^3+\cdots).$

$$=\pi d + \frac{\delta}{1-\delta}$$

$$\frac{2}{1-\delta} > \frac{\delta}{1-\delta} + \pi d. \qquad \delta \in (0,1)$$

$$\frac{2-\delta}{1-\delta} > \pi d.$$

$$2-\delta > \pi d - \pi d \cdot \delta.$$

$$(\pi d - 1)\delta > \pi d - 2.$$

$$\therefore \delta > \frac{\pi d - 2}{\pi d - 1} = 1 - \frac{1}{\pi d - 1}$$

When $\delta > \frac{\pi d - 2}{\pi d - 1}$ (Collude, Collude) can be supported as a SPNE.