

# Midterm #6

a) Convert the following 2 numbers to binary, octal, & hex  
 $2.875_{10}$  &  $0.1796875_{10}$

$$2.875_{10} = 10.111_2 = 2.7_8 = 2.E_{16}$$

$$\begin{array}{r} 0.875 \\ * 2 \\ \hline 1.750 \\ 0.750 \\ \hline 2 \\ \hline 1.500 \\ 0.5 \\ \hline 2 \\ \hline 1.0 \end{array}$$

$$\begin{array}{r} 0.875 \\ * 8 \\ \hline 7.000 \end{array}$$

$$\begin{array}{r} 0.875 \\ * 16 \\ \hline 5.250 \\ 8.750 \\ \hline 14.000 \\ \downarrow \\ E \end{array}$$

$$10.111_2 = 2.875_{10} = 2.7_8 = 2.E_{16}$$

$$1*2^1 + 0*2^0 + 1*2^{-1} + 1*2^{-2} + 1*2^{-3}$$

$$= 2 + 0 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$

$$= 2 + \frac{4}{8} + \frac{2}{8} + \frac{1}{8}$$

$$= 2 + \frac{7}{8} = \frac{23}{8}$$

$$= 2.875_{10} \checkmark$$

$$\begin{array}{r} 8 \overline{) 7.000} \\ 6.4 \\ \hline 0.60 \\ 0.56 \\ \hline 40 \end{array}$$

$$010.111 \rightarrow 2.7 \checkmark$$

$$2*8^0 + 7*8^{-1} = 2 + \frac{7}{8} = \frac{23}{8} = 2.875_{16} \checkmark$$

$$0010.1110 \rightarrow 2.E \checkmark$$

$$2*16^0 + E*16^{-1} = 2*16^0 + 14*16^{-1} = 2 + \frac{14}{16} = 2 + \frac{7}{8} = \frac{23}{8} = 2.875_{10} \checkmark$$

Now  
do the  
next one

$$0.1796875_{10} = 0.0010111_2 = 0.134_8 = 0.2\varepsilon_{16}$$

$$0 \times 2^0 = 0$$

$$+ 0 \times 2^{-1} = 0$$

$$+ 0 \times 2^{-2} = 0$$

$$+ 1 \times 2^{-3} = \frac{1}{8}$$

$$+ 0 \times 2^{-4} = 0$$

$$+ 1 \times 2^{-5} = \frac{1}{32}$$

$$+ 1 \times 2^{-6} = \frac{1}{64}$$

$$+ 1 \times 2^{-7} = \frac{1}{128}$$

$$x = \frac{1}{8} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128}$$

$$= \frac{16}{128} + \frac{4}{128} + \frac{2}{128} + \frac{1}{128}$$

$$= \frac{23}{128} \checkmark$$

$$\approx 0.1796875 \checkmark$$

$$0.0001011100$$

$$0. \quad 2 \quad 14$$

$$0. \quad 2 \quad \varepsilon \checkmark$$

$$0.1796875$$

$$\times 2$$

$$0.3593750$$

$$\times 2$$

$$0.7187500$$

$$\times 2$$

$$1.4375000$$

$$0.4375$$

$$\times 2$$

$$0.8750$$

$$\times 2$$

$$1.750$$

$$0.75$$

$$\times 2$$

$$1.50$$

$$0.50$$

$$\times 2$$

$$1.00$$

$$0.1796875$$

$$\times 8$$

$$1.4375000$$

$$0.4375$$

$$\times 8$$

$$3.5000$$

$$0.5$$

$$\times 8$$

$$4.0$$

$$0 \times 8^0 = 0$$

$$+ 1 \times 8^{-1} = \frac{1}{8}$$

$$+ 3 \times 8^{-2} = \frac{3}{64}$$

$$+ 4 \times 8^{-3} = \frac{4}{512}$$

$$= \frac{11}{64} + \frac{1}{128}$$

$$= \frac{22}{128} + \frac{1}{128}$$

$$= \frac{23}{128} \checkmark$$

$$0.1796875$$

$$\times 16$$

$$1.0781250$$

$$1.7968750$$

$$2.8750000$$

$$0.875$$

$$\times 16$$

$$5.250$$

$$8.750$$

$$14.000$$

$$\downarrow$$

$$\varepsilon$$

$$0 \times 16^0 = 0$$

$$2 \times 16^{-1} = \frac{2}{16}$$

$$14 \times 16^{-2} = \frac{14}{256}$$

$$= \frac{16}{128} + \frac{7}{128} = \frac{23}{128} \checkmark$$

a part 1

$$2.875_{10} = 10.111_2 = 2.7_8 = 2.\varepsilon_{16}$$

$$0.1796875_{10} = 0.0010111_2 = 0.134_8 = 0.2\varepsilon_{16}$$

apart 2) When Done, Convert the following to a float representation by the definition in class. In other words, an 8 digit Hex representation using the 4 byte float specification. Defined in class.

$$2.875_{10} = 2 + \frac{7}{8} = \frac{16}{8} + \frac{7}{8} = \frac{23}{8} = 23 * 8^{-1} = 23 * 2^{-3}$$

$$23_{10} = 10111_2 \rightarrow 0.10111 * 2^5 * 2^{-3} = 0.10111 * 2^2$$

5 4 3 2 1      Now Represent in 32 Bits

$$0.10111_2$$

$$* 2^2$$

$$\underline{01011100} \underline{00000000} \underline{00000000} \underline{00000010}$$

Hex: 5      C      0      0      0      0      0      2

$$2.875_{10} = 5C000002_{16}$$

$$0.1796875_{10} = \frac{23}{128} = 23_{10} * 128^{-1} = 10111_2 * 2^{-7} = 0.10111_2 * 2^{-2}$$

Now Represent in 32 Bits

$$0.10111_2$$

$$* 2^{-2} \begin{matrix} 00000010 \\ 11111101 \end{matrix}$$

$$\underline{01011100} \underline{00000000} \underline{00000000} \underline{11111101}$$

Hex: 5      C      0      0      0      0      F      E

$$0.1796875_{10} = 5C0000FE_{16}$$



b) Do the same for a) given they are negative values.  
 Flip the bits. All of the ones in the mantissa/fractional part

$2.875_{10}$  or  $5C000002_{16}$

Power stays the same

0	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
1	0	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0

10  
A 4 0 0 0 0 0 0 2

$$00 \boxed{-2.875_{10} = A400002_{16}}$$

power does not change

0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0
1	0	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

11  
+1

1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0
1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0

10  
A 4 0 0 0 0 0 F E

$$00 \boxed{-0.1796875_{10} = A40000FE}$$

c) Convert the Float Representations of the following into the decimal number given the definition in class  
59999901 & 59999902 & A66667FE

59999901:

Find where it repeats

$$\begin{array}{ccccccc}
 5 & 9 & 9 & 9 & 9 & 9 & 0 & 1 \\
 0101 & 1001 & 1001 & 1001 & 1001 & 1001 & 0000 & 0001
 \end{array}$$

$$\begin{aligned}
 &= 0.1011001_2 * 2^1 = 1.011001_2 \\
 &= 1 * 2^0 + 0 * 2^{-1} + 1 * 2^{-2} + 1 * 2^{-3} + 0 * 2^{-4} + 1 * 2^{-5} + 1 * 2^{-6} \\
 &= 1 + \frac{1}{4} + \frac{1}{8} + \frac{1}{64} = \frac{64}{64} + \frac{16}{64} + \frac{8}{64} + \frac{1}{64} = \frac{89}{64} = 1.390625_{10}
 \end{aligned}$$

59999902

$$\begin{array}{ccccccc}
 5 & 9 & 9 & 9 & 9 & 9 & 0 & 2 \\
 0101 & 1001 & 1001 & 1001 & 1001 & 1001 & 0000 & 0010
 \end{array}$$

Find where it repeats

$$\begin{aligned}
 &0.1011001_2 * 2^2 = 10.11001_2 \\
 &1 * 2^1 + 0 * 2^0 + 1 * 2^{-1} + 1 * 2^{-2} + 0 * 2^{-3} + 0 * 2^{-4} + 1 * 2^{-5} \\
 &= 2 + \frac{1}{2} + \frac{1}{4} + \frac{1}{32} \\
 &= \frac{64}{32} + \frac{16}{32} + \frac{8}{32} + \frac{1}{32} = \frac{89}{32} = 2.78125_{10}
 \end{aligned}$$

A66667FE

$$\begin{array}{ccccccc}
 A & 6 & 6 & 6 & 6 & 7 & F & E \\
 1010 & 0110 & 0110 & 0110 & 0110 & 0111 & 1111 & 1110 \\
 1010 & 0110 & 0110 & 0110 & 0110 & 0110 & 1111 & 1101 \\
 0101 & 1001 & 1001 & 1001 & 1001 & 1001 & 0000 & 0010
 \end{array}$$

$$\begin{aligned}
 &-0.1011001_2 * 2^{-2} \\
 &= (0 * 2^0 + 1 * 2^{-1} + 0 * 2^{-2} + 1 * 2^{-3} + 1 * 2^{-4} + 0 * 2^{-5} + 1 * 2^{-6} + 1 * 2^{-7}) * 2^{-2} \\
 &= (\frac{1}{2} + \frac{1}{8} + \frac{1}{16} + \frac{1}{128}) * \frac{1}{4} = (\frac{64}{128} + \frac{16}{128} + \frac{8}{128} + \frac{1}{128}) * \frac{1}{4} \\
 &= -(\frac{89}{128}) * \frac{1}{4} = -\frac{89}{512} = -0.173828125_{10}
 \end{aligned}$$