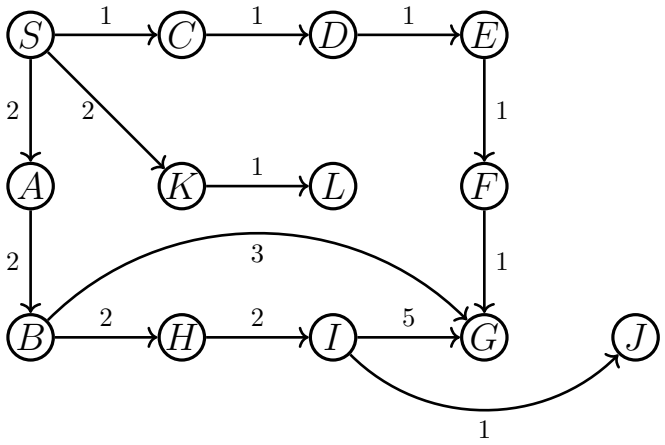
COMP 4475 *Assignment One (Soln)*

1. (**30 marks**) Consider the graph in the following



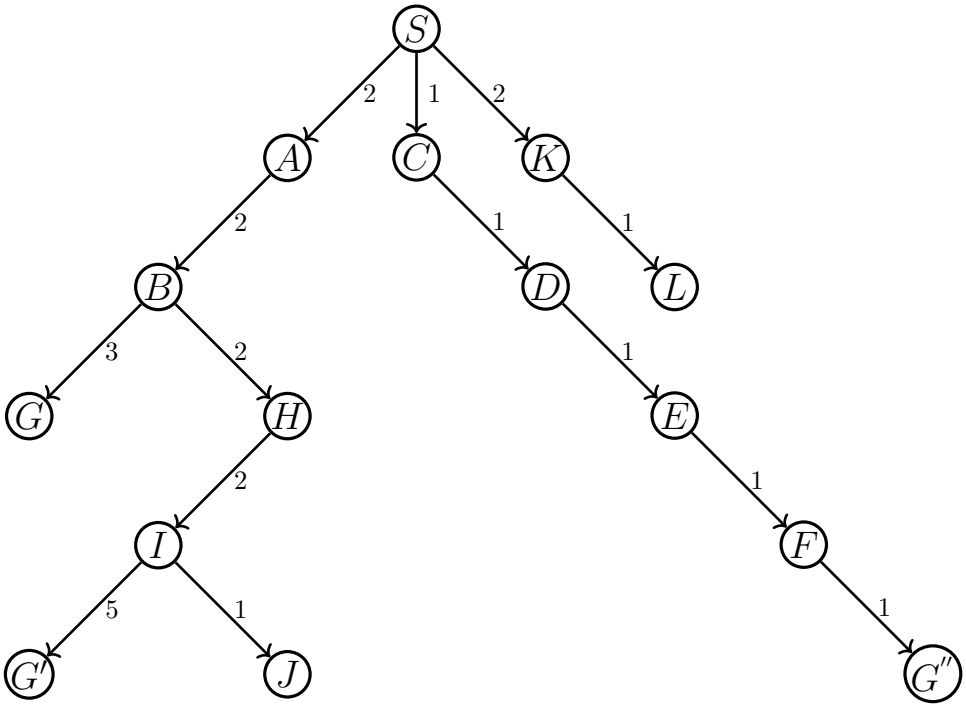
The heuristic estimate of the distance to G are:

$$h(A,2),\ h(B,3),\ h(C,4), h(D,3),\ h(E,2),\ h(F,1),\ h(G,0)$$

$$h(H,4),\ h(I,5),\ h(J,6), h(K,5),\ h(L,6),\ h(S,4).$$

(**4 marks**) Draw the corresponding search tree for the graph. The tree is rooted at S, and the goal node is G.

soln:



Find a path from S to G using each of the following search strategies, from (a) to (e). For each one of the strategies, report whether the returned solution path is an optimal one. Give your explanation and remarks on "why-optimal" or "why-non-optimal". For A* search in particular, your discussion will have to check with the admissibility properties of the heuristics.

(a) (**4 marks**) depth-first search
soln: $s \to a \to b \to h \to i \to g$. Non-optimal, as DFS ignores the cost of edges.

(b) (**4 marks**) breadth-first search
soln: $s \to a \to b \to g$. Non-optimal, as BFS also ignores the cost of edges (but note that with BFS, a path with smallest total number of edges will be found).

(c) (**4 marks**) least-cost first search
soln: $s \to c \to d \to e \to f \to g$. Optimal, as this is just Dijkstra's algorithm.

(d) (**4 marks**) best-first search
soln: $s \to a \to b \to g$. Non-optimal, as DFS ignores the cost of edges.

(e) (**10 marks**) A* search
soln: $s \to c \to d \to e \to f \to g$. Optimal. Note that A* does not necessarily return an optimal path. However for this example, we have heuristics which are admissible, which ensure the optimality of the returned path.

2. (**15 marks**) Consider the language that contains the predicates of symbols *degree*, *student*, *pass*, *test*, *goodscore*, and *subscribe*, all of arty 0. Given the following knowledge base built from this language:

$$
\begin{aligned}
&\mathit{degree} \leftarrow \mathit{student} \wedge \mathit{pass}.\\
&\mathit{pass} \leftarrow \mathit{test} \wedge \mathit{goodscore}.\\
&\mathit{test} \leftarrow \mathit{subscribe}.\\
&\mathit{student}.\\
&\mathit{subscribe}.
\end{aligned}
$$

(a) (**5 marks**) How many interpretations exist for this propositional language?
soln: Each proposition (in total 6 of them) can be either true of false, hence, there are in total $2^6 = 64$ different interpretations.

(b) (**5 marks**) Consider the following interpretations:

	$\pi(\mathit{student})$	$\pi(\mathit{subscribe})$	$\pi(\mathit{test})$	$\pi(\mathit{goodscore})$	$\pi(\mathit{pass})$	$\pi(\mathit{degree})$
I_1	F	T	T	F	F	F
I_2	T	T	F	T	F	T
I_3	T	T	T	T	F	F
I_4	T	T	T	T	T	T
I_5	T	T	T	F	F	F

Which interpretations are models of the knowledge-base? Which are not? Why (not)?
soln:

- I_1 is not a model, as *student* is false in I_1.
- I_2 is not a model, as *subscribe* is true, but *test* is false, hence the sentence $\mathit{test} \leftarrow \mathit{subscribe}$ is not satisfied in I_2.
- I_3 is not a model. As the sentence $\mathit{degree} \leftarrow \mathit{student} \wedge \mathit{pass}$ is equivalent to $\neg\mathit{goodscore} \vee \neg\mathit{test} \vee \mathit{pass}$, but in I_3, *goodscore* is true, *test* is true, and *pass* is false.
- I_4 and I_5 are models of the knowledge-base, as each one of them satisfies all the five sentences.

(c) (**5 marks**) Give all logical consequences of the knowledge-base.
soln: *student* and *subscribe*, which are given. In addition, *test*, which can be derived from the easy reasoning: since *subscribe* is true, *test* is true, as we have $\mathit{test} \leftarrow \mathit{subscribe}$. In other words, *test* is a logical consequence of the knowledge-base. For any interpretation, if it is model of the knowledge-base, it is also a model of *test*.

3. (**15 marks**) Which of the following are correct?

(a) $A \equiv B \models (A \vee B)$.
soln: False. As $A \equiv B$ is equivalent to a KB of two clauses $(\neg A \vee B)$ and $(\neg B \vee A)$. An interpretation where both A and B are false, will satisfy the KB (thus a model), but in this interpretation, the sentence $A \vee B$ is false.

(b) $(A \equiv B) \models \neg A \vee B$.
soln: True. Again $A \equiv B$ is equivalent to a KB of two clauses $(\neg A \vee B)$ and $(\neg B \vee A)$. An interpretation to be a model of the KB of course satisfies $(\neg A \vee B)$.

(c) $(A \wedge B) \rightarrow C \models (A \rightarrow C) \vee (B \rightarrow C)$.
soln: True. When A and B are both true, C must be true. Hence it must be the case that, either if A is true, C must be true, or if B is true, C must be true. In fact, LHS and RHS can be converted into one same clause $(\neg A \vee \neg B \vee C)$.

(d) $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$.
soln: False. $(A \vee B)$ is shared by both sides. Meanwhile, any interpretation that satisfies $(\neg D \vee E)$ will have to satisfy $(\neg C \vee \neg D \vee E)$. But the conversed direction does not hold.

(e) $(A \equiv B) \wedge (\neg A \vee B)$ is satisfiable.
soln: True. Same argument for (b) can be applied here.