Comp 4433 Assignment 2

Each student is required to do this assignment individually and to hand in the answer sheet through D2L on the due date. You may use Microsoft Word or other software to type out your answer. The solutions should be neat and easy to read.

The score of the assignment will depend on:

Specification and documentation: 20 %
Correctness: 80 %

Late assignments will be penalized (-10% with 1-day delay; -20% with 2-day delay and -30% with 3-day delay) and will not be accepted after 3 days.

Problem 1. Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant for $n\leq 2$. Make your bounds as tight as possible, and justify your answers:

- $T(n) = 7T(n/3) + n^2$.
- $T(n) = 7T(n/2) + n^2$.
- $T(n) = 2T(n/4) + \sqrt{n}$.
- $T(n) = 4T(n/3) + nlgn$.
- $T(n) = T(n-2) + n^2$.

Problem 2. Can the master method be applied to the recurrence $T(n) = 4T(n/2) + n^2 lg\, n$? Why or why not? Give an asymptotic upper bound for this recurrence.

Problem 3. Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 3T(\lfloor n/2 \rfloor) + n$. Use the substitution method to verify your answer.

Problem 4. Consider a modification of the rod cutting problem, in which, in addition to a price p_i for each rod, each cut incurs a fixed cost of c. The revenue associated with a solution is now the sum of the prices of the pieces minus the costs of making the cuts.

- Characterize the structure of an optimal solution.
- Give a recursive solution.
- Give a dynamic-programming algorithm to solve this modified problem. Use pseudo codes to write the procedure or procedures. By input p, n, c, your algorithm should finally print out the optimal cutting.

Problem 5. Develop a **dynamic programming algorithm** for the knapsack problem: given n items of know weights $w_1, \ldots, w_n$ and values $v_1, \ldots, v_n$ and a knapsack of capacity W, find the most valuable subset of the items that fit into the knapsack. We assume that all the weights and the knapsack's capacity are positive integers, while the item values are positive real numbers. (This is the 0-1 knapsack problem).

- Analyze the structure of an optimal solution.
- Give the recursive solution.
- Give a solution to this problem by writing pseudo code procedures.
- Analyze the running time for your algorithms.