COMP 4475 *Assignment Two*

Due: March 8^{th}, Before Class (10AM)

1. (**20 marks**) (Ex.2 in Ch2 of the book KRR) This question involves formalizing the properties of mathematical groups in FOL. Recall that a set is considered to be a group relative to a binary function f and an object e if and only if

   - f is associative;
   - e is an identity element for f, that is for any x, $f(e, x) = f(x, e) = x$; and
   - every element has an inverse, that is, for any x, there is an i such that $f(x, i) = f(i, x) = e$.

   (a) Formalize these as sentences of FOL with two nonlogical symbols, a function symbol f, and a constant symbol e, and show using interpretations that the sentences logically entail the following property of groups: For every x and y, there is a z such that $f(x, z) = y$.

   (b) Repeat the entailment proof using Resolution. To do so, you will need to treat equality as a predicate and add to the sentences of part (a) some or all of the axioms of equality (Section 4.2.4 in KRR): reflexibility, symmetry, transitivity. In addition, add the axiom of the substitution of equals for equals, that is for every x, y and z, if $x = y$, then $f(x, z) = f(y, z)$ and $f(z, x) = f(z, y)$.

2. (**30 marks**) Suppose we use the following KB (where x, y, z are variables and *r1, r2, r3, goal* are constants) to determine whether a particular robot can score.

   (a) $Open(x) \wedge HasBall(x) \rightarrow CanScore(x)$
   (b) $Open(x) \wedge CanAssist(y, x) \wedge HasBall(y) \rightarrow CanScore(x)$
   (c) $PathClear(x, y) \rightarrow CanAsist(x, y)$
   (d) $PathClear(x, z) \wedge CanAssist(z, y) \rightarrow CanAssist(x, y)$
   (e) $PathClear(x, goal) \rightarrow Open(x)$
   (f) $PathClear(y, x) \rightarrow PathClear(x, y)$
   (g) $HasBall(r3)$
   (h) $PathClear(r1, goal)$
   (i) $PathClear(r2, r1)$
   (j) $PathClear(r3, r2)$
   (k) $PathClear(r3, goal)$

   Intuitively, $CanScore(x)$ means x can score on goal. $CanAssist(x, y)$ means there exists some series of passes that can get the ball from x to y. $Open(x)$ means x can shoot on goal directly. And $PathClear(x, y)$ means the path between x and y is clear.

   - Provide a SLD-derivation for the query $CanScore(x)$ in which the answer provided is $r1$.
   - Provide a SLD-derivation for the query $CanScore(x)$ in which the answer provided is $r3$.
   - How many "distinct" derivations (i.e., involving different pass sequences) are there for the fact $CanScore(r3)$?

3. (**10 marks**) What is the result of the following applications of substitution?

   - $P(x, y, z)$ $\{x/c, y/f(a)\}$, where a and c are constants, x, y, z are variables.
   - $Q(x, y)$ $\{x/z, y/z\}$, where x,y,z are all variables.

4. (**10 marks**) Find a most general unifier (if one exists) of the following pairs.

   - $P(y, a, b, y)$ and $P(c, f, g, f)$ where a and b are constants, and f,g,y are variables.
   - $P(f(x), r(x), c)$ and $P(w, r(q), q)$, where c is a constant, and x, w, q are variables.

5. (**30 marks**) (Ex.4 in Ch5 of the book KRR) In this question, we will explore the semantic properties of propositional Horn clauses. For any set of clauses S, define $\mathcal{I}_S$ to be the interpretation that satisfies an atom p if and only if $S \models p$.

   - Show that if S is a set of positive Horn clauses, then $\mathcal{I}_S \models S$.
   - Give an example of a set of clauses S where $\mathcal{I}_S \not\models S$.
   - Suppose that S is a set of positive Horn clauses and that c is a negative Horn clause. Show that if $\mathcal{I}_S \not\models c$ then $S \cup \{c\}$ is unsatisfiable.
   - Suppose that S is a set of positive Horn clauses and that T is a set of negative ones. Using part (c), show that if $S \cup \{c\}$ is satisfiable for every $c \in T$, then $S \cup T$ is satisfiable also.
   - In the propositional case, the normal Prolog interpreter can be thought of as taking a set of positive Horn clauses S (the program) and a single negative clause c (the query) and determining whether or not $S \cup \{c\}$ is satisfiable. Use part (d) to conclude that Prolog can be used to test the satisfiability of an arbitrary set of Horn Clauses.