Quiz 2 Computational Physics

October 24, 2025

1 Quiz 2

Welcome to your quiz. This is 15% of your grade. Each problem is worth 5 points, but you only have to do two problems (or your lowest problem score will be dropped.)

Problem 3 has a bonus section worth 2 additional points. It's also (in my opinion) the most interesting problem.

Correctness of solution is all that matters, but FYO my solution to each problem is ≤ 15 lines. If you find yourself writing far more than that, there may be an easier solution.

[1]:
```
import numpy as np
from matplotlib import pyplot as plt
```

1.0.1 1. Numerical Differentiation

Let's test your understanding of numerical differentiation. The function at hand is $y(x) = \sin(x)$ and I would like you to compute $y'(x)$ at $x = \pi/4$. This is to be done numerically without the use of libraries that know the derivatives – you are computing the derivative numerically yourself.

1. Compute this derivative for all values of `dxs = np.array([10**-k for k in range(8)])`.
2. Compute the absolute error between your numerical result and $\cos(\pi/4)$ as a function of the dx value.
3. Using `plt.plot` to demonstrate that the error is power law in dx.

1.0.2 2. Classical Mechanics

Consider a ball with mass $m = 1$ with initial position and velocity $y_0 = 314.15$, $v_0 = 27.18$, acted on by the force of gravity. Using the Euler update method from class, numerically compute the trajectory $y(t)$. Do this in a way that stops the simulation when it hits the ground (i.e. don't overshoot by more than one time step). When does the ball hit the ground?

1.0.3 3. Entropy and 1d Ising

In class we did a 2d Ising model. Here we'll do a 1d ising model with 3 sites and periodic boundary conditions. The energy is

$E = -\sum_{\langle ij \rangle} \sigma_i \sigma_j$

where the sum is over nearest neighbors. Please: 1. Compute the possible states. 2. Compute their energies. 3. Compute their *normalized* probabilities for `betas =`

`.1*np.array(list(range(1,31)))`, where each β is a different inverse temperature and unnormalized we have Boltzmann $p_i \sim e^{-\beta E_i}$. 4. For each β, compute the entropy $S = -\sum_i p_i \log(p_i)$ where the sum is over states. 5. Plot entropies as a function of β.

Bonus 2 points: As β gets large the result asymptotes to a non-zero number. Explain why it asymptotes to a non-zero value, and why to this specific value of the entropy. Each is worth 1 point.